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Intermediate Macro
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Lecture 9

Short Run Inflation/Ouput Dynamics and Policy

1 Baseline Fixed and Flex Price Models

There are seven variables to be determined: $(Y_t, C_t, N_t, P_t, M_t, \mu_t, R_t^n)$. In the fixed price equilibrium, P_t is fixed and μ_t varies. The reverse is true in the flex price equilibrium.

1.1 Fixed Price Model

Aggregate Demand

$$Y_t = C_t \tag{1}$$

$$C_t = E_t\left\{R_t^n \frac{P_t}{P_{t+1}}\beta\frac{1}{C_{t+1}}\right\}^{-1} \tag{2}$$

Aggregate Supply

$$Y_t = A_t N_t \tag{3}$$

$$A_t = (1+\mu_t)\frac{W_t}{P_t} = (1+\mu_t)\frac{N_t^{\gamma_n}}{(1/C_t)} \tag{4}$$

$$P_t = \overline{P} \tag{5}$$

Monetary Policy Rule

$$\frac{M_t}{P_t} = a_m\left(1-\frac{1}{R_t^n}\right)^{-1} C_t \tag{6}$$

$$M = \overline{M}$$

Simplified System:

IS Curve:

$$Y_t = E_t\left\{R_t^n \beta\frac{1}{Y_{t+1}}\right\}^{-1}$$

LM Curve:

$$\frac{\overline{M}}{\overline{P}} = a_m \left(1 - \frac{1}{R_t^n}\right)^{-1} C_t$$

AS Curve:

$$\frac{1}{(1+\mu_t)} = (\frac{Y_t}{A_t})^{1+\gamma_n}$$

1.2 Flex Price Model

$$Y_t^* = E_t \left\{ R_{t+1}^* \beta \frac{1}{Y_{t+1}^*} \right\}^{-1}$$

$$\frac{\overline{M}}{P_t} = a_m \left(1 - \frac{1}{R_t^{n^*}}\right)^{-1} Y_t^*$$

$$\frac{1}{(1+\mu)} = (\frac{Y_t}{A_t}^*)^{1+\gamma_n}$$

with

$$R_{t+1}^* = R_t^{n^*} E_t(\frac{P_t}{P_{t+1}})$$

2 Price Adjustment:

fixed price case, with no adjustment

$$A_t = (1+\mu_t)\frac{W_t}{\overline{P}_{t-1}}$$

flex price case

$$A_t = (1+\mu)\frac{W_t}{P_t^*}$$

If $\mu_t > \mu$, $\overline{P}_{t-1} > P_t^*$.

Rule for aggregate price adjustment:

$$\frac{P_t}{P_{t-1}} = (\frac{1+\mu}{(1+\mu_t)})^{\kappa}$$

with $0 < \kappa < 1$

$$\begin{aligned}\frac{P_t}{P_{t-1}} &= ((\frac{Y_t}{Y_t^*})^{1+\gamma_n})^{\kappa} \\ &= (\frac{Y_t}{Y_t^*})^{\lambda}\end{aligned}$$

3 Loglinear Approximation

Taking loglinear approximation of the IS/LM model around the flex price equilibrium with the steady state value of A_t, yields:
IS:

$$y_t = -(r_t^n - E_t\pi_{t+1}) + E_t y_{t+1} + \chi_t$$

LM:

$$\bar{m}_t - p_t = y_t - \nu r_t^n$$

where $\nu = 1/(R^n - 1)$ and where χ_t is an exogenous demand disturbance.

AS:

$$p_t = \pi_t + p_{t-1}$$

$$\pi_t = \kappa(-\mu_t)$$

$$\mu_t = -(1+\gamma_n)(y_t - y_t^*)$$

$$\pi_t = \lambda(y_t - y_t^*)$$

Determination of y_t^*:

$$\begin{aligned} y_t^* &= a_t + n_t^* \\ a_t &= \gamma_n n_t^* + c_t^* \\ &= \gamma_n(y_t^* - a_t) + y_t^* \\ y_t^* &= a_t \end{aligned}$$

Thus we can collapse the model to three equations::

IS:

$$y_t = -(r_t^n - E_t\pi_{t+1}) + E_t y_{t+1} + \chi_t$$

LM:

$$\bar{m}_t - p_t = y_t - \nu r_t^n$$

AS:

$$\pi_t = \lambda(y_t - y_t^*)$$

Note that the model delivers a positive relation between the output gap and inflation. This captures well the 1960s and more recent data. But it cannot explain the 1970s, where there was simultaneously high inflation and a low output gap.

