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A STOCHASTIC MODEL FOR DRAINAGE PATTERNS INTO AN INTRAMONTANE TREINCH

A.E. SCHEIDEGGER*

ABSTRACT

The typical drainage patterns observed on the sides of intramontane trenches can be explained by a random walk model of fluvial erosion.

INTRODUCTION

When an essentially straight, primary valley runs rougly parallel to a continental divide in high mountain country, it has been observed by Gerber (1944) that a characteristic drainage pattern develops as displayed on figure 1, because the tributaries and their drainage areas have a regular arrangement so that one can attach an order to them. The basins that reach to the main divide are I order tributary drainage basins; they will naturally touch each other which necessarily gives rise to "residual or remainder areas" nearer the main rivers. These "residual areas" are again subdivided: the largest drainage basin therein is a II order tributary basin, etc. Finally, the lowest order "basins" constitute only triangular segments on the slope of the main

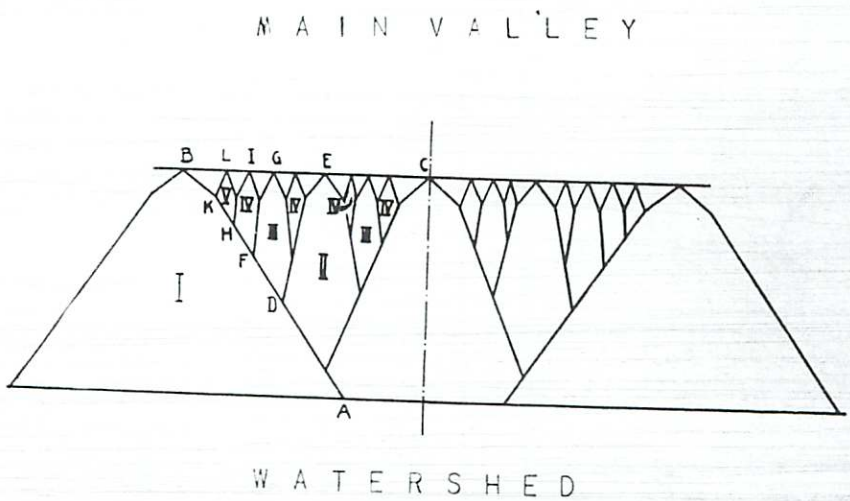

Fig. 1 — Gerber's (1944) scheme for the ordering of drainage areas.

valley ("valley sectors"), so that the general scheme of subdivision of the total drainage area between its parallel boundaries is as shown schematicaly in figure 1. This type of subdivision expresses itself in a very characteristic appearance of e.g. Alpine valleys where the postulated circumstances ("main" valley parallel to divide) prevail.

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It is the purpose of the present paper to first present Gerber's (1944) observations of a special case in a statistical form and then to show that the observed drainage patterns could be constituted by a stochastic process, which can be modeled by a Monte Carlo technique on a computer. This probabilistic approach to the development of drainage patterns as recently advocated by Leopold, Langbein, and the writer (Leopold and Langbein, 1962; Langbein and Scheidegger, 1966) produces, at least qualitatively, the observations that were made by Gerber (1944) in the Swiss Alps.

Observations

The general geometry is as follows: A principal divide must run roughly parallel to and at some distance from a main drainage channel. (See fig. 1.) Although one might think of a "piedmont" arrangement for this to occur, the most striking cases of the geometrical arrangement discussed here occur when an intramontane trench exists roughly parallel to the strike of the mountain range. The Rhone Valley in the Swiss Alps, the case examined here, is only one example of such intramontane trenches. The Rocky Mountain trench in Canada is another.

