Professor Mark Gertler
Intermediate Macroeconomic Theory
Spring 2011
March 22

**Lecture 7**

# Money, Inflation and Nominal Interest Rates: Long Run Trend Behavior

In the first part of the course, we developed a theory of the long run trend behavior of "real economic variables", including quantity variables such as real output, employment, investment and consumption, etc., and including relative prices such as the real wage and the real rate of interest. The models we studied, however, had nothing to say about nominal phenomena such as the price level, the rate of inflation, and the nominal rate of interest. We turn to this issue now. For the time being, we continue to focus on long run behavior.

We begin with some notation and some definitions:

Let the price level at $t$, $P_t$, be the cost in units of domestic currency of a representative basket of consumption goods. $P_t$ is thus an index of prices in the economy. The rate of inflation, $\pi_t$ is the percent change in the price index:

$$\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} \approx \log P_t - \log P_{t-1}$$

The nominal interest rate $i_t$ is the rate of return in dollars on an asset, as opposed to the real rate of interest, $r_t$, which is the rate of return in units of consumption goods. For a discount bond that sells for dollar price $S_t$ today and yields a dollar tomorrow, the net nominal interest rate is given by

$$i_t = \frac{1 - S_t}{S_t}$$

Conversely, the "ex post" real rate $\widetilde{r}_t$ is given by

$$\begin{aligned}
\widetilde{r}_t &= \frac{(1/P_{t+1}) - (S_t/P_t)}{(S/P_t)} \\
&= \frac{1+i_t}{1+\pi_{t+1}} - 1 \\
&= \frac{i_t - \pi_{t+1}}{1}(1 - \frac{\pi_{t+1}}{1+\pi_{t+1}}) \\
&\approx i_t - \pi_{t+1}
\end{aligned}$$

assuming the inflation rate is "not too large". That is, the ex post real rate of interest is approximately the nominal rate of interest minus the rate of inflation. We use the term ex post, because the realized inflation rate at $t+1$, $\pi_{t+1}$, is in general not know with certainty at time $t$. This identity linking real and nominal rates is sometimes known as the "Fisher" identity, after Irving Fisher who was the first to characterize it.

We make the distinction between $\widetilde{r}_t$ and the expected real rate $r_t$, which is based in the forecast or "expection" of inflation, $E_t\pi_{t+1}$, as follows

$$r_t \approx i_t - E_t\pi_{t+1}$$

Note that as long is the bond is riskless in nominal terms, the nominal rate $i_t$ is known with certainty.

In the long run, there is a link between the behavior of the money supply and nominal variables. Periods where money growth has been high for a sustained period have also been periods where inflation has been relatively high, This has been true not only across time but also across countries.

The first to formalize such a relationship was (again) Irving Fisher, who wrote the *Purchasing Power of Money* in 1912. He begins with the observation that the purchasing power of money, i.e., the number of baskets of consumption goods that a unit of money can buy, equals $\frac{1}{P}$. Hence there is an link between between the purchasing power of money and the price level. It follows that theory of the price level is really a theory of what determines the real value of unit of money. Fisher then proceed to develop the Quantity Theory of Money to explain the purchasing power of money and the price level.

Fisher begins with the follwing definition of money: Anything generally acceptable in exchange. Money is used as vehicle of exchange because it solves the double co-incidence of wants problem, which is prevalent in barter societies. There are two other important roles. First because it is used in exchange it also serves as a unit of account (i.e. transactions are typically specified in dollars). Second, money is also an asset held in portfolios.

The net nominal return on holding money, however, is typically zero. The real return is typically negative when inflation is positive, as is the norm. In particular, following the logic from calculating the real return on bonds, the real return on a unit of money, $r_t^m$, is given by

$$r_t^m = -\pi_{t+1}$$

This relation follows since the nominal interest rate on a unit of money is zero.

A key question is: why do individuals hold money when its return is dominated by other assets?. The answer is that there is a convenience yield from holding money in a portfolio that is associated with its role as a medium of exchange. Assuming that it is costly to adjust a portolio, it is opitmal to hold a inventory of money to meet transactions needs.

For an advanced economy, there are two basic types of money: First outside (or fiat) money, which is a liability of the government (currency); Second, inside money, which is a liability of priivate fiancial institutions (e.g.,checking accounts).

Outside money typically enters the economy when the Federal Reserve purchases government bonds. The currency is a liability of the Federal Reserve, while government bonds are an asset. (The government bonds are liabilities of the U.S. government, with most held by the private sector and some by the Federal Reserve.)

How does inside money enter the economy? Roughly speaking, the capital stock of the U.S. economy is financed with equity (i.e., stock), corporate bonds, and bank loans. Banks hold loans and reserves of outside money as assets and funds these assets by issuing time deposits and checking accounts. Checking accounts are inside money. Banks reserves along with currency equal the base money supply that the Federal Reserve has issued. The main measure of the money supply, however, is M1, which consists of currency and checking accounts. A broader measure called M2 includes saving, time, and money market mutual fund deposits along with M1.

