Lab Report #4: Asset Pricing Fundamentals

Revised: October 12, 2014

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *State prices and related objects.* Consider an economy with three states. State prices and probabilities are

State z	State Price $Q(z)$	Probability $p(z)$	Dividend $d(z)$
1	1/2	1/3	1
2	1/3	1/3	2
3	1/4	1/3	3

   (a) What is the pricing kernel in each state?

   (b) What is the price of a one-period bond? What is its return?

   (c) What are the risk-neutral probabilities? Why are they different from the true probabilities?

   (d) Suppose equity is a claim to the dividend in the last column. What is its price? What is the return on equity in each state?

   (e) What is the expected return on equity? The risk premium?

2. *Pricing kernels and risk-neutral probabilities with Poisson risk.* Consider a representative agent economy with a power utility agent facing "Poisson risk." Utility is $u(c) = c^{1-\alpha}/(1-\alpha)$ with risk aversion parameter $\alpha > 0$. Log consumption growth is $z = \log g = \log c_1 - \log c_0$ for $z = 0, 1, 2, \ldots$ with probabilities $p(z) = e^{-\omega}\omega^z/z!$ for some parameter $\omega > 0$.

   You may want to use the power series expansion

$$e^x \;=\; 1 + x + x^2/2! + x^3/3! + \cdots \;=\; \sum_{j=0}^{\infty} x^j/j!,$$

   which is sometimes used to define the exponential function e^x.

   (a) What is the pricing kernel $m(z)$ in each state z?

   (b) What are the state prices $Q(z)$?

   (c) What are the risk-neutral probabilities $p^*(z)$?

   (d) Show that the risk-neutral distribution is Poisson.

   (e) How do the risk-neutral probabilities $p^*(z)$ differ from the true probabilities $p(z)$?

(f) Set $\omega = 4$ and $\alpha = 1$ and plot $p(z)$ and $p^*(z)$ for z between zero and 10. How do they differ? Why?

Matlab mini-tutorial on bar charts. Suppose we have vectors `z`, `p`, and `pstar`. We can plot probabilities against `z` with the commands

```
bar(z, p)           % just p
bar(z, [p pstar])   % p and pstar together
```

The order of inputs in Matlab plot commands is x variable first (horizontal axis), then the y variable (vertical axis): `plot(x,y)`, `bar(x,y)`, etc.

3. *Option prices.* We're going to explore the risk-neutral approach to valuing options. A call option gives the owner the right to purchase an asset one period from now at a price k — the so-called *strike price* — determined now. If the future price is $s(z)$, the owner will exercise the option and purchase the stock only if s is greater than (or equal to?) k. That gives rise to the option cash flow

$$d(z) \quad = \quad \max\{0, s(z) - k\}.$$

Given this cash flow, we value the option as we would any other asset. We'll use specifically the risk-neutral valuation equation

$$q^c \quad = \quad q^1 \sum_z p^*(z) d(z) \quad = \quad q^1 \sum_z p^*(z) \max\{0, s(z) - k\},$$

where q^c is the price of the call option and q^1 is the price of a one-period riskfree bond.

The risk-neutral environment we'll work with is lognormal. A state z has a standard normal risk-neutral distribution. The log of the future price is connected to the state (again, in risk-neutral terms) by $\log s(z) = \mu + \sigma z$.

One last thing: we're going to use a discrete approximation to the distribution of z, which is easier to work with than the real thing (numerical integration is neither pretty nor efficient). In Matlab terms, set up a grid of points for z and assign probabilities to them from the standard normal pdf:

```
zmax = 4;
dz = 0.5;
z = [-zmax:dz:zmax]';
pstar = exp(-z.^2/2)*dz/sqrt(2*pi);
```

We can make this approximation as close to the original as we want by shrinking `dz`.

(a) One check on the approximation is the sum of the probabilities. Do they sum to one here?

(b) Set up a similar grid of values for $s(z)$. Use $q^1 = 0.95$, $\sigma = 0.1$, and

$$\mu \quad = \quad \log(100/q1) - \sigma^2/2.$$

More on this later. What value of μ do you get?

(c) Compute the cash flows $d(z)$ for an option with strike price $k = 110$. Use the risk-neutral pricing equation to compute its value.

You may find these Matlab commands helpful:

```
d_positive = s >= k
d = d_positive.*(s-k);
```

The first line generates a vector that equals one if $s \geq k$ and zero otherwise.

(d) We usually require the risk-neutral probabilities to value the underlying asset correctly, a requirement we call the *no-arbitrage condition*. With this in mind, apply the risk-neutral pricing equation to the asset itself, a claim (so to speak) to the value of the asset one period from now: $d(z) = s(z)$. What is the value of this asset? Can you connect it to the condition we used to derive μ?