

## Business Cycle Indicators

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Probably the leading use of macroeconomic data (and macroeconomists) is forecasting: predicting future movements in economic variables so that businesses can decide how much to produce, investors can decide how to allocate their assets, and households can decide how much to spend. The good news is that forecasting is possible; we're not simply throwing darts at a board. The bad news is that it's not easy — even the best forecasters are far from perfect.

This set of notes is devoted to short-term “business cycle” indicators — variables that indicate changes in near-term economic conditions — and how to use them. In principle we could be interested in many features of the economy: output, inflation, interest rates, exchange rates, and so on. We'll focus on output, but the methods can easily be applied to other variables. We look at the US, but similar ideas and methods apply to any country with reliable data.

### Terminology

We refer to the properties of economic indicators with two related sets of terms. One set of terms describes whether an indicator's movements tend to come before or after movements in output. We say an indicator *leads* output if its ups and downs typically precede those of output, and *lags* output if they come after. An indicator whose movements are contemporaneous with those of output is referred to as *coincident*. Thus the adjectives leading, lagging, and coincident describe the timing of an indicator's movements relative to those of output. Looking ahead, you might guess that leading indicators are most useful in forecasting. The stock market, for example, is a common leading indicator; it leads output by 6-8 months, as we'll see shortly.

A second set of terms refers to whether an indicator's movements are positively or negatively correlated with output. If the correlation is positive, we say it is *proccyclical*. If the correlation is negative, we say it is *countercyclical*. Most indicators are procyclical: employment, stock prices, housing starts, and so on. The most common countercyclical indicators have to do with unemployment: both the unemployment rate and new claims for unemployment insurance rise during recessions.

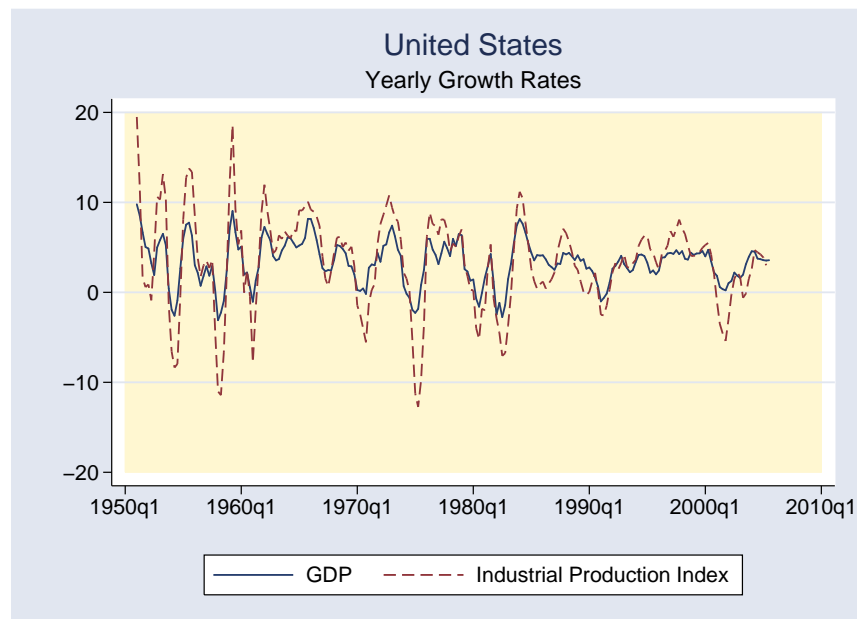


Figure 1: GDP and Industrial Production.

### On forecasting

The classic forecasting problem goes something like this: What do we expect the value of [some economic variable] to be  $k$  periods in the future? Here  $k$  is any period of time you like, but we're usually interested in anything from next week to a few years in the future.

If we're forecasting GDP, there's an extra difficulty: we don't know the present or the recent past, much less the future. We've seen, for example, that fourth-quarter GDP is first reported near the end of the following January, and even that number is a preliminary estimate. From the perspective of mid-January, then, we need to "forecast" the previous quarter.

We're going to shortcut this difficulty (somewhat) by using the monthly Industrial Production (IP) index as a substitute for real GDP, but the issue is a general one: the time lag in getting data is both an issue in its own right and a constraint on forecasting the future. IP measures output in manufacturing, mining, and utilities. More important, its fluctuations are strongly correlated with those in GDP. You can see that in Figure 1, which compares year-on-year growth rates in GDP and IP (aggregated to a quarterly frequency). You will notice that IP is more volatile than GDP but otherwise follows its ups and downs reasonably well. You may also notice some differences between them in the recent past, which have been traced to the rising importance of services in the US economy. In the US, IP is reported by the Federal Reserve in the middle of the following month. Data for December, for example, are available in mid-January. Using IP therefore gives us a shorter information lag

than GDP. In addition, the monthly frequency gives us a finer time interval for near-term forecasting. For both reasons, we will focus our discussion of forecasting on IP rather than GDP, although the same principles apply to both, as well as to other macroeconomic and financial variables.

