Goodness-of-Fit Tests for the Akash Distribution

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Abstract:

The Akash distribution is a simple, but flexible, one-parameter distribution that is appropriate for the analysis of lifetime data. It has been shown to be superior to various competing one-parameter distributions in several studies. However, to date, no appropriate goodness-of-fit tests for the Akash distribution have been constructed. This paper employs the "biased transformation" methodology proposed by Raschke (2009) to adapt several goodness-of-fit tests, based on the empirical distribution function, for use with the Akash distribution. We undertake a simulation experiment that demonstrates that these (modified) tests perform well in the context of the Akash distribution, in terms of both low size distortion, and desirable power against a range of alternatives. We find that the "biased transformation" Anderson-Darling test dominates the other tests that are considered.

Key Words: Akash distribution; Goodness-of-fit testing; empirical distribution function; lifetime data

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1. Introduction

The modelling of "lifetime data" is an important topic in statistical analysis. Examples of such data arise naturally in a wide range of disciplines, including medicine, engineering, biology, environmental studies, economics, *etc*., and they form the basis for reliability analysis. The measurement of such data may be "complete", in the sense that the full lifetime of each item is available. Alternatively, some measurements may be "incomplete" as they may involve left or right censoring. In this paper we will be concerned only with complete measurements.

There are many continuous distributions, with support on the positive real half-line, that are routinely used in reliability studies. Many of the most widely used such distributions belong to the "generalized gamma" family (Stacy, 1962), including the exponential, gamma, Weibull, half-normal, and Nakagami distributions. Many variations and generalizations of these and other distributions (such as the log-normal distribution) have also been applied to lifetime data, and their density functions are capable of modelling a wide range of shapes. Often, this shape-flexibility comes at the expense of additional parameters being part of the distribution's structure.

Simple probability distributions with a single parameter, but relatively flexible densities, are very attractive. One such distribution is the Lindley distribution (Lindley, 1958), whose density function can assume a wider range of shapes than the exponential distribution. Recently, Shanker (2015a) proposed the Akash distribution, the original form of which has a single (scale) parameter, and whose density, hazard, and mean life functions allow for greater shape flexibility than the Lindley distribution counterparts. These characteristics make the Akash distribution an attractive choice when modelling lifetime data and conducting reliability studies.

In this paper we focus on the currently unaddressed problem of testing the Akash distribution for goodness-of-fit to a set of data. Specifically, we consider several well-known such tests, based on the empirical distribution function (EDF) of the sample. These tests need to be customized to the Akash distribution, and we achieve this by adopting a transformation proposed by Rasche (2009). The next section describes the Akash distribution itself, and the EDF tests and Raschke's transformation are introduced in section 3. Section 4 discusses a Monte Carlo simulation study that addresses the sizes and powers of the various tests in finite samples. This study allows us to recommend one test as being superior in this respect to the other tests. Some empirical applications of the goodness-of-fit testing are presented in section 5, and section 6 provides some concluding remarks and suggestions for further research on this topic.

2. The Akash distribution

If X follows the Akash distribution with a scale parameter, λ, its cumulative distribution function (c.d.f.) is

$$F(x) = 1 - \left[1 + \frac{\lambda x(\lambda x+2)}{(\lambda^2+2)}\right] \exp(-\lambda x) \,, \tag{1}$$

its probability density function (p.d.f.) is

$$f(x) = \frac{\lambda^3}{(\lambda^2+2)}(1+x^2)exp(-\lambda x) \qquad ; \qquad x > 0, \lambda > 0 \tag{2}$$

and the hazard function is

$$h(x) = \lambda^3(1+x^2)/[\lambda x(\lambda x+2) + (\lambda^2+2)] \quad . \tag{3}$$

Illustrative plots of the c.d.f., p.d.f., and hazard function, for various values of λ, are provided by Shanker (2015a). He also shows that the r'th. moment of X about the origin is

$$\mu_r' = r!\,[\lambda^2 + (r+1)(r+2)]/[\lambda^r(\lambda^2+2)] \qquad ; \quad r = 1, 2, 3, \ldots. \tag{4}$$

implying that $E[X] = [\lambda^2+6]/[\lambda(\lambda^2+2)]$; and $Var.[X] = [\lambda^4 + 16\lambda^2 + 12]/[\lambda(\lambda^2+2)]^2$.

Given a sample of n independent observations, with a sample mean of $\bar{x}$, the log-likelihood function is

$$l = 3nlog(\lambda) - nlog(\lambda^2+2) - \lambda n\bar{x} + \sum_{i=1}^{n} \log(1+x_i^2). \tag{5}$$

So,

$$\frac{\partial l}{\partial \lambda} = \left(\frac{3n}{\lambda}\right) - \frac{2n\lambda}{(\lambda^2+2)} - n\bar{x} \,, \tag{6}$$

and the maximum likelihood estimator (MLE) of λ, say $\tilde{\lambda}$, is obtained by (numerically) solving the equation,

$$\bar{x}\lambda^3 - \lambda^2 + 2\bar{x}\lambda - 6 = 0 \tag{7}$$

for λ. (Equating $\bar{x}$ and $E[X]$, we see that the method of moments estimator and the MLE of λ coincide.)

Shanker (2015b) and Shamket *et al.* (2018) extended the Akash distribution to a two-parameter counterpart. Shanker *et al*. (2015) and Shanker and Fesshaye (2016) provided a detailed comparative discussion of the Akash distribution, together with numerous applications using "real-life" data. Although these authors report Kolmogorov-Smirnov test statistics in their applications, these statistics are not constructed in the manner described in the next section, so as to be applicable to the Akash distribution. Together with log-likelihood values and information criteria, they are used in a purely ordinal manner to rank the results associated with different distributions. No "goodness-of-fit testing is undertaken. However, these applications provide convincing evidence of the superiority of the Akash distribution in many contexts, and serve to motivate the current study.

