Lab Report #5: Excess Returns

Revised: April 6, 2013

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

1. *Risk and return in US equity portfolios.* Modern economies issue a wide range of assets, whose returns can be wildly different. Here we summarize the properties of returns on some common equity portfolios.

   We'll start with data input. Go to Ken French's data site, a standard source for data on financial returns for a broad range of equity-related portfolios:

   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

   Download the files associated with, respectively, the "Fama-French Factors" and "Portfolios Formed on Size" at the links

   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors.zip
   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/Portfolios_Formed_on_ME.zip.

   In both cases, you'll find a txt file inside a zip file. Copy the first table in each txt file into a spreadsheet with the dates aligned. In the first file ("factors") you want the first column (the date), the second (the excess return on the market), and the fifth (the short-term or riskfree interest rate RF). In the second file ("size") you want the first column (the date again) and columns three to five (returns on portfolios of small, medium, and large firms).

   When you're done, read the data into Matlab. At this point you should have the riskfree rate and excess returns on the market and three size portfolios (small, medium, and large) — five variables all together. The numbers are monthly, July 1926 to (last I looked) January 2013. I believe they are percentages, but you should check.

   (a) Compute the mean, standard deviation, skewness, and excess kurtosis for each excess return series. Which portfolio has the highest excess return? Lowest?

   (b) The Sharpe ratio for any asset or portfolio is the mean of its excess return over its standard deviation. Which portfolio has the highest Sharpe ratio? Lowest?

   (c) Do the same for log excess returns; that is, compute the mean, standard deviation, skewness, and excess kurtosis for each excess return series. (To get this right, you should add the riskfree rate to each excess return, divide by 100, add one, then take logs. Think about why all this is called for.)

   (d) Is there a clear link here between risk, measured here by the standard deviation of the excess return, and return, measured by the mean excess return? Should there be?

Solution: The idea here is to look at some popular equity portfolios and see how their returns differ from broad-based equity indexes. We see (i) mean excess returns in some cases are larger than the equity premium and (ii) skewness and kurtosis are a standard feature. Skewness differs between returns and log returns, as you might expect: the log is a concave function, so it moderates large positive returns in levels. Finally, we remind ourselves that $E(mr) = 1$ does not imply any particular relation between mean and standard deviation of excess returns. In that respect, the model is similar to the CAPM, where the mean is connected to covariance with the market, not the standard deviation or Sharpe ratio or other simple risk measure.

(a) Last year's numbers below. You can get the current ones by running the posted Matlab program.

Properties of monthly excess returns

	Portfolio			
Statistic	market	small	medium	big
Mean	0.6172	0.9738	0.8532	0.5987
Std dev	5.4571	8.5584	6.8520	5.2757
Skewness	0.1685	2.1813	0.9816	0.1860
Excess kurtosis	7.3997	21.8628	11.7830	7.1738
Sharpe ratio	0.1131	0.1138	0.1245	0.1135

The highest mean excess return is small firms at about 1% a month.

(b) Sharpe ratios also reported above. You can compute them from the ratio of the first row to the second. Here medium-size firms are highest. The larger mean return of small firms is undone by a large standard deviation.

(c) Similar table for log excess returns below, also last year's. Note that we need to need to the riskfree rate to excess returns and take logs.

Properties of monthly log excess returns

	Portfolio			
Statistic	market	small	medium	big
Mean	0.4669	0.6329	0.6224	0.4583
Std dev	5.4556	8.1132	6.7205	5.2682
Skewness	−0.5298	0.4937	−0.0850	−0.4779
Excess kurtosis	6.5282	9.6576	7.5655	6.3824

(d) There's some connection between the mean and the standard deviation in levels, not so much in logs. Neither is all that informative: we know risk has to do with the connection to the pricing kernel, anything else would be purely accidental.

