3rd Exam

November 16 2022

General indications

- This is an **individual** exam with duration **180 minutes: from 9:00 a.m. to 12:00 m.**.
- Cellphones must be turned off during the exam.
- Only the use of a single sheet with notes is allowed for the 1st part of the exam. The use of books and/or other analog resources is forbidden as well as the use of electronic calculators.
- Any form of noncompliance with the rules above will result in annulment.
- Answers must be fully justified.
- When you are finished you can upload your practical solution to eaulas as a single .zip file.
- Good luck!

Part 1. Theoretical exercises:

1. (10 pts) Prove that if A and B are similar $n \times n$ matrices ($A = MBM^{-1}$ for an invertible $n \times n$ matrix M), then their eigenvalues are equal. What happens with the eigenvectors? Justify.

2. (10 pts) Find the values of $a,\ b \in \mathbb{R}$ for which the following matriz is diagonalizable.

$$\begin{bmatrix} a & b & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

   Find the Jordan canonical form for two values (of your choice) of a and b for which the matrix is not diagonalizable.

3. (15 pts) **To do at last:** Pinky and the Brain have built a machine that, using strong electric and magnetic fields, makes a particle oscillate rapidly. This is useful because a rapidly oscillating particle can generate large amounts of power. Brain has modeled the behavior of the position of the particle using the following dynamical system:

$$x(k+1) = x(k) - y(k) + 2z(k), \quad y(k+1) = -y(k) + z(k), \quad z(k+1) = z(k),$$

   where $x(k)$, $y(k)$, and $z(k)$ are the coordinates of the particle at time k, k given in chronons (a very small unit of time). The Brain asks Pinky to locate the particle in an initial position such that it oscillates (that is, it goes from (x, y, z) to $(-x, -y, -z)$), but Pinky has no idea where to put it and asks for your help. Find this initial position, and the state transition matrix. You can use Matlab for the matrix operations.

Part 2. Practical exercise:

1. (15 pts) Simulate the system in problem 3 using Matlab. Describe the behavior for different initial conditions. What qualitatively different behaviors does the system exhibit? (oscillatory behavior, cycles of any order, fixation, etc.) Describe this behaviors in terms of the spectral properties of the system matrix.