3.1 Expectations-Augmented Phillips Curve

To capture the the experience of the 1970s, it is important to take into account the influence of inflation expectations of price setting. Firms set prices in nominal terms but the care about the implied relative price (nominal price divided by overall price level.) Accordingly the following relation for inflation was suggested to take this phenomenon into account:

$$\pi_t = \lambda(y_t - y_t^*) + E_t\pi_{t+1} + u_t$$

where u_t is an exogenous disturbance to inflation. This relation for inflation is called the "expectations-augmented" Phillips curve.

The dynamics of inflation accordingly depend on how inflation expectations are formed. We will consider two polar cases: backward looking (or "adaptive") and forward looking (or "rational".)

3.1.1 adaptive expectations:

Here firm's use past history of inflation to forecast the future. A simple example is:

$$E_t\pi_{t+1} = \pi_{t-1}$$

accordingly:

$$\pi_t = \lambda(y_t - y_t^*) + \pi_{t-1} + u_t$$

Note that in this case, the output gap determines the change in inflation. One can see how an inflation spiral - like the kind that occurred in the 1970s - could arise in this framework. If there is a period of excess demand, inflation will drift up, how far is drifts depends on both the magnitude and persistence fo the positive output gap.

The complete model in this case is given by

IS::

$$y_t = -(r_t^n - \pi_{t-1}) + E_t y_{t+1} + \chi_t$$

LM:

$$\bar{m}_t - p_t = y_t - \nu r_t^n$$

AS:

$$\pi_t = \lambda(y_t - y_t^*) + \pi_{t-1} + u_t$$

The only way the central bank can get inflation back down is the tighten monetary policy to push output below the natural level y_t^*. If it chooses not to do so, it must accept higher inflation.

In the steady state:

steady state

$$y_t = y_t^*$$

$$\begin{aligned} \bar{m}_t - p_t &= -\nu r^n \\ \bar{m}_t - \bar{m}_{t-1} &= \pi \end{aligned}$$

The Fed "accommodates" inflation by having having money grwoth equal to steady state inflation. This keeps real money balances unchanged. If it wants to reduces inflation, it must reduce inflation below.

Interest Rate Rules:
A policy that seems to work well in stabilizing inflation is an interest rate rule of the form:

$$r_t^n = \phi_\pi \pi_{t-1}$$

with $\phi_\pi > 1$. This rule is know as "Taylor rule" after John Taylor of Stanford who first recommended it. In this case the system becomes:

IS::

$$y_t = -(r_t^n - \pi_{t-1}) + E_t y_{t+1} + \chi_t$$

LM:

$$r_t^n = \phi_\pi \pi_{t-1}$$

AS:

$$\pi_t = \lambda(y_t - y_t^*) + \pi_{t-1} + u_t$$

Combining the interest rate rule with the IS curve yields

$$y_t = -(\phi_\pi - 1)\pi_{t-1} + E_t y_{t+1} + \chi_t$$

Thus, natural mechanism is in place to contract the economy when inflation is above target, and vice-versa when it is below. Note that it is critical that $\phi_\pi > 1$. During the 1970s this was not the case. Estimates suggest that it was slightly below unity in this case.

3.2 Rational Expectations

We now suppose firms have rational expectation. That is, they generate the same forecast the underlying economic model does, given the information available at the time of the forecast.
This approach yields an explanation for the high inflation of the 1970s, based on central bank credibility. The argument begins by noting that, everything else equal, the central bank would like to push the level of output y_t above than the flexible price equilibrium y_t^*. Since there is imperfect competition, the flexibile price equilibrium level of output is below the socially optimal level y_t^0 (due to the markup.)l:

$$y_t^0 = y_t^* + k \tag{7}$$

where $k > 0$ depends on the (constant) desired mark-up. But then $y_t - y_t^0 = y_t - y_t^* - k$. Thus the bliss point for the output gap $y_t - y_t^*$ is positive. In this instance, unless the central bank can credibly promise to control inflation, steady state inflation may be inefficiently high, as originally emphasized by Kydland and Prescott (1977) and Barro.and Gordon (1983) and many others.
Let $x_t = (y_t - y_t^*)$. Then suppose the central bank cares about both squared deviations of output and inflation from target. Then its policy problem is given by::,

$$\max_{\{x_{t+i},\pi_{t+i}\}_{i=0}^{\infty}} -\frac{1}{2} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\alpha(x_{t+i} - k)^2 + \pi_{t+i}^2 \right] \right\} \tag{8}$$

subject to

$$\pi_{t+i} = \lambda x_{t+i} + E_{t+i}\pi_{t+i+1}$$

In this case the central bank takes expectations of the future as given, so that the problem becomes a simple static one:.

