Lab Report #7: Dynamics in Theory and Data

(Started: April 11, 2012; Revised: October 16, 2013)

Due at the start of class. You may speak to others, but whatever you hand in should be your own work.

1. (dynamics of interest rates) We'll look at the autocorrelations of interest rates to get a sense of their dynamics. The first step is to download some data from the Fed. Go to

   http://www.federalreserve.gov/releases/h15/data.htm

   and download monthly data for Treasury constant maturities, specifically the 3-month and 10-year maturities, for the period 1985 to present. Read them into Matlab and:

   (a) Compute the mean, standard deviation, and autocorrelation function (acf) for the 1-month interest rate. (You may recall that we used the program `acf.m` for the latter in class. It's our program, not part of Matlab, although Matlab's Econometrics Toolbox has a similar function. It works on time series objects, which is something I'd prefer to avoid for now. But by all means do whatever you wish.)

   (b) Describe the acf for an AR(1):

   $$x_t \;=\; (1-\varphi)\mu + \varphi x_{t-1} + \theta w_t,$$

   where $\{w_t\} \sim \text{NID}(0,1)$. How do the acf's compare for the data and a suitably estimated AR(1)?

   (c) Compute the mean, standard deviation, autocorrelation function (acf) for the 10-year interest rate. How do they compare to the 1-month rate?

2. (two-state Markov chain) We can get a sense of how Markov chains work with a two-state example. A two-state chain is characterized by a 2 by 2 transition matrix P. Because the rows sum to one, P has (essentially) two parameters. A convenient parameterization is

   $$P \;=\; (1-\varphi)\begin{bmatrix} \omega & 1-\omega \\ \omega & 1-\omega \end{bmatrix} + \varphi \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad (1)$$

   where the two parameters are ω and φ.

   (a) Under what conditions on (ω,φ) is P a legitimate transition matrix?

   (b) What are the two-period transitions P^2? You can either do this by hand or get Matlab to do it. Either way, the key is to arrange the terms into a form similar to (1).

   (c) What about the k-period transitions?

   (d) What happens as we continue to increase k? What is the equilibrium distribution?

   (e) (extra credit) What are the eigenvalues of P?

3. (state-space representations) State-space models have a similar mathematical structure in the sense that dynamics follow from powers of a matrix. The canonical version is

$$x_{t+1} \quad = \quad Ax_t + Bw_{t+1}. \tag{2}$$

Here x is a vector, $w \sim \mathcal{N}(0, I)$ is also a vector (of possibly different dimension), and (A, B) are matrices.

(a) Consider the ARMA(1,1):

$$y_t \quad = \quad \varphi_1 y_{t-1} + \theta_0 w_t + \theta_1 w_{t-1}.$$

Show that this can be expressed in the same form as (2).

(b) Ditto for the ARMA(2,1):

$$y_t \quad = \quad \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \theta_0 w_t + \theta_1 w_{t-1}.$$

(c) (extra credit) For the general model (2), what is the distribution of x_{t+2} given x_t?

(d) (extra credit) Ditto for x_{t+k}. Under what conditions does this converge as k gets large?

Matlab program: