5
Sources of Economic Growth

Tools: Cobb-Douglas production function; level and growth accounting; continuously-compounded growth rates.

Big Ideas:

- Level and growth accounting allow us to *quantify* the sources of growth: the contributions of capital, labor, and total factor productivity (TFP) to growth in real GDP.
- TFP accounts for most of the cross-country differences in output per worker and in differences in the growth rate of output per worker.

If saving rates aren't responsible for the enormous differences we see in living standards, what is? The answer is productivity, but our purpose here is to develop a tool that will give us the answer, whatever it might be. Our ingredients are data (always a good thing) and a little bit of theory (the production function). The combination allows us to attribute differences in output and its growth rate to differences in inputs (capital and labor) and total factor productivity (everything else). The answer, as noted, is mostly productivity: Rich countries are rich because they're productive, and countries that are growing quickly typically have rapid productivity growth, as well. Robert Solow gets credit for this line of thought, too.

5.1 Cross-country differences in output per worker

The production function gives us some insight into cross-country differences in GDP per worker. You'll recall that the production function connects an economy's output (real GDP) to the quantity of inputs used in production (capital and labor) and the efficiency with which those inputs are used (productivity). In equation form:

$$Y \; = \; AF(K,L) \; = \; AK^{\alpha}L^{1-\alpha}, \tag{5.1}$$

where (as before) Y is real GDP or output, A is total factor productivity (TFP), K is the capital stock, and L is the quantity of labor (typically employment). More commonly, we divide both sides by L and express output per worker as

$$Y/L \; = \; A(K/L)^{\alpha}, \tag{5.2}$$

so that output per worker depends on total factor productivity (A) and capital per worker (K/L). For most countries, we have reasonably good data for GDP, employment, and the capital stock, and productivity can be found as a residual:

$$A \; = \; Y/(K^{\alpha}L^{1-\alpha}). \tag{5.3}$$

We'll continue to use $\alpha = 1/3$, so there is nothing about equations (5.1) and (5.2) that we don't know. In this sense, the production function is no longer an abstract idea, but a practical tool of analysis.

Table 5.1: Data for Mexico and the US.

	Employment	Education	Capital	GDP
Mexico	46.94	7.61	4,278	1,293
US	155.45	12.27	42,238	12,619

Aggregate data for 2009 (education for 2007). Employment is expressed in millions, education in years, and capital and GDP in billions of 2005 US dollars.

The production function allows us to make explicit comparisons across countries. If we apply equation (5.2) to two countries and take the ratio, we get

$$\frac{(Y/L)_1}{(Y/L)_2} \; = \; \left[\frac{A_1}{A_2}\right] \left[\frac{(K/L)_1}{(K/L)_2}\right]^{\alpha}, \tag{5.4}$$

where the subscripts 1 and 2 refer to the two countries. The ratio of output per worker is, thus, attributed to some combination of the ratios of TFP and capital per worker. Exercises based on (5.4) are referred to as *level comparisons*. If we have data, we can say which of these factors is most important.

If we did this in logarithms, the components would add rather than multiply. As a result, the contribution of each component can be expressed as a fraction the total. We were tempted to do this, but worried it would unduly try your patience.

Example (Mexico and US). You occasionally hear people in the US say that Mexican workers are paid so much less that they pose a threat to American jobs. (In Mexico, you hear the same thing about Chinese workers.) We can't address that issue (yet) but we can say something about the source of differences in output per worker, which is closely related to differences in wages. The data in Table 5.1 imply that output per worker is 2.95 times higher in the US, but why? We'll use the data in Table 5.1 to come up with an answer.

Let's start with TFP. For Mexico, the data in the table imply that

$$A_M \;=\; 1293/[4278^{1/3} 46.94^{2/3}] \;=\; 6.12.$$

A similar calculation for the US gives us $A_{US} = 12.53$. Thus, TFP is 2.05 ($= 12.53/6.12$) times higher in the US. Similarly, the capital-labor ratio is 2.98 times higher in the US. The impact on output per worker is summarized by

$$\begin{aligned}\frac{(Y/L)_{US}}{(Y/L)_M} &= \frac{A_{US}}{A_M}\left[\frac{(K/L)_{US}}{(K/L)_M}\right]^{1/3} \\ &= (2.05)(2.98)^{1/3} \\ &= (2.05)(1.44) \;=\; 2.95.\end{aligned}$$

It seems, therefore, that both TFP and capital per worker play a role in accounting for the 2.95-to-1 ratio of US to Mexican output per worker. So the reason why output per worker is higher in the US is a combination of higher productivity and higher capital per worker.

This is your chance for speculation: Why do you think the capital-labor ratio is lower in Mexico? Why do you think productivity is lower?

