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Intermediate Macro
Spring 2011

Lecture 8
A Model of Short Run Fluctuations

We now turn to developing a macroeconomic model of the business cycle. Our approach is to start with the baseline competitive equilibrium model and then add frictions that permit the framework to explain business cycles. The framework will also be useful for study how monetary and fiscal policies might be used to tame the cycle.
There key departures from the baseline competitive model: 1. Money is introduced; 2. There is imperfect competitive; 3 There rigidities in price setting (known as nominal rigidities. (1) allows nominal variables to be introduced and also permits an analysis of monetary policy; (2) permits introducing imperfect nominal price adjustment and also implies that the flexible price equilibrium level of output (the "natural" level of output) will lie below is socially efficient level; (3) implies that variations in the nominal money stock can affect real economic activity and that fluctuations in output about the natural level are possible.
For ease of exposition we start with a model that has consumption goods only. Later we add investment and government spending. Eventually we also introduce financial market frictions.

1 Baseline Model

The model includes households, firms and a government sector:

- The representative household consumes a final good C_t, supplies labor N_t, holds real money balances M_t/P_t, saves in the form of private bonds B_t (which, in equilibrium will be in zero net supply, since everyone is the same.)
- Firms (assume they have mass unity) are monopolistic competitors and each produce a differentiated product $Y_t(f)$ using labor $N_t(f)$. These firms set nominal prices $P_t(f)$.
- The government sector consists of a central bank that conducts monetary policy and a fiscal authority that pays lump sum transfers.

1.1 The household's problem

The representative household chooses $\{C_{t+i}, N_{t+i}, \frac{M_{t+i}}{P_{t+i}}, \frac{B_{t+i+1}}{P_{t+i}}\}_{i=0}^{\infty}$ to maximize

$$E_t\left\{\sum_{i=0}^{\infty}\beta^i\left[\log C_{t+i} + a_m \log\left(\frac{M_{t+i}}{P_{t+i}}\right) - \frac{a_n}{1+\gamma_n}N_{t+i}^{1+\gamma_n}\right]\right\} \tag{1}$$

subject to

$$C_t = \frac{W_t}{P_t} N_t + +\Pi_t + TR_t - \frac{M_t - M_{t-1}}{P_t} - \frac{\left(\frac{1}{1+i_t}\right) B_{t+1} - B_t}{P_t} \tag{2}$$

where $\gamma_n > 0$. Let $\sigma = 1/\gamma$ W_t/P_t be the real wage, the profits from owning the monopolistic competitive firms, TR_t government transfers, and i_t the nominal interest rate..
The FONCs for this maximization problem are:

$$\frac{W_t}{P_t} = \frac{a_n N_t^{\gamma_n}}{1/C_t} \text{ (labor supply)} \tag{3}$$

$$1 = E_t \left\{ \frac{P_t}{P_{t+1}} \beta \frac{1/C_{t+1}}{1/C_t} \right\} + \frac{a_m/(M_t/P_t)}{1/C_t} \text{ (money demand)} \tag{4}$$

$$1 = E_t \left\{ R_{t+1} \beta \frac{1/C_{t+1}}{1/C_t} \right\} \text{ (consumption/saving)} \tag{5}$$

where R_{t+1} is the real interest rate, defined as

$$R_{t+1} \equiv (1 + i_t) \frac{P_t}{P_{t+1}} \tag{6}$$

Combining the money and bond pricing equations (4) and (5), yields the money demand equation:

$$\frac{M_t}{P_t} = a_m \left(1 - \frac{1}{(1 + i_t)}\right)^{-1} C_t \tag{7}$$

1.2 Firms

There is a continuum of intermediate good firms owned by consumers, indexed by $f \in [0, 1]$. Each firm uses both labor $N_t(f)$ and capital $K_t(f)$ to produce output according to the following constant returns to scale technology[1]:

$$Y_t(f) = A_t N_t(f) \tag{8}$$

where A_t is a technology parameter. Each firm faces the following demand functin, where $\varepsilon > 1$, is the relative price elasticity.

