Lab Report #2: Equity Returns & Certainty Equivalents

Revised: September 23, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Use Matlab where possible and attach your code to your answer.

Solution: Brief answers follow, but see also the attached Matlab program; download the pdf, open, click on pushpin:

1. *Properties of US equity returns.* Gene Fama is the 2013 recipient of the Nobel Prize in economics and one of the giants of modern finance. His focus has largely been on asset returns — why some assets have higher returns, on average, than others. His long-time coauthor Ken French kindly posts the "Fama-French datasets" on his his website. I've attached some of their data to this document in spreadsheet form (download the pdf, open in the Adobe Reader or the equivalent, click on pushpin):

   The variables are

   - Column 1: year
   - Column 2: excess return on equity (return on a broad-based portfolio of stocks minus the riskfree rate)
   - Column 3: one-year riskfree rate of return

   Returns are expressed as percentages.

   Your first task is to read the data into Matlab. Once you've done that:

   (a) Use the command `hist` to plot a histogram for the excess return.

   (b) Compute the mean, standard deviation, skewness, and excess kurtosis for the excess return series.

   (c) In what respects do excess returns look normal? In what respects not?

   (d) Speculate about why the mean excess return is positive.

Solution:

(a) See the Matlab link.

(b) The numbers are

Series	Mean	Std Dev	Skewness	Ex Kurtosis
Excess return	8.527	20.50	–0.33	–0.11
Riskfree rate	3.505	3.13	0.98	0.86

Note that you have to subtract 3 to get excess kurtosis from the kurtosis command in Matlab.

(c) The histogram looks very roughly normal, in the sense that the distribution is peaked in the middle and sort of symmetric. It's hard to tell whether the differences are noise or something more systematic. Skewness and excess kurtosis aren't wildly different from zero, which would be another indication of non-normality.

This is for annual data. With monthly data, there's greater negative skewness and positive excess kurtosis.

(d) Hmmm... Maybe reward for investing in risk? Note the standard deviation of the excess return: it's much larger than the mean.

2. *Concave and convex functions.* Consider the following functions defined over positive values of x:

(a) $f(x) = \log x$

(b) $f(x) = x^{\alpha}$ for $\alpha = 2$

(c) $f(x) = x^{1-\alpha}/(1-\alpha)$ for $\alpha = 2$

For each one:

- State whether it's concave, convex, or something else.
- Given your answer to (a), what does Jensen's inequality say about the relative magnitudes of $E[f(x)]$ and $f[E(x)]$?
- Verify your answer to (b) by computing $E[f(x)]$ and $f[E(x)]$ for the Bernoulli random variable

$$x = \begin{cases} 100 & \text{with probability } 1/2 \\ 200 & \text{with probability } 1/2. \end{cases}$$

Solution: This is a check on Jensen's inequality.

- (a) Concave, (b) convex, (c) concave.
- If concave $E[f(x)] < f[E(x)]$. If convex the reverse.
- Summary:

	(a)	(b)	(c)
$E[f(x)]$	4.95	25,000	–0.0075
$f[E(x)]$	5.01	22,500	–0.0067

3. *Certainty equivalents with power utility.* Consider an agent with power utility,

$$u(c) = c^{1-\alpha}/(1-\alpha),$$

facing Bernoulli consumption risk,

$$c = \begin{cases} a & \text{with probability } 1-\omega \\ a+b & \text{with probability } \omega. \end{cases}$$

Here a and b are both positive, ω is between zero and one, and $\alpha = 5$.

(a) If $\omega = 0.2$, what values of (a, b) give c a mean of 150 and a standard deviation of 50?

(b) With these numbers, what is the agent's expected utility?

(c) What is the agent's certainty equivalent? Risk penalty?

(d) How do your answers change if $\omega = 0.8$?

(e) Why do the certainty equivalents differ between the two cases?

Solution:

(a) The mean and variance of c are

$$\begin{aligned} E(c) &= a + \omega b \\ \text{Var}(c) &= (1-\omega)\omega b^2. \end{aligned}$$

You can find this directly from the definitions, or through the cgf. If we set the mean equal to 150 and the variance equal to 50^2, that gives us $a = b = 125$.

(b) I get expected utility of $-8.3200e - 10$. Why such a small number? Because we're taking large numbers to the power -4.

(c) The certainty equivalent is in consumption units and is better behaved. In this case we get $\mu = 131.66$. The risk penalty is $rp = \log(\bar{c}/\mu) = 0.1304$.

(d) If we switch to $\omega = 0.8$, that calls for $a = 50$ and $b = 125$. The certainty equivalent is $\mu = 74.28$ and the risk penalty is $rp = \log(\bar{c}/\mu) = 0.7028$.

The risk penalty is a lot bigger than we saw before. Since the mean and variance haven't changed, this must be the result of higher-order cumulants.