The Global Economy

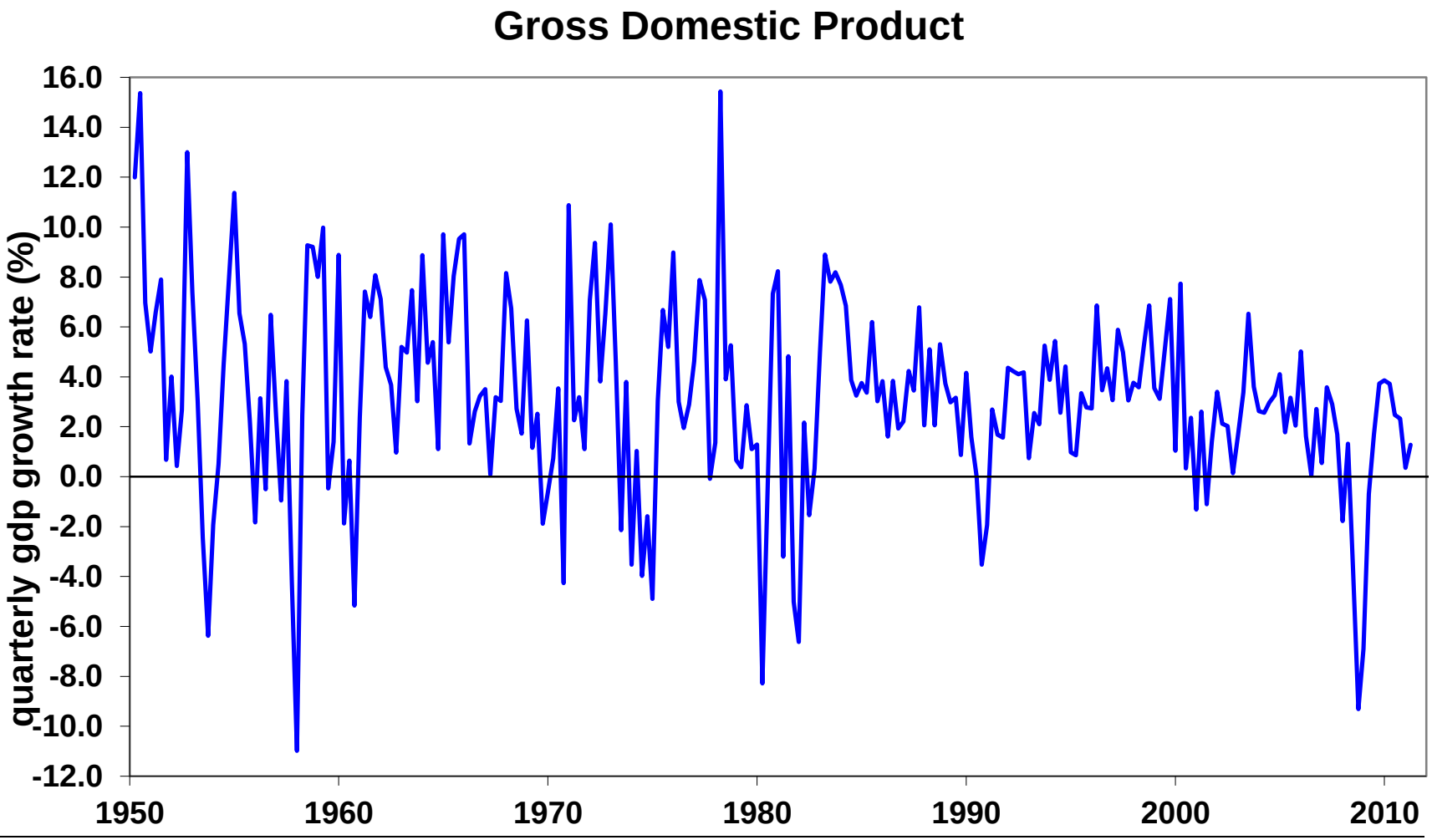
The Production Function

NYU STERN

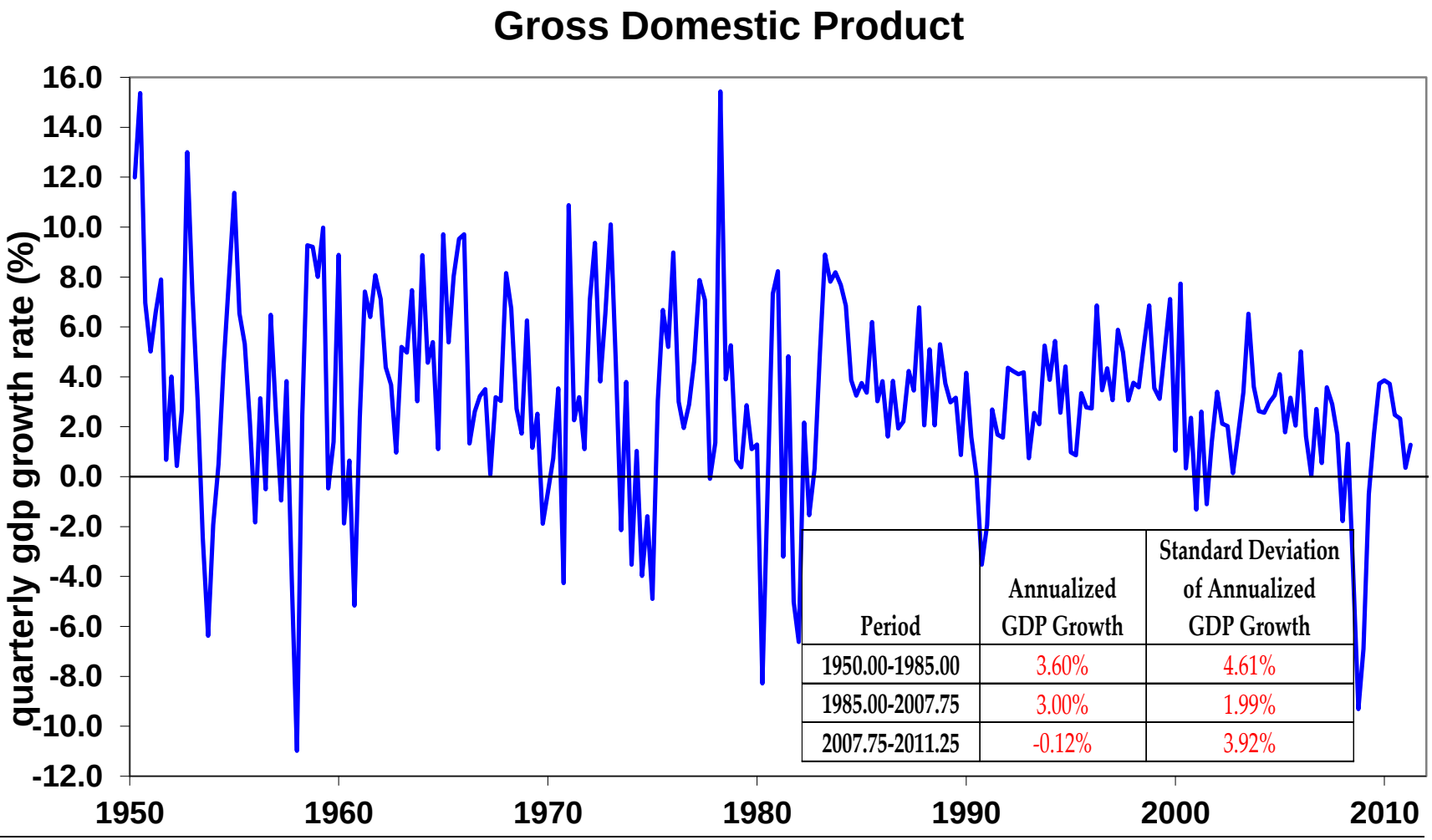
Roadmap

- PS0 Answers
- In the news
- Long run growth facts
- Production function theory
- Capital and labor inputs
- Productivity
- GDP per capita revisited

Practice Problems I

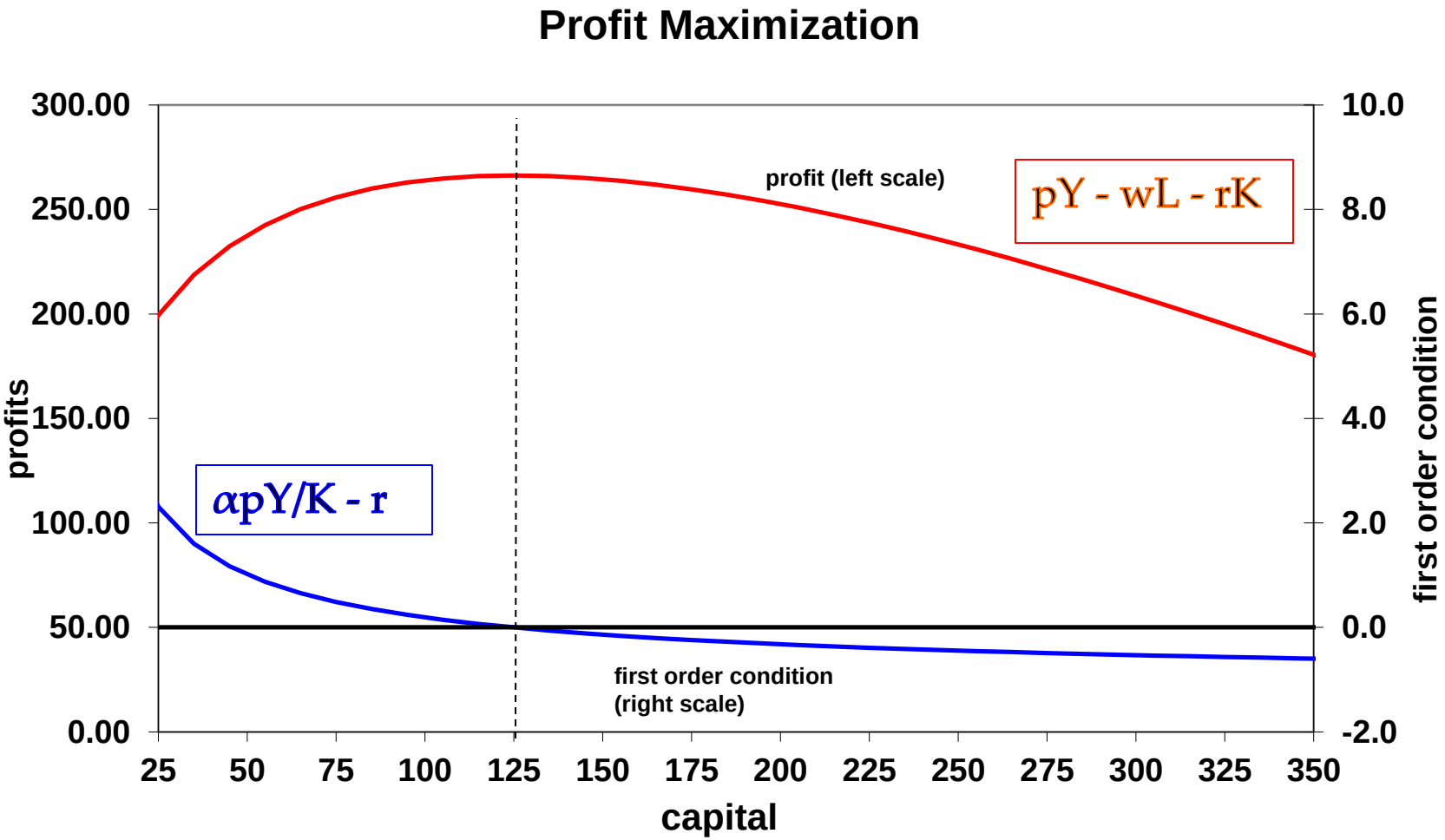


Practice Problems I



Period	Annualized GDP Growth	Standard Deviation of Annualized GDP Growth
1950.00-1985.00	3.60%	4.61%
1985.00-2007.75	3.00%	1.99%
2007.75-2011.25	-0.12%	3.92%

Practice Problems II



Submitting Assignments

- Electronic submission under Assignments
- All answers in one MS WORD file
- Can insert Excel graphs into WORD file
- "Compute XYZ and turn in a plot"
  - Just submit the plot, not the computations
- Neatness helps!

