Quiz #2

(Revised: August 27, 2013)

Please write your name below. Then complete the exam in the space provided. There are THREE questions. You may refer to one page of notes: standard paper, both sides, any content you wish.

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(Name and signature)

1. (Sharpe ratios) (40 points) We'll look at Sharpe ratios in a two-period representative agent economy. Endowment growth g is Bernoulli:

$$g \;=\; \begin{cases} 1.00 & \text{with probability } 1-\omega \\ 1.10 & \text{with probability } \omega \end{cases}$$

with $\omega = 0.3$. The representative agent has power utility with discount factor $\beta = 0.98$ and risk aversion $\alpha = 5$. Equity is a claim to g.

   (a) What is the pricing kernel for this economy? What are the state prices? (10 points)

   (b) What are the price and return of a one-period riskfree bond? (10 points)

   (c) What is the price of equity? What are the mean and standard deviation of its excess return? What is its Sharpe ratio? (10 points)

   (d) What is the maximum Sharpe ratio for this economy? (10 points)

2. (entropy bound revisited) (30 points) The idea here is to derive the entropy bound from a maximization problem. We'll do this in an arbitrary two-period economy with a finite set of states. Each state z has probability $p(z)$ and pricing kernel $m(z)$. An asset has returns $r(z)$ that satisfy the pricing relation

$$\sum_z p(z)m(z)r(z) \;=\; 1. \tag{1}$$

Our mission is to characterize the asset with the highest expected log return,

$$\sum_z p(z)\log r(z).$$

We'll refer to this as the "high-return asset."

   (a) What is the entropy of the pricing kernel? Express it in terms of $m(z)$ and $p(z)$. (10 points)

   (b) Use Lagrangian methods to find the returns $r(z)$ (one number for each state) that maximize the expected log return while satisfying the pricing relation (1). How is the return on the high-return asset related to the pricing kernel? (10 points)

(c) Show that the high-return asset attains the entropy bound. (10 points)

3. (put and call options) (30 points) We'll look, once again, at the prices of one-year options when the risk-neutral distribution of the underlying is lognormal. If the future value of the underlying is s_{t+1}, then $\log s_{t+1} \sim \mathcal{N}(\kappa_1, \kappa_2)$. We've seen, in this case, that the price of a European put option with strike price k is

$$
\begin{aligned}
q_t^p &= q_t^1 k N(d) - q_t^1 e^{\kappa_1 + \kappa_2/2} N(d - \kappa_2^{1/2}) \\
d &= (\log k - \kappa_1)/\kappa_2^{1/2}.
\end{aligned}
$$

(a) What is the price of a call option with the same strike price? (15 points)

(b) What is the no-arbitrage condition for this environment? Use it to derive the BSM formula for call options from your answer to (a). (15 points)