Professor Mark Gertler
Intermediate Macroeconomic Theory
Spring 2009
March 4

Midterm 1

Answer each of the following three problems:

ai

Problem 1
(40 points)

It is often argued that increases in government debt will raise interest rates and crowd out private investment. Evaluate this argument in the context of the following model:

Consider a simple two period closed economy with the following features: a representative household that consumes, saves and receives dividend income (from ownership of firms): a representative firm that produces output using capital, invests in new capital, and pays dividends to households; and a government that spends each period, and finances this spending through taxes and borrowing.

The household and the firm act competitively. The key details about the economy are as follows

The Representative Household:

Let S^i be household i's saving, Π_k^i dividends in period $_k$, R the gross real interest rate (equal to one plus the net interest rate), all in units of consumption goods. Further, we normalize the price of consumption goods at unity. The representative household chooses C_1^i, C_2^i, and S^i,to solve

$$\max \log C_1^i + \beta \log C_2^i \tag{1}$$

subject to:

$$C_1^i = \Pi_1^i - T_1 - S^i \tag{2}$$

$$C_2^i = \Pi_2^i - T_2 + RS^i \tag{3}$$

The household takes as given R, Π_1^i and Π_2^i. Capital markets are perfect.

.

The Representative Firm:

The firm maximizes the discounted stream of profits returned to the household. Given that there is no uncertainty, the firm discounts future profits at the rate $1/R$. Accordingly, the firm chooses investment I^j, Y_2^j, andK_2^j to solve

$$\max \Pi_1^j + \frac{\Pi_2^j}{R} \tag{4}$$

subject to:

$$\Pi_1^j = Y_1^j - I^j + B^j \tag{5}$$

$$\Pi_2^j = Y_2^j + (1-\delta)K_2^j - RB^j \tag{6}$$

$$Y_k^j = A_k (K_k^j)^\alpha, \qquad k = 1, 2$$

$$K_2^j = (1-\delta)K_1^j + I^j$$

taking K_1^j and R as given and where B^j is firm borrowing. We abstract from adjustment costs of investment.

Government

Let G_k be government expenditures on goods and services in period $k = 1, 2$, T_k taxes in period k, and B^g the stock of government issued in period 1.

$$B^g + T_1 = G_1 \tag{7}$$

$$T_2 = G_2 + RB^g \tag{8}$$

Economy-Wide Resource Constraint

In period 1, output is divided between consumption, investment: and government expenditures:

$$Y_1 = C_1 + I + G_1 \tag{9}$$

In period 2, output and the remaining capital stock is divided between consumption and government expenditures.

$$(1-\delta)K_2 + Y_2 = C_2 + G_2 \tag{10}$$

Questions

1. Derive the first order condition for the household consumption/saving relation and then solve for the household's choice of consumption in period 1.

2. Derive the first order condition for the firm investment decision.

3. Present the set of equations that describe the equilibrium.

4. Suppose that the government increases the stock of government debt by reducing taxes in period 1.What is the impact on the market equilbrium interest rate and investment? Explain.

5. Now suppose that households cannot borrow, i.e,

$$C_1 \leq \Pi_1 - T_1$$

   How does this modification affect your answer to the previous question?

Problem 2

(30 points)

Consider the investment problem of a firm that operates for two periods (this is a partial equilibrium model; do not worry about the household). Output Y in periods one and two for the firm are given by

$$Y_1 = A_1 K_1^{\alpha}$$

$$Y_2 = A_2 K_2^{\alpha}$$

where capital K is given in the first period, but K_2 depends on investment I, as follows:

$$K_2 = (1-\delta)K_1 + I$$

The firm borrows at the rate R^b and can save at the rate R, with

$$R^b = \Omega R$$

where $\Omega > 1$.

The firm chooses I to maximize the present discounted returns to shareholders, given by,

$$Y_1 + B - I - \frac{c}{2}(\frac{I}{K_1})^2 K_1 + \frac{Y_2 + (1-\delta)K_2 - R^b B}{R}$$

where B is the amount the firm borrows.

Questions:

1. Under what circumstances does the firm need to borrow to finance its investment?.
2. Show that if the firm needs to borrow, it is opitmal to borrow the minimum amount necessary.
3. Derive the firm's optimal choice of investment. Does it depend on whether or not the firm has to borrow? If so, why.
4. During a financial crisis, financial instituitons have greater difficulty transferring funds from lenders to borrowers, which leads to an increase in Ω. What is the impact of an increase in Ω on investment. Does it depend on whether the firms needs to borrow?
5. If the central bank can adjust R in response to the financial crisis, how should it do so? Explain.

Problem 3

(30 points)

Consider a competitive equilibrium economy with households that have an infinite horizon. In addition, households suppy labor endogenously.

Representative Household

Suppose there is a representative family with the following objective.

$$\max_{N_t, C_t} \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i})]$$

where C_{t+i} is consumption.with $0 < \beta < 1$. The household supplies labor exogenously.

The budget constraint is given by

$$C_t = W_t N_t + R_t K_t - K_{t+1}$$

Representative Firm

The firm hires labor and rents capital from households in order to produce output. It's optimization problem each period is given by

$$\max Y_t - W_t N_t - (R_t - 1 + \delta) K_t$$

subject to

$$Y_t = K_t^{\alpha} (A N_t)^{1-\alpha}$$

where $R_t - 1 + \delta$ is the user cost of capital (net interest $R_t - 1$ plus depreciation δ).

Law of Motion for Capital:

$$K_{t+1} = (1 - \delta) K_t + I_t$$

Resource Constraint

$$Y_t = C_t + I_t$$

Evolution of Productivity

$$\frac{A_{t+1}}{A_t} = 1 + a$$

This size of the labor force is fixed, i.e.

$$N_t = N$$

Questions

1. Derive the household's first order condition for consumption and saving.
2. Derive the firm's first order conditions for labor and capital demand.
3. Present the set of equations that determine the balanced growth path equilibrium values of: $\frac{Y}{A}, \frac{C}{A}, \frac{I}{A}, \frac{K}{A}$. (Note we are not deflating these values by N since N is not growing.)
4. Now suppose the household choose labor supply endogenously. The households objective is now given by:

   $$\max_{N_t, C_t} \sum_{i=0}^{\infty} \beta^i [\log(C_{t+i}) - \nu N_{t+i}]$$

   Derive the household's first order condition for labor supply N_t. (Hint: the labor supply decision is a static period-by-period decision, i.e., you can solve for the optimal choice of N_t ignoring the future.) Then present the set of equations that determine the balanced growth path equilibrium values of $\frac{Y}{A}, \frac{C}{A}, \frac{I}{A}, \frac{K}{A}, N$.
5. Suppose there are two economies that are identical, except that in economy 1, productivity is a multiple of productivity in country 2, with $A_t^1 = \theta A_t^2$., with $\theta > 1$. Suppose the two economies are along balanced growth paths. Then at any time t, how do Y_t^1, C_t^1 and N_t^1 compare with Y_t^2, C_t^2 and N_t^2.? (Hint: you should be able to answer this questions mostly by inspecting your answer to the previous question.)