Presentation Contents

1. Summary of data model and correlation metrics. (2 slides)
2. Top-level description of correlator and overview of uarch. (2 slides)
3. Scaling counter for interactive zoom in/out (2 slides)
4. Window function “Logdrop”, and use in scaling counter (3 slides)
5. Jittery strobe approach to find delays (1 slide)
6. Outputting correlation results with dimmable LED (1 slide)
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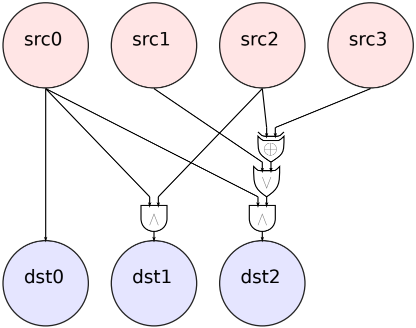
Connections Between Measurement Streams

- Only high level data is available due to physical constraints.
- "relationship" $\implies$ There is a logic function connecting two streams of data.
- Knowledge of a relationship existence is more important than the exact details of the relationship.



Correlation Metrics

$$\mathrm{cov}(X,Y) = \mathbb{E}[(X - \mathbb{E}[X])\,(Y - \mathbb{E}[Y])] = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y] \quad (1)$$

$$X, Y \in [0,1] \implies \frac{-1}{4} \leqslant \mathrm{cov}(X,Y) \leqslant \frac{1}{4} \quad (2)$$

$$\dot{\mathrm{C}}\mathrm{ov}(f_x, f_y) := 4\left|\mathbb{E}[f_x \odot f_y] - \mathbb{E}[f_x]\mathbb{E}[f_y]\right| \quad (3)$$

$$X \perp\!\!\!\perp Y \iff \Pr(X) = \Pr(X|Y) \quad (4)$$

$$\dot{\mathrm{D}}\mathrm{ep}(f_x, f_y) := \left|\frac{\mathbb{E}[f_x|f_y] - \mathbb{E}[f_x]}{\mathbb{E}[f_x|f_y]}\right| = 1 - \frac{\mathbb{E}[f_x]\mathbb{E}[f_y]}{\mathbb{E}[f_x \odot f_y]} \quad (5)$$

More Correlation Metrics

$$\dot{\mathrm{Ham}}(f_x, f_y) = 1 - \mathbb{E}[|f_x - f_y|] \tag{6}$$

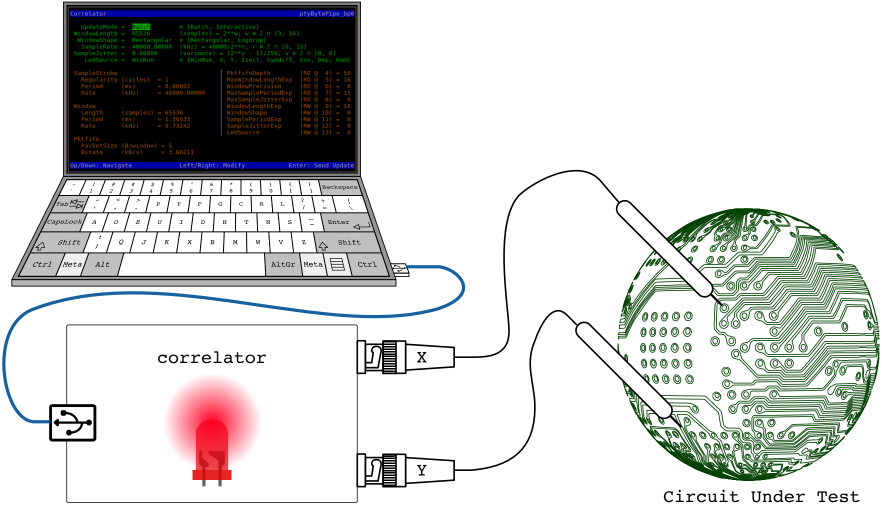
$$\dot{\mathrm{Cls}}(f_x, f_y) = 1 - \frac{\sqrt{\mathbb{E}\left[|f_x - f_y|^2\right]}}{\sqrt{2}} \tag{7}$$

$$\dot{\mathrm{Cos}}(f_x, f_y) = \frac{\mathbb{E}[f_x \odot f_y]}{\sqrt{\mathbb{E}[f_x^2]}\sqrt{\mathbb{E}\left[f_y^2\right]}} \tag{8}$$