## Good indicators

Good forecasts require good inputs. One way to forecast a variable is with its own past. Future growth rates of IP, for example, might be related to current and past growth rates. We can usually do better than that by adding other indicators to our analysis. Speaking generally, a good indicator should have one or more of these properties:

- Correlation. A good indicator is correlated with the variable we are forecasting.
- Lead. A good indicator leads the variable we are forecasting.
- Timeliness. A good indicator is available quickly.
- Stability. A good indicator does not undergo major revisions subsequent to its initial release, and its relation with the variable we are forecasting doesn't change over time.

On the whole, measures of economic activity (employment, for example) tend to be strong on correlation, weak on timeliness (see the discussion of GDP above) and stability (many economic series are revised frequently). The best ones lead the business cycle. In contrast, financial indicators (equity prices, interest rates) are weaker on correlation but stronger on the other three properties: they're typically available immediately, often lead the cycle, and are not revised. Various indexes of leading indicators combine multiple series with the hope of getting the best from each. The Conference Board's quasi-official index of leading indicators is the most common example.

## Identifying good indicators: the cross-correlation function

How do we identify indicators with high potential? We'll use another bit of terminology that leads to an extremely useful graphical representation of the dynamic relation between two variables: the *cross-correlation function* (ccf).

You may recall that the correlation between two variables ( $x$  and  $y$ , say) is a measure of how closely they are related in a statistical sense. If the correlation is (say) 0.8, then

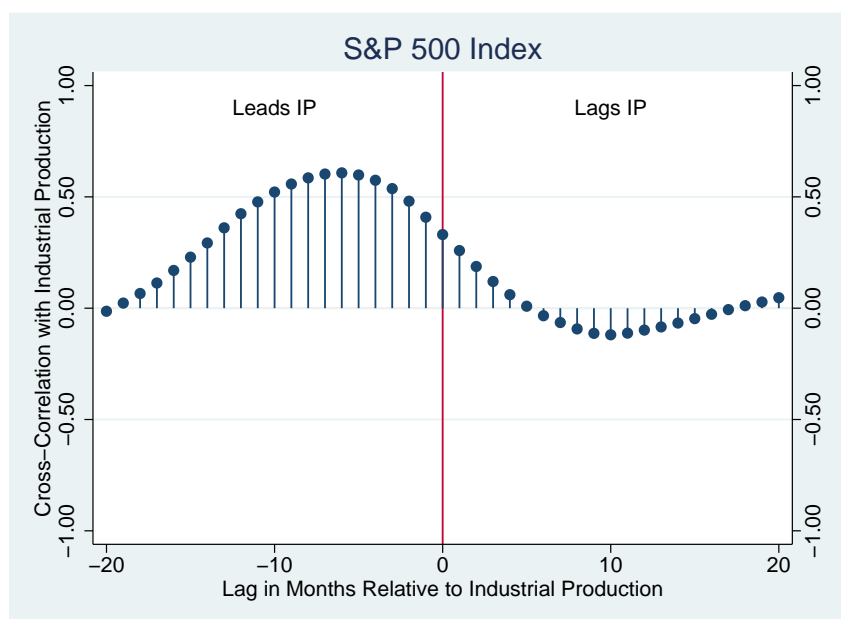


Figure 2: Cross-correlation function for the S&P 500 index and industrial production. Both series are year-on-year growth rates for the period 1960-present. The large correlations to the left tell us that the S&P 500 index is a good indicator of future industrial production. Why? Because the correlation is high (over 0.5) to the left (which indicates that the S&P 500 “leads” industrial production).

observations with large values of  $x$  tend also to have large values of  $y$ . If the correlation is 0.4, this association is weaker. And if the correlation is  $-0.8$ , observations with large values of  $x$  tend to have small values of  $y$  — and vice versa.

The cross-correlation function extends the concept of correlation to the timing of two indicators. Specifically, consider the correlation between  $x$  at date  $t$  and  $y$  at date  $t - k$ . If  $k$  is negative, then we’re talking about the correlation between  $x$  now and  $y$   $k$  periods in the future. If  $k$  is positive, we have the correlation between  $x$  now and  $y$   $k$  periods in the past. By looking at the pattern of correlations, we can identify indicators  $x$  that tend to lead the variable  $y$ . We refer to  $k$  as the lag of  $y$  vs  $x$ , but if  $k$  is negative it evidently refers to a lead. Mathematically, we write

$$\text{ccf}(k) = \text{corr}(x_t, y_{t-k}).$$

Typically, we would graph this against  $k$ , with  $k$  starting with a negative number and moving to positive numbers. The pattern of correlations tells us whether an indicator  $x$  leads or lags (on average) a variable  $y$ .

Let’s move from the abstract to the concrete to make sure we understand what the ccf represents. [You might want to work your way through this paragraph slowly, it’s important.] We calculate the year-on-year growth rates of the S&P 500 index and

industrial production and compute their ccf using the S&P 500 for  $x$  and industrial production for  $y$ . Figure 2 is a plot of their correlations against the lag  $k$ . There's a lot of information here, so let's go through it one dot at a time. The dot at  $k = 0$  (on the vertical line at the center of the figure) shows that the contemporaneous correlation is about 0.2. Contemporaneous means that we're looking at the two variables at the same time: March 2001 industrial production is lined up with March 2001 S&P 500, and so on. Next consider the dot corresponding to  $k = -10$  on the left side of the figure. The correlation of (roughly) 0.5 pictured in the figure shows the growth rate of industrial production with the growth rate of the S&P 500 index dated 10 months earlier. Evidently high growth in equity prices now is associated with high growth in IP 10 months later. Finally, consider a dot on the right side of the figure. The dot at  $k = +10$  suggests that the correlation of industrial production growth with equity price growth 10 months later is about  $-0.2$ .

