

Lab Report #7: Dynamics in Theory and Data

Revised: November 23, 2014

Due at the start of class. You may speak to others, but whatever you hand in should be your own work.

1. *ARMA(1,1) models.* Consider the models

$$x_t = \sum_{j=0}^{\infty} a_j w_{t-j}$$

with iid standard normal innovations w_t and coefficients a_0, a_1 , and $a_{j+1} = \varphi a_j$ for $j \geq 1$ and parameter $0 < \varphi < 1$.

- (a) What is the variance of x ?
- (b) What is the covariance of x_t and x_{t-1} ?
- (c) What is the autocovariance function? The autocorrelation function?
- (d) What configuration of parameter values gives us negative autocorrelations?
- (e) *Extra credit.* Show that the model can be expressed in traditional ARMA(1,1) form,

$$x_t = \varphi x_{t-1} + \sigma(w_t + \theta w_{t-1}).$$

2. *Forward-looking equity prices.* Suppose equity prices are given by the forward-looking difference equation

$$q_t = d_t + \delta E_t(q_{t+1})$$

where E_t means the expectation conditional on the state at date t .

- (a) How is the price related to future dividends? (Ignore bubbles here.)
- (b) Take the AR(1) state variable

$$x_t = \varphi x_{t-1} + \sigma w_t,$$

with the usual iid standard normal w_t 's. What is the price of equity if $d_t = x_t$?

- (c) What is the price of equity if $d_t = x_{t-1}$?

Maturity n	Price q^n
1 year	0.9800
2 years	0.9600
3 years	0.9400
4 years	0.9200
5 years	0.9000

Table 1. Bond prices.

3. *Bond basics.* Consider the bond prices in Table 1.
- (a) What are the yields y^n ?
 - (b) What are the forward rates f^{n-1} ?
 - (c) How are the yields and forward rates related? Verify for y^3 .