Problem 1

(1) Using the equation for expectation formation, we have

IS: $y_t = -(i_t - \pi_{t-1}) + E_t y_{t+1} + x_t$

LM: $i_t = \bar{i}_t$

AS: $\pi_t = \lambda(y_t - y_t^*) + \pi_{t-1}$

We assume that the nominal interest rate is kept fixed, that initially $\pi_{t-1} = 0$, and that the economy is hit with a demand shock at time t only.

Our system of equations at t and t+1 then becomes:

time t:

$$y_t = -\bar{i} + E_t y_{t+1} + x_t$$

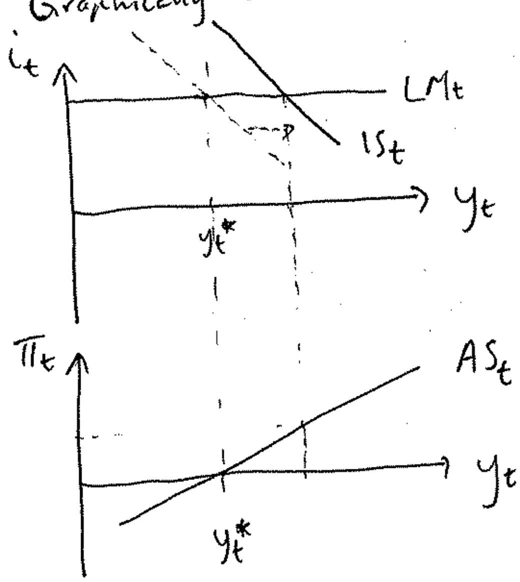
$$\pi_t = \lambda(y_t - y_t^*)$$

time t+1:

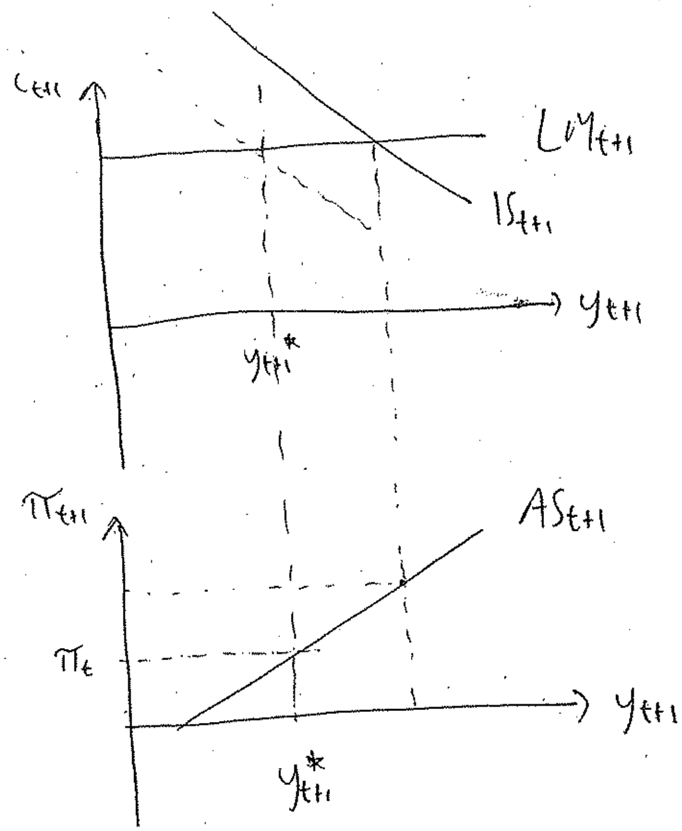
$$y_{t+1} = -\bar{i} + \pi_t + E_{t+1} y_{t+2}$$

$$\pi_{t+1} = \lambda(y_{t+1} - y_{t+1}^*) + \pi_t$$

Graphically we have



x_t shifts IS at t to the right. This raises the output gap and ~~to~~ leads to inflation at time t.