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t \tag{9}$$

$$\Pi_t = \frac{P_t(f)}{P_t} Y_t(f) - \frac{W_t}{P_t} N_t(f)$$

Combining equations yields the following unconstrained problem, with $P_t(f)$ as the choice variable.

$$\Pi_t = \frac{P_t(f)}{P_t} \left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t - \frac{W_t}{P_t} \frac{\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t}{A_t}$$

The first order necessary conditions are given by

[1] Recall that capital at the aggregate level is predetermined: $K_t = \int_0^1 K_t(f)\, df$.

$$(1-\varepsilon)\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon} Y_t - \varepsilon\frac{\frac{W_t}{P_t}}{A_t}\left[\frac{P_t(f)}{P_t}\right]^{-\varepsilon-1} Y_t = 0$$

which yields

$$\frac{P_t(f)}{P_t} = \frac{1}{1-1/\varepsilon}\frac{\frac{W_t}{P_t}}{A_t}$$

$$\frac{P_t(f)}{P_t} = (1+\mu)\frac{\frac{W_t}{P_t}}{A_t}$$

where μ is the net markup. Note that in the flex price case, the firm adjust price to hit a desired net markup μ.

By contrast, in the fixed price case, the firm produces to meet demand so long as it is doesn't lose money at the margin; so long as $\mu_t(f) \geqq 0$. In this case, the markup $\mu_t(f)$ is determined as a residual by the relation between relative price $\frac{\overline{P}_t(f)}{P_t}$ and marginal cost.

$$\frac{\overline{P}_t(f)}{P_t} = (1+\mu_t(f))\frac{\frac{W_t}{P_t}}{A_t};$$

We restrict attention to symmetric equilibria where all firms behave the same way. Thus, $\frac{P_t(f)}{P_t} = 1$ and $\frac{\overline{P}_t(f)}{P_t} = 1$. It follows that:

Symmetric Equilibrium: fixed price case:

$$1 = (1+\mu_t(f))\frac{\frac{W_t}{P_t}}{A_t}$$

$$A_t = (1+\mu_t)\frac{W_t}{P_t}$$

Symmetric equlibrium: Flex price case:

$$A_t = (1+\mu)\frac{W_t}{P_t}$$

1.3 Aggregation, resource constraints and government policy

Aggregate Production

$$\begin{aligned} Y_t &= \int_0^1 Y_t(f)df && (10) \\ &= A_t \int_0^1 N_t(f)df && (11) \\ &= A_t N_t && (12) \end{aligned}$$

Resource constraints

Since we assumed that capital was in fixed supply, we have:

$$Y_t = C_t \tag{13}$$

Government policy

The central bank sets the money supply exogenously:

$$M_t = \overline{M}_t \tag{14}$$

where Y_t^* denotes the natural (i.e. flexible price equilibrium) level of output.
Any seigniorage revenue is rebated lump-sum to the households, so that the government budget constraint is given by:

$$\frac{M_t - M_{t-1}}{P_t} = TR_t \tag{15}$$

2 Equilibrium

There are seven variables to be determined: $(Y_t, C_t, N_t, P_t, M_t, \mu_t, i_t)$. In the fixed price equilibrium, P_t is fixed and μ_t varies. The reverse is true in the flex price equilibrium.

Aggregate Demand

$$Y_t = C_t \tag{16}$$

$$C_t = E_t \left\{ (1+i_t) \frac{P_t}{P_{t+1}} \beta \frac{1}{C_{t+1}} \right\}^{-1} \tag{17}$$

Equation (17) describes the IS curve relating aggregate demand (which is equal only to consumption in this model) to the inverse of the interest rate.

