## Problem 1

(1) IS curve:

Fixed price ($p_t = \bar{p}$): $y_t = -i_t + \mathbb{E}_t\, y_{t+1} + x_t$

Flex price ($\mu_t = 0$): $y_t^* = \underbrace{-i_t + \mathbb{E}_t\, p_{t+1} - p_t}_{= -r_t^*} + \mathbb{E}_t\, y_{t+1}^* + x_t$

LM curve:

Fixed price ($p_t = \bar{p}$): $\overline{m}_t - \bar{p} = y_t - \nu r_t^n = y_t - \nu i_t$

Flex price ($\mu_t = 0$): $\overline{m}_t - p_t = y_t^* - \nu r_t^n = y_t^* - \nu i_t$

AS curve:

Fixed price ($p_t = \bar{p}$): $0 = \mu_t + (1+\gamma_n)(y_t - a_t)$

Flex price ($\mu_t = 0$): $y_t^* = a_t$

② Fixed price model

1) $y_t = -i_t + E_t y_{t+1} + x_t$

2) $\bar{m}_t - \bar{p} = y_t - \nu i_t$

3) $\mu_t = -(1+\gamma_n)(y_t - a_t)$

$$2) \Rightarrow \quad -i_t = \left(\bar{m}_t - \bar{p} - y_t\right) \cdot \frac{1}{\nu}$$

$$\text{into } 1) \Rightarrow \quad y_t = \frac{1}{\nu}\left(\bar{m}_t - \bar{p}\right) - \frac{1}{\nu} y_t + E_t y_{t+1} + x_t$$

$$\Rightarrow \quad y_t = \frac{1}{1+\nu}\left(\bar{m}_t - \bar{p}\right) + \frac{\nu}{1+\nu} E_t y_{t+1} + \frac{\nu}{1+\nu} \cdot x_t$$

Substitute in for $y_{t+1}$ assuming future demand shocks are 0 (only consider drop in $x_t$ and set $x_{t+\tau} = 0$ for $\tau \geq 1$).

$$\Rightarrow \quad y_t = \frac{1}{1+\nu}\left(\bar{m}_t - \bar{p}\right) + \frac{\nu}{1+\nu} E_t\left(\frac{1}{1+\nu}\left(\bar{m}_{t+1} - \bar{p}\right) + \frac{\nu}{1+\nu} E_{t+1} y_{t+2}\right) + \frac{\nu}{1+\nu} x_t$$

$$= \frac{1}{1+\nu}\left(\bar{m}_t - \bar{p}\right) + \frac{1}{1+\nu} E_t\left(\frac{\nu}{1+\nu}\right)\left(\bar{m}_{t+1} - \bar{p}\right) + \left(\frac{\nu}{1+\nu}\right)^2 E_t y_{t+2} + \frac{\nu}{1+\nu} x_t$$

$$\Rightarrow \quad \boxed{y_t = \frac{1}{1+\nu} \cdot \sum_{\tau=0}^{\infty} \left(\frac{\nu}{1+\nu}\right)^{\tau} E_t\left(\bar{m}_t - \bar{p}\right) + \frac{\nu}{1+\nu} \cdot x_t}$$

Fixed price model

We see that with a fixed monetary policy, a drop in demand ($x_t \downarrow$) reduces output in the fixed price model.

Employment : $y_t = a_t + n_t$

$\Rightarrow$ a drop in $y_t$ must imply a drop in employment $n_t$:
Reduced demand with no response in monetary policy or prices leads firms to reduce supply and hence reduce employment.

Mark-up : $\mu_t = -(1+\gamma_n)(y_t - a_t)$

$\Rightarrow$ a drop in output increases the mark-up $\mu_t$.
Equilibrium in the labor market lowers the firm's marginal cost as the real wage must drop. With fixed prices the firm's mark-up over marginal cost increases.

Interest rate : $y_t = \overline{m}_t - \overline{p} + \nu i_t$

$\Rightarrow$ a drop in output reduces the interest rate.
With fixed prices this reduces the real interest rate (since monetary policy is fixed).

Flex price model:

1) $$y_t^* = \overbrace{-i_t + E_t p_{t+1} - p_t}^{-r_t^*} + E_t y_{t+1}^* + \chi_t$$

2) $$\overline{m}_t - p_t = y_t^* - \nu i_t$$

3) $$y_t^* = a_t$$

Mark-up: $\mu_t = 0$

In the flex price model prices are free to adjust, so firms always set the mark-up equal to its optimal level. Hence, there is no change in the mark-up.

Output: $y_t^* = a_t$

In the flex price model output is determined by fluctuations in total factor productivity, so there is no change in output.

Employment: $y_t^* = a_t + n_t^*$

No change in employment.