2. *Disaster risk and the equity premium.* We'll add a third "disaster" state to our analysis

of the equity premium and see how it changes our view of it. The key input is the distribution of log consumption growth,

$$\log g \quad = \quad \begin{cases} \mu + \sigma & \text{with probability } (1-\omega)/2 \\ \mu - \sigma & \text{with probability } (1-\omega)/2 \\ \mu - \delta & \text{with probability } \omega \end{cases}$$

What's the idea? If $\omega = 0$, we're back to our symmetric two-state distribution. But if we introduce a small positive value of ω and a "largish" $\delta > 0$, we have a "disaster" state that changes the distribution dramatically.

The question is what this does to the equity premium. We'll define equity as a claim to consumption growth g. We define the equity premium in logs,

$$E(\log r^e - \log r^1),$$

and aim at a target value of 0.0400 (4%).

(a) If $\omega = 0$, what values of μ and σ deliver the observed mean and variance of log consumption growth, namely 0.0200 and 0.0350^2?

(b) Suppose $\beta = 0.99$ and $\alpha = 10$. What are $\log r^1$ and the equity premium, $E \log r^e - \log r^1$?

(c) What is entropy? How does it relate to the equity premium you computed above?

(d) Now consider $\omega = 0.01$ and $\delta = 0.30$. (These numbers are based on a series of studies by Robert Barro and his coauthors.) With these numbers, what values of μ and σ reproduce the observed mean and variance of log consumption growth?

(e) With (again) $\beta = 0.99$ and $\alpha = 10$, what are $\log r^1$ and the equity premium, $E \log r^e - \log r^1$? How does it compare to your previous calculation?

(f) How does entropy differ between the disaster and no-disaster cases?

(g) How does entropy change if $\delta = -0.30$, so that the extreme state is good news rather than bad? Can you guess why?

Solution: The idea is to show how changing the distribution in ways that produce negative skewness can increase risk premiums even if we keep the standard deviation of log consumption growth the same. It's like a partial derivative result: vary skewness while holding the standard deviation constant. The question in practice is how much of this is reasonable.

(a) The expressions for the mean and variance of $\log g$ are

$$\begin{aligned} E(\log g) &= \mu + \omega\delta = 0.0200 \\ \mathrm{Var}(\log g) &= (1-\omega)\sigma^2 + \omega(1-\omega)\delta^2 = 0.0350^2. \end{aligned}$$

When $\omega = 0$, the mean is $\mu = 0.0200$ and the standard deviation is $\sigma = 0.0350$.

(b) With these values, we have $r^1 = 1.1618$, $\log r^1 = 0.1500$, and $E \log r^e - \log r^1 = 0.0112$.

(c) Entropy here is 0.0600. This is an upper bound on expected excess returns, so the model is evidently able to generate risk premiums greater than the equity premium.

(d) When $\omega = 0.01$, we need to set $\mu = 0.0230$ and $\sigma = 0.0183$ to maintain the mean and variance at their sample values. See (a).

(e) With these values, we have $r^1 = 1.0529$, $\log r^1 = 0.0516$, and $E \log r^e - \log r^1 = 0.0438$. We have a success! The equity premium goes up and is now above our target. In that respect, the disaster state is a useful innovation, but we still need a large risk aversion parameter for it to work.

(f) Entropy rises, too, to 0.1585. Yaron's bazooka!

(g) If we switch the sign of δ, entropy falls to 0.0371. Evidently positive skewness in consumption and dividend growth isn't helpful. This is connected to our discussion of skewness and entropy: positive skewness in $\log m$ increases entropy. With power utility, that requires negative skewness in $\log g$. Changing the sign of δ gives us positive skewness and reduces entropy.

