Professor Mark Gertler
Intermediate Macroeconomic Theory
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Lecture 5

Two-Period Competitive Equilibrium Model: Part 2

We now extend the two-period model of the previous lecture to allow for fiscal policy, and also consider the extension to the open economy. We again abstract from labor (though note we considered this extension in the previous lecture.)

We start by adding fiscal policy to a closed economy and then subsequently consider an extension to the open economy. Keep in mind that for the most part we are studying an economy with perfectly competitive markets that operates at full capacity. Thus, the conclusions we derive apply to this kind of setting, unless stated otherwise..

1 Environment

Assume:

(i) Two periods: 1 and 2.

(ii) One representative household that: consumes, saves and receives dividend income (from ownership of firms) and pays taxes.

(iii) One representative firm that produces output using capital, invests in new capital, and pays dividends to households.

(iv) The household and the firm act competitively, i.e., each takes market prices as given

(v) Physical capital is the only productive input.

(vi) A government that consumers, levies taxes (lump sum) and may borrow in the first period.

We next characterize preferences, technology and resource constraints, and government fiscal policy.

Preferences

The household consumes in both periods. Let C^i_k be household i's consumption in period k. Then household preferences are given by

$$\frac{1}{1-\sigma}(C^i_1)^{1-\sigma} + \frac{1}{1-\sigma}\beta(C^i_2)^{1-\sigma} \quad : \quad \text{if } \sigma \neq 1 \tag{1}$$

$$\log C^i_1 + \beta \log C^i_2 \quad : \quad \text{if } \sigma = 1 \tag{2}$$

with $\sigma > 0$ and $0 < \beta < 1$.

Technology

Firm j produces output in period 1 using capital input, K^j_1, and also produces output in period 2 using capital.input K^j_2 Captial in period 2 depends upon the amount of period 1 investment, I. Let Y^j_k be output in period k and A_k total factor productivity. Then each period, production is given by

$$Y^j_1 = A_1(K^j_1)^\alpha \tag{3}$$

$$Y^j_2 = A_2(K^j_2)^\alpha \tag{4}$$

The link between capital and investment is given by

$$K^j_2 = (1-\delta)K^j_1 + I^j \tag{5}$$

where δ is the rate of depreciation. Capital in period 2 depends on capital leftover from period 1.plus period 1 investment net adjustment costs.

Adding I^j units of capital costs

$$I^j + \frac{c}{2}(\frac{I^j}{K^j_1})^2 K^j_1$$

where $\frac{c}{2}(\frac{I^j}{K^j_1})^2 K^j_1$ reflects costs to the firm of adjusting its capital stock. Capital in period 2 depends on capital leftover from period 1.plus period 1 investment net adjustment costs. Note that we take K^j_1 as given in period 1.

Government

Let G_k be government expenditures on goods and services in period $k = 1.2$, T_k taxes in period k, and B^g the stock of government issued in period 1.

$$B^g + T_1 = G_1 \tag{6}$$

$$T_2 = G_2 + RB^g \tag{7}$$

Note that the government finances any excess of expenditures over taxes in period one by issuing debt. In period 2, however, taxes must be sufficiently large to cover the interest and principle obligation in the debt.

We can combine the two period budget constraints into a single budget constraint:

$$T_1 + \frac{T_2}{R} = G_1 + \frac{G_2}{R} \tag{8}$$

The governments intertemporal budget constraint state the prsesent value of taxes must equal the present value of government expenditures. Note that this is independent of the mix between taxes and borrowing that the government uses to finance period one government expenditures. If the government reduces taxes a dollar this period, it must raise them by R dollars next period to pay off the additional debt. The present value of tax collections is unchanged.

