

Economics V31.036

Math quiz

1. Assume that $Y = AK^\alpha N^{1-\alpha}$ where $\alpha \in (0, 1)$. Please compute $\frac{\partial Y}{\partial N}$ and $\frac{\partial Y}{\partial K}$.

2. Let $f(t) = e^{at}$. Please compute $f'(t) \equiv \frac{df(t)}{dt}$ and $\frac{d}{dt} \log f(t)$.

3. The time 0 price of an asset with an income stream $f(t), t \geq 0$ is $a(r) = \int_0^\infty e^{-rt} f(t) dt$, where $r > 0$ is the instantaneous interest rate and t is time. Please compute $a(r)$ for $f(t) = Be^{\delta t}$ where $0 < \delta < r$.

4. Please compute $a'(r) = \frac{d}{dr}a(r)$ where $a(r) = \int_0^\infty e^{-rt}f(t)dt$. After computing $a'(r)$, please give a formula for the time 0 price $\int_0^\infty e^{-rt}g(t)dt$ of an asset having payout stream $g(t) = te^{-\delta t}, t \geq 0$.

5. Please evaluate the following infinite series:

$$1 + .5 + .5^2 + .5^3 + \cdots .$$

6. The probability density function for a univariate standardized normal distribution is $\phi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}x^2}$. Please compute (a) $\int xf(x)dx$, and (b) $\int x^2f(x)dx$. Please briefly interpret the quantities you computed in (a) and (b).