Lab Report #0: Risk and Return

Revised: April 5, 2013

I used the old template, ignore and go to the problems.

1. *Asset returns.* The canonical asset pricing relation can be expressed, variously, as

$$\begin{aligned} q^j &= E(md^j) \\ &= \sum_s p(s)m(s)d^j(s) \\ &= \sum_s \theta(s)d^j(s). \end{aligned}$$

   Here q^j is the price "today" of asset j, a claim to the dividend d^j "tomorrow." States s occur with probability $p(s)$. In each state s, the pricing kernel is $m(s)$ and the state price is $\theta(s)$.

   Suppose we have two states, $s = 1$ and $s = 2$, which occur with equal probabilities $\theta(1) = 2/3$ and $\theta(2) = 1/3$. State prices are $\theta(1) = 1$ and $\theta(2) = 2$.

   (a) Draw the event tree that corresponds to this environment.
   (b) What are the pricing kernel's values in the two states?
   (c) Consider an asset with $d(1) = 1$ and $d(2) = 2$. Compute its price q using state prices and the pricing kernel. Verify that you get the same answer.
   (d) How is the (gross) return r related, in general, to q and d? The mean return?
   (e) What are the returns in each state on the asset in part (c)? What is the mean return?
   (f) Consider a second asset with dividends $d(1) = 2$ and $d(2) = 1$. What are its price and mean return? Why is its mean return different from the asset in (b)?

2. *Risk and risk aversion.* In the same setting as the previous equation, we'll consider the pricing implied by a representative agent with power utility. Here the pricing kernel is related to consumption growth g by $m = \beta g^{-\alpha}$. Here m is the intertemporal marginal rate of substitution, β is the discount factor, and $\alpha \geq 0$ is the coefficient of relative risk aversion.

   (a) Suppose $\beta = 0.95$ and $\alpha = 2$. Using the pricing kernel from the previous question, what is consumption growth in the two states?
   (b) In addition to the other assets, compute the riskfree return: the return on an asset with dividends $d(1) = d(2) = 1$.
   (c) What is the risk premium on the asset in part (c) above?
   (d) Now increase risk aversion to $\alpha = 5$. What is the expected return on the asset on part (c)? Its risk premium? What do you think is happening here?