3. Raschke's "biased transformation" tests

Raschke (2009) proposed a transformation to map the problem of testing the null hypothesis that the data follow a beta (or gamma) distribution into the problem of testing the null hypothesis of normality. This then allows the use of standard EDF tests. A corresponding transformation can be used for other distributions. For example, Giles (2024) uses this approach to construct EDF tests for the hypothesis that data follow the Kumaraswamy distribution. To construct such "biased transformation" tests for the Akash distribution we follow these steps (Raschke, 2011, p.80):

(i) Assuming that the data, *X*, follow the Akash distribution, obtain the MLE ($\tilde{\lambda}$) of the scale parameter, λ.

(ii) Generate a sample of *Y*, where $Y = \Phi^{-1}(F(X))$, where Φ is the c.d.f. for the standard Normal distribution, and $F(.)$ is the Akash distribution's c.d.f., given in (1)

(iii) Compute the MLEs of the two parameters of the normal distribution for *Y*.

(iv) Apply an EDF test for normality to the *Y* data.

(v) For a chosen significance level, α, the null hypothesis, reject H_0: "X is Akash" if H_0': "Y is Normal" is rejected. The (upper tail) critical values will be those for the case where both of the normal distribution's parameters are unknown.

Following Raschke (2011) and Giles (2024), we consider five common EDF tests for normality at step (iv). These tests are appropriate here, as the Akash distribution is continuous. Let Y_i (i = 1, 2, 3, …., *n*) be the ordered *Y* values; and let $\tilde{G}(Y_i)$ denote the normal c.d.f. with the MLEs of the mean and variance, evaluated at Y_i. The first two tests are based on the quantities $D^+ = \max_{(i)} [i/n - \tilde{G}(Y_i)]$, $D^- = \max_{(i)} [\tilde{G}(Y_i) - (i-1)/n]$, and $D = \max [D^+, D^-]$. The Kolmogorov-Smirnov test statistic is $D^* =$

$D(\sqrt{n} - 0.01 + 0.85/\sqrt{n})$; and Kuiper's test statistic is $V^* = V(\sqrt{n} + 0.05 + 0.82/\sqrt{n})$, where $V = (D^+ + D^-)$. See Stephens (1986) for further details.

Defining $W^2 = \sum_{i=1}^{n}[\tilde{G}(Y_i) - (2i-1)/(2n)]^2$, the Cramér-von Mises test statistic is $W^{2*} = W^2(1.0 + 0.5/n))$. Similarly, if $U^2 = W^2 - n\{\sum_{i=1}^{n}(\tilde{G}(Y_i)/n) - 0.5\}^2$, Watson's test statistic is defined as $U^{2*} = U^2(1.0 + 0.5/n)$. Finally, the Anderson-Darling test statistic is $A^{2*} = A^2(1.0\_0.75/n/ + 2.25/n^2)$, where $A^2 = -n - \sum_{i=1}^{n}\{(2i-1)\left(ln\left[\tilde{G}(Y_i)\right] + \ln\left[1 - \tilde{G}(Y_{n+1-i})\right]\right)\}/n$. For each test, the null hypothesis is rejected if the test statistic exceeds the appropriate critical value, and the latter are provided by Stephens (1986, p.123, Table 4.7), as reported in the last row of Table 1 in the next section.

4. A simulation experiment

We have conducted an extensive Monte Carlo simulation experiment to investigate the properties of the various tests outlined in the last section. The purpose of this study is to evaluate the true sizes (significance levels) and powers of the tests for a range of values of the scale parameter, λ, and for different sample sizes. In the case of the power evaluations, a selection of appropriate alternative "lifetime" distributions is considered. These include the exponential, gamma, Weibull, Lindley, Nakagami, half-Normal and log-Normal distributions. The size and power computations are undertaken for both 5% and 10% "nominal" significance levels for the tests, and one of our primary interests lies with ranking the performances of the various EDF tests in terms of size distortion and power.

All of the simulations used 10,000 Monte Carlo replications using the R software (R Core Team, 2024). Random variates for the Akash distribution were generated by the acceptance-rejection method, using the R package 'AcceptReject' (Marinho, 2024). Random variates for the other distributions considered in the power simulations were generated using the R packages 'VGAM' (Yee, 2024), 'nakagami' (Moss, 2024), 'fdrtool' (Klaus and Strimmer, 2024), and 'stats' (R Core Team, 2024). The 'GoFKernel package for R (Pravia, 2024) was used to invert the c.d.f. in step (ii) outlined in the previous section. The author's code for the simulations is available at https://github.com/DaveGiles1949/r-code.

The results relating to the true sizes of the tests are summarized in Table 1. The range of values considered for the scale parameter, λ, allows for different shapes for the Akash p.d.f., and the sample sizes are compatible with various applications. We see from that table that the true sizes of the tests generally exceed the nominal significance levels slightly, at least for $n < 50$. As expected, given the asymptotic validity of the tests, these sizes converge to the nominal significance levels as n increases. There is minimal size distortion in all cases when n = 100. Comparing the various tests, there is no evidence of one dominating all of the others in terms of minimal size distortion.

Table 2 provides a selection of results relating to the powers of the EDF tests against various alternative distributions. These powers are computed as "raw" rejection rates of the null hypothesis that the data are Akash-distributed when this hypothesis is false. That is, the asymptotic critical values are used, with no "size-adjustment" to allow for the fact that there is a small amount of size distortion exhibited in Table 1. This provides information that is in accord with the way in which the tests would be used in practice. "Size-adjusted" power is more appropriate when making theoretical comparisons between the powers of competing tests, but to compute such powers the simulated adjustments that need to be made to the critical values depend not only on the choice of test and sample size, but also on the unknown value of λ. This implies that for this particular testing problem, "size-adjusted" powers are of less interest than "raw" powers. The latter inform the practitioner as to the ability of the tests to reject various false hypotheses. Giles (2024) discusses this issue in the context of adapting Raschke's procedure for goodness-of-fit testing for the Kumaraswany distribution. He provides some simulated "size-adjusted" powers for a range of situations, and finds that the rankings of the EDF tests under consideration are the same as if "raw" powers are compared.