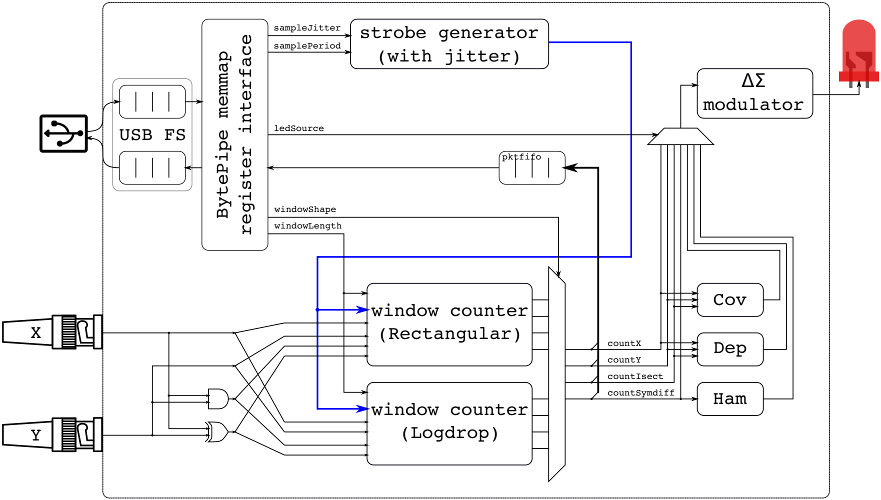
$$\dot{\mathrm{Tmt}}(f_x, f_y) = \frac{\mathbb{E}[f_x \odot f_y]}{\mathbb{E}[f_x] + \mathbb{E}[f_y] - \mathbb{E}[f_x \odot f_y]} \tag{9}$$

- NOTE: Positive results only $\in [0, 1]$

Correlator Usage



Correlator Microarchitecture



Timebase (`windowLength` and `sampleRate`)

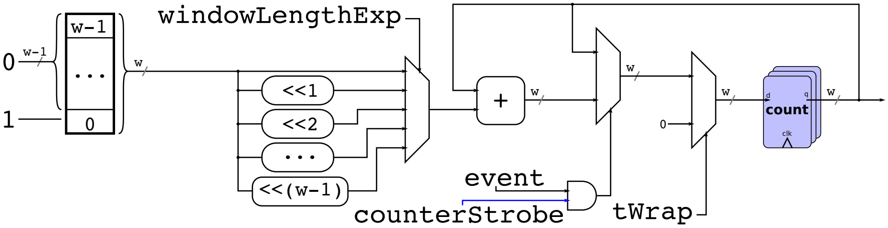
Balance output data rate with how fast you can detect a change of input behaviour.

Options for implementation:

1. IEEE754 floats. Easy to understand, trivial to integrate with other systems. Costly in area, power, fmax, implementation time.
2. "Standard" counter. Simple, small area, low power, high fmax. Output requires scaling.
3. Rectangular counter. Similar size, power, and fmax as standard counter. Output always appears on the same bits, no scaling required.

Correlator moves on exponential scales with $\texttt{windowLength} = 2^{[3,16]}$, $\texttt{sampleRate} = \frac{48\,\text{MHz}}{2^{[0,15]}}$.

Rectangular Counter



Windowing (windowShape)

Modelling attention/focus or performing Fourier analysis necessitates some sort of bell curve.

Options for implementation:

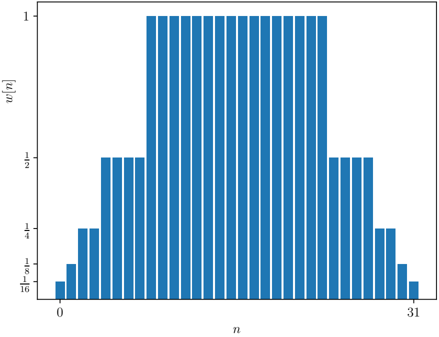
1. Pre-computed coefficient table, indexed by t, with a multiplier. The usual method, flexible with window shapes, almost arbitrary frequency response. Requires significant memory and supporting peripheral circuitry.
2. Logdrop counter. No memory required, only a small number of gates/Look Up Table (LUT)s. Fixed frequency response.

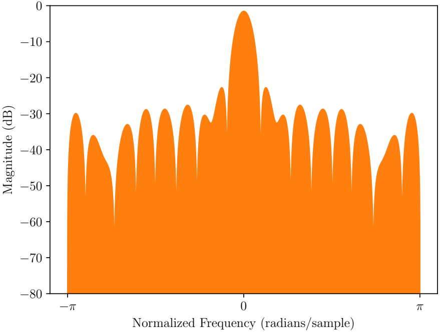
Correlator selects `windowShape` as either Rectangular or Logdrop.

Logdrop Window Function

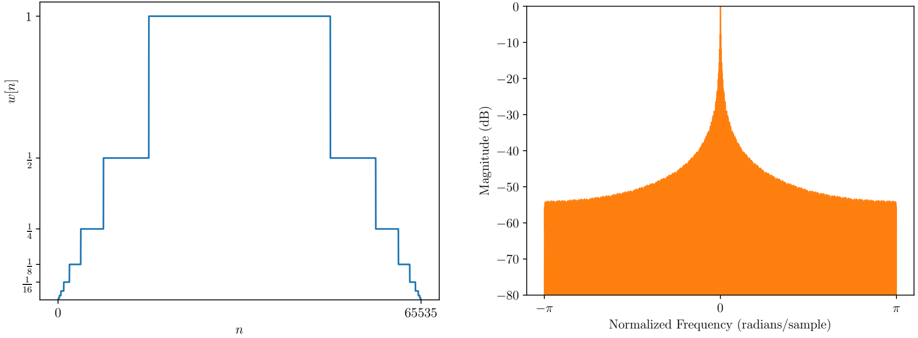
$$t \in \mathbb{Z}\ [0, N); \quad N = 2^p; \quad p \in \mathbb{Z} > 2 \tag{10}$$

$$w[n] = 2^{\lceil \log_2 \min(n+1, N-n) \rceil - \log_2 \frac{N}{2}} \quad \in \mathbb{R} \cap (0, 1] \tag{11}$$

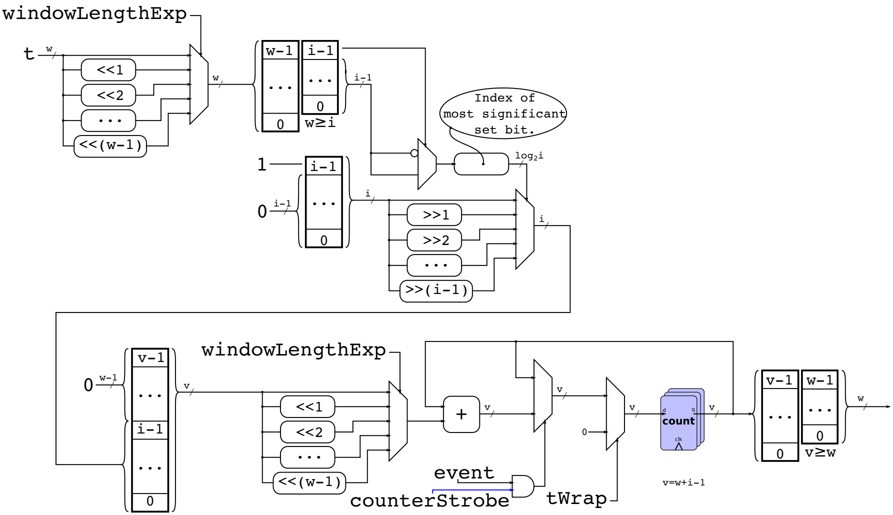




Logdrop Window Function (N=64k)



Logdrop Counter



Relations with delays (`sampleJitter`)

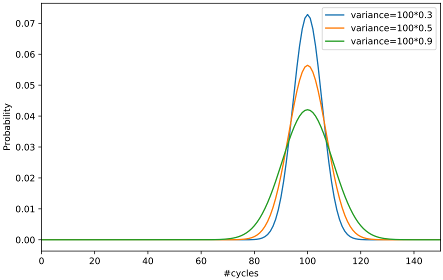
Relationship may have a delay of some cycles.
Options for implementation:

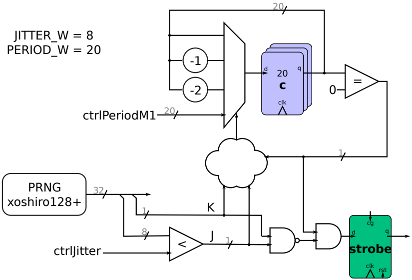
1. Buffer both X and Y, then compute relationships for all values of delay. All results available for all data. Extremely resource intensive, both compute and memory.
2. Randomized testing with jittery sample clock. Can detect relationships within a range of delay values with small amount of resource, and that range may be very large. Requires searching in real-time to pick out exact delay values.