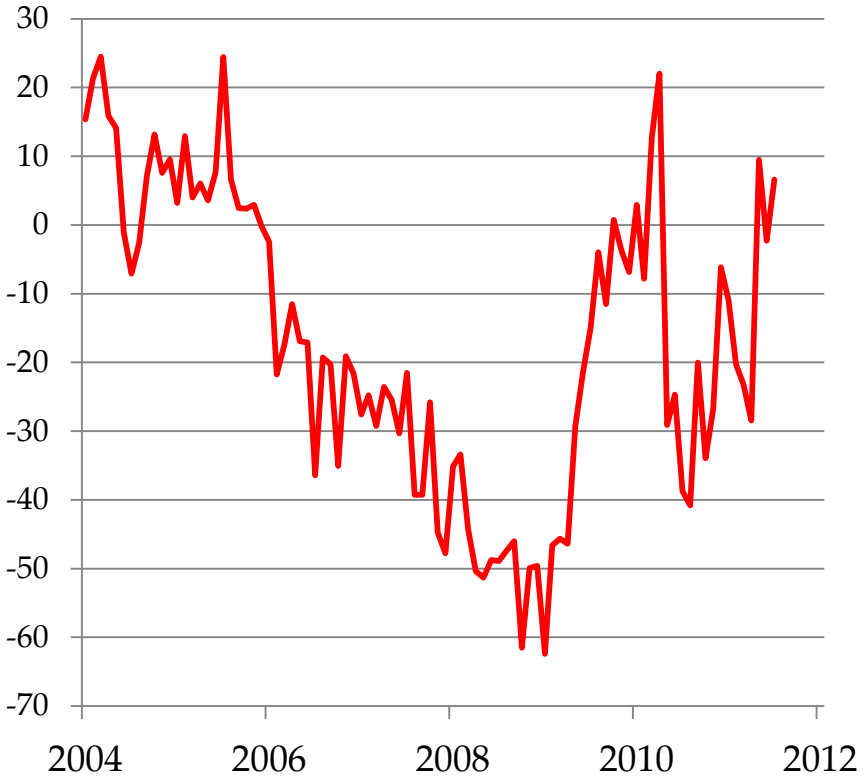
In the news

- Housing Market
  - Case-Shiller House Prices
  - New Home Sales
  - Consumer Confidence
- What's special about housing?
  - Forward-looking (leading indicator)
  - Leverage
  - Role in crisis

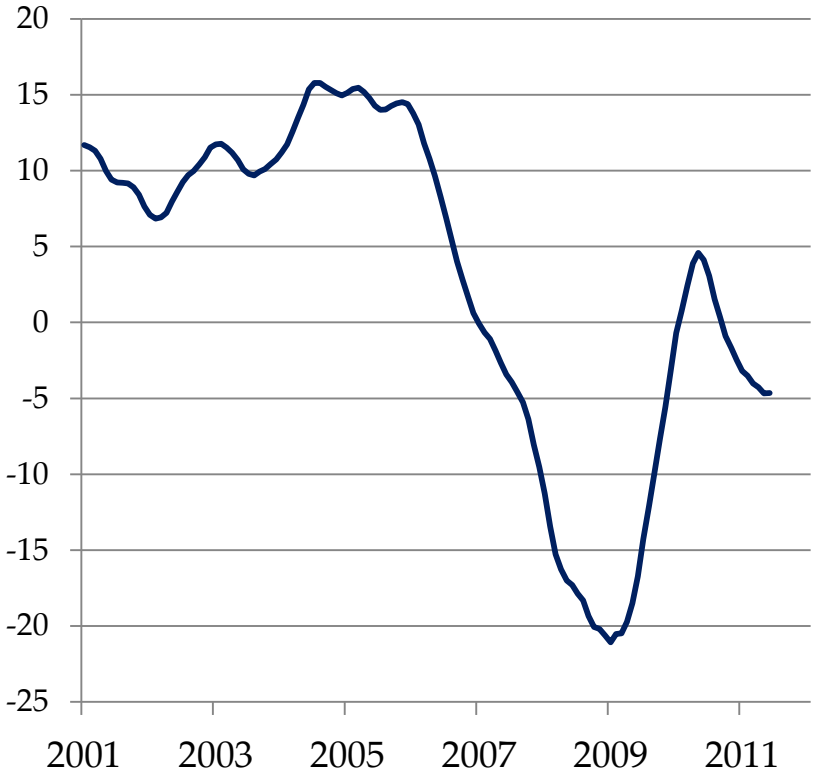
Home Sales and Prices

Annual Percentage Change

New Single-family Home Sales



Home Prices

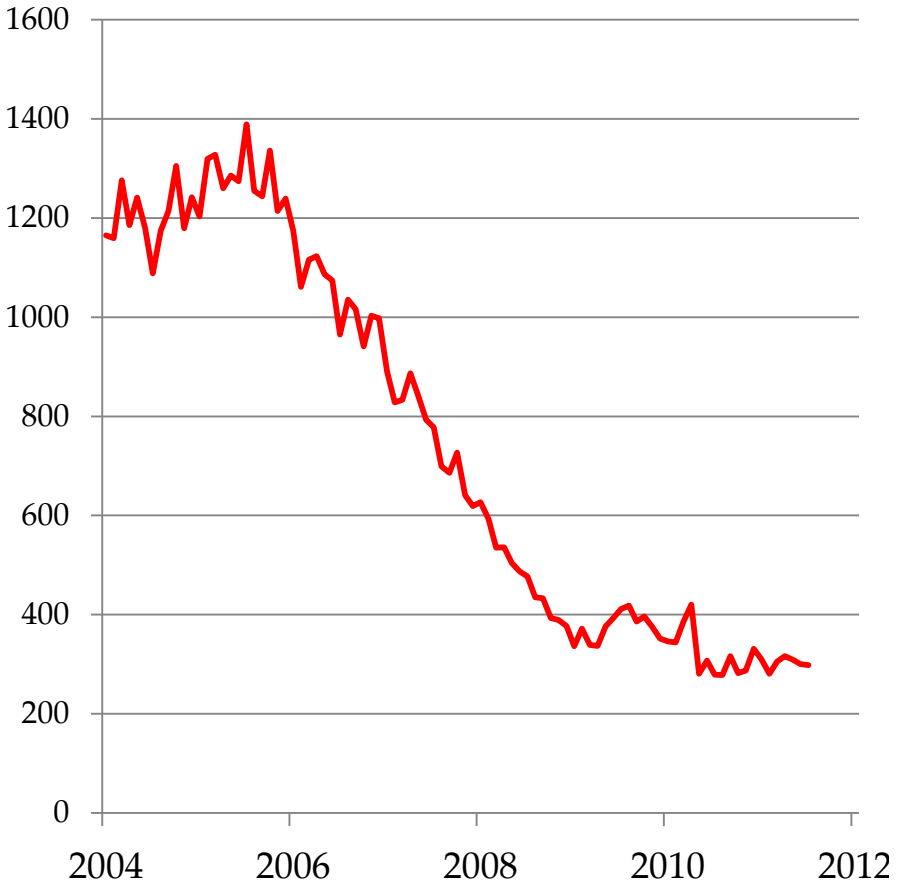


Sources: Standard and Poors Case-Shiller Index and Dept. of Commerce.

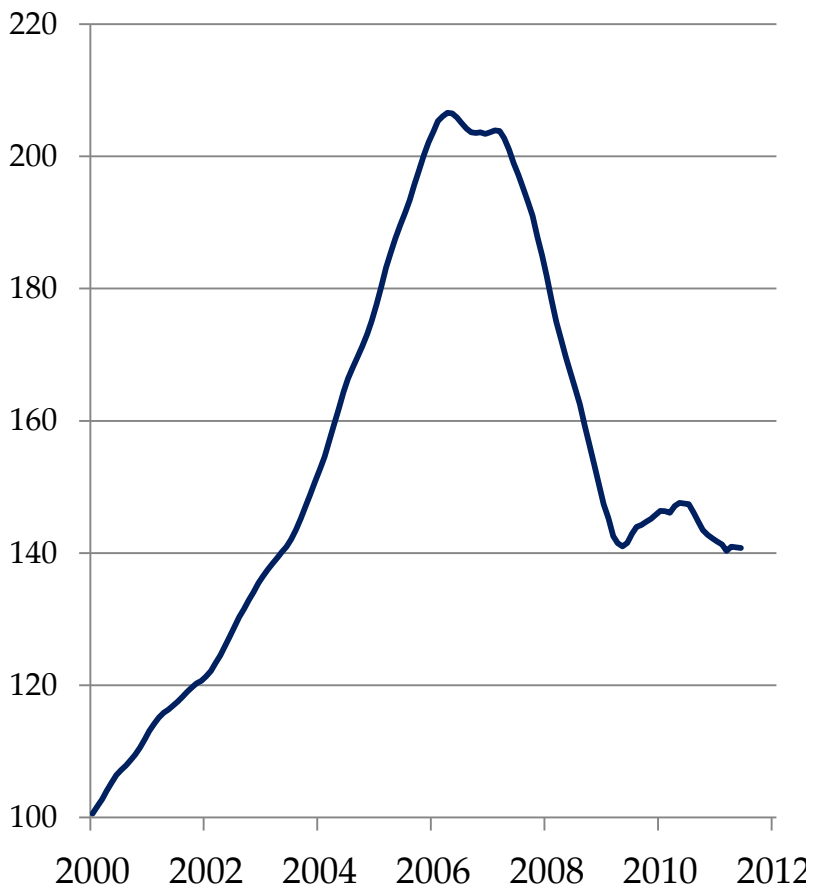
Home Sales and Prices

Levels

New Single-family Home Sales, Thousands



Home Prices, 2000=100



Sources: Standard and Poor's Case-Shiller Housing Prices and Dept. of Commerce.

Roadmap

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- Capital and labor inputs
- Productivity
- GDP per capita revisited

What can growth theory tell us?

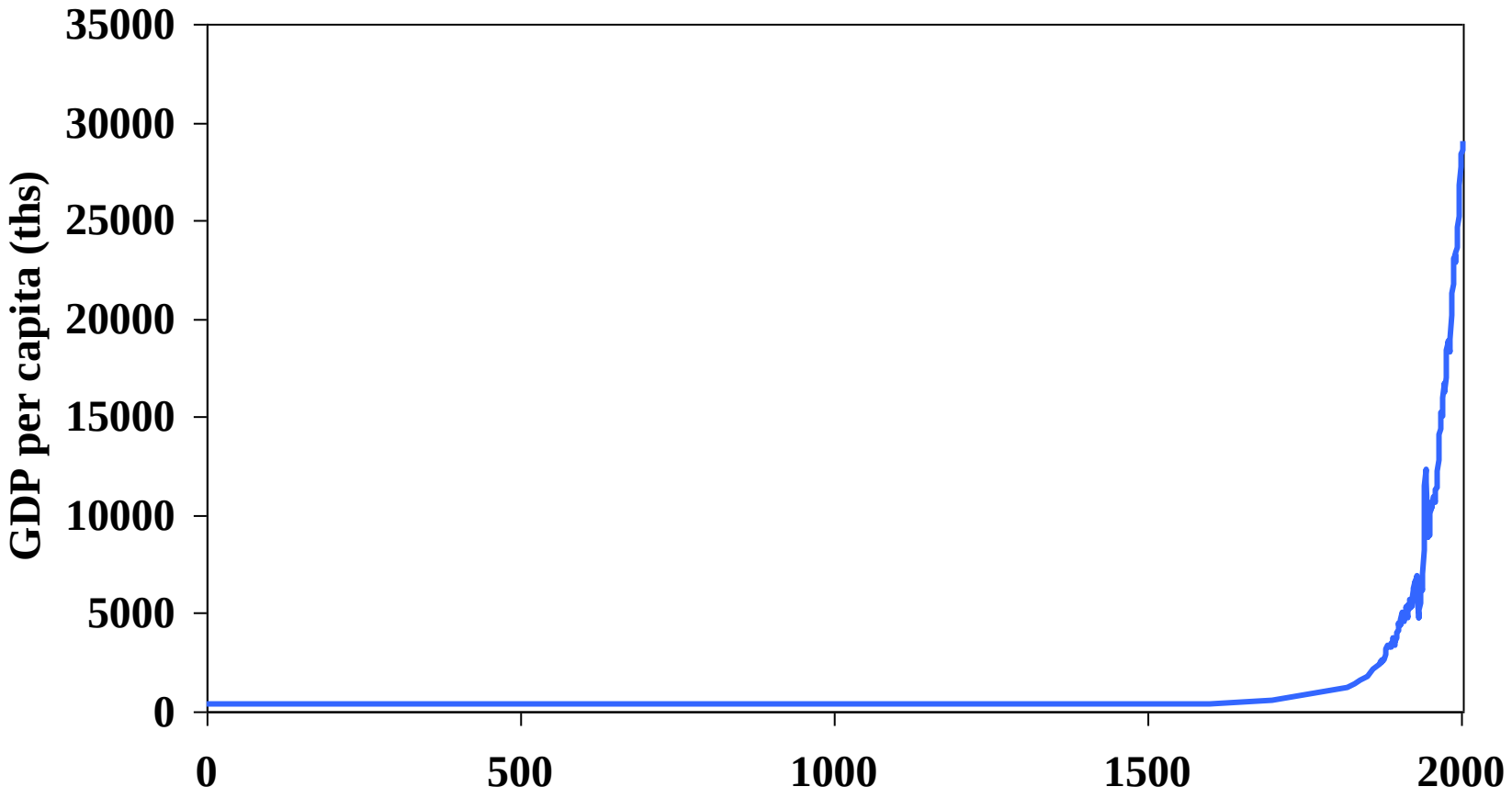
- Why do some countries grow quickly?
- What produces a growth "miracle?"
- Can miracles be replicated?
- Does a country save "too much?"
- How can we forecast long-run growth?

