Professor Mark Gertler
New York University
Intermediate Macro
Spring 2009

Lecture 8

A Model of Short Run Fluctuations

Consider a dynamic general equilibrium model with money, imperfect competition and nominal rigidities.

1 Baseline Model

The model includes households, firms and a government sector:

- The representative household consumes a final good C_t, supplies labor N_t, holds real money balances M_t/P_t, saves in the form of private bonds B_t (which, in equilibrium will be in zero net supply, since everyone is the same.)
- Firms (assume they have mass unity) are monopolistic competitors and each produce a differentiated product $Y_t(f)$ using capital $K_t(f)$ and labor $N_t(f)$. These firms set nominal prices $P_t(f)$.
- The government conducts fiscal and monetary policy.

1.1 The household's problem

The representative household chooses $\{C_{t+i}, N_{t+i}, \frac{M_{t+i}}{P_{t+i}}, \frac{B_{t+i+1}}{P_{t+i}}\}_{i=0}^{\infty}$ to maximize

$$E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\log C_{t+i} + a_m \log\left(\frac{M_{t+i}}{P_{t+i}}\right) - \frac{a_n}{1+\gamma_n}N_{t+i}^{1+\gamma_n}\right]\right\} \tag{1}$$

subject to

$$C_t = \frac{W_t}{P_t}N_t + +\Pi_t + TR_t - \frac{M_t - M_{t-1}}{P_t} - \frac{\left(\frac{1}{R_{t+1}^n}\right)B_{t+1} - B_t}{P_t} \tag{2}$$

where $\gamma_n > 0$. Let $\sigma = 1/\gamma$, W_t/P_t be the real wage, the profits from owning the monopolistic competitive firms, TR_t government transfers, and R_{t+1}^n is the nominal interest rate..
The FONCs for this maximization problem are:

$$\frac{W_t}{P_t} = \frac{a_n N_t^{\gamma_n}}{1/C_t} \text{ (labor supply)} \tag{3}$$

$$1 = E_t\left\{\frac{P_t}{P_{t+1}}\beta\frac{1/C_{t+1}}{1/C_t}\right\} + \frac{a_m/(M_t/P_t)}{1/C_t} \text{ (money demand)} \tag{4}$$

$$1 = E_t\left\{R_{t+1}\beta\frac{C_t}{C_{t+1}}\right\} \text{ (consumption/saving)} \tag{5}$$

where R_{t+1} is the real interest rate, defined as

$$R_{t+1} \equiv R^n_{t+1} \frac{P_t}{P_{t+1}} \tag{6}$$

Combining the money and bond pricing equations (4) and (5), yields the money demand equation:

$$\frac{M_t}{P_t} = a_m \left(1 - \frac{1}{R^n_{t+1}}\right)^{-1} C_t \tag{7}$$

1.2 Firms

There is a continuum of intermediate good firms owned by consumers, indexed by $f \in [0,1]$. Each firm uses both labor $N_t(f)$ and capital $K_t(f)$ to produce output according to the following constant returns to scale technology[1]:

$$Y_t(f) = A_t N_t(f) \tag{8}$$

where A_t is a technology parameter. Each firm faces the following demand functin, where $\varepsilon > 1$, is the relative price elasticity.

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t \tag{9}$$

$$\Pi_t = \frac{P_t(f)}{P_t} Y_t(f) - \frac{W_t}{P_t} N_t(f)$$

Combining equations yields the following unconstrained problem, with $P_t(f)$ as the choice variable.

$$\Pi_t = \frac{P_t(f)}{P_t}\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t - \frac{W_t}{P_t} \frac{\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t}{A_t}$$

The first order necessary conditions are given by

$$(1-\varepsilon)\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t + \varepsilon \frac{\frac{W_t}{P_t}}{A_t}\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon-1} Y_t = 0$$

which yields

$$\frac{P_t(f)}{P_t} = \frac{1}{1-1/\varepsilon} \frac{\frac{W_t}{P_t}}{A_t}$$

$$\frac{P_t(f)}{P_t} = (1+\mu) \frac{\frac{W_t}{P_t}}{A_t}$$

where μ is the net markup. Note that in the flex price case, the firm adjust price to hit a desired net markup μ.