This pattern of correlations tells us a lot about the timing of movements in the two variables. In general, negative values of  $k$  (the left side of the figure) indicate correlations of the S&P 500 with future industrial production; we would say they reflect the tendency of stock prices to lead output. Positive values of  $k$  (the right side of the figure) indicate correlations of the S&P 500 with past industrial production; they reflect the tendency of stock prices to lag output. What we see in the figure is a strong correlation of the S&P 500 index with industrial production 7-8 months later. Evidently the stock price index is a leading indicator of industrial production.

A digression. We snuck something in here that we should mention again, although it's not particularly important for our purposes: We used year-on-year growth rates instead of monthly growth rates. We could use either, but the year-on-year pictures are smoother and therefore more attractive. With monthly growth rates we'd see a similar pattern but the correlations would be both smaller and choppier.

Let's look at some other indicators and see which ones lead IP. Some of the most common indicators are labor market variables, constructed by the Bureau of Labor Statistics. Cross-correlation functions for four of them are pictured in Figure 3. Nonfarm payroll employment (a measure of employment constructed from a survey of firm payrolls) is (apparently) a slightly lagging indicator, since the ccf peaks with a lag of 1-2 months. It is nevertheless useful, because the correlation (over 0.8) is unusually strong. And even a 2-month lag is more timely than the GDP numbers. The unemployment rate is countercyclical (note the negative correlations) and lags IP in the sense that the largest correlation comes at a lag of 3-4 months. It seems that a rise (fall) in output is associated with a fall (rise) in the unemployment rate 3-4 months later. New applications ("claims") for unemployment insurance are also countercyclical, but the correlation is stronger than for the overall unemployment rate and it leads industrial production by 2-3 months. Another popular indicator (not shown) is average hours worked per week in manufacturing. This indicator is strongly procyclical and leads industrial production by 2-4 months. The labor market, in short,

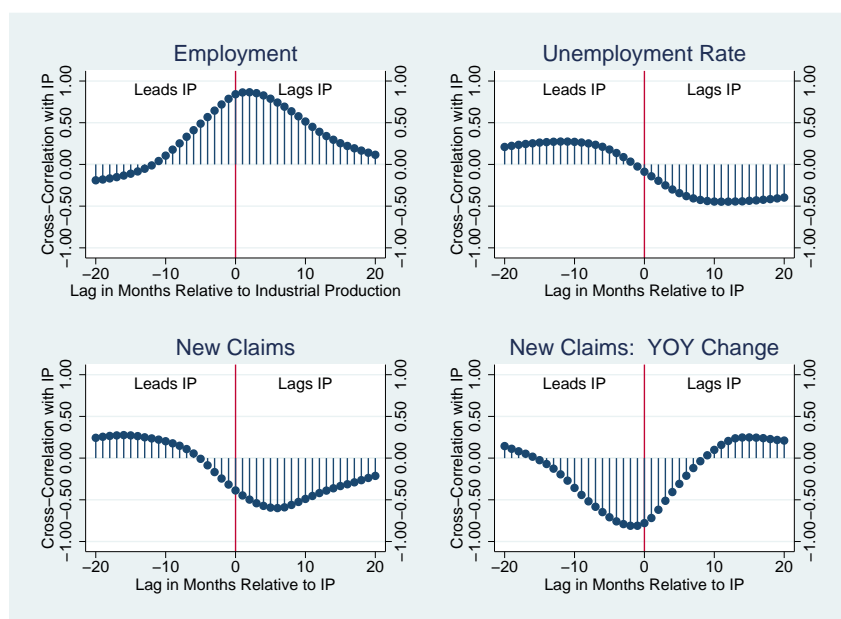


Figure 3: Cross-correlation functions for labor market indicators and industrial production.

provides a good overall picture of the economy, and in some cases supplies indications of future movements in industrial production. The leading variables (“new claims” and “average weekly hours”) are more highly correlated with industrial production than the S&P 500 index, but the leads are shorter.

Other sources of useful information are various measures and surveys of economic activity conducted by the Bureau of the Census and private organizations. Cross-correlation functions for four common ones are pictured in Figure 4. The first two are building permits and housing starts, two indicators of new home construction reported by the Census. Two ideas lie behind their use: that construction of new capital is more volatile than other sectors of the economy, and that decisions to build new homes reflect optimism about the future. The cross-correlation functions suggest they work: while the correlations are smaller than with (say) employment, the leads are substantial (10 months or so). The next two are popular private surveys. Consumer sentiment is based on a survey of consumers collected by the University of Michigan. It reflects consumers’ optimism about current and future economic conditions. The purchasing managers index is what we call a “diffusion index.” It’s based on a survey of purchasing managers who report whether they see economic activity increasing or decreasing. Each is used as is. We see in the figure that both are procyclical leading indicators.

