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Lecture 3

One Period Competitive Equilibrium Model of Output and Employment

The canonical model of long run equilibrium is the competitive equilibrium model. Here I present a static competitive equilibrium model and illustrate how it determines the aggregate value of output and employment. After finishing this we will turn to an intertemporal model where saving and investment are determined as well.

1 Environment

Assume:

(i) One periods: Only consumption goods produced.

(ii) One representative household that: consumes, supplies labor, and receives dividend income (from ownership of firms.)

(iii) One representative firm that produces output, demands labor, and pays dividends to households.

(iv) The household and the firm act competitively, i.e., each takes market prices as given.

We next characterize preferences, technology and resource constraints:

Preferences

The household consumes and works in period 1. Let C^i be household consumption and N^i be household labor supply in period. Then household preferences are given by

$$u(C^i) - \nu(N^i) \tag{1}$$

with :

$u(0) = 0$, $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'(0) = \infty$, $u'(\infty) = 0$,

$\nu(0) = 0$, $\nu'(\cdot) > 0$, $\nu''(\cdot) > 0$, $\nu'(0) = 0$, $\nu'(\infty) = \infty$,

These properties imply that $u(\cdot)$ is increasing and concave and that $\nu(\cdot)$ is increasing and convex. [1] Concavity of $u(\cdot)$ implies diminishing marginal utility of consumption, while convexity of $\nu(\cdot)$ suggests increasing marginal disutility from labor supply.

[1] The conditions $u'(0) = \infty$, $u'(\infty) = 0$, $\nu'(0) = 0$, $\nu'(\infty) = \infty$ are known as Inada conditions and guarantee an interior solution in equilibrium with positive and finite values of C_i and N.

Technology

The firm produces output in period 1 using labor input. Let Y_j be output by firm j and N^j employment. Then each period, production is given by

$$Y^j = Af(N^j) \tag{2}$$

The production function has the following properties:

$f(0) = 0,\ f'(\cdot) > 0,\ f''(\cdot) < 0,\ f'(0) = \infty,\ f'(\infty) = 0$,where A is total factor productivity. Observe that the production function exhibits diminishing marginal product: Output is increasing in employment, but at a decreasing rate as the level of employment rises. One interpretation is that output depends on both capital and labor input (i.e., $Y^j = Af(K^j, N^j)$, but that capital is fixed in the short run at $\overline{K}$. Since capital is fixed, we may suppress it, (i.e, $Y^j = Af(\overline{K}., N^j) = Af(N^j)$). Diminshing marginal product of labor then arises in the short run because capital is fixed in supply.

Economy-Wide Resource

Aggregate output equals aggregate consumption:

$$\int_0^1 Y^j dj = Y = C = \int_0^1 C^i di \tag{3}$$

2 Household and Firm Behavior

The Household's Decision Problem

Let Π be lump sum dividends (all households receive the same), and W the real wage. The household chooses C^i, to solve N^i.

$$\max u(C^i) - \nu(N^i) \tag{4}$$

subject to:

$$C^i = WN^i + \Pi \tag{5}$$

The household takes as given W and Π...

To be concrete, let:

$$\begin{aligned} u(C^i) &= \frac{1}{1-\sigma}(C^i)^{1-\sigma};\ \sigma > 0 \text{ and } \neq 1 \\ &= \log C \text{ if } \sigma = 1 \end{aligned}$$

$$\nu(N^i) = b\frac{1}{1+\varphi}(N^i)^{1+\varphi}$$

To solve the household's decision problem, it is simplest to turn the constrained problem into an unconstrained one by plugging (5) into (4). This is possible as long as the budget constraints are always binding, so that there is no unused income. This latter condition is ensured by the assumptions we made on $u(\cdot)$.

The representative household accordingly chooses and N^i to solve

$$\max \frac{1}{1-\sigma}(WN^i + \Pi_1)^{1-\sigma} + \frac{1}{1+\varphi}(N^i)^{1+\varphi},$$

given W and Π..

The first order necessary condition with respect to N^i is given by:

$$\underbrace{W(C^i)^{-\sigma}}_{\text{MB of Labour Supply}} - \underbrace{b(N^i)^{\varphi}}_{\text{MC of Labour Supply}} = 0 \tag{6}$$

The household adjusts labor supply until marginal benefit in utility terms, the real wage times the marginal utility of consumption, equals the marginal disutility of labor effort. Our restrictions on preferences guarantee that equation (6) describes a local optimum (i.e., that the second order condition holds.) To express marginal benefit and cost in units of consumption goods, divide by $u'(C_1^d)$ to obtain:

$$W = b\frac{(N^i)^{\varphi}}{(C^i)^{-\sigma}}. \tag{7}$$

The marginal benefit in units of consumption goods is simply the real wage. The marginal cost in units of consumption goods is $\nu'(N^S)$ normalized by the marginal utility of consumption, $u'(C_1^d)$.

The labor supply curve is defined as combinations of W and N^i that satisfy equation (7), given C^i.. Note that there are two "channels" via which a shift in W may affect N^S. The first is a substitution effect: a rise in W raises the marginal benefit of working (the left side). The second is a wealth effect that raises the marginal cost by reducing the marginal utility of an additional unit of consumption (the denominator on the right side.) The substitution effect induces a rise in labor effort, while the income effect induces a decline.

The parameter φ^{-1} is known as the Frisch elasticity of labor supply. Taking logs of each side of (7) yields,

$$\log W = \log b + \varphi \log N^i + \sigma \log C^i$$

Rearranging:

$$\log N^i = \varphi^{-1}[\log W - \log b - \sigma \log C^i]$$

Holding constant consumptio, the percent response of hours to a percent change in wages is φ^{-1}.

