NYU STERN

**Fall 2011**

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# Practice Problems – Math Refresher
## September 17, 2011
## ANSWER KEY

1. **GDP Statistics:** For this question, use the Excel workbook "PS0.xls" which can be downloaded from Blackboard. The data in the workbook are real Gross Domestic Product (GDP) from the Bureau of Economic Analysis, the government agency tasked with computing GDP. The data is quarterly; the dating convention is that 1950.00 is first quarter of 1950, 1950.25 is second quarter of 1950, etc.

   a. Step one: In column C, compute the *annualized* quarter-on-quarter growth rate (multiply the continuously compounded rate of change by four). (Hint: Page 4 of the Math Refresher has an example of continuously compounded growth rates for Korea.)

   b. Step two: Plot the growth rates you computed in part a. Label the axes and give the chart a title.

   c. Step three: Using Excel's statistical functions, complete columns 2 and 3 of the table below.

| Period | Annualized GDP Growth | Standard Deviation of Annualized GDP Growth |
|---|---|---|
| **1950.00-1985.00** | 3.60% | 4.61% |
| **1985.00-2007.75** | 3.00% | 1.99% |
| **2007.75-2011.25** | -0.12% | 3.92% |

   d. Step four: In words, briefly describe how GDP growth has changed over these three periods.

The most striking difference between the first and second periods is the reduced amplitude of fluctuations in real GDP at short-term intervals. Business cycles were notably milder in the second period, which is known to economists as "the Great Moderation." The brief third period is dominated by the deepest and longest post-World War II recession, combined with a remarkably sluggish recovery. The U.S. economy still has not returned to the peak level of GDP reached in the third quarter of 2007. Moreover, the rebound of economic volatility suggests that the Great Moderation has ended.

**2. Optimization:** The production function (which we will study in detail in the next class) is a mathematical model of the relationship between inputs, in this case labor and capital, and output. Our production function is

$$Y = K^{\alpha} L^{1-\alpha}$$

where $Y$ is output, $K$ is capital, $L$ is labor, and $a = 1/3$ is a parameter. Output is sold at price $p = 10$, the wage rate is $w = 1.25$, and the price of capital is $r = 1.20$. For this question we will assume that the number of hours of work you have available is fixed at 27, so your job is to choose the right amount of capital to use in order to maximize profits.

Profits can be expressed as revenue minus costs:

$$\text{profit} = pK^{\alpha} L^{1-\alpha} - wL - rK \, .$$

a. In column A of an Excel worksheet, create a column of capital values ranging from 5 to 445, in increments of 10. Next, for each value of capital in column A, compute the revenue associated with using that amount of capital, given the values of the other variables specified above. Is revenue increasing, or decreasing as you add capital?

Revenue increases at a diminishing pace.

b. For each value of capital in column A, compute the costs ($wL + rK$) associated with using that amount of capital, given the values of the other variables specified above. Are costs increasing or decreasing as you add capital?

Costs increase linearly with additions to capital.

c. Compute profit for each value of capital. Create a plot with capital on the x-axis and profit on the y-axis. At what value of the capital stock is profit maximized?

K = 125

d. An alternative to the plot-and-check method in part c. is to use calculus. To find the value of capital that maximizes profits, take the derivative of the profit function ($pK^{\alpha} L^{1-\alpha} - wL - rK$) with respect to $K$. This derivative is called the "first order condition." The derivative is $\alpha p K^{\alpha-1} L^{1-\alpha} - r$. (If you do not know why this is the derivative, see the Math Refresher.) In your workbook, compute the value of the first order condition for each value of capital. At which value of capital is the first order condition equal to 0?

K = 125