decided to hire Milan Consulting Group Inc. (MCG) to help address some of the company's ongoing problems. MCG's team was composed of Maurizio Donato, a junior partner, and Roberto Benello and Sofia Cominetti, two young associates in their first engagements. The three consultants spent four days at Filatoi Riuniti touring the production facilities, reviewing operations, interviewing managers, and studying mounds of data. After a weekend of hard thinking back in Milan, Maurizio Donato started the next Monday's project meeting with his two young associates with the words: "Our first priority is to find ways to reduce the costs that our client faces which are jeopardizing the future of the factory and the jobs of the almost 200 employees. Our goal is to come up with smart workable ideas, *e subito*!" He then outlined what he thought were four areas for obvious cost reduction:

(a) reducing machine down-time through improved inspection strategies,

(b) differential scheduling of machine maintenance,

(c) different management of overtime on production shifts, and

(d) improved outsourcing strategies from the six local spinning mills.

Roberto and Sofia immediately went to work on the analysis of these four proposals. They found that the total expected cost savings from the first three proposals would probably amount to roughly $200,000 per year. This was not a particularly large sum of money in comparison to Filatoi Riuniti's sales of $15 million (although it easily justified their consulting fees!). However, when they started to work on the fourth proposal, they immediately saw the potential for very large savings through more optimal outsourcing strategies with the six local spinning mills.

Optimizing the Outsourcing of Spinning Production

Filatoi Riuniti produces four different sizes of yarn (coarse, medium, fine, and extra fine) and there is an autonomous demand for each of the four sizes. Filatoi Riuniti can prepare enough raw cotton to meet their total demand, but they lack sufficient machine capacity to spin the four sizes of yarn, and as discussed above, they have been outsourcing part of the spinning production to six local mills: Ambrosi, Bresciani, Castri, De Blasi, Estensi and Giuliani. Table 7.30 shows February's spinning production schedule. Of the total demand of 104,500 Kg of yarn, 32,000 Kg were outsourced to the six local mills. All of the spinning of Coarse yarn was outsourced (29,000 Kg), and 13% of the spinning of Medium yarn was outsourced. The exhibit also shows the prices charged by the six mills to Filatoi Riuniti. The head of the purchasing department at Filatoi Riuniti said "We spin the finer sizes in-house and outsource the rest of the work. We outsource each yarn size to the lowest-price mill and then meet demand with the next-lowest-price mill." Roberto and Sofia thought that this outsourcing strategy could easily lead to sub-optimal outsourcing decisions, since outsourcing decisions were optimized only one at a time as opposed to optimizing all outsourcing simultaneously. In order to analyze the potential savings from optimizing the outsourcing of spinning production, they started to work with the client to identify the decision variables, the constraints, and the objective function to optimize.

Decision variables. Given the amount of each yarn size that Filatoi Riuniti needs to deliver to meet demand, the problem was how to allocate spinning production (both at Filatoi Riuniti and at the six local mills) in order to minimize costs. The decision variables of the optimization model are denoted X_{ij}, which represents the amount of yarn of size i that the company j would be assigned to produce. In this context,

i = 1, 2, 3, and 4 means "extra fine," "fine," "medium," and "coarse," respectively. Similarly, j = A, B, C, D, E, F, G mean Ambrosi, Bresciani, Castri, De Blasi, Estensi, Filatoi Riuniti, and Giuliani. See the blank table "Decision Variables" of the spreadsheet FILATOIR.XLS as a guide. Each X_{ij} must of course be nonnegative because none of the mills can produce negative amounts of spun yarn!

Variable Costs of Production. Roberto and Sofia knew the prices charged to Filatoi Riuniti by the six local mills (see Table 7.30). For internal purposes, they also needed to know Filatoi Riuniti's internal production costs in order to determine how much of each yarn size should optimally be produced internally versus externally. After a couple of days spent with the plant managers and the chief accountant, they came up with a fair estimate of the production cost for each of the four yarn sizes. See Table 7.31. The two blanks in the table indicate that Ambrosi and De Blasi cannot produce extra fine yarn.

Transportation costs. The yarn that is spun by the six local mills needs to be transported from Filatoi Riuniti to the mills for spinning and then be transported back to the production plant of Filatoi Riuniti in order to refine it and store it prior to delivery to customers. Sofia realized that they needed to obtain accurate data on transportation costs. One of the operations managers explained to her: "We have an agreement with a local truck company which takes care of all the transportation. The

Decision Variables X_{ij}
Yarn produced by each factory (Kg/month)

	Size			
Supplier	*Extrafine*	*Fine*	*Medium*	*Coarse*
Ambrosi				
Bresciani				
Castri				
De Blasi				
Estensi				
Filatoi R.				
Giuliani				

Machine Hours Required for Production (Hours/Kg)

