NYU | STERN

The Global Economy
Professor David Backus

Problem Set #0: Math & Spreadsheet Review

Revised: September 9, 2014

This is an individual assignment: you may speak to others about it, but what you hand in should be your own work.

Suggestions:

- *Read the Math Review before you start.*
- *Start Question 3 first to make sure you're able to download the data from FRED.*
- *Read the syllabus about handing in a professional product.*

1. *Exponents and logarithms (30 points).* A *production function* (next class) is a mathematical model of the relation between a firm or country's output and its inputs, which here will be capital and labor. We'll use the function

$$Y = K^{\alpha} L^{1-\alpha}, \tag{1}$$

where Y is the quantity of output, K is the quantity of capital input (plant and equipment), L is the quantity of labor input (number of employees), and $\alpha = 1/3$ is a parameter (take this as given, always). We'll set $K = 100$ and $L = 50$.

   (a) Compute output Y by entering the following formula in a spreadsheet:

```
= 100^(1/3)*50^(2/3)
```

   Make sure you understand what this does. What value do you get? (10 points)

   (b) What is the natural logarithm of Y? (Remember: use the spreadsheet function `LN`.) (5 points)

   (c) Show that the production function can be written

$$\log Y = \alpha \log K + (1 - \alpha) \log L,$$

   where $\log x$ means the natural logarithm of the variable x. What properties of logarithms do we need to derive this from equation (1)? (10 points)

   (d) Compute $\log Y$ using the expression you verified in (c), which translates into the spreadsheet formula

```
= (1/3)*LN(100) + (2/3)*LN(50)
```

   Verify that your answer is the same as the one you computed in (b). (5 points)

2. *How much capital (40 points)?* Our mission here is to use the production function and market prices to decide how much capital a firm should "rent." Let us say that a firm sells output at price p per unit, rents/hires capital K and labor L at prices r and w, and produces output Y according to the production function (1). Its profit is therefore revenue minus cost:

$$\text{Profit} \;=\; pY - (rK + wL) \;=\; pK^{\alpha}L^{1-\alpha} - (rK + wL). \tag{2}$$

   How much capital does the firm want? To make this concrete, let us say that $p = 1$, $w = 1/2$, $r = 3/16 = 0.1875$ (18.75%), $L = 27$, and (always) $\alpha = 1/3$. We'll solve the problem two ways: using a spreadsheet, and using calculus.

   (a) In column A of a spreadsheet, create a column of values for capital K running from 1 to 100 in increments of 1. Then, in column B, compute revenue $pY = pK^{\alpha}L^{1-\alpha}$ for each value of K. Does revenue increase or decrease as you increase capital? (10 points)

   (b) In column C, compute cost $(rK+wL)$ for each value of K. Does cost increase or decrease as you increase capital? (10 points)

   (c) In column D, compute profit for each value of K. Turn in a graph with profit on the y-axis and capital on the x-axis, with the axes clearly labeled. At which value of capital is profit highest? (10 points)

   (d) Now we do the same thing using calculus. Think of profit — equation (2) — as a function of capital K. Compute its derivative and set it equal to zero. For what value of K is the derivative equal to zero? Why does this give us the profit-maximizing value of K? (10 points)

3. *Macroeconomic volatility (30 points).* The term "business cycle" refers to the periodic ups and downs of the economy, evident in GDP (a measure of the total output of the economy) and many other things (employment, retail sales, stock prices, and so on). Years of experience tells us that many things go up and down together, but some components go up and down more. We say they're more volatile, in the sense that the standard deviations of their growth rates are larger.

   Our mission is to verify both facts using quarterly data on GDP and two of its expenditure components, consumption and investment. The first step is to download the data from FRED: http://research.stlouisfed.org/fred2/.

Table 1. FRED Data Codes.

Series or Variable	FRED Code
GDP	GDPC96
Consumption	PCECC96
Investment	GPDIC96

The FRED series codes are given in Table 1. Most people use FRED's Excel add-in to download the data straight from FRED to a spreadsheet on their computer. If you have difficulty with this, the following somewhat more cumbersome method is close to foolproof:

- Go to FRED.
- Type GDPC96 in the search box in the upper right corner. That will generate a graph of real GDP.
- Below the graph, you'll see graph options labeled GRAPH SETTINGS, ADD DATA SERIES, and EDIT SERIES. Click on ADD DATA SERIES.
- Type the code for consumption in the search box directly below the words ADD DATA SERIES and press enter. Then click on the blue button labeled Add Series below the search box. You should now have two lines, one for GDP, the other for consumption.
- Repeat the last two steps with investment.
- Once you have all three series on the graph, click on the Export tab just below the graph. Click on Download as XLS to download the data in a spreadsheet.

See also our FRED videos at http://www.youtube.com/user/NYUSternGE. Whatever method you use, you should end up with a spreadsheet that has dates in the first column and the three series in the next three columns.

Once you have the data, compute, for each series, discretely-compounded annual rates of growth over the period 1950Q1 to 2013Q2. The formula is

$$g_t \;=\; \left[(x_t - x_{t-1}) / x_{t-1} \right] \times 100. \;=\; \left[(x_t / x_{t-1}) - 1 \right] \times 100.$$

The 100 at the end converts the (quarterly) growth rate to a percentage. Place each growth rate in its own column. Note that the growth rate for 1950Q1 requires data for 1949Q4.

Now you're ready to do some analysis:

(a) For each of the three growth rates, compute the mean and standard deviation. How do the standard deviations compare? (10 points)

(b) What are the correlations among the three variables? Do you agree that these variables "move up and down together but have different volatilities"? (A graph would be useful here, too, but is not required.) (10 points)

(c) Compute the continuously-compounded growth rate of GDP using the formula

$$\gamma_t \;=\; (\log x_t - \log x_{t-1}) \times 100.$$

How do the mean and standard deviation compare to those of the discretely-compounded growth rate? Why are the two growth rates so similar? (10 points)