Economy-Wide Resource Constraints

In period 1, output is divided between consumption, investment: and government expenditures:

$$Y_1 = C_1 + I + G_1 \tag{9}$$

In period 2, output and the remaining capital stock is divided between consumption and government expenditures.

$$(1-\delta)K_2 + Y_2 = C_2 + G_2 \tag{10}$$

2 Household and Firm Behavior

2.0.1 The household decision problem

Let S^i be household i's saving, Π_k^i dividends in period $_k$, R the gross real interest rate (equal to one plus the net interest rate), all in units of consumption goods. Further, we normalize the price of consumption goods at unity. The representative household chooses C_1^i, C_2^i, and S^i,to solve

$$\max \frac{1}{1-\sigma}(C_1^i)^{1-\sigma} + \frac{1}{1-\sigma}\beta(C_2^i)^{1-\sigma} \tag{11}$$

subject to:

$$C_1^i = \Pi_1^i - T_1 - S^i \tag{12}$$

$$C_2^i = \Pi_2^i - T_2 + RS^i \tag{13}$$

The household takes as given R, Π_1^i and Π_2^i. Note that S^i may be positive or negative. Negative values of S^i imply borrowing. Implicit in our formulation, however, is the assumption of perfect capital markets; i.e., the households is able to borrow at the same rate R for which it is able to lend.

It is instructive to combine the two period budget constraints given by (12) and (13) into a single intertemporal budget constraint given by

$$C_1^i + \frac{C_2^i}{R} = \Pi_1^i + \frac{\Pi_2^i}{R} - T_1 - .\frac{T_2}{R} \tag{14}$$

According to equation (14), the households lifetime consumption plan must satisfy the constraint that the present value of consumption is equal to the present value of income after tax. Accordingly, consumption depends on discounted after tax lifetime income as opposed to after-tax income in the current period. This implication, however, depends on the assumption of perfect capital markets. What permitted collapsing the period budget constraints into the single intertemporal constraint for negative as well as positive values of S is that the value of R is independent of the sign of S; i.e., the individual can freely borrow or lend at the gross rate R.

To solve the household's decision problem, it is simplest to turn the constrained problem into an unconstrained one by plugging (12) and (13) into (**??**). This is possible as long as the two one-period budget constraints are always binding, so that there is no unused income. This latter condition is ensured by the assumptions we made on the utility function..

The representative household accordingly chooses S^i to solve

$$\max\left(\frac{1}{1-\sigma}(\Pi_1 - T_1 - S^i)^{1-\sigma} + \beta\frac{1}{1-\sigma}(\Pi_2 - T_2 + RS^i)^{1-\sigma}\right.,$$

given R, Π_1, and Π_2.

As before, The first order necessary condition with respect to saving is given by:

$$(C_1^i)^{-\sigma} = R\beta(C_2^i)^{-\sigma} \tag{15}$$

From the first order condition and the intertemporal budget constraint, one can obtain an explicit solution for consumption in period 1

$$C_1 = \frac{1}{1+\beta^{\frac{1}{\sigma}}R^{\frac{1}{\sigma}-1}}\left(\Pi_1 - T_1 + \frac{\Pi_2 - T_2}{R}\right) \tag{16}$$

Assuming perfect capital markets, consumption depends on the present value of lifetime after tax income. Note that the timing of tax payments doesn't matter, only the present value does.

2.0.2 The Firm Decision Problem

The firm's decision problem is the same as in the case without fiscal policy. The firm maximizes the discounted stream of profits returned to the household. Given that there is no uncertainty, the firm discounts future profits at the rate $1/R$. Accordingly, the firm chooses I, Π_1, and Π_2 to solve

$$\max \Pi_1^j + \frac{\Pi_2^j}{R} \tag{17}$$

subject to:

$$\Pi_1 = A_1 K_1^\alpha - I^j - \frac{c}{2}(\frac{I^j}{K_1^j})^2 K_1^j + B^j \tag{18}$$

$$\Pi_2 = A_2 K_2^\alpha + (1-\delta)K_2 - RB^j \tag{19}$$

taking as given R. R is the opportunity cost of funds that the firms faces in period 1. Further,

$$K_2^j = (1-\delta)K_1^j + I^j$$

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Again, we can convert the problem into an unconstrained maximization problem by plugging the constraints (18) and (19) into (17). The firm then chooses I to solve

$$\max A_1 K_1^\alpha - I^j - \frac{c}{2}(\frac{I^j}{K_1^j})^2 K_1^j + \frac{1}{R}\{A_2[(1-\delta)K_1^j + I^j]^\alpha + (1-\delta)[(1-\delta)K_1^j + I^j]\}$$

