Course Overview and Introduction to LP

DSO 570: The Analytics Edge
Data, Models, and Effective Decisions
(MW 5-6:20pm)

Session 02 (01/20/21)

Agenda and Announcement

- Please complete the survey flashcard
  - Link available on BB under Lectures > Session 02
- Course overview
- The Chocolate Game
  - Excel spreadsheet in available on BB
  - We will play this game on Monday (01/25)
  - One of you will get a chance to win up to $200!
- Introduction to Linear Programming: NBT Problem
- Next Session: Intro to LP in refinery optimization
  - Please read the handout on refinery optimization before coming to class (available in BB under Lectures > Session 03)
- HW #1 is on BB and is due on on Wed 2/3 at 5pm

Student Survey

- IMPORTANT: Please complete the survey and upload it to BB by this Thursday (1/21) at 5PM
- I will try my best to learn all of your names as soon as possible
- I want to try and tailor the course to help your career goals as much as possible

Student Survey: Due by Thursday (1/21) at 5PM

Attached Files: StudentSurvey.xlsx (9.516 KB)

I want to learn more about your background and inter
Please complete this short survey. There are 9 questio

NOTE: The last question (Question #9) is completely o
your bid in Question #9. We will play the game in class
your bid.

INSTRUCTION: Please complete the attached Excel fil
the above text "Student Survey: Due by Thursday (1/2
"Browse My Computer" to select your file to upload, a
the file, you MUST also press the "Submit" button.

You can save your work-in-process (WIP) within Blackb
back, click on "Continue Current Submission" to retrie
"Submit". You should see Attempt #1 under Submissic
the file here, then I cannot see the file either.

Question	Description	Your Answer
1	First and Last Name	
2	If different from your first name, please indicate the name that you would like to be called	
3	Linkedin (if applicable)	
4	Major (MBA/BUAN/Mkt/Other) and Year	
5	Hometown (City, State, & Country)	
6	Briefly describe your work experience and industry	
7	Briefly describe your career goal	
8	Funny or interesting tidbits about yourself that you would like me to know. Examples: favorite foods, snacks, music, sports, hobbies, internships, or accomplishments.	
9 (Optional)	If you are intersted in playing the Chocoloate Game, please indicate your bid (in US Dollars)	

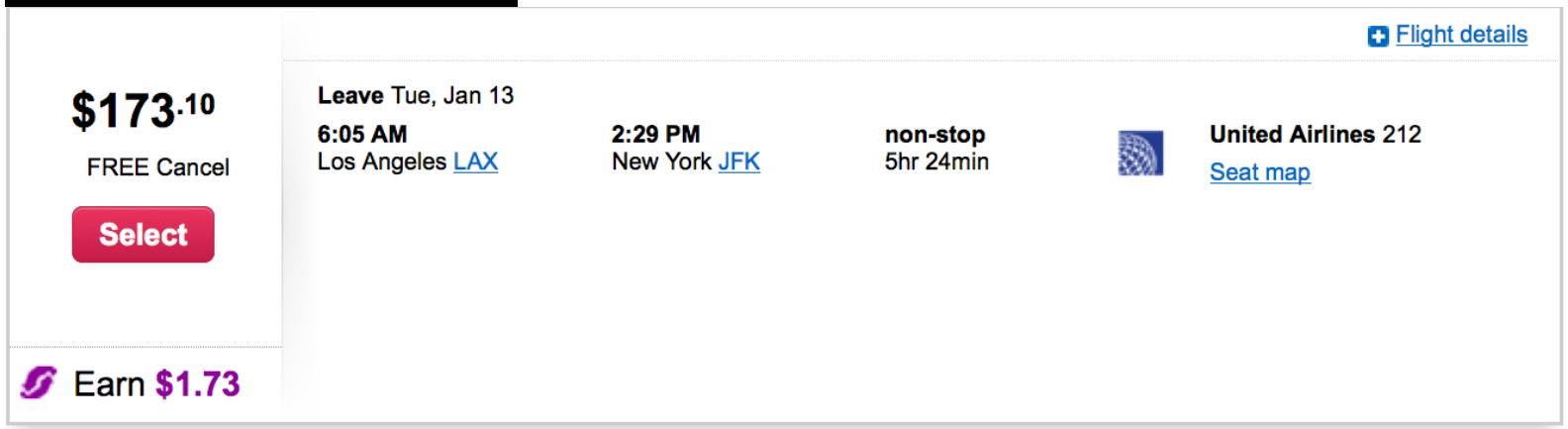
Syllabus: Please review carefully!