The results in Table 2 show that each of the tests is "unbiased' in all of the situations considered. That is, their power always exceeds the nominal significance level, which is a highly desirable property. The tests' powers increase with the sample size, reflecting their asymptotic validity. We also see that the Anderson-Darling test has the highest overall (raw) power among those investigated. In part, this reflects the fact that it also has the largest size in approximately 75% of the cases considered in Table 1. The dominance of the A-D test here is consistent with the results obtained by Raschke (2011) for the beta distributions, and Giles (2024) for the Kumaraswamy distribution, each using the "biased transformation" testing framework.

Of the other tests under study, the Cramér-von Mises test clearly ranks second in terms of raw power, which is also consistent with the results reported by Raaschke (2011) and Giles (2024). Watson's test and the Kolmogorov-Smirnov test share third ranking, and Kuiper's test is generally the least powerful. It is also clear that the Cramér-von Mises test out-performs Watson's test in terms of size-adjusted power – it has lower size and higher raw power than the latter test. Finally, the power performances of the tests depend very much on the form of the alternative distributions, and the values of their parameters.

5. Applications

We illustrate the application of the EDF goodness-of-fit tests for the Akash distribution with two quite different sets of data. The empirical and (estimated) theoretical distribution functions are compared for these two data-sets in Figure 1. The two data-sets can be downloaded from https://github.com/DaveGiles1949/Data.

The first application uses the fourth data-set employed by Shanker and Fesshaye (2016), and originating from Picciotto (1970). The data measure the quality of yarn by recording the number of stress cycles needed before the yarn breaks. The sample of 100 observations takes values ranging from 15 to 829 cycles, with a mean and standard deviation of 222 and 144.6 cycles respectively. When the Akash distribution is fitted to the data, the MLE of the scale parameter is $\tilde{\lambda} = 0.0135$. The values of the Kolmogorov-Smirnov, Kuiper, Watson, Cramér-von Mises, and Anderson-Darling test statistics are 0.9425,1.4451, 0.1148, 0.1207, and 0.6635 respectively. From the critical values reported in the last row of Table 1, we see that the first of these tests implies rejection of the null hypothesis (that the data follow an Akash distribution) at the 5% significance level. All of the other tests imply rejection of the null hypothesis at the 10% level, but not at the 5% significance level. So, if the preferred Anderson-Darling test is adopted, the Akash distribution is supported at the 5% significance level.

The second application uses a sample of 26 observations on the survival time for patients with ovarian cancer. The data are reported by Edmunson *et al*. (1979), and are used as an example in the R package, 'survival' (Thereau, 2024). The data take values between 59 and 1,227 days, with a mean of 599.5 days, and a standard deviation of 399.7 days. In this example, is $\tilde{\lambda} = 0.3754$, and the Kolmogorov-Smirnov, Kuiper, Watson, Cramér-von Mises, and Anderson-Darling test statistics take the values 0.5191, 1,0412, 0.0462, 0.0501, and 0.3754 respectively. These values all support the hypothesis that the data follow an Akash distribution at the 10% significance level, or lower.

6. Conclusions

In this paper we have provided several appropriate goodness-of-fit tests for the Akash distribution. Based on the empirical distribution of the sample data, and the "biased transformation" proposed by Raschke (2009), these tests fill an important gap in the reliability literature associated with this interesting distribution. The five tests all perform well in terms of low size-distortion, even in samples of modest size. Moreover, they exhibit levels of power that range from acceptable to excellent, against a wide range of alternative distributions that are commonly employed in the study of lifetime data. The Anderson-Darling test – modified to be appropriate for the Akash distribution – performs best in terms of power. This is consistent with similar studies involving the beta distribution (Raschke, 2011) and the Kumaraswamy distribution (Giles, 2024).

The tests that are studied in this paper warrant further research. In particular, their power properties against an even wider range of alternative distributions could be considered. In addition, corresponding tests could be developed for use with left-censored and/or right-censored data.

Table 1: Simulated sizes of the EDF tests*

		α = 5%					α = 10%				
n	***λ***	**K-S**	**Kuiper**	**C-M**	**Watson**	**A-D**	**K-S**	**Kuiper**	**C-M**	**Watson**	**A-D**
	0.05										
10		0.0518	0.0557	0.0456	0.0465	0.0525	0.1106	0.1090	0.1006	0.1086	0.1141
15		0.0497	0.0515	0.0511	0.0530	0.0574	0.1094	0.1080	0.1086	0.1113	0.1137
25		0.0446	0.0476	0.0468	0.0485	0.0486	0.0971	0.0944	0.0930	0.0966	0.0961
50		0.0490	0.0511	0.0489	0.0501	0.0490	0.1017	0.1000	0.0992	0.1031	0.0993
100		0.0549	0.0540	0.0524	0.0515	0.0497	0.1050	0.1056	0.1014	0.1066	0.1023
	0.1										
10		0.0540	0.0566	0.0533	0.0553	0.0592	0.1130	0.1128	0.1085	0.1131	0.1223
15		0.0541	0.0523	0.0541	0.0546	0.0583	0.1097	0.1087	0.1095	0.1110	0.1174
25		0.0486	0.0469	0.0494	0.0484	0.0511	0.1064	0.1002	0.1001	0.1050	0.1055
50		0.0489	0.0475	0.0504	0.0506	0.0506	0.1036	0.0969	0.1003	0.1042	0.1036
100		0.0480	0.0482	0.0526	0.0506	0.0521	0.1007	0.0996	0.0972	0.0996	0.1013
	0.5										
10		0.0616	0.0602	0.0583	0.0585	0.0684	0.1218	0.1178	0.1154	0.1183	0.1288
15		0.0555	0.0543	0.0544	0.0547	0.0585	0.1139	0.1071	0.1113	0.1140	0.1170
25		0.0480	0.0470	0.0516	0.0512	0.0557	0.1001	0.0993	0.0998	0.1036	0.1039
50		0.0535	0.0523	0.0525	0.0530	0.0556	0.1051	0.0999	0.1006	0.1021	0.1062
100		0.0456	0.0483	0.0487	0.0481	0.0488	0.0999	0.0948	0.0951	0.1000	0.0942
	1.0										
10		0.0578	0.0583	0.0553	0.0562	0.0626	0.1168	0.1196	0.1117	0.1197	0.1272
15		0.0506	0.0529	0.0520	0.0517	0.0548	0.1064	0.1060	0.1053	0.1108	0.1132
25		0.0502	0.0513	0.0549	0.0536	0.0587	0.1062	0.1050	0.1057	0.1081	0.1107
50		0.0499	0.0494	0.0500	0.0495	0.0507	0.1033	0.0950	0.0986	0.1003	0.1016
100		0.0533	0.0517	0.0509	0.0495	0.0509	0.1066	0.1023	0.0982	0.0993	0.1045
Crit.		0.895	1.489	0.126	0.117	0.752	0.819	1.386	0.104	0.096	0.631