We could go on. There are hundreds of indicators, more all the time. The most common one we’ve skipped is the slope of the yield curve: flat or downward-sloping yield curves are associated with slower-than-usual future growth in output. More on

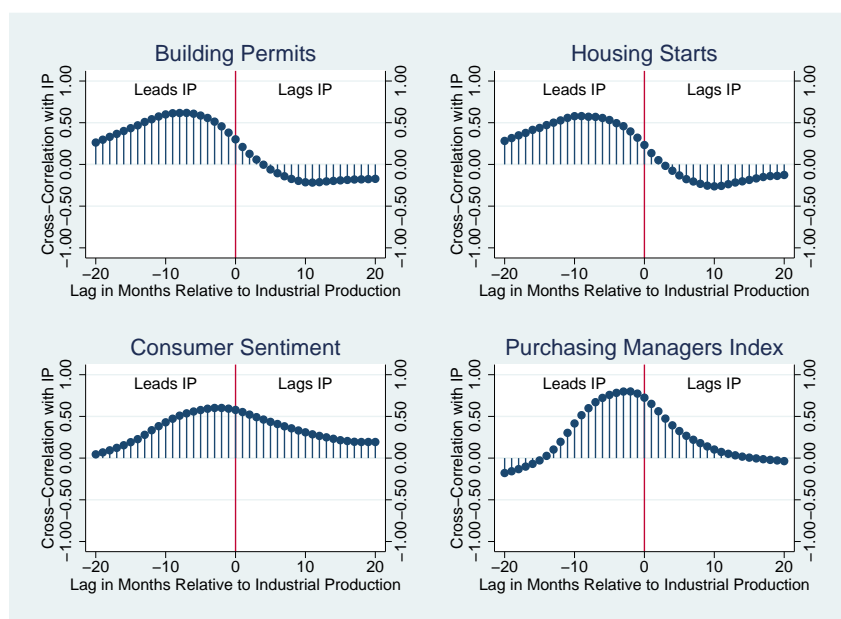


Figure 4: Cross-correlation functions for surveys of economic activity and industrial production.

this in the Appendix.

### Informal assessment of economic conditions: the business-cycle scorecard

Now that we understand how to identify good indicators, the question is how to put them to work. The question is how to combine the inputs of multiple indicators. One way to do that is to summarize them informally, which is what we do here. Another is to use multivariate regression, which is the next topic.

The business cycle scorecard is a summary of what selected indicators tell us about near-term economic conditions. We'll use the four monthly indicators pictured in Figures 5 and 6. In the first figure we see the monthly growth rate of IP (top panel) and the change in (nonfarm) employment for the period 1960 to the present. They show similar patterns, with the major postwar downturns evident in each. Evidently employment is procyclical: it rises in good times and falls in bad times. Industrial production is a “noisier” series, which is one reason many analysts prefer employment as a measure of current economic conditions. More on this later. The lines show us the mean value (the solid line) and plus and minus one standard deviation (the dashed lines). The lines are useful benchmarks for telling how strong the value of an indicator is relative to past experience.

In the second figure (Figure 6), we see similar data for new claims for unemployment insurance and housing starts. New claims are reported weekly; the figure is based

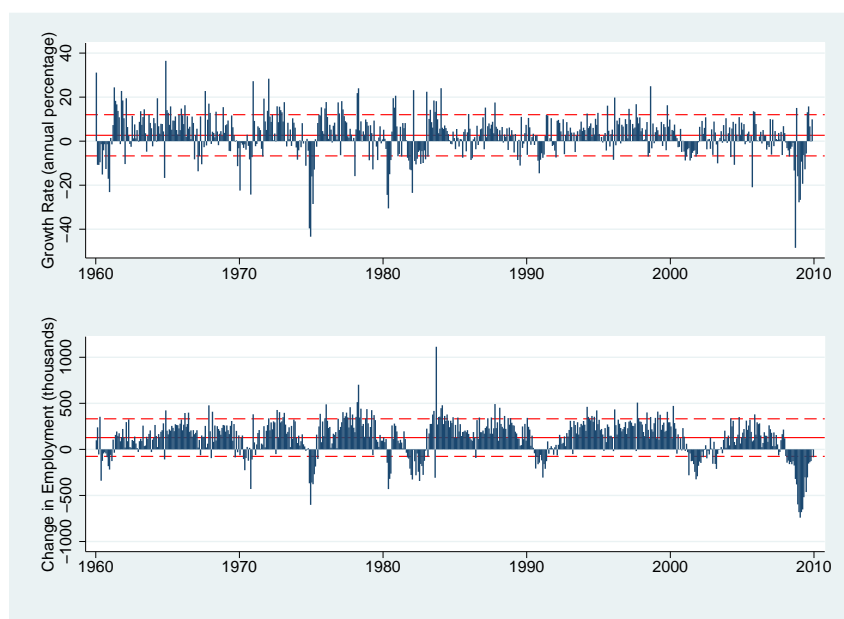


Figure 5: Industrial Production and Employment. The two panels show, respectively, the growth rate of industrial production and the change in the number of people employed.

