Lab Report #6: Options & Volatility

Revised: October 31, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *Bond yields.* Our mission is to explore the relation between the price of a bond and its yield. Suppose we have a 5-year bond with annual coupons of c and a principle of 100. Thus an owner gets a cash flow of c after years one to four and $100 + c$ after year five. If the bond sells for price q, it's common to express it as the discounted cash flow

$$q = c/(1+y) + c/(1+y)^2 + c/(1+y)^3 + c/(1+y)^4 + (c+100)/(1+y)^5.$$

The discount rate y is referred to as the yield or *yield to maturity*. Equally common is to use y as a way to report the price, since knowing y is enough to compute the price (plug it into the equation).

(a) Plot the price q against a grid of points y between 0.00 and 0.10. How does the price vary as we change the yield?

(b) Suppose the price is $q = 102$. Use your graph to estimate the yield y.

(c) Write a bisection program to find the yield y associated with price $q = 102$. *Comment: See the Matlab guide to anonymous functions and the root-finding code posted on the course outline for examples.*

(d) How does your answer change if the price is $q = 99$. Why?

(e) *Optional (for aficionados only).* If we define $d = 1/(1+y)$, we see that the bond price is a polynomial in d:

$$-q + cd + cd^2 + cd^3 + cd^4 + (c+100)d^5 = 0.$$

Since it's a polynomial of degree 5, it has five roots. What happened to the other ones when we computed the yield earlier?

2. *Finding calls from puts.* If we know put prices at given strikes k, we can compute the call prices at the same strikes from the put-call parity relation. And vice versa.

Here's an example. Consider a one-year put option on a stock currently selling for 100. The option with strike price $k = 95$ has a price of 2. The one-period bond price is $q_t^1 = 0.99$. What is the price of a call option at the same strike?

3. *Black-Scholes-Merton volatility.* Our mission here is to examine the role of the mysterious volatility parameter σ in the BSM formula. The calculations refer to put options on the S&P 500 exchange-traded fund, ticker symbol `SPY`. You can look up prices at Yahoo Finance for various strikes and maturities. Or use a Bloomberg machine.

We'll use these inputs: The current price of the underlying is $s_t = 208$. The price of a one-year bond is $q_t^1 = 1.00$.

(a) Consider a put option with strike price $k = 180$. If volatility $\sigma = 0.10$, what is the price of the option? If $\sigma = 0.20$?

(b) Plot the option price against volatility for σ between 0.01 and 0.30. What does it look like? Can you say why?

(c) Suppose the price of the option is 6. What value of σ does that correspond to?