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The Global Economy
Professor David Backus

Practice Problems A: Production & Capital Formation

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This will not be collected or graded, but it's a good way to make sure you're up to speed. We recommend you do it before the next class.

> **Solution:** Brief answers follow, but see also the attached spreadsheet: download this pdf file, open it with the Adobe Reader or the equivalent, and click on the pushpin:

1. *Labor productivity.* Consider our production function,

$$Y = AK^{\alpha}L^{1-\alpha}.$$

When needed, we'll use these numerical values: $\alpha = 1/3$, $A = 1$, $L = 100$, and $K = 250$.

(a) What are the components of the production function? What do they tell us?

(b) A common summary number is the average product of labor, Y/L, often referred to simply as labor productivity. Using the numbers given, what are output Y and labor productivity Y/L?

(c) Suppose A rises from 1 to 1.25. What happens to output? Labor productivity?

(d) Consider a similar country in which $Y = 150$, $K = 150$, and $L = 100$. Which country has higher total factor productivity A? Which has higher labor productivity Y/L?

> **Solution:**
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> (a) In order: Y is output, total production in a country, usually measured by real GDP; A is total factor productivity (TFP), a measure of how effectively inputs are used to produce output; K is the quantity of physical capital, plant and equipment; L is the number of employees working to produce output; α is a parameter that we set equal to 1/3 (one-third of output is paid to capital, two-thirds to labor).
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> (b) We get $Y = 135.721$ and $Y/L = 1.357$.

(c) If A rises 25%, output and output-per-worker rise proportionately: $Y = 169.651$ and $Y/L = 1.697$.

(d) Total factor productivity A in the new country is 1.310, which is higher than the previous country. Labor productivity Y/L is 1.500, which is lower. The difference reflects the roles of A and K in labor productivity.

2. *Dynamics of the capital-output ratio.* The investment rate is very high in China — over 40% of GDP — which suggests that China's capital intensity must also be high. But is it? This will take some work, but we'll see that a country that grows rapidly has to invest a lot just to keep up with output. Could that be the case in China?

We typically express capital intensity with the ratio of the capital stock to GDP, the so-called *capital-output ratio*. In this ratio, both numbers are "real:" we want to know many machines are used to produce a given quantity of output. Most countries — rich, poor, and in between — have ratios around 2 or 3.

(a) Our starting point for thinking about how the ratio changes over time is the relation connecting capital next year to capital today:

$$K_{t+1} \;=\; (1-\delta)K_t + I_t.$$

What is δ here? I_t?

(b) Divide the whole equation by current output Y_t:

$$\frac{K_{t+1}}{Y_t} \;=\; (1-\delta)\,\frac{K_t}{Y_t} + \frac{I_t}{Y_t}.$$

We almost have the connection between the capital-output ratio today and next year, but the left side has a timing mismatch: K_{t+1} is divided by Y_t. We know, however, how current and future output are connected: $Y_{t+1} = (1+g_{t+1})Y_t$, where g_{t+1} is the (simple) growth rate. Use this to show

$$(1+g_{t+1})\frac{K_{t+1}}{Y_{t+1}} \;=\; (1-\delta)\,\frac{K_t}{Y_t} + \frac{I_t}{Y_t}.$$

(c) Now suppose the growth rate is constant at g and the investment rate is constant at $I_t/Y_t = s$. How does the equation change?

(d) Finally, suppose the capital-output ratio settles down to a constant, so that $K_{t+1}/Y_{t+1} = K_t/Y_t = K/Y$. What is this "steady state" value of K/Y?

(e) We're now ready to answer our questions. If $s = 0.18$, $g = 0.03$, and $\delta = 0.06$, what is the steady capital-output ratio? (These are ballpark US numbers.)

Now suppose the growth rate rises to $g = 0.10$, which is roughly what we've seen in China. What is the capital-output ratio if the investment rate doesn't change? How much does the investment rate s have to rise to produce the same steady state?

(f) Does this help to explain why China's investment rate is so high?

Solution:

(a) By convention δ is the depreciation rate on capital (the fraction lost in a year from wear and tear) and I is real investment (purchases of new plant and equipment).

(b) Yours to do.

(c) That gives us

$$(1+g)\frac{K_{t+1}}{Y_{t+1}} \;=\; (1-\delta)\,\frac{K_t}{Y_t} + s.$$

(d) If K/Y is constant, we get $K/Y = s/(g+\delta)$. In words, an increase in the investment rate s raises K/Y. But an increase in depreciation δ or GDP growth g reduces K/Y. The growth rate effect comes from the denominator: If g is high, we need to invest more to keep the capital-output ratio at a given level.

(e) The steady state capital-output ratio with ballpark US numbers is $s/(g+\delta) = 0.18/(0.03+0.06) = 2$. If we raise g to 0.1, this falls to 1.125. Alternatively, if we ask how much s would have to rise to get $K/Y = 2$, we set $s = (K/Y)(g+\delta) = 2(0.1+0.06) = 0.32$.

The point is that when we look at changes in K/Y (or similar ratios), a lot of the action comes from the denominator. We'll see the same thing when we look at the ratio of government debt to GDP. In this case, China needs a high investment rate just to keep the capital-output ratio at the US level because it's growing so rapidly.

3. *Recovering from war in the Solow model.* Solow's analysis tells us that productivity growth is more important than capital formation for the long-term performance of countries, but it's a mistake to disregard capital formation altogether. The standard example is war, which can destroy a lot of the physical capital. The subsequent recoveries typically follow the Solow model pretty well, with growth rates high at first, then declining as capital catches up to its prewar level.

We'll consider an economy that starts at its long-run "stationary" level of capital, then go on to study its recovery following a 20% drop.

(a) Consider an economy whose parameters include $A = 1$, $L = 100$, $\alpha = 1/3$, $\delta = 0.1$, and $s = 0.2$. Explain what each of these does.

(b) Suppose we start out with $K = 250$. If you adapt the spreadsheet and compute K for many periods, what value does it seem to be heading toward? What about K/Y?

(c) Suppose you start at the steady state computed in (b), but the capital stock falls suddenly by 20% (think: war). Enter this in the spreadsheet and see what happens. What happens to the growth rate of GDP in subsequent years? How many periods does it take for the capital stock to recover to 90% of its steady state value?

Solution:

(a) Most of these were covered in Question 1. The new elements are the depreciation rate δ (depreciation equals δ times the current stock of capital) and the saving rate (the fraction of income that's saved, which equals the fraction of output devoted to investment in this model).

(b) After 100 years, $K = 282.81$. [Not required, but the steady state calculation gives us $K = (sA/\delta)^{1/(1-\alpha)}L = 282.84$, so we're pretty close.]

(c) If we start at 226 (about 20% lower than the steady state), we get to 254 (halfway to the steady state) in 10-11 years. The GDP growth rate declines steadily, much as we saw in Japan and Germany following World War II.