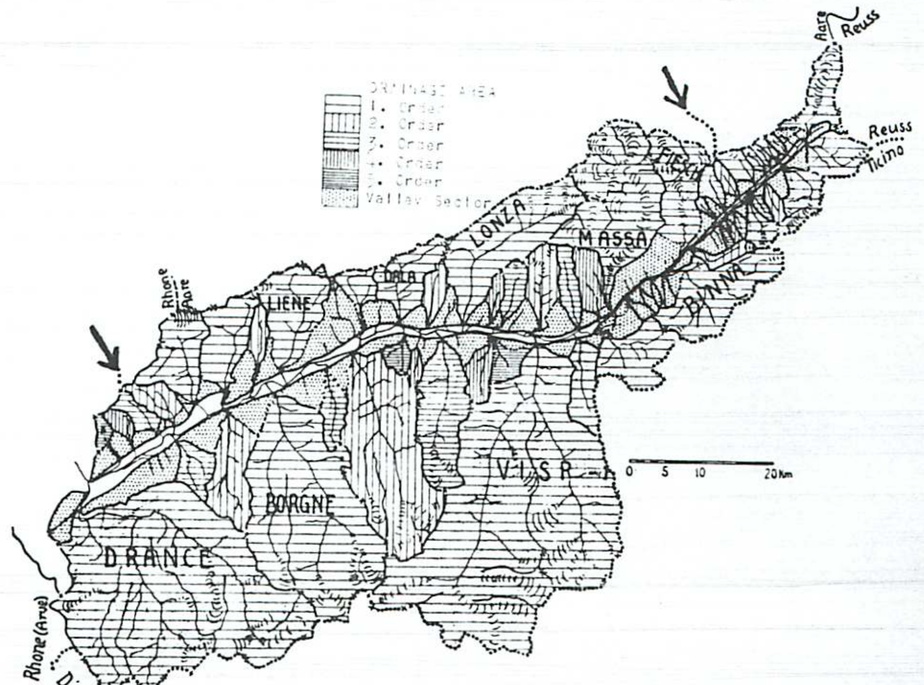

Fig. 2 — Gerber's (1944) subdivision of the drainage into the Rhone Valley.

Gerber (1944) has given a detailed description of the upper part of the Rhone Valley in Switzerland with a view toward analyzing the drainage domains of the individual tributaries. Upon a detailed analysis of the pertinent topographic maps and after field examination, he arrived at the division of the total drainage area of the Rhone into component basins shown in figure 2.

Gerber ordered the component basins according to the scheme outlined in the introduction. The "valley sectors" (no order) are the "remainder-surfaces" on the side slope of the main valley.

On the southern (left) side of the Rhone Valley there are three large drainage areas of the first order (Drance, Borgne, Visp) which is not sufficient to make a statistical summary. However, on the north (right) side the main divide runs for a long interval at about a constant distance from the main valley (the pertinent interval is indicated on figure 2 by arrows). Within this interval, there are 9 first-order, 6 second-order, 2 third-order, 2 fourth-order drainage basins and 18 valley sectors (see also table 1). With these numbers, it is possible to make an analysis in terms of a probabilistic model.

In his analysis of the Rhone tributaries, Gerber maintains the thesis that the pertinent drainage network is due to fluvial erosion, not to glacial scouring. Crickmay (1964), analyzing the geomorphology and genesis of the Rocky Mountain trench, comes to a similar conclusion. Naturally, glaciers will modify the detailed geomorphology, but the general pattern is created in periods of fluvial erosion. Any explanation of the observed facts, therefore, is to be sought in a model of branching patterns of rivers.

Theory

a) *The abstract model*

An abstract model of the physical processes that are involved assumes that the formation of the landforms in question is solely due to fluvial erosion. Moreover, we shall assume that the main valley and the watershed are straight and run parallel to one another, and that every point in the strip between the main valley and watershed has to be drained to the main valley.

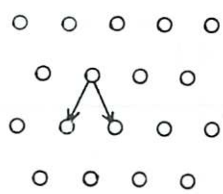

Fig. 3 — The model grid showing the possibilities for drainage at each point.

We further suppose, in analogy with the theory of plains rivers by Leopold and Langbein (1962), that the drainage occurs in the manner of a random walk. However, because of the rather special boundary conditions prevailing in the present case under consideration, the fundamental stochastic assumptions in our model will be different from those in the Leopold-Langbein model. It is clear that there is, in general, a rather high gradient in the elevation between the watershed and the main valley, so that all drainage will be essentially in the direction of this gradient, and never backwards (as is possible in a plains river). We thus assume that the drainage in a unit step 1 (time interval) is always "forward" (toward the main valley), but that it randomly may go one-half unit to the left or to the right. Thus, it is possible to model the drained strip by a grid of points; the points are arranged in rows where each subsequent row is displaced sideways by one-half of the lattice distance with regard to the former row. For the drainage of each grid point, there are two possibilities : to the nearest downward point to the left or to the right. The choice as to which of the two possibilities actually takes place is completely random. The grid, the "down" direction, and the possible choices (these may take place at each point) are illustrated in figure 3.

TABLE 1

Order	North Side Rhone Valley	Monte Carlo model		
		First trial	Second trial	Third trial
I	9	10	12	11
II	6	7	8	6
III	2	3	4	2
IV	2	2	1	0
Valley sector	18	18	18	16

TABLE 2

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It is clear that the above grid, with the random drainage possibilities at each point as stipulated, has the same boundary conditions as observed in nature in the cases discussed above. The problem is now to determine whether the drainage patterns produced on the grid by the stipulated stochastic process will have any similarity with the drainage patterns observed in nature, so that the process postulated here can be regarded as the explanation of the observations.

b) *A Monte Carlo solution*

The simplest way for a deduction of the drainage patterns induced by the stochastic process described above is by simulating the latter directly on a computer; this is commonly referred to as "Monte-Carlo" method and was applied to plains rivers e.g. by Schenck (1963).

Accordingly, random numbers were generated on the computer (Burroughs 220) of the U.S. Geological Survey. The computer was instructed to print "1" when the number was larger than the mean, "0" when it was smaller. Twenty rows of 50 numbers each were printed, subsequent rows were staggered by one character so as to make the correspondence with the grid of figure evident. A typical result of this procedure is shown in table 2.