By contrast, in the fixed price case, the firm produces to meet demand so long as it is doesn't lose money at the margin; so long as $\mu_t(f) \geqq 0$. In this case, the markup $\mu_t(f)$ is determined as a residual by the relation between relative price $\frac{P_t(f)}{P_t}$ and marginal cost.

[1]Recall that capital at the aggregate level is predetermined: $K_t = \int_0^1 K_t(f)\, df$.

$$\frac{\overline{P}_t(f)}{P_t} = (1 + \mu_t(f))\frac{\frac{W_t}{P_t}}{A_t};$$

We restrict attention to symmetric equilibria where all firms behave the same way. Thus, $\frac{P_t(f)}{P_t} = 1$ and $\frac{\overline{P}_t(f)}{P_t} = 1$. It follows that:
Symmetric Equilibrium: fixed price case:

$$1 = (1 + \mu_t(f))\frac{\frac{W_t}{P_t}}{A_t}$$

$$A_t = (1 + \mu_t)\frac{W_t}{P_t}$$

Symmetric equlibrium: Flex price case:

$$A_t = (1 + \mu)\frac{W_t}{P_t}$$

1.3 Aggregation, resource constraints and government policy

Aggregate Production

$$Y_t = A_t N_t \tag{10}$$

Resource constraints

Since we assumed that capital was in fixed supply, we have:

$$Y_t = C_t \tag{11}$$

Government policy

The central bank sets the money supply exogenously:

$$M_t = \overline{M}_t \tag{12}$$

where Y_t^* denotes the natural (i.e. flexible price equilibrium) level of output.
Any seigniorage revenue is rebated lump-sum to the households, so that the government budget constraint is given by:

$$\frac{M_t - M_{t-1}}{P_t} = TR_t \tag{13}$$

2 Equilibrium

There are seven variables to be determined: $(Y_t, C_t, N_t, P_t, M_t, \mu_t, R_t^n)$. In the fixed price equilibrium, P_t is fixed and μ_t varies. The reverse is true in the flex price equilibrium.

Aggregate Demand

$$Y_t = C_t \tag{14}$$

$$C_t = E_t \left\{ R_{t+1}^n \frac{P_t}{P_{t+1}} \beta \frac{1}{C_{t+1}} \right\}^{-1} \tag{15}$$

Equation (15) describes the IS curve relating aggregate demand (which is equal only to consumption in this model) to the inverse of the interest rate.

Aggregate Supply

$$Y_t = A_t N_t \tag{16}$$

$$A_t = (1 + \mu_t) \frac{N_t^{\gamma_n}}{(1/C_t)} \tag{17}$$

$$P_t = \overline{P} \tag{18}$$

Monetary Policy Rule

$$\frac{M_t}{P_t} = a_m \left(1 - \frac{1}{R_{t+1}^n}\right)^{-1} C_t \tag{19}$$

$$M = \overline{M}$$

Simplified System:
IS Curve:

$$Y_t = E_t \left\{ R_{t+1}^n \beta \frac{1}{Y_{t+1}} \right\}^{-1}$$

LM Curve:

$$\frac{\overline{M}}{\overline{P}} = a_m \left(1 - \frac{1}{R_{t+1}^n}\right)^{-1} C_t$$

AS Curve:

$$\frac{1}{(1+\mu_t)} = (\frac{Y_t}{A_t})^{1+\gamma_n}$$

Flex Price Model

$$Y_t^* = E_t \left\{ R_{t+1}^{n*} \beta \frac{1}{Y_{t+1}^*} \right\}^{-1}$$

$$\frac{\overline{M}}{P} = a_m \left(1 - \frac{1}{R_{t+1}^{n^*}} \right)^{-1} Y_t^*$$

$$\frac{1}{(1+\mu)} = (\frac{Y_t}{A_t}^*)^{1+\gamma_n}$$

so simplified system becomes:
IS Curve:

$$Y_t = E_t \left\{ R_{t+1}^{n} \beta \frac{1}{Y_{t+1}} \right\}^{-1}$$

LM Curve:

$$\frac{\overline{M}}{\overline{P}} = a_m \left(1 - \frac{1}{R_{t+1}^{n}} \right)^{-1} C_t$$