A brief history of time

Statistic	Year			
	1	**1000**	**1820**	**2008**
Population (millions)	225	267	1,042	6,694
GDP Per Capita	467	425	666	7,614
Life expectancy	24	24	26	66

Source: Maddison website. For life expectancy, Maddison, 2001. Life expectancy shown in 2008 is for the year 2000.

Trends are important



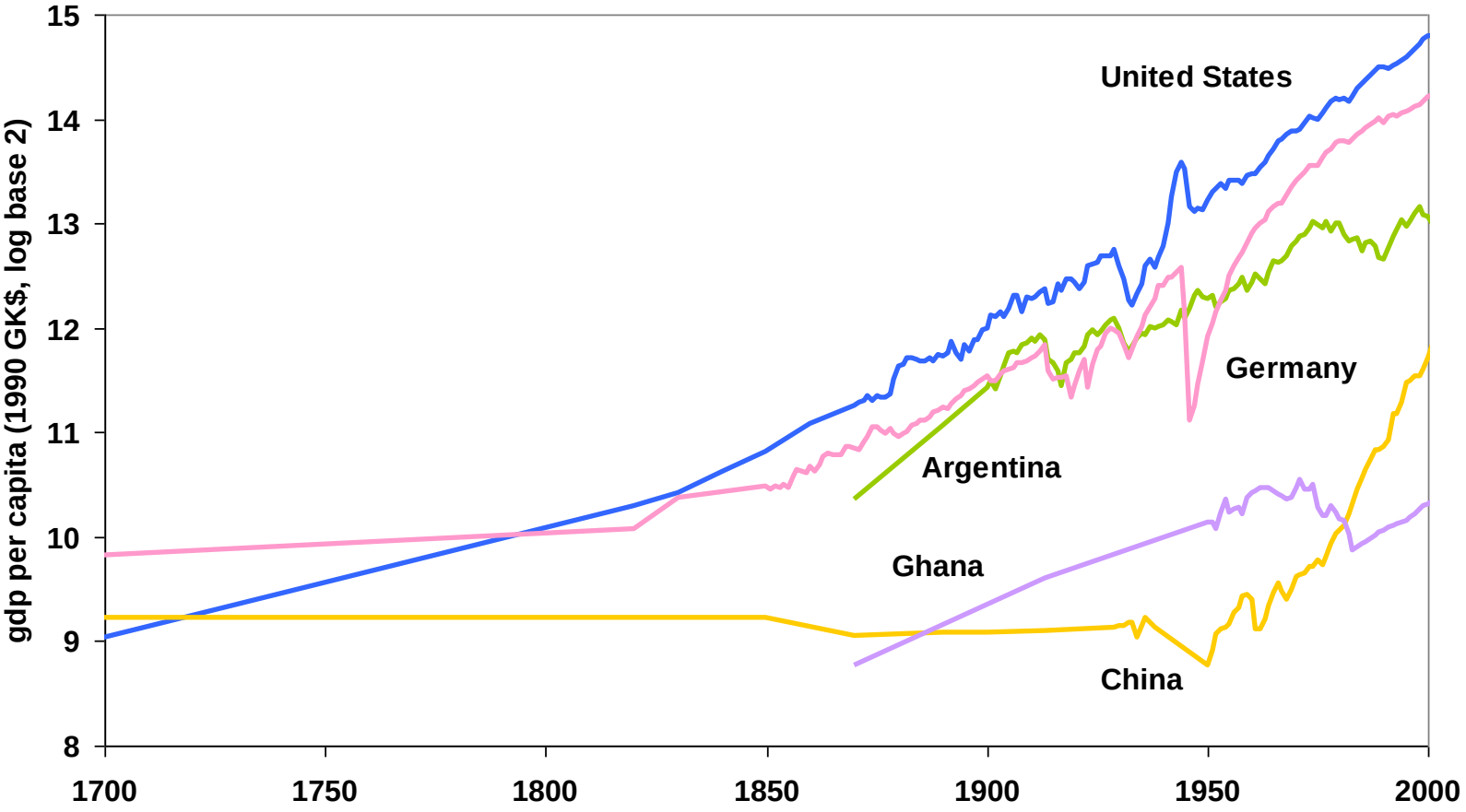
Source: Maddison, "Historical Statistics for the World Economy"

GDP per capita

Region	Year			
	0	1000	1820	2008
Western Europe	599	425	1,218	**21,672**
Western offshoots	400	400	1,202	**30,152**
Japan	400	425	669	**22,816**
Latin America	400	400	691	6,973
Former "USSR"	400	400	688	7,904
China	450	466	600	6,725
Africa	472	425	420	1,760
World Average	467	453	666	7,614

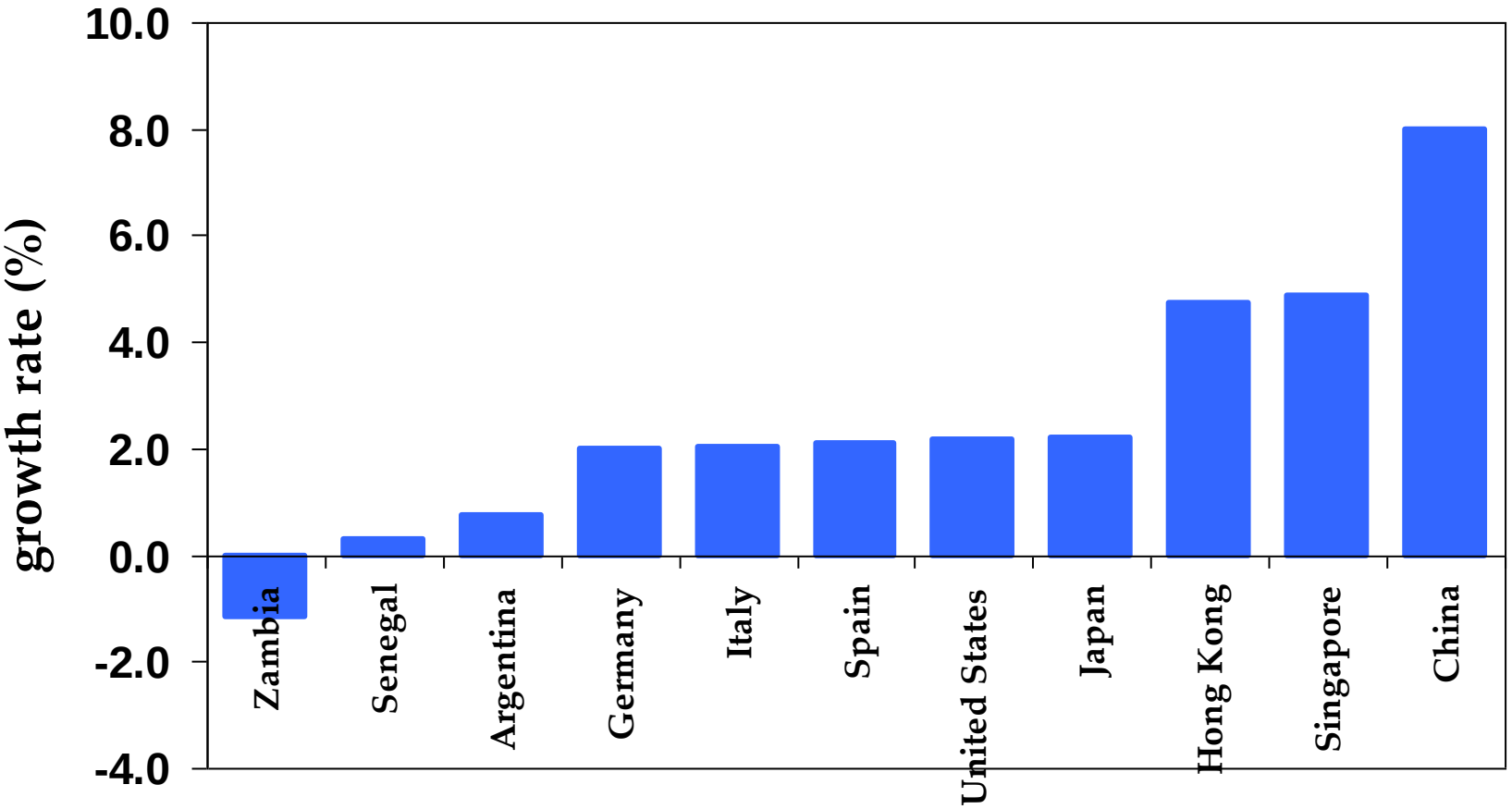
Source: Maddison website.

Trends are important



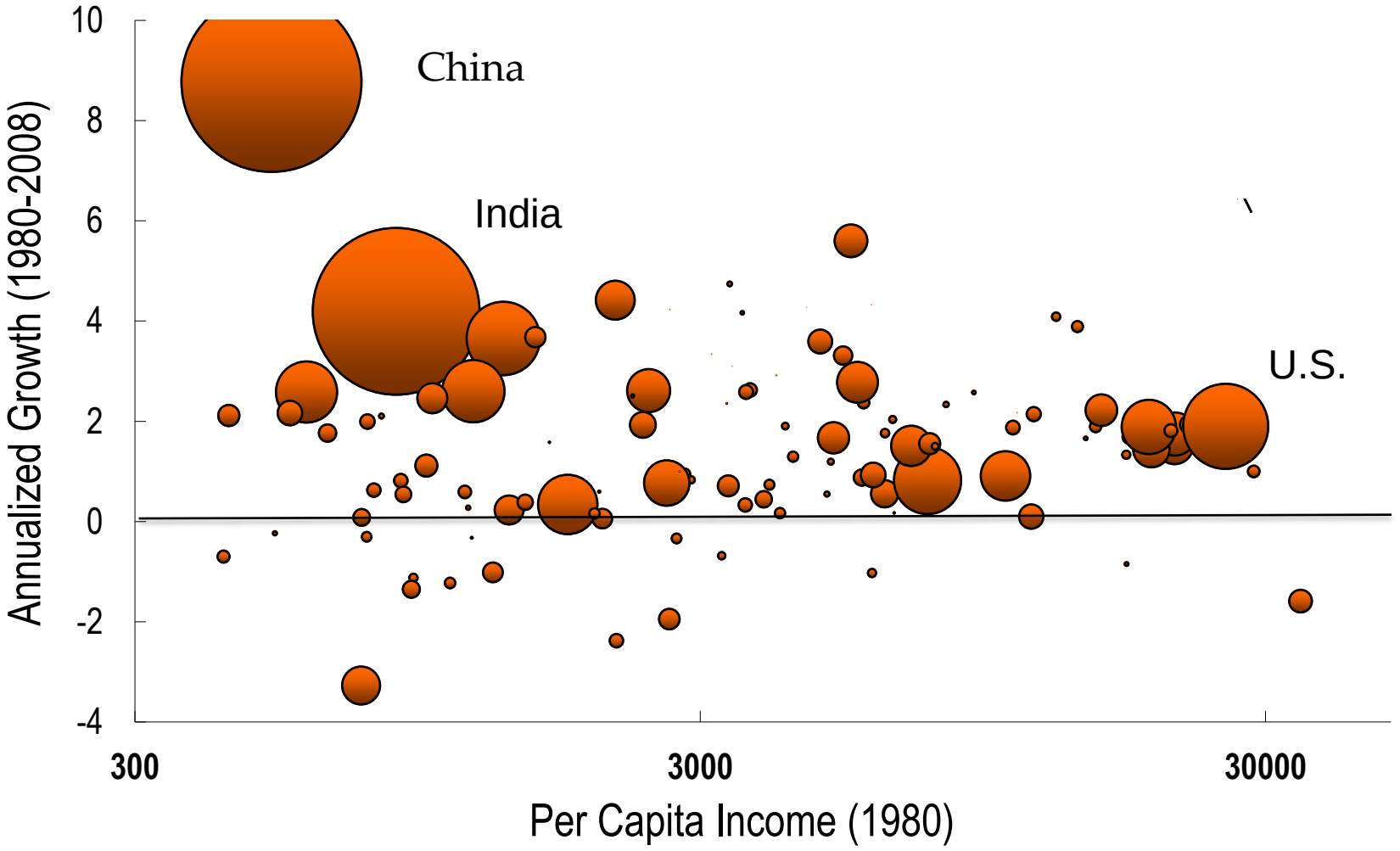
Source: Maddison, "Historical Statistics for the World Economy," 2001.