5.2 Cross-country differences in growth rates

Our next task is to apply similar methods to account for cross-country differences in *growth rates* rather than levels.

Warning, growth rates ahead: Before you continue, you might want to go back and review continuously-compounded growth rates in the Mathematics Review, Chapter 1.

As before, the starting point is the production function. If we take the natural logarithm of both sides of the production function (5.1), we find that

$$\ln Y_t \;=\; \ln A_t + \alpha \ln K_t + (1-\alpha) \ln L_t$$

for any date t. This follows from two properties of logarithms: $\ln(xy) = \ln x + \ln y$ and $\ln x^a = a \ln x$. If we take the difference between two adjacent periods t and $t-1$ we get

$$\ln Y_t - \ln Y_{t-1} \;=\; (\ln A_t - \ln A_{t-1}) + \alpha(\ln K_t - \ln K_{t-1}) + (1-\alpha)(\ln L_t - \ln L_{t-1}).$$

Notice that each of the components should be recognizable as continuously-compounded growth rates discussed in the growth-rate discussion.

If we consider differences over n periods, we can divide each term by the number of periods to get

$$\begin{aligned} \left(\frac{\ln Y_t - \ln Y_{t-n}}{n}\right) &= \left(\frac{\ln A_t - \ln A_{t-n}}{n}\right) + \alpha \left(\frac{\ln K_t - \ln K_{t-n}}{n}\right) \\ &\quad + (1-\alpha)\left(\frac{\ln L_t - \ln L_{t-n}}{n}\right). \end{aligned}$$

Notice that we have expressed the average, continuously-compounded growth rate of GDP into the average, continuously-compounded growth rate of each component of the production function (i.e., TFP, capital, and labor). Using our notation convention that continuously-compounded growth rates are represented by γ, we can express the formula above more succinctly as

$$\gamma_Y \;=\; \gamma_A + \alpha\gamma_K + (1-\alpha)\gamma_L. \tag{5.5}$$

In words, this equation says that the growth rate of output can be attributed to growth in TFP, capital, and labor. Moreover, the terms add up because of our use of logarithms and continuously compounded growth rates. Additivity is nice, as it allows us to make statements about the contributions of each component to the growth of GDP.

As with levels, we can do the same for the growth rate of output per worker:

$$\begin{aligned} \gamma_{Y/L} &= \gamma_Y - \gamma_L \\ &= \gamma_A + \alpha(\gamma_K - \gamma_L) \\ &= \gamma_A + \alpha\gamma_{K/L}. \end{aligned} \tag{5.6}$$

Exercises based on (5.5) and (5.6) are referred to as *growth accounting*, which allows us to make statements about the contributions of each component in accounting for the growth of GDP (5.5) or GDP per worker (5.6).

Both versions of growth accounting give us some insight into the sources of economic growth, as the example below shows.

Example (Chile between 1965 and 2009). GDP increased by almost a factor of five between 1965 and 2009. Can we say why? The relevant data are reported in Table 5.2.

Table 5.2: Chilean aggregate data for 1965 and 2009.

	Employment	Education	Capital	GDP
1965	2.71	4.77	65.63	33.62
2009	7.52	7.97	819.81	199.2

The first step is to compute growth rates. Over this period, the average annual growth rate of real GDP was

$$\gamma_Y \;=\; \frac{\ln Y_{2009} - \ln Y_{1965}}{44} \;=\; (5.29 - 3.52)/44 \;=\; 0.0404,$$

or 4.04 percent. Using the same method, we find that the growth rates of the other variables we need are $\gamma_K = 5.74$ percent and $\gamma_L = 2.32$ percent. The growth rate of total factor productivity is the residual in equation (5.5):

$$\gamma_A \;=\; \gamma_Y - [\alpha\gamma_K + (1-\alpha)\gamma_L] \;=\; 0.58\%.$$

(You could also compute A for each period and calculate the growth rate directly.) So why did output grow? Our numbers indicate that of the 4.04 percent growth in output, 0.58 percent was due to TFP; 1.91 percent $[= 5.74 \times \frac{1}{3}]$ was due to increases in capital; and 1.55 percent $[= 2.32 \times (2/3)]$ was due to increases in employment.

What about output per worker? That seems to be the more interesting comparison, because it's closer to an average living standard. The growth rate of output per worker is $\gamma_{Y/L} = 1.72$ percent. Its components are

$$\begin{aligned} \gamma_{Y/L} &= \gamma_A + \alpha\gamma_{K/L} \\ 1.72 &= 0.58 + (1/3)3.42. \end{aligned}$$

In this case, most of the growth in output per worker came from capital per worker, rather than TFP.