AS Curve:

$$\frac{1+\mu}{(1+\mu_t)} = (\frac{Y_t}{Y_t^*})^{1+\gamma_n}$$

3 Loglinear Approximation

Consider a loglinear approximation of the function $f(X_t)$ about $f(X)$. Start with the first order approximation:

$$\begin{aligned} F(X_t) &\approx f(X) + \frac{\partial f}{\partial X} dX_t \\ &\approx f(X) + \frac{\partial f}{\partial X} X \frac{dX_t}{X} \end{aligned}$$

where $\frac{\partial f}{\partial X}$ is evaluated at $X_t = X$. Next, let x_t denote the log deviation of X_t from X, i.e., $x_t = \log X_t - \log X$. Then for small percent changes in X_t:

$$x_t \approx \frac{dX_t}{X}$$

Thus we can write:

$$f(X_t) \approx f(X) + \frac{\partial f}{\partial X} X x_t$$

or equivalently:

$$f(X_t) - f(X) \approx \frac{\partial f}{\partial X} X x_t$$

Taking loglinear approximation of the IS/LM model around the flex price equilibrium with the steady state value of A_t, yields:

IS:

$$y_t = -r_t^n + E_t y_{t+1}$$

LM:

$$\bar{m}_t - \bar{p} = y_t - \nu r_t^n$$

where $\nu = 1/(R^n - 1)$. AS:

$$\mu_t = -(1+\gamma_n)(y_t - y_t^*)$$

Determination of y_t^*:

$$\begin{aligned}
y_t^* &= a_t + n_t^* \\
a_t &= \gamma_n n_t^* + c_t^* \\
&= \gamma_n (y_t^* - a_t) + y_t^* \\
y_t^* &= a_t
\end{aligned}$$

Real Business Cycle Theory - Cycle represent fluctuations in y_t^*. Not true of many cycles.

3.1 Adding disturbances and solving the model:

$$\begin{aligned}
y_t &= -r_t^n + E_t y_{t+1} + \chi_t \\
\bar{m}_t - \bar{p} &= y_t - \nu r_t^n
\end{aligned}$$

Disturbances are stationary mean zero: $\lim E_t\{\epsilon_{t+i}\} = 0$ as $i \to \infty$.

$$\begin{aligned}
r_t^n &= -\frac{1}{\nu}(\bar{m}_t - \bar{p}) + \frac{1}{\nu} y_t \\
y_t &= \frac{1}{\nu}(\bar{m}_t - \bar{p}) - \frac{1}{\nu} y_t + E_t y_{t+1} + \chi_t \\
y_t &= \frac{1}{1+\nu}(\bar{m}_t - \bar{p}) + \frac{\nu}{1+\nu} E_t y_{t+1} + \frac{\nu}{1+\nu}\chi_t \\
y_t &= \sum_{i=0} \left(\frac{\nu}{1+\nu}\right)^i \left(\frac{1}{1+\nu}(\bar{m}_{t+i} - \bar{p}) + \frac{\nu}{1+\nu}\chi_{t+i}\right)
\end{aligned}$$

to set $y_t = 0$, and set $\bar{m}_{t+i} = \bar{p} - \nu\chi_{t+i} \to r_t^n = \chi + \frac{1}{\nu} y_t$

$$y_t = \sum_{i=0} (-r_{t+i}^n + \chi_{t+i})$$

that gives complete stabilization: i.e. under this rule, $y_t = 0$.

