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Lecture 3

One Period Competitive Equilibrium Model of Output and Employment

The canonical model of long rune equilibrium is the competitive equilibrium model. Here I present a static competitive equilibrium model and illustrate how it determines the aggregatve value of output and employment. After finishing this we will turn to an intertermporal model where saving and investment are determined as well. .

1 Environment

Assume:

(i) One periods: Only consumption goods produced.

(ii) One representative household that: consumes, supplies labor, and receives dividend income (from ownership of firms.)

(iii) One representative firm that produces output, demands labor, and pays dividends to households.

(iv) The household and the firm act competitively, i.e., each takes market prices as given.

We next characterize preferences, technology and resource constraints:

Preferences

The household consumes in both periods, but only works in period 1. Let C^i be household consumption and N^i be household labor supply in period. Then household preferences are given by

$$u(C) - \nu(N) \tag{1}$$

with :

$u(0) = 0$, $u'(\cdot) > 0$, $u''(\cdot) < 0$, $u'(0) = \infty$, $u'(\infty) = 0$,

$\nu(0) = 0$, $\nu'(\cdot) > 0$, $\nu''(\cdot) > 0$, $\nu'(0) = 0$, $\nu'(\infty) = \infty$,

and

$0 < \beta < 1$.[1]

These properties imply that $u(\cdot)$ is increasing and concave and that $\nu(\cdot)$ is increasing and convex. Concavity of $u(\cdot)$ implies diminishing marginal utility of consumption, while convexity of $\nu(\cdot)$ suggests

[1] The conditions $u'(0) = \infty$, $u'(\infty) = 0$, $\nu'(0) = 0$, $\nu'(\infty) = \infty$ are known as Inada conditions and guarantee an interior solution in equilibrium with positive and finite values of C_i and N.

increasing marginal disutility from labor supply. The parameter β is known as the household's subjective discount factor. It reflects how the household weights consuming in the future relative to consuming today.

Technology

The firm produces output in period 1 using labor input. Let Y_j be output by firm j and N^j employment. Then each period, production is given by

$$Y^j = Af(N^j) \tag{2}$$

The production function has the following properties:

$f(0) = 0,\ f'(\cdot) > 0,\ f''(\cdot) < 0,\ f'(0) = \infty,\ f'(\infty) = 0,$

$g(0) = 0,\ g'(\cdot) > 0,\ g''(\cdot) < 0,\ g'(0) = \infty,\ g'(\infty) = 0.$

where A is total factor productivity.

Economy-Wide Resource

Aggregate output equals aggregate consumption:

$$\int_0^1 Y^j dj = Y = C = \int_0^1 C^i di \tag{3}$$

2 Household and Firm Behavior

The Household's Decision Problem

Let Π be lump sum dividends (all households receive the same), and W the real wage. The household chooses C^i, to solve N^i.

$$\max u(C^i) - \nu(N^i) \tag{4}$$

subject to:

$$C^i = WN^i + \Pi \tag{5}$$

The household takes as given W and Π...

To be concrete, let:

$$\begin{aligned} u(C^i) &= \frac{1}{1-\sigma}(C^i)^{1-\sigma};\ \sigma > 0 \text{ and } \neq 1 \\ &= \log C \text{ if } \sigma = 1 \end{aligned}$$

$$\nu(N^i) = b\frac{1}{1+\varphi}(N^i)^{1+\varphi}$$

To solve the household's decision problem, it is simplest to turn the constrained problem into an unconstrained one by plugging (5) into (4). This is possible as long as the two one-period budget constraints are always binding, so that there is no unused income. This latter condition is ensured by the assumptions we made on $u(\cdot)$.

The representative household accordingly chooses and N^i to solve

$$\max \frac{1}{1-\sigma}(WN^i + \Pi_1)^{1-\sigma} + \frac{1}{1+\varphi}(N^i)^{1+\varphi},$$

given W and Π..

The first order necessary condition with respect to N^i is given by:

$$\underbrace{W(C^i)^{-\sigma}}_{\text{MB of Labour Supply}} - \underbrace{b(N^i)^{\varphi}}_{\text{MC of Labour Supply}} = 0 \tag{6}$$

The household adjusts labor supply until marginal benefit in utility terms, the real wage times the marginal utility of consumption, equals the marginal disutility of labor effort. Our restrictions on preferences guarantee that equation (6) describes a local optimum (i.e., that the second order condition holds.) To express marginal benefit and cost in units of consumption goods, divide by $u'(C_1^d)$ to obtain:

$$W = b\frac{(N^i)^{\varphi}}{(C^i)^{-\sigma}}. \tag{7}$$

The marginal benefit in units of consumption goods is simply the real wage. The marginal cost in units of consumption goods is $\nu'(N^S)$ normalized by the marginal utility cost of working, $u'(C_1^d)$.

The labor supply curve is defined as combinations of W and N^i that satisfy equation (7), given C^i.. Note that there are two "channels" via which a shift in W may affect N^S. The first is a substitution effect: a rise in W raises the marginal benefit of working (the left side). The second is a wealth effect that raises the marginal cost by reducing the marginal utility of an additional unit of consumption (the denominator on the right side.) Since the marginal cost of working is increasing in N^s, the substitution effect induces a rise in labor effort, while the income effect induces a decline. The parameter φ is known as the Frisch elasticity of labor supply.

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The Firm

The firm maximizes the discounted stream of profits returned to the household. Given that there is no uncertainty, the firm discounts future profits at the rate $1/R$. Accordingly, the firm chooses N^d, I, Π_1, and Π_2 to solve

$$\max \Pi^j \tag{8}$$

subject to:

$$\Pi^j = Y^j - WN^j \tag{9}$$
$$= Af(N^j) - WN^j \tag{10}$$

taking as given W .

For convenience let

$$f(N^j) = (N^j)^{1-\alpha}$$

Again, we can convert the problem into an unconstrained maximization problem by plugging the constraints (9) into (8). The firm then chooses N^j to solve:

$$\max A(N^j)^{1-\alpha} - WN^j$$

The first order necessary conditions for labor is given by:

$$(1-\alpha)A(N^j)^{-\alpha} = W \tag{11}$$

According to equation (11), the firm adjusts labor demand until the marginal benefit, given by the marginal physical product of labor, equals the marginal cost, given by the real wage.

From the firm's decision problem we obtain a demand curves for labor: The labor demand curve is given by combinations of W and N^d that satisfy equation (11). .

Competitive Market Equilibrium

Since all households are identical and all firms are identical, we can restrict attention to symmetric equilibrium (where consumption and labor supply is the same for each household and employment and output are the same for each firms.

A competitive equilibrium for this economy is an allocation (N, Y, C) and a relative price vector $(W,)$ such that the household and the firm is each maximizing its respective objective, markets clear, and the economy resource constraints are satisfied. In practice, to determine the three quantity variables: N, Y, C, we need three independent relations:

Labor Market:

$$(1-\alpha)A(N)^{-\alpha} = b\frac{(N)^{\varphi}}{(C)^{-\sigma}}. \tag{12}$$

Resource Constraint

$$Y = C \tag{13}$$

Technology Constraints:

Wages can be read off the market equilibrium.

$$W = (1-\alpha)A(N)^{-\alpha} = b\frac{(N)^{\varphi}}{(C)^{-\sigma}} \tag{14}$$

The equilbirum is simple enough to solve by hand:

$$(1-\alpha)A(N)^{-\alpha} = b\frac{(N)^{\varphi}}{(AN^{1-\alpha})^{-\sigma}}$$

$$\begin{aligned}(1-\alpha)A^{1-\sigma} &= bN^{\alpha+\varphi+\sigma(1-\alpha)}\\ N &= (\frac{1-\alpha}{b}A^{1-\sigma})^{\frac{1}{\alpha+\varphi+\sigma(1-\alpha)}}\end{aligned}$$

It is then easy to solve for C and Y.