Sources of Economic Growth

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So if saving rates aren't responsible for the enormous differences we see in living standards, what is? The answer is productivity, but our purpose here is to develop a tool that will give us the answer, whatever it might be. Our ingredients are data (always a good thing) and a little bit of theory (the production function). The combination allows us to attribute differences in output and its growth rate to differences in inputs (capital and labor) and total factor productivity (everything else). The answer, as noted, is mostly productivity: rich countries are rich because they're productive, and countries that are growing quickly typically have rapid productivity growth as well. Robert Solow gets credit for this line of thought, too.

Cross-country differences in output per worker

In class we've looked at per capita GDP as measure of aggregate performance: output/income per person. Here we'll look at GDP per worker. The two are connected by

$$Y/POP \;=\; (Y/L)(L/POP).$$

Any differences in GDP per capita reflect either GDP per worker or the ratio of workers to population. We'll focus on the former now, but return to the latter when we get to labor markets.

The production function gives us some insight into GDP per worker. You'll recall that the production function connects an economy's output (real GDP) to the quantity of inputs used in production (capital and labor) and the efficiency with which those inputs are used (productivity). In equation form:

$$Y \;=\; AF(K,L) \;=\; AK^{\alpha}L^{1-\alpha}, \tag{1}$$

where (as before) Y is real GDP or output, A is total factor productivity (TFP), K is the capital stock, and L is the quantity of labor (typically employment). More commonly, we divide both sides by L and express output per worker as

$$Y/L \;=\; A(K/L)^{\alpha}, \tag{2}$$

so that output per worker depends on total factor productivity (A) and capital per worker (K/L). For most countries, we have reasonably good data for GDP, employment, and the capital stock, and productivity can be found as a residual:

$$A \;=\; Y/(K^{\alpha}L^{1-\alpha}).$$

We'll continue to use $\alpha = 1/3$, so there is nothing about equations (1,2) we don't know. In this sense, the production function is no longer an abstract idea, It's a practical tool of analysis.

The production function allows us to make explicit comparisons across countries. If we apply equation (2) to two countries and take the ratio, we get

$$\frac{(Y/L)_1}{(Y/L)_2} = \left[\frac{A_1}{A_2}\right]\left[\frac{(K/L)_1}{(K/L)_2}\right]^{\alpha}, \tag{3}$$

where the subscripts 1 and 2 refer to the two countries. The ratio of output per worker is thus attributed to some combination of the ratios of TFP and capital per worker. Exercises based on (3) are referred to as *level comparisons*. If we have data, we can say which of these factors is most important. If we did this in logarithms, the components would add rather than multiply, but that may be pushing you too far.

Example (Mexico and US). You occasionally hear people in the US say that Mexican workers are paid so much less that they pose a threat to American jobs. (In Mexico, you hear the same thing about Chinese workers.) We can't address that issue — yet — but we can say something about the source of differences in output per worker, which is closely related to differences in wages. The data in Table 1 imply that output per worker is 2.62 times higher in the US, but why? We'll use the data in Table 1 to come up with an answer.

Let's start with TFP. For Mexico, the data in the table imply

$$A_M = 852/[1617^{1/3}34.65^{2/3}] = 6.83.$$

A similar calculation for the US gives us $A_{US} = 12.49$. Thus TFP is 1.83 ($= 12.49/6.83$) times higher in the US. Similarly, the capital-labor ratio is 2.96 times higher in the US. The impact on output per worker is summarized by

$$\begin{aligned}
\frac{(Y/L)_{US}}{(Y/L)_M} &= \frac{A_{US}}{A_M}\left[\frac{(K/L)_{US}}{(K/L)_M}\right]^{1/3} \\
&= (1.83)(2.96)^{1/3} \\
&= (1.83)(1.44) = 2.62.
\end{aligned}$$

	Employment	Education	Capital	GDP
Mexico	34.65	7.23	1,617	852
US	142.08	12.05	19,600	9,169

Table 1: Mexico and US: Aggregate data for 2000. Employment is expressed in millions, education in years, and capital and GDP in billions of 2000 US dollars.