3.2 Interest Rate as the Policy Instrument

Modern central banks in industrialized economies use the short term interest rate as the policy instrument, as opposed to the money supply. The reason is that under money supply targeting, instability in money demand can create gyrations inh interest rates that affect the real economy.

To see, suppose that money demand is now given by:

$$\bar{m}_t - \bar{p} = y_t - \nu r_t^n + \varepsilon_t$$

where ε_t is a stationary disturbance to money demand. Proceeding as in the previous section:

$$y_t = \sum_{i=0} \left(\frac{\nu}{1+\nu}\right)^i \left(\frac{1}{1+\nu}(\bar{m}_{t+i} - \bar{p} - \varepsilon_{t+i}) + \frac{\nu}{1+\nu}\chi_{t+i}\right)$$

Note that shocks to money demand will affect output, unless the central bank is able to adjust the money supply to perfectly offset them.

For this reason, central banks tend to choose target values for the path on interest rates:

$$r_t^n = \overline{r_t^n}$$

They then adjust the money supply to hit this target. Note that in doing so, they are adjusting the money supply to offset any money demand disturbance. Note the this equation defines the LM curve under an interest rate rule (that is, the LM curve is horizonal at $r_t^n = \overline{r_t^n}$.

In this simple model, how should then central bank go about setting the interest rate?

Note first that the flexible price interest rate - or natural rate of interest - r_t^{n*}, is given by

$$y_t^* = -r_t^{n*} + E_t y_{t+1}^* + \chi_t$$

Given $y_t^* = a_t$:

$$r_t^{n*} = (E_t a_{t+1} - a_t) + \chi_t$$

It follows that we can express the output gap, $y_t - y_t^*$ as:

$$y_t - y_t^* = -(r_t^n - r_t^{n*}) + E_t(y_t - y_{t+1}^*)$$

Iterating forward:

$$y_t - y_t^* = \sum_{i=0}^{\infty} -(r_{t+i}^n - r_{t+i}^{n*})$$

Thus, if the central bank's objective is to stabilize the output gap, it should set $r_{t+i}^n = r_{t+i}^{n*}$, for all i. (Note that it has to not only do this in the current period, but also commit to doing it in the future. r_{t+i}^{n*} is also know as the "neutral rate.", i.e., the rate at which yield output demand equal to the flexible price equilibrium level of supply.

Note also that in this simple model, the neutral rate summarizes all the relevant information about demand and supply shocks.

4 Fiscal Policy

$$Y_t = C_t + G_t$$

suppose further that capital markets are perfect, and thus Ricardian equivalence holds (i.e. , the timing of taxes doesn't matter.)

Loglinear model with fiscal policy

Aggregate demand.

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t$$
$$c_t = -r_t^n + E_t c_{t+1} + \chi_t$$

Aggregate supply

$$y_t = a_t + n_t$$

$$a_t = \mu_t + \gamma_n n_t + c_t$$
$$p_t = \overline{p}$$

Monetary sector

$$m_t - \bar{p} = y_t - \nu r_t^n + \varepsilon_t$$

$$r_t^n = \overline{r_t^n}$$

as before, the system can be reduced to IS/LM and AS curves.

Simplified System:

IS curve

$$y_t = -\frac{C}{Y}r_t + E_t y_{t+1} + \frac{C}{Y}\chi_t + \frac{G}{Y}[g_t - E_t g_{t+1}]$$

LM curve

$$r_t^n = \overline{r_t^n}$$

AS curve

$$\mu_t = -(1+\gamma_n)(y_t - y_t^*)$$

with

$$y_t^* = \frac{1+\gamma_n}{\frac{Y}{C}+\gamma_n}a_t + \frac{G/C}{\frac{Y}{C}+\gamma_n}g_t$$
$$r_t^{n*} = \frac{Y}{C}[E_t y_{t+1}^* - y_t^*] + \frac{G}{C}[g_t - E_t g_{t+1}] + \chi_t$$

The effect of g_t on y_t^* reflects a wealth effect on labor supply -

The effect of a temporary increase in g_t on y_t is:

$$dy_t = \frac{G}{Y}dg_t$$

or,

$$\frac{dY_t}{Y} = \frac{G}{Y}\frac{dG_t}{G}$$

or

$$dY_t = dG_t$$

that is, the multiplier effect of a temporary increase in government expenditures is unity. Note that a persistent effect will have an even weaker effect (i.e., if $E_t g_{t+1}$ rises at least part of the way with g_t). Future increases in G induce househodls to save instead of consume, in anticipation of higher taxes (as we discussed much earlier.)

