

Quiz #3

Revised: August 27, 2013

Please write your name below. Then complete the exam in the space provided. There are TWO questions. You may refer to one page of notes: standard paper, both sides, any content you wish.

(Name and signature)

1. *Linear models (60 points).* Consider the linear time series model

$$x_t = \varphi x_{t-1} + w_t,$$

with $\{w_t\}$ independent normal random variables with mean zero and variance one. Now consider a second random variable y_t built from x_t and the same disturbance w_t by

$$y_t = x_t + \theta w_t.$$

The question is how this combination behaves.

- (a) Is there a state variable for which x_t is Markov? What is the distribution of x_{t+1} conditional on the state at date t ? (10 points)
 - (b) Express x_t as a moving average. What are its coefficients? (5 points)
 - (c) Is there a state variable for which y_t is Markov? What is the distribution of y_{t+1} conditional on the state at date t ? (10 points)
 - (d) Express y_t as a moving average. What are its coefficients? (5 points)
 - (e) Under what conditions is y_t stable? That is: under what conditions does the distribution of y_{t+k} , conditional on the state at t , converge as k gets large? (10 points)
 - (f) What is the equilibrium or stationary distribution of y_t ? (10 points)
 - (g) What is the first autocorrelation of y_t ? (10 points)
2. *Stochastic volatility and equity pricing (40 points).* The Cox-Ingersoll-Ross model of bond pricing consists of the equations

$$\begin{aligned} \log m_{t+1} &= -(1 + \lambda^2/2)x_t + \lambda x_t^{1/2} w_{t+1} \\ x_{t+1} &= (1 - \varphi)\delta + \varphi x_t + \sigma x_t^{1/2} w_{t+1}, \end{aligned}$$

with the usual independent standard normal disturbances w_t . We add to it an equation governing the dividend d_t paid by a one-period equity-like claim:

$$\log d_{t+1} = \alpha + \beta x_t + \gamma x_t^{1/2} w_{t+1}.$$

Here x_t plays the role of the state: if we know x_t , we know the conditional distributions of $(m_{t+1}, x_{t+1}, d_{t+1})$.

- (a) Conditional on x_t , what are the mean and variance of $\log m_{t+1}$? What is its distribution? (10 points)
- (b) What is the price q_t^1 of a one-period bond? What is its return $r_{t+1}^1 = 1/q_t^1$? (10 points)
- (c) What is the price q_t^e of equity, a claim to the dividend d_{t+1} ? What is its return r_{t+1}^e ? (10 points)
- (d) Conditional on x_t , what is the expected log excess return on equity, $E_t(\log r_{t+1}^e - \log r_{t+1}^1)$? (10 points)