- Course e-mail Address: paat.dso570@gmail.com
- Office Hours: Tue and Wed 2-4pm, Thu 11 - 1pm, and by appt.
  - Rationale: HWs and Cases are due on Wednesday
  - Other times? I want to accommodate as many students as possible.
- All lecture notes, HW questions, and case descriptions are on BB
  - Optional textbook: Data, Models, and Decisions: The Fundamental of Management Science, 2nd Edition, by D. Bertsimas and R. Freund
- We will have 6 assignments: You can work individually or with a partner.
  - Assignment with the lowest score will be dropped
  - Assignment #2, #3, and #4 involve case analysis. I will ask student teams to present their solution during the lecture
- We will have 2 exams (midterm & final), but we will put more emphasis on the exam with the highest score
  - Combined Exam Score: (80% x highest score) + (20% x lowest score)
- If you put in a good faith effort and complete all the assignments and exams in the class, you are guaranteed at least at "B+" in the course.

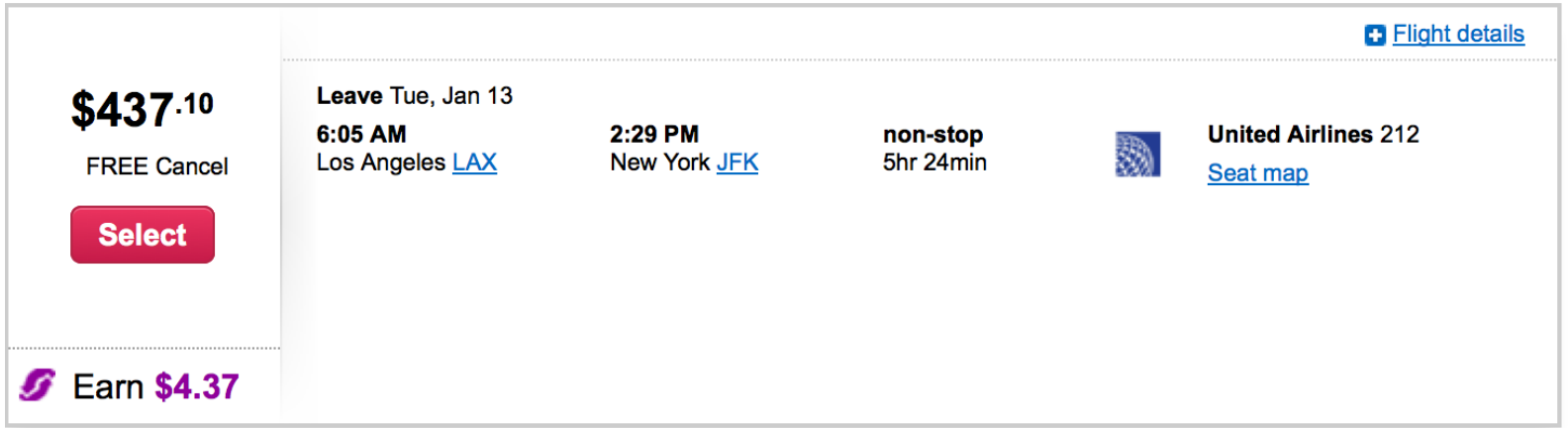
What is the goal of this course?

- How can companies make effective decisions under uncertainty through models and optimization?
- Overview of the modules:
  1. Linear Programming
  2. Nonlinear Programming
  3. Discrete Optimization (aka Integer Programming)
  4. Dynamic Optimization

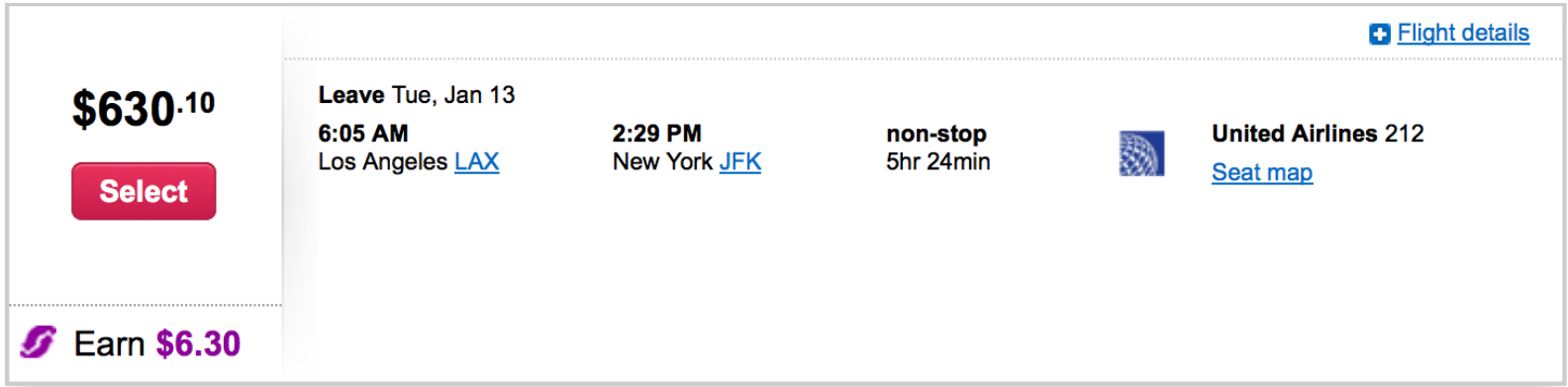
Optimization in Action



Economy ticket from LAX to JFK departing on 1/13, on 1/5



Price for the same flight on 1/7



Price for the same flight on 1/12

Why do airlines frequently change ticket prices?

The Chocolate Game

- A simplified version of the optimization problem faced every single day by airlines, hotels, cruise lines, retailers, and rental car companies
- **ONE** student will be chosen as a chocolate seller.
- The seller will have an opportunity to purchase **10** boxes of chocolate as your supply
- The seller randomly chooses 30 customers sequentially.
  - The first 20 customers are labeled as "Leisure" customers
    - Tend to book their flights early (possibly bargain hunters)
  - The last 10 customers are labeled as "Business" customers
    - Tend to book their flights at the last minute (business meetings,etc.)
  - Observation: The two types of customers have different price points, or distribution of willingness-to-pays (WTP)

How do we model each customer's WTP ?

Modeling the WTP of Each Customer

- EACH "Leisure" customer (first 20 customers) has the following WTP distribution

WTP for Leisure Customer	$ 5	$ 7	$ 9	$ 11	$13	$15	$17	$20
Probability	6/31	5/31	5/31	5/31	4/31	3/31	2/31	1/31

- EACH "Business" customer (the last 10 customers) has the following WTP distribution

WTP for Business Customer	$ 5	$ 7	$ 9	$ 11	$13	$15	$17	$20
Probability	0	0	1/31	2/31	2/31	2/31	14/31	10/31

We will use the Chocolate Game Simulator to generate the random WTP! (Available on BB)

Seller's Decision and Sequence of Events

- Step 1: **Before each** customer's WTP is revealed, the seller ("you") must set the price of the chocolate box.
- Step 2: The customer's WTP is drawn from an appropriate distribution based on the customer's type.
- Step 3: If the WTP drawn (in Step 2) is **greater than or equal to** the price that you have set (in Step 1), then you make a sale.
  - The revenue is equal to *the price of the chocolate box that you have just set*.
  - But, if the WTP is **less than** the price that you have set, then there is no sale.
- Step 4: Move on the next customer and repeat Step 1 again.
- Check out the Chocolate Game simulator together
- ***The seller is GUARANTEED to earn between $90 and $200!***
  - ***You can always set the price at $9 per box.***

What is the expected value of this game?

https://PollEv.com/paatrusmevic433

What is the optimal expected revenue of the Chocolate Game?

Over $160	**A**
$141 - $160	**B**
$121 - $140	**C**
$101 - $120	**D**
$90 - $100	**E**

Determining the Seller

- We will play this game on Monday (1/25)
- Unfortunately, there can only be ONE seller.
- We will use a sealed-bid 2nd price auction (like eBays).
  - If you want to play the Chocolate Game on Monday (1/25), please submit your bid (Question #9) in the Student Survey by Thursday (1/21) at 5pm!
  - The student with the highest bid will be chosen as the seller.
- The cost to the seller is equal to **the 2nd highest bid**
- **Payment to the Seller:**
  - Either a) zero or b) the difference between the revenue you earn from the game and the the 2nd highest bid, whichever is larger!
  - I can pay the seller using check, Venmo, or Paypal!
- CAVEAT: You will NOT lose any money, but you may "lose your face". Please be careful with you bid.

