ECON-UB 233 Dave Backus @ NYU

Quiz #1

(Revised: February 27, 2012)

Please write your name below. Then complete the exam in the space provided. There are THREE questions. You may refer to one page of notes: standard paper, both sides, any content you wish.

(Name and signature)

1. (3-state "Bernoulli") (30 points) Consider a 3-state distribution in which we can explore the impact of high-order cumulants. This is sometimes called a "categorical distribution."

   Let us say, to be concrete, that the state z takes on the values $\{-1, 0, 1\}$ with probabilities $\{\omega, 1-2\omega, \omega\}$. A random variable x is defined by $x(z) = \delta z$.

   (a) Does x have a legitimate probability distribution? (5 points)

   (b) What is the moment generating of x? The cumulant generating function? (10 points)

   (c) What are the mean and variance of x? (5 points)

   (d) What are the measures of skewness and excess kurtosis, γ_1 and γ_2? Under what conditions is γ_2 large? (5 points)

   (e) Is there a value of ω that reproduces the values of γ_1 and γ_2 of a normal random variable? (5 points)

2. (3-state "Bernoulli," continued) (20 points) For a representative agent economy based on the same state and random variable, suppose x is log consumption growth, $x(z) = \log c_1(z) - \log c_0$, and utility has the additive form,

$$u(c_0) + \beta \sum_z p(z) u[c_1(z)],$$

   with $u(c) = c^{1-\alpha}/(1-\alpha)$.

   (a) What is the pricing kernel? (5 points)

   (b) What are the state prices? (5 points)

   (c) What are the risk-neutral probabilities? How do they vary with risk aversion α? (5 points)

   (d) Which is more valuable, a claim to one unit of the good in state $z = -1$ or one unit in state $z = +1$? Why? (5 points)

3. (short answers) (40 points) Give short answers to the following questions:

(a) Suppose $x \sim \mathcal{N}(\kappa_1, \kappa_2)$. What are κ_1 and κ_2? What is the distribution of $\log \beta - \alpha x$? (5 points)

(b) How would you compute skewness in a sample of data, x_t for $t = 1, 2, \ldots, T$? (5 points)

(c) What determines whether skewness raises or lowers expected utility? Which applies to power utility? (5 points)

(d) What is a resource constraint? How does it differ from a budget constraint? (5 points)

(e) What are the prices of Arrow securities for this collection of asset prices and dividends?

$$\begin{array}{ll} \text{Asset 1:} & q^1 = 1.0, \;\; d^1(1) = 2, \;\; d^1(2) = 1 \\ \text{Asset 2:} & q^2 = 1.5, \;\; d^2(1) = 2, \;\; d^2(2) = 2. \end{array}$$

Is this a "no-arbitrage" environment? How do you know? (10 points)

(f) In our usual two-period setup, suppose the representative agent has power utility and $x(z) = \log c_1(z) - \log c_0$ has an exponential distribution:

$$p(x) \;\; = \;\; \lambda e^{-\lambda x},$$

for $x \geq 0$ and $\lambda > 0$. What is the risk-neutral distribution of x? (10 points)