The first order necessary conditions for investment is given by:

$$R = \frac{A_2 \alpha K_2^{\alpha-1} + 1 - \delta}{1 + c\frac{I}{K}} \tag{20}$$

Taking into account that K_2^j is increasing in I^j yields an inverse relation between K_2^j and R.

3 Competitive Market Equilibrium

A competitive equilibrium for this economy is an allocation (I, C_1, C_2, Y_1, Y_2, K_2) and a relative price vector (R) such that the household and the firm is each maximizing its respective objective, markets clear, and the economy resource constraints are satisfied.

Capital market:

$$\frac{A_2 \alpha K_2^{\alpha-1} + 1 - \delta}{1 + c\frac{I}{K}} = R = \frac{(C_1)^{-\sigma}}{\beta(C_2)^{-\sigma}} \tag{21}$$

Resource Constraints:

$$Y_1 = C_1 + I + \frac{c}{2}(\frac{I}{K_1})^2 K_1 + G_1 \tag{22}$$

$$Y_2 + (1-\delta)K_2 = C_2 + G_2 \tag{23}$$

Technology Constraints:

$$Y_1 = A_1(K_1)^\alpha \tag{24}$$

$$Y_2 = A_2(K_2)^\alpha \tag{25}$$

Evolution of Capital

$$K_2 = (1-\delta)K_1 + I$$

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4 Understanding the Effects of Fiscal Policy

Let's begin with the second period. Total resources available, $A_1(K_1)^\alpha + (1-\delta)K_2$ depends on the capital stock in period 2, which is determined by investment in period 1. Thus an unanticipated change in G_2 does not affect output in period 2 (since it will not affect K_2). The impact will be an exact offsetting movement in C_2. A change in G_2 anticipated in period 1 could affect period 2 output by affecting period 1 investment. We now turn to period 1 behavior.

It is first useful to consider the link between saving and investment in period 1. By definition, investment expenditures must equal the sum of private and national saving:

$$\begin{aligned} I(R) &= (Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - T_1 - C_1) + (T_1 - G_1) \\ &= Y_1 - C_1 - G_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 \end{aligned}$$

where $\partial I/\partial R < 0$, as implied by equation (20). Using the solution for consumption in equilibrium:

$$I(R) = Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - \frac{1}{1+\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}(\Pi_1 - G_1 + \frac{\Pi_2 - G_2}{R}) - G_1$$

with

$$\Pi_1 + \Pi_2/R = A_1 K_1^{\alpha} - I - \frac{c}{2}(\frac{I}{K_1})^2 K_1 + (A_2 K_2^{\alpha} + (1-\delta)K_2)/R$$

In the special case $\sigma = 1$ and $R\beta = 1$

$$\begin{aligned} I(R) &= Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - \frac{\Pi_1 + \beta\Pi_2}{1+\beta} + \frac{G_1 + \beta G_2}{1+\beta} - G_1 \\ &= \{Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - \frac{\Pi_1 + \beta\Pi_2}{1+\beta}\} - \frac{\beta}{1+\beta}[G_1 - G_2] \end{aligned}$$

Observe that investment is decreasing in G_1 and increasing in G_2. Intuitively, an increase in G_1 reduces national saving. Holding constant, C_1 and G_2,a one unit increase in G_1 reduces national saving. C_1 falls by $\frac{1}{1+\beta} < 1$, which leads to a partially offsetting increase in national saving. On net, however, national saving decreases, leading to a decrease in investment.The real rate R increases to insure investment equals national saving. (Note that $\frac{\Pi_1+\beta\Pi_2}{1+\beta}$ depends on investment,. However under reasonable parameter assumptions, this does change the sign of the effect of government spending on investment.)