The Obama plan is based on a multiplier in the range of 1.5 to 2.0 How can one rationalize this? Borrowing constraints are key.

4.1 Fiscal Policy with Liquidity-Constrained Consumers

Let c_t^u be consumption (in log deviations from the steady-state) by unconstrained consumers and c_t^c consumption by constrained consumers: Then

$$c_t = (1-\nu)c_t^u + \nu c_t^c$$

where ν is the fraction of consrtained consumers.

$$c_t^u = -r_t^n + E_t c_{t+1}^u$$

$$c_t^c = \frac{Y}{C}y_t - \frac{T}{C}\tau_t$$

where τ_t is taxes. Combining equations yields:

$$c_t = -(1-\nu)r_t^n + \nu(\frac{Y}{C}y_t - \frac{T}{C}\tau_t) + E_t[c_{t+1} - \nu(\frac{Y}{C}y_{t+1} - \frac{T}{C}\tau_{t+1})]$$

Rearranging and iterating yields

$$c_t = \nu(\frac{Y}{C}y_t - \frac{T}{C}\tau_t) - (1-\nu)E_t\sum_{i=0}^{\infty} r_{t+i}^n$$

Combine with the resource constraint $y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t$ to obtain

$$y_t = \upsilon(y_t - \frac{T}{Y}\tau_t) - \frac{C}{Y}(1-\nu)E_t\sum_{i=0}^{\infty} r_{t+i}^n + \frac{G}{Y}g_t$$

$$y_t = \frac{1}{1-\upsilon}[-\upsilon\frac{T}{Y}\tau_t - \frac{C}{Y}(1-\nu)E_t\sum_{i=0}^{\infty} r_{t+i}^n + \frac{G}{Y}g_t]$$

where $\frac{1}{1-\upsilon}$ is the multipler. The evidence suggests assuming $\nu = .4$, implying $\frac{1}{1-\upsilon} \sim 1.7$.

Note that government expenditure multiplier exceeds the tax multiplier:

$$dY_t = Y dy_t = \frac{1}{1-v} dG_t$$

implying the government expenditure multiplier is $\frac{1}{1-v}$.

The tax multiplier is

$$dY_t = Y dy_t = -\frac{v}{1-v} d\tau_t$$

The tax multiplier is $-\frac{v}{1-v}$ (implying that tax reductions have a positive impact on spending). It is smaller than the expenditure multiplier since tax changes affect spending indirectly, through their impact on consumption spending. In our example, a dollar of tax cuts leads to about 66 cents worth of spending (.4/.6).

5 Investment

We next add investment:

$$Y_t = C_t + I_t + G_t$$

In loglinear form:

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} i_t + \frac{G}{Y} g_t$$

Investment: Competitive capital producers make new investment goods to sell at the price Q_t. Each investment good costs one unit of output to make plus adjustment costs. Firm profits are given by

$$Q_t I_t - I_t - \frac{1}{2} b (I_t - z)^2$$

where $Q_t I_t$ are revenues from the sale of I_t new capital goods,and $I_t - \frac{1}{2} b (I_t - z)^2$ is the cost of producing them, where the latter term is adjustment costs. z is the "steady state" level of investment, so adjustment costs are in terms of deviations from the steady state.
The first necessary conditions yield a relation between Q_t and I_t.

$$Q_t - 1 - b(I_t - z) = 0$$

Rearranging

$$I_t = z + \frac{1}{b}(Q_t - 1)$$

I_t is thus increasing in Q_t. How much depends inversely on the adjustment cost parameter. Note also that in steady state, where $I_t = z$, Q_t equals one.
We next obtain a relation for Q_t.

