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Lecture 9

Extensions to the Baseline Model: Fiscal Policy and Investment

1 Fiscal Policy

We now introduce government spending, which is financed by lump sume taxes.
Now suppose that output is divided between consumption and government expenditures:

$$Y_t = C_t + G_t$$

with G_t financed by lump sum taxes T_t :

$$G_t = T_t$$

Suppose further that capital markets are perfect. Thus, Ricardian equivalence holds (i.e , the timing of taxes doesn't matter to consumption spending, only the present value.) We take G_t as exogenous.
Appending fiscal policy to the loglinear fixed price model of Lecture 8 yields the following system of seven equations in seven endogenous variables:

aggregate demand.

$$y_t = \frac{C}{Y}c_t + \frac{G}{Y}g_t$$

$$c_t = -i_t + E_t c_{t+1} + \chi_t$$

aggregate supply

$$y_t = a_t + n_t$$

$$a_t = \mu_t + \gamma_n n_t + c_t$$

$$p_t = \overline{p}$$

monetary sector

$$m_t - \bar{p} = y_t - \nu i_t + \varepsilon_t$$

$$i_t = \bar{i}_t$$

where we are assuming that the central bank uses the nominal interest rate as the instrument of monetary policy. As before, χ_t and ε_t are exogenous random distrubances.

Also as before, the system can be reduced to IS/LM and AS curves. (As before, to derive the IS curve, combine the consumption equaton with the resource constraint.)

IS curve

$$y_t = -\frac{C}{Y} i_t + E_t y_{t+1} + \frac{C}{Y} \chi_t + \frac{G}{Y}[g_t - E_t g_{t+1}]$$

LM curve

$$i_t = \bar{i}_t$$

AS curve

$$\mu_t = -(1+\gamma_n)(y_t - y_t^*)$$

where y_t^* is the flexible price equilibrium value of output. As in lecture 8, to derive the specific form of the aggregate supply curve (that relates the markup inversely to the output gap), we make use of the fact that in the flexible price equilibrium $\mu_t = 0$.

Fiscal policy enters via the IS curve. Holding constant the interest rate, a temporary increase in government spending raises output demand. Note that an expected increase in future government spending reduces current demand. An expected future increase in government spending induces households to save instead of consume in order to be able to pay for for the rise in future taxes.

An expression for y_t^* comes from the following three equations

$$y_t^* = \frac{C}{Y} c_t^* + \frac{G}{Y} g_t$$

$$y_t^* = a_t + n_t^*$$

$$a_t = \gamma_n n_t^* + c_t^*$$

Combining yields

$$y_t^* = \frac{1+\gamma_n}{\frac{Y}{C} + \gamma_n} a_t + \frac{G/C}{\frac{Y}{C} + \gamma_n} g_t$$

The positive effect of g_t on y_t^* reflects a "wealth effect" on labor supply : With higher g_t, household's consume less, holding constant labor supply. Since marginal utility is thus higher, household's are willing to work more. Note that with $G = 0$, $y_t^* = a_t$, as in the model without fiscal policy (see lecture 8).

We can the expression for the flexible price equilibrium real interest rate, r_t^*, by also making use of the consumption/saving relation:

$$r_t^* = \frac{Y}{C}[E_t y_{t+1}^* - y_t^*] + \frac{G}{C}[g_t - E_t g_{t+1}] + \chi_t$$

the fiscal "multplier" in the baseline model.

Holding constant i_t, the effect of a temporary increase in g_t on y_t is:

$$dy_t = \frac{G}{Y} dg_t$$

Since $dy_t = \log y_t - \log y \approx \frac{dY_t}{Y}$ and $dg_t = \log g_t - \log g \approx \frac{dG_t}{G}$:

$$\frac{dY_t}{Y} = \frac{G}{Y}\frac{dG_t}{G}$$

or

$$dY_t = dG_t$$

That is, the multiplier effect of a temporary increase in government expenditures is unity. Note that a persistent effect will have an even weaker effect (i.e., if $E_t g_{t+1}$ rises at least part of the way with g_t). Future increases in G induce households to save instead of consume, in anticipation of higher taxes (as we discussed much earlier.)

The recent Obama fiscal stimulus plan is based on a multiplier in the range of 1.5 to 2.0 How can one rationalize this? Borrowing constraints are key.

1.1 Fiscal Policy with Liquidity-Constrained Consumers

Let c_t^u be consumption (in log deviations from the steady-state) by unconstrained consumers and c_t^c consumption by constrained consumers: Then

$$c_t = (1-\nu)c_t^u + \nu c_t^c$$

where ν is the fraction of constrained consumers.