Why do we study this game?

- The chocolate game represents a multi-period optimization problem with significant uncertainty that is common in many industries.
  - Airlines: Chocolate boxes = seats on a (direct) flight, say from LA to JFK
    - Leisure travelers often book their flight early, while business travelers tend to book closer to the departure date.
    - In airline applications, things become more complicated when we have connecting flights (more on this later in the course)
  - Hotel: Chocolate boxes = room nights
  - Cruise Line: Chocolate boxes = cabins on the ship
  - Rental Car: Chocolate boxes = car-day
  - Retail: It's very common for retailers to adjust prices over the selling season based on the (evolving) demand forecast and remaining inventory

Why is this game interesting?

- Multi-period decision making
  - Your decision today needs to take into account what will happen in the future!
  - Simple myopic/one-period optimization will do poorly
- Need to account for uncertainty
- Sophisticated models and optimization will gives you *big advantages* in this problem
  - Those of you who are thinking of doing a simple "simulation", please be very careful!
- Demonstrate the power of the analytics edge!
  - Doing simple things much better!
  - By modeling and analyzing this problem properly, we can make a lot of money!
  - We will solve this problem in Sessions 25 – 28.
    - This is a very difficult problem, and we need to cover a lot of background materials before we can present the solution.

How much should you bid?

- How much to bid depends crucially on what you think is a good strategy?
  - If you are the chocolate seller, how should you price each box of chocolate?
- Factors to consider in setting the price:
  - Probability of making a sale (this is similar to demand forecast)
  - Expected revenue
  - Remaining number of inventory
  - Remaining number of customers
  - Characteristics of customers: Business vs. leisure
  - **Timing is very important!**
    - You have to sell to the leisure customers first (first 20). Business customers arrive later (last 10)

Application of LP to Advertising Allocation*

Orencia is a medication for Rhematoid Arthritis

- NBT is a small newspaper with a growing web site
  - Pre-sells "impressions" on its web site to companies
- Whenever a visitor to the NBT website is shown an ad, this counts as an *impression for the corresponding advertiser*
- Cost-per-impression ("CPM") are the same across companies and ads

* D. Chickering and D. Heckerman, "Targeting Advertising on the Web with Inventory Management," *Interfaces* 33:71-77, 2003.

NBT Traffic Volumes and Ad Sales

- NBT site has 3 sections: **N**ews, **T**ravel, and **S**ports
  - Forecasted page views (in thousands per month):

Section	News	Travel	Sports	Total
Page Views (thousands/month)	2,000	1,200	1,800	5,000

- 5 primary advertisers: **A**pple Cruises, **B**ankBoston, **C**oolTickets, **D**-Mobile Wireless, **E**-Cooking
  - NBT has pre-sold the impressions to advertisers, and it must deliver certain # of impressions to each advertiser

Advertiser	Apple Cruises	BankBoston	CoolTickets	D-Mobile	E-Cooking	Total
Ads Sold (thousands/month)	500	1,800	900	1,500	300	5,000

How should NBT allocate its impressions in different sections to different advertisers?

NBT Advertising Problem (cont.)

- Key Observation: Probability that a visitor to the NBT website will click on an ad depends on the advertiser and the section of the site
  - Often called the click-through rate
  - Crucial metric of the quality of impressions for advertisers
  - Estimated from data (using possibly Logistic Regression or Advanced Choice Modeling)

Click-through Rate	Apple Cruises	Bank Boston	CoolTickets	D-Mobile	E-Cooking
News	0.02	0.05	0.03	0.03	0.01
Travel	0.05	0.01	0.01	0.03	0.04
Sports	0.01	0.04	0.04	0.01	0.01

Goal: Allocate ad views to sections of the website to meet the contractual obligations and simultaneously *maximize the click-throughs*

LP Decision Variables

- 15 variables: 5 advertisers x 3 sections
- AppleCruise (3 variables): A_N, A_T, A_S -- # of impressions per month from the News, Travel, and Sports sections, respectively (in thousands per month)
- BankBoston (3 variables): B_N, B_T, B_S -- # of impressions per month from the News, Travel, and Sports sections, respectively (in thousands per month)
- CoolTickets(3 variables): C_N, C_T, C_S -- # of impressions per month from the News, Travel, and Sports sections, respectively (in thousands per month)
- D-Mobile (3 variables): D_N, D_T, D_S -- # of impressions per month from the News, Travel, and Sports sections, respectively (in thousands per month)
- E-Cooking (3 variables): E_N, E_T, E_S -- # of impressions per month from the News, Travel, and Sports sections, respectively (in thousands per month)