GDP growth rates, 1975-2005



Source: World Bank, World Development Indicators, 2000 prices in USD, PPP adjusted.

Take-off Pattern: 1980-2008



Note: Size of circle is proportional to population. Data source: World Bank.

Long run growth facts

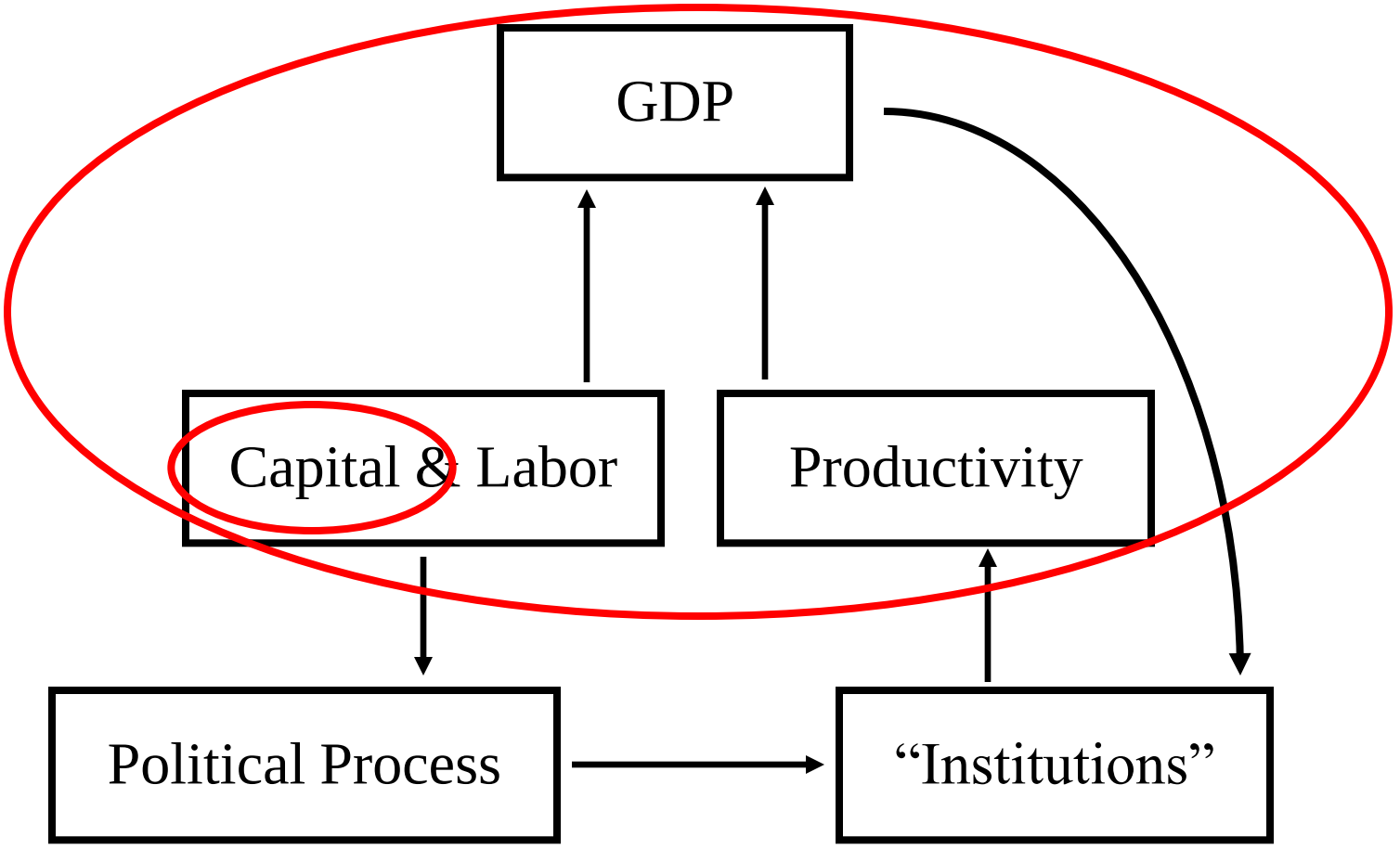
- Huge variation in cross-country income
- Large variation in growth rates
  - Poor countries grow slowly
  - Rich countries grow moderately
  - Some poor countries grow very fast

Production Function Theory

Why theory?

- Theory is a tool to help organize thoughts
- Theory is a framework for analyzing data
- Theory helps simplify a complex world

Theory: the moving parts



Production function

- Idea: relate output to inputs
- Mathematical version:

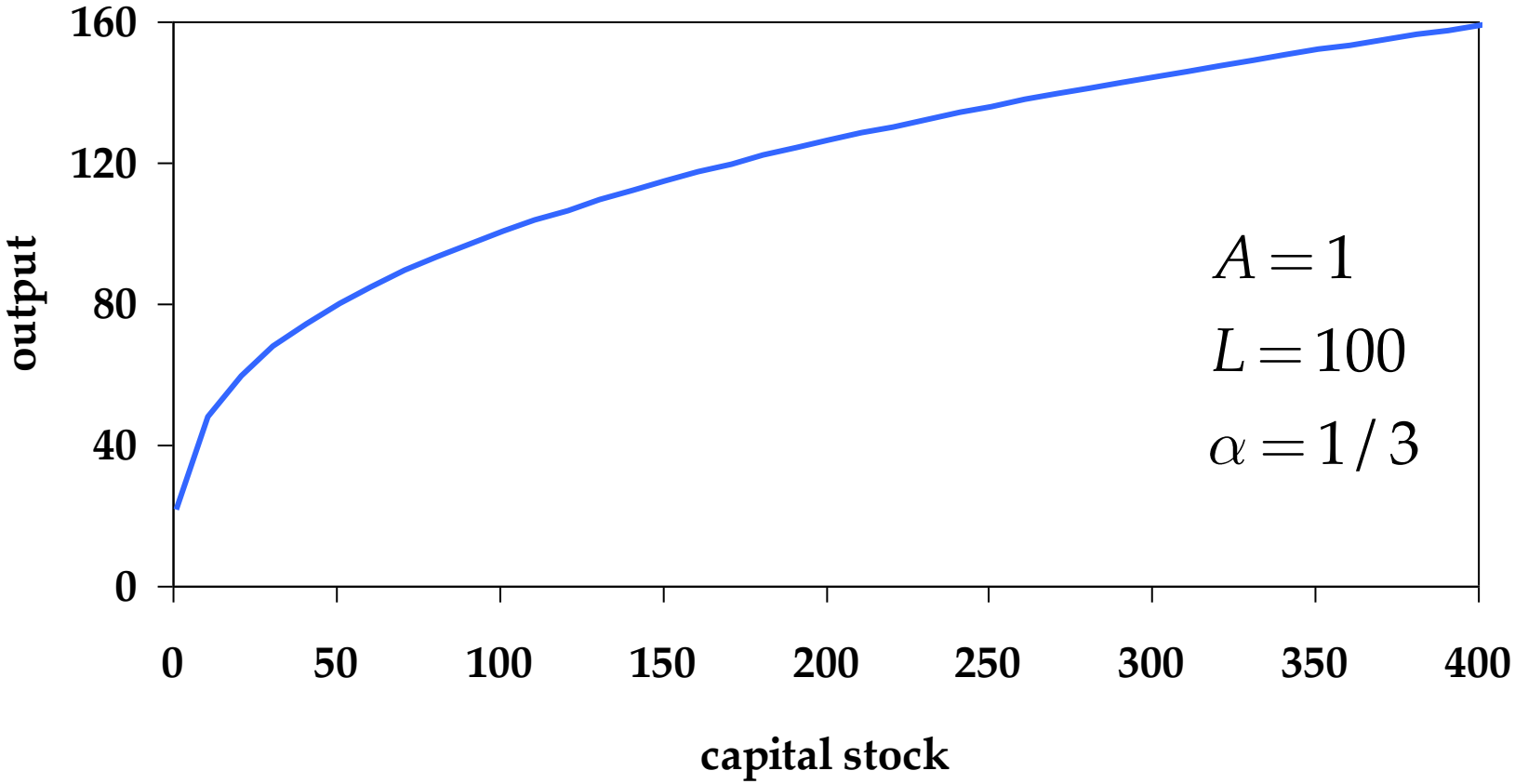
$$Y = AF(K, L)$$
$$= AK^{\alpha}L^{1-\alpha}$$

- Definitions:
  - K = quantity of physical capital used in production (plant and equipment)
  - L = quantity of labor used in production
  - A = total factor productivity (everything else)

Production function properties

- More inputs lead to more output
  - Positive marginal products of capital and labor
- Diminishing marginal products
  - If we increase one input, each increase leads to less additional output
  - Marginal product = partial derivative of production function
- Constant returns to scale
  - If we double ****both**** inputs, we double output

Production function properties



What is labor's income share?

- Profit maximization

$$\max_{K,L} \ \pi = AK^{\alpha}L^{1-\alpha} - rK - wL$$

- First order conditions (marg. rev = marg. cost)

$$\frac{d\pi}{dK} = \alpha AK^{\alpha-1}L^{1-\alpha} - r = 0$$

$$\frac{d\pi}{dL} = \ 1-\alpha \ \ AK^{\alpha}L^{-\alpha} - w = 0$$

What is labor's income share?

- Work with the second condition

$$(1-\alpha)AK^{\alpha}L^{-\alpha} = w$$

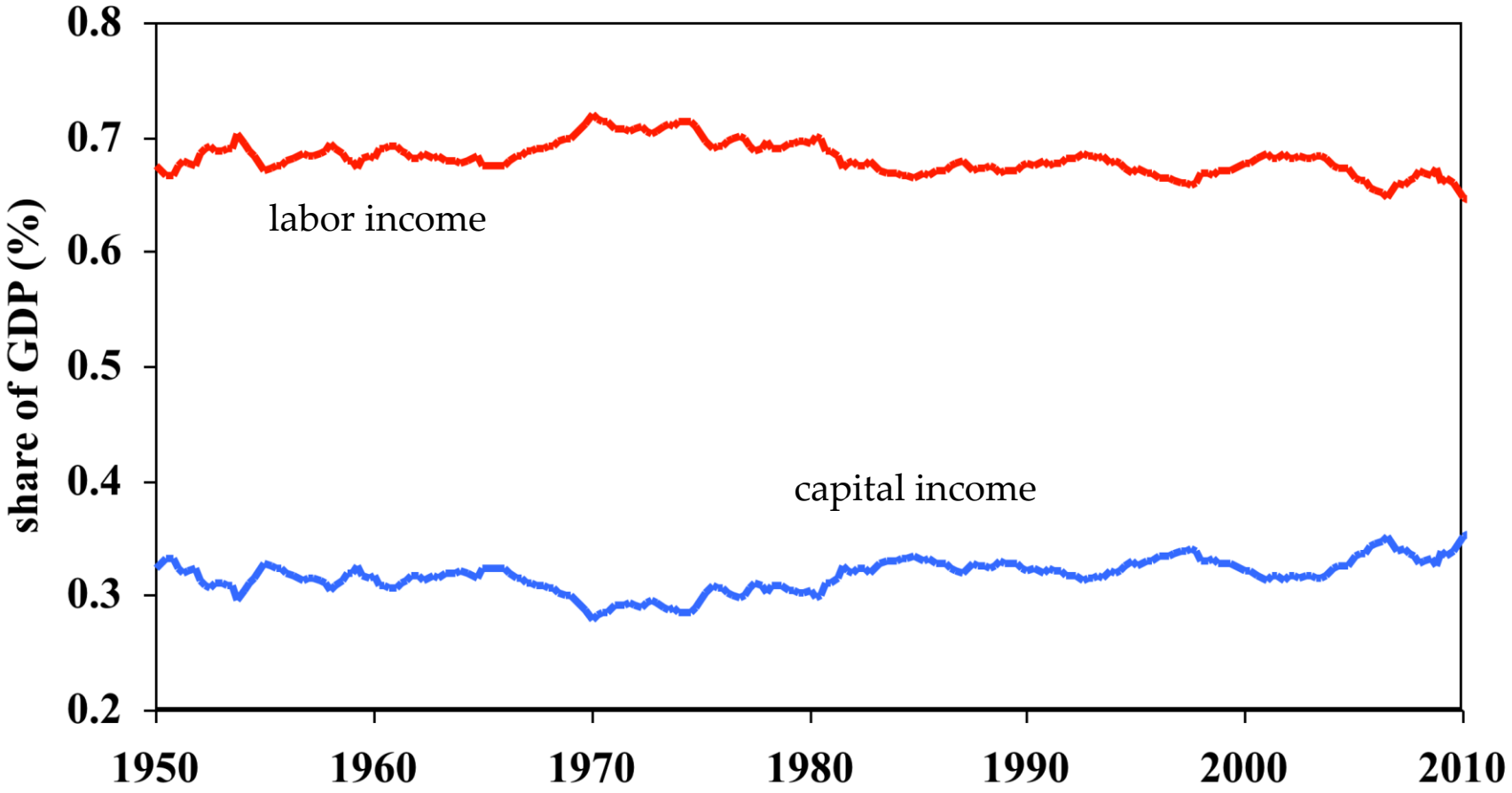
- Multiply each side by L

$$(1-\alpha)AK^{\alpha}L^{1-\alpha} = wL$$

- Substitute Y on left side and note that right side is payments to labor to estimate alpha:

$$\alpha = 1 - \frac{wL}{Y} \approx 0.33$$

GDP by income type



Inputs: capital

- Meaning: physical capital, plant and equipment
- Why does it change?
  - Depreciation/destruction
  - New investment
- Mathematical version:

$$K_{t+1} = K_t - \delta K_t + I_t$$
$$= (1-\delta)K_t + I_t$$

- Adjustments for quality?