It seems, therefore, that both TFP and capital per worker play a role in accounting for the 2.6 to 1 ratio of US to Mexican output per worker. So the reason why output per worker is higher in the US labor is a combination of higher productivity and higher capital per worker.

This is your chance for speculation: Why do you think the capital-labor ratio is lower in Mexico? Why do you think productivity is lower?

[Note: differences between the US and Mexico are smaller with this data, which has been PPP-adjusted, than if we had simply multiplied Mexican GDP by the exchange rate to express it in dollars. The reason: many goods and services are cheaper in Mexico than the US, so when we apply the same prices to both countries the differences are smaller. See the discussion of PPP adjustment in the notes on national income and product accounts.]

Growth rates

Warning: Read the next two paragraphs carefully!

Our next task is to apply similar methods to account for differences growth, but first we need to be clear about what we mean by growth rates. For many purposes in this course, we will define the growth rate of a variable x between dates t and $t+1$ as

$$\gamma \;=\; \log x_{t+1} - \log x_t \;=\; \Delta \log x_{t+1}.$$

The expression "log" here means the natural log, the function LN in a spreadsheet. We refer to γ as the *continuously-compounded growth rate* for reasons we will ignore. (See the "Math Review" if you're interested.) Typically the dates are years, so γ is an annual growth rate. If we want to express it as a percentage, we multiply by 100. The average growth rate between t and $t+n$ has a similar definition:

$$\gamma \;=\; \left(\log x_{t+n} - \log x_t\right)/n.$$

The terminology here is that γ is the average continuously compounded growth rate.

You might want to know why aren't using the traditional definition of a growth rate, say g in
$$1 + g \;=\; x_{t+1}/x_t \quad \Leftrightarrow \quad g \;=\; (x_{t+1} - x_t)/x_t.$$
Why use γ rather than g? The answer is that what follows works out more neatly with γ. Otherwise we get the kinds of annoying compounding terms you might recall from bond pricing.

You can stop here if you like; in fact, we order you to go immediately to the next section unless you are reasonably comfortable with mathematics. If you are, then here's a more elaborate explanation:

- There's little difference if the growth rates are small. This isn't an argument in favor of our definition, but it's good to know. Suppose $x_t = 100$ and $x_{t+1} = 110$. Then $g = 0.100$ and $\gamma = \log 110 - \log 100 = 0.0953$, so the growth rates are 10% and 9.53%. If the growth rate was smaller, the difference would be smaller, too. If you're not mathematically inclined, go immediately to the next point. If you are, we would tell you that g is a first-order Taylor series approximation to γ. Note that γ is a function of g:

$$\gamma \;=\; \log x_{t+1} - \log x_t \;=\; \log(x_{t+1}/x_t) \;=\; \log(1+g).$$

  This follows from a property of logarithms: $\log x - \log y = \log(x/y)$. A first-order approximation of the function around the point $g = 0$ is

$$\gamma \;\approx\; \log(1) + (1)(g - 0) \;=\; g,$$

  where "$\approx$" means "approximately equal to." Higher-order terms are $g^2/2$, $g^3/6$, and so on, which are very small if g is small.

- Growth rates are additive. Suppose you're interested in the growth rate of a product xy. For example, x might be the price deflator and y real output, so that xy is nominal output. With the traditional measure, the growth rate of xy is

$$1 + g_{xy} \;=\; \frac{x_{t+1}y_{t+1}}{x_t y_t} \;=\; (1+g_x)(1+g_y).$$

  If $g_x = g_y = 0.10$, then $g_{xy} = 0.21$. But note what happens with our definition:

$$\gamma_{xy} \;=\; \log\left(\frac{x_{t+1}y_{t+1}}{x_t y_t}\right) \;=\; \log\left(\frac{x_{t+1}}{x_t}\right) + \log\left(\frac{y_{t+1}}{y_t}\right) \;=\; \gamma_x + \gamma_y.$$

  They add up! Thus the growth rate of a product is the sum of the growth rates. That's not quite true for traditional growth rates, because of the "compound interest" effect: $(1+g_x)(1+g_y) = 1 + g_x + g_y + g_x g_y$. The last term is small if the growth rates are, but it's not zero. This additive feature of growth rates is the primary reason we use them. For similar reasons, the growth rate of x/y equals the growth rate of x minus the growth rate of y.

