CS2035b Data Analysis and Visualization - Lab 3

General Lab Instructions to Help Labs Run Smoothly

- Read through the lab instructions **before** coming to the lab.
- Do any required pre-lab preparation.
- Bring a **printed** copy of the lab instructions to the lab.

Overview and Preparation

This (and all subsequent) labs will be using MatLab 2016b as installed in the Health Sciences 14 & 16 general computing labs. You must attend this lab in HSB14 or HSB16 in order to get assistance from the TA. Attendance is mandatory and you must sign the attendance sheet. You can use your UWO login/password to login to these machines. Lab submission is to be done via Owl. Remember, labs are worth 10% of the total grade for this course (there are 11 labs in total, you must do 8 to receive full marks).

Upon completion of this lab, you should have done the following in the MatLab environment:

- Created a file containing the output to the file output03.txt for the MatLab code run in this lab.
- Submit this file (output03.txt) via the course Owl page.

Exercise 1: Simple Matrix Calculations

In this exercise, you do some simple matrix arithmetic in MatLab. Do the following:

1. Create a 4×4 matrix containing random numbers using the MatLab function **rand** and save the results in variable `A`. Do not end your statement with a semi-colon, this will echo the contents of `A`. Note that the numbers are printed in short floating format (4 digits to the right of the decimal point). Label this output as follows:

```
fprintf('A, a 4 by 4 matrix of random numbers\n');
A=rand(4,4)
```

2. Create variable `B` containing the transpose of `A` (i.e. `A'`). Again label and echo the contents of `B`.

3. Compute variables `C1` and `C2`: `C1=A*B` and `C2=A.*B`, again labelling and echoing the output. Do you see the difference between `*` and `.*` and why they are different. `*` does matrix multiplication and `.*` does element by element multiplication of the two equal sized matrices.

Exercise 2: Matrix Construction and Vector Products

Run the following MatLab code:

1. Create `E` as a 4×4 identity matrix, labelling and echoing the output. You can use MatLab function **eye** to do this.

2. Construct a 2×2 matrix `q`= $\begin{bmatrix} 4 & 7 \\ 8 & 3 \end{bmatrix}$ and insert it into `E` at upper-left coordinates (2,3), labelling and echoing the output.

3. Insert the transpose of `q` at upper left hand coordinates (1,1), labelling and echoing the output.

4. Compute row vector `a` as the first 3 elements of the 2^{nd} row of this matrix, labelling and echoing the output.

5. Compute column vector `b` as the first 3 elements of the 3^{rd} column of this matrix, labelling and echoing the output.

6. Compute the inner product of `a` and `b` and `a*b`, labelling and echoing the output. Notice you obtain a single number (a scalar). This calculation is also called a **dot** product (MatLab function **dot** also computes this). Verify that your output is the same as the **dot** function.

7. Compute the outer product of vectors `a` and `b` as `b*a`. Notice that you obtain a 4×4 rank-deficient matrix as the result.

8. Compute vectors `c1` and `c2` as the cross (vector) product of `a` and `b` as `c1=cross(a,b)` and `c2=cross(b,a)`. Label and echo the output.

9. Compute the dot products of `a` and `c1` and `b` and `c1`. Note that dot product values of 0 mean the two vectors are perpendicular. Using vector `c2` gives the same results as `c1`, so there is no need to do that calculation. Label and echo the output. We have already computed the dot product of `a` and `b` and the result is not 0, so `a` and `b` are not perpendicular. If `a` and `b` lie in the same plane, then `cross(a,b)` is the vector normal to the plane containing these vectors (often this is a very useful thing to be able to compute, especially in Computer Graphics).

Exercise 3: Eigenvectors and Eigenvalues

1. Consider a real 3×3 symmetric matrix:

$$A = \begin{pmatrix} 124.6 & 95.3 & 42.7 \\ 95.3 & 55.33 & 2.74 \\ 42.7 & 2.74 & 33.33 \end{pmatrix}$$

2. Using **google** (or any other search engine) find out how to compute the eigenvalues and eigenvectors of this matrix in MatLab. Compute the eigenvalues, $\lambda_1 \leq \lambda_2 \leq \lambda_3$ and their corresponding eigenvalues $\hat{e}_1$, $\hat{e}_2$ and $\hat{e}_3$.

3. Verify that all the eigenvalues are mutually perpendicular. Hint: the **dot** product might be useful here.

4. Verify that the definition of the eigenvectors and eigenvalues holds:

$$A\hat{e}_i - \lambda_i \hat{e}_i \approx 0$$

   for i equal to 1 to 3. For a real, symmetric matrix all eigenvalues are positive numbers and the eigenvectors for a **basis** for the matrix.

5. Compute the condition number of A as $\kappa = cond(A)$ and as $\kappa = \frac{\lambda_3}{\lambda_1}$. What conclusion can you draw?