Problem 1)

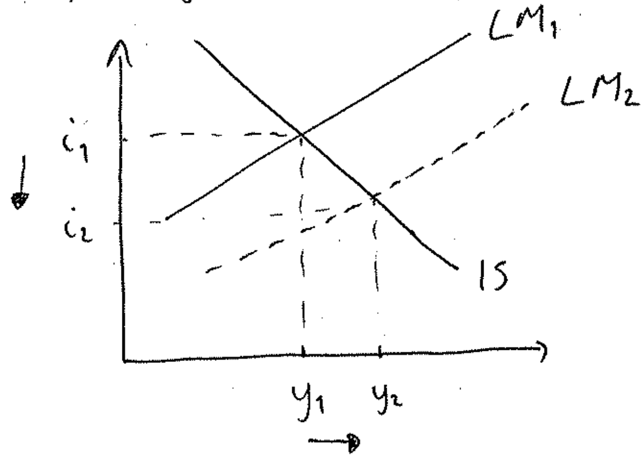
1)
$$y_t = -i_t + E_t\, y_{t+1} \qquad (IS)$$

$$\bar{m}_t - \bar{p} = y_t - \nu i_t \qquad (LM)$$

$$a_t = \mu_t + \gamma_n n_t + y_t = \mu_t + \gamma_n (y_t - a_t) + y_t$$

$$\Rightarrow \quad \mu_t = -(1+\gamma_n)(y_t - a_t) \qquad (AS)$$

2) Graphically:



nominal interest rate, i_t ↓

output, y_t ↑

markup, μ_t ↓

Intuitively: increasing the money supply $\bar{m}_t$ makes more money available and hence, lowers the price of money, that is, decreases the interest rate. Lower interest rates stimulate aggregate demand so output increases. Prices are fixed so when employment and the real wage increase, the mark-up charged by firms decreases.

3) Following the hint:

$$LM \Rightarrow \quad i_t = \frac{1}{\nu} y_t - \frac{1}{\nu}(\bar{m}_t - \bar{p})$$

Insert into IS:

$$IS \Rightarrow \quad y_t = -\frac{1}{\nu} y_t + \frac{1}{\nu}(\bar{m}_t - \bar{p}) + E_t y_{t+1}$$

$$y_t = \frac{1}{1+\nu}(\bar{m}_t - \bar{p}) + \frac{\nu}{1+\nu} E_t y_{t+1}$$

using L.I.E.

$$= \frac{1}{1+\nu}(\bar{m}_t - \bar{p}) + \frac{\nu}{1+\nu}\left(\frac{1}{1+\nu} E_t (\bar{m}_{t+1} - \bar{p}) + \frac{\nu}{1+\nu} E_t y_{t+2}\right)$$

$$= \frac{1}{1+\nu}\left((\bar{m}_t - \bar{p}) + \frac{\nu}{1+\nu} E_t (\bar{m}_{t+1} - \bar{p})\right) + \left(\frac{\nu}{1+\nu}\right)^2 E_t y_{t+2}$$

$$\Rightarrow \quad \boxed{y_t = \frac{1}{1+\nu} E_t \sum_{\tau=0}^{\infty} \left(\frac{\nu}{1+\nu}\right)^{\tau} (\bar{m}_{t+\tau} - \bar{p})} \quad \circledast$$

Consider a one-period increase $\bar{m}_t \to \bar{m}_t + \Delta$.

This implies $\quad y_t \to y_t + \frac{\Delta}{1+\nu} \quad$ (from $\circledast$)

$$\Rightarrow \quad i_t \to i_t + \frac{1}{\nu}\cdot\left(\frac{\Delta}{1+\nu}\right) - \frac{1}{\nu}\cdot\Delta \quad \text{(from LM)}$$

$$= i_t - \frac{\Delta}{1+\nu}$$

$$\mu_t \to \mu_t - (1+\gamma_n)\frac{\Delta}{1+\nu} \quad \text{(from AS)}$$

Consider a two-period increase in $\bar{m}_t$: $\bar{m}_t + \Delta$, $\bar{m}_{t+1} + \Delta$

$$y_t \to y_t + \frac{\Delta}{1+\nu} + \frac{\nu}{(1+\nu)^2}\cdot\Delta = y_t + \frac{(1+2\nu)}{(1+\nu)^2}\Delta$$

So the effect on output (and hence on the markup) is larger when the change in the money supply is persistent. The expected future money supply in addition to today's money supply affects output today.