5.3 Extensions

We will sometimes use modifications of these tools. Two of the more common ones are based on (i) more-refined measures of labor and/or (ii) GDP per capita rather than GDP per worker. The logic is the same as before, but we gain an extra term or two.

Labor measures. Consider a measure of labor that includes adjustments for hours worked h and human capital H. If the labor input is hHL (with L the number of people employed), the production function becomes

$$Y = AF(K, hHL) = AK^{\alpha}(hHL)^{1-\alpha}. \tag{5.7}$$

How does this change our analysis of levels and growth rates? In a level comparison, this leads to

$$\frac{Y_1}{Y_2} = \left[\frac{A_1}{A_2}\right] \left[\frac{K_1}{K_2}\right]^{\alpha} \left[\frac{L_1}{L_2}\right]^{1-\alpha} \left[\frac{h_1}{h_2}\right]^{1-\alpha} \left[\frac{H_1}{H_2}\right]^{1-\alpha}.$$

The subscripts 1 and 2 again represent countries. You can derive further modifications for output per worker (Y/L) and output per hour worked (Y/hL). In a growth-rate analysis, the augmented production function (5.7) leads to

$$\gamma_Y = \gamma_A + \alpha\gamma_K + (1-\alpha)(\gamma_h + \gamma_H + \gamma_L)$$

for output and

$$\begin{aligned} \gamma_{Y/L} &= \gamma_A + \alpha\gamma_{K/L} + (1-\alpha)(\gamma_h + \gamma_H) \\ \gamma_{Y/hL} &= \gamma_A + \alpha\gamma_{K/hL} + (1-\alpha)\gamma_H \end{aligned}$$

for output per worker and output per hour, respectively. If this sounds complicated, remember that the choice of tool depends on the question we're trying to answer.

We have some choices when it comes to measuring human capital capital. One simple choice is to equate human capital capital with years of school: $H = S$ if we want to give it mathematical form. A more sophisticated choice is to give education a rate of return, so that

$$H = \exp(\sigma S), \tag{5.8}$$

where σ is kind of a rate of return on school, as each year raises human capital capital proportionately. Estimates of σ are in the range of 0.07, so that each year of school raises human capital capital by about 7 percent.

Per capita GDP. The analysis above concerned GDP per worker (rather than per capita). How can we adapt our analysis to account for GDP per capita? Here's a trick: start with equation (5.2) and multiply both sides by the ratio of employment to population:

$$Y/POP = (L/POP)(Y/L) = (L/POP)A(K/L)^{\alpha}.$$

In a level comparison, this gives us an extra term: the ratio of L/POP across countries. In growth rates, we'd add an extra term for the growth rate of the employment ratio:

$$\gamma_{Y/POP} \;=\; \gamma_{L/POP} + \gamma_A + \alpha\gamma_{K/L}.$$

And if you want to get fancy, you can add hours and human-capital terms, as we did above.

Example (Mexico and US, revisited). How does our analysis of the US and Mexico change if we incorporate differences in human capital? We set human capital H equal to years of school and redo our earlier analysis. TFP is now

$$A_M \;=\; 1293/[4278^{1/3}(7.61 \times 46.94)^{2/3}] \;=\; 1.58$$

for Mexico and $A_{US} = 2.36$ for the US. Note that the ratio has fallen from 2.05 to 1.49. Why? Because part of the previous difference now shows up in human capital. [Reminder: A is a residual, so any change in the analysis changes our measure of it.] We now attribute some of the difference in output per worker to a difference in education:

$$\begin{aligned}
\frac{(Y/L)_{US}}{(Y/L)_M} &= \frac{A_{US}}{A_M}\left[\frac{(K/L)_{US}}{(K/L)_M}\right]^{1/3}\left[\frac{H_{US}}{H_M}\right]^{2/3} \\
&= (1.49)(2.98)^{1/3}(1.61)^{2/3} \\
&= (1.49)(1.44)(1.38) \;=\; 2.95.
\end{aligned}$$

It appears that more than half of our earlier difference in TFP stems from differences in education. We amend our previous analysis to add: A substantial part of the difference between output per worker in the US and Mexico stems from differences in education.

An alternative is to measure human capital using our rate-of-return formula, equation (5.8). If we do this, the ratio of human capital is 1.39, which is less than we had before. This choice makes an even bigger difference with countries like India, which have low average education. If years of school go from two to three, is that a 50-percent increase in human capital or a seven-percent increase? You be the judge. Of course, it may depend on what they learn in school, too.

Executive summary

1. Recall that a production function links output to inputs and productivity.

2. Therefore, differences in output and growth rates across countries stem from differences in the levels and growth rates of inputs and productivity (TFP).

