Lab Report #4: Excess Returns

(Started: July 19, 2011; Revised: March 23, 2012)

Due at the start of class. You may speak to others, but whatever you hand in should be your own work.

Solution: Short answers follow. See Matlab code at the end for complete answers.

1. (disaster risk and the equity premium) We'll add a third "disaster" state to our analysis of the equity premium and see how it changes our view of it. The key input is the distribution of log consumption growth,

$$\log g \;=\; \begin{cases} \mu+\sigma & \text{with probability } (1-\omega)/2 \\ \mu-\sigma & \text{with probability } (1-\omega)/2 \\ \mu-\delta & \text{with probability } \omega \end{cases}$$

What's the idea? If $\omega = 0$, we're back to our symmetric two-state distribution. But if we introduce a small positive value of ω and a "largish" $\delta > 0$, we have a "disaster" state that changes the distribution dramatically.

The question is what this does to the equity premium. For reasons that should be clear from class, we define the equity premium in logs,

$$E(\log r^e - \log r^1),$$

and aim at a target value of 0.0400 (4%). As before, we'll define equity in this model as a claim to consumption growth g.

The structure is roughly similar to that in the Matlab program we used in class, but I encourage you to write your own. (Why? I usually find it harder to adapt someone else's code than write my own. You're welcome to do either, but don't say I didn't warn you.)

(a) If $\omega = 0$, what values of μ and σ deliver the observed mean and variance of log consumption growth, namely 0.0200 and 0.0350^2?

(b) Suppose $\beta = 0.99$ and $\alpha = 5$. What are r^1, $\log r^1$, and the equity premium, $E \log r^e - \log r^1$?

(c) What is the equity premium if α equals 10? 20?

(d) Now consider $\omega = 0.01$ and $\delta = 0.30$. (These numbers are based on a series of studies by Robert Barro and his coauthors.) With these numbers, what values of μ and σ reproduce the observed mean and variance of log consumption growth?

(e) With (again) $\beta = 0.99$ and $\alpha = 5$, what are r^1, $\log r^1$, $E \log r^e$, and the equity premium, $E \log r^e - \log r^1$?

(f) What is the equity premium if α equals 10? 20? How does it compare to your previous calculations?

(g) (extra credit) How does the equity premium change if $\delta = -0.30$, so that the extreme state is good news rather than bad? Why?

(h) (extra credit) How does entropy differ between the disaster and no-disaster cases?

Solution:

(a) The expressions for the mean and variance of $\log g$ are

$$\begin{aligned} E(\log g) &= \mu + \omega\delta = 0.0200 \\ \mathrm{Var}(\log g) &= (1-\omega)\sigma^2 + \omega(1-\omega)\delta^2 = 0.0350^2. \end{aligned}$$

When $\omega = 0$, the mean is $\mu = 0.0200$ and the standard deviation is $\sigma = 0.0350$.

(b) With these values, we have $r^1 = 1.0995$, $\log r^1 = 0.0948$, and $E \log r^e - \log r^1 = 0.0055$.

(c) The equity premium is 0.0112 when $\alpha = 10$ and 0.0208 when $\alpha = 20$. Both are well below our target value of 0.0400.

(d) When $\omega = 0.01$, we need to set $\mu = 0.0230$ and $\sigma = 0.0183$ to maintain the mean and variance at their sample values. See (a).

(e) With these values, we have $r^1 = 1.0907$, $\log r^1 = 0.0868$, and $E \log r^e - \log r^1 = 0.0097$. That's partial success: the equity premium went up, even though we held the mean and variance of log consumption growth constant. In that respect, the disaster state is a useful innovation, but we still need risk aversion above 5 to hit the equity premium.

(f) The equity premium is 0.0438 when $\alpha = 10$ and 0.2285 when $\alpha = 20$. Evidently a much smaller value of α suffices when we have a disaster state.

(g) If we switch the sign of δ, the equity premium is below even the two-state version: 0.0037. Apparently positive skewness in consumption and dividend growth isn't helpful. It's not part of the question, but this is connected to the discussion of skewness and entropy: positive skewness in $\log m$ increases entropy. With power utility, that requires negative skewness in $\log g$.

(h) We compute entropy directly from m:

$$H(m) = \log E(m) - E \log m.$$

With $\alpha = 5$, entropy is 0.0232 with the disaster state, 0.0152 without. So the disaster increases entropy. The difference between the two increases with risk aversion. When $\alpha = 20$, entropy is 1.5675 with the disaster state, 0.2273 without. Yaron's bazooka!

It's not part of the question, but changing the sign of δ reverses all this: we need disasters, not booms.

2. (risk and return) Modern economies issue a wide range of assets, whose returns can be wildly different. We typically attribute this to "risk," somehow defined, but before we get to that point, it's helpful to have some idea of the properties of returns on some common assets: their sample moments, for example.

   We'll start with data input. Go to Ken French's data site, a standard source for data on financial returns for a broad range of equity-related portfolios:

   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

   Download the files associated with, respectively, the "Fama-French Factors" and "Portfolios Formed on Size" at the links

   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors.zip
   http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/Portfolios_Formed_on_ME.zip.

   In both cases, you'll find a txt file inside a zip file. Copy the first table in each txt file into a spreadsheet with the dates aligned. In the first file ("factors") you want the first column (the date), the second (the excess return on the market), and the fifth (the short-term or riskfree interest rate RF). In the second file ("size") you want the first column (the date again) and columns three to five (returns on portfolios of small, medium, and large firms).

   When you're done, read the data into Matlab and convert returns to excess returns, where necessary, by subtracting the one-period rate. At this point you should have excess returns on the market and three size portfolios (small, medium, and large) — four variables all together.

   (a) Compute the mean, standard deviation, skewness, and excess kurtosis for each excess return series. Which portfolio has the highest excess return? Lowest?

   (b) Do the same for log excess returns. (Make sure you add one to them before taking logs.)

   (c) Is there a clear link here between risk, measured here by the standard deviation of the excess return, and return, measured by the mean excess return? Does it matter whether we use levels or logs?

   (d) The Sharpe ratio for any asset or portfolio is the mean of its excess return over its standard deviation. Is there a clear link between Sharpe ratios and mean excess returns?

   Comment: Returns here are monthly.

   Solution: The essential step here is to understand the units. The data are monthly returns, expressed as percentages (that is, multiplied by 100). That's not a problem with returns and excess returns, the change in units carries through the calculations. But with log returns, it's essential that we divide by 100 before adding one and taking logs. I multiplied by 100 after taking logs to make the numbers comparable to those in levels, but that's a subtle issue.

   (a) I added skewness and kurtosis to maintain our theme, but it's obviously not expected of you. The results are included in the table below.

Properties of monthly excess returns

Statistic	Portfolio market	small	medium	big
Mean	0.6172	0.9738	0.8532	0.5987
Std dev	5.4571	8.5584	6.8520	5.2757
Skewness	0.1685	2.1813	0.9816	0.1860
Excess kurtosis	7.3997	21.8628	11.7830	7.1738
Sharpe ratio	0.1131	0.1138	0.1245	0.1135

The highest mean excess return is small firms.

(b) Similar table for log excess returns below. We need to add one to returns and take logs.

Properties of monthly log excess returns

Statistic	Portfolio market	small	medium	big
Mean	0.4669	0.6329	0.6224	0.4583
Std dev	5.4556	8.1132	6.7205	5.2682
Skewness	−0.5298	0.4937	−0.0850	−0.4779
Excess kurtosis	6.5282	9.6576	7.5655	6.3824

(c) There's some connection between the mean and the standard deviation in levels, not so much in logs. Neither is all that informative: we know risk has to do with the connection to the pricing kernel, anything else would be purely accidental.

