New York University December 2011 Econ UA.367 Econ UB.234
Final Examination

**Instructions:** Please answer all questions. This is a closed book exam.

**1. 30 points** There are two periods, today and tomorrow, and one good that people value. A bank can attract an unlimited quantity of deposits $D$ if and only if it pays a risk free gross interest rate of (at least) $(1+r) > 1$. That is, to attract funds, if the bank takes in $D$ deposits today, it must repay at least $(1+r)D$ units of the good to depositors tomorrow regardless of the state of the world (this is what it means to be risk free). There are two possible states of the world $s$ tomorrow, $s = A$ and $s = B$, that occur with probability $\pi_A \in (0,1)$ and $\pi_B = 1 - \pi_A$. A bank can invest in two kinds of risky state-contingent claims. It can purchase $Q_A$ units of the good tomorrow contingent on $s = A$ for a total cost of $p_A Q_A$ where $p_A > 0$. It can purchase $Q_B$ units of the good tomorrow contingent on $s = B$ for a total cost of $p_B Q_B$. The bank takes the state-contingent prices $p_A, p_B$ as given (i.e., the bank is a price taker).

**a.** Give a formula for the cost of purchasing one unit of the good tomorrow *for sure*, i.e., regardless of whether $s = A$ or $s = B$. Explain the reasoning that leads you to assert this formula.

**b.** Describe a portfolio choice $(Q_A, Q_B)$ that allows the bank to return $(1+r)D$ to depositors tomorrow regardless of whether $s = A$ or $s = B$.

**c.** Now assume that the government insures the bank at zero cost *to the bank* (but not necessarily to the government). That is, if payoffs on the bank's portfolio are not big enough to cover $(1+r)D$ tomorrow, the government liquidates the bank and uses the proceeds plus tax payer's money to pay off the depositors. Assume that the bank knows the probabilities $\pi_A, \pi_B$ and wants to maximize its expected profits. Please compute the bank portfolio today and size of deposits that maximize expected profits of the bank.

**d.** Please compute the probability that the bank fails under the optimal bank portfolio you computed in part **c**.

**e.** Please compute the expected costs of the deposit insurance to the tax payer.

**f.** Please use this little model to interpret the situation that policy makers face today in deciding whether to bail out banks' creditors when banks' portfolios are under water.

**2. (20 points)** For times $t = 1, 2, \ldots, T$ where $T$ is large, an economist has data on excess returns $\zeta_{t+1}^i$ on assets $i = i, \ldots, I$, where

$$\zeta_{t+1}^i \equiv R_{t+1}^i - R_{t+1}^f$$

where $R_{t+1}^f$ is the risk-free gross rate of return between $t$ and $t+1$. The economist uses the long time series to estimate

$$\frac{E\zeta_{t+1}^i}{\sigma(\zeta_{t+1}^i)} \equiv \text{Sharpe ratio for asset i}$$

where he/she estimates $E\zeta_{t+1}^i$ as the sample average and $\sigma(\zeta_{t+1}^i)$ as the sample standard deviation of excess returns on asset $i = 1, \ldots, 9$. The economist has computed the following Sharpe ratios:

Table 1: Sharpe ratios for nine assets

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|
| -.01 | .25 | .05 | .1 | .08 | .02 | -.03 | .24 | .10 |

From these data, use the theory of asset pricing to say as much as you can about $E(m)$ and $\sigma(m)$, the mean and standard deviation of a stochastic discount factor that prices all assets. Please justify your statements explicitly in terms of the relevant economic theory.

**3. (30 points)** Let $'$ denote a transpose and let $R_t' = \begin{bmatrix} R_{1t+1} & R_{2t+1} & \cdots & R_{nt+1} \end{bmatrix}$, so $R_t$ is an $n \times 1$ vector of gross returns on assets $i = 1, \ldots, n$. Let $m_{t+1}$ be a stochastic discount factor.

**a.** The fundamental equation of asset pricing is $E_t m_{t+1} R_{t+1} = 1$. Please justify this equation.

Assume that we observe time series for the vector $R_{t+1}$ for $t = 1, \ldots, T$ but *don't* observe $m_{t+1}$. Nevertheless, we want to extract implications of $EmR = 1$ for the statistical behavior of $m$.

**b.** Find a formula for the population regression of the hidden random variable $m$ on a constant (intercept) and the random vector $R$, being sure to give formulas for the regression coefficients in terms of the moments that you can observe or estimate.