	Size				*Production Capacity (Machine hours per month)*
Supplier	*Extrafine*	*Fine*	*Medium*	*Coarse*	
Ambrosi		0.400	0.375	0.250	2,500
Bresciani	0.700	0.500	0.350	0.250	3,000
Castri	0.675	0.450	0.400	0.250	2,500
De Blasi		0.450	0.350	0.200	2,600
Estensi	0.650	0.450	0.400	0.250	2,500
Filatoi R.	**0.625**	**0.500**	**0.425**	**0.425**	**38,000**
Giuliani	0.700	0.450	0.350	0.400	2,500

Cost of Production ($/Kg)

	Size			
Supplier	*Extrafine*	*Fine*	*Medium*	*Coarse*
Ambrosi		13.00	10.65	9.60
Bresciani	17.40	14.10	11.20	9.45
Castri	17.40	14.22	11.00	9.50
De Blasi		14.30	11.25	9.60
Estensi	17.50	13.80	11.40	9.60
Filatoi R.	**18.25**	**13.90**	**11.40**	**8.90**
Giuliani	19.75	13.90	10.75	9.40

Cost of Transportation ($/Kg)

	Size				*Round trip distance (km)*
Supplier	*Extrafine*	*Fine*	*Medium*	*Coarse*	
Ambrosi	0.30	0.30	0.45	0.45	30
Bresciani	0.40	0.40	0.60	0.60	40
Castri	0.80	0.80	1.20	1.20	80
De Blasi	0.70	0.70	1.05	1.05	70
Estensi	0.70	0.70	1.05	1.05	70
Filatoi R.	—	—	—	—	—
Giuliani	0.50	0.50	0.75	0.75	50
($/Kg/Km)	0.010	0.010	0.015	0.015	

Demand to Meet (Kg/month)

Extrafine	*Fine*	*Medium*	*Coarse*
25,000	26,000	28,000	28,000

TABLE 7.31
Production schedule, costs, and constraints—March.

contract with the truck company is very simple. They charge a fixed amount per kilometer per unit volume." Each product has a different density and therefore takes up a different volume per Kg. One Kg of finer product is more dense and so is less expensive to transport on a per Kg basis. Of course, each local mill is located at a different distance from Filatoi Riuniti. Armed with the contract with the truck company, a road map with the location of the six local mills, and product specification data, Sofia was able to estimate the transportation cost per Kg of each product for all the local mills. These numbers are shown in the table "Cost of Transportation" in Table 7.31. For example, it costs \$0.01 per Kg per Km to transport fine yarn, and the round trip distance from Filatoi Riuniti to the Giuliani mill is $2 \times 25 = 50$ Km. Therefore, the table shows that it costs $(0.01 \times 50) = \$0.50$ to transport one Kg of fine yarn to Giuliani and back.

Resource consumption. Another important task was to understand the actual spinning machine production capacity of the six local mills and of Filatoi Riuniti itself. During the time spent with the plant manager, Roberto learned that production capacity is measured in machine hours per month and each product size requires a different amount of machine hours per Kg of product. He spent some more time with the plant engineer trying to estimate the six local mills' capacity in terms of machine hours per month and their production rate in terms of hours needed to produce one Kg of a given product size. Because each mill has different types of machines in different configurations, the number of machine hours required to produce one Kg of product differs among the mills. After a full day of work and very many telephone calls, fax, and email messages, Roberto and the plant engineer produced a table containing the production capacity and production rate per product for each of the six mills plus Filatoi Riuniti itself. These capacity and production rate numbers are shown in the two tables "Machine Hours Required for Production" and "Production Capacity" in Table 7.31. For example, at the Bresciani mill, it takes 0.70 hours to produce one Kg of extra fine yarn and there are at total of 3,000 machine hours per month available.

Product Demand. After talking to the marketing and sales manager at Filatoi Riuniti, Sofia estimated the demand for the four spun yarn sizes for March, which is shown in the table "Demand to Meet" in Table 7.31.

Armed with all of this data, Roberto and Sofia felt that they had enough information to solve for the outsourcing production strategy that would minimize the costs of producing spun yarn.

Assignment:

(a) Formulate Filatoi Riuniti's purchasing problem for the coming month (March):

1. Write down the formula for the objective function of your model.
2. Your model must have a capacity constraint for each local spinning mill. Write down the capacity constraints for the Ambrosi mill, for example.
3. Filatoi Riuniti must meet demand for each of the four sizes of yarn. Your model must have a constraint for the demand for each of the four sizes of yarn. Write down the constraint for the demand for extra fine yarn, for example.

(b) Construct the spreadsheet for your optimization model. First, open the spreadsheet FILATOIR.XLS. Then complete your model and optimize it using the Solver. You will need to:

- create the objective function and the constraints in the appropriate place in your spreadsheet, and
- launch the Solver and optimize your model. Can you assume a linear model? What is the optimal supply strategy?

(c) Filatoi Riuniti should obviously consider increasing its spinning machine capacity. They could slightly expand the production capacity of the existing machines by renting an upgrade. This would increase their spinning production capacity by 600 hours/month. The monthly rental cost is \$1,500/month. Would you recommend that they rent the upgrade? (Try to answer this question without re-optimizing your model.)

