Quiz #2

(Revised: August 27, 2013)

Please write your name below. Then complete the exam in the space provided. There are THREE questions. You may refer to one page of notes: standard paper, both sides, any content you wish.

(Name and signature)

1. Approaches to asset pricing (40 points). Consider a two-period representative agent economy. Consumption growth g is Bernoulli:

$$g = \begin{cases} 1.00 & \text{with probability } 1 - \omega & \text{(state 1)} \\ 1.10 & \text{with probability } \omega & \text{(state 2)} \end{cases}$$

with $\omega = 1/2$. The representative agent has power utility with discount factor $\beta = 0.98$ and risk aversion $\alpha = 5$. Equity is a claim to g.

- (a) What are the mean and standard deviation of $\log g$? (10 points)
- (b) What are the state prices? Which state has greater value? Why? (10 points)
- (c) What are the price and return of a one-period riskfree bond? (10 points)
- (d) What are the risk-neutral probabilities? How do they compare to the true probabilities? Why? (10 points)
- 2. The high-return asset (10 points). Consider a similar representative agent economy. Log consumption growth $\log g$ is normal with mean κ_1 and variance κ_2 . The representative agent has power utility with $\beta = 1$ and $\alpha = 10$. Equity is a claim to g^{λ} .
 - (a) What is the price of a claim to equity? What is the expected return? (10 points)
 - (b) Denote the expected log return on equity by $E \log r^e$. For what value of λ is this return highest? (10 points)
 - (c) What is the entropy of the pricing kernel in this economy? (10 points)
- 3. Digital options (30 points). A digital or binary option either pays some fixed amount or not. Consider, for example, a digital option on the underlying s_{t+1} . A digital call with strike k pays 100 if $s_{t+1} > k$. A digital put with strike k pays 100 if $s_{t+1} \le k$.

Such options can be attacked using the same methods we've used to price traditional options. Suppose $\log s_{t+1}$ has a normal risk-neutral distribution with mean κ_1 and variance κ_2 .

- (a) What is the no-arbitrage condition for this situation? (10 points)
- (b) What is the analog of put-call parity? (10 points)
- (c) What is the price of a put with strike k? (10 points)