- Averages are easy to compute. Suppose we want to know the average growth rate of x over n periods:

$$\gamma \;=\; \frac{(\log x_{t+1} - \log x_t) + (\log x_{t+2} - \log x_{t+1}) + \cdots + (\log x_{t+n} - \log x_{t+n-1})}{n}.$$

  If you look at this for a minute, you might notice that most of the terms cancel. The term $\log x_{t+1}$, for example, shows up twice, once with a positive sign, once with a negative sign. If we eliminate the redundant terms, we find that the average growth rate is

$$\gamma \;=\; \frac{\log x_{t+n} - \log x_t}{n} \;=\; \frac{\log(x_{t+n}/x_t)}{n}.$$

  We can compute it, then, from the initial and final values of x.

Finally, to go from growth rates back to levels, we need to use a method that corresponds to the growth rate we are using. For a traditional growth rate, we update levels by $x_{t+1} = (1 + g)x_t$. For continuously-compounded growth rates, we use $x_{t+1} = \exp(\gamma)x_t$.

Cross-country differences in growth rates

We are now ready to apply the methods of the first section to growth rates. As before, the starting point is the production function. If we take the natural logarithm of both sides of the production function (1), we find that

$$\log Y_t = \log A_t + \alpha \log K_t + (1 - \alpha) \log L_t$$

for any date t. This follows from two properties of logarithms: $\log(xy) = \log x + \log y$ and $\log x^a = a \log x$. If we take the difference between two adjacent periods, we get

$$\Delta \log Y_t = \Delta \log A_t + \alpha \Delta \log K_t + (1 - \alpha)\Delta \log L_t,$$

whose components should be recognizable as continuously-compounded growth rates. If we consider differences over several periods, we can divide each term by the number of periods to get

$$\left(\frac{\log Y_{t+n} - \log Y_t}{n}\right) = \left(\frac{\log A_{t+n} - \log A_t}{n}\right) + \alpha \left(\frac{\log K_{t+n} - \log K_t}{n}\right)$$
$$+ (1 - \alpha)\left(\frac{\log L_{t+n} - \log L_t}{n}\right)$$

or

$$\gamma_Y = \gamma_A + \alpha\gamma_K + (1 - \alpha)\gamma_L, \tag{4}$$

where γ_X is the average continuously-compounded growth rate of the variable X. Draw a box around this equation, it's important and we'll use it repeatedly. It says that the growth rate of output can be attributed to growth in productivity, capital, and labor. Thanks to our clever use of logarithms, the terms add up.

As with levels, we can do the same for the growth rate of output per worker:

$$\gamma_{Y/L} = \gamma_Y - \gamma_L$$
$$= \gamma_A + \alpha(\gamma_K - \gamma_L)$$
$$= \gamma_A + \alpha\gamma_{K/L}. \tag{5}$$

Exercises based on (4) and (5) are referred to as *growth accounting*. We refer to the terms on the right of (4) and (5) as *contributions* to the growth rates of Y and Y/L, respectively.

	Employment	Education	Capital	GDP
1965	2,751.4	4.77	65,192.41	34,988.5
2000	6,019.1	7.89	258,200.21	150,896.0

Table 2: Chile: Aggregate data for 1965 and 2000.

Both versions give us some insight into the sources of economic growth. For example:

Example (Chile between 1965 and 2000). GDP increased by almost a factor of five between 1965 and 2000. Can we say why? The relevant data are reported in Table 2. The first step is to compute growth rates. Over this period, the average annual growth rate of real GDP was

$$\gamma_Y = \frac{\log Y_{2000} - \log Y_{1965}}{35} = (11.93 - 10.46)/35 = 0.0417,$$

or 4.17%. Using the same method, we find that the growth rates of the other variables we need are $\gamma_K = 3.93\%$ and $\gamma_L = 2.24\%$. The growth rate of total factor productivity is the residual in equation (4):

$$\gamma_A = \gamma_Y - [\alpha\gamma_K + (1-\alpha)\gamma_L] = 1.37\%.$$

(You could also compute A for each period and calculate the growth rate directly.) So why did output grow? Our numbers indicate that of the 4.17% growth in output, 1.37% was due to TFP, 1.31% [= 3.93/3] was due to increases in capital, and 1.49% [= 2.24 × (2/3)] was due to increases in employment.