[Note the effect on the interest rate however:

$$i_t \rightarrow i_t + \frac{1}{\nu}\left(\frac{1+2\nu}{(1+\nu)^2}\right)\cdot\Delta - \frac{1}{\nu}\cdot\Delta$$

$$= i_t + \frac{1}{\nu}\cdot\Delta\left(\frac{1+2\nu-(1+\nu)^2}{(1+\nu)^2}\right)$$

$$= i_t - \frac{\nu}{(1+\nu)^2}\cdot\Delta$$

It decreases <u>less</u> than when the change was temporary.]

4) The larger effect on output from a persistent increase in the money supply comes from the effect of expectations of future policy on demand today (see eqn. ⊛). This effect only occurs if the Federal Reserve clearly communicates its plans for future policy so that expectations are formed in accordance with the actual future plans of the Federal Reserve.

[A related issue is the credibility of the Federal Reserve's communication. By communicationg future plans to the market and later acting according to those plans, the Federal Reserve can build credibility over time.]

Problem 2

1) $y_t - y_t^* = -(i_t - r_t^*) + E_t(y_{t+1} - y_{t+1}^*)$

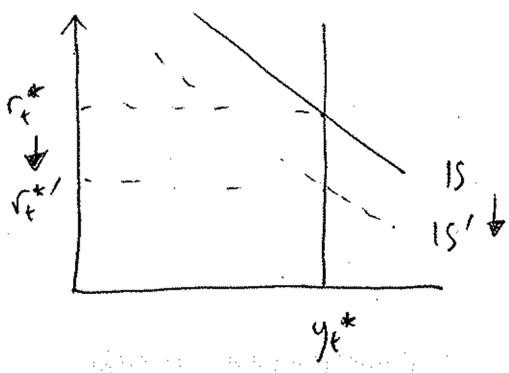
2) $y_t - y_t^* = -(i_t - r_t^*) + E_t\left(-(i_{t+1} - r_{t+1}^*) + E_{t+1}(y_{t+2} - y_{t+2}^*)\right)$

$\Rightarrow \quad y_t - y_t^* = E_t \sum_{\tau=0}^{\infty} -(i_{t+\tau} - r_{t+\tau}^*)$

set $i_t = r_t^*$ and $i_{t+\tau} = r_{t+\tau}^* \quad \forall \tau \geq 1$

3) $y_t^* = -r_t^* + E_t y_{t+1}^* + \chi_t$

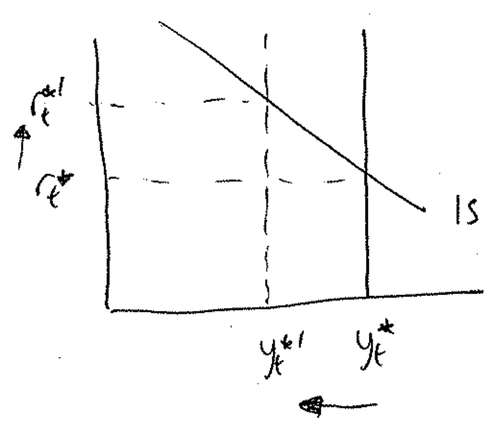
$\chi_t \downarrow \Rightarrow r_t^* \downarrow$ so C.B. set $i_t \downarrow = r_t^*$



A drop in demand is countered by the central bank who cuts interest rates to stimulate demand.

$y_t^* = a_t^* = -r_t^* + E_t y_{t+1}^* + \chi_t$

$a_t \downarrow \Rightarrow r_t^* \uparrow$ so C.B. set $i_t \uparrow = r_t^*$



A drop in productivity leads to a drop in the natural level of output y_t^*. The central bank cannot affect y_t^* through monetary policy, so it raises interest rates to cut demand (and hence match the new natural rate y_t^*).

4)

$$y_t = -i_t + x_t + E_t y_{t+1} = -i_t + x_t + E_t\left(-i_{t+1} + x_{t+1} + E_{t+1} y_{t+2}\right)$$

$$\Rightarrow \quad y_t = E_t \sum_{\tau=0}^{\infty} \left(-i_{t+\tau} + x_{t+\tau}\right)$$

$x_{t+3} \downarrow$

$$y_t^* = E_t \sum_{\tau=0}^{\infty} \left(-r_{t+\tau}^* + x_{t+\tau}\right)$$

$$y_{t+1} = E_{t+1} \sum_{\tau=0}^{\infty} \left(-i_{t+1+\tau} + x_{t+1+\tau}\right)$$

$x_{t+1+2} \downarrow$

A drop in demand 3 periods from today affects output today and in periods ~~1 and 2~~ $t+1$, $t+2$. The central bank wants to keep $y_t, y_{t+1}, y_{t+2}, y_{t+3}$ all at the natural levels of output $y_t^*, y_{t+1}^*, y_{t+2}^*, y_{t+3}^*$. To do so it must create the expectation that when the demand shock hits at $t+3$ it will be countered by a decrease in interest rates at $t+3$. By managing expectations appropriately the central bank can eliminate the negative effects of the news of a future demand drop.