* Crit. = Upper-tail critical values. Source: Stephens (1986, p.123), Table 4.7.

Table 1 (continued): Simulated sizes of the EDF tests*

		α = 5%					α = 10%				
n	***λ***	**K-S**	**Kuiper**	**C-M**	**Watson**	**A-D**	**K-S**	**Kuiper**	**C-M**	**Watson**	**A-D**
	1.5										
10		0.0602	0.0565	0.0523	0.0528	0.0613	0.1228	0.1186	0.1118	0.1178	0.1262
15		0.0531	0.0492	0.0493	0.0500	0.0562	0.1092	0.1031	0.1057	0.1082	0.1170
25		0.0490	0.0494	0.0499	0.0507	0.0543	0.1006	0.0966	0.1047	0.1059	0.1106
50		0.0513	0.0481	0.0516	0.0495	0.0527	0.1015	0.0941	0.0966	0.0982	0.1047
100		0.0519	0.0476	0.0510	0.0491	0.0509	0.1024	0.0966	0.0990	0.1006	0.0984
	2.0										
10		0.0599	0.0579	0.0544	0.0543	0.0623	0.1190	0.1173	0.1103	0.1129	0.1275
15		0.0567	0.0538	0.0547	0.0554	0.0586	0.1100	0.1097	0.1063	0.1091	0.1167
25		0.0446	0.0459	0.0457	0.0446	0.0479	0.0979	0.0912	0.0970	0.1012	0.1010
50		0.0504	0.0487	0.0517	0.0504	0.0534	0.1031	0.0974	0.1005	0.1027	0.1033
100		0.0472	0.0428	0.0484	0.0465	0.0483	0.0976	0.0899	0.0944	0.0973	0.0973
	2.5										
10		0.0568	0.0577	0.0549	0.0536	0.0631	0.1190	0.1204	0.1098	0.1156	0.1270
15		0.0514	0.0496	0.0509	0.0516	0.0561	0.1072	0.1038	0.1037	0.1076	0.1105
25		0.0476	0.0463	0.0485	0.0482	0.0499	0.0985	0.0969	0.091	0.0988	0.1023
50		0.0506	0.0480	0.0499	0.0497	0.0522	0.1080	0.1019	0.1028	0.1044	0.1065
100		0.0498	0.0493	0.0524	0.0520	0.0522	0.1028	0.0990	0.1025	0.1053	0.1016
	3.0										
10		0.0604	0.0608	0.0595	0.0600	0.0681	0.1191	0.1222	0.1173	0.1213	0.1301
15		0.0517	0.0541	0.0519	0.0532	0.0573	0.1043	0.1043	0.1057	0.1089	0.1167
25		0.0488	0.0508	0.0547	0.0528	0.0562	0.1023	0.0996	0.1041	0.1087	0.1117
50		0.0722	0.0752	0.0550	0.0596	0.0835	0.1469	0.1449	0.1198	0.1298	0.1697
100		0.0469	0.0438	0.0464	0.0466	0.0487	0.0976	0.0926	0.0940	0.0956	0.0972
Crit.		0.895	1.489	0.126	0.117	0.752	0.819	1.386	0.104	0.096	0.631

* Crit. = Upper-tail critical values. Source: Stephens (1986, p.123), Table 4.7.

