

Lab Report #5: Returns & Risk Premiums

Revised: October 28, 2014

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *Disaster risk and the equity premium.* We add a “disaster” state to our analysis of the equity premium and see how it changes our perspective. We know power utility agents dislike negative skewness, so we conjecture that an asset with a disaster outcome will have a large risk premium. The key input is the distribution of log consumption growth,

$$\log g = \begin{cases} \mu - \sigma & \text{with probability } (1 - \omega)/2 \\ \mu + \sigma & \text{with probability } (1 - \omega)/2 \\ \mu - \theta & \text{with probability } \omega. \end{cases}$$

If $\omega = 0$, we’re back to our symmetric two-state distribution. But if we choose a small positive value of ω and a “largish” $\theta > 0$, we have a “disaster” state that changes the distribution dramatically.

We’ll define equity as a claim to consumption growth g and explore the impact of a disaster state on the equity premium. Suppose, as usual, that we have a representative agent with power utility. Preference parameters are $\beta = 0.99$ and $\alpha = 10$ throughout.

- (a) Verify that the mean and variance of log consumption growth are

$$\begin{aligned} E(\log g) &= \mu - \omega\theta \\ \text{Var}(\log g) &= (1 - \omega)(\sigma^2 + \omega\theta^2). \end{aligned}$$

- (b) If $\omega = 0$, what values of μ and σ deliver the observed mean and variance of log consumption growth, namely 0.0200 and 0.0350²?
- (c) With these parameter values, compute the equity premium and its inputs.
- (d) Now consider $\omega = 0.01$ and $\theta = 0.30$. (These numbers are based on a series of studies by Robert Barro and his coauthors.) With these numbers, what values of μ and σ reproduce the observed mean and variance of log consumption growth?
- (e) Compute the equity premium and its components with these parameter values. How does it compare to your previous calculation?
- (f) What are the maximum Sharpe ratios when $\omega = 0$ and when $\omega = 0.01$? How do they compare to the Sharpe ratio we observe for equity in US data?
- (g) What is entropy when $\omega = 0$ and when $\omega = 0.01$?
- (h) Summarize what you’ve found. How does a disaster state affect the magnitude of risk premiums in this economy?

2. *Valuing put options* . We can value options the same way we value any other asset, it's simply a matter of getting the cash flows right. With future topics in mind, we use a discrete approximation to a normal mixture, which we use to produce departures from normality: skewness, excess kurtosis, and so on. (No, that's not an oxymoron; see the notes on random variables.)

To be specific, we'll use two normal distributions as inputs. The first one has mean zero and variance one, the second has mean θ and variance δ . We denote their pdf's by p_1 and p_2 , respectively. The pdf of our state variable z is

$$p(z) = (1 - \omega)p_1(z) + \omega p_2(z)$$

for some mixing parameter ω between zero and one. Roughly speaking, θ governs skewness and δ governs excess kurtosis. We'll set $\omega = 0.2$, $\theta = -1$, and $\delta = 2$. We construct a discrete approximation with the Matlab commands

```
% grid
zmax = 6;
dz = 0.1;
z = [-zmax:dz:zmax]';
% mixture
omega = 0.2; theta = -1; delta = 2;
p1 = exp(-z.^2/2)*dz/sqrt(2*pi);
p2 = exp(-(z-theta).^2/(2*delta))*dz/sqrt(2*pi*delta);
p = (1-omega)*p1 + omega*p2;
```

We'll use the usual representative agent setup. Log consumption growth is

$$\log g(z) = \mu + \sigma z.$$

The mean and variance are then

$$\begin{aligned} E(\log g) &= \mu + \omega\theta \\ \text{Var}(\log g) &= \sigma^2[(1 - \omega) + \omega(\theta^2 + \delta)]. \end{aligned}$$

With power utility, the pricing kernel is

$$m(z) = \beta \exp(-\alpha \log g) = \beta \exp[-\alpha(\mu + \sigma z)].$$

We'll set $\beta = 0.99$ and $\alpha = 1$.

- Plot p (the mixture) and p_1 (the standard normal). How do they differ?
- Use the commands

```
meanz = sum(p.*z)
stdz = sqrt(sum(p.*(z-meanz).^2))
skewz = sum(p.*(z-meanz).^3)/stdz^3
xkurz = sum(p.*(z-meanz).^4)/stdz^4 - 3
```

to compute properties of the mixture distribution. What indications do they give us that the distribution is not normal? What are the skewness and excess kurtosis of $\log g$?

- (c) Choose values of μ and σ to reproduce the mean (0.0200) and standard deviation (0.0350) of $\log g$.
- (d) Define equity as a claim to the dividend $d(z) = 100 g(z)$. What is its price? Its expected return?
- (e) A put option with “strike price” k gives the owner the right to sell the dividend $d(z)$ at date 1 for price k . The owner of the option will only do this if $d(z) < k$, which gives rise to the cash flow

$$f(z) = \max\{0, k - d(z)\}.$$

You can compute this in Matlab with the commands

```
k = 125;
f = (k-d).*(k>d);
```

(Ask yourself what $(k>d)$ does. Try it if you're not sure.) Graph the cash flow $f(z)$ versus the dividend $d(z)$ for $k = 125$.

- (f) What is the price (at date 0) of the option? (Note: This is a claim to the cash flow f and priced as we would any other cash flow.) What is its expected return?