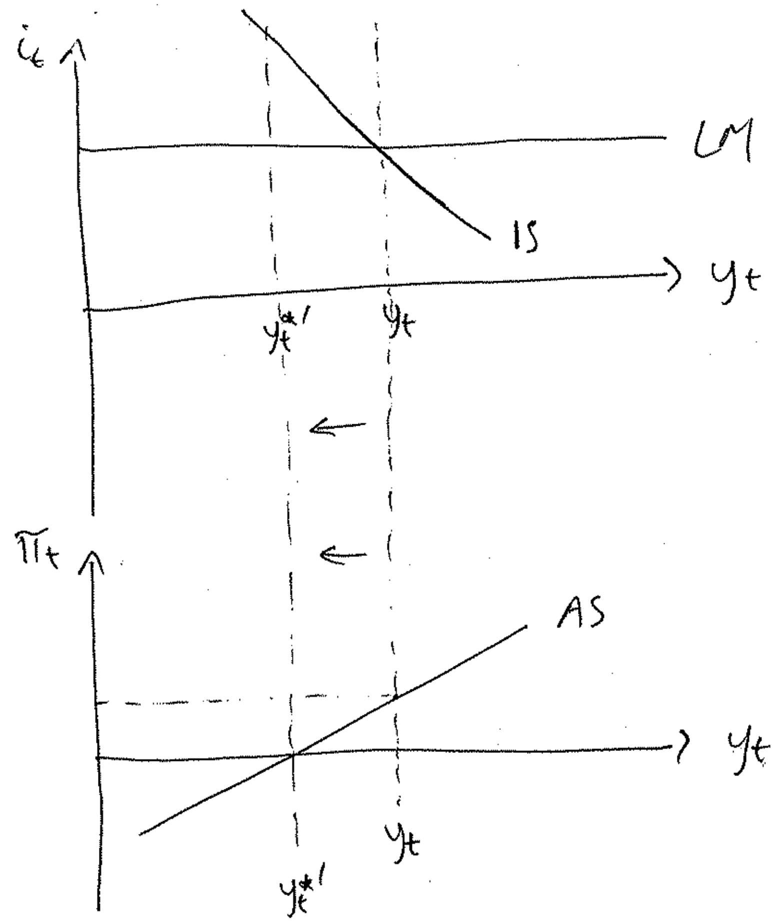


Inflation at t affects expectations of future inflation. This shifts IS at t+1 to the right making the increase in the output gap persistent.

Inflation π_t also shifts the AS curve up at t+1.

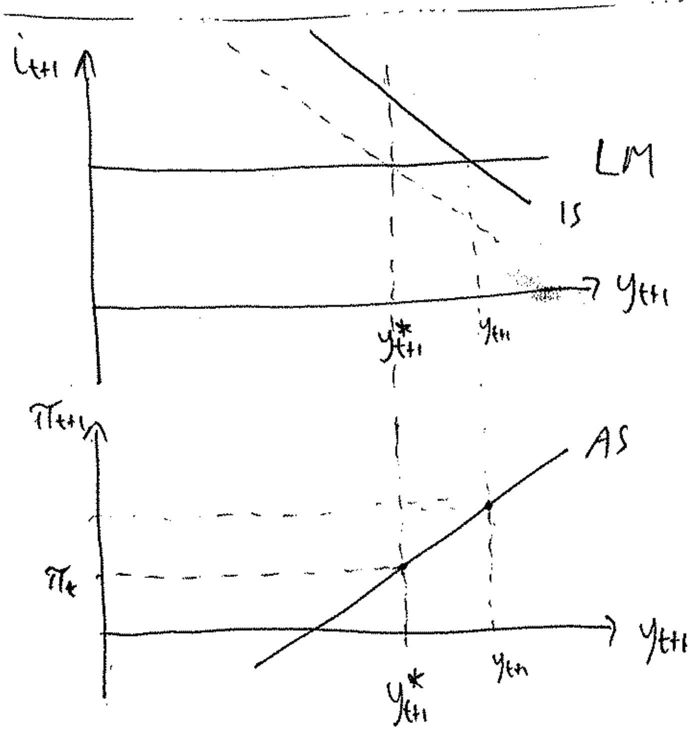
So inflation at t+1 is previous inflation + an additional term from the persistent output gap.

(2) Assume that the output gap is initially zero, but a negative supply shock reduces y_t^*.



A shift left in y_t^* leads to a positive output gap and hence inflation at time t.

(Assume that the negative supply shock only lasts one period.)

