Professor Mark Gertler
Intermediate Macroeconomic Theory
Spring 2011

# Problem Set 4

Answer each of the following two problems:

## Problem 1

Consider a competitive equilibrium economy with households that have an infinite horizon. In addition, households suppy labor endogenously.

Suppose there is a representative family with the following objective.

$$\max_{N_t, C_t} \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i})]$$

where $C_{t+i}$ is consumption.with $0 < \beta < 1$. The household supplies labor exogenously.

The budget constraint is given by

$$C_t = W_t N_t + R_t K_t - K_{t+1}$$

The representative firm hires labor and rents capital from households in order to produce output. It's optimization problem each period is given by

$$\max Y_t - W_t N_t - (R_t - 1 + \delta) K_t$$

subject to

$$Y_t = K_t^{\alpha} (A N_t)^{1-\alpha}$$

where $R_t - 1 + \delta$ is the user cost of capital (net interest $R_t - 1$ plus depreciation $\delta$).

**Law of Motion for Capital:**

$$K_{t+1} = (1 - \delta) K_t + I_t$$

**Resource Constraint**

$$Y_t = C_t + I_t$$

**Evolution of Productivity**

$$\frac{A_{t+1}}{A_t} = 1 + a$$

This size of the labor force is fixed, i.e.

$$N_t = N$$

## Questions

1. Derive the household's first order condition for consumption and saving.
2. Derive the firm's first order conditions for labor and capital demand.
3. Present the set of equations that determine the balanced growth path equilibrium values of: $\frac{Y}{A}, \frac{C}{A}, \frac{I}{A}, \frac{K}{A}$. (Note we are not deflating these values by $N$ since $N$ is not growing.)
4. Now suppose the household choose labor supply endogenously. The households objective is now given by:

$$\max_{N_t, C_t} \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i}) - \nu N_{t+i}]$$

   Derive the household's first order condition for labor supply $N_t$. (Hint: the labor supply decision is a static period-by-period decision, i.e., you can solve for the optimal choice of $N_t$ ignoring the future.) Then present the set of equations that determine the balanced growth path equilibrium values of $\frac{Y}{A}, \frac{C}{A}, \frac{I}{A}, \frac{K}{A}, N$.

5. Suppose there are two economies that are identical, except that in economy 1, productivity is a multiple of productivity in country 2, with $A_t^1 = \theta A_t^2$., with $\theta > 1$. Suppose the two economies are along balanced growth paths. Then at any time $t$, how do $Y_t^1, C_t^1$ and $N_t^1$ compare with $Y_t^2, C_t^2$ and $N_t^2$.? (Hint: you should be able to answer this questions mostly by inspecting your answer to the previous question.)

## Problem 2

Consider a competitive equilibrium economy with households that have an infinite horizon. In addition, households suppy labor endogenously.

Suppose there is a representative family with the following objective.

$$\max_{N_t, C_t} \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i})]$$

where $C_{t+i}$ is consumption.with $0 < \beta < 1$. The household supplies labor exogenously.

The budget constraint is given by

$$C_t = W_t N_t + R_t K_t - K_{t+1}$$

The representative firm hires labor and rents capital from households in order to produce output. It's optimization problem each period is given by

$$\max Y_t - W_t N_t - (R_t - 1 + \delta) K_t$$

subject to

$$Y_t = K_t^{\alpha} (A N_t)^{1-\alpha}$$

where $R_t - 1 + \delta$ is the user cost of capital (net interest $R_t - 1$ plus depreciation $\delta$).

**Law of Motion for Capital:**

$$K_{t+1} = (1 - \delta) K_t + I_t$$

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## Questions

1. Derive the household's first order condition for consumption and saving.

2. Derive the firm's first order conditions for labor and capital demand.

3. Present the set of equations that determine the balanced growth path equilibrium values of: $\frac{Y}{A}$, $\frac{C}{A}$, $\frac{I}{A}$, $\frac{K}{A}$. (Note we are not deflating these values by $N$ since $N$ is not growing.)

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Derive the household's first order condition for labor supply $N_t$. (Hint: the labor supply decision is a static period-by-period decision, i.e., you can solve for the optimal choice of $N_t$ ignoring the future.) Then present the set of equations that determine the balanced growth path equilibrium values of $\frac{Y}{A}$, $\frac{C}{A}$, $\frac{I}{A}$, $\frac{K}{A}$, $N$.

5. Suppose there are two economies that are identical, except that in economy 1, productivity is a multiple of productivity in country 2, with $A_t^1 = \theta A_t^2$., with $\theta > 1$. Suppose the two economies are along balanced growth paths. Then at any time $t$, how do $Y_t^1, C_t^1$ and $N_t^1$ compare with $Y_t^2, C_t^2$ and $N_t^2$.? (Hint: you should be able to answer this questions mostly by inspecting your answer to the previous question.)