Lab Report #2: Sums, Mixtures, & Certainty Equivalents

Revised: August 20, 2014

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Use Matlab where possible and attach your code to your answer.

1. *Properties of US equity returns.* Gene Fama is one of the giants of modern finance. His coauthor over the past twenty years or so has been Ken French. In this problem, we'll compute sample properties of US equity returns using one of the so-called Fama-French datasets, which French kindly posts on his website. Go to French's website and download the files labelled "Fama/French factors" at this link. Then follow these instructions:

   - Read the enclosed txt file into a spreadsheet program.
   - Strip off the header (the three lines at the top).
   - Label the columns `date xs smb hml rf`.
   - Delete the annual data at the bottom.
   - Save in a convenient format (xls?).

   When you're done, read the whole thing into Matlab using `xlsread` or other command appropriate to the format of your file. You now have monthly data going back to 1926 on various equity portfolios, expressed as percentages. They haven't been annualized: a monthly return of 1.0 in the data corresponds to roughly 12% annually. We'll focus on `xs`, the excess return on a broad-based equity portfolio. Excess return means, in this context, the return minus that on the riskfree bond, `rf`. Also there but not needed for now: `smb` is the return on small firms minus big firms and `hml` is the return on firms with high book-to-market minus those with low book-to-market.

   (a) Use the command `hist` to plot a histogram for `xs`.

   (b) Compute the mean, standard deviation, skewness, and excess kurtosis for the excess return series.

   (c) Does the mean look high or low to you?

   (d) In what respects do these excess returns look normal? In what respects not?

2. *Sums and mixtures.* More fun with cumulant generating functions (cgfs). The ingredients include two independent normal random variables, $x_1 \sim \mathcal{N}(0,1)$ and $x_2 \sim \mathcal{N}(\theta,1)$. Do the calculations in Matlab and submit your code with your answer.

   (a) Consider the sum $y = x_1 + x_2$. What is its cgf? Its first four cumulants? What are its measures of skewness and excess kurtosis, γ_1 and γ_2?

   (b) Now consider the mixture z:

   $$z \;=\; \begin{cases} x_1 & \text{with probability } 1-\omega \\ x_2 & \text{with probability } \omega. \end{cases}$$

   What is its cgf? Its first four cumulants? What are its measures of skewness and excess kurtosis, γ_1 and γ_2? What determines the sign of γ_1?

(c) In what ways does z differ from a normal random variable?

3. *Concave and convex functions.* Consider the following functions defined over positive values of x:
   (a) $f(x) = \log x$
   (b) $f(x) = \exp(x)$
   (c) $f(x) = x^\alpha$ for $\alpha = 2$
   (d) $f(x) = -x^\alpha$ for $\alpha = 2$
   (e) $f(x) = x^\alpha$ for $\alpha = 1$

   For each one:

   - State whether it's concave, convex, or something else.
   - State whether $E[f(x)]$ is greater or less than $f[E(x)]$.
   - Verify your previous answer by computing $E[f(x)]$ and $f[E(x)]$ for the Bernoulli random variable

$$x = \begin{cases} 100 & \text{with probability } 1/2 \\ 200 & \text{with probability } 1/2. \end{cases}$$

4. *Certainty equivalents for quadratic utility.* There's a long history of risk preference with quadratic utility:

$$u(c) = c - c^2.$$

   It's seldom used these days, but is a convenient starting point for thinking about risk. Let us say, to be concrete, that c is random with mean μ and variance σ^2.
   (a) What is expected utility? How does it depend on the distribution of consumption outcomes?
   (b) What is the certainty equivalent?
   (c) How does the certainty equivalent compare to expected consumption?