ECON-UB 233 Dave Backus @ NYU

Quiz #3

(Revised: August 27, 2013)

Please write your name below. Then complete the exam in the space provided. There are TWO questions. You may refer to one page of notes: standard paper, both sides, any content you wish.

(Name and signature)

1. (moving averages) (50 points) Consider the MA(2),

$$x_t = \delta + w_t + \theta_1 w_{t-1} + \theta_2 w_{t-2},$$

with $\{w_t\} \sim \text{NID}(0,1)$ (the w's are independent normals with mean zero and variance one). Our mission is to explore its properties.

   (a) What is the mean of x? The variance? (10 points)

   (b) What are the conditional means, $E_t(x_{t+1})$, $E_t(x_{t+2})$, and $E_t(x_{t+3})$? (10 points)

   (c) What are the conditional variances, $\text{Var}_t(x_{t+1})$, $\text{Var}_t(x_{t+2})$, and $\text{Var}_t(x_{t+3})$? (10 points)

   (d) What is the autocovariance function,

$$\gamma(k) = \text{Cov}(x_t, x_{t-k}),$$

   for $k = 0, 1, 2, 3$? (10 points)

   (e) What is the autocorrelation function? Under what conditions are $\rho(1)$ and $\rho(2)$ positive? (10 points)

2. (moving average bond pricing) (50 points) Consider the bond pricing model

$$\begin{aligned} \log m_{t+1} &= -\lambda^2/2 - x_t + \lambda w_{t+1} \\ x_t &= \delta + \sigma(w_t + \theta w_{t-1}). \end{aligned}$$

   (a) What is the short rate f_t^0? (10 points)

   (b) Suppose bond prices take the form

$$\log q_t^n = A_n + B_n w_t + C_n w_{t-1}.$$

   Use the pricing relation to derive recursions connecting $(A_{n+1}, B_{n+1}, C_{n+1})$ to (A_n, B_n, C_n). What are (A_n, B_n, C_n) for $n = 0, 1, 2, 3$? (20 points)

   (c) Express forward rates as functions of the state (w_t, w_{t-1}). What are f_t^1 and f_t^2? (10 points)

   (d) What is $E(f^1 - f^0)$? What parameters govern its sign? (10 points)