Unconstrained consumers obey the conventional consumption/saving relation:

$$c_t^u = -i_t + E_t c_{t+1}^u$$

Constrained consumers just consume their disposable income:

$$\upsilon C c_t^c = \upsilon Y y_t - \upsilon T \tau_t$$

where υC is steady state consumption by constrained consumers, υY is steady state income and υT is steady state taxes.
We can rearrange to obtain

$$c_t^c = \frac{Y}{C}y_t - \frac{T}{C}\tau_t$$

We can also iterate to obtain the following expression for c_t^u :

$$c_t^u = -\sum_{i=0}^{\infty} i_{t+i}$$

We can thus derive the IS curve as follows:

$$y_t = \frac{C}{Y}((1-\nu)c_t^u + \nu c_t^c) + \frac{G}{Y}g_t$$

which implies

$$y_t = \frac{C}{Y}[(1-\nu)(-\sum_{i=0}^{\infty} i_{t+i}) + \nu(\frac{Y}{C}y_t - \frac{T}{C}\tau_t)] + \frac{G}{Y}g_t$$

Rearranging yields the following IS curve:

$$y_t = \frac{1}{1-\upsilon}\{\frac{C}{Y}[(1-\nu)(-\sum_{i=0}^{\infty} i_{t+i})\} + \frac{1}{1-\upsilon}\{\frac{G}{Y}g_t - \nu\frac{T}{Y}\tau_t\}$$

Thus:

$$dy_t = \frac{1}{1-\upsilon}\frac{G}{Y}dg_t$$

which implies:

$$\frac{dY_t}{Y} = \frac{1}{1-\upsilon}\frac{G}{Y}\frac{dG_t}{G}$$

which leads to:

$$\frac{dY_t}{dG_t} = \frac{1}{1-\upsilon} > 1$$

where $\frac{1}{1-\upsilon}$ is the multipler.

The evidence suggests assuming $\nu = .4$, implying $\frac{1}{1-\upsilon} \sim 1.7$. The Obama administration economists use this kind of scenario to rationalize a government expenditure mulitplier in the range of 1.5 to 2.0.

By similar reasoning the tax multpilier is given by:

$$\frac{dY_t}{dT_t} = -\frac{\upsilon}{1-\upsilon}$$

The tax multiplier is $-\frac{\upsilon}{1-\upsilon}$,implying that tax reductions have a positive impact on spending. It is smaller than the government expenditure multiplier since tax changes affect spending indirectly, through their impact on consumption spending (by contsrained households). In our example, a dollar of tax cuts leads to about 66 cents worth of spending (.4/.6).

Note that the tax cut would have a bigger effect if it was targted toward constrained households. Indeed, if the tax cut was given only to constrained households the multiplier would be the same absolute value as the government expenditure multiplier, since each dollar of the tax cut would be directly spent. Indeed, this was the reason for including an extension of unemployment benefits in the fiscal stimulus package (since unemployed individuals were likely constrained.)

Finally, note that in this model it is possible to stimulate demand either through monetary policy (reducing i_t) or by fiscal policy (raising g_t or reducing τ_t). However, once the nominal rate reaches the zero lower bound, fiscal policy is the only option. (Later we will consider "financial" policy.)

2 Investment

We next add investment. The resource constraint becomes:

$$Y_t = C_t + I_t + G_t$$

In loglinear form:

$$y_t = \frac{C}{Y}c_t + \frac{I}{Y}inv_t + \frac{G}{Y}g_t$$

Investment:

Competitive capital producers make new investment goods to sell at the price Q_t. Each investment good costs one unit of output to make plus adjustment costs. A capital producer's profits are given by

$$Q_t I_t - I_t - \frac{1}{2}b(I_t - z)^2$$

where $Q_t I_t$ are revenues from the sale of I_t new capital goods,and $I_t + \frac{1}{2}b(I_t - z)^2$ is the cost of producing them, where the latter term is adjustment costs.. z is the "steady state" level of investment, so adjustment costs are in terms of deviations from the steady state.

The first necessary conditions yield a relation between Q_t and I_t.

$$Q_t - 1 - b(I_t - z) = 0$$

Rearranging

$$I_t = z + \frac{1}{b}(Q_t - 1)$$

I_t is thus increasing in Q_t. How much depends inversely on the adjustment cost parameter. Note also that in steady state, where $I_t = z$, Q_t equals one.

Q:.

We next obtain a relation for Q_t.