Fisher begins the Quantity theory with the following accounting identity:

$$Expenditures \equiv Receipts$$

Accordingly, let $M$ be the supply of outside money, $V$ the velocity of money (the number of times a unit of money turns over within a give time period) and $Y$ real output. Then

$$\begin{aligned} MV &= \exp enditures \\ PY &= receipts \end{aligned}$$

which implies

$$MV = PY$$

Note that since the value of a unit of outside money equals the value of a unit of inside money, we can abstract from the latter in developing a theory of what determines $P$.

From the quantity identity.

$$\frac{\Delta M}{M} + \frac{\Delta V}{V} = \frac{\Delta P}{P} + \frac{\Delta Y}{Y}$$

$$\frac{\Delta P}{P} = \frac{\Delta M}{M} + \frac{\Delta V}{V} - \frac{\Delta Y}{Y}$$

Inflation thus depends on the sum of money and velocity growth, net the growth rate of real output.

To move from the identity to the theory, Fisher makes two assumptions:

1. $\frac{\Delta Y}{Y}$ is independent of $\frac{\Delta M}{M}, \frac{\Delta V}{V}, \frac{\Delta P}{P}$ – (*i.e, monetary neutrality*)

2. $\frac{\Delta V}{V} = 0$,

   which implies,

$$\frac{\Delta P}{P} = \frac{\Delta M}{M} - \frac{\Delta Y}{Y}$$

Given these two assumptions, inflation varies exactly with money growth net output growth.

The assumption of monetary neutrality is based on the premise that only real factors (preferences, technology, factors supplies) determine real economic quantities and relative prices, along the lines of the growth model we developed earlier. (For the long run this is a reasonable hypothesis, but as we will see going forward, it is a questionable assumption for short run behavior.)

To motivate why V constant, note that there is a relationship between V and money demand.

$$M = kPY$$

$$k = \frac{1}{V}$$

Thus, the assumption that V is constant is really an assumption that money demand is a stable function of expenditures. Fisher argued that $k$ (and hence $V$) likely depended on the state of financial institutons which do not change much in the short run. For example the easier it is to obtain money on short notice (e.g. via ATM machines), the less money that individuals need to hold for a given level of expenditures.

Given assumptions 1 and 2, the price level changes proportionately with the money suppy. Intuitively, if $M$ increases there is excess supply of money at the existing price level (since money demand depends on $PY$ which has yet to change). This leads to an attempt to buy goods, which prices up $P$ to the point where money demand equals money suppy, with real output Y unchanged.

As I noted earlier, the quanitty theory is not an unreasonable description of behavior in the long run or across countries. In the short run, however, it tends to fail badly. In particular

1. V is not constant - it is highly procyclical.

2. Y is not independent of M. Periods where the Federal Reserve has sharply contract the money supply have been followed by real output declines.

Here we discuss the first issue: accounting for variable velocity. In particular, we allow money demand to depend not only on nominal expenditures but also its opportunity cost, the nominal interest rate:

$$\frac{M}{P} = cYe^{-\nu i}$$

Equivalently,

$$\frac{M}{P} = \frac{1}{V}Y;$$

with

$$\frac{1}{V} = ce^{-\nu i}$$

Velocity thus varies positively with the nominal interest rate.

Taking the logs yields the following relationship for the demand for real money balances:

$$\begin{aligned} \log M - \log P &= \log Y - \log V \\ &= \log Y + \log c - \nu i \\ &= \log Y + \log c - \nu(r + E\pi_{+1}) \end{aligned}$$

since $i = (r + E\pi_{+1})$ and where $\pi_{+1} = p_{+1} - p$.

We are now in a position to characterize the dynamics of the price level and the inflation rate. Suppose that output and the real interest rate corresponds to the steady state of frictionless competitive economy absent growth (so that real output is constant). Let $Y^*$ and $r^*$ correspond to the "natural" values of output and the real interest rate. As in the conventional competitive equilibrium, these variables are independent of nominal variables such as the money and the price level. In addition, let $x_t$ denote $\log X_t$ Then we can write:

$$m_t - p_t = y^* + \log c_t - \nu r^* - \nu E_t \pi_{+1}$$

which can be rewritten as

$$m_t - p_t = k - \nu(E_t p_{t+1} - p_t)$$

with

$$k = y^* + \log c_t - \nu r^*$$

We can then solve for the price level:

$$\begin{aligned} p_t &= \frac{1}{1+\nu}(m_t - k) + \frac{\nu}{1+\nu} E_t p_{t+1} \\ &= (1-\alpha)(m_t - k) + \alpha E_t p_{t+1} \end{aligned}$$

where $\alpha = \frac{\nu}{1+\nu}$, with $0 < \alpha < 1$. Note that the price level depends on the money suppy and the expectation of next period's price level.

