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Intermediate Macroeconomic Theory
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Problem Set 3

Answer each of the following two questions:

1 Problem 1

Consider a simple two period economy with the following features: a representative household that consumes, saves and receives dividend income (from ownership of firms); a representative firm that produces output using capital, invests in new capital, and pays dividends to households; and a government that spends each period, and finances this spending through taxes and borrowing. In addition, the economy is open and takes the world interest rate as given.

The household and the firm act competitively. The key details about the economy are as follows

The Representative Household:

Let S^i be household i 's saving, Π_k^i dividends in period k , R the gross real interest rate (equal to one plus the net interest rate), all in units of consumption goods. Further, we normalize the price of consumption goods at unity. The representative household chooses C_1^i , C_2^i , and S^i , to solve

$$\max \log C_1 + \beta \log C_2 \tag{1}$$

subject to:

$$C_1^i = \Pi_1^i - T_1 - S^i \tag{2}$$

$$C_2^i = \Pi_2^i - T_2 + RS^i \tag{3}$$

The household takes as given R, Π_1^i and Π_2^i . Capital markets are perfect.

The Representative Firm:

The firm maximizes the discounted stream of profits returned to the household. Given that there is no uncertainty, the firm discounts future profits at the rate $1/R$. Accordingly, the firm chooses I^j, Y_2^j , and K_2^j to solve

$$\max \Pi_1^j + \frac{\Pi_2^j}{R} \quad (4)$$

subject to:

$$\Pi_1^j = Y_1^j - I^j - \frac{1}{2}c \left(\frac{I^j}{K_1^j} \right)^2 K_1^j \quad (5)$$

$$\Pi_2^j = Y_2^j + (1 - \delta)K_2^j \quad (6)$$

$$Y_k^j = A_k K_k^j, \quad k = 1, 2$$

$$K_2^j = (1 - \delta)K_1^j + I^j$$

taking K_1^j and R as given.

Government

Let G_k be government expenditures on goods and services in period $k = 1, 2$, T_k taxes in period k , and B^g the stock of government issued in period 1.

$$B^g + T_1 = G_1 \quad (7)$$

$$T_2 = G_2 + RB^g \quad (8)$$

Economy-Wide Resource Constraints and World Real Interest Rate, R^w

In period 1, output is divided between consumption, investment, government expenditures, and net exports:

$$Y_1 = C_1 + I + \frac{1}{2}c \left(\frac{I^j}{K_1^j} \right)^2 K_1^j + G_1 + XM \quad (9)$$

In period 2, output, the remaining capital stock, and the earnings on foreign assets are divided between consumption and government expenditures.

$$(1 - \delta)K_2 + Y_2 + RXM = C_2 + G_2 \quad (10)$$

$$R = R^w$$

Questions

1. Derive the first order condition for the household consumption/saving relation and then solve for the household's choice of consumption in period 1.

2. Derive the first order condition for the firm investment decision.

3. Present the set of equations that describe the equilibrium.

4. Now suppose that the government increases G_1 in the first period but then promises to reduce G_2 next period by an amount that's equal in present value. (i.e, $\Delta G_1 + \Delta G_2/R = 0$.)

What is the impact on I, C_1, C_2 , and XM ? Explain. Does the equilibrium depend on B^g ? Explain.

5. Now suppose markets are imperfect. Households cannot borrow. In this case, could the effect of the fiscal experiment described in 4. depend on whether the increase in G_1 is financed by government borrowing or taxes? Explain.

2 Problem 2

Consider the investment problem of a firm that operates for two periods. Output Y in periods one and two for the firm are given by

$$Y_1 = A_1 K_1^\alpha$$

$$Y_2 = A_2 K_2^\alpha$$

where capital K is given in the first period, but depends on investment I in the second period, as follows:

$$K_2 = (1 - \delta)K_1 + I$$

The firm chooses I to maximize the present discounted returns to shareholders, given by,

$$Y_1 + B - I - \frac{c}{2} \left(\frac{I}{K} \right)^2 K_1 + \frac{Y_2 + (1 - \delta)K_2 - RB}{R}$$

where B is the amount the firm borrows.

Questions:

1. Assuming perfect capital markets, describe and interpret the firm's first order condition. Does the firm's investment decision depend on the quantity B that it borrows.
2. Now suppose capital markets are imperfect, which has the effect of limiting B . In particular suppose that the firm begins the period with some tangible assets (e.g. real estate) of value V_t , which can be used as collateral for borrowing (i.e. the lender can seize these collateral assets if the borrower defaults). The borrowing constraint is of the form:

$$B_t \leq V_t$$

If the constraint is binding, how is investment determined. How would you determine whether the constraint is binding?

Now suppose that there is a sharp drop in the value of the firm's assets. What is the impact on I_t ? In this case, what happens to the marginal return to investment relative to the interest rate R ?