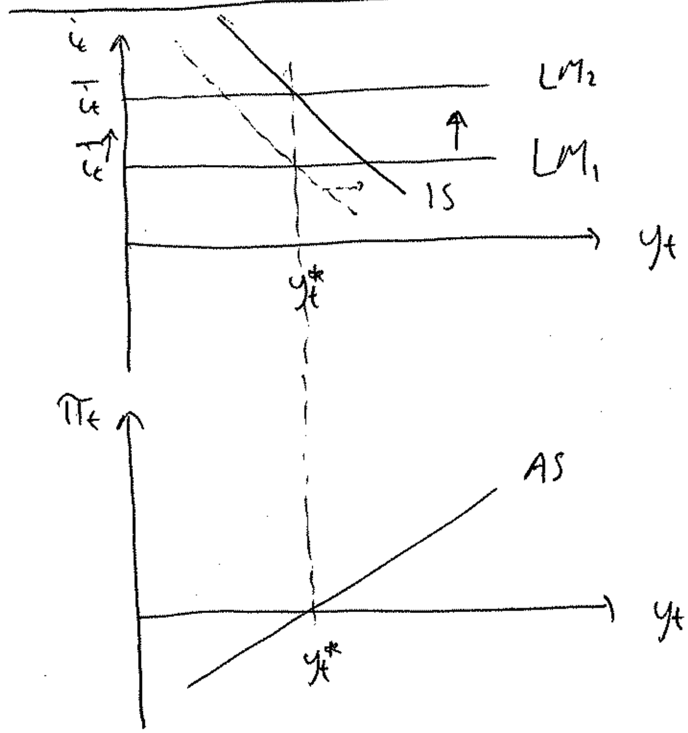


Inflation at t shifts the IS-curve at t+1 to the right. This leads to persistence in the output gap.

Inflation π_t also shifts the AS curve up. This shift + the persistence in the output gap leads to even higher inflation in period t+1.

(3) In both cases: increase i_t to prevent a positive output gap and hence inflation.

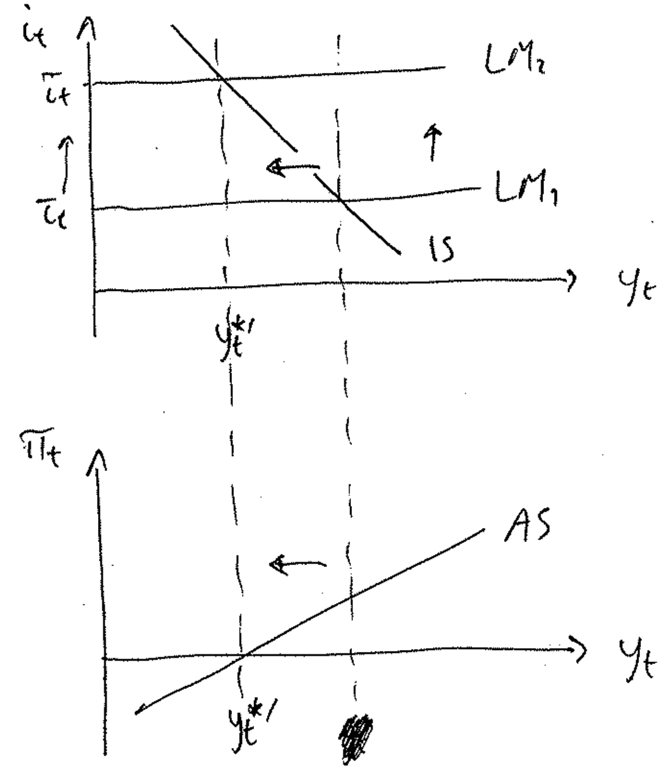
Case 1: demand shock



Demand shock causes IS to shift right. Raise i_t to prevent a positive output gap.

W/o a positive output gap, there is no inflation at t.

Case 2: supply shock



A negative supply shock shifts y_t^* to the left. Demand is too high (positive output gap) so the Fed raises i_t to prevent inflationary pressure. When i_t is at LM$_2$ the output gap is 0 and there is no inflation.

Note:

In answering this question I assumed that we started from a steady state with $\pi_{t-1} = 0$ and some shock at time t. The central bank can then set the interest rate to counter the effect of a shock and prevent any inflation from occurring. It does so by raising the interest rate so that IS and LM intersect where $y_t = y_t^*$.

