

Final Exam

Problem 1

Let $y_t \equiv$ real output, $i_t \equiv$ nominal interest rate, $\pi_t \equiv$ inflation, $\chi_t \equiv$ demand shock, and $y_t^* =$ natural level of output. All variables are percent deviations from their respective steady state values. Then consider the following simple economy

IS:

$$y_t = -(i_t - E_t \pi_{t+1}) + E_t y_{t+1} + \chi_t$$

LM:

$$i_t = \bar{i}_t$$

AS:

$$\pi_t = \lambda(y_t - y_t^*) + E_t \pi_{t+1}$$

Assume further that inflationary expectations are formed adaptively, as follows:

$$E_t \pi_{t+1} = \pi_{t-1}$$

1. Explain how a positive demand shock (increase in χ_t) could produce a inflationary spiral. (You may want to use graphs.)
2. Explain how a negative supply shock (fall in y_t^*) could produce an inflationary spiral.
3. Suppose you are running the Fed and your goal is to keep y_t close to y_t^* and π_t close to zero. Describe how you would adjust i_t in each scenario (diagrams will suffice).

Problem 2

Let $x_t = y_t - y_t^*$ and $r_t^* \equiv$ the natural real rate of interest. Then consider the following simple model:
IS:

$$x_t = -[(i_t - E_t \pi_{t+1}) - r_t^*] + E_t x_{t+1}$$

AS:

$$\pi_t = \lambda(x_t) + E_t \pi_{t+1}$$

Suppose further that each period the central bank cares about x_t and π_t according to the following objective function:

$$-[\alpha(x_t - k)^2 + \pi_t^2]$$

where k is the target value of the output gap and zero is the target value of inflation, and where α is the relative weight on output gap deviations from target. According to this objective, the central bank would like to keep the output gap and inflation as close as possible to their respective targets.

Assume further that individuals have rational expectations, i.e., $E_t \pi_{t+1}$ is the same forecast that the model generates.

1. Show how, within this framework, steady state inflation will be inefficiently above target if the central bank cannot commit to keeping it at target.
2. How does the steady state nominal interest rate i behave in this case? (Hint: use the IS curve at steady state to back out i .)
3. Suppose that instead of the rule given by 2., the central bank can commit to the following rule:

$$i_t = r_t^* + E_t x_{t+1} + (1 + \phi) E_t \pi_{t+1}$$

with $\phi > 0$. First, show that if the central bank can commit to following this rule, then the only steady state for π_t is zero. (Hint: combine the rule with the IS and AS curves and then examine the steady state.) Second, show that so long as $\phi > 0$, inflation will always revert to the zero steady state. (Hint: use the IS, AS and policy rule equations to find an expression for the expected change of inflation, $E_t \pi_{t+1} - \pi_t$ as a linear function of the current level π_t and then show that whether inflation is falling or rising when $\pi_t > 0$ depends on the sign of ϕ .) Explain intuitively why $\phi > 0$ helps keep inflation stable.

Problem 3

Let inv_t denote investment, g_t government spending and q_t the relative price of capital. Then consider the following macro model with investment.

Aggregate Demand

$$\begin{aligned}y_t &= \frac{C}{Y}c_t + \frac{I}{Y}inv_t + \frac{G}{Y}g_t \\c_t &= -i_t + E_t c_{t+1} \\inv_t &= \frac{1}{zb}q_t\end{aligned}$$

$$q_t = E_t \sum_{i=0}^{\infty} \left(\frac{1}{R_k}\right)^i \left[\frac{R_k - 1}{R_k} y_{t+1+i} - \frac{1+i}{R_k} ((i_{t+i} - \pi_{t+1+i}) + \psi_{t+i}) \right]$$

Aggregate Supply

$$\begin{aligned}y_t &= a_t + n_t \\ \pi_t &= \lambda(y_t - y_t^*) + E_t \pi_{t+1}\end{aligned}$$

Monetary sector

$$m_t - p_t = y_t - \nu i_t + \varepsilon_t$$

$$i_t = \overline{i_t}$$

1. First collapse the model to IS/LM and AS curves (note that we are assuming the central bank follows an interest rate rule.).
2. Use the IS/LM/AS curves to describe how the recent financial crisis (manifested by an increase in ψ_{t+i}) produced a severe recession. You may also supplement your analysis with algebraic relations, but a cogent use of diagrams along with precise explanations can suffice.
3. Describe how policy-makers stimulated the economy when the nominal interest rate was at the lower bound. In what way was the Federal Reserve's promise to keep interest rates low for "an extended period" helpful?