Lab Report #7: Dynamics in Theory and Data

Revised: November 19, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work.

1. *Is the light on?* Consider Hairer's example. A state variable z_t equals one if the light is on at date t, zero if it's off. Between dates, the probability that the state stays the same is $(1+\varphi)/2$.
   (a) What is the probability that the state changes? How do you know?
   (b) How does the probability distribution over next period's state depend on this period's?
   (c) If the light is on at date t, what is the probability that it's on at $t+1$? At $t+2$? At $t+k$ for $k > 2$?
   (d) How does your uncertainty about the light's state change with the forecast horizon? Your ability to forecast?

2. *MA(1).* Consider the MA(1)

$$x_t \;=\; \theta_0 w_t + \theta_1 w_{t-1}$$

   with iid standard normal innovations w_t and coefficients (θ_0, θ_1).
   (a) What is the variance of x?
   (b) What is the covariance of x_t and x_{t-1}? How does it change if $\theta_1 = 0$? $\theta_0 = 0$?
   (c) What is the autocorrelation function?
   (d) We estimate the autocorrelation to be 0.4. What does that tell us about the coefficients (θ_0, θ_1)?

3. *ARMA(1,1).* Consider the ARMA(1,1) model

$$x_t \;=\; \sum_{j=0}^{\infty} a_j w_{t-j}$$

   with iid standard normal innovations w_t and coefficients a_0, a_1, and $a_{j+1} = \varphi a_j$ for $j \geq 1$ and parameter $0 < \varphi < 1$.
   (a) What is the variance of x?
   (b) What is the covariance of x_t and x_{t-1}?
   (c) What is the autocovariance function? The autocorrelation function?
   (d) What configuration of parameter values gives us negative autocorrelations?
   (e) *Extra credit.* Show that the model can be expressed in traditional ARMA(1,1) form,

$$x_t \;=\; \varphi x_{t-1} + \sigma(w_t + \theta w_{t-1}).$$