**c.** Describe how to construct a lower bound on the standard deviation $\sigma(m)$ as a function of the mean $E(m)$.

**4. (20 points)** A representative consumer has preferences over consumption streams ordered by

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \quad , 0 < \beta < 1, \gamma > 0$$

where $\beta \equiv \frac{1}{1+\rho}$ and $c_t$ is consumption per worker. The consumer supplies one unit of labor inelastically. The technology is

$$y_t = f(k_t) = z k_t^{\alpha}, \quad 0 < \alpha < 1$$

where $y_t$ is output per worker, $k_t$ is capital per worker, $x_t$ is gross investment per unit of labor, $g_t$ = government expenditures per unit of labor, and

$$\begin{aligned} y_t &= c_t + x_t + g_t \\ k_{t+1} &= (1-\delta)k_t + x_t, \quad 0 < \delta < 1 \end{aligned}$$

The government finances its expenditures stream $\{g_t\}$ by levying a stream of flat rate taxes $\{\tau_{ct}\}$ on the value of the consumption good purchased at $t$, a stream of flat rate taxes $\{\tau_{kt}\}$ on earnings from capital at $t$, and a stream of lump sum taxes $\{\tau_{ht}\}$. There is a competitive equilibrium with the price system being $\{q_t, r_t, w_t\}_{t=0}^{\infty}$, where $q_t$ is the price of time $t$ consumption and investment goods, $r_t$ is the price of renting capital at time $t$, and $w_t$ is the price of renting labor at time $t$. All trades occur at time 0 and all prices are measured in units of the time 0 consumption good. The initial capital stock $k_0$ is given.

Assume that $\{\tau_{ct}, \tau_{kt}\}_{t=0}^{\infty}$ are all constant sequences (their values don't change over time) but that $\{g_t\}_{t=0}$ follows the path described in panel **c** of figure 1 – it takes a once and for all jump at time $t = 10$. Panels **a** and **b** give consumption paths for two economies that are identical except in one respect. In one of the economies, the time 10 jump in $g$ had been anticipated since time 0, while in the other, the jump in $g$ that occurs at time 10 is completely unanticipated at time 10. Please tell which panel corresponds to which view of the arrival of news about the path of $g_t$. Please say as much as you can about how $\{k_{t+1}\}_{t=0}^{\infty}$ and the interest rate behave in these two economies.

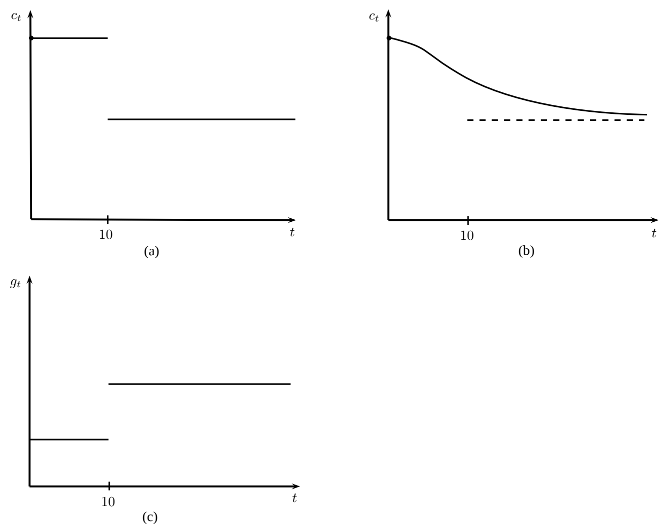


Figure 1: Panels **a** and **b**: consumption $c_t$ as function of time in two economies. Panel **c**: government expenditures $g_t$ as a function of time.

**5. (20 points)** A representative consumer has preferences over consumption streams ordered by

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma}, \quad , 0 < \beta < 1, \gamma > 0$$

where $\beta \equiv \frac{1}{1+\rho}$ and $c_t$ is consumption per worker. The consumer supplies one unit of labor inelastically. The technology is

$$y_t = f(k_t) = z k_t^{\alpha}, \quad 0 < \alpha < 1$$

where $y_t$ is output per worker, $k_t$ is capital per worker, $x_t$ is gross investment per unit of labor, $g_t$ = government expenditures per unit of labor, and

$$\begin{aligned} y_t &= c_t + x_t + g_t \\ k_{t+1} &= (1-\delta) k_t + x_t, \quad 0 < \delta < 1 \end{aligned}$$

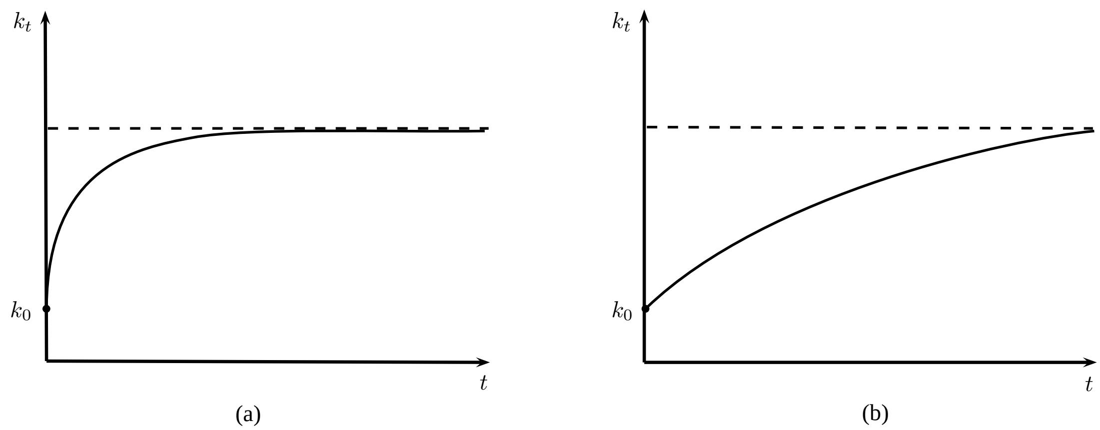


Figure 2: Capital stock as function of time in two economies with different values of $\gamma$.

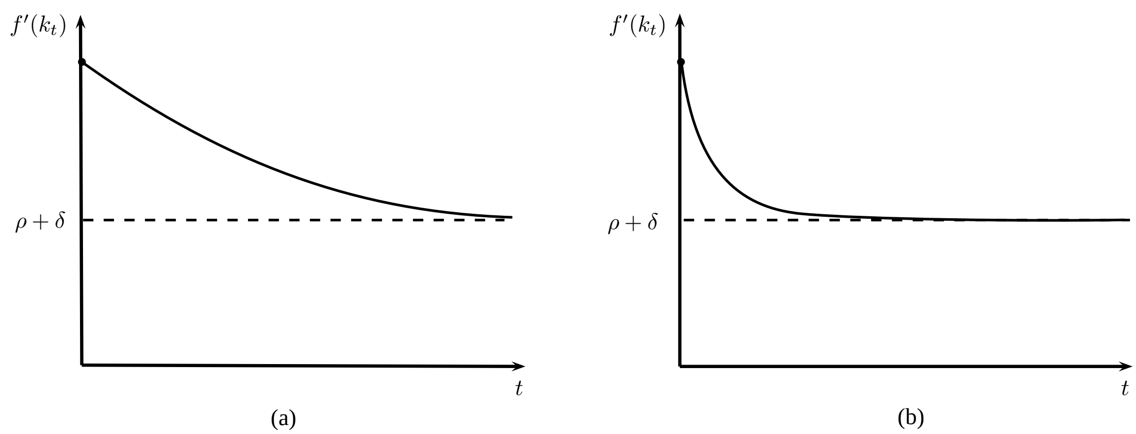


Figure 3: Marginal product of capital as function of time in two economies with different values of $\gamma$.

The government finances its expenditures stream $\{g_t\}$ by levying a stream of flat rate taxes $\{\tau_{ct}\}$ on the value of the consumption good purchased at $t$, a stream of flat rate taxes $\{\tau_{kt}\}$ on earnings from capital at $t$, and a stream of lump sum taxes $\{\tau_{ht}\}$. There is a competitive equilibrium with the price system being $\{q_t, r_t, w_t\}_{t=0}^{\infty}$, where $q_t$ is the price of time $t$ consumption and investment goods, $r_t$ is the price of renting capital at time $t$, and $w_t$ is the price of renting labor at time $t$. All trades occur at time 0 and all prices are measured in units of the time 0 consumption good. The initial capital stock $k_0$ is given.

Assume that $\{g_t, \tau_{ct}, \tau_{kt}\}_{t=0}^{\infty}$ are all constant sequences (their values don't change over time). In this problem, we ask you to infer differences across two economies in which all aspects of the economy are identical *except* the parameter $\gamma$ in the utility function. In both economies, $\gamma > 0$, but in one economy $\gamma$ is high and in the other it is low. Among other identical features, the two economies have identical government policies and identical initial capital stocks.

**a.** Please look at figure 2. Please tell which outcome for $\{k_{t+1}\}_{t=0}^{\infty}$ describes the low $\gamma$ economy, and which describes the high $\gamma$ economy. Please explain your reasoning.

**b.** Please look at figure 3. Please tell which outcome for $\{f'(k_t)\}_{t=0}^{\infty}$ describes the low $\gamma$ economy, and which describes the high $\gamma$ economy. Please explain your reasoning.