Table 2: Simulated powers of the EDF tests

n	K-S	Kuiper	C-M	Watson	A-D	K-S	Kuiper	C-M	Watson	A-D
					Exponential (rate = 0.5)					
			α = 5%					**α = 10%**		
10	0.0748	0.0731	0.0761	0.0722	0.0899	0.1443	0.1389	0.1444	0.1452	0.1672
15	0.0838	0.0778	0.0954	0.0889	0.1058	0.1577	0.1446	0.1630	0.1618	0.1823
25	0.1114	0.1035	0.1319	0.1193	0.1426	0.1969	0.1748	0.2135	0.2018	0.2283
50	0.1811	0.1514	0.2193	0.1917	0.2336	0.2808	0.2402	0.3134	0.2859	0.3338
100	0.3280	0.2745	0.4048	0.3449	0.4297	0.4598	0.3956	0.5202	0.4689	0.5384
250	0.7003	0.6453	0.7953	0.7277	0.8177	0.8035	0.7492	0.8643	0.8185	0.8814
500	0.9425	0.9242	0.9769	0.9588	0.9833	0.9740	0.9609	0.9890	0.9802	0.9916
1000	0.9986	0.9978	0.9997	0.9992	0.9999	0.9997	0.9994	0.9999	0.9997	1.0000
					Exponential (rate = 1.0)					
			α = 5%					**α = 10%**		
10	0.0564	0.0619	0.0598	0.0576	0.0686	0.1219	0.1200	0.1190	0.1202	0.1358
15	0.0582	0.0577	0.0649	0.0609	0.0713	0.1190	0.1153	0.1230	0.1203	0.1331
25	0.0658	0.0630	0.0718	0.0677	0.0787	0.1278	0.1201	0.1347	0.1318	0.1449
50	0.0806	0.0761	0.0891	0.0825	0.0972	0.1460	0.1351	0.1559	0.1514	0.1669
100	0.1088	0.1001	0.1221	0.1101	0.1318	0.1845	0.1714	0.1986	0.1886	0.2109
250	0.2036	0.1830	0.2400	0.2159	0.2604	0.3178	0.2845	0.3424	0.3194	0.3676
500	0.3629	0.3358	0.4368	0.3941	0.4743	0.4958	0.4579	0.5558	0.5202	0.5872
1000	0.6490	0.6099	0.7337	0.6902	0.7724	0.7608	0.7242	0.8183	0.7849	0.8493
					Gamma (shape = 1.0, rate = 0.5)					
			α = 5%					**α = 10%**		
10	0.0832	0.0796	0.0881	0.0822	0.1020	0.1528	0.1445	0.1570	0.1554	0.1729
15	0.0860	0.0798	0.0966	0.0887	0.1062	0.1599	0.1481	0.1688	0.1649	0.1844
25	0.1176	0.1036	0.1333	0.1222	0.1505	0.2040	0.1787	0.2175	0.2067	0.2340
50	0.1767	0.1566	0.2204	0.1902	0.2355	0.2807	0.2436	0.3195	0.2908	0.3427
100	0.3223	0.2741	0.3952	0.3415	0.4235	0.4509	0.3892	0.5142	0.4631	0.5388
250	0.6918	0.6318	0.7947	0.7250	0.8151	0.8085	0.7486	0.8643	0.8186	0.8820
500	0.9443	0.9279	0.9793	0.9601	0.9838	0.9743	0.9619	0.9895	0.9806	0.9924
1000	0.9990	0.9982	0.9997	0.9995	0.9998	0.9994	0.9991	0.9998	0.9997	0.9999

Table 2 (continued): Simulated powers of the EDF tests

n	K-S	Kuiper	C-M	Watson	A-D	K-S	Kuiper	C-M	Watson	A-D
Gamma (shape = 2.0, rate = 0.5)										
			***α* = 5%**					***α* = 10%**		
10	0.0644	0.0668	0.0656	0.0645	0.0752	0.1293	0.1290	0.1291	0.1331	0.1479
15	0.0696	0.0687	0.0751	0.0706	0.0832	0.1421	0.1348	0.1418	0.1422	0.1552
25	0.0850	0.0826	0.0988	0.0908	0.1075	0.1585	0.1444	0.1697	0.1630	0.1849
50	0.1271	0.1080	0.1493	0.1271	0.1662	0.2194	0.1879	0.2403	0.2182	0.2721
100	0.2105	0.1710	0.2613	0.2141	0.3050	0.3287	0.2740	0.3794	0.3280	0.4348
250	0.4921	0.4256	0.6203	0.5140	0.7109	0.6431	0.5717	0.7393	0.6504	0.8150
500	0.8260	0.7978	0.9262	0.8580	0.9640	0.9076	0.8849	0.9625	0.9225	0.9854
1000	0.9891	0.9897	0.9993	0.9947	1.0000	0.9967	0.9964	0.9999	0.9987	1.0000
Lindley (shape = 0.5)										
			***α* = 5%**					***α* = 10%**		
10	0.0624	0.0642	0.0628	0.0636	0.0721	0.1292	0.1250	0.1252	0.1292	0.1437
15	0.0665	0.0665	0.0709	0.0700	0.0811	0.1336	0.1273	0.1339	0.1366	0.1456
25	0.0798	0.0718	0.0852	0.0816	0.0947	0.1502	0.1388	0.1574	0.1568	0.1691
50	0.1078	0.0921	0.1227	0.1077	0.1320	0.1843	0.1618	0.2012	0.1886	0.2180
100	0.1731	0.1422	0.2111	0.1796	0.2303	0.2797	0.2343	0.3148	0.2867	0.3393
250	0.3805	0.3245	0.4637	0.3929	0.5041	0.5138	0.4521	0.5794	0.5244	0.6258
500	0.6683	0.6190	0.7855	0.7072	0.8226	0.7850	0.7417	0.8630	0.8110	0.8927
1000	0.9355	0.9218	0.9791	0.9548	0.9883	0.9724	0.9634	0.9908	0.9803	0.9946
Lindley (shape = 1.0)										
			***α* = 5%**					***α* = 10%**		
10	0.0631	0.0619	0.0650	0.0613	0.0733	0.1294	0.1215	0.1231	0.1247	0.1399
15	0.0644	0.0608	0.0677	0.0642	0.0745	0.1238	0.1195	0.1253	0.1252	0.1398
25	0.0655	0.0602	0.0719	0.0678	0.0794	0.1276	0.1133	0.1302	0.1277	0.1399
50	0.0780	0.0731	0.0860	0.0813	0.0923	0.1426	0.1322	0.1514	0.1457	0.1575
100	0.1057	0.0957	0.1256	0.1124	0.1316	0.1865	0.1684	0.2032	0.1941	0.2139
250	0.2061	0.1932	0.2466	0.2245	0.2581	0.3199	0.2902	0.3488	0.3265	0.3639
500	0.3618	0.3438	0.4402	0.4033	0.4601	0.4950	0.4678	0.5558	0.5284	0.5751
1000	0.6363	0.6242	0.7388	0.6996	0.7567	0.7561	0.7430	0.8239	0.7968	0.8386