What about output per worker? That seems to be the more interesting comparison, because it's closer to an average living standard. The growth rate of output per worker is $\gamma_{Y/L} = 1.93\%$. Its components are

$$\begin{aligned} \gamma_{Y/L} &= \gamma_A + \alpha\gamma_{K/L} \\ 1.93 &\quad 1.31 + (1/3)1.69, \end{aligned}$$

so most of the growth in output per worker comes from productivity.

Extensions (optional)

We will sometimes use modifications of these tools. Two of the more common ones are based on (i) more refined measures of labor and/or (ii) GDP per capita rather than GDP per worker. The logic is the same as before, but we gain an extra term or two.

Labor measures. Consider a measure of labor that includes adjustments for hours worked h and human capital H. If the labor input is hHL (with L the number of people employed), the production function becomes

$$Y = AF(K, hHL) = AK^{\alpha}(hHL)^{1-\alpha}. \tag{6}$$

How does this change our analysis of levels and growth rates? In a level comparison, this leads to

$$\frac{Y_1}{Y_2} = \left[\frac{A_1}{A_2}\right]\left[\frac{K_1}{K_2}\right]^{\alpha}\left[\frac{L_1}{L_2}\right]^{1-\alpha}\left[\frac{h_1}{h_2}\right]^{1-\alpha}\left[\frac{H_1}{H_2}\right]^{1-\alpha}.$$

The subscripts 1 and 2 again represent countries. You can derive further modifications for output per worker (Y/L) and output per hour worked (Y/hL). In a growth rate analysis, the augmented production function (6) leads to

$$\gamma_Y = \gamma_A + \alpha\gamma_K + (1-\alpha)(\gamma_h + \gamma_H + \gamma_L)$$

for output and

$$\gamma_{Y/L} = \gamma_A + \alpha\gamma_{K/L} + (1-\alpha)(\gamma_h + \gamma_H)$$
$$\gamma_{Y/hL} = \gamma_A + \alpha\gamma_{K/hL} + (1-\alpha)\gamma_H$$

for output per worker and output per hour, respectively. If this sounds complicated, remember that the choice of tool depends on the question we're trying to answer.

We have some choices when it comes to measuring human capital. One simple choice is to equate human capital and years of school: $H = S$ if we want to give it mathematical form. A more sophisticated choice is to give education a rate of return, so that

$$H = \exp(\sigma S), \tag{7}$$

where σ is kind of a rate of return on school, as each year raises human capital proportionately. Estimates of σ are in the range of 0.07, so that each year of school raises human capital by about 7%. (The reason for the word "about" is that it's a continuously-compounded rate of return, something you're free to ignore.)

Per capita GDP. We often start with GDP per capita, rather than GDP per worker. How can we adapt our analysis to account for the former? Here's a trick: start with equation (2) and multiply both sides by the ratio of employment to population:

$$Y/POP = (L/POP)(Y/L) = (L/POP)A(K/L)^{\alpha}.$$

In a level comparison, this gives us an extra term: the ratio of L/POP across countries. In growth rates, we'd add an extra term for the growth rate of the employment ratio:

$$\gamma_{Y/POP} = \gamma_{L/POP} + \gamma_A + \alpha\gamma_{K/L}.$$

And if you want to get fancy, you can add hours and human capital terms, as we did above.