3. Level accounting and growth accounting allows us to quantify the differences in output and growth arising from differences in inputs and total factor productivity.

Review questions

1. Growth rates. Take the following data:

	Output Y	Employment L
1950	10	2
2000	50	3

   (a) What are the average annual continuously-compounded growth rates of Y and L?

   (b) What is the analogous growth rate of Y/L?

   Answer.

   (a) The growth rate of output is

$$\gamma_Y = [\ln(50) - \ln(10)]/(2000 - 1950) = 0.0322,$$

   or 3.22% per year. A similar calculation gives us $\gamma_L = 0.0081 =$ 0.81% per year.

   (b) We can do this two ways. The easiest is

$$\gamma_{Y/L} = \gamma_Y - \gamma_L = 0.0241.$$

   You can also compute it by dividing Y by L and applying the same method we used in (a).

2. France and the UK. In 2007, the data were:

	Employment	Education	Capital	GDP
France	29.51	8.48	6,478	1,986
UK	31.79	9.88	5,243	2,070

   Which country had higher output per worker? Why? You should assume that human capital is equal to years of school.

Answer. Ratios were as follows:

$$\begin{aligned}\left(\frac{(Y/L)_F}{(Y/L)_{UK}}\right) &= \left(\frac{A_F}{A_{UK}}\right)\left(\frac{(K/L)_F}{(K/L)_{UK}}\right)^{1/3}\left(\frac{H_F}{H_{UK}}\right)^{2/3}\\ 1.03 &= (1.04)(1.33)^{1/3}(0.86)^{2/3}.\end{aligned}$$

That is, France had slightly higher TFP and more capital per worker, but a lower level of education than the UK.

3. US and Japan. Explain why output grew faster in Japan between 1970 and 1985. Data:

	United States			Japan		
	1970	1985	Growth	1970	1985	Growth
GDP	2083	3103	2.66	620	1253	4.69
Capital	8535	13039	2.83	1287	3967	7.50
Labor	78.6	104.2	1.88	35.4	45.1	1.61

Employment is measured in millions of workers, GDP and capital in billions of 1980 US dollars. Growth rates are continuously-compounded average annual percentages.

Answer. In levels (as opposed to growth rates), we see that the US had much greater output per worker in 1970: 26.5 (thousand 1980 dollars per worker) vs 17.5. Where did this differential come from? One difference is that American workers in 1970 had three times more capital to work with: K/L was 108.6 in the US, 36.4 in Japan. If we use our production function, we find that total factor productivity A was also slightly higher in the US in 1970: 5.55 v. 5.29. Thus, the major difference between the countries in 1970 appears to have been in the amount of capital: American workers had more capital and, therefore, produced more output, on average.

By 1985, much of the difference had disappeared. For the US, the output growth rate of 2.66 percent per year can be divided into 0.94 percent due to capital and 1.26 percent due to employment growth. That leaves 0.47 percent for growth in total factor productivity. For Japan, the numbers are 2.48 percent for capital, 1.08 percent for labor, and 1.13 percent for productivity. Evidently, the largest difference between the two countries was in the rate of capital formation: Japan's capital stock grew much faster, raising its capital-labor ratio from 36.4 in 1970 to 88.0 in 1985.

If you're looking for more

Our calculations are based on various editions of the Penn World Table,

`http://www.rug.nl/research/ggdc/data/penn-world-table`,

which includes data on GDP per worker and related variables constructed on a consistent basis for most countries in the world. We typically post a spreadsheet of the latest version for our classes, but this is the source.

The tools of growth accounting are widely used by industry analysts. Some of the most interesting applications have been done by McKinsey, whose studies have connected cross-country differences in TFP to government regulation, management practices, and the competitive environment. Some of this work is summarized in William Lewis's *The Power of Productivity* (University of Chicago Press, 2004). Other examples are available on McKinsey's web site; search "mckinsey productivity."

Symbols used in this chapter

Table 5.3: Symbol table.

Symbol	Definition
Y	Output (real GDP)
POP	Population
A	Total factor productivity (TFP)
K	Stock of physical capital (plant and equipment)
L	Quantity of labor (number of people employed)
$F(K, L)$	Function of inputs K and L in production function
α	Exponent of K in Cobb-Douglas production function (= capital share of income)
γ_x	continuously-compounded growth rate of variable x
g_x	Discretely-compounded growth rate of variable x
ln	Natural logarithm (inverse operation of exp)
exp	Exponential function (inverse operation of ln)
h	Hours worked per worker
H	human capital
σ	Value of an extra year of schooling (= rate of return on schooling)
Y/POP	Output per capita
L/POP	Ratio of employment to population

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