Lab Report #5: Options & Volatility

(Started: August 20, 2011; Revised: October 16, 2013)

Due at the start of class. You may speak to others, but whatever you hand in should be your own work.

1. (BSM formula) We'll examine the BSM formula in some purely theoretical numerical examples. In what follows, the current price of the underlying is 100, the option maturity is one year, and the one-year bond price is 0.98.
   (a) If volatility $\sigma = 0.10$, what are the prices of call options at strike prices of 90, 100, and 110?
   (b) What are the prices of put options with the same strikes?
   (c) If volatility rises to $\sigma = 0.15$, what happens to the prices of calls?
   (d) For strikes of 90, 100, and 110, graph the call price against volatility σ using a grid between (roughly) 0.01 and 0.50. (This gives you three lines, one for each strike.) How do call prices vary with volatility? Does the pattern vary with the strike price?
   (e) For a strike of 110 and a call price of 2.00, what is the implied value of σ?

2. (volatilities on S&P 500 E-mini options) For the S&P 500 E-mini options, the prices of options are more conveniently expressed in terms of their implied volatilities. We'll compute them here for quotes reported on March 15, 2012:

	Call Price		Put Price	
Strike	Bid	Ask	Bid	Ask
1340	82.50	85.75	28.25	30.25
1350	75.25	78.25	30.75	32.75
1360	68.00	71.00	33.25	35.50
1370	61.25	64.00	36.25	38.75
1380	54.50	57.25	39.50	42.25
1390	48.25	50.75	43.25	45.75
1400	42.25	44.75	47.00	50.00
1410	36.75	39.25	51.25	54.50
1420	31.75	33.75	56.00	59.25
1430	27.00	29.00	61.25	64.50

The price of the underlying contract was 1395.75. The interest rate was essentially zero, so the appropriate bond price was one. The options expire June 15, so $\tau = 3/12 = 1/4$.

   (a) Compute "mid" quotes as averages of bid and ask. Use put-call parity to compute call prices from mid puts. Plot call prices — bid, ask, and implied by puts — against the strike. How do they compare?

(b) Write a program using, say, the secant method to compute implied volatilities for mid quotes of call options. Graph them against the strike. What shape does the resulting "smile" have? What does the shape suggest to you?

Matlab program: