Lab Report #8: Bond Prices & Predictable Returns

Revised: December 2, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code, if any.

Maturity n	Price q^n
1 year	0.9900
2 years	0.9800
3 years	0.9600
4 years	0.9300
5 years	0.8900

Table 1. Bond prices at various maturities. Here q^n is the price now of a claim to one dollar in n years.

1. *Equity prices and dividends.* Suppose the ex-dividend price of equity is

$$q_t = \delta E_t \left(d_{t+1} + q_{t+1} \right) \tag{1}$$

with discount factor $0 < \delta < 1$.

   (a) Express the price as a function of expected future dividends.

   (b) Suppose dividends follow

$$d_{t+1} = (1 - \varphi)\mu + \varphi d_t + \sigma(w_{t+1} + \theta w_t),$$

   where $\{w_t\}$ is a sequence of independent standard normal random variables. What definition of the state is enough to describe the conditional distribution of d_{t+1} at date t?

   (c) How is the price q_t related to the state?

   (d) Optional, extra credit. What are the variances of q and d? How do they relate to Shiller's observation that prices are more variable than dividends?

2. *Bond basics.* Consider the bond prices in Table 1.

   (a) What are the yields y^n?

   (b) What are the forward rates f^{n-1}?

   (c) How are the yields and forward rates related? Verify for y^3.

3. *Bond prices with a moving average pricing kernel.* Consider the bond pricing model

$$\log m_{t+1} = -\lambda^2/2 + \lambda w_{t+1} + \sigma w_t.$$

(a) What kind of process is $\log m$?

(b) What is the short rate f_t^0?

(c) Suppose bond prices take the form

$$\log q_t^n \quad = \quad A_n + B_n w_t.$$

Use the pricing relation to derive recursions connecting (A_{n+1}, B_{n+1}) to (A_n, B_n). What are (A_n, B_n) for $n = 0, 1, 2, 3$?

(d) Express forward rates as functions of the state w_t. What are f_t^1 and f_t^2?

(e) What is $E(f^1 - f^0)$? What parameters govern its sign?