Lab Report #5: Returns & Risk Premiums

Revised: October 20, 2014

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *Disaster risk and the equity premium.* We add a "disaster" state to our analysis of the equity premium and see how it changes our perspective. We know power utility agents dislike negative skewness, so we conjecture that an asset with a disaster outcome will have a large risk premium. The key input is the distribution of log consumption growth,

$$\log g \;=\; \begin{cases} \mu - \sigma & \text{with probability } (1-\omega)/2 \\ \mu + \sigma & \text{with probability } (1-\omega)/2 \\ \mu - \theta & \text{with probability } \omega. \end{cases}$$

   If $\omega = 0$, we're back to our symmetric two-state distribution. But if we choose a small positive value of ω and a "largish" $\theta > 0$, we have a "disaster" state that changes the distribution dramatically.

   We'll define equity as a claim to consumption growth g and explore the impact of a disaster state on the equity premium. Suppose, as usual, that we have a representative agent with power utility. Preference parameters are $\beta = 0.99$ and $\alpha = 10$ throughout.

   (a) Verify that the mean and variance of log consumption growth are

$$\begin{aligned} E(\log g) &= \mu - \omega\theta \\ \mathrm{Var}(\log g) &= (1-\omega)(\sigma^2 + \omega\theta^2). \end{aligned}$$

   (b) If $\omega = 0$, what values of μ and σ deliver the observed mean and variance of log consumption growth, namely 0.0200 and 0.0350^2?

   (c) With these parameter values, compute the equity premium and its inputs.

   (d) Now consider $\omega = 0.01$ and $\theta = 0.30$. (These numbers are based on a series of studies by Robert Barro and his coauthors.) With these numbers, what values of μ and σ reproduce the observed mean and variance of log consumption growth?

   (e) Compute the equity premium and its components with these parameter values. How does it compare to your previous calculation?

   (f) What are the maximum Sharpe ratios when $\omega = 0$ and when $\omega = 0.01$? How do they compare to the Sharpe ratio we observe for equity in US data?

   (g) What is entropy when $\omega = 0$ and when $\omega = 0.01$?

   (h) Summarize what you've found. How does a disaster state affect the magnitude of risk premiums in this economy?

2. *Valuing put options* . We can value options the same way we value any other asset, it's simply a matter of getting the cash flows right. With future topics in mind, we use a discrete approximation to a normal mixture, which we use to produce departures from normality: skewness, excess kurtosis, and so on. (No, that's not an oxymoron; see the notes on random variables.)

   To be specific, we'll use two normal distributions as inputs. The first one has mean zero and variance one, the second has mean θ and variance δ. We denote their pdf's by p_1 and p_2, respectively. The pdf of our state variable z is

   $$p(z) \quad = \quad (1-\omega)p_1(z) + \omega p_2(z)$$

   for some mixing parameter ω between zero and one. Roughly speaking, θ governs skewness and δ governs excess kurtosis. We'll set $\omega = 0.01$, $\theta = -1$, and $\delta = 2$. We construct a discrete approximation with the Matlab commands

```
% grid
zmax = 6;
dz = 0.1;
z = [-zmax:dz:zmax]';
% mixture
omega = 0.2; theta = -1; delta = 2;
p1 = exp(-z.^2/2)*dz/sqrt(2*pi);
p2 = exp(-(z-theta).^2/(2*delta))*dz/sqrt(2*pi*delta);
p  = (1-omega)*p1 + omega*p2;
```

   We'll use the usual representative agent setup. Log consumption growth is

   $$\log g(z) \quad = \quad \mu + \sigma z.$$

   The mean and variance are then

   $$\begin{aligned} E(\log g) &= \mu + \omega\theta \\ \mathrm{Var}(\log g) &= \sigma^2[(1-\omega) + \omega(\theta^2 + \delta)]. \end{aligned}$$

   With power utility, the pricing kernel is

   $$m(z) \quad = \quad \beta \exp(-\alpha \log g) \quad = \quad \beta \exp[-\alpha(\mu + \sigma z)].$$

   We'll set $\beta = 0.99$ and $\alpha = 1$.

   (a) Plot p (the mixture) and p_1 (the standard normal). How do they differ?

   (b) Use the commands

```
meanz = sum(p.*z)
stdz  = sqrt(sum(p.*(z-meanz).^2))
skewz = sum(p.*(z-meanz).^3)/stdz^3
xkurz = sum(p.*(z-meanz).^4)/stdz^4 - 3
```

   to compute properties of the mixture distribution. What indications do they give us that the distribution is not normal? What are the skewness and excess kurtosis of $\log g$?

(c) Choose values of μ and σ to reproduce the mean (0.0200) and standard deviation (0.0350) of $\log g$.

(d) Define equity as a claim to the dividend $d(z) = 100\ g(z)$. What is its price? Its expected return?

(e) A put option with "strike price" k gives the owner the right to sell the dividend $d(z)$ at date 1 for price k. The owner of the option will only do this if $d(z) < k$, which gives rise to the cash flow

$$f(z) \quad = \quad \max\{0, k - d(z)\}.$$

You can compute this in Matlab with the commands

```
k = 125;
f = (k-d).*(k>d);
```

(Ask yourself what `(k>d)` does. Try it if you're not sure.) Graph the cash flow $f(z)$ versus the dividend $d(z)$ for $k = 125$.

(f) What is the price (at date 0) of the option? (Note: This is a claim to the cash flow f and priced as we would any other cash flow.) What is its expected return?