Objective Function

Click-through Rate	Apple Cruises	Bank Boston	CoolTickets	D-Mobile	E-Cooking
News	0.02	0.05	0.03	0.03	0.01
Travel	0.05	0.01	0.01	0.03	0.04
Sports	0.01	0.04	0.04	0.01	0.01

- Maximize total expected ad clicks per month
- Maximize:

$$0.02\,A_N + 0.05\,B_N + 0.03\,C_N + 0.03\,D_N + 0.01\,E_N +$$
$$0.05\,A_T + 0.01\,B_T + 0.01\,C_T + 0.03\,D_T + 0.04\,E_T +$$
$$0.01\,A_S + 0.04\,B_S + 0.04\,C_S + 0.01\,D_S + 0.01\,E_S$$

For a linear program (LP), the objective function must be a LINEAR function of the decision variables!

You can multiply each decision variables by a number (possibly zero). You can add or subtract two decision variables. However, you CANNOT multiply/divide decision variables.

LP Constraints and Requirements

- The total number of impressions per month assigned from each section is at most the number of the impressions per month received by that section
  - News: $A_N + B_N + C_N + D_N + E_N \leq 2000$
  - Travel: $A_T + B_T + C_T + D_T + E_T \leq 1200$
  - Sports: $A_S + B_S + C_S + D_S + E_S \leq 1800$

  Forecasted Page Views in Each Section
- Total number of ads delivered per month for an advertiser is at least the number of ads that were "pre-sold" to the the advertiser per month
  - Apple Cruises: $A_N + A_T + A_S \geq 500$
  - BankBoston: $B_N + B_T + B_S \geq 1800$
  - CoolTickets: $C_N + C_T + C_S \geq 900$
  - D-Mobile: $D_N + D_T + D_S \geq 1500$
  - E-Cooking: $E_N + E_T + E_S \geq 300$

  # of ads view sold to each advertiser = minimum # of impression NBT needs to deliver to each advertiser

Non-negativity Condition

- The decision variables are of course nonnegative
  - $A_N \geq 0$, $B_N \geq 0$, $C_N \geq 0$, $D_N \geq 0$, $E_N \geq 0$
  - $A_T \geq 0$, $B_T \geq 0$, $C_T \geq 0$, $D_T \geq 0$, $E_T \geq 0$
  - $A_S \geq 0$, $B_S \geq 0$, $C_S \geq 0$, $D_S \geq 0$, $E_S \geq 0$
- NOTE: In practice, the solutions are required to be whole numbers (you cannot deliver 1.2 impressions)
  - We will ignore this issue for now
  - Will come back to revisit this again after the midterm when we consider discrete optimization

For a LP, every constraint must be of the form:

LINEAR function of decision variables ≤, = ,≥ number

Complete NBT Model

Maximize

$0.02\,A_N + 0.05\,B_N + 0.03\,C_N + 0.03\,D_N + 0.01\,E_N +$
$0.05\,A_T + 0.01\,B_T + 0.01\,C_T + 0.03\,D_T + 0.04\,E_T +$
$0.01\,A_S + 0.04\,B_S + 0.04\,C_S + 0.01\,D_S + 0.01\,E_S$

Subject to

News: $A_N + B_N + C_N + D_N + E_N \le 2000$
Travel: $A_T + B_T + C_T + D_T + E_T \le 1200$
Sports: $A_S + B_S + C_S + D_S + E_S \le 1800$

Apple Cruises: $A_N + A_T + A_S \ge 500$
BankBoston: $B_N + B_T + B_S \ge 1800$
CoolTickets: $C_N + C_T + C_S \ge 900$
D-Mobile: $D_N + D_T + D_S \ge 1500$
eCooking: $E_N + E_T + E_S \ge 300$