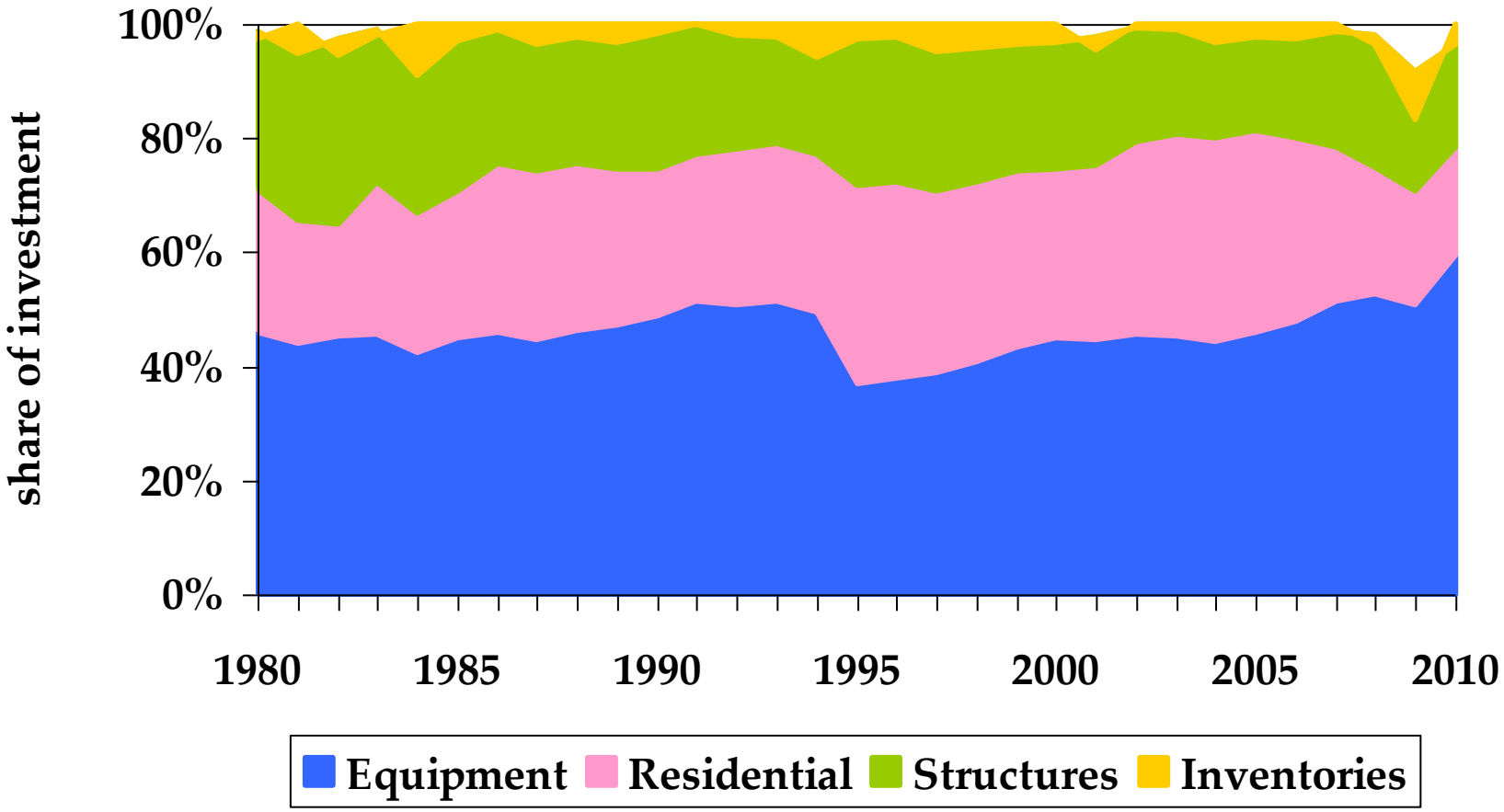
Measuring Capital

- Option #1: direct surveys of plant and equipment
- Option #2: perpetual inventory method
  - Pick an initial value K_0
  - Pick a depreciation rate (or measure depreciation directly)
  - Measure K like this:

$$K_{t+1} = (1-\delta)K_t + I_t$$

- In practice, #2 is the norm:
  - Get I from NIPA
  - Set $\delta = 0.06$ [ballpark number]
  - Example: $K_{2010} = 100$, $\delta = 0.06$, $I = 12 \rightarrow K_{2011} = ??$

Investment composition



Note: Data prior to 1995 on difference base year.

Inputs: labor

- Meaning: units of work effort
- Why does it change?
  - Population growth
  - Fraction of population employed (extensive margin)
  - Hours worked per worker (intensive margin)

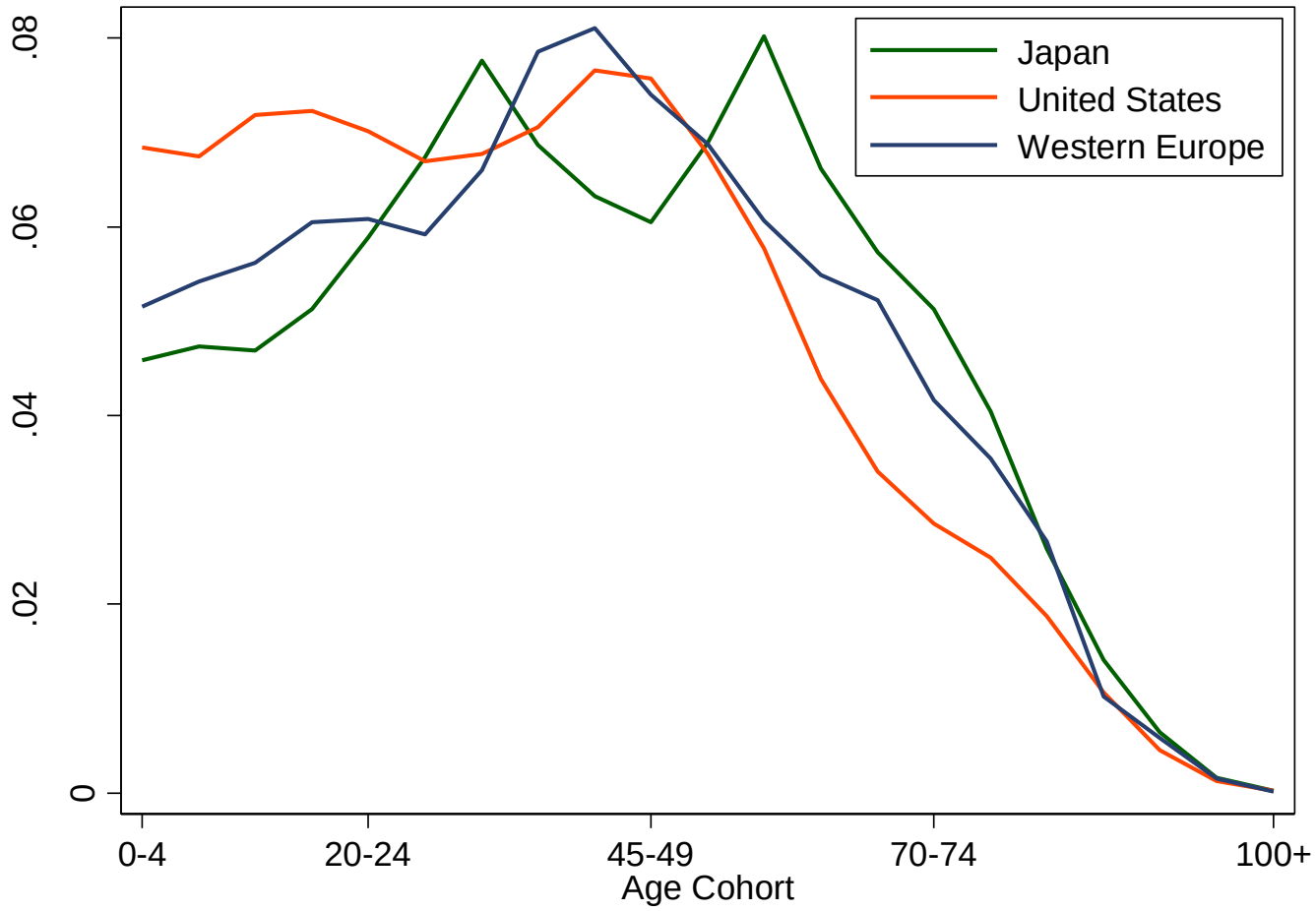
Inputs: labor

- Adjustments for quality? Quantity?
  - Skill: education? other?
- Mathematical version
  - Human capital = H (years of school?)