Once the random numbers are printed out, it is a simple matter to construct the drainage net. We start at the top of the sheet, then "0" represents drainage to the left (seen from below) "1" to the right. The drainage net that evolves is shown in figure 4.

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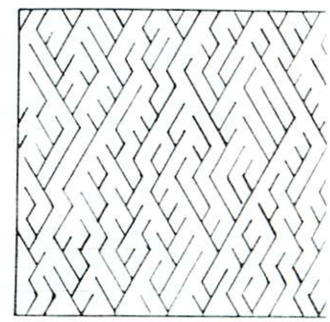

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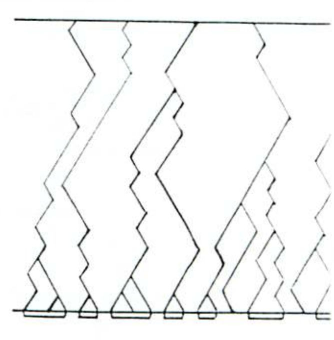

Fig. 5 — F

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ACKNOWLEDGMENT

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The writer wishes to thank Messrs. C
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The next step is to draw the boundaries between the drainage areas generated by our random numbers. If this is done, the result is as shown in figure 5. The procedure was repeated three times with approximately the same results.

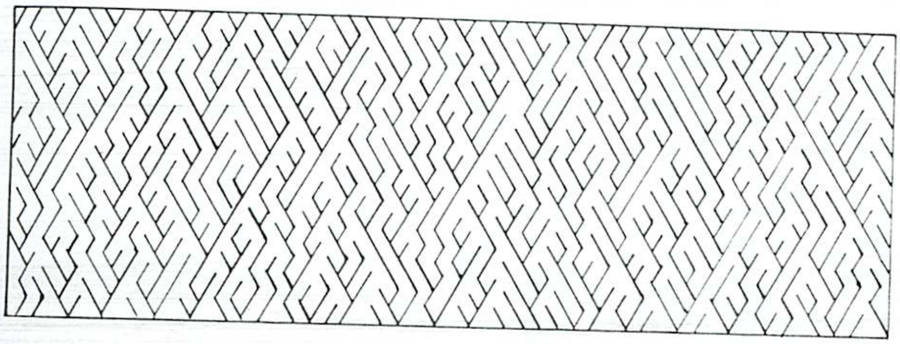

Fig. 4 — Random—generated drainage net.

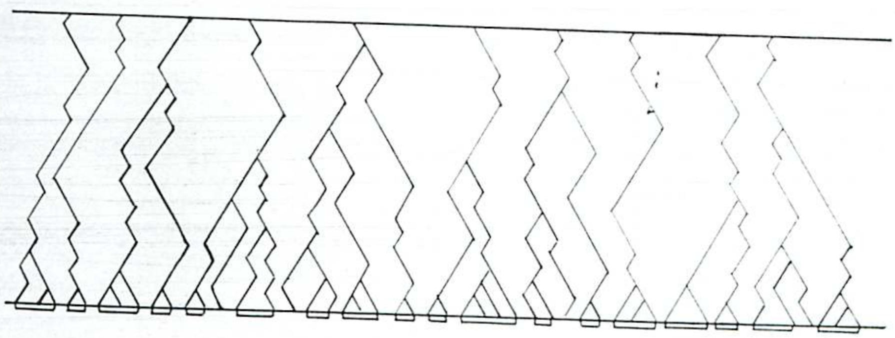

Fig. 5 — Random — generated drainage basins.

One can now classify the various drainage areas according to order, in the same manner as this was done by Gerber. In this fashion, the valley sectors became more divided up than inherent in Gerber's scheme. In order to be in conformity with Gerber's procedure, one must count into one valley sector all areas containing only single narrow channels; this convention of designating valley sectors has been indicated in figure 5 by brackets underneath the drained strip. Counting the various drainage areas pertaining to table 2 gives the result that there are 10 basins of first, 7 of second, 3 of third, 2 of fourth order and 18 valley sectors (see also table 1). The procedure was also carried out for 2 further cases. The results are tabulated in table 2 as "second" and "third" trials. These results suggest strongly that the observed distribution is essentially random.

Acknowledgment

The program for the random number arrangement (table 2 and also the other trials) was written and executed by Mr. P.B. Cawood of the U.S. Geological Survey in Washington. Mr. W.B. Langbein of the U.S. Geological Survey has read the paper and made valuable comments. The writer wishes to thank Messrs. Cawood and Langbein for their efforts.

The writer's attention was drawn to the problem of valley sectors discussed here during many

personal conversations and excursions into the Swiss Alps with Dr. E. Gerber. The writer wishe to take this opportunity to acknowledge his indebtedness to the hospitality and stimulatin influence of Dr. Gerber.

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(1) Nous remercions M. le Profess
muniqué cette documentation.