Lab Report #8: Bond Pricing

(Started: April 11, 2012; Revised: October 16, 2013)

Due at the start of class. You may speak to others, but whatever you hand in should be your own work.

1. (estimating parameters for the Vasicek model) Consider the Vasicek model, which for our purposes will be the pricing kernel

$$\log m_t \;=\; \delta + \sum_{j=0}^{\infty} a_j w_{t-j}$$

with parameters $(\delta, a_0, a_1, \varphi)$ and $a_{j+1} = \varphi a_j$ for $j \geq 1$. This is, of course, the ARMA(1,1) version of our infinite moving average.

We'll use the data in Table 3 of

http://pages.stern.nyu.edu/~dbackus/233/BFMW_JFE_01.pdf,

specifically the properties of forward rates, to estimate the parameters of the model. The same table was distributed in class with the notes on bond pricing. And remember: the data are reported as annual percentages, but the time interval here is monthly.

(a) What parameter values reproduce the reported standard deviation and autocorrelation of the short rate (the forward rate of maturity 0)?

(b) Choose a_0 to reproduce the mean difference between forwards of maturity $n = 60$ (5 years) and $n = 0$.

(c) Given the other parameters, what value of δ reproduces the mean short rate?

(d) What do your parameter values imply for the mean difference between forwards of maturity $n = 120$ (10 years) and $n = 0$? $n = 24$ (2 years)?

(e) Given your parameter values, how does the standard deviation of the n-period forward rate vary with n? How do your numbers compare to those in the table?

(f) How would you summarize the differences between the model's implications and the evidence?

2. (another affine model) Consider the bond pricing model characterized by

$$\begin{aligned}
\log m_{t+1} &= -(\lambda_0 + \lambda_1 x_t)^2/2 - x_t + (\lambda_0 + \lambda_1 x_t) w_{t+1} \\
x_{t+1} &= (1-\varphi)\delta + \varphi x_t + \sigma w_{t+1}.
\end{aligned}$$

This is sometimes referred to as an "essentially affine model," but the reasons for that escape me. It is, however, a third convenient example of an affine model — in addition to Vasicek and CIR.

(a) What is the one-period bond price? What role does the term $-(\lambda_0 + \lambda_1 x_t)^2/2$ play?

(b) Suppose bond prices take the form

$$\log q_t^n \quad = \quad A_n + B_n x_t.$$

What are the recursions that generate (A_{n+1}, B_{n+1}) from (A_n, B_n)? What initial values (A_0, B_0) would you use?

(c) Suppose λ_0 is given to you. Using the same logic as the previous question, use features of the short rate f^0 and the five-year forward rate f^{60} to determine the other parameters, $(\varphi, \sigma, \lambda_1)$.

(d) Plot mean forward rates $E(f^n)$ versus maturity n. Comment on any differences between the model and the evidence.

Matlab program: