Lab Report #1: Moments & Cumulants

Revised: September 17, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Use Matlab where possible and attach your code to your answer.

1. *Moments of normal random variables.* This should be review, but will get you started with moments and generating functions.

   Suppose x is a normal random variable with mean $\mu = 0$ and variance σ^2.

   (a) What is x's standard deviation?

   (b) What is x's probability density function?

   (c) What is x's moment generating function (mgf)? (Don't derive it, just write it down.)

   (d) What is $E(e^x)$?

   (e) Let $y = a + bx$. What is $E(e^{sy})$? How does it tell you that y is normal?

2. *Sample moments of normal random variables.* It's often helpful to experiment with artificial test problems, just to remind ourselves how the code works. Here we compute sample moments of artificial data generated in Matlab and verify that calculations of various moments do what we think they do.

   This generates the data we'll use:

```
format compact                % single-spacing of output
nobs = 1000;                  % number of observations
rng('default');               % sets "seed" so you can replicate the output
x = -1 + 2*randn(nobs,1);     % vector of normals with mean -1 and sd 2
```

   These commands generate "pseudo-random" numbers from a normal distribution with mean -1 and standard deviation 2 and puts them in the vector `x`. As always, you can find out more about Matlab commands by typing `help command` at the prompt; for example, `help rng` or `help randn`.

   (a) Our first check is to see if the sample moments correspond, at least approximately, to our knowledge of normal random variables. Run the commands:

```
xbar = mean(x)
moments = mean([(x-xbar).^2 (x-xbar).^3 (x-xbar).^4])
```

   What do you get? How do your calculations compare to the analogous true (or population) moments?

   (b) Our second check is on the Matlab commands `std(x)`, `skewness(x)`, and `kurtosis(x)`. Can you reproduce them with the sample moments computed in (a)?

3. *Sums of independent random variables.* Consider the sum of n random variables, say $y = x_1 + x_2 + \cdots + x_n$ with the x's "iid" (independent and identically distributed) Poisson random variables. That is, each x_i takes on values $x = 0, 1, 2, \ldots$ with probability $e^{-\omega}\omega^x/x!$. The parameter ω ("intensity") is positive.

   (a) What is the cumulant generating function (cgf) of x? (Don't derive it, just write it down.)

   (b) Use the cgf to derive the first four cumulants of x.

   (c) Now consider y. What is its cgf? How do we know that y is Poisson? What is its intensity parameter?

   (d) What are y's first four cumulants? How do our measures of skewness and excess kurtosis vary with n?

4. *Squared normal random variable.* Suppose x is standard normal (normal with mean zero and variance one). What are the mgf and cgf of $y = x^2$? What are y's mean and variance?

   Hints. (a) Apply the definition of the mgf. (b) Then combine the exponential terms — similar to what we did to derive the mgf of a normal random variable.