Aggregate Supply

$$Y_t = A_t N_t \tag{18}$$

$$A_t = (1+\mu_t) \frac{a_n N_t^{\gamma_n}}{(1/C_t)} \tag{19}$$

$$P_t = \overline{P} \tag{20}$$

Monetary Policy Rule

$$\frac{M_t}{P_t} = a_m \left(1 - \frac{1}{(1+i_t)} \right)^{-1} C_t \tag{21}$$

$$M = \overline{M}$$

Simplified System:
IS Curve:

$$Y_t = E_t \left\{ (1+i_t) \beta \frac{1}{Y_{t+1}} \right\}^{-1}$$

LM Curve:

$$\frac{\overline{M}}{\overline{\overline{P}}} = a_m \left(1 - \frac{1}{(1+i_t)}\right)^{-1} Y_t$$

AS Curve:

$$\frac{1}{(1+\mu_t)} = a_n(\frac{Y_t}{A_t})^{1+\gamma_n}$$

Flex Price Model

$$Y_t^* = E_t \left\{ [(1+i_t)\frac{P_t}{P_{t+1}}]^* \beta \frac{1}{Y_{t+1}^*} \right\}^{-1}$$

$$\frac{\overline{M}}{P_t} = a_m \left(1 - \frac{1}{(1+i_t)}\right)^{-1} Y_t^*$$

$$\frac{1}{(1+\mu)} = a_n(\frac{Y_t}{A_t}^*)^{1+\gamma_n}$$

So the simplified fixed-price system becomes:
IS Curve:

$$Y_t = E_t \left\{ (1+i_t)\beta \frac{1}{Y_{t+1}} \right\}^{-1}$$

LM Curve:

$$\frac{\overline{M}}{\overline{\overline{P}}} = a_m \left(1 - \frac{1}{(1+i_t)}\right)^{-1} Y_t$$

AS Curve:

$$\frac{1+\mu}{(1+\mu_t)} = (\frac{Y_t}{Y_t^*})^{1+\gamma_n}$$

3 Loglinear Approximation

Consider a loglinear approximation of the function $f(X_t)$ about $f(X)$. Start with the first order approximation:

$$\begin{aligned} F(X_t) &\approx f(X) + \frac{\partial f}{\partial X} dX_t \\ &\approx f(X) + \frac{\partial f}{\partial X} X \frac{dX_t}{X} \end{aligned}$$

where $\frac{\partial f}{\partial X}$ is evaluated at $X_t = X$. Next, let x_t denote the log deviation of X_t from X, i.e., $x_t = \log X_t - \log X$. Then for a small percent changes in X_t:

$$x_t \approx \frac{dX_t}{X}$$

Thus we can write:

$$f(X_t) \approx f(X) + \frac{\partial f}{\partial X} X x_t$$

or equivalently:

$$f(X_t) - f(X) \approx \frac{\partial f}{\partial X} X x_t$$

Taking a loglinear approximation of the IS/LM model around the flex price equilibrium with the steady state value of A_t, yields:
IS:

$$y_t = -i_t + E_t y_{t+1}$$

LM:

$$\bar{m}_t - \bar{p} = y_t - \nu i_t$$

where $\nu = 1/i$.
AS:

$$\mu_t = -(1+\gamma_n)(y_t - y_t^*)$$

(Note: $i_t = \log(1+i_t) - \log(1+i)$ and $\mu_t = \log(1+\mu_t) - \log(1+\mu)$)
Determination of y_t^*:

$$\begin{aligned}
y_t^* &= a_t + n_t^* \\
a_t &= \gamma_n n_t^* + c_t^* \\
&= \gamma_n (y_t^* - a_t) + y_t^* \\
y_t^* &= a_t
\end{aligned}$$

Thus fluctuations in the natural level of output are driven by fluctuations in total factor productivity. Real business cycle theory argues that fluctuations in output are in fact mainly fluctuations in y_t^*, implying that fluctuations in a_t are the main business cycle driving force. However, this view appears inconsistent with most of the evidence on business cycles.