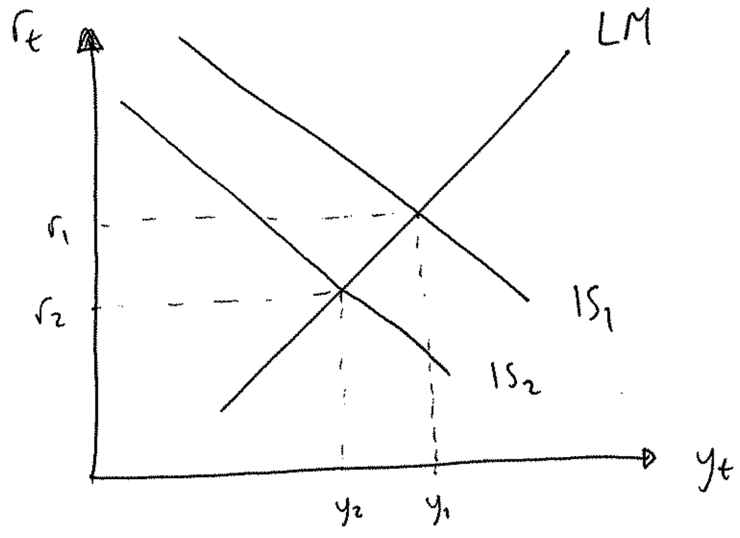
Interest rate: $$i_t = r_t^* + E_t p_{t+1} - p_t$$

The downward shift in the IS curve leads to a reduction in the real interest rate.

$$\cancel{\overline{m}_t - p_t = y_t^* - \nu r_t^* - \nu(E_t p_{t+1} - p_t)}$$

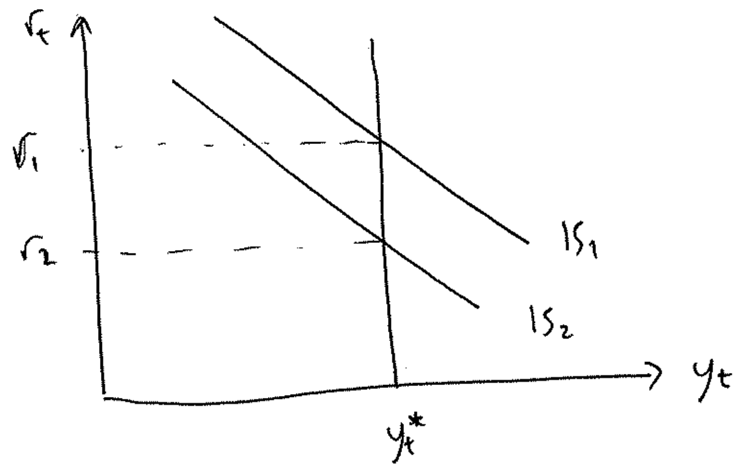
~~Prices are flexible~~ $y_t^* = -r_t^* + E_t y_{t+1}^* + \chi_t$

## Graphically




Fixed price model:

$X_t \downarrow \Rightarrow$ IS shift down and left $\Rightarrow$ output and employment drops, interest rate drops, mark-up increases




Flex price model:

$X_t \downarrow \Rightarrow$ IS shifts down and to the left $\Rightarrow$ prices adjust immediately, and there is no change in the mark-up ($\mu_t = 0$). $\Rightarrow$ no change in output and employment

$\Rightarrow$ interest rate drops since reduced demand reduces the demand for money (so the price of money drops).

# Problem 2

① Fixed price IS curve:

$$y_t = x_t - i_t + \mathbb{E}_t\, y_{t+1}$$
$$= x_t - i_t + \mathbb{E}_t\left(x_{t+1} - i_{t+1} + \mathbb{E}_{t+1} y_{t+2}\right)$$
$$= x_t - i_t + \mathbb{E}_t\left(x_{t+1} - i_{t+1}\right) + \mathbb{E}_t\, y_{t+2}$$
$$= x_t - i_t + \mathbb{E}_t\left(x_{t+1} - i_{t+1}\right) + \mathbb{E}_t\left(x_{t+2} - i_{t+2} + \mathbb{E}_{t+2} y_{t+3}\right)$$

$$\Rightarrow \boxed{y_t = \sum_{\tau=0}^{\infty} \mathbb{E}_t\left(x_{t+\tau} - i_{t+\tau}\right)}$$

② Consider separately the shocks at $t$ and their expected future values:

$$y_t = x_t - i_t + \sum_{\tau=1}^{\infty} \mathbb{E}_t\left(x_{t+\tau} - i_{t+\tau}\right)$$

Now consider a large drop in $x_t$: $x_t \to x_t - \Delta$

~~An increase~~ A decrease in $i_t$ by $\Delta$ would leave output unchanged.

But if $\Delta$ is too large $i_t$ cannot drop enough.

$i_t = \log(1+\hat{i}_t) - \log(1+\hat{i})$ (see notes 8, page 6)

The nominal rate $\hat{i}_t \geq 0$ (zero lower bound), so the percentage deviation from steady state $i_t \geq -\log(1+\hat{i})$.

(3) Say that lowering interest rates each period by $\Delta/10$ does not violate the zero lower bound. Then if the central bank commits to lowering the interest rates by this amount for 10 periods we have

$$y_t = (x_t - \Delta) - (i_t - \Delta/10) + \sum_{\tau=1}^{9} E_t\left(x_{t+\tau} - (i_{t+\tau} - \Delta/10)\right) + \sum_{\tau=10}^{\infty} E_t\left(x_{t+\tau} - i_{t+\tau}\right)$$

Since $-\Delta + \frac{\Delta}{10} + 9 \cdot \frac{\Delta}{10} = 0$, the central bank can (in principle) stimulate output by creating the expectation that future nominal rates will be lower as well.