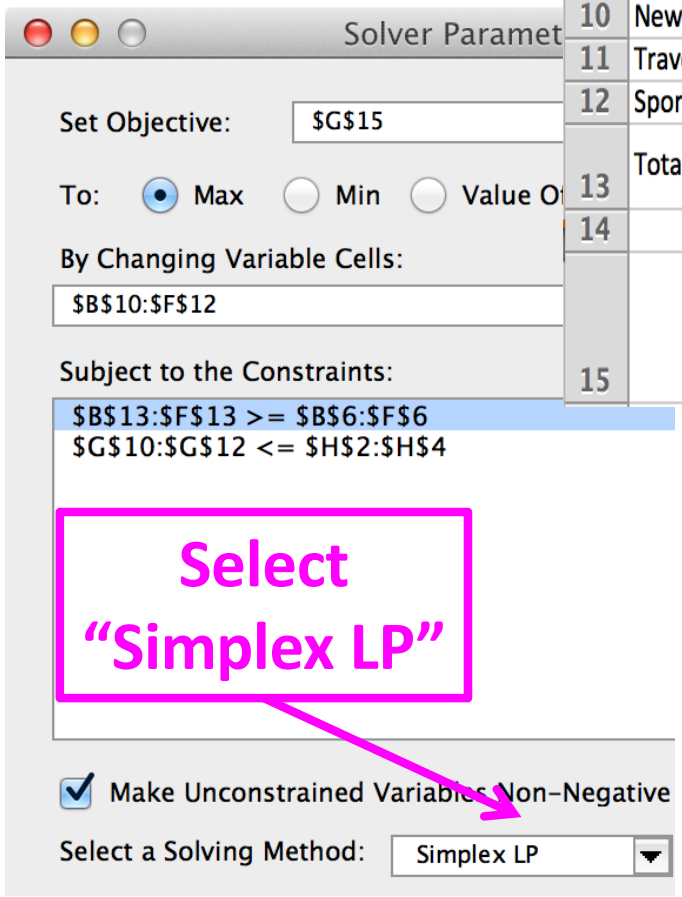
Nonnegativity: $A_N, B_N, C_N, D_N, E_N,$
$A_T, B_T, C_T, D_T, E_T,$
$A_S, B_S, C_S, D_S, E_S \ge 0$

In Excel, it is important to think of how to structure the decision variables, so that we can exploit the SUMPRODUCT command, and express the constraints succinctly

Instruction for installing Excel Solver is available on Blackboard

Tips for Structuring Excel

	A	B	C	D	E	F	G	H
1	**Data**	Apple Cruises	BankBoston	CoolTickets	D-Mobile	E-Cooking		Page Views
2	News	0.02	0.05	0.03	0.03	0.01		2000
3	Travel	0.05	0.01	0.01	0.03	0.04		1200
4	Sports	0.01	0.04	0.04	0.01	0.01		1800
5								
6	Ads Sold:	500	1800	900	1500	300		
7								
8								
9	**Decision Variables:**	Apple Cruises	BankBoston	CoolTickets	D-Mobile	E-Cooking	Total by Section	
10	News	0	900	0	1100	0	=SUM(B10:F10)	
11	Travel	500	0	0	400	300	=SUM(B11:F11)	
12	Sports	0	900	900	0	0	=SUM(B12:F12)	
13	Total by Advertiser	=SUM(B10	=SUM(C	=SUM(D	=SUM(E1(	=SUM(F10:F12)		
14								
15						Objective: Total Clicks	=SUMPRODUCT(B10:F12,B2:F4)	



- **Decision variables are in contiguous (green) cells**
- **The objective can be expressed using SUMPRODUCT**
- **Constraints can be express succinctly**

Excel Outputs

	A	B	C	D	E	F	G	H
1	**DATA**	Apple Cruises	BankBoston	CoolTickets	D-Mobile	eCooking		Page Views
2	News	0.02	0.05	0.03	0.03	0.01		2000
3	Travel	0.05	0.01	0.01	0.03	0.04		1200
4	Sports	0.01	0.04	0.04	0.01	0.01		1800
5								
6	Ads Sold:	500	1800	900	1500	300		
7								
8								
9	Decision Variables:	Apple Cruises	BankBoston	CoolTickets	D-Mobile	eCooking	Total by Section	
10	News	0	900	0	1100	0	2000	
11	Travel	500	0	0	400	300	1200	
12	Sports	0	900	900	0	0	1800	
13	Total Impression	500	1800	900	1500	300		
14								
15						Objective: Total Clicks	199	

IMPORTANT: Please study the 2-page handout on refinery optimization. We will start working through this problem together in-class on Monday (1/25).