Table 2 (continued): Simulated powers of the EDF tests

n	K-S	Kuiper	C-M	Watson	A-D	K-S	Kuiper	C-M	Watson	A-D
					Lindley (shape = 2.0)					
			α = 5%					**α = 10%**		
10	0.0565	0.0572	0.0537	0.0528	0.0630	0.1201	0.1173	0.1113	0.1148	0.1296
15	0.0562	0.0546	0.0615	0.0588	0.0675	0.1109	0.1075	0.1126	0.1148	0.1225
25	0.0572	0.0542	0.0600	0.0571	0.0628	0.1120	0.1030	0.1115	0.1101	0.1159
50	0.0540	0.0534	0.0578	0.0565	0.0595	0.1060	0.1034	0.1126	0.1158	0.1171
100	0.0653	0.0624	0.0684	0.0655	0.0723	0.1207	0.1144	0.1242	0.1235	0.1290
250	0.0862	0.0919	0.0951	0.0912	0.0980	0.1566	0.1562	0.1653	0.1636	0.1712
500	0.1152	0.1250	0.1330	0.1296	0.1412	0.2027	0.2068	0.2139	0.2123	0.2301
1000	0.1900	0.2074	0.2303	0.2281	0.2500	0.2982	0.3195	0.3426	0.3422	0.3598
					Nakagami (shape = 2.0, scale = 1.0)					
			α = 5%					**α = 10%**		
10	0.0687	0.0665	0.0686	0.0668	0.0793	0.1331	0.1308	0.1315	0.1330	0.1480
15	0.0697	0.0637	0.0760	0.0695	0.0856	0.1331	0.1218	0.1383	0.1369	0.1518
25	0.0770	0.0686	0.0882	0.0802	0.0982	0.1492	0.1335	0.1578	0.1500	0.1724
50	0.1136	0.0891	0.1303	0.1117	0.1435	0.1931	0.1594	0.2084	0.1890	0.2309
100	0.1902	0.1459	0.2246	0.1781	0.2514	0.3003	0.2359	0.3273	0.2828	0.3595
250	0.4332	0.3440	0.5216	0.4266	0.5731	0.5609	0.4735	0.6325	0.5518	0.6804
500	0.7269	0.6374	0.8307	0.7325	0.8722	0.8309	0.7560	0.8933	0.8273	0.9241
1000	0.9585	0.9364	0.9885	0.9646	0.9944	0.9828	0.9687	0.9951	0.9841	0.9975
					Nakagami (shape = 1.0, scale = 1.0)					
			α = 5%					**α = 10%**		
10	0.0631	0.0624	0.0635	0.0605	0.0738	0.1230	0.1177	0.1239	0.1243	0.1427
15	0.0632	0.0607	0.0674	0.0632	0.0760	0.1241	0.1150	0.1229	0.1226	0.1366
25	0.0699	0.0657	0.0732	0.0659	0.0791	0.1339	0.1207	0.1325	0.1286	0.1484
50	0.0855	0.0732	0.0964	0.0816	0.1052	0.1567	0.1345	0.1639	0.1524	0.1764
100	0.1323	0.1018	0.1483	0.1210	0.1672	0.2191	0.1741	0.2354	0.2058	0.2604
250	0.2542	0.1936	0.3093	0.2413	0.3515	0.3809	0.3010	0.4287	0.3571	0.4712
500	0.4653	0.3656	0.5596	0.4462	0.6283	0.5976	0.4937	0.6738	0.5754	0.7325
1000	0.7646	0.6774	0.8623	0.7625	0.9095	0.8587	0.7886	0.9184	0.8472	0.9489

Table 2 (continued): Simulated powers of the EDF tests

n	K-S	Kuiper	C-M	Watson	A-D	K-S	Kuiper	C-M	Watson	A-D
				Weibull (shape = 1.5, scale = 1.0)						
			α = 5%					**α = 10%**		
10	0.0652	0.0628	0.0600	0.0582	0.0698	0.1257	0.1243	0.1230	0.1241	0.1338
15	0.0575	0.0574	0.0591	0.0564	0.0645	0.1185	0.1118	0.1164	0.1148	0.1268
25	0.0561	0.0517	0.0574	0.0548	0.0621	0.1134	0.1068	0.1128	0.1103	0.1225
50	0.0602	0.0565	0.0602	0.0567	0.0637	0.1179	0.1072	0.1142	0.1119	0.1241
100	0.0714	0.0622	0.0738	0.0677	0.0791	0.1303	0.1178	0.1333	0.1279	0.1465
250	0.0977	0.0863	0.1055	0.0920	0.1151	0.1776	0.1514	0.1763	0.1654	0.1946
500	0.1481	0.1238	0.1722	0.1441	0.1982	0.2438	0.2068	0.2619	0.2403	0.2941
1000	0.2477	0.2074	0.2943	0.2465	0.3432	0.3706	0.3136	0.4097	0.3607	0.4648
				Weibull (shape = 0.5, scale = 1.0)						
			α = 5%					**α = 10%**		
10	0.1646	0.1502	0.1819	0.1644	0.2056	0.2591	0.2379	0.2723	0.2612	0.3026
15	0.2147	0.1945	0.2533	0.2286	0.2833	0.3243	0.2921	0.3542	0.3318	0.3837
25	0.3393	0.3071	04031	0.3628	0.4325	0.4540	0.4112	0.4994	0.4668	0.5278
50	0.5988	0.5574	0.6804	0.6263	0.7142	0.7075	0.6582	0.7647	0.7221	0.7908
100	0.8859	0.8592	0.9323	0.9051	0.9449	0.9315	0.9122	0.9591	0.9409	0.9675
250	0.9985	0.9980	0.9997	0.9992	0.9997	0.9997	0.9992	0.9999	0.9995	0.9999
				Weibull (shape = 2.0, scale = 1.0)						
			α = 5%					**α = 10%**		
10	0.0687	0.0674	0.0645	0.0638	0.0744	0.1334	0.1280	0.1319	0.1315	0.1495
15	0.0682	0.0637	0.0693	0.0649	0.0785	0.1299	0.1213	0.1305	0.1290	0.1440
25	0.0697	0.0626	0.0744	0.0688	0.0850	0.1369	0.1215	0.1396	0.1343	0.1517
50	0.0875	0.0721	0.0970	0.0823	0.1107	0.1592	0.1338	0.1660	0.1517	0.1837
100	0.1264	0.1002	0.1500	0.1228	0.1716	0.2150	0.1746	0.2337	0.2029	0.2571
250	0.2566	0.1933	0.3147	0.2442	0.3582	0.3808	0.2963	0.4277	0.3565	0.4756
500	0.4645	0.3660	0.5626	0.4489	0.6332	0.6058	0.4942	0.6745	0.5763	0.7408
1000	0.7647	0.6760	0.8687	0.7611	0.9144	0.8626	0.7899	0.9228	0.8540	0.9517