To solve for the expected price level, we impose "rational expectations," which in this context means that the forecast of the price level by individuals with the model is the same that the model would generate, given information available at time $t$. In particular, iterating forward yields.

$$p_{t+1} = (1-\alpha)(m_{t+1} - k) + \alpha E_{t+1} p_{t+2}$$

Then using the law of iterated expectations, which implies,

$$E_t E_{t+1} x_{t+2} = E_t x_{t+2}$$

we obtain an expectation of $p_{t+1}$ that is consistent with the model:

$$E_t p_{t+1} = (1-\alpha)(E_t m_{t+1} - k) + \alpha E_t p_{t+2}$$

which implies

$$p_t = (1-\alpha)(m_t - k) + (1-\alpha)\alpha(E_t m_{t+1} - k) + \alpha^2 E_t p_{t+2}$$

Note that $p_t$ now depends on $m_t$ and $E_t p_{t+2}$. Note that since $\alpha^2 < \alpha$, $E_t p_{t+2}$ affects the current price level by less than does $E_t p_{t+1}$.

By repeatedly using the same kind of forward iteration procedure to solve out for the expected future price level we obtain

$$p_t = E_t \sum_{i=0} (1-\alpha)\alpha^i (m_{t+i} - k)$$

which simplifies to

$$p_t = E_t \sum_{i=0} (1-\alpha)\alpha^i m_{t+1} - k$$

since $\sum_{i=0}(1-\alpha)\alpha^i = \frac{1-\alpha}{1-\alpha} = 1.$

$$\pi_t = p_t - p_{t-1} = E_t \sum_{i=0} (1-\alpha)\alpha^i (E_t m_{t+i} - E_{t-1} m_{t-1+i}).$$

In the special case of perfect foresight (where $E_t x_{t+i} = x_{t+i}$).

$$\pi_t = p_t - p_{t-1} = \sum_{i=0} (1-\alpha)\alpha^i \mu_{t+i}$$

with $\mu_t = \frac{M_t - M_{t-1}}{M_{t-1}}$. Note that inflation depends not only current money growth but also on what money growth is expected to be in the future.

Let us first consider the case where the money growth rate is fixed: $\mu_t = \mu$ for all $t$ :

$$\pi_t = \sum_{i=0} (1-\alpha)\alpha^i \mu$$

which implies

$$\pi = \frac{1-\alpha}{1-\alpha}\mu = \mu$$

Now suppose that from $t = 0$ to $T-1$, money growth is fixed at a low value $\underline{\mu}$ and then at $T$ it jumps to a higher value, $\overline{\mu}$.:

$$\begin{aligned}\pi_t &= \sum_{i=0}^{T-1}(1-\alpha)\alpha^i\underline{\mu} + \sum_{i=T}(1-\alpha)\alpha^i\overline{\mu} \\ &= \sum_{i=0}(1-\alpha)\alpha^i\underline{\mu} + \alpha^T\sum_{i=0}(1-\alpha)\alpha^i(\underline{\mu}-\overline{\mu})\end{aligned}$$

which implies:

$$\pi = \underline{\mu} + \alpha^T(\underline{\mu}-\overline{\mu})$$

Thus, if money growth is perceived to increase in the future, the current inflation rate will exceed the current money growth rate. A scenario where this might arise is when a current is running large deficits and it is perceived by the public that the government will eventually try to take care of its indebtedness problem in the future by printing money (to buy back the debt), as opposed to raising taxes.

We note that such a framework was used to explain the hyperinflation in Germany in the 1920a and in particular why over this period that the inflation rate exceeded the money growth for a period of time.

To see the possible link between government expenditures and money growth, note that we can express the government budget constraint as

$$G + (1+r)B^g = T + B^g_+ + \frac{M_+ - M}{P}$$

where $B^g$ is the stock of government debt. We rewrite as the following relation for government debt

$$B^g = \frac{1}{1+r}[T + \mu_+(\frac{M}{P}) - G + B^g_+]$$

where $\mu_+ = \frac{M_+ - M}{M}$ and where $\mu_+(\frac{M}{P})$ is the reveneue or "seignorage" from money creation. Iterating forward yields

$$B^g_t = \sum_{i=0}(\frac{1}{1+r})^{1+i}[T_{t+i} + \mu_{t+1+i}(\frac{M}{P})_{t+i} - G]$$

where we assume for convenience that both $r$ and $G$ are fixed. What the relation says is that the present value of government revenues from taxes and money creation must exceed the present value of government expenditures by an amount that is sufficient to pay for the existing government debt. Note that both taxes and money creation are options.

In steady state

$$B = \frac{1}{r}[T + \mu\frac{M}{P} - G]$$

or, equivalently,

$$rB = T + \mu\frac{M}{P} - G$$

We note that advanced economies (e.g. the U.S) typically do not resort to money creation to finance it debt, given the costs of inflation. Governments that do not have their fiscal house in order have quite frequently in the past resort to money creation.

Finally, keep in mind that the analysis still applies to long run trend behavior. We move to the short run next.