Let D_t be the dividend on a capital. Then the return from ownership of a unit of capital from t to $t+1$, R^k_{t+1}, is given by

$$R^k_{t+1} = \frac{D_{t+1} + Q_{t+1}}{Q_t}$$

Let Ψ be the risk premim for the return to capital, which we take to be constant. Then by arbitrage

$$E_t R^k_{t+1} = E_t R^n_t \frac{P_t}{P_{t+1}} + \Psi$$

In the fixed price equilibrium accordingly,

$$E_t R^k_{t+1} = R^n_t + \Psi$$

In steady state, further,

$$R^k = \beta^{-1} + \Psi$$

Combining equations yields a relation between R^n_t and Q_t.

$$E_t \frac{D_{t+1} + Q_{t+1}}{Q_t} = R^n_t + \Psi$$

Rearranging:

$$Q_t = E_t \frac{D_{t+1} + Q_{t+1}}{R^n_t + \Psi}$$

Iterating forward:

$$Q_t = E_t \sum_{i=0}^{\infty} D_{t+1+i} / \Pi^i_{j=0}(R^n_{t+j} + \Psi)$$

We can obtain to following loglinear equaton for q_t from the relation $Q_t = E_t \frac{D_{t+1}+Q_{t+1}}{R^n_t+\Psi}$:

$$q_t = E_t\{\frac{R_k - 1}{R_k} d_{t+1} + \frac{1}{R_k} q_{t+1} - \frac{R_n}{R_k} r^n_t\}$$

Iterating forward

$$q_t = E_t \sum_{i=0}^{\infty} (\frac{1}{R_k})^i [\frac{R_k - 1}{R_k} d_{t+1+i} - \frac{R_n}{R_k} r^n_{t+i}]$$

q_t thus depends positively on the future dividend stream and negatively on the path of short term interest rates.

Monetary policy accordingly affects investment in the following way: an increase in r^n_t reduces q_t, and thus reduces investment. The strength of the effect depends on the persistence of the interest rate change.

To close the model, we assume dividends are proportionate to output

$$D_t = \alpha Y_t$$

We can now construct the complete loglinear model

Aggregate Demand

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}i_t + \frac{G}{Y}g_t$$

$$c_t = -r_t^n + E_t c_{t+1} + \chi_t$$

$$i_t = \frac{z}{b}q_t$$

$$q_t = E_t \sum_{i=0}^{\infty} (\frac{1}{R_k})^i [\frac{R_k - 1}{R_k}\alpha y_{t+1+i} - \frac{R_n}{R_k} r_{t+i}^n]$$

Aggregate Supply

$$y_t = a_t + n_t$$

$$a_t = \mu_t + \gamma_n n_t + c_t$$

$$p_t = \overline{p}$$

monetary sector

$$m_t - \bar{p} = y_t - \nu r_t^n + \varepsilon_t$$

$$r_t^n = \overline{r_t^n}$$

We collapse to:

IS

$$y_t = \frac{C}{Y} \cdot E_t \sum_{i=0}^{\infty} (-r_{t+i}^n + \chi_{t+i}) + \frac{I}{Y}\frac{z}{b}q_t + \frac{G}{Y}g_t$$

with

$$q_t = E_t \sum_{i=0}^{\infty} (\frac{1}{R_k})^i [\frac{R_k - 1}{R_k}\alpha y_{t+1+i} - \frac{R_n}{R_k} r_{t+i}^n]$$

LM

$$r_t^n = \overline{r_t^n}$$

AS

$$\mu_t = -(1 - \gamma_n)(y_t - y_t^*)$$

Note that the addition of investment makes the solution for y_t^* more complicated.

Overall, however, the IS/LM/AS structure remains similar to before. Interest rates, though, now affect investment as well as consumption.