ECON-UB 233 Dave Backus @ NYU

Lab Report #6: Options & Volatility

Revised: November 10, 2015

Due at the start of class. You may speak to others, but whatever you hand in should be your own work. Please include your Matlab code.

Solution: Brief answers follow, but see also the attached Matlab program; download the pdf, open, click on pushpin:

1. *Bond yields.* Our mission is to explore the relation between the price of a bond and its yield. Suppose we have a 5-year bond with annual coupons of c and a principle of 100. Thus an owner gets a cash flow of c after years one to four and $100 + c$ after year five. If the bond sells for price q, it's common to express it as the discounted cash flow

$$q \;\;=\;\; c/(1+y) + c/(1+y)^2 + c/(1+y)^3 + c/(1+y)^4 + (c+100)/(1+y)^5.$$

The discount rate y is referred to as the yield or *yield to maturity*. Equally common is to use y as a way to report the price, since knowing y is enough to compute the price (plug it into the equation).

Set the coupon $c = 5$ and do the following:

(a) Plot the price q against a grid of points y between 0.00 and 0.10. How does the price vary as we change the yield?

(b) Suppose the price is $q = 102$. Use your graph to estimate the yield y.

(c) Write a bisection program to find the yield y associated with price $q = 102$. *Comment: See the Matlab guide to anonymous functions and the root-finding code posted on the course outline for examples.*

(d) How does your answer change if the price is $q = 99$. Why?

(e) *Optional (for aficionados only).* If we define $d = 1/(1 + y)$, we see that the bond price is a polynomial in d:

$$-q + cd + cd^2 + cd^3 + cd^4 + (c+100)d^5 \;\;=\;\; 0.$$

Since it's a polynomial of degree 5, it has five roots. What happened to the other ones when we computed the yield earlier?

Solution:

(a) The bond price is a decreasing function of the yield in this interval.

(b) The bond price of 102 corresponds to a yield of about 0.045. See the figure generated by the Matlab code.

(c) Bisection gives us a more accurate answer: $y = 0.0454$.

(d) If $q = 99$ we get $y = 0.0523$.

(e) In terms of d, we see that we have a polynomial of degree 5. The fundamental theorem of algebra tells us it has 5 roots. The other ones in this case are negative or complex, which doesn't make sense in this context. Descartes' rule of signs tells us, in general, that problems of this form have a single positive root because there's only one sign change in the coefficients.

2. *Finding calls from puts.* If we know put prices at given strikes k, we can compute the call prices at the same strikes from the put-call parity relation. And vice versa.

   Here's an example. Consider a one-year put option on a stock currently selling for 100. The option with strike price $k = 95$ has a price of 2. The one-period bond price is $q_t^1 = 0.99$. What is the price of a call option at the same strike?

Solution: Put-call parity is

$$\underbrace{q_t^c}_{\text{buy call}} \;-\; \underbrace{q_t^p}_{\text{sell put}} \;+\; \underbrace{q_t^1 k}_{\text{present value of strike}} \;=\; \underbrace{s_t}_{\text{buy stock}},$$

so the price of the call is

$$q_t^c \;=\; s_t + q_t^p - q_t^1 k \;=\; 100 + 2 - 0.99 \cdot 95 \;=\; 7.95.$$

3. *Black-Scholes-Merton volatility.* Our mission here is to examine the role of the mysterious volatility parameter σ in the BSM formula. The calculations refer to put options on the S&P 500 exchange-traded fund, ticker symbol `SPY`. You can look up prices at Yahoo Finance for various strikes and maturities. Or use a Bloomberg machine.

   We'll use these inputs: The current price of the underlying is $s_t = 208$. The price of a one-year bond is $q_t^1 = 1.00$.

   (a) Consider a put option with strike price $k = 180$ and a maturity of one year. If volatility $\sigma = 0.10$, what is the price of the option? If $\sigma = 0.20$?

   (b) Plot the option price against volatility for σ between 0.01 and 0.30. What does it look like? Can you say why?

   (c) Suppose the price of the option is 6. What value of σ does that correspond to?

Solution:

(a) The put price is 0.64 with $\sigma = 0.1$ and 5.30 with $\sigma = 0.2$.

(b) See figure generated by Matlab program. The put price is increasing in volatility σ. The big picture answer is that option payoffs are convex functions of the underlying, so an increase in risk raise their expected value. We haven't nailed that down completely, but that's what's going on.

(c) From the picture, it looks to be a bit over 0.2. Looking more closely, it's 0.2116.