Correlator selects `sampleJitter` variance $\frac{2^{[0,8]}-1}{256}$.

Sampling Strobe

$$\Pr(\text{period is } n \text{ cycles}) \approx \frac{1}{\sqrt{sj}\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{n-s}{\sqrt{sj}}\right)^2\right) \qquad (12)$$





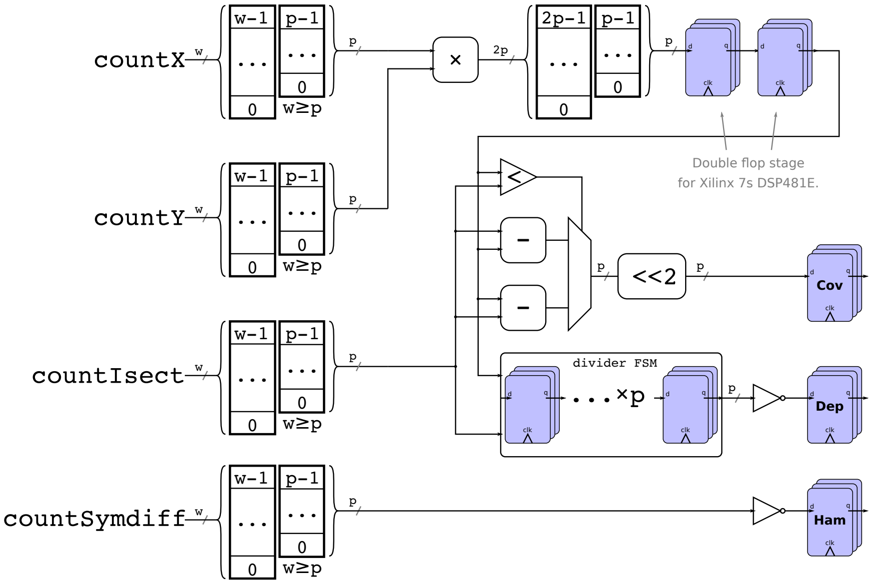
Outputting results (`ledSource`)

Options for output:

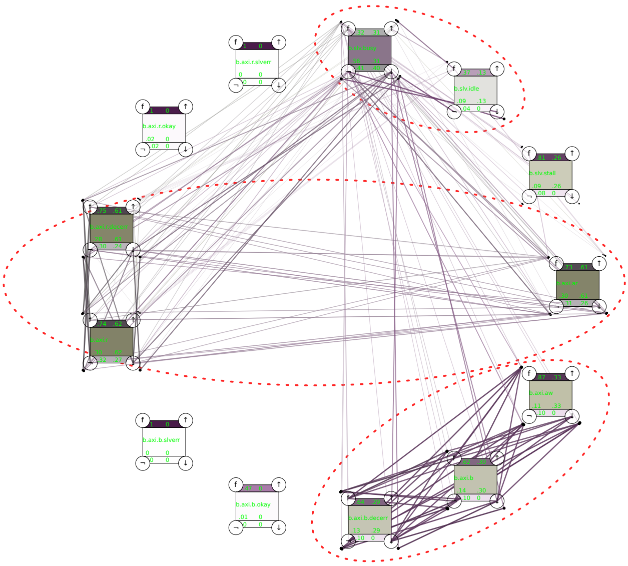
1. Exact values for all results over data link. Allows arbitrary offline processing. May require too much bandwidth and storage.
2. Approximate value for results as fast as humans can consume in real-time. View result as brightness of an LED. Good for interactive searching but not offline processing.

Correlator selects `ledSource` to be one of $\dot{\mathrm{C}}\mathrm{ov}$, $\dot{\mathrm{D}}\mathrm{ep}$, or $\dot{\mathrm{H}}\mathrm{am}$. The first two because they are demonstrably useful for the general case, and the last because it's trivial and may be useful for specific cases.

Correlator Metrics



Visualizing Correlation Example



Summary

- Assist SoC development via correlation analysis using low-cost hardware.
- Proof-of-concept integration to UltraSoC Status Monitor.
- Full details in the thesis (soon).

Questions