# Lab Report #5: Options & Volatility

(Started: August 20, 2011; Revised: March 26, 2012)

*Due at the start of class. You may speak to others, but whatever you hand in should be your own work.*

**Solution:** Answers follow. See Matlab code at the end for calculations.

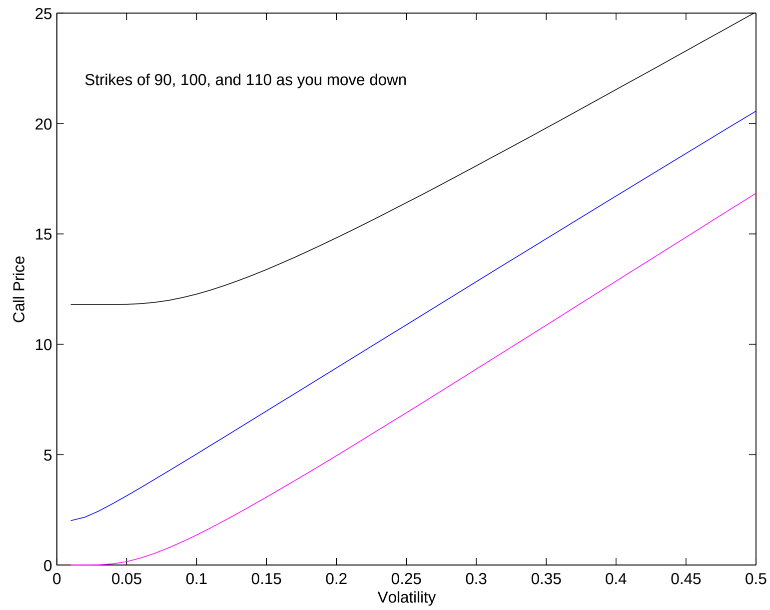
1. (BSM formula) We'll examine the BSM formula in some purely theoretical numerical examples. In what follows, the current price of the underlying is 100, the option maturity is one year, and the one-year bond price is 0.98.
   (a) If volatility $\sigma = 0.10$, what are the prices of call options at strike prices of 90, 100, and 110?
   (b) What are the prices of put options with the same strikes?
   (c) If volatility rises to $\sigma = 0.15$, what happens to the prices of calls?
   (d) For strikes of 90, 100, and 110, graph the call price against volatility $\sigma$ using a grid between (roughly) 0.01 and 0.50. (This gives you three lines, one for each strike.) How do call prices vary with volatility? Does the pattern vary with the strike price?
   (e) For a strike of 110 and a call price of 2.00, what is the implied value of $\sigma$?

**Solution:**

(a,b,c) Answers below (more accuracy than you need). Put prices were computed from calls using put-call parity. See Matlab code for details.

| Strike | Call (0.10) | Call (0.15) | Put (0.10) |
|---|---|---|---|
| 90.0000 | 12.2691 | 13.3828 | 0.4691 |
| 100.0000 | 5.0281 | 6.9722 | 3.0281 |
| 110.0000 | 1.3584 | 3.0726 | 9.1584 |

(d) You can see the results in the figure below. Call prices increase with volatility in all cases. (Puts, too, for that matter.) For at-the-money options, the relation is linear. For others, there's some curvature to it. You can't see it in the figure, but it's S-shaped. You can see how this works if you look at the formula for the derivative (the "vega"), but that's more than we need here.




(e) From the values computed for the figure, $\sigma = 0.12$ is about it.

2. (volatilities on S&P 500 E-mini options) For the S&P 500 E-mini options, the prices of options are more conveniently expressed in terms of their implied volatilities. We'll compute them here for quotes reported on March 15, 2012:

| | Call Price | | Put Price | |
|---|---|---|---|---|
| Strike | Bid | Ask | Bid | Ask |
| 1340 | 82.50 | 85.75 | 28.25 | 30.25 |
| 1350 | 75.25 | 78.25 | 30.75 | 32.75 |
| 1360 | 68.00 | 71.00 | 33.25 | 35.50 |
| 1370 | 61.25 | 64.00 | 36.25 | 38.75 |
| 1380 | 54.50 | 57.25 | 39.50 | 42.25 |
| 1390 | 48.25 | 50.75 | 43.25 | 45.75 |
| 1400 | 42.25 | 44.75 | 47.00 | 50.00 |
| 1410 | 36.75 | 39.25 | 51.25 | 54.50 |
| 1420 | 31.75 | 33.75 | 56.00 | 59.25 |
| 1430 | 27.00 | 29.00 | 61.25 | 64.50 |

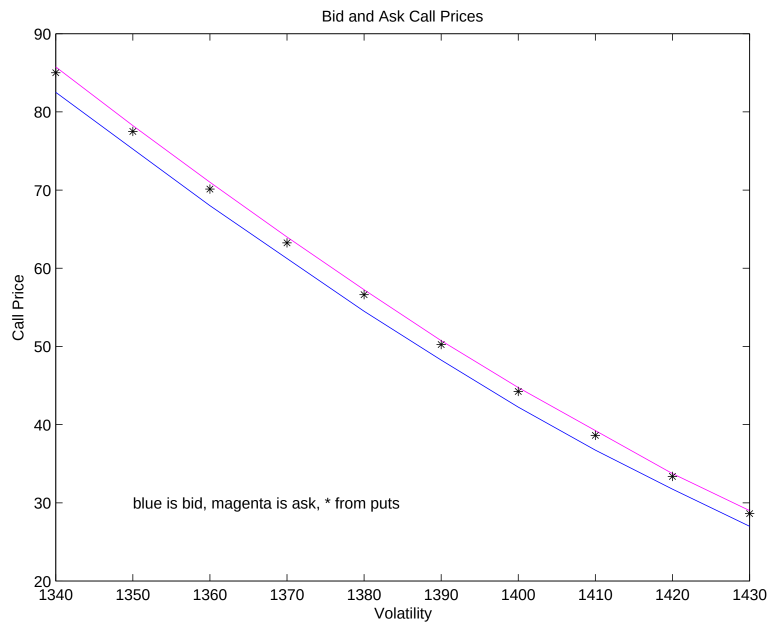
The price of the underlying contract was 1395.75. The interest rate was essentially zero, so the appropriate bond price was one. The options expire June 15, so $\tau = 3/12 = 1/4$.

(a) Compute "mid" quotes as averages of bid and ask. Use put-call parity to compute call prices from mid puts. Plot call prices — bid, ask, and implied by puts — against the strike. How do they compare?

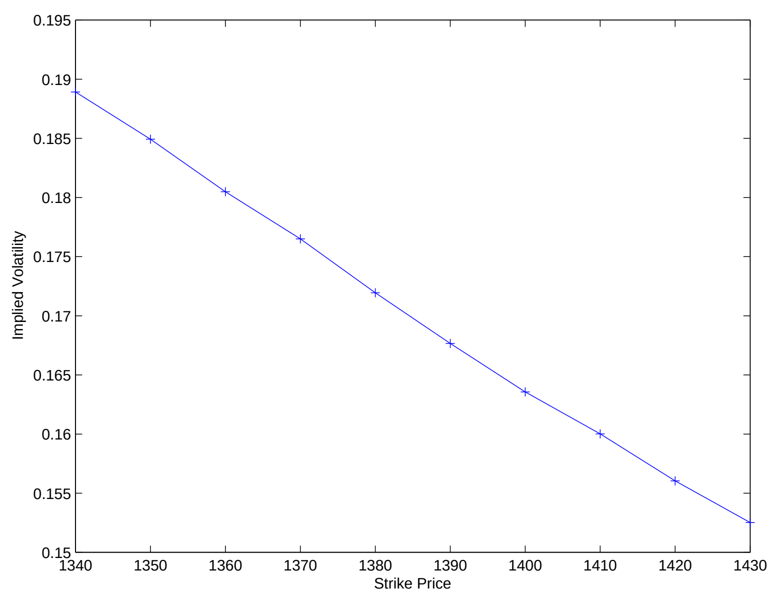
(b) Write a program using, say, the secant method to compute implied volatilities for mid quotes of call options. Graph them against the strike. What shape does the resulting "smile" have? What does the shape suggest to you?

**Solution:**

(a) See the figure below. Call prices computed from mid puts (asterisks) are within the bid-ask spread for call prices.




(b) Another figure:



I used the secant method on $\log(\sigma)$, but the log is probably overkill. We see that implied vols decline with strike. That means that prices at low strikes are relatively more expensive than the BSM formula with constant $\sigma$ would suggest. This isn't really a smile, but it's what you see for equity index options. You would also usually see, at least at shorter maturities, some convexity in the smile that isn't apparent here.

Matlab program:

```
%  hw5_s12.m
%  Matlab program for Lab Report #5, Spring 2012
%  NYU course ECON-UB 233, Macro foundations for asset pricing, Mar 2012.
%  Written by:  Dave Backus, March 2012
format compact
format short
clear all

%%
disp('Answers to Lab Report 5')

disp(' ')
disp('-----------------------------------------------------------------')
disp('Question 1 (option prices)')

disp(' ')
disp('Inputs')
tau = 1
q_tau = 0.98;
q = 100.00
b = [90; 100; 110];

% BSM formula
% define price as function of sigma, two steps for clarity (or not?)
d = @(sigma,b) (log(q./(q_tau.*b))+tau*sigma.^2/2)./(sqrt(tau)*sigma);
call = @(sigma,b) q*normcdf(d(sigma,b)) - q_tau.*b.*normcdf(d(sigma,b)-sqrt(tau)*sigma);

q_call_10 = call(0.10,b);
q_call_15 = call(0.15,b);

q_put_10 = q_call_10 + q_tau*b - q;

disp(' ')
disp('Strike, Call Prices at vol 0.10 and 0.15, Put Prices at 0.10')
[b q_call_10 q_call_15 q_put_10]

% call price v sigma
sigma = [0.01:0.01:0.50]';

q_call_90 = call(sigma,90);
q_call_100 = call(sigma,100);
q_call_110 = call(sigma,110);

disp(' ')
```

```
[dummy,i] = min(abs(q_call_110-2.00));
sigma_implied_parte = sigma(i)

figure(1)
clf
plot(sigma,q_call_90,'k')
hold on
plot(sigma,q_call_100,'b')
plot(sigma,q_call_110,'m')
axis([0 0.5 0 25])
xlabel('Volatility')
ylabel('Call Price')
text(0.02,22,'Strikes of 90, 100, and 110 as you move down')

print -depsc hw5_q1d.eps

return

%%
disp(' ')
disp('------------------------------------------------------------------')
disp('Question 2 (implied volatilities)')
clear all
format compact

disp(' ')
disp('Inputs')
tau = 3/12
q_tau = 1.00
q = 1395.75

data = [
    1340  82.50 85.75 28.25 30.25;
    1350  75.25 78.25 30.75 32.75;
    1360  68.00 71.00 33.25 35.50;
    1370  61.25 64.00 36.25 38.75;
    1380  54.50 57.25 39.50 42.25;
    1390  48.25 50.75 43.25 45.75;
    1400  42.25 44.75 47.00 50.00;
    1410  36.75 39.25 51.25 54.50;
    1420  31.75 33.75 56.00 59.25;
    1430  27.00 29.00 61.25 64.50];

b = data(:,1);
call_bid = data(:,2);
call_ask = data(:,3);
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```
put_bid = data(:,4);
put_ask = data(:,5);

call_mid = (call_bid+call_ask)/2;
put_mid = (put_bid+put_ask)/2;

disp(' ')
disp('(a) Calls from puts')

call_fromputs = q - q_tau*b + put_mid;

figure(1)
clf
plot(b, call_bid, 'b')
hold on
plot(b, call_ask, 'm')
plot(b, call_fromputs, 'k*')
title('Bid and Ask Call Prices')
xlabel('Strike Price')
ylabel('Call Price')
text(1350,30,'blue is bid, magenta is ask, * is from puts')

print -depsc hw5_q2a.eps

disp(' ')
disp('(b) Implied vols for mid calls via secant method')
clear functions

% BSM formula
% define f = call price as function of sigma, two steps for clarity (or not?)
d = @(sigma,b) (log(q./(q_tau.*b))+tau*sigma.^2/2)./(sqrt(tau)*sigma);
f = @(sigma,b) q*normcdf(d(sigma,b)) - q_tau.*b.*normcdf(d(sigma,b)-sqrt(tau)*sigma) ...
        - call_mid;

% convergence parameters
tol = 1.e-8;
maxit = 50;

% starting values
% NB:  we do this for log(sigma), which makes sure sigma is positive
x_before = log(0.08) + zeros(size(b));
x_now = log(0.12) + zeros(size(b));
f_before = f(exp(x_before),b);
f_now = f(exp(x_now),b);

% compute implied vol
```

```
t0 = cputime;
for it = 1:maxit
    fprime = (f_now - f_before)./(x_now - x_before);
    x_new = x_now - f_now./fprime;
    f_new = f(exp(x_new),b);
    diff_x = max(abs(x_new - x_now));
    diff_f = max(abs(f_new));
%    [it diff_x diff_f]

    if max(diff_x,diff_f) < tol, break, end

    x_before = x_now;
    x_now = x_new;
    f_before = f_now;
    f_now = f_new;
end

% display results
it
time = cputime - t0
diffs = [diff_x diff_f]
disp(' ')
disp('Strike/1000 and Vol')
vol = exp(x_new);
[b/1000 vol]

figure(2)
clf
plot(b, vol, 'b')
hold on
plot(b, vol, 'b+')
xlabel('Strike Price')
ylabel('Implied Volatility')

print -depsc hw5_q2b.eps

return
```