(d) Alternatively, Filatoi Riuniti could increase its spinning machine capacity by renting another spinning machine for the production of only medium size yarn, for a monthly rental cost of \$3,000. The machine has a production capacity of 300 hours per month (the machine would run at the same rate of 0.425 hours/Kg). Suppose that the estimated production cost of running this machine is less than for Filatoi Riuniti's existing machines and is estimated to be \$5.70/Kg (as opposed to \$11.40/Kg for their existing machines according to Table 7.31). Would you recommend that Filatoi Riuniti rent the machine? (Try to answer this question without re-optimizing your model.)

(e) A new client is interested in purchasing up to 6,000 Kg/month of medium size yarn. What is the minimum price that Filatoi Riuniti should quote to this new client? Would it be a fixed price per Kg? Which additional question(s) might you ask this client? (In answering this question, assume that Filatoi Riuniti has not decided to expand its spinning machine capacity, and that Filatoi Riuniti does not want to change the prices that they currently charge their existing clients.)

(f) Your outsourcing production strategy optimization model is based in part on the prices charged by the local mills to Filatoi Riuniti and on an estimate of Filatoi Riuniti's internal production costs. The plant manager, the accounting department, and you estimate that Filatoi Riuniti's internal production costs could vary within a 5% range of the figures shown in Table 7.31. Would your recommendations change in the extreme cases? Why or why not?

(g) You estimate that the production capacity of one of your local mills, De Blasi, could vary within a 20% range of the figures shown in Table 7.31. Would your recommendations change in the extreme cases? Why or why not?

(h) Suppose that you present your proposed outsourcing plan to the owners of the Ambrosi mill. They complain to you that their mill cannot easily produce fine size yarn; in fact they presently can only produce medium and coarse size yarn, and they would incur substantial one-time set-up costs to ramp up for the production of fine size yarn. However, the optimal solution of the model indicates that it would be in Filatoi Riuniti's interests for the Ambrosi mill to produce fine size yarn. The owners want to maintain good business relations with Filatoi Riuniti, but they do not want to bear the full cost of ramping up for production of fine yarn. The contracts that Filatoi Riuniti currently has with its customers will not expire for at least another 12 months. Up to what amount would you be willing to share the one-time set-up costs for production of fine yarn with the owners of the Ambrosi mill?

(i) Suppose that you find out that one of the local mills, Giuliani, has the possibility of running an overtime shift (which would double their capacity) by paying its

workers only 13% more the normal wage (it is a family-owned business). You know that the workers' salaries contribute to approximately 50% of the prices that the Giuliani mill charges Filatoi Riuniti for spinning yarn. The transportation cost component of the objective function would not change, of course. Modify the model in order to take into account this possibility and re-optimize. Does the optimal solution change? Why? [Helpful modeling hint: Think of the "overtime" part of this mill as a new mill with higher product costs.]

7.10 EXERCISES

EXERCISE 7.1 A computer parts manufacturer produces two types of monitors—monochrome and color. There are two production lines, one for each type of monitor. The monochrome monitor line has a daily capacity of 700 units per day. The color monitor line has a daily capacity of 500 units per day. In department A, the tubes are produced for both monitor lines. In department A, the production of a monochrome tube requires one hour of labor, and a color monitor requires two hours of labor. Total daily labor hours in department A is 1,200 hours. In department B, the monitors are inspected. The monochrome monitor requires 3 hours of labor for inspection. The color monitor requires 2 hours of labor for inspection. A total of 2,400 hours of labor are available in department B. The monochrome monitor nets an earnings contribution of \$40 per unit. The color monitor nets an earnings contribution of \$30 per unit.

In order to maximize the net earnings of the company, we set up a linear optimization model with decision variables M for the daily production of monochrome monitors (in hundreds of monitors), and C for the daily production of color monitors (in hundreds of monitors). The linear optimization model is:

$$\begin{aligned}
&\text{maximize} && 40M + 30C \\
&\text{subject to:} && \\
&\text{M capacity:} && M \leq 7 \\
&\text{C capacity:} && C \leq 5 \\
&\text{A labor:} && M + 2C \leq 12 \\
&\text{B labor:} && 3M + 2C \leq 24 \\
&\text{Nonnegativity:} && M, C \geq 0.
\end{aligned}$$

(a) Solve the linear optimization model graphically. Show each constraint, the feasible region, and identify the optimal solution.

(b) Which two constraints are binding at the optimal solution? Solve these two constraints in the two unknowns to compute the optimal production plan exactly. What is M? What is C? What is the contribution to earnings?

(c) Consider the labor constraint for department A: $M + 2C \leq 12$. Suppose the number 12 was changed to 13, i.e., we had an additional 100 labor hours in department A. Re-solve the two equations in two unknowns to compute the new values of M and C, and the new optimal contribution to earnings.

(d) Compare the new contribution to earnings to the old contribution to earnings of part (b). What does the difference in the earnings indicate about the marginal value of labor in department A?