Matlab program:

```
%  hw5_s13
%  Matlab program for Lab Report #4, Spring 2012
%  NYU course ECON-UB 233, Macro foundations for asset pricing, Mar 2012.
%  Written by:  Dave Backus, March 2012

disp('Answers to Lab Report 5')

%%
disp(' ')
disp('-----------------------------------------------------------------')
disp('Question 1')
format compact
clear all

% read spreadsheet with Fama-French returns
% dates monthly from 1926 07 to end of 2011
% series:  Date1, MKTXS, SMB, HML, RF, Date2, DUMMY, LO30, MED40, HI30

FF_returns = xlsread('FamaFrench_returns.xlsx', 'Sheet1');
[nobs,nvars] = size(FF_returns)

disp(' ')
disp('Extra stuff to make sure things are lined up with the right units')
disp('Mean of inputs (mktxs, rf, small, med, big)')
mean(FF_returns(:,[2 5 6 7 8]))

rf = FF_returns(:,5);
rfbig = FF_returns(:,[5 5 5 5]);     % expand to make subtraction easy
returns = FF_returns(:,[2 6 7 8]);
returns(:,1) = returns(:,1) + rf;
xs_returns = returns - rfbig;

disp(' ')
disp('Moments of xs returns (mean, std, skew, kurt, sharpe)')
disp('(mkt, rf, small, med, big)')
[mean(xs_returns); std(xs_returns); skewness(xs_returns); ...
    kurtosis(xs_returns) - 3;  mean(xs_returns)./std(xs_returns)]

disp(' ')
disp('Moments of log xs returns (mean, std, skew, kurt, sharpe)')
disp('(mkt, rf, small, med, big)')

lxs_returns = log(1+returns/100) - log(1+rfbig/100);
lxs_returns = 100*lxs_returns;
```

```
[mean(lxs_returns); std(lxs_returns); skewness(lxs_returns); ...
    kurtosis(lxs_returns) - 3]

figure(1)
clf
plot(std(xs_returns),mean(xs_returns),'k*')
hold on
plot(std(lxs_returns),mean(lxs_returns),'b*')
ylabel('Mean Excess Return')
xlabel('Std Dev of Excess Return')
text(7.5, 0.45, 'black is levels, blue is logs')

%print -depsc hw4_q2c.eps

figure(2)
clf
plot(mean(xs_returns)./std(xs_returns),mean(xs_returns),'b*')
ylabel('Mean Excess Return')
xlabel('Sharpe Ratio')
text(0.45,8.5,'black is levels, blue is logs')

%print -depsc hw5_q2d.pdf

%%
disp(' ')
disp('-----------------------------------------------------------------')
disp('Question 2 (disaster risk)')
format compact
clear all

disp(' ')
disp('Analytics for cumulants')
syms s omega mu sigma delta
mgf = omega*exp(s*(mu-delta)) + 0.5*(1-omega)*(exp(s*(mu+sigma))+exp(s*(mu-sigma)));
cgf = log(mgf);

disp(' ')
disp('Cumulants')
kappa1 = subs(diff(cgf,s,1),s,0);    %  mean
kappa2 = subs(diff(cgf,s,2),s,0);    % variance
kappa3 = subs(diff(cgf,s,3),s,0);
kappa4 = subs(diff(cgf,s,4),s,0);

kappa1 = simplify(kappa1)
kappa2 = simplify(kappa2)
```

```
gamma1 = simplify(kappa3/kappa2^(3/2))
gamma2 = simplify(kappa4/kappa2^2)

disp(' ')
disp('Asset prices and returns')
clear all

% consumption process
omega = 0.01;   % choose a value
delta = -0.3;
mu = 0.0200 + omega*delta
sigma = sqrt(0.0350^2 - omega*(1-omega)*delta^2)

p = [(1-omega)/2; (1-omega)/2; omega];
logg = [mu + sigma; mu-sigma; mu-delta];
g = exp(logg);

% preferences
beta = 0.99;
alpha = 10;

% asset prices
m = beta*g.^(-alpha);
d = g;

q1 = sum(p.*m);
r1 = 1/q1
logr1 = log(r1)

qe = sum(p.*m.*d)
Elogre = sum(p.*log(d)) - log(qe);

eq_prem = Elogre - logr1

% entropy
H = log(sum(p.*m)) - sum(p.*log(m))

return
```