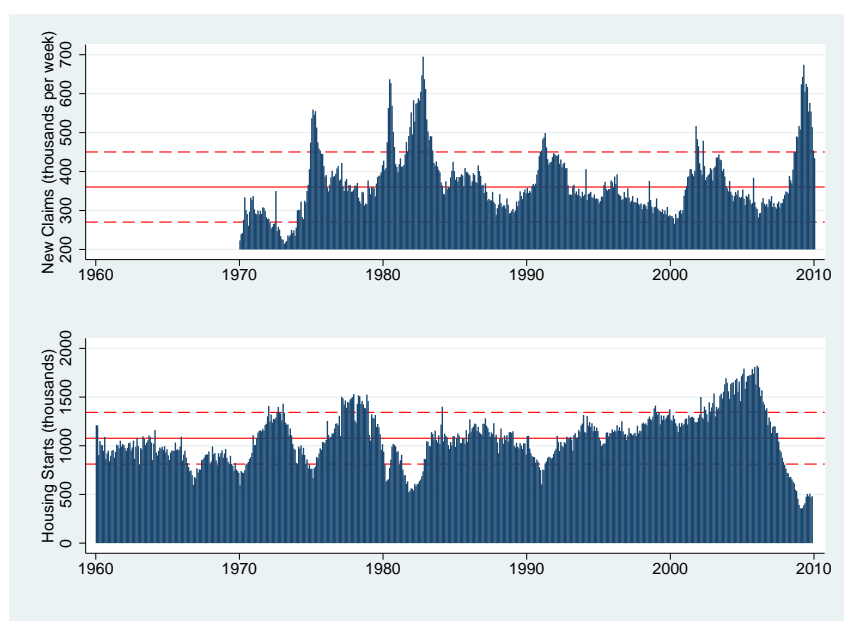


Figure 6: Industrial Production and Employment. The two panels show, respectively, new claims for unemployment insurance and housing starts, two popular indicators of economic conditions.

on the 4-week moving average. The second panel is housing starts. You can see in the figure that housing starts don't always go up and down with the economy. In



the 2001 recession, housing starts fell only slightly. In 2008 we made up for that, with housing starts falling to their lowest point since (at least) 1960. None of that will come as a surprise to you. These four indicators come from the Federal Reserve (industrial production), the Bureau of Labor Statistics (employment, new claims), and the Bureau of the Census (housing starts). These government agencies are the primary sources of economic indicators in the US. There are private indicators as well, but the government indicators widely used and publicly available.

In the business cycle scorecard, we rate each indicator as strong positive if the current value of the indicator is above the “mean plus one standard deviation” line, weak positive if it’s between the mean line and the one above it, weak negative if it’s below the mean line but above the “mean minus one standard deviation” line, and strong negative if it’s below the bottom line. That’s a rough cut, to be sure, but a useful one. That leads to this summary of economic conditions as of January 2010 based on the four indicators we have seen so far:

- Industrial production: growth has been positive for several months, a marked improvement on the recent past. Assessment: weak positive. With a couple strong positive months behind us, we could see this improve.
- Employment: the most recent number remains negative, but is a huge improvement over the recent past and (slightly) above the lower line. Assessment: weak positive.
- New claims: they remain high but have fallen dramatically over the last 6 months. Assessment: weak negative.
- Housing starts: they remain very low by historical standards, although there has been modest improvement in the recent past. Assessment: strong negative.

Indicator	Strong Negative	Weak Negative	Weak Positive	Strong Positive
Industrial production			x	
Employment			x	
New Claims		x		
Housing starts	x			
Summary	1	1	2	0

Table 1: Business-Cycle Scorecard: US Economic Conditions in January 2010.

These assessments are collected in Table 1. Overall, we see two negatives and two positives, a mixed set of signals that’s pretty common late in a downturn. A more extensive analysis would use more indicators, decide how much weight to give each one, assess how far into the future they point, and so on.

### Formal assessment of economic conditions: regression-based forecasting (optional)

A more formal statistical approach is to include as many indicators as we like in a multivariate regression. We estimate the regression by some appropriate method and use it to forecast the future. Here are the steps we might follow in constructing a forecast of (say) industrial production  $k$  months in the future.

The first step is to construct the variable we're forecasting. Let us say that we're interested in the growth rate of industrial production between now and  $k$  months in the future. You can do what you want, but we compute the (annualized) growth rate this way:

$$\gamma_{t,t+k} = \log(IP_{t+k}/IP_t) \times (12/k).$$

We refer to  $k$  (here measured in months) as the *forecast horizon*. The adjustment factor “ $12/k$ ” converts the growth rate to annual units. For a one-year forecast, then, we would set  $k = 12$  and compute the year-on-year growth rate.

The second step is to find some variables you think would be useful in forecasting. The previous section might give you some ideas. There's a half-step that sneaks in about here, too: what form of the indicator to use. In most cases we use growth rates of the indicators, too, either over one period or a year, whichever you think works best. But some variables are used as is. In Figure 3, for example, the cross-correlation for the unemployment rate is for the rate, period, not its growth rate, change, or other transformation.

Third, you put all the ingredients into a statistical package and run a regression. For example, to forecast IP growth, we would estimate the regression

$$\gamma_{t,t+k} = a + bx_t + \text{residual},$$

where  $x_t$  is the value of the indicator we have chosen. We use a sample of data to estimate the parameters  $a$  and  $b$ . Note well: the growth rate is between now (date  $t$ ) and a future date ( $t + k$ ), but the indicator is observed now (at  $t$ ). This is central to the exercise: we use what we know now to predict the future. It's not kosher to use future variables to predict the future, because we don't know the future when we make the forecast (duh!).

