New York University | November 7, 2011
Econ UB:234 - Econ UA:367 | Thomas J. Sargent

Midterm Examination

Instructions: Please answer all three questions. Points possible are stated at the beginning of each question.

1. 25 points A consumer has multi period utility function

$$\sum_{t=0}^{\infty} \beta^t \ln c_t, \quad 0 \leq \beta < 1. \tag{1}$$

The consumer chooses $\{c_t, A_{t+1}\}_{t=0}^{\infty}$ to maximize (1) subject to $c_t \geq 0$ for $t \geq 0$, $A_0 = 0$, $\lim_{T \to +\infty} \beta^T c_T^{-1} A_{T+1} = 0$, and

$$A_{t+1} = R_t [A_t + y_t - c_t]. \tag{2}$$

Here R_t is the gross interest rate on financial assets between t and $t+1$, y_t is labor income, c_t is consumption, and A_t is financial assets at the beginning of t.

It happens that the consumer's income $\{y_t\}_{t=0}^{\infty}$ and the interest rate $\{R_t\}_{t=0}^{\infty}$ are such that the consumer wants to set $c_t = y_t$ and $A_{t+1} = 0$ for all $t \geq 0$, where

$$y_t = \begin{cases} y_0 \delta^t, & \text{if } t = 0, \ldots, T; \delta > 0, \\ y_0 \delta^T \phi^{t-T} & \text{for } t = T+1, \ldots, \infty; \phi > 0. \end{cases}$$

a. Find a formula for R_t at each $t \geq 0$.

b. Interpret your formula for R_t in terms of the consumer's *impatience* and her *income growth*.

2. 25 points A government wants to choose a sequence of tax collections to *minimize* the following measure of tax distortions:

$$\sum_{t=0}^{\infty} \beta^t D(T_t), \quad 0 < \beta < 1$$

where $D(T_t)$ is a measure of the costs of distortions at date t, T_t is total tax revenues at t, and D is a twice continuously differentiable function with $D' >$

$0, D'' > 0$. The government confronts an exogenous stream of expenditures $\{G_t\}_{t=0}^{\infty}$ and faces a sequence of government budget constraints

$$B_{t+1} = R[B_t + G_t - T_t], B_0 = 0,$$

where B_{t+1} is government debt issued at t and due at $t+1$. The government can borrow or lend. Here $R = (r+1)$ is the gross rate of return on government debt. Assume that $R = \beta^{-1} > 1$. Assume that

$$\lim_{s\to\infty} \beta^s D'(T_s) B_s = 0,$$

a condition that rules out "Ponzi schemes".

a. Consider the government expenditure process

$$G_t = \begin{cases} 1 & \text{if } t \text{ even,} \\ 0 & \text{if } t \text{ odd.} \end{cases}$$

Find the optimal setting for taxes $\{T_t\}_{t=0}^{\infty}$.

b. Consider the expenditure process

$$G_t = \begin{cases} 1 & \text{if } t \neq 10j,\ j = 0, 1, 2, \ldots, \\ 10 & \text{if } t = 10j,\ j = 0, 1, 2, \ldots \end{cases}$$

Here a 'war' happens in periods $0, 10, 20, \ldots$ and peace prevails otherwise. Find the optimal tax sequence $\{T_t\}_{t=0}^{\infty}$.

c. Does this analysis remind you of any other type of model that you know? Please explain.

3. 25 points A consumer's optimal decision rule for consumption satisfies

$$c_t = (1-\beta)\big[F_t + E_t \sum_{j=0}^{\infty} \beta^j y_{t+j}\big] \tag{3}$$

where c_t is consumption, β^{-1} is the gross one-period interest rate, which is constant over time, y_t is the consumer's income at time t, F_t is the consumer's financial assets at the beginning of t, and $E_t(\cdot)$ means the best forecast of $(\cdot)$ (whatever $(\cdot)$ is), conditional on information that the consumer knows at t. At time t, assume that the consumer knows current and past values of

y_t's, but not future values. The consumer's labor income follows the random process

$$y_{t+1} = \delta_0 + \delta_1 y_t + \delta_2 y_{t-1} + \sigma \epsilon_{t+1}$$

where $\{\epsilon_{t+1}\}_{t=0}^{\infty}$ is an independently and identically distributed (iid) sequence of scalar normally distributed scalar random variables, each with mean 0 and variance 1. (Please make whatever assumptions you want about δ_1 and δ_2 in order to make the subsequent questions meaningful.)

a. Find an expression for the consumer's decision rule of the form

$$c_t = (1-\beta)\big[F_t + \alpha_0 + \alpha_1 y_t + \alpha_2 y_{t-1}\big].$$

Please describe how to find formulas for $\alpha_0, \alpha_1, \alpha_2$.

b. Measured in constant 2005 dollars, the changes in consumption for this consumer over 2008 (which started out better than it ended) were as follows:

quarter	$c_t - c_{t-1}$
2008I	1000
2008II	0
2008III	0
2008IV	-4000

What can you infer from these consumption change numbers, if anything, about the consumer's past, present, and future labor income?