(d) There's less variation in Sharpe ratios than mean returns, because the high return assets also have high standard deviations.

Matlab program:

```
%  hw4_s12
%  Matlab program for Lab Report #4, Spring 2012
%  NYU course ECON-UB 233, Macro foundations for asset pricing, Mar 2012.
%  Written by:  Dave Backus, March 2012

disp('Answers to Lab Report 4')

%%
disp(' ')
disp('----------------------------------------------------------------')
disp('Question 1 (disaster risk)')
format compact
clear all

syms s omega mu sigma delta

disp(' ')
disp('Analytics for cumulants')
mgf = omega*exp(s*(mu-delta)) + 0.5*(1-omega)*(exp(s*(mu+sigma))+exp(s*(mu-sigma)));
cgf = log(mgf);

disp(' ')
disp('Cumulants')
kappa1 = subs(diff(cgf,s,1),s,0);    %  mean
kappa2 = subs(diff(cgf,s,2),s,0);    % variance
kappa3 = subs(diff(cgf,s,3),s,0);
kappa4 = subs(diff(cgf,s,4),s,0);

kappa1 = simplify(kappa1)
kappa2 = simplify(kappa2)

gamma1 = simplify(kappa3/kappa2^(3/2))
gamma2 = simplify(kappa4/kappa2^2)

disp(' ')
disp('Asset prices and returns')
clear all

% consumption process
omega = 0.01;
delta = 0.3;
mu = 0.0200 + omega*delta
sigma = sqrt(0.0350^2 - omega*(1-omega)*delta^2)
```

```
p = [(1-omega)/2; (1-omega)/2; omega];
logg = [mu + sigma; mu-sigma; mu-delta];
g = exp(logg);

% preferences
beta = 0.99;
alpha = 20;

% asset prices
m = beta*g.^(-alpha);
d = g;

q1 = sum(p.*m);
r1 = 1/q1
logr1 = log(r1)

qe = sum(p.*m.*d)
Elogre = sum(p.*log(d)) - log(qe);

eq_prem = Elogre - logr1

% entropy
H = log(sum(p.*m)) - sum(p.*log(m))

return

%%
disp(’ ’)
disp(’------------------------------------------------------------------’)
disp(’Question 2’)
format compact
clear all

% read spreadsheet with Fama-French returns
% dates monthly from 1926 07 to end of 2011
% series:  Date1, MKTXS, SMB, HML, RF, Date2, DUMMY, LO30, MED40, HI30

FF_returns = xlsread(’FamaFrench_returns.xlsx’);
[nobs,nvars] = size(FF_returns)

disp(’ ’)
disp(’Extra stuff to make sure things are lined up with the right units’)
disp(’Mean of inputs (mktxs, rf, small, med, big)’)
mean(FF_returns(:,[2 5 8:10]))

rf = FF_returns(:,5);
```

```
rfbig = FF_returns(:,[5 5 5 5]);     % expand to make subtraction easy
returns = FF_returns(:,[2 8:10]);
returns(:,1) = returns(:,1) + rf;
xs_returns = returns - rfbig;

disp(' ')
disp('Moments of xs returns (mean, std, skew, kurt, sharpe)')
disp('(mkt, rf, small, med, big)')
[mean(xs_returns); std(xs_returns); skewness(xs_returns); ...
    kurtosis(xs_returns) - 3;  mean(xs_returns)./std(xs_returns)]

disp(' ')
disp('Moments of log xs returns (mean, std, skew, kurt, sharpe)')
disp('(mkt, rf, small, med, big)')

lxs_returns = log(1+returns/100) - log(1+rfbig/100);

[mean(lxs_returns); std(lxs_returns); skewness(lxs_returns); ...
    kurtosis(lxs_returns) - 3;  mean(lxs_returns)./std(lxs_returns)]


return
```