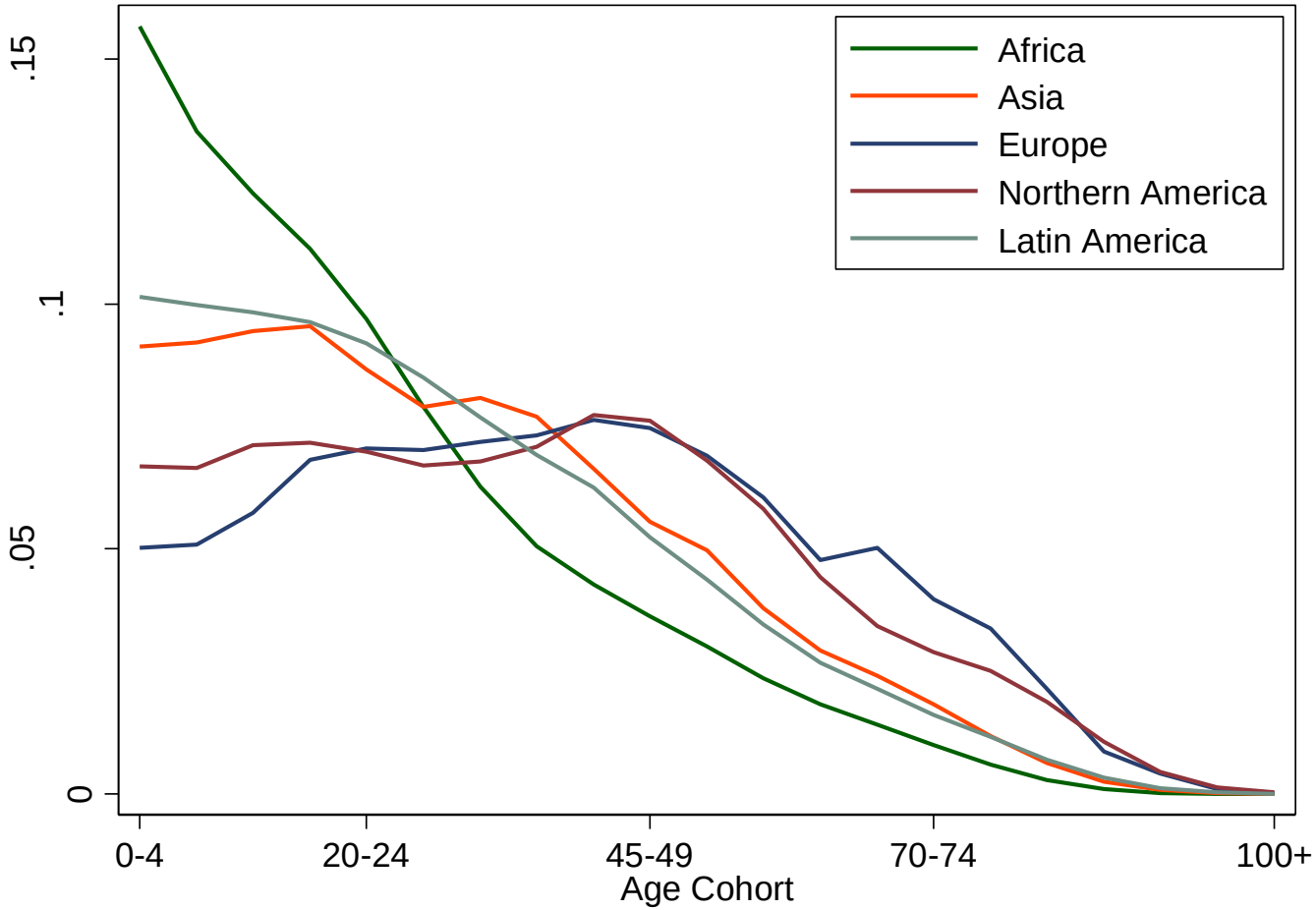
$$Y = AF\ K, H, L$$

$$Y = AK^{\alpha}\ HL^{\ 1-\alpha}$$

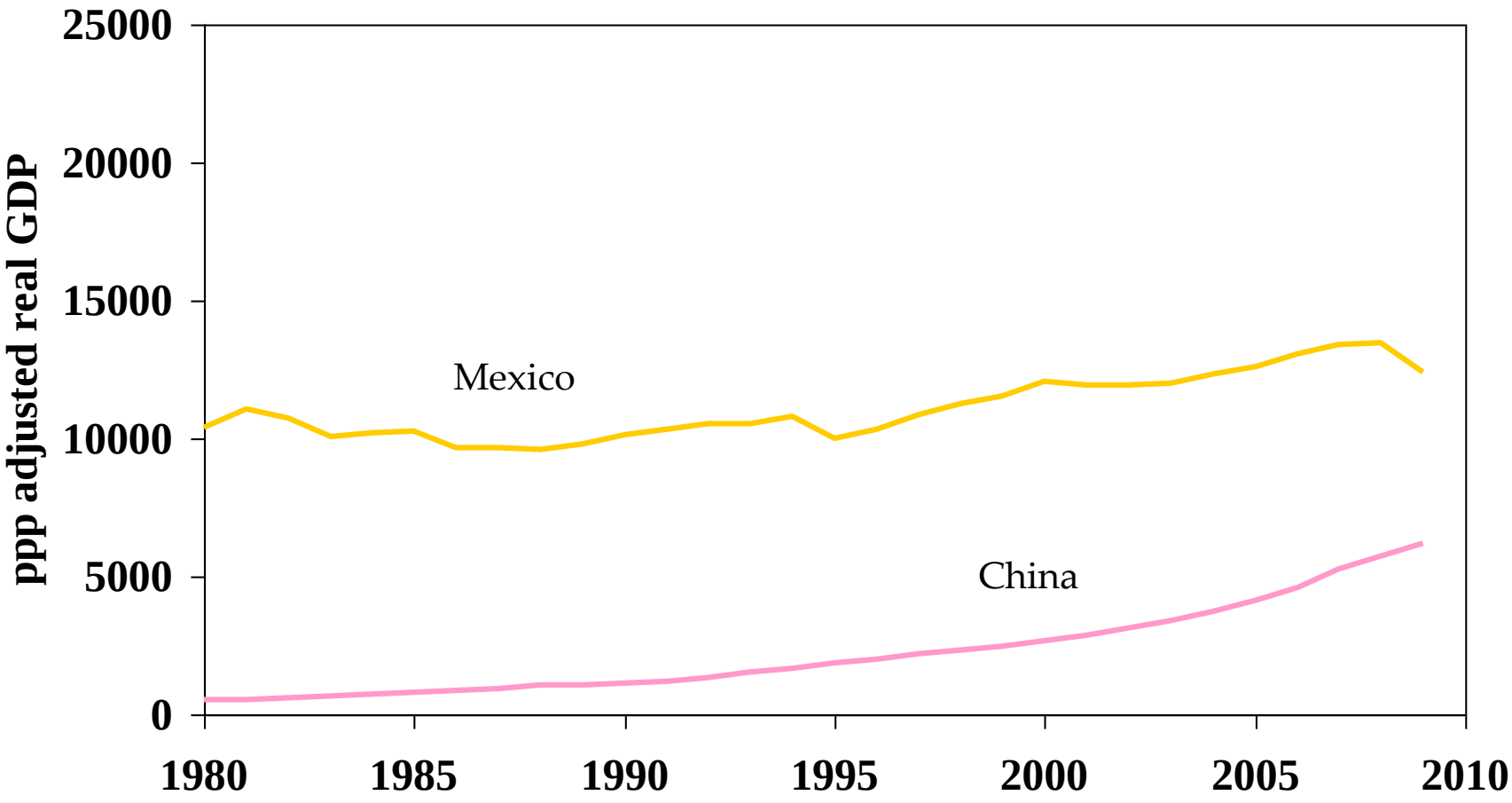
Age distribution



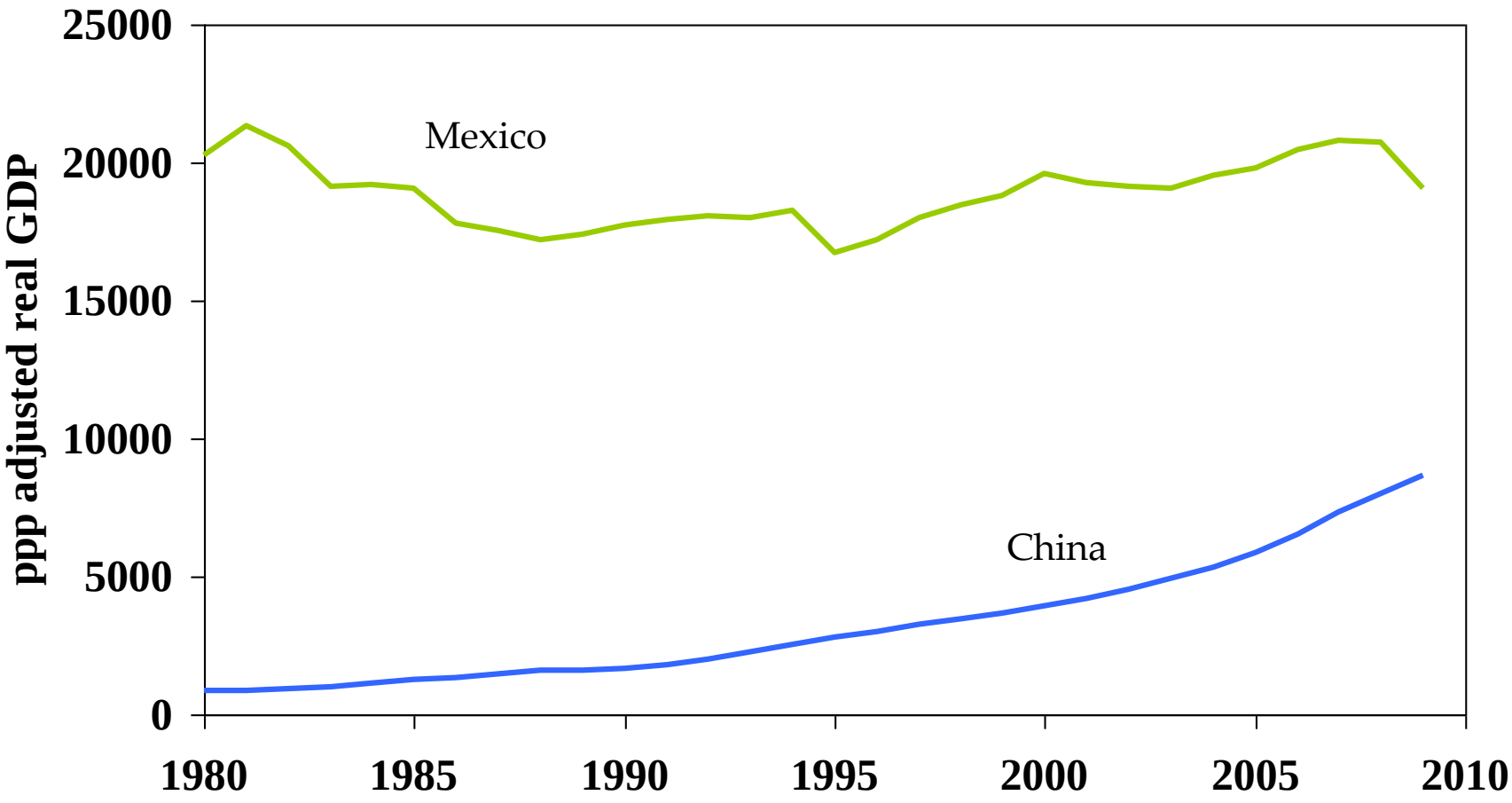
Age distribution



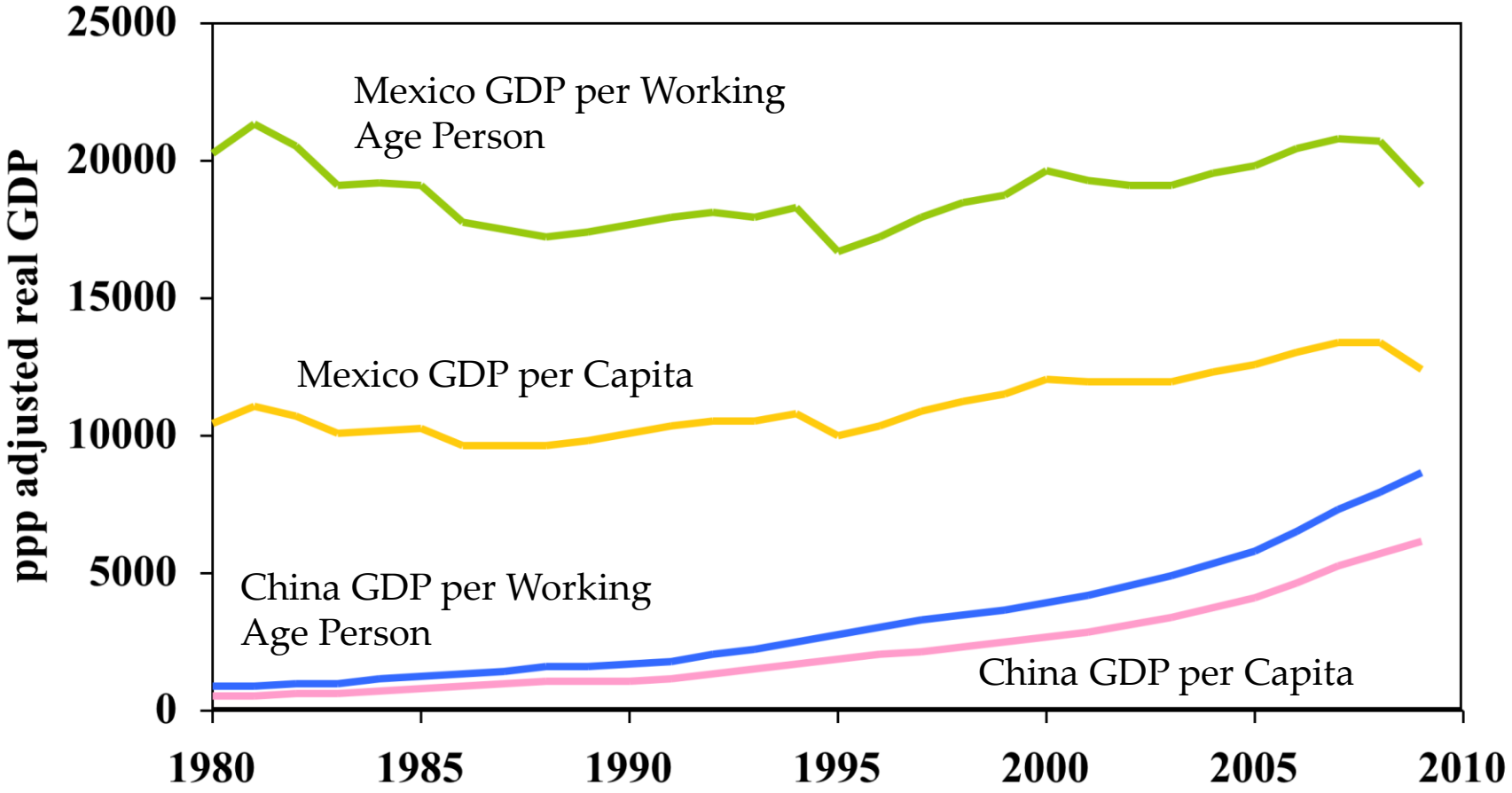
GDP per capita



GDP per working age person



GDP


25000
20000
15000
10000
5000
0
ppp adjusted real GDP
1980
1985
1990
1995
2000
2005
2010
Mexico GDP per Working Age Person
Mexico GDP per Capita
China GDP per Working Age Person
China GDP per Capita

Productivity

- Standard number:
  - Average product of labor: Y/L
- Our number:
  - Total Factor Productivity: Y/F(K,L)
- How do we measure it?
  - Solve the production function for A

$$Y = AK^{\alpha}L^{1-\alpha}$$

$$A = \frac{Y}{K^{\alpha}L^{1-\alpha}}$$

Productivity

$$A = \frac{Y}{K^{\alpha} L^{1-\alpha}} = \frac{Y}{L} \times \left(\frac{K}{L} \right)^{-\alpha}$$

- Example (US): Y/L = 33, K/L = 65

$$A = 33 \times 65^{-0.33} = 8.21$$

- Note: the TFP number by itself is meaningless, but comparisons across countries or time are useful.

