# Lab Report #6: Options & Volatility
Revised: November 6, 2014

*Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.*

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**Solution:** Brief answers follow, but see also the attached Matlab program; download the pdf, open, click on pushpin: 

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1. *Root-finding.* Consider the function $f(x) = \sin(x) + x\cos(x)$. Our mission is to find values of $x$ in the interval $[0.5, 2.5]$ for which $f(x) = 0$.

   (a) Plot $f$ against a grid of points $x$ between 0.5 and 2.5. How many solutions do you see?

   (b) Write or adapt a bisection or Newton's method program to find a solution. What is it?

   (c) Change the function to $f(x) = sin(x) + x\cos(x) - c$. Modify your program to find solutions in the same interval for $c = \{1/2, 0, -1/2\}$. What are they? Can you get your program to find them all at once?

   *Comment: See the Matlab guide to* anonymous functions *or the* root-finding code *posted on the course outline for examples.*

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   **Solution:**

   (a) The function is monotonic in this interval and crosses the axis once.

   (b) The solution is 2.0288.

   (c) For the vector of values for $c$, the solutions are $(1.8289, 2.0288, 2.2047)$.

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2. *Black-Scholes-Merton formula.* Our mission here is to examine the role in the BSM formula of the mysterious volatility parameter $\sigma$. The calculations refer to call options on the S&P 500 exchange-traded fund, ticker symbol `SPY`. On October 27, 20014 (today), the price of the underlying is $s_t = 197.07$. We'll consider prices of call options with maturities of 5 months or $\tau = 5/12$ years. On the same date, the price of a bond of similar maturity was $q_t^\tau = 1.0000$ (yes, the interest rate is essentially zero).

   (a) Consider a call option with strike price $k = 197$ ("at the money"). If volatility $\sigma = 0.10$, what is the price of the option? If $\sigma = 0.20$?

   (b) Plot the call option price against volatility for $\sigma$ between 0.01 and 0.30.

(c) On the same graph, do the same for options with strikes of 180 and 220.

(d) Consider call prices of 19.68 ($k = 180$), 7.28 ($k = 197$), and 0.32 ($k = 220$). Use your graph to estimate the values of $\sigma$ that deliver these prices. How do they compare? If $\sigma$ is a "parameter," why are they different?

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**Solution:**

(a) Call prices are 5.11 and 10.18.

(b,c) See Matlab program.

   d The vols are 0.1728, 0.1428, and 0.1057.

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3. *Volatilities for S&P 500 options.* We continue our examination of option prices and volatility: same date and underlying as the previous question.

Yahoo finance reports the following quotes:

| Strike | Call Price Bid | Call Price Ask | Put Price Bid | Put Price Ask |
|---|---|---|---|---|
| 150 | 47.25 | 47.62 | 0.73 | 0.79 |
| 160 | 37.61 | 37.95 | 1.26 | 1.32 |
| 170 | 28.28 | 28.60 | 2.15 | 2.22 |
| 180 | 19.56 | 19.80 | 3.63 | 3.69 |
| 190 | 11.83 | 11.93 | 5.96 | 6.07 |
| 200 | 5.57 | 5.66 | 9.87 | 9.99 |
| 210 | 1.69 | 1.76 | 16.03 | 16.37 |
| 220 | 0.29 | 0.34 | 24.70 | 25.11 |
| 230 | 0.04 | 0.08 | 34.46 | 34.93 |

(a) Compute "mid" quotes as averages of bid and ask. Plot these mid-market put and call prices against the strike price.

(b) Use put-call parity to compute call prices from mid puts. Plot call prices — bid, ask, and implied by puts — against the strike. How do they compare?

(c) Write a bisection or Newton's method program to compute implied volatilities for mid quotes of call options. Graph them against the strike. What shape does the resulting "smile" have?

*Comments:*

- *I use this two-step definition of the BSM formula:*

```
d = @(sigma,k) (log(s./(q_tau.*k))+tau*sigma.^2/2)./(sqrt(tau)*sigma);
f = @(sigma,k) s*normcdf(d(sigma,k)) - ...
        q_tau.*k.*normcdf(d(sigma,k)-sqrt(tau)*sigma) - call_mid;
```

*Here I've subtracted the call price from the formula to give us a function* f *whose value is zero when we find the right volatility.*

- *If you're lazy like me, you can look up the derivative in Wikipedia, where they refer to it as the "vega," a nonexistent Greek letter. Or you could use bisection and skip the whole thing. We can calculate the derivative with an anonymous function, too:*

```
fp = @(d) s*sqrt(tau)*exp(-d.^2/2)/sqrt(2*pi);
```

**Solution:**

(a) See Matlab program.

(b) Ditto. The call prices implied by puts are above the bid and ask of the calls.

(c) See Matlab program. The "smile" is downward sloping, with a little curvature to the right.