However, as in HW7, if there is some initial inflation $\pi_{t-1} > 0$, then the central bank wants to counter the effect of a shock and eliminate the initial inflation. It would then raise the interest rate by more to produce a negative output gap and hence reduce inflation.

Agents in this qn. have adaptive expectations. $E_t \pi_{t+1} = \pi_{t-1}$. Expectations only depend on observed inflation. Hence, a discussion of rules vs. discretion (commitment vs. no commitment) does not really apply.

When agents have rational expectations $E_t \pi_{t+1}$ has to be consistent with the model and the behavior of the central bank. In that case it is potentially useful for the central bank to commit to a rule since it helps anchor expectations for the future. This is illustrated in qn. 2.

Problem 2:

(1) The central bank solves

$$\max_{x_t, \pi_t} \quad -\frac{1}{2}\left[\alpha (x_t - k)^2 + \pi_t^2\right]$$

$$\text{s.t.} \quad \pi_t = \lambda \cdot x_t + E_t \pi_{t+1}$$

$$\Rightarrow \max_{x_t} \quad -\frac{1}{2}\left[\alpha (x_t - k)^2 + (\lambda x_t + E_t \pi_{t+1})^2\right]$$

The central bank solves this problem taking $E_t \pi_{t+1}$ as given:

$$\text{FOC}(x_t): \quad -\alpha (x_t - k) - (\lambda x_t + E_t \pi_{t+1}) \cdot \lambda = 0$$

$$\Rightarrow \quad x_t - k = -\frac{\lambda}{\alpha} \cdot (\lambda x_t + E_t \pi_{t+1})$$

$$\Rightarrow \quad x_t - k = -\frac{\lambda}{\alpha} \cdot \pi_t$$

In steady state $x_t = 0$, so the constant inflation rate in steady state is

$$\boxed{\pi_t = \frac{\alpha}{\lambda} \cdot k}$$

An inflation bias arises from the fact that the central bank cares about the output gap ($\alpha > 0$) and wants to target an output gap that is positive ($k > 0$).

The central bank cannot commit to an inflation target, so will solve this maximization problem taking expectations as given. The private sector will in equilibrium expect positive inflation at a rate which makes the central bank choose $x_t = 0$.

(2) In steady state, in order for inflation to be constant, $x_t = 0$. Using the IS curve to solve for i_t:

(and $E_t x_{t+1} = 0$ in ss.)

$$i_t = r_t^* + E_t \pi_{t+1}$$

So in steady state we have

$$i = r^* + \frac{\alpha}{\lambda} \cdot k.$$

The steady state nominal interest rate is higher than the steady state natural rate of interest due to the inefficiently high inflation.

(3) Now the central bank follows the rule

$$i_t = r_t^* + E_t x_{t+1} + (1+\phi) E_t \pi_{t+1}$$

instead of solving the optimization problem in part 1).

(i) The rule implies that

$$i_t - E_t \pi_{t+1} - r_t^* = E_t x_{t+1} + \phi E_t \pi_{t+1}$$

Inserting this into the IS curve $\Rightarrow$

$$x_t = -\cancel{E_t x_{t+1}} - \phi E_t \pi_{t+1} + \cancel{E_t x_{t+1}}$$

Inserting this into the AS curve $\Rightarrow$

$$\pi_t = \lambda \cdot (-\phi E_t \pi_{t+1}) + E_t \pi_{t+1}$$

$$\Rightarrow \quad \pi_t = (1 - \lambda\phi) E_t \pi_{t+1} \qquad \circledast$$

In steady state with constant inflation $\pi_t = E_t \pi_{t+1}$, we therefore must have that $\pi_t = 0$.

(ii) The key here is that when $\phi > 0$ ~~and~~ an increase in expected inflation leads to a greater increase in the nominal interest rate.
The real interest rate is

$$r_t = i_t - E_t \pi_{t+1}$$

If the increase in i_t is larger than the increase in $E_t \pi_{t+1}$ it ensures that the real rate increases.
The rule helps anchor inflation expectations by enforcing a response in the real rate. A higher real ~~and~~ interest rate reduces the output gap and brings inflation down.