Table 2 (continued): Simulated powers of the EDF tests

n	K-S	Kuiper	C-M	Watson	A-D	K-S	Kuiper	C-M	Watson	A-D
					Half-Normal (mean = 0.5)					
			α = 5%					α = 10%		
10	0.0803	0.0781	0.0825	0.0798	0.0934	0.1517	0.1487	0.1495	0.1501	0.1683
15	0.0854	0.0787	0.0955	0.0906	0.1056	0.1586	0.1450	0.1631	0.1598	0.1812
25	0.1034	0.0895	0.1168	0.1048	0.1295	9.1800	0.1593	0.1943	0.1836	0.2116
50	0.1714	0.1383	0.2052	0.1700	0.2256	0.2728	0.2232	0.3022	0.2744	0.3249
100	0.3109	0.2416	0.3758	0.3057	0.4174	0.4413	0.3605	0.4979	0.4354	0.5376
250	0.6603	0.5781	0.7714	0.6727	0.8226	0.7738	0.7031	0.8526	0.7897	0.8888
500	0.9276	0.8987	0.9785	0.9422	0.9885	0.9653	0.9480	0.9898	0.9733	0.9952
1000	0.9987	0.9983	0.9998	0.9990	0.9998	0.9995	0.9995	0.9998	0.9997	1.0000
					Half-Normal (mean = 1.0)					
			α = 5%					α = 10%		
10	0.0685	0.0673	0.0674	0.0659	0.0791	0.1337	0.1318	0.1296	0.1328	0.1463
15	0.0674	0.0649	0.0696	0.0670	0.0780	0.1281	0.1217	0.1309	0.1316	0.1411
25	0.0668	0.0638	0.0739	0.0679	0.0786	0.1283	0.1196	0.1343	0.1310	0.1441
50	0.0878	0.0748	0.0984	0.0850	0.1047	0.1646	0.1375	0.1677	0.1528	0.1813
100	0.1321	0.1035	0.1522	0.1269	0.1652	0.2195	0.1804	0.2362	0.2090	0.2599
250	0.2632	0.1955	0.3131	0.2456	0.3613	0.3858	0.2961	0.4275	0.3568	0.4790
500	0.4849	0.3758	0.5689	0.4562	0.6405	0.6165	0.5086	0.6876	0.5868	0.7505
1000	0.7826	0.6934	9.8840	0.7822	0.9274	0.8750	0.8060	0.9314	0.8683	0.9605
					Half-Normal (mean = 1.5)					
			α = 5%					α = 10%		
10	0.0757	0.0749	0.0781	0.0765	0.0882	0.1472	0.1438	0.1411	0.1443	0.1603
15	0.0796	0.0741	0.0862	0.0833	0.0970	0.1477	0.1368	0.1527	0.1496	0.1668
25	0.0911	0.0809	0.1021	0.0924	0.1117	0.1626	0.1442	0.1730	0.1657	0.1880
50	0.1447	0.1167	0.1673	0.1405	0.1877	0.2357	0.1957	0.3583	0.2300	0.2770
100	0.2483	0.1926	0.2986	0.2425	0.3352	0.3735	0.2964	0.4127	0.3579	0.4481
250	0.5401	0.4431	0.6537	0.5397	0.7062	0.6785	0.5832	0.7525	0.6717	0.7977
500	0.8467	0.7874	0.9279	0.8584	0.9548	0.9138	0.8706	0.9630	0.9220	0.9780
1000	0.9915	0.9853	0.9981	0.9940	0.9993	0.9968	0.9946	0.9994	0.9978	0.9998