Fourth and last: Once we have estimates of the regression parameters ( $\hat{a}$  and  $\hat{b}$ , say), we use them and the current value(s) of the indicator(s) ( $x$ , say) to compute the forecast:

$$\hat{\gamma}_{t,t+k} = \hat{a} + \hat{b} x_t.$$

The “hats” remind us we are using estimates;  $\hat{\gamma}_{t,t+k}$  is our forecast of future growth. There are lots of variants of this approach: you can add multiple indicators, lags

of the indicators  $(x_{t-1}, x_{t-2}, \dots)$ , and even (esp?) past values of the growth rate of industrial production. We recommend all of the above.

The result of such an exercise is generally a useful forecast — useful in the sense that it tells us something about the future. Something, but not everything! Over periods of a year or two, forecast accuracy is usually modest. Even in-sample, the regressions rarely have  $R^2$ s above 0.25, which tells us that most of the future (at least 75% of it) is unpredictable. Some people see a lesson in this: it might be more important to know how to respond when the unexpected occurs than to have better forecasts. In practice, both are useful: knowing something about the future, and having backup plans to deal with the inevitable forecasting errors.

### Aggregation and prediction markets (optional)

There's another appealing approach to forecasting: let markets do the work. Most of the best forecasts aggregate information from multiple indicators and sources. Indexes of leading indicators do this one way: they combine multiple indicators to produce an index, which is then used to forecast the future. Or we could use multiple indicators in regression-based forecasts, as we suggested above.

Another approach is to aggregate the forecasts themselves: take several forecasts, perhaps based on different indicators, and average them. The Business-Cycle Scorecard is a simple version of this. The so-called “Blue Chip” forecast is an average of forecasts generated by experts, and it performs better than any single forecaster. Some statistical forecasters do the same sort of thing on their own: they generate multiple forecasts with methods like our forecasting regression, then average them to generate a final aggregate forecast. Again, the aggregate tends to do better than the individual forecasts.

A related idea is to rely on markets, which aggregate information from the people using them. Presidential futures markets, for example, have predicted the popular vote in the last four elections more accurately than any of the major polls. In the economic arena, there is a growing number of markets in which you can trade futures contracts whose payoffs are tied to the value of specific economic numbers: the consumer price index, the fed funds rate, and so on. These markets are increasingly used as forecasts themselves, with one wrinkle. The simplest interpretation is that the futures price is a market forecast of the relevant economic number. For example, if we are interested in the value of an economic number  $y$  to be released in 6 months ( $y_{t+6}$ , say), we might use its current futures price ( $f_t$ , say):

$$f_t = \text{Market's Current Forecast of } y_{t+6}.$$

Experience (and possibly some insight) tells us that we may want to make a correction for the risk of the contract:

$$f_t = \text{Market's Current Forecast of } y_{t+6} + \text{Risk Premium}.$$

There's no limit to the amount of sophistication we can bring to bear on the last term, but for now you can simply note that we probably want to address it in some way. Once you do, markets are an extremely useful source of information about the future. An application to the yield curve is included in an (optional) appendix.

### Executive summary

1. Fluctuations in economic activity can be (partially) predicted by a number of indicators.
2. The cross-correlation function is a tool for describing the timing of the relation between two indicators: for example, whether one indicator leads another.
3. Markets are useful aggregators of information — and increasingly popular sources of economic forecasts.

### Review questions

1. Give two examples each of procyclical, countercyclical, leading, and lagging indicators. Base your answers on the figures above.

Answer. Up to you.

2. We mentioned housing starts as an indicator of future economic activity. In what ways do you think it's a good indicator? A bad one? (For further information, see the US Census Bureau's [web site](#).)

Answer. Good: connected to housing, which as a durable good should be cyclically sensitive and volatile; available quickly; it leads the cycle (as you can see from its ccf). Bad: based on a sample, which leads to short-term noise; revised periodically; housing is a small part of the economy.

3. The unemployment rate is widely reported in the press, but professionals rarely use it. Why do you think that is?

Answer. One reason is that the unemployment rate understates the change in employment in a downturn. Some people who lose jobs leave the labor force, so they're not included in the unemployment rate. Another reason is that the unemployment rate is a lagging indicator. It falls slowly well after the economy turns around. Employment (the number of people actually working) is the preferred indicator.

4. The term spread (a long yield minus a short yield) is positively correlated with future output growth. Why do you think that is?

Answer. This is speculation; nothing we've done so far provides a clear answer. But: We've seen that a steep yield curve means that we expect short-term interest rates to rise. Short-term interest rates may rise because the economy is growing quickly (interest rates are typically higher in booms) and/or because we expect higher inflation (sometimes associated with a booming economy).

5. In 2002, a government agency recommended that we establish a futures market in terrorist attacks, on the grounds that it would give us a useful public indicator of their likelihood. The idea was widely criticized. Do you think it was a good idea or a bad one? What would you need to do to implement it?

Answer. Another case of a good idea thrown out because it sounded bad to politicians. It's not clear such attacks are predictable, but if they are we'd expect futures markets to do as well as any other method. To implement the idea, you'd need to define (and possibly quantify) a terrorist event.

