Lab Report #6: Options & Volatility

Revised: November 3, 2014

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *Root-finding.* Consider the function $f(x) = \sin(x) + x\cos(x)$. Our mission is to find values of x in the interval $[0.5, 2.5]$ for which $f(x) = 0$.

   (a) Plot f against a grid of points x between 0.5 and 2.5. How many solutions do you see?

   (b) Write or adapt a bisection or Newton's method program to find a solution. What is it?

   (c) Change the function to $f(x) = sin(x) + x\cos(x) - c$. Modify your program to find solutions in the same interval for $c = \{1/2, 0, -1/2\}$. What are they? Can you get your program to find them all at once?

   Comment: See the Matlab guide to anonymous functions or the root-finding code posted on the course outline for examples.

2. *Black-Scholes-Merton formula.* Our mission here is to examine the role in the BSM formula of the mysterious volatility parameter σ. The calculations refer to call options on the S&P 500 exchange-traded fund, ticker symbol `SPY`. On October 27, 20014 (today), the price of the underlying is $s_t = 197.07$. We'll consider prices of call options with maturities of 5 months or $\tau = 5/12$ years. On the same date, the price of a bond of similar maturity was $q_t^\tau = 1.0000$ (yes, the interest rate is essentially zero).

   (a) Consider a call option with strike price $k = 197$ ("at the money"). If volatility $\sigma = 0.10$, what is the price of the option? If $\sigma = 0.20$?

   (b) Plot the call option price against volatility for σ between 0.01 and 0.30.

   (c) On the same graph, do the same for options with strikes of 180 and 220.

   (d) Consider call prices of 19.68 ($k = 180$), 7.28 ($k = 197$), and 0.32 ($k = 220$). Use your graph to estimate the values of σ that deliver these prices. How do they compare? If σ is a "parameter," why are they different?

3. *Volatilities for S&P 500 options.* We continue our examination of option prices and volatility: same date and underlying as the previous question.

   Yahoo finance reports the following quotes:

	Call Price		Put Price	
Strike	Bid	Ask	Bid	Ask
150	47.25	47.62	0.73	0.79
160	37.61	37.95	1.26	1.32
170	28.28	28.60	2.15	2.22
180	19.56	19.80	3.63	3.69
190	11.83	11.93	5.96	6.07
200	5.57	5.66	9.87	9.99
210	1.69	1.76	16.03	16.37
220	0.29	0.34	24.70	25.11
230	0.04	0.08	34.46	34.93

(a) Compute "mid" quotes as averages of bid and ask. Plot these mid-market put and call prices against the strike price.

(b) Use put-call parity to compute call prices from mid puts. Plot call prices — bid, ask, and implied by puts — against the strike. How do they compare?

(c) Write a bisection or Newton's method program to compute implied volatilities for mid quotes of call options. Graph them against the strike. What shape does the resulting "smile" have?

Comments:

- *I use this two-step definition of the BSM formula:*

```
d = @(sigma,k) (log(s./(q_tau.*k))+tau*sigma.^2/2)./(sqrt(tau)*sigma);
f = @(sigma,k) s*normcdf(d(sigma,k)) - ...
        q_tau.*k.*normcdf(d(sigma,k)-sqrt(tau)*sigma) - call_mid;
```

  Here I've subtracted the call price from the formula to give us a function `f` *whose value is zero when we find the right volatility.*

- *If you're lazy like me, you can look up the derivative in Wikipedia, where they refer to it as the "vega," a nonexistent Greek letter. Or you could use bisection and skip the whole thing. We can calculate the derivative with an anonymous function, too:*

```
fp = @(d) s*sqrt(tau)*exp(-d.^2/2)/sqrt(2*pi);
```