## Problem 3

(1) • Following the hint:

$$c_t^u = -i_t + E_t c_{t+1}^u$$
$$= -i_t + E_t\left(-i_{t+1} + E_{t+1} c_{t+2}^u\right) = -i_t - E_t i_{t+1} + E_t c_{t+2}^u$$
$$= -i_t - E_t i_{t+1} + E_t\left(-i_{t+2} + E_{t+2} c_{t+3}^u\right)$$

$$\Rightarrow \quad \boxed{c_t^u = -\sum_{\tau=0}^{\infty} E_t i_{t+\tau}}$$

• Combining the first two equations:

$$y_t = \frac{C}{Y}\left((1-\nu) c_t^u + \nu c_t^c\right) + \frac{G}{Y}\cdot g_t$$

• Insert the two expressions for $c_t^u$ and $c_t^c$:

$$y_t = \frac{C}{Y}(1-\nu)\left(-\sum_{\tau=0}^{\infty} E_t i_{t+\tau}\right) + \frac{C}{Y}\cdot\nu\cdot\left\{\frac{Y}{C}\cdot y_t - \frac{T}{C}\tau_t + \frac{\theta}{\nu}\frac{TR}{C}\cdot \tau r_t\right\}$$
$$+ \frac{G}{Y}\cdot g_t$$

• Solve for $y_t$:

$$\cancel{y_t = -\frac{C}{Y}\sum_{\tau=0}^{\infty} E_t i_{t+\tau} - \frac{\nu T}{Y}\cdot\tau_t}$$

$$y_t = -\frac{C}{Y}\sum_{\tau=0}^{\infty} E_t i_{t+\tau} + \frac{1}{1-\nu}\left(-\frac{\nu T}{Y}\cdot\tau_t + \theta\frac{TR}{Y}\tau r_t + \frac{G}{Y}\cdot g_t\right)$$

→ IS curve.

(2) Multipliers:

- On $g_t$: $dy_t = \frac{1}{1-\nu} \cdot \frac{G}{Y} \cdot dg_t$

This expression holds for the variables in percentage deviations from steady state. Now for every variable we have

$$x_t = \log X_t - \log X = \log\left(1 + \frac{X_t - X}{X}\right) \approx \frac{X_t - X}{X}$$

Hence: $dx_t = \frac{dX_t}{X}$

Thus we get $\frac{dY_t}{Y} = \frac{1}{1-\nu} \frac{G}{Y} \cdot \frac{dG_t}{G}$

$$\Rightarrow \boxed{\frac{dY_t}{dG_t} = \frac{1}{1-\nu}} \Rightarrow \text{multiplier on gov. spending.}$$

- On $\tau_t$: $dy_t = -\frac{\nu}{1-\nu} \cdot \frac{T}{Y} \, d\tau_t$

$$\Rightarrow \frac{dY_t}{Y} = \frac{-\nu}{1-\nu} \cdot \frac{T}{Y} \cdot \frac{dT_t}{T}$$

$$\Rightarrow \boxed{\frac{dY_t}{dT_t} = \frac{-\nu}{1-\nu}} \Rightarrow \text{multiplier on taxes.}$$

- On $\tau r_t$: $dy_t = \frac{\theta}{1-\nu} \frac{TR}{Y} \cdot d\tau r_t$

$$\Rightarrow \frac{dY_t}{Y} = \frac{\theta}{1-\nu} \cdot \frac{TR}{Y} \cdot \frac{dTR_t}{TR}$$

$$\Rightarrow \boxed{\frac{dY_t}{dTR_t} = \frac{\theta}{1-\nu}} \Rightarrow \text{multiplier on transfers.}$$

Intuition:

We see that the parameters that determine the size of the multipliers are $\nu$ - the fraction of constrained consumers - and $\theta$ - the fraction of transfers that goes to constrained consumers.

If $\nu = 0$ (so $\theta = 0$ as well) then there are no constrained consumers and the multipliers on taxes and transfers are 0. Unconstrained consumers do not make consumption decisions based on the timing of taxes and transfers. The multiplier on gov. spending is 1.

If $\nu \neq 0$ then a fraction of consumers are constrained, and for them the timing of taxes and transfers matter. Changing taxes and/or transfers changes their disposable income which they consume immediately. This also increases the multiplier on gov. spending since gov. spending also increases the disposable income of constrained consumers.

3. To maximize the effect of increasing transfers, the government should set $\theta = 1$. Transfers should go to the constrained consumers to raise their disposable income, since they consume it rather than save it.

For $\theta = 1$ the multipliers on transfers and gov. spending are the same. In that case the effect of transfering money to constrained consumers (who spend it) or increasing gov. spending which also leads to an increase in the income of constrained consumers, has the same effect of stimulating the economy.