3.1 Adding disturbances and solving the model:

Now suppose there is an autonomous source of spending by households, captured by the random disturbance χ_t. The IS/LM system may then be expressed as

$$\begin{aligned}
y_t &= -i_t + E_t y_{t+1} + \chi_t \\
\bar{m}_t - \bar{p} &= y_t - \nu i_t
\end{aligned}$$

where χ_t is a stationary mean zero random variable; i.e.,: $\lim E_t\{\chi_{t+i}\} = 0$ as $i \to \infty$.
We can then solve for output as follows

$$\begin{aligned}
i_t &= -\frac{1}{\nu}(\bar{m}_t - \bar{p}) + \frac{1}{\nu} y_t \\
y_t &= \frac{1}{\nu}(\bar{m}_t - \bar{p}) - \frac{1}{\nu} y_t + E_t y_{t+1} + \chi_t \\
y_t &= \frac{1}{1+\nu}(\bar{m}_t - \bar{p}) + \frac{\nu}{1+\nu} E_t y_{t+1} + \frac{\nu}{1+\nu}\chi_t \\
y_t &= \sum_{i=0} \left(\frac{\nu}{1+\nu}\right)^i \left(\frac{1}{1+\nu}(\bar{m}_{t+i} - \bar{p}) + \frac{\nu}{1+\nu}\chi_{t+i}\right)
\end{aligned}$$

Output thus depending on both the current end expected values of the real money supply and the demand disturbance.
Suppose that the goal of monetary policy is the stabilized output at its long run trend value, i.e, to to set $y_t = 0$. Then a monetary policy that accomplishes this goal sets $\bar{m}_{t+i} = \bar{p} - \nu\chi_{t+i}$. Doing so has the interest rate adjust to offset the effect of the demand disturbance on output. That is, under the policy

$$i_t = -\chi + \frac{1}{\nu} y_t$$

$$y_t = \sum_{i=0} (-i_{t+i} + \chi_{t+i})$$

which implies complete stabilization: i.e. under this rule

$$y_t = 0$$

3.2 Interest Rate as the Policy Instrument

Modern central banks in industrialized economies use the short term interest rate as the policy instrument, as opposed to the money supply. The reason is that under money supply targeting, instability in money demand can create gyrations in interest rates that affect the real economy.
To see, suppose that money demand is now given by:

$$\bar{m}_t - \bar{p} = y_t - \nu i_t + \varepsilon_t$$

where ε_t is a stationary disturbance to money demand. Proceeding as in the previous section:

$$y_t = \sum_{i=0} \left(\frac{\nu}{1+\nu}\right)^i \left(\frac{1}{1+\nu}(\bar{m}_{t+i} - \bar{p} - \varepsilon_{t+i}) + \frac{\nu}{1+\nu}\chi_{t+i}\right)$$

Note that shocks to money demand will affect output, unless the central bank is able to adjust the money supply to perfectly offset them.
For this reason, central banks tend to choose target values for the path on interest rates:

$$i_t = \bar{i}_t$$

They then adjust the money supply to hit this target. Note that in doing so, they are adjusting the money supply to offset any money demand disturbance. Note the this equation defines the LM curve under an interest rate rule (that is, the LM curve is horizonal at $i_t = \bar{i}_t$.)
In this simple model, how should then central bank go about setting the interest rate?
Note first that the flexible price interest rate - or natural rate of interest - r_t^*, is given by

$$y_t^* = -r_t^* + E_t y_{t+1}^* + \chi_t$$

Given $y_t^* = a_t$:

$$r_t^* = (E_t a_{t+1} - a_t) + \chi_t$$

It follows that we can express the output gap, $y_t - y_t^*$ as:

$$y_t - y_t^* = -(i_t - r_t^*) + E_t(y_t - y_{t+1}^*)$$

Iterating forward:

$$y_t - y_t^* = \sum_{i=0}^{\infty} -(i_t - r_{t+i}^*)$$

Thus, if the central bank's objective is to stabilize the output gap, it should set $i_t = r_{t+i}^*$, for all i. (Note that it has to not only do this in the current period, but also commit to doing it in the future. r_t^* is also know as the "neutral rate.", i.e., the rate at which yield output demand equal to the flexible price equilibrium level of supply.

Note also that in this simple model, the neutral rate summarizes all the relevant information about demand and supply shocks.