### If you're looking for more

There are many sources of leading indicators around the world and lots of guides to them. Among them:

- This is the best one I've seen: Bernard Baumohl, *The Secrets of Economic Indicators*. If you use economic indicators in your job, buy this book.
- Jim Stock and Mark Watson are the two leading academics working on economic indicators. Their 2003 paper in the Richmond Fed *Economic Review* is an interesting look back at the 2001 recession and why many didn't see it coming. Here's a [link](#).
- The National Bureau of Economic Research runs an email alert service for major macroeconomic data releases, primarily for the US. See [link](#).

To do it right, this topic requires some knowledge of time series statistics. If you'd like to learn more about forecasting economic and financial variables specifically, we recommend "Forecasting Times Series Data" (B90.2302), taught in alternate years by Professors Deo and Hurvich, two of our best statisticians.

## Appendix: Reading the yield curve (optional)

We describe how the yield curve can be used to infer market expectations of future interest rates. It's a concrete example of the kind of market-based forecasting described in an earlier section.

### The yield curve

We use the yield curve to infer “the market’s” forecast of future short-term interest rates. Roughly speaking, the slope of the yield curve tells us what the market expects of future short-term interest rates. But before we explain how this works, we need to review yields and forward rates. Most of this should be familiar if you’ve taken “Foundations of Finance.”

Yields and forward rates are subspecies of interest rates. *Yields* are simply a way of reporting bond prices. If you go into the bond business you’ll find there are lots of details to worry about (accrued interest, day count conventions, etc), but we’ll keep it simple and look at yields on bonds with no coupons — zero-coupon bonds or “zeros.” A one-year zero is a claim to (say) \$100 in one year. A two-year zero is a claim to \$100 in two years. An  $m$ -year zero is a claim to \$100 in  $m$  years, and so on. If the price of an  $m$ -year bond is  $p_m$ , its yield  $y_m$  is defined by the present value formula:

$$p_m = 100/(1 + y_m)^m. \quad (1)$$

This formula is based on annual compounding, which is the most convenient convention for our purposes; more on this shortly. Note that if we know prices, we can find yields, and vice versa. The *yield curve* for zero-coupon bonds is a graph of yield  $y_m$  versus maturity  $m$ .

The yield curves you see in the newspaper are usually based on yields of coupon bonds, and in the case of US treasuries they use semi-annual compounding. They capture similar information, but they’re a little harder to make sense of. Why? Because the yield depends not only on maturity, it also depends on the coupon. Since the coupons vary over time and across maturities, it’s never exactly clear what we’re talking about. For that reason, most formal fixed income analysis starts with prices and yields of zeros.

*Example.* Suppose prices of zero-coupon bonds are

Maturity	Price
1	94.24
2	87.70
3	81.22
4	75.16
5	69.66

What are the yields?

Answer. The yields are 6.11, 6.78, 7.18, 7.40, and 7.50, respectively, expressed as annual percentages. The third one solves the equation  $81.22 = 100/(1 + y_3)^3$ , namely  $1 + y_3 = (100/81.22)^{1/3}$ . The others are similar.

Yields are interest rates that apply over a number of periods. For example, the  $m$ -year yield at date  $t$  applies over the period of time between  $t$  (now) to  $t + m$  ( $m$  years from now). One way to think about the return on the bond is that the investor invests  $p$  and gets a rate of return equal to the yield  $y$  for every period until maturity. In the two-period case, this leads to

$$100 = p_2 \times (1 + y_2)(1 + y_2),$$

a variant of our present value formula, equation (1). Another way to think about the return is that the rates vary across periods. In the two-period case, the bond might have different rates of return in the first and second periods. But what are these returns? We know the return on a one-period bond is  $y_1$ , so the first-period return should be  $y_1$ . What about the second period? We need an interest rate that applies to the second period alone but that remains consistent with the price of the bond. That is, a rate  $f_1$  that satisfies

$$100 = p_2 \times (1 + y_1)(1 + f_1).$$

Putting these two equations together tells us

$$(1 + y_2)^2 = (1 + y_1)(1 + f_1)$$

or

$$1 + f_1 = p_1/p_2.$$

We refer to  $f_1$  as the one-period-ahead *forward rate*, since it applies at date  $t$  to the time period between  $t + 1$  and  $t + 2$ . It's the rate of return on a *forward contract* in which we agree at  $t$  to invest a fixed amount at  $t + 1$  for one period. For our purposes, you can think of forward contracts as identical to the more common futures. The latter differ (a little) in having a mark-to-market requirement, which aficionados will take seriously but the rest of us can safely ignore.

The same logic applies to bonds with maturities greater than two. The bond price can be expressed as

$$\begin{aligned} p_m &= 100/(1 + y_m)^m \\ &= 100/[(1 + y_1)(1 + f_1) \cdots (1 + f_{m-1})]. \end{aligned}$$

The two relations together imply

$$1 + f_{m-1} = p_{m-1}/p_m, \tag{2}$$

where  $f_m$  is the  $m$ -period-ahead forward rate. Draw a circle around this equation: you'll be using it soon. What we're doing here is using bonds of two consecutive maturities to "pick off" the forward rate that applies over the last period.