Table 2 (continued): Simulated powers of the EDF tests

n	K-S	Kuiper	C-M	Watson	A-D	K-S	Kuiper	C-M	Watson	A-D
					Log-Normal (0, 1)					
			α = 5%					**α = 10%**		
10	0.1965	0.1823	0.2146	0.1965	0.2499	0.2936	0.2762	0.3143	0.2999	0.3506
15	0.2706	0.2589	0.3253	0.2929	0.3618	0.3855	0.3593	0.4261	0.4023	0.4649
25	0.4228	0.4086	0.5155	0.4652	0.5548	0.5423	0.5156	0.6134	0.5729	0.6559
50	0.7077	0.6923	0.8066	0.7492	0.8430	0.8042	0.7820	0.8722	0.8308	0.8967
100	0.9438	0.9443	0.9798	0.9607	0.9877	0.9731	0.9685	0.9890	0.9794	0.9935
250	1.000	1.000	1.000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
					Log-Normal (0, 2)					
			α = 5%					**α = 10%**		
10	0.4143	0.4210	0.4801	0.4532	0.5197	0.5364	0.5260	0.5835	0.5667	0.6213
15	0.5951	0.6069	0.6815	0.6464	0.7125	0.7060	0.6987	0.7634	0.7381	0.7919
25	0.8286	0.8447	0.8994	0.8735	0.9141	0.8916	0.8905	0.9318	0.9157	0.9460
50	0.9877	0.9901	0.9961	0.9927	0.9970	0.9944	0.9945	9.9977	0.9963	0.9983
100	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
					Log-Normal (1, 1)					
			α = 5%					**α = 10%**		
10	0.1965	0.1858	0.2198	0.2027	0.2578	0.3026	0.2885	0.3294	0.3137	0.3737
15	0.2743	0.2702	0.3398	0.3037	0.3840	0.3999	0.3786	0.4532	0.4266	0.5015
25	0.4331	0.4392	0.5461	0.4899	0.6009	0.5639	0.5532	0.6545	0.6086	0.7097
50	0.7391	0.7648	0.8591	0.8031	0.9007	0.8396	0.8441	0.9162	0.8771	0.9438
100	0.9650	0.9791	9.9930	0.9824	0.9972	0.9865	0.9884	0.9975	0.9928	0.9990
250	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.000	1.0000
					Log-Normal (1, 2)					
			α = 5%					**α = 10%**		
10	0.3893	0.4103	0.4646	0.4410	0.5131	0.5216	0.5266	0.5866	0.5646	0.6377
15	0.5627	0.6032	0.6803	0.6407	0.7213	0.6877	0.7060	0.7666	0.7421	0.8090
25	0.8043	0.8515	0.9026	0.8743	0.9261	0.8838	0.9028	0.9386	0.9216	0.9551
50	0.9858	0.9941	0.9973	0.9949	0.9985	0.9957	0.9974	0.9988	0.9978	0.9995
100	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Figure 1 (a): Distribution Functions
(Yarn Failure Data)

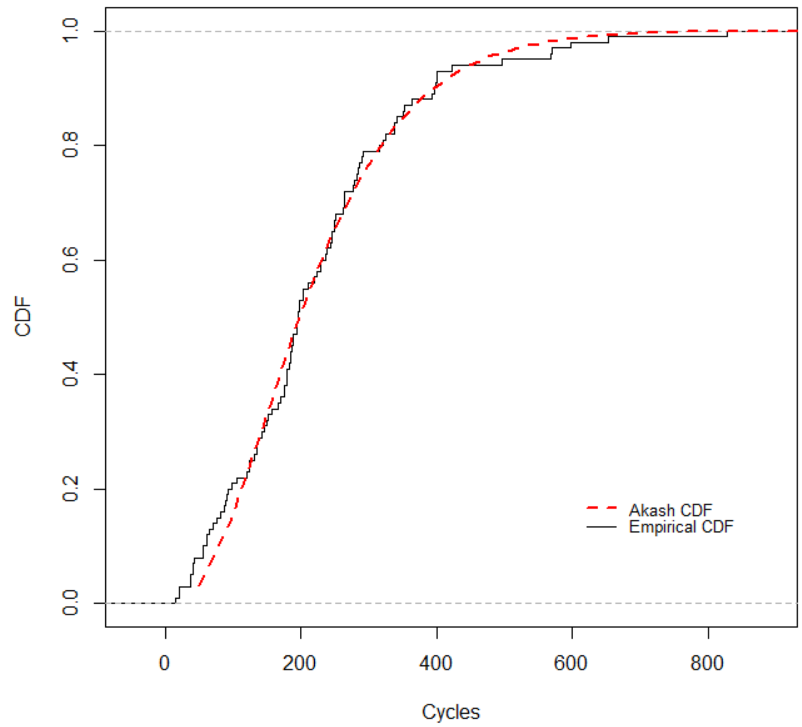
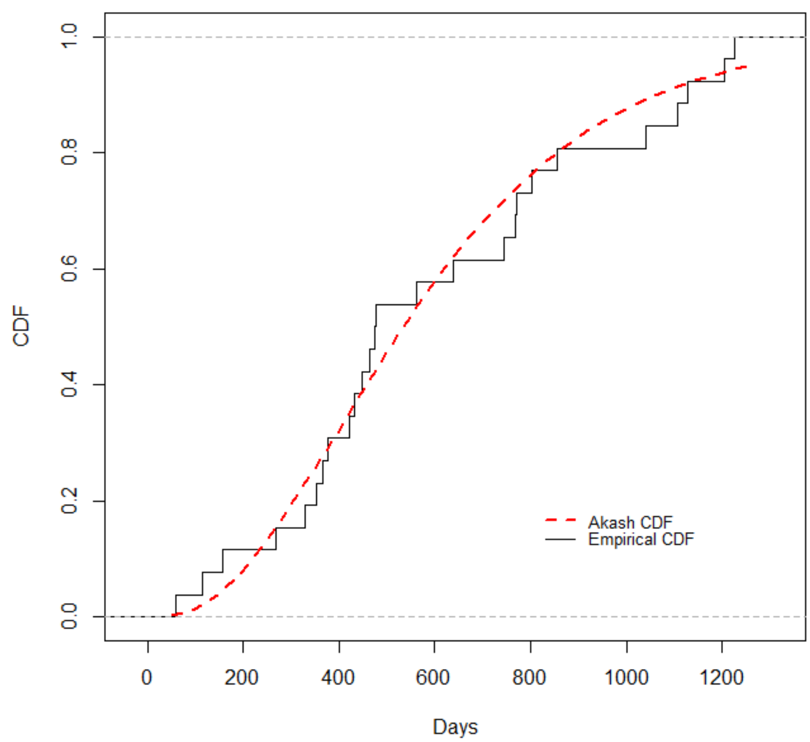


Figure 1 (b): Distribution Functions
(Ovarian Cancer Data)



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