Example (Mexico and US, revisited). How does our analysis of the US and Mexico change if we incorporate differences in human capital? We set human capital H equal to years of school and redo our earlier analysis. TFP is now

$$A_M = 852/[1617^{1/3}(7.23 \times 34.65)^{2/3}] = 1.827$$

for Mexico and $A_{US} = 2.376$ for the US. Note that the ratio has fallen from 1.83 to 1.30. Why? Because part of the previous difference now shows up in human capital. [Reminder: A is a residual, so any change in the analysis changes our measure of it.] We now attribute some of the difference in output per worker to a difference in education:

$$\begin{aligned}
\frac{(Y/L)_{US}}{(Y/L)_M} &= \frac{A_{US}}{A_M} \left[\frac{(K/L)_{US}}{(K/L)_M}\right]^{1/3} \left[\frac{H_{US}}{H_M}\right]^{2/3} \\
&= (1.30)(2.96)^{1/3}(1.67)^{2/3} \\
&= (1.30)(1.44)(1.41) = 2.62.
\end{aligned}$$

It appears that more than half of our earlier difference in TFP stems from differences in education. We amend our previous analysis to add: a substantial part of the difference between output per work in the US and Mexico stems from differences in education.

An alternative is to measure human capital using our rate of return formula, equation (7). If we do this, the ratio of human capitals is 1.40, which is less than we had before. This choice makes an even bigger difference with countries like India that have low average education. If years of school go from 2 to 3, is that a 50% increase in human capital or a 7% increase? You be the judge. Of course, it may depend on what they teach them, too.

Executive summary

1. Recall: a production function links output to inputs and productivity.

2. Therefore: differences in output and growth rates stem from differences in the levels and growth rates of inputs and productivity.

3. Bottom line (illustrated by examples): most large differences in output per worker reflect large differences in productivity.

Review questions

1. France and the UK. In 2000, the data were

	Employment	Education	Capital	GDP
France	27.497	7.86	3,852	1,351
UK	29.697	9.42	2,873	1,326

   Which country had higher output per worker? Why? You should assume that human capital is equal to years of school.

   Answer. Ratios were as follows:

   $$\left(\frac{(Y/L)_F}{(Y/L)_{UK}}\right) = \left(\frac{A_F}{A_{UK}}\right)\left(\frac{(K/L)_F}{(K/L)_{UK}}\right)^{1/3}\left(\frac{H_F}{H_{UK}}\right)^{2/3}$$

   $$1.10 = (1.10)(1.45)^{1/3}(0.83)^{2/3}.$$

   That is: France had higher TFP and more capital per worker, but a lower level of education than the UK.

2. US and Japan. Explain why output grew faster in Japan between 1970 and 1985. Data:

	United States			Japan		
	1970	1985	Growth	1970	1985	Growth
GDP	2083	3103	2.66	620	1253	4.69
Capital	8535	13039	2.83	1287	3967	7.50
Labor	78.6	104.2	1.88	35.4	45.1	1.61

   Employment is measured in millions of workers, GDP and capital in billions of 1980 US dollars. Growth rates are continuously-compounded average annual percentages.

   Answer. In levels (as opposed to growth rates) we see that the US had much greater output per worker in 1970: 26.5 (thousand 1980 dollars per worker) vs 17.5. Where did this differential come from? One difference is that American workers in 1970 had three times more capital to work with: K/L was 108.6 in the US, 36.4 in Japan. If we use our production function, we find that total factor productivity A was also slightly higher in the US in 1970: 5.64 vs 5.35. Thus, the major difference between the countries in 1970 appears to be in the amount of capital: American workers had more capital and therefore produced more output, on average.

   By 1985, much of the difference had disappeared. For the US, the output growth rate of 2.66% per year can be divided into 0.94% due to capital and

1.26% due to employment growth. That leaves 0.47% for productivity growth. For Japan the numbers are 2.48% for capital, 1.08% for labor, and 1.13% for productivity. Evidently the largest difference between the two countries was in the rate of capital formation: Japan's capital stock grew much faster, raising its capital-labor ratio from 36.4 in 1970 to 88.0 in 1985.

If you're looking for more

Many macroeconomics textbooks cover similar material. The tools are widely used by analysts. Some of the most interesting applications have been done by McKinsey, whose studies have connected cross-country differences in TFP to government regulation, management practices, and the competitive environment. Some of this work is summarized in William Lewis's *The Power of Productivity* (University of Chicago Press, 2004). Other examples are available on McKinsey's web site.