Lab Report #8: Bond Prices & Predictable Returns

Revised: December 12, 2013

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *Bond basics.* Consider the bond prices at some date t:

Maturity n	Price q^n
0	1.0000
1 year	0.9512
2 years	0.8958
3 years	0.8353
4 years	0.7788
5 years	0.7261

   (a) What are the yields y^n?

   (b) What are the forward rates f^{n-1}?

   (c) Suppose f_t^1 rises by 1 percent. How does y_t^1 change? How does y_t^2 change?

2. *Exponential-affine equity valuation.* We explore variation in expected excess returns ("risk premiums") in an environment like the one we used for bond pricing. We say a return or expected return is predictable if it depends on the state. If we know the state, we can use it to "predict" the return. In the asset management business, you might imagine strategies to exploit such information. For example, you might invest more in assets when their expected returns are high, and less when they're low.

   Here's the model. The pricing kernel is

$$\begin{aligned} \log m_{t+1} &= -(\lambda_0 + \lambda_1 x_t)^2/2 - x_t + (\lambda_0 + \lambda_1 x_t) w_{t+1} \\ x_{t+1} &= (1-\varphi)\delta + \varphi x_t + \sigma w_{t+1}, \end{aligned}$$

   where $\{w_t\}$ is (as usual) a sequence of independent standard normal random variables. This model differs from Vasicek in having a price of risk λ that depends on the state x_t. The dependence is carefully designed to deliver loglinear ("exponential-affine") bond prices.

   (a) What is the (stationary) mean of x_t? The variance?

   (b) What is the price q_t^1 of a one-period bond? What is its (gross) return r_{t+1}^1?

   (c) What is the mean log return $E(\log r_{t+1}^1)$? What parameter(s) govern its value in the model?

(d) Now consider the price of "equity," a claim to the dividend

$$\log d_{t+1} \quad = \quad \alpha + \beta w_{t+1}.$$

What is its price q_t^e? Its (gross) return r_{t+1}^e?

(e) What is the expected excess return $E_t(r_{t+1}^e - r_{t+1}^1)$ in state x_t? In what states is it high? In what states low?

(f) Which equation is responsible for the predictability of the expected return?

Comment: most of these calculations are simpler in logs. Eg, compute $\log q_t^1$ rather than q_t^1.

Matlab program: