

Quiz #1

Spring 2012

Please write your name below. Then complete the exam in the space provided. There are THREE questions. You may refer to one page of notes: standard paper, both sides, any content you wish.

(Name and signature)

1. (3-state “Bernoulli”) (30 points) Consider a 3-state distribution in which we can explore the impact of high-order cumulants. This is sometimes called a “categorical distribution.”

Let us say, to be concrete, that the state z takes on the values $\{-1, 0, 1\}$ with probabilities $\{\omega, 1 - 2\omega, \omega\}$. A random variable x is defined by $x(z) = \delta z$.

- (a) Does x have a legitimate probability distribution? (5 points)
 - (b) What is the moment generating of x ? The cumulant generating function? (10 points)
 - (c) What are the mean and variance of x ? (5 points)
 - (d) What are the measures of skewness and excess kurtosis, γ_1 and γ_2 ? Under what conditions is γ_2 large? (5 points)
 - (e) Is there a value of ω that reproduces the values of γ_1 and γ_2 of a normal random variable? (5 points)
2. (3-state “Bernoulli,” continued) (20 points) For a representative agent economy based on the same state and random variable, suppose x is log consumption growth, $x(z) = \log c_1(z) - \log c_0$, and utility has the additive form,

$$u(c_0) + \beta \sum_z p(z) u[c_1(z)],$$

with $u(c) = c^{1-\alpha}/(1-\alpha)$.

- (a) What is the pricing kernel? (5 points)
 - (b) What are the state prices? (5 points)
 - (c) What are the risk-neutral probabilities? How do they vary with risk aversion α ? (5 points)
 - (d) Which is more valuable, a claim to one unit of the good in state $z = -1$ or one unit in state $z = +1$? Why? (5 points)
3. (short answers) (40 points) Give short answers to the following questions:

- (a) Suppose $x \sim \mathcal{N}(\kappa_1, \kappa_2)$. What are κ_1 and κ_2 ? What is the distribution of $\log \beta - \alpha x$? (5 points)
- (b) How would you compute skewness in a sample of data, x_t for $t = 1, 2, \dots, T$? (5 points)
- (c) What determines whether skewness raises or lowers expected utility? Which applies to power utility? (5 points)
- (d) What is a resource constraint? How does it differ from a budget constraint? (5 points)
- (e) What are the prices of Arrow securities for this collection of asset prices and dividends?

$$\begin{aligned} \text{Asset 1:} \quad & q^1 = 1.0, \quad d^1(1) = 2, \quad d^1(2) = 1 \\ \text{Asset 2:} \quad & q^2 = 1.5, \quad d^2(1) = 2, \quad d^2(2) = 2. \end{aligned}$$

*Is this a “no-arbitrage” environment? How do you know? (10 points)

- (f) *In our usual two-period setup, suppose the representative agent has power utility and $x(z) = \log c_1(z) - \log c_0$ has an exponential distribution:

$$p(x) = \lambda e^{-\lambda x},$$

for $x \geq 0$ and $\lambda > 0$. What is the risk-neutral distribution of x ? (10 points)