Next note that an increase in G_2 increases current investment. Intuitively, consumption declines as households expect higher taxes to finance the increase in government expenditures. The decline in consumption increase national saving and hence current investment.

Finally, note that government borrowing does not affect the economy's real equilibirium. This because any change in government borrowing that affects current taxes, leads to an offsetting adjustment in present value of future taxes to cover the interest and prinicple on the debt. Thus changes in government borrowing do not affect the present value of taxes that households face. This tax burdent, as we saw earlier, depends on the present value of real government expenditures.

The independence of real activity from whether the government finances current expenditures from debt or current taxes is know as the Ricardo Equivalence theorem, named after a famous 19th century economist David Ricardo. As Ricardo noted, however, the theorem does not hold if individuals face borrowing constraints. In this instance, as we saw earlier, the timing of taxes matters, as well as the present value.

5 Open Economy Issues

In the open economy, the real interest rate adjusts to equate global saving and investment. Since countries may borrow from one another, at any point in time saving need not equal investment within a country's borders.

Here we consider an open economy that is small enough to take the world interest rate R^was given

$$R = R^w$$

With perfect international financial markets, households, firms and the government can lend and borrow at the world interest rate R^w.

A portion of period 1 output now goes to net exports XM.

$$Y_1 = C_1 + I + G_1 + XM$$

Accumulation of foreign assets FA equals net exports.

$$FA = XM$$

Country income in period 2 includes the interest and principal from earnings on foreign assets, RFA.

$$Y_2 + (1 - \delta)K_2 + RFA = C_1 + G_2$$

Competitive Equilibrium

Capital market:

saving supply

$$R^w = \frac{(C_1)^{-\sigma}}{\beta(C_2)^{-\sigma}} \tag{26}$$

investment demand

$$R^w = \frac{A_2\alpha K_2^{\alpha-1} + 1 - \delta}{1 + c\frac{I}{K}}$$

<u>Resource Constraints</u>:

$$Y_1 = C_1 + I, +G_1 + XM \tag{27}$$

$$Y_2 + (1-\delta)K_2 + RXM = C_2 + G_2 \tag{28}$$

<u>Technology Constraints</u>:

$$Y_1 = A_1(K_1)^\alpha \tag{29}$$

$$Y_2 = A_2(K_2)^\alpha \tag{30}$$

<u>Evolution of Capital</u>

$$K_2 = (1-\delta)K_1 + I$$

Note that relative to the closed economy, there is one extra variable, XM, but also one extra restriction: $R = R^w$.

saving and investment:

$$\begin{aligned} XM + I(R^w) &= (Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - T_1 - C_1) + (T_1 - G_1) \\ &= Y_1 - (\frac{I}{K_1})^2 K_1 - C_1 - G_1 \end{aligned}$$

where $\partial I/\partial R < 0$, as implied by equation (20). Using the solution for consumption in equilibrium:

$$XM + I(R^w) = Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - \frac{1}{1+\beta^{\frac{1}{\sigma}} R^{\frac{1}{\sigma}-1}}(\Pi_1 - G_1 + \frac{\Pi_2 - G_2}{R}) - G_1$$

with

$$\Pi_1 + \Pi_2/R = A_1 K_1^\alpha - I - \frac{c}{2}(\frac{I}{K_1})^2 K_1 + (A_2 K_2^\alpha + (1-\delta)K_2)/R$$

In the special case $\sigma = 1$,

$$\begin{aligned} XM + I(R) &= Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - \frac{\Pi_1 + \beta\Pi_2}{1+\beta} + \frac{G_1 + \beta G_2}{1+\beta} - G_1 \\ &= \{Y_1 - \frac{c}{2}(\frac{I}{K_1})^2 K_1 - \frac{\Pi_1 + \beta\Pi_2}{1+\beta}\} - \frac{\beta}{1+\beta}[G_1 - G_2] \end{aligned}$$

In this case a rise in G_1 reduces XM.