Finally, note that we can graph in (7) in (W, N) space, holding C constant. The collection of points where (7) is satisfed is know as the "labor supply curve." Since $b\frac{(N^i)^{\varphi}}{(C^i)^{-\sigma}}$ is increasing in N^i, holding C^i constant, the labor supply curve is upward sloping.

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The Firm

The firm maximizes profits returned to the household. Accordingly, the firm chooses N^j, Y^j, and Π^j to solve

$$\max \Pi^j \tag{8}$$

subject to:

$$\begin{aligned} \Pi^j &= Y^j - WN^j && (9) \\ &= Af(N^j) - WN^j && (10) \end{aligned}$$

taking as given W .

For convenience let

$$f(N^j) = (N^j)^{1-\alpha}$$

Note this implies $f' = (1-\alpha)(N^j)^{-\alpha} > 0$ and $f'' = -\alpha(1-\alpha)(N^j)^{-\alpha} < 0$.

Again, we can convert the problem into an unconstrained maximization problem by plugging the constraints (9) into (8). The firm then chooses N^j to solve:

$$\max A(N^j)^{1-\alpha} - WN^j$$

The first order necessary conditions for labor is given by:

$$(1-\alpha)A(N^j)^{-\alpha} = W \tag{11}$$

According to equation (11), the firm adjusts labor demand until the marginal benefit, given by the marginal physical product of labor, equals the marginal cost, given by the real wage.

From the firm's decision problem we obtain a demand curves for labor: The labor demand curve is given by combinations of W and N^d that satisfy equation (11). .Note that the demand curve is downward sloping in (W, N) space, due to diminishing marginal product of labor.

Competitive Market Equilibrium

Since all households are identical and all firms are identical, we can restrict attention to symmetric equilibrium (where consumption and labor supply is the same for each household and employment and output are the same for each firms.). Further, we normal the number of households and firms each to unity.

A competitive equilibrium for this economy is an allocation (N, Y, C) and a relative price vector $(W,)$ such that the household and the firm is each maximizing its respective objective, markets clear, and the economy resource constraints are satisfied. In practice, to determine the three quantity variables: N, Y, C, we need three independent relations:

Labor Market:

$$(1-\alpha)A(N)^{-\alpha} = b\frac{(N)^{\varphi}}{(C)^{-\sigma}}. \tag{12}$$

Resource Constraint

$$Y = C \tag{13}$$

Technology Constraints:

$$Y = AN^{1-\alpha}$$

Wages can be read off the market equilibrium.

$$W = (1-\alpha)A(N)^{-\alpha} = b\frac{(N)^{\varphi}}{(C)^{-\sigma}} \tag{14}$$

The equilibrium is simple enough to solve by hand:

$$(1-\alpha)A(N)^{-\alpha} = b\frac{(N)^{\varphi}}{(AN^{1-\alpha})^{-\sigma}}$$

$$\begin{aligned}
(1-\alpha)A^{1-\sigma} &= bN^{\alpha+\varphi+\sigma(1-\alpha)} \\
N &= (\frac{1-\alpha}{b}A^{1-\sigma})^{\frac{1}{\alpha+\varphi+\sigma(1-\alpha)}}
\end{aligned}$$

It is then easy to solve for C and Y.

3 Extensive versis Intensive Margin

Let $H \equiv$ total hours and $N \equiv$ total employment. Then by definition

$$H = \frac{H}{N} \cdot N$$

That is, total hours equals hours per worker time the total number of workers. Over the business cycle, most of the movement in hours is due to changes in employment (the extensive margin) as opposed to hours per worker (the intensive margin). Roughly two thirds of cyclical hours movement is due to employment changes and one third to hours per worker changes. Employment changes play an even biggger role in large recessions.

That hours is driven mostly by employment explains why hours and unemployment U have a strong inverse relation over the cycle:

$$U = \frac{L - N}{L} = \frac{L - \frac{N}{H} \cdot H}{L}$$

where L is labor force participation. Given that $\frac{N}{H}$ is relative stable, U varies inversely with H.

In the model thus far, employment is varies along the intensive margin (hours per worker). Here we illustrate how, under certain assumptions, the model may capture adjustment along the extensive margin.

I: Labor Supply Adjustment Along the Intensive Margin

Let C_t and N_t denote consumption and hours worked, respectively. Assume a representative agent with preferences given by

$$\frac{1}{1-\sigma}\ C_t^{1-\sigma} - \frac{1}{1+\varphi}\ N_t^{1+\varphi}$$

It follows that the F.O.NC. are given by

$$W_t = MRS_t = C_t^{\sigma}\ N_t^{\varphi}$$

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Case II: Labor Supply Adjustment Along the Extensive Margin

Now assume that individuals either do not work or work a fixed amount of hours per week. Suppose there is a representative household with a continuum of members represented by the unit interval, and who differ according to their disutility of work. Specifically, let us assume that z^{φ} is the disutility of work for member j. Under perfect consumption insurance within the household, and interpreting N_t as the fraction of working household members in period t, total household utility will be given by

$$\int_0^1 \frac{1}{1-\sigma}\ C_t^{i1-\sigma} dz - \int_0^{N_t} z^{\varphi}\ dz$$
$$\frac{1}{1-\sigma}\ C_t^{1-\sigma} - \int_0^{N_t} z^{\varphi}\ dz$$

Note that

$$\int_0^{N_t} j^{\varphi} dj = \frac{1}{1+\varphi} N_t^{1+\varphi}$$

Accordingly, the utility function for the family in this case is isomorphic to case of adjustment along the intensive margin. It follows that F.O.NC. will be the same as in the previous case..