Putting all the pieces together, we have three ways to express the same information: bond prices, yields, and forward rates. Given data on a complete set of maturities, we can compute any one of the three from any other.

*Example (continued).* For the bond prices and yields reported earlier, verify that the forward rates are

Maturity ( $m$ )	Price ( $p_m$ )	Yield ( $y_m$ )	Forward Rate ( $f_{m-1}$ )
1	94.24	6.11	6.11
2	87.70	6.78	7.45
3	81.22	7.18	7.98
4	75.16	7.40	8.06
5	69.66	7.50	7.90

Note that the maturities of forward rates are one less than those of bond prices and yields. We do this because they naturally start at zero, for reasons that may be clearer to you shortly. The 0-period-ahead forward rate is simply the current one-period yield:  $f_0 = y_1$ .

Answer. We compute forward rates from prices. For  $m = 5$ , the calculation implied by equation (2) is

$$1 + 0.0790 = 75.16/69.66.$$

The others are similar.

## Reading the yield curve

Now that we've done all this work, we can use it to predict future one-period yields. Put simply but somewhat imprecisely, we're going to say that forward rates are the market's expectation of future one-period yields, plus an adjustment for risk that depends on maturity but doesn't vary over time. Once we've done the adjustment, we can read expected future one-period yields from the forward rate curve. The idea is referred to as the *expectations hypothesis* because it's based on the premise that forward rates include expectations of future one-period yields — *short rates* in conventional terminology.

To be more precise we need to add a little notation. Since we'll be dealing a lot with the one-period yield or short rate, let us label it  $i$ , so that  $i_t$  is the yield on a one-period bond bought at date  $t$ . We label (one-period) forward rates by  $f_{m,t}$ , meaning

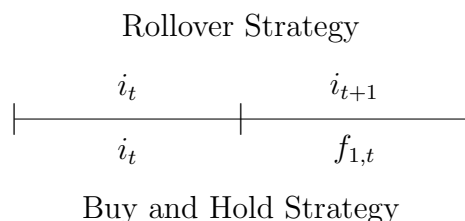


the rate on a forward contract arranged at date  $t$  but applying to the period between dates  $t + m$  and  $t + m + 1$ . The expectations hypothesis states that the forward rate is the market's expectation or forecast of the future short rate plus a risk premium:

$$f_{m,t} = E_t(i_{t+m}) + \text{risk premium}_m. \quad (3)$$

The risk premium is assumed to be constant across time but may vary by maturity. The notation  $E_t$  is meant to convey the expectation made at date  $t$ . If we know forward rates and risk premiums, we can compute expected future short rates.

That's what the expectations hypothesis is, but where does it come from? Let's think about what an investor would demand of a forward rate. An investor with a two-period time horizon has (at least) two choices. She could buy and hold a two-period bond at date  $t$ , thus getting (according to our second interpretation) returns of  $i_t$  in the first period and  $f_{1,t}$  in the second. Alternatively, she could roll over a one-period bond, getting (again)  $i_t$  the first period and the short rate  $i_{t+1}$  in the second. These two possibilities can be pictured like this:



If  $i_{t+1}$  were known, then we would expect the market to drive the forward rate  $f_{1,t}$  and the future short rate  $i_{t+1}$  together:  $f_{1,t} = i_{t+1}$ . If the future short rate were higher, then no one would buy the long bond. Its price would fall and the forward rate would rise. If it were lower, everyone would buy the long bond and bid up its price. In an uncertain world, similar logic might lead us to guess that the forward rate is the market's forecast of the future short rate, with some adjustment for risk:

$$f_{1,t} = E_t(i_{t+1}) + \text{risk premium}.$$

Similar reasoning for longer maturities leads us to equation (3).

The only remaining issue is estimating the risk premiums. There are as many ways to do this as there are finance professors, but a relatively simple method is to use the average difference between the short rate and the  $n$ -period forward rate as an estimate of its risk premium; that is, we use

$$\text{risk premium}_m = E(f_{m,t} - i_t) \quad (4)$$

and estimate the expectation on the right with the mean. This should work as long as forecast errors (the difference between the future short rate and its expectation)

are zero, on average, and the distribution of the short rate doesn't change over time (the means of the short rate and future short rate are the same). Table 2 summarizes mean forward rates for US treasuries for the period 1970-2000. Since the short rate is the first forward rate, we compute risk premiums by subtracting it from the other forward rates.

Maturity	Mean Forward Rate	Risk Premium
1	7.51	0.00
2	8.10	0.59
3	8.39	0.88
4	8.57	1.06
5	8.67	1.16

Table 2: Mean Forward Rates for US Treasuries, 1970-2000.

*Example (continued again).* For the forward rates reported earlier, what are the implied expected future short rates?

Answer. The calculations are summarized by

Maturity ( $m$ )	Forward Rate ( $f_{m-1}$ )	Risk Premium	Future Short Rate ( $i_{t+m-1}$ )
1	6.11	0.00	6.11
2	7.45	0.59	6.86
3	7.98	0.88	7.10
4	8.06	1.06	7.00
5	7.90	1.16	6.74

In short, we can read expected future short-term interest rates from the forward rate curve, which we compute from the yield curve for zeros. If you find this interesting, there's a lot more on related topics in the "Debt Instruments" course.