$$\max_{x_t,\pi_t} -\frac{1}{2}[\alpha(x_t - k)^2 + \pi_t^2] + F_t \tag{9}$$

subject to

$$\pi_t = \lambda x_t + f_t$$

where

$$\begin{aligned} F_t &= -\frac{1}{2}E_t[\sum_{i=1}^{\infty}\beta^i(\alpha(x_{t+i} - k)^2 + \pi_{t+i}^2)] \\ f_t &= E_t\pi_{t+1} + u_t \end{aligned}$$

The FOC is then:

$$x_t - k = -\frac{\lambda}{\alpha}\pi_t \tag{10}$$

Proceding as before we obtain the equilibrium values for π_t and x_t under discretion:

$$\pi_t = \frac{\alpha}{\lambda}k \tag{11}$$

$$x_t = 0 \tag{12}$$

Thus, there is a positive inflationary bias ($\pi_t > 0$ as u_t goes to zero), but no difference in output from the natural level.

Intuitively, if the central bank desires to push output above potential (the presence of k in the optimality condition 10 reflects this temptation), a rational private sector recognizes this incentive, and in equilibrium inflation rises to the point where the central bank is no longer tempted to expand output. As a consequence, inflation is forced in the long run above target (trend inflation is $\bar{\pi} = \frac{\alpha}{\lambda}k$). The resulting equilibrium is strictly Pareto dominated by the equilibrium for $k = 0$.

Following this analysis, a large literature stresses the possible gains from making binding commitments over future monetary policies. Rogoff (1985) suggests to appoint a "conservative" central banker, who assigns little if not zero weight to the output gap ($\alpha \approx 0$), in order to reduce the inflationary bias (to zero at the limit)..

Alternatively, the central bank can commit only to $k = 0$ in order to achieve the equilibrium allocation in the baseline model under discretion, or equivalently commit to an interest rate rule that targets $\bar{\pi} = 0$ (that is a policy that can be checked by the private sector)..

There are other reasons why the central bank may prefer to set an inflation target above zero. One is, in particular, the deflationary trap: if deflation is expected, $E_t\pi_{t+1} < 0$, the central bank is constrained by the lower bound on nominal interest rate, $i_t \geq 0$, and needs to commit to be inflationary in the future. We turn to this next.

3.3 The Zero Lower Bound, Deflation Traps and Credibiliy.

Lets write the baseline model as follows:

IS

$$y_t - y_t^* = -[(r_t^n - E_t\pi_{t+1}) - r_{t+1}^*] + E_t(y_{t+1} - y_{t+1}^*)$$

LM

$$r_t^n = \overline{r_t^n} > 0$$

AS

$$\pi_t = \lambda(y_t - y_t^*) + E_t\pi_{t+1}$$

Suppose that negative demand shocks have pushed that natural rate of interest below zero for $k+1$ periods (the IS curve has moved downward to the point where it intersects the aggregate supply curve at an interest rate below zero.) In this case the central bank will push nominal interest rates to zero: It cannot reduce them any further.
Accordingly:

$$\begin{aligned} y_t - y_t^* &= -E_t\sum_{i=0}^{\infty}[(r_{t+i}^n - \pi_{t+1+i}) - r_{t+1+i}^*] \\ &= -E_t\sum_{i=0}^{k}[-\pi_{t+1+i} - r_{t+1+i}^*] - E_t\sum_{j=k+1}^{\infty}[(r_{t+j}^n - \pi_{t+1+j}) - r_{t+1+j}^*] \end{aligned}$$

$$\pi_t = E_t\sum_{i=0}^{\infty}(y_{t+i} - y_{t+i}^*)$$

First suppose that once the natural rate become positive, the private sector expects the central bank will set the nominal so that the real rate rate equals the natural rate. In this case, it is expected that after period $t+k$, the interest rate gap $[(r_{t+i}^n -_t \pi_{t+1+i}) - r_{t+1+i}^*]$ is zero. Accordingly, in this case, the output gap will be negative and deflation will emerge, making the output gap even lower. To see, first suppose that inflation is zero for k periods. In this case the output gap will clear be negative for k periods. It follows that inflation will be negative, and so to will expected inflation - i.e. there will be expected deflation. The rise in expected deflation will further reduce the output gap, and so on.
What is a way out? If the central bank can credibly committ to keep the interest rate gap positive for a period after the liquidity trap ends, this will stimulate current output (which depends on future interest rate gaps.) This is tantamount to the central bank committing to having some inflation in the future once the economy emerges from the liquidity tarp