$$\frac{E_t(D_{t+1} + Q_{t+1})}{R} = Q_t$$

$$\Rightarrow \quad Q_t = E_t\left(\frac{D_{t+1}}{R}\right) + \frac{1}{R} \cdot E_t(Q_{t+1})$$

$$Q_{t+1} = E_{t+1}\left(\frac{D_{t+2}}{R}\right) + \frac{1}{R} E_{t+1}(Q_{t+2})$$

$$\Rightarrow \quad Q_t = E_t\left(\frac{D_{t+1}}{R}\right) + \frac{1}{R} E_t\left\{ E_{t+1}\left(\frac{D_{t+2}}{R}\right) + \frac{1}{R} E_{t+1}(Q_{t+2})\right\}$$

$$= E_t\left(\frac{D_{t+1}}{R}\right) + \frac{1}{R} E_t E_{t+1}\left(\frac{D_{t+2}}{R}\right) + \frac{1}{R^2} E_t E_{t+1}(Q_{t+2})$$

$$\overset{\text{L.I.E.}}{=} E_t\left(\frac{D_{t+1}}{R}\right) + E_t\left(\frac{D_{t+2}}{R^2}\right) + \frac{1}{R^2} E_t(Q_{t+2})$$

$$Q_{t+2} = E_{t+2}\left(\frac{D_{t+3}}{R}\right) + \frac{1}{R} E_{t+2}(Q_{t+3})$$

$$\Rightarrow \quad Q_t = E_t\left(\frac{D_{t+1}}{R}\right) + E_t\left(\frac{D_{t+2}}{R^2}\right) + E_t\left(\frac{D_{t+3}}{R^3}\right) + \frac{1}{R^3} E_t(Q_{t+3})$$

$$\vdots$$

$$Q_t = E_t\left(\sum_{i=\cancel{0}\,1}^{\infty} \left(\frac{1}{R}\right)^i D_{t+i}\right) + \underbrace{\lim_{i\to\infty} \left(\frac{1}{R^i}\right) \cdot E_t(Q_{t+i})}_{=0}$$

Key concepts: • linearity of expectation operator

• Law of Iterated expectations.

Problem 2

Money demand:

$$m_t - p_t = k - \alpha E_t(p_{t+1} - p_t)$$

Starting point: variable velocity

- Money demand depends on opportunity cost of holding money $\Rightarrow$ the nominal interest rate.
- Specific choice of functional form:

$$\frac{M_t}{P_t} = c \cdot Y_t \cdot e^{-\nu \cdot i_t} = \frac{1}{V_t} \cdot Y_t$$

- Velocity: $\frac{1}{V_t} = c \cdot e^{-\nu \cdot i_t}$ or $V_t = \frac{1}{c} \cdot e^{\nu \cdot i_t}$

- Logs:

$$\begin{aligned} m_t - p_t &= \log c + y_t - \nu i_t \\ &= \log c + y_t - \nu \cdot (r_t + E_t \cdot \pi_{t+1}) \end{aligned}$$

- Let Y^*, r^* be the natural levels of output and the real interest rate $\Rightarrow$ independent of nominal variables like money demand and inflation

$$\Rightarrow \quad m_t - p_t = \log c + y^* - \nu r^* - \nu E_t(p_{t+1} - p_t)$$

$$\Rightarrow \quad \boxed{m_t - p_t = k - \nu \cdot E_t(p_{t+1} - p_t)}$$

Next, perfect foresight $\Rightarrow$ drop expectation

$$m_t - p_t = k - \nu(p_{t+1} - p_t)$$

$$m_t - p_t = k - \nu p_{t+1} + \nu p_t$$

$$p_t = \frac{m_t - k}{1+\nu} + \frac{\nu}{1+\nu} \cdot p_{t+1}$$

$\Rightarrow$

$$p_t = (1-\alpha)(m_t - k) + \alpha\, p_{t+1}$$

$$p_{t+1} = (1-\alpha)(m_{t+1} - k) + \alpha\, p_{t+2}$$

$$\Rightarrow \quad p_t = (1-\alpha)(m_t - k) + \alpha(1-\alpha)(m_{t+1} - k) + \alpha^2 p_{t+2}$$

$$= (1-\alpha)(m_t - k) + \alpha(1-\alpha)(m_{t+1} - k) + \alpha^2(1-\alpha)(m_{t+2} - k) + \alpha^3 p_{t+3}$$

$$\Rightarrow \quad p_t = (1-\alpha) \cdot \sum_{i=0}^{\infty} \alpha^i (m_{t+i} - k) = (1-\alpha) \sum_{i=0}^{\infty} \alpha^i m_{t+i} - (1-\alpha) k \cdot \sum_{i=0}^{\infty} \alpha^i$$

$$\Rightarrow \quad \boxed{p_t = (1-\alpha) \sum_{i=0}^{\infty} \alpha^i m_{t+i} - k}$$

$$p_{t+1} = (1-\alpha) \sum_{i=0}^{\infty} \alpha^i m_{t+1+i} - k$$

$$\Pi_{t+1} = p_{t+1} - p_t = (1-\alpha) \sum_{i=0}^{\infty} \alpha^i (m_{t+1+i} - m_{t+i})$$

$$\Rightarrow \quad \boxed{\Pi_{t+1} = (1-\alpha) \sum_{i=0}^{\infty} \alpha^i \mu_{t+1+i}}$$

Constant $\mu \Rightarrow \Pi_{t+1} = \mu \quad \forall\, t+1$

Permanent drop in $\mu \rightarrow \underline{\mu} \Rightarrow$ permanent drop in $\Pi_{t+1} = \underline{\mu}$

$$\Pi_{t} = (1-\alpha) \sum_{i=0}^{T-1} \alpha^i \underline{\mu} + (1-\alpha) \sum_{i=T}^{\infty} \alpha^i \mu$$

$$= \cancel{(1-\alpha)}\underline{\mu} \cdot \frac{(1-\alpha)}{\cancel{1-\alpha}} (1 + \alpha + \alpha^2 + \ldots + \alpha^{T-1}) + (1-\alpha)\alpha^T \sum_{i=0}^{\infty} \alpha^i \mu$$

$$\boxed{\Pi_{t} = \underline{\mu} \cdot (1 - \alpha^T) + \alpha^T \mu} = \underline{\mu} + \alpha^T (\mu - \underline{\mu})$$

Path of inflation: