

The University of Western Ontario

Computer Science 2035b

Solutions for Midterm Examination - Friday, February 27th, 2015

Surname	
Given Name	
Student Number	

This exam consists of 4 questions (7 pages including this page) worth a total of 100%. It is an open book exam, course notes and any MatLab book(s) are allowed. All answers are to be written in this booklet. Scrap work may be done on the back of each page; this will not be marked. No laptops or cell phones are allowed. The exam is 50 minutes long and comprises 20% of your final mark. Please print your full name and student number in the space provided above before you start this exam.

(1) 40%	
(2) 10%	
(3) 30%	
(4) 20%	
Total	

Professor: John Barron

(40%) Consider the following MatLab matrices A, B and C:

```
A= [19  5  6  8;
    30 22 12 13;
    28  2  5  9;
    4  6 24 9];
B=[3 1; 1 3];
C=[1; 3];
```

1. (4%) Using the original A above, if $A(1:2,3:4)=\text{eye}(2)$ what is the value of A?

```
A = 19      5      1      0
    30     22      0      1
    28      2      5      9
     4      6     24      9
```

2. (4%) Using the original A above, if $A(3:4,:) = A(:,3:4)'$ what is the value of A?

```
A = 19      5      6      8
    30     22     12     13
     6     12      5     24
     8     13      9      9
```

3. (4%) What is B*B:

```
B*B=| 3*3+1*1 3*1+1*3 | = | 10      6 |
    | 1*3+3*1 1*1+3*3 |   |  6     10 |
```

4. (4%) What is $B.*B$:

$$B.*B = \begin{bmatrix} 3*3 & 1*1 \\ 1*1 & 3*3 \end{bmatrix} = \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix}$$

5. (4%) What is the value of $[B; C]$?

Error using vertcat

Dimensions of matrices being concatenated are not consistent.

6. (4%) What is the value of $[C; C]$?

$$[C;C] = \begin{bmatrix} 1 \\ 3 \\ 1 \\ 3 \end{bmatrix}$$

7. (4%) Consider a 4 element column vector s . How would you solve the system of equations $A*s=D$, where $D=[1 \ 2 \ 3 \ 4]'$? Do not try to solve this system of equations!!!

$$s=A \backslash D; \text{ giving } s = \begin{bmatrix} 0.4148 \\ -0.0417 \\ 0.5854 \\ -1.2731 \end{bmatrix}$$

You were not expected to solve this system of equations!!!

8. (4%) What happens when we execute `B*C`?

```
B*C = 6
```

```
10
```

9. (4%) What happens when we execute `C*B`?

```
Error using *
```

```
Inner matrix dimensions must agree.
```

10. (4%) Using the original array `A`, what is the value of `reshape(A,2,8)`?

```
Take the elements of A column by column (i.e. A(:)'):
```

```
19 30 28 4 5 22 2 6 6 12 5 24 8 13 9 9
```

```
Now put them in the new array column by column
```

```
reshape(A,2,8) is
```

```
19    28    5    2    6    5    8    9
```

```
30     4   22    6   12   24   13    9
```

```
[Take the elements of A column by column.]
```

(2) (10%) This is the Lab question.

1. (3%) Consider the following expression: $6+1^2\cdot 3^4\cdot 5+1^2/3^4\cdot 5-6$. Parenthesize this expression according to the precedence of the operators. You do not need to compute its value. What does $1^2\cdot 3^4\cdot 5$ evaluate to?
Parenthesized expression:

$((6+(((1^2)^3)^4)^5)+(((1^2)/(3^4))*5))-6$

You do not have to compute the answer, which is 1.0617

Value of $1^2\cdot 3^4\cdot 5$:

1 - as 1 raised to anything is still 1.

2. (7%) Draw the graph plotted by:

```
x=[1:10]
y=x.^3;
plot(x,y);
title('y=x^3');
print triple.jpg -djpeg
```

Label the x and y axes and show the title printed.

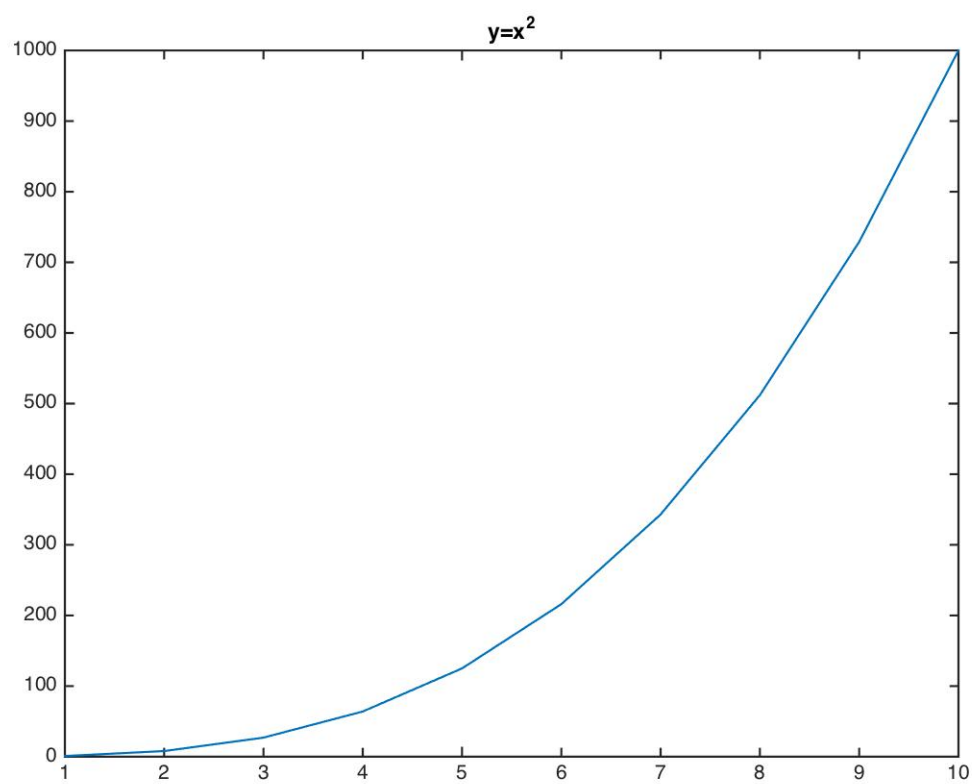


Figure 1: $y = x^3$ for $x \in [1, 10]$.

(3) (30%) This question is loosely related to Assignment 2. Consider the evaluation a polynomial $p(x) = x^9 + x^6 + x^3 + 1$ for x being a n component column vector $\mathbf{x} = \text{linspace}(0, 1, n)$.

1. (10%) Give the straightforward (brute force) vectorization of this polynomial:

$$\mathbf{p} = \mathbf{x}.^9 + \mathbf{x}.^6 + \mathbf{x}.^3 + 1$$

2. (10%) Give the vectorization of this polynomial that does the least number of multiplications. One solution:

$$\begin{aligned} \mathbf{x3} &= \mathbf{x} * \mathbf{x} * \mathbf{x}; \\ \mathbf{x6} &= \mathbf{x3} * \mathbf{x3}; \\ \mathbf{x9} &= \mathbf{x6} * \mathbf{x3}; \\ \mathbf{p} &= \mathbf{x9} + \mathbf{x6} + \mathbf{x3} + 1; \end{aligned}$$

The straightforward solution does 15 element by element multiplications. The optimized solution does 4 element by element multiplications. Another solution uses Horner's rule:

$$\mathbf{p} = \mathbf{x}.^9 + \mathbf{x}.^6 + \mathbf{x}.^3 + 1$$

can be re-written as:

$$\begin{aligned} &= \mathbf{x} * \mathbf{x} * \mathbf{x} * (\mathbf{x} * \mathbf{x} * \mathbf{x} * (\mathbf{x} * \mathbf{x} * \mathbf{x} + 1) + 1) + 1; \\ &= \mathbf{x3} * (\mathbf{x3} * (\mathbf{x3} + 1) + 1) \end{aligned}$$

where $\mathbf{x3} = \mathbf{x} * \mathbf{x} * \mathbf{x}$. This also requires 4 element by elements multiplications.

3. (10%) Consider the following serialized loop that computes the above polynomial:

```
for i=1:n
    x(i)=i;
    p(i)=x(i)^9+x(i)^6+x(i)^3+1;
end
```

Rewrite the original loop so that MatLab's JIT compiler can compile the loop. Improve on the loop efficiency using the ideas in questions (1) and (2).

```
x=zeros(1,n,'double');
p=zeros(1,n,'double');
for i=1:n
    x(i)=i;
    x3=x(i)^3;
    x6=x3*x3;
    x9=x6*x3;
    p(i)=x9+x6+x3+1;
end
```

If you use `x3(i)`, `x6(i)` or `x9(i)` then these arrays have to be pre-allocated.

(4) (20%) Consider the following matrix $Q = \begin{bmatrix} 42 & 0 & -42 \\ 38 & 0 & -38 \\ 19 & 0 & -19 \end{bmatrix}$;

1. (5%) What does `L=Q>-2` print?

```
L=Q>5
```

```
1      1      0
1      1      0
1      1      0
```

2. (5%) What does `C=find(Q>-2)` print? (Hint: `find` returns the coordinates of `Q>-2` as if `Q` were reshaped to be a column vector).

Note that `L(:)'` is a 1D row vector: 1 1 1 1 1 1 0 0 0

`C=find(Q>-2)` gives the 1D coordinates:

```
C =
```

```
1
2
3
4
5
6
```

3. (5%) What does `size(Q,2)` print?

```
size(Q,2)
```

```
ans = 3
```

4. (5%) What does `sum(L(:))` compute?

`L=Q>-2` is `1 1 0`

`1 1 0`

`1 1 0`

So `L(:)` has values `1 1 1 1 1 1 0 0 0` and the `sum(L(:))=6`.