Let D_t be the dividend on equity claim to a unit of capital. Then the return from ownership of a unit of capital from t to $t+1$, R^k_{t+1}, is given by

$$R^k_{t+1} = \frac{D_{t+1} + Q_{t+1}}{Q_t}$$

Let $\Psi_t > 1$ reflect the required excess return to capital, which we take as given. (In general, the excess return depends on risk factors and the degree of frictions in capital markets). Then by aribitrage, investors must be indifferent between holding capital and a nominal bond:

$$E_t R^k_{t+1} = \Psi_t E_t(1 + i_t)\frac{P_t}{P_{t+1}}$$

In the fix price equilibrium accordingly,

$$E_t R^k_{t+1} = \Psi_t(1 + i_t)$$

In steady state, further,

$$R^k = \Psi(1 + i)$$

Combining equations yields a relation between $(1 + i_t)$ and Q_t.

$$E_t \frac{D_{t+1} + Q_{t+1}}{Q_t} = \Psi_t(1 + i_t)$$

Rearranging:

$$Q_t = E_t \frac{D_{t+1} + Q_{t+1}}{\Psi_t(1 + i_t)}$$

Iterating forward:

$$Q_t = E_t \sum_{i=0}^{\infty} D_{t+1+i} / \Pi_{j=0}^{i} (\Psi_{t+j}(1 + i_{t+j}))$$

We can obtain to following loglinear equaton for q_t from the relation $Q_t = E_t \frac{D_{t+1}+Q_{t+1}}{\Psi_t(1+i_t)}$:

$$q_t = E_t\{\frac{R^k - 1}{R^k} d_{t+1} + \frac{1}{R_k} q_{t+1} - \frac{(1+i)}{R_k}[i_t + \psi_t]\}$$

where $\psi_t = \log \Psi_t - \log \Psi$.
Iterating forward

$$q_t = E_t \sum_{i=0}^{\infty} (\frac{1}{R_k})^i \{[\frac{R_k - 1}{R_k} d_{t+1+i} - \frac{(1+i)}{R_k}[i_{t+i} + \psi_{t+i}]\}$$

q_t thus depends positively on the future dividend stream and negatively on the path of short term interest rates and the required excess return.

Monetary policy accordingly affects investment in the following way: An increase in i_t reduces q_t, and thus reduces investment. The strength of the effect depends on the persisitence of the interest rate change.

To close the model, we assume dividends are proportionate to output

$$D_t = \alpha Y_t$$

We can now construct the complete loglinear model

Aggregate Demand

$$y_t = \frac{C}{Y} c_t + \frac{I}{Y} inv_t + \frac{G}{Y} g_t$$

$$c_t = -r_t^n + E_t c_{t+1} + \chi_t$$

$$i_t = \frac{1}{zb} q_t$$

$$q_t = E_t \sum_{i=0}^{\infty} (\frac{1}{R^k})^i [\frac{R^k - 1}{R^k} y_{t+1+i} - \frac{(1+i)}{R^k}(i_{t+i} + \psi_{t+i})]$$

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Aggregate Supply

$$y_t = a_t + n_t$$

$$a_t = \mu_t + \gamma_n n_t + c_t$$

$$p_t = \overline{p}$$

Monetary Sector

$$m_t - \bar{p} = y_t - \nu i_t + \varepsilon_t$$

$$i_t = \bar{i}_t$$

Note that we are abstracting from the effect of capital on production. This can be justified by the fact the percent changes in the capital stock over the near term are quite small. We are also assuming that there no diminish returns from increasing employment in the short run. This can be justified if in the short run firms are able to vary the intensity they use the capital stock along with labor (e.g. by increasing the number of shifts in a plant.)

We can then collapse the model to:

IS

$$y_t = \frac{C}{Y} \cdot E_t \sum_{i=0}^{\infty} -(i_{t+i} + \chi_{t+i}) + \frac{I}{Y}\frac{1}{zb}q_t + \frac{G}{Y}g_t$$

with

$$q_t = E_t \sum_{i=0}^{\infty} (\frac{1}{R^k})^i [\frac{R^k - 1}{R^k}\alpha y_{t+1+i} - \frac{(1+i)}{R^k}(i_{t+i} + \psi_{t+i})]$$

LM

$$i_t = \bar{i}_t$$

AS

$$\mu_t = -(1-\gamma_n)(y_t - y_t^*)$$

Note that the addition of investment makes the solution for y_t^* a bit more complicated, but for our purposes there is no harm in ignoring this complication.

Overall, however, the IS/LM/AS structure remains similar to before. Interest rates, though, now affect investment as well as consumption. In addition, "excess return" shocks may now affect investment and output.