Computational and Differential Geometry 2021-1



MIDTERM ASSIGNMENT 1

February 26, 2021

Indicaciones generales

- 1. Fecha de publicación: 26 de febrero de 2021 desde las 9:00.
- 2. Fecha de entrega: 6 de marzo de 2021 hasta las 7:00.
- 3. Único medio de entrega: https://e-aulas.urosario.edu.co.
- 4. Formato de entrega: código en Python.
- 5. Importante: no use acentos ni deje espacios en los nombres de los archivos que cree.
- 6. La actividad debe realizarse individualmente.
- 7. Los grupos pueden consultar sus ideas con los profesores para recibir orientación; sin embargo, la solución y detalles del ejercicio debe realizarlos **individualmente**. Cualquier tipo de fraude o plagio es causa de anulación directa de la evaluación y correspondiente proceso disciplinario.
- 8. El grupo de trabajo debe indicar en su entrega de la solución a la actividad cualquier asistencia que haya recibido.
- 9. El grupo no debe consultar ninguna solución a la actividad que no sea la suya.
- 10. El grupo no debe intentar ocultar ningún código que no sea propio en la solución a la actividad (a excepción del que se encuentra en las plantillas).
- 11. Las entregas están sujetas a herramientas automatizadas de detección de plagio en códigos.
- 12. e-aulas se cerrará a la hora acordada para el final de la evaluación. La solución de la actividad debe ser subida antes de esta hora. El material entregado a través de e-aulas será calificado tal como está. Si ningún tipo de material es entregado por este medio, la nota de la evaluación será 0.0.

Introduction

The ministry of defense wants to build a server farm that is going to keep sensitive information. The server farm must fulfill a special requirement: there can be no traces of humidity or water infiltration; otherwise data may be compromised. Therefore, the government hires a hydrology company to explore the construction site so they can locate all water and humidity sources; like those coming from rain, for instance.

In order to map out the terrain the company has randomly sampled different parts of it, registering *location and elevation*. The final goal is to provide information on where to locate drain pipes and design a drainage network so the farm can stay dry with no humidity issues that could damage the servers.

A drainage network keeps information on the hydrologic properties of the terrain; for example, water resource locations and potential flood areas. The collection of water streams, sources, basins, and sinks forms such a network, see Fig. 1.

There is a problem, however. The hydrology company has no software to post-process the information and construct the whole drainage network. So they have decided to outsource this part of the job to you! It is going to be your duty, then, to solve the problem of the drainage network.

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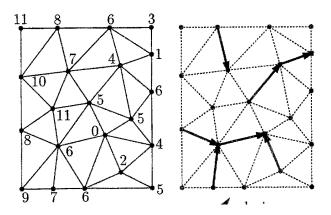


Figure 1: Left. Top view of the construction site, with the randomly sampled points —vertices—, modeled as a triangulated irregular network with elevations at its vertices (see below for details). Right. Top view of the drainage network of the construction site showing water sources, water pits, and the flow of water.

Problem

Your task is to design and implement an application, along with a test suite, that specifically addresses the functionality needed by the hydrology company. The functionality required evaluates aspects of the hydrologic features of the construction site. Such features are local properties of drainage network such as points, lines, and regions of special interest to the flow of water due to gravity and obstacles present on the terrain.

The functionality of the application must be implemented as a class and its corresponding test suite, with the following specifications:

- 1. Given the *planar* location of a point find its corresponding elevation value. Do this by means of a linear interpolation using the sample data.
- 2. Given the planar location of a point find its largest drainage basin. The drainage basin is a quadrilateral, composed of two sampled triangles, that contains the planar location.
- 3. Given the planar location of a point report the quality of the triangulation around it. The quality of a triangle can be measured by reporting the triangle's largest and smallest angles.
- 4. Given the planar location of an initial point, find water path down the terrain. This is a series of points (or edges) that show, given a point, the descending flow of the water.
- 5. Plot the elevation profile. Given the set of sample points on the terrain, plot its elevation profile as a three dimensional graph.

Notice that the entry points and the initial point need not be one of the sampled points in the terrain, but it cannot lie on an edge of the triangulation. In order to find the path followed by the water use the following assumptions: (i) at any point, water follows the steepest descent; (ii) at every point, the direction of flow is unique, streams cannot split; (iii) watercourses end only at local minima or at the boundary of the terrain.

Strategy

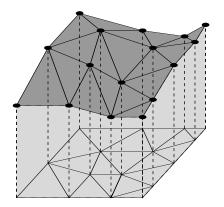
Let us now analyze how to proceed. The input to the problem is the randomly sampled data of the terrain recorded as *location: a point in the plane, and elevation: just a number*.

Next, remembering what you learned in Computational Geometry, you know that you need to build a computational model of the sampled terrain so that you can efficiently process the

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information. A good candidate for this is a digital elevation model (DEM), which is a computational model for representing terrain relief based on a finite number of points or samples. There are different types of DEMs, but since the sample points on the terrain are not uniformly distributed you know that the best DEM for the job is a triangulated irregular network, see Fig. 2.



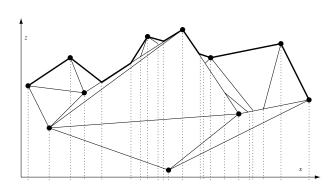


Figure 2: Left: Three dimensional view of the DEM of the construction site modeled as a TIN. Right: Side view of the DEM showing an elevation profile, ridges, and valleys.

A triangulated irregular network (TIN) is a polyhedral terrain; that is, the image of a piecewise-defined linear function with domain in two dimensions. A TIN is a triangulation of the location points, as vertices of the triangulation, augmented with the elevation data. This data structure gives rise to a piecewise-linear function that allows for interpolation of the elevations at any location (point), even though such location is not in the sample set, see Fig. 2.

Hence, a TIN is generated from the sampled points on the terrain by computing the Delaunay triangulation having vertices at the sample points. Thus, a TIN is constructed using algorithms for Delaunay triangulations.

Now that we have a clear picture of how to proceed with the construction of the data structure, it is possible to start implementing the functionality required by the hydrology company specified above. Start by analyzing what the prototype of the class methods should look like, implement them, and finally, test them.

References

L. De Floriani, P. Magillo, and E. Puppo. Applications of Computational Geometry to Geographic Information Systems. In J.-R. Sack and J. Urrutia, editors, *Handbook of Computational Geometry*, chapter 7, pages 333-388. North Holland, 2000.

L. Floriani and P. Magillo. Algorithms for Visibility Computation on Terrains: A Survey. *Environment and Planning B: Planning and Design* **30**, issue 5, pages 709-728 (2003).