Problem 3

1)

$$y_t = -\frac{C}{Y}\cdot \bar{i}_t + E_t y_{t+1} + \frac{C}{Y} X_t + \frac{G}{Y}\left(g_t - E_t g_{t+1}\right)$$

$$= \frac{C}{Y}\left(X_t - \bar{i}_t\right) + \frac{G}{Y}\left(g_t - E_t g_{t+1}\right) + E_t y_{t+1}$$

$$= \frac{C}{Y}\left(X_t - \bar{i}_t\right) + \frac{G}{Y} g_t - \cancel{\frac{G}{Y} E_t g_{t+1}}$$

$$+ E_t \frac{C}{Y}\left(X_{t+1} - \bar{i}_{t+1}\right) + \cancel{\frac{G}{Y} E_t g_{t+1}} - \frac{G}{Y} E_t g_{t+2} + E_t y_{t+2}$$

$$\vdots$$

$$\Rightarrow \quad y_t = E_t \sum_{\tau=0}^{\infty} \frac{C}{Y}\left(X_{t+\tau} - \bar{i}_{t+\tau}\right) + \frac{G}{Y}\cdot g_t$$

A drop in X_t should be countered by a drop in the interest rate $\bar{i}_t$ to keep y_t unchanged. However, if the drop in X_t is large the necessary drop in $\bar{i}_t$ ~~must~~ is large as well. Therefore, a large enough drop in X_t implies that the required drop in $\bar{i}_t$ is too large as interest rates cannot drop below 0.

2) If $X_t \to X_t - \Delta$ and the interest rate $\bar{i}_t$ cannot change since it is at the lower bound, then the government can increase $g_t \to g_t + \frac{C}{G}\Delta$ to counter the effect on y_t $\left(y_t \to y_t - \frac{C}{Y}\Delta + \frac{G}{Y}\cdot\frac{C}{G}\Delta = y_t\right)$. By raising g_t even further, the government can raise interest rates and hence move out of the liquidity trap.

$(\Delta x > 0,\ x_t \to x_t - \Delta x)$

3) Let's say x_t drops by amount Δx, and we want policy that leaves y_t at y_t^* (by changing i_t and g_t).

$$y_t = y_t - \frac{C}{Y} \cdot \Delta x - \frac{C}{Y} \cdot \Delta i + \frac{G}{Y} \Delta g$$

where Δi is the change in i_t and Δg is the change in g_t.

What matters is that $-\frac{C}{Y} \Delta x - \frac{C}{Y} \Delta i + \frac{G}{Y} \Delta g = 0$

$$\Rightarrow \quad \Delta x = -\Delta i + \frac{G}{C} \cdot \Delta g$$

So we can for example set $\Delta i = -\Delta x$ and $\Delta g = 0$. That is, reduce interest rate to counter fall in demand. Or we can set $\Delta i = 0$ and set $\Delta g = \frac{C}{G} \Delta x$. That is, increase gov. spending to counter fall in demand.

We can also do both, and we see that the larger the drop in interest rates the smaller is the required increase in spending. The required spending is given by

$$\Delta g = \frac{C}{G} \left(\Delta x + \Delta i \right)$$

So the closer Δi is to $-\Delta x$ the smaller is Δg.

4) With no fiscal policy:

$$y_t = \mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{c}{Y} \left(x_{t+\tau} - \bar{\iota}_{t+\tau} \right)$$

Large drop $x_t \to x_t - \Delta x$, $\Rightarrow$ cannot reduce $\bar{\iota}_t$ enough.

Monetary policy can be used to move the economy out of the liquidity trap by committing to maintaining low interest rates for several periods.

If $\bar{\iota}_{t+\tau} \to \bar{\iota}_{t+\tau} - \frac{\Delta x}{T}$ for T periods, then as long as T is large enough $\frac{\Delta x}{T}$ will be small enough that the zero lower bound never binds, and the effect of the drop in demand on output will be cancelled out.

$$y_t \to y_t - \frac{c}{Y} \Delta x - T \cdot \frac{c}{Y} \left(- \frac{\Delta x}{T} \right) = y_t .$$