Productivity with human capital

- How do we measure it?
  - Solve the production function for A

$$Y = AK^{\alpha}\ \ HL\ ^{1-\alpha}$$

$$A = \frac{Y}{K^{\alpha}\ \ HL\ ^{1-\alpha}} = \frac{Y}{L} \times \left(\frac{K}{L}\right)^{-\alpha}\ \ H\ ^{\alpha-1}$$

- Example (US): Y/L = 33, K/L = 65, H = 12.05

$$A = 33 \times 65^{-0.33} \times 12.05^{-0.67} = 1.56$$

GDP per capita, revisited

- What accounts for GDP per capita?

$$\frac{Y}{N} = \frac{AK^{\alpha} \; HL \;^{1-\alpha}}{N} \frac{L}{L}$$

$$\frac{Y}{N} = \left(\frac{AK^{\alpha} \; HL \;^{1-\alpha}}{L} \right) \times \left(\frac{L}{N} \right)$$

$$\frac{Y}{N} = A \left(\frac{K}{L} \right)^{\alpha} \; H \;^{1-\alpha} \times \left(\frac{L}{N} \right)$$

GDP per capita, revisited

$$\frac{Y}{N} = A\left(\frac{K}{L}\right)^{\alpha} H^{1-\alpha} \times \left(\frac{L}{N}\right)$$

- Reasons for high GDP per capita:
  - More workers: L/N
  - More productivity: A
  - More capital: K/L
  - More skill: H
  - Not present but could be added: hours worked

What have we learned today?

- The production function links output to inputs and productivity:

$$Y = AK^{\alpha} \; HL^{1-\alpha}$$

- The capital input (K):
  - Plant and equipment, a consequence of investment (I)
- The labor input (HL):
  - Population growth, age distribution, participation and hours, skill (H)
- TFP (A) can be inferred from data on output and inputs

The Global Economy

Solow Growth Model

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Capital accumulation and growth

- Can capital accumulation sustain growth?
- Can capital accumulation explain miracles?
- How does population growth affect growth?
- How do saving rates affect growth?
- Recurring themes in politics

Simple model

- Production function

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

- Capital accumulation equation

$$K_{t+1} = K_t + I_t - \delta K_t$$

- Saving flow equation

$$S_t = I_t$$

Savings rates

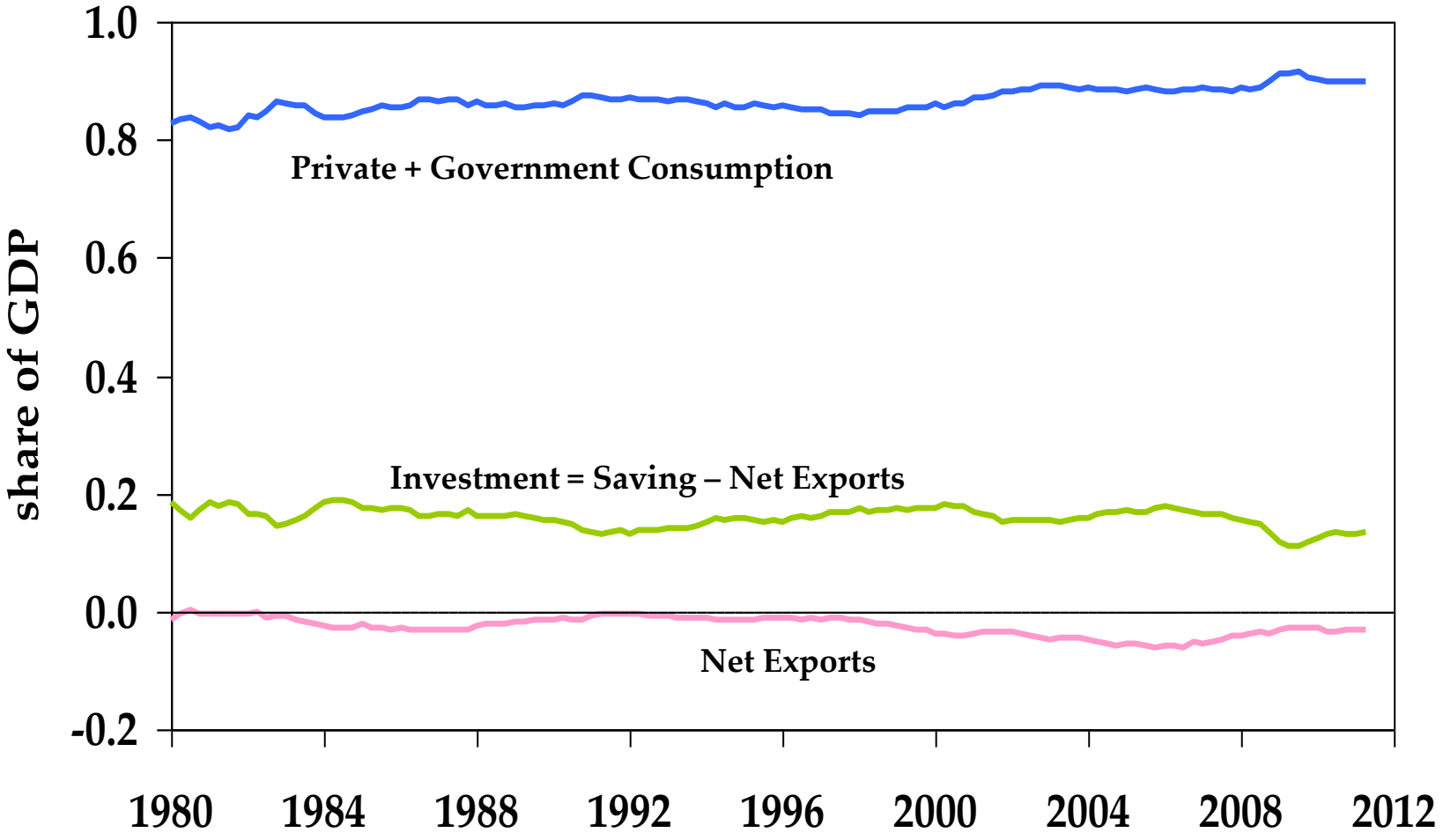
- Assumption: constant saving rate

$$S_t = sY_t$$

- Is this a good assumption?
  - How would we check?
- Savings identity

$$S_t = Y_t - C_t - G_t$$

Savings


1.0
0.8
0.6
0.4
0.2
0.0
-0.2
share of GDP
Private + Government Consumption
Investment = Saving – Net Exports
Net Exports
1980
1984
1988
1992
1996
2000
2004
2008
2012

Solow model

- Production function

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}$$

- Capital accumulation equation

$$K_{t+1} = K_t + I_t - \delta K_t$$

- Saving flow equation

$$S_t = I_t$$

- Saving rate

$$S_t = sY_t$$

Solow model

- Combining the equations

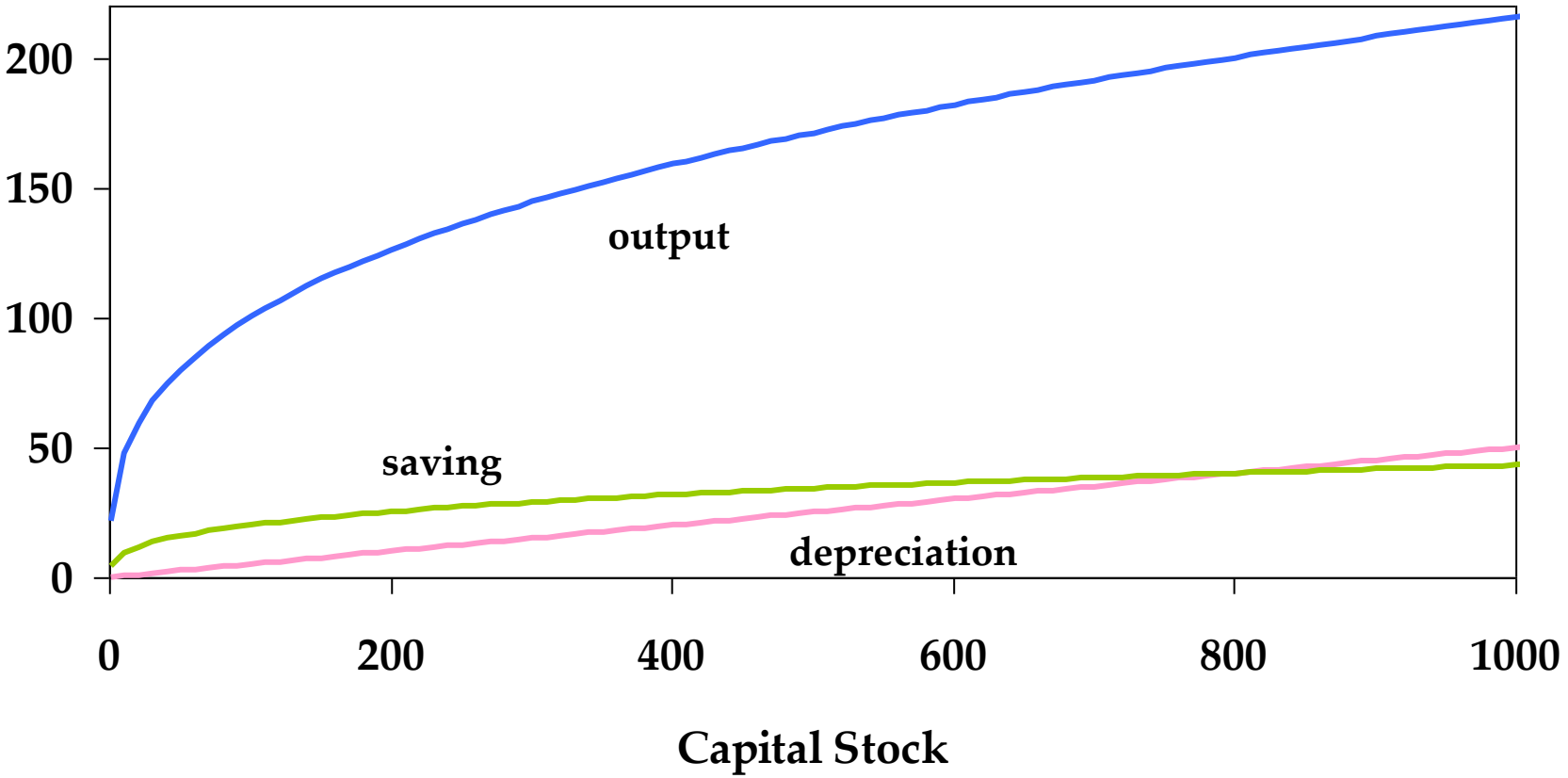
$$K_{t+1} = K_t + sAK_t^{\alpha} L^{1-\alpha} - \delta K_t$$

- Evolution of the capital stock

$$\Delta K = sAK^{\alpha} L^{1-\alpha} - \delta K$$

- Two forces: investment, depreciation

Solow model: dynamics



Solow model: convergence

- Eventually: constant K, => Y, K/L, and Y/L
- Countries with low K and Y grow faster
- Eventually K stops changing

$$\Delta K = sAK^{\alpha}L^{1-\alpha} - \delta K = 0$$

- This is the *steady state*

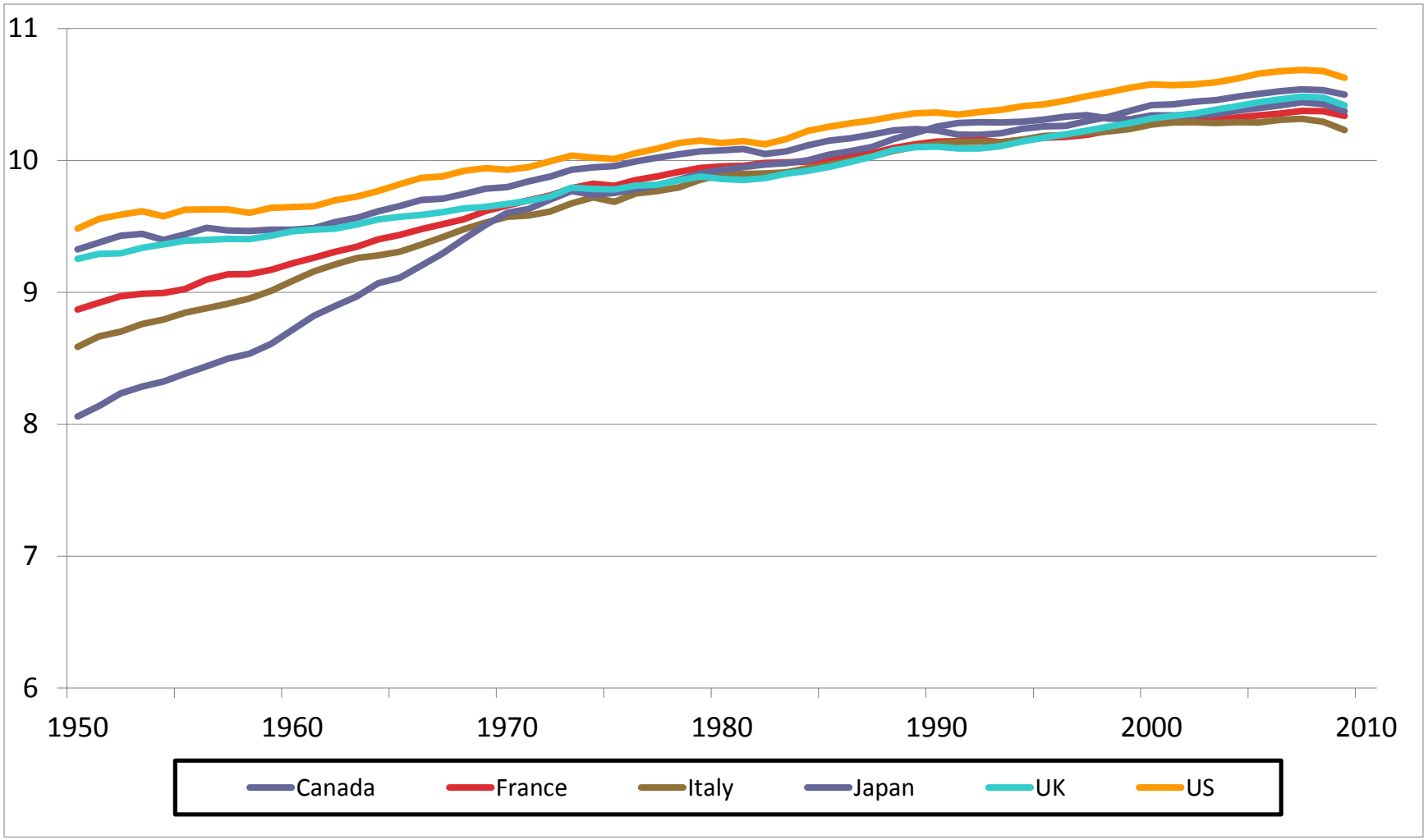
$$K_{ss} = \left(\frac{As}{\delta}\right)^{\frac{1}{1-\alpha}} L \qquad Y_{ss} = A\left(\frac{sA}{\delta}\right)^{\frac{\alpha}{1-\alpha}} L$$

Solow model: convergence

- Do we see convergence?
- What data could we use to test this implication of the theory?
  - GDP per capita
  - GDP per capita vs. growth rates
  - What would we see?

Convergence?

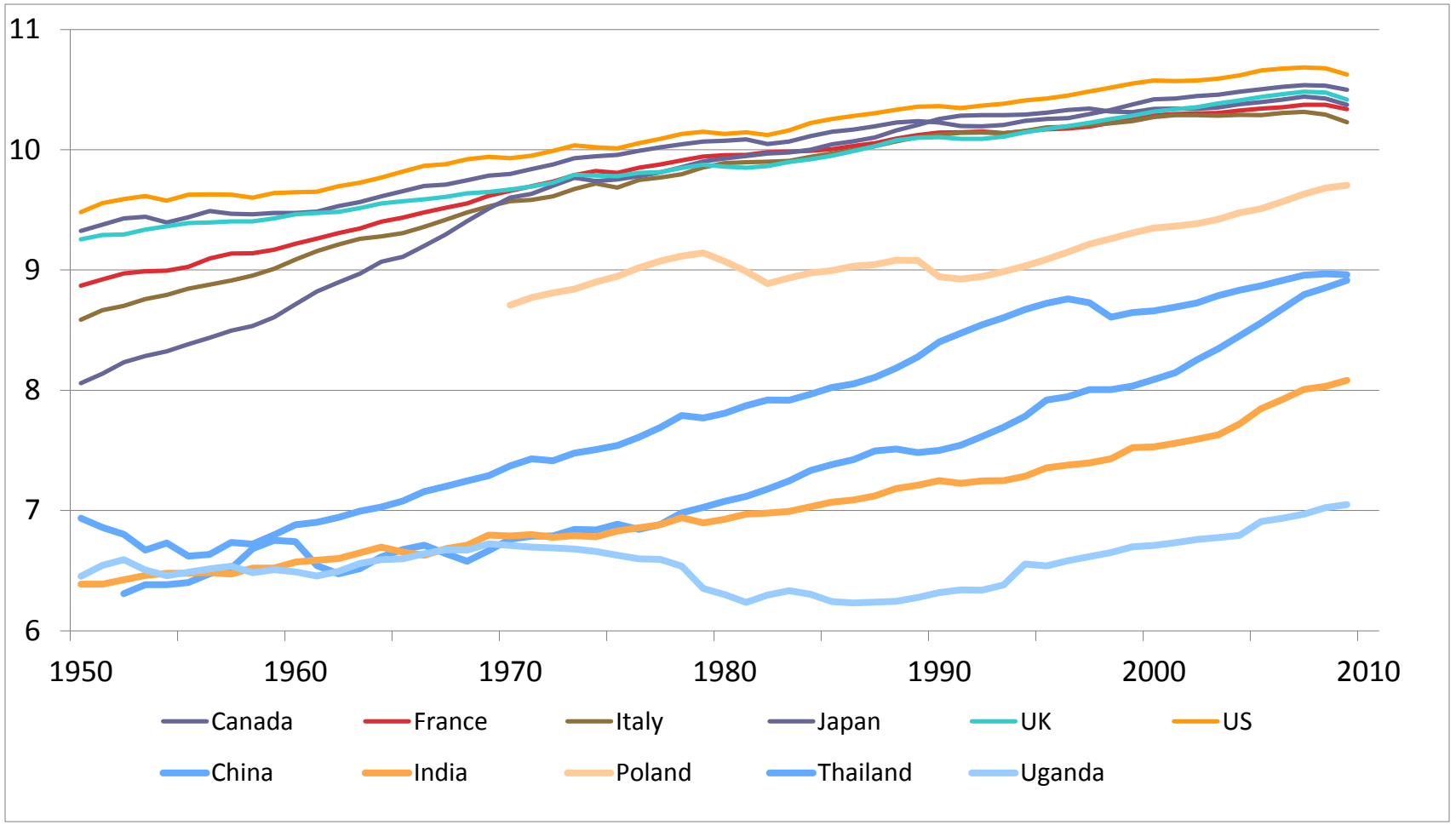
Log of Real Per Capita GDP (PPP, 2005 Chained US$)



Source: Penn World Tables 7.0

Convergence?

Log of Real Per Capita GDP (PPP, 2005 Chained US$)



Source: Penn World Tables 7.0

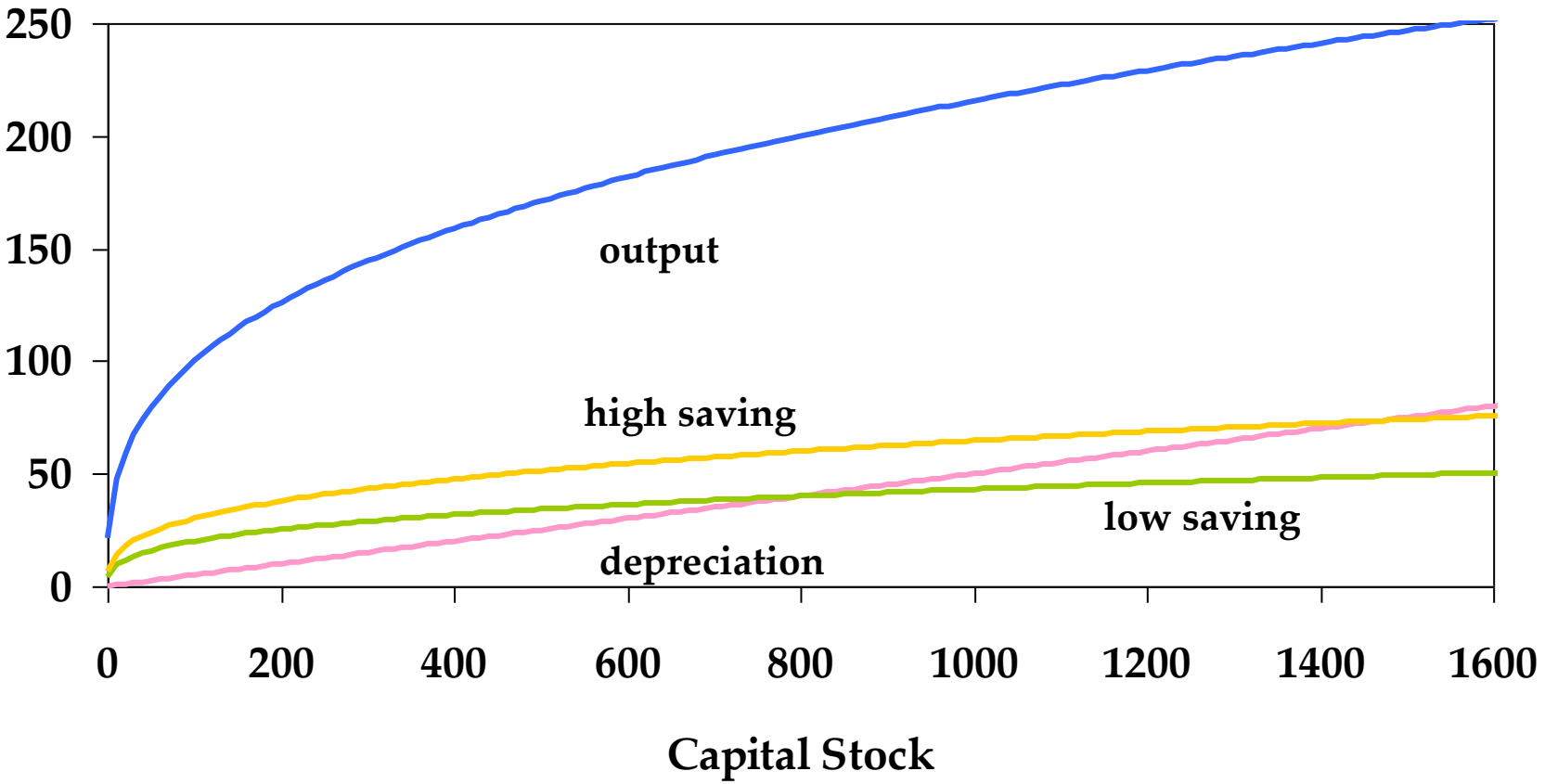
Level effects vs. growth effects

- *Level effect*: change in GDP level
  - Temporary change in growth rate
- *Growth effect:* permanent change in growth rate
- Level or growth rate?
  - Saving rate
  - Population growth

Saving?

- How does the saving rate affect growth?
- See Solow.xls

Solow model: saving



Population growth?

- More on this next week, but…
- Increases GDP, decreases GDP per capita
- Must invest more to provide capital for new workers
- In data, doesn't vary enough to account for growth experiences

Making a "miracle"

- Solow model
  - High saving rate
  - Low capital-labor ratio
- Are these enough?

What have we learned today?

- Solow model
  - Growth from saving-financed increases in capital
  - Convergence property: growth eventually stops
  - Conclusion: capital not the key to sustained growth
  - Value: tool for exploring growth of emerging economies
- What are we missing?
  - TFP growth

For the ride home

- Is aid ($$) good for developing countries?
  - Bono: yes
  - Bill Easterly (NYU faculty): maybe, proof?
- In the Solow model
  - Is money/capital helpful?
  - What else might be?
- Discussion board topic: Housing story