

The University of Western Ontario

Computer Science 2035b

Final Examination - Friday, April 25th, 2014

Professor: John Barron

Surname	
Given Name	
Student Number	

This exam consists of 10 questions (25 pages including this page) worth a total of 180 marks (which will be scaled to 100%). It is an open book exam, course notes and any MatLab book(s) are allowed. No laptops or cell phones are allowed. All answers are to be written in this booklet. Scrap work may be done on the back of each page; this will not be marked. The exam is 180 minutes long (3 hours) and comprises 35% of your final mark. Should your final exam grade be higher than your midterm exam grade (worth 20% of your final grade), your final exam grade in this course will count for the full 55% of your exam grade.

Please print you full name and student number in the space provided below before you start this exam.

(1) 20 marks		(6) 20 marks	
(2) 15 marks		(7) 15 marks	
(3) 20 marks		(8) 30 marks	
(4) 25 marks		(9) 30 marks	
(5) 25 marks		(10) 10 marks	
Left Total		Right Total	
Total out of 210		Total out of 100	

Question 1 (20%) Choose **one** answer for each question.

1. (1 mark) MatLab stands for **Math**matics **Lab**oratory and was invented by the Russians in the 1960's during their space race with the Americans.

(a) true

(b) false

2. (1 mark) The transpose of Matrices $A*B$ is $A'*B'$, where $'$ is the MatLab transpose operator.

(a) true

(b) false

3. (1 mark) For the systems of equations $A*x=B$, both $x=A\backslash B$ and $x'=B'/A'$ (where $'$ is the MatLab transpose operator) give the same numerical solutions.

(a) true

(b) false

4. (1 mark) For the 2×2 matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A(:)$ yields 1, 2, 3 and 4.

(a) true

(b) false

5. (1 mark) For $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $A = \text{reshape}(A, 2, 2)$ yields $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.

(a) true

(b) false

6. (1 mark) Least Squares means having more unknowns than equations.

(a) true

(b) false

7. (1 mark) Handle graphics allows us to change the appearance of plots that were plotted earlier if the handles for those plots are known.
 - (a) true
 - (b) false
8. (1 mark) For GUIs in MatLab, *.fig files give the layout of the GUI.
 - (a) true
 - (b) false
9. (1 mark) Multi-core solutions always run faster than single core solutions.
 - (a) true
 - (b) false
10. (1 mark) A GPU (Graphics Processing Unit) with compute capacity ≥ 1.0 can do floating point arithmetic.
 - (a) true
 - (b) false
11. (1 mark) Symbolic integration means MatLab may be able to compute the indefinite integral of a function symbolically.
 - (a) true
 - (b) false
12. (1 mark) Symbolic integration means MatLab may be able to compute the definite integral of a function numerically.
 - (a) true
 - (b) false
13. (1 mark) `randn(1000,1)` yields a 1000 component column vector of random numbers having a standard deviation of approximately $\sigma = 1$ and means of approximately $\mu = 0.0$.
 - (a) true
 - (b) false

14. (1 mark) Kurtosis is a measure of the symmetry in a distribution of numbers.
- (a) true
 - (b) false
15. (1 mark) MatLab function `polyval` does a least squares fit of the polynomial coefficients of order n to data of size $\geq n + 1$.
- (a) true
 - (b) false
16. (1 mark) An edgel is a pixel representing an intensity discontinuity at that pixel.
- (a) true
 - (b) false
17. (1 mark) MatLab 2013b can run all its edge detectors on the GPU.
- (a) true
 - (b) false
18. (1 mark) Histogram equalization is a form of contrast enhancement on images.
- (a) true
 - (b) false
19. (1 mark) The edge maps of the 3 colour planes of a colour image are identical.
- (a) true
 - (b) false
20. (1 mark) Any calculation on a variable with a `nan` value yields a `nan` value.
- (a) true
 - (b) false

(2) (15%) Consider the evaluation of the polynomial:

$$f(x, y) = x^6y + y^6x + x^3y + y^3x.$$

Give the most efficient vectorized solution for the polynomial when:

```
x = linspace(0,1,100000);  
y = linspace(1,0,100000);
```

Efficiency is judged by the number of multiplications required.

Answer:

```
% x3 and y3 require 4 multiplications  
x3=x.^3;  
y3=y.^3;  
% This solution requires 6+4 multiplications  
f=x3.*x3.*y+y3.*y3.*x + x3.*y + y3.*y  
% This solution requires 4+4 multiplications  
f=x3.*(x3.*y+y)+y3.*(y3.*x + x)
```

(3) (20 marks) Consider the following system of equations:

$$5x + 6y + 7z = 8$$

$$7x + 6y + 5z = 3$$

$$9x + 7y + 3z = 10$$

$$14x + 15y + 16z = 30$$

$$16x + 15y + 14z = 28$$

$$9x + 3y + 27z = 30$$

1. (8 marks) Set up the matrices for this system of equation as $\mathbf{A}*\mathbf{s}=\mathbf{B}$.

Answer:

$$A = \begin{bmatrix} 5 & 6 & 7 \\ 7 & 6 & 5 \\ 9 & 7 & 3 \\ 14 & 15 & 16 \\ 16 & 15 & 14 \\ 9 & 3 & 27 \end{bmatrix},$$

$$B = \begin{bmatrix} 8 \\ 3 \\ 10 \\ 30 \\ 28 \\ 30 \end{bmatrix} \quad \text{and}$$

$$s = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

2. (3 marks) What is the **best** MatLab solution for s using least squares?

Answer:

$$s=A\backslash B$$

3. (3 marks) The “least squares integration matrix”, A , is not a square matrix in the equation $A * s = B$. Rewrite this equation with a new “least squares integration matrix” that is made square by multiplying both A and B by a certain matrix (that is derived from matrix A).

Answer:

$$A' * A * s = A' * B$$

4. (3 marks) Solve this system of equations using the inverse of the new least squares integration matrix.

Answer:

$$s=\text{inv}(A'*A)*A'*B$$

5. (3 marks) Do you expect the exact same solution for all systems of equations using $A\backslash B$ or your inverse solution in 4. Why or why not?

Answer:

$A\backslash B$ is better $\text{inv}(A'*A)*A'*B$ as the computation of the inverse of a matrix is more effected by roundoff error. In general, we would get the same solution for most systems of equations. But if the system of equations is numerically unstable (high condition number) $A\backslash B$ is the better solution.

(4) (25 marks) Consider the following 2D graphs and the incomplete MatLab code used to generate them. Fill in the single missing MatLab statement (after the `figure` statement) in each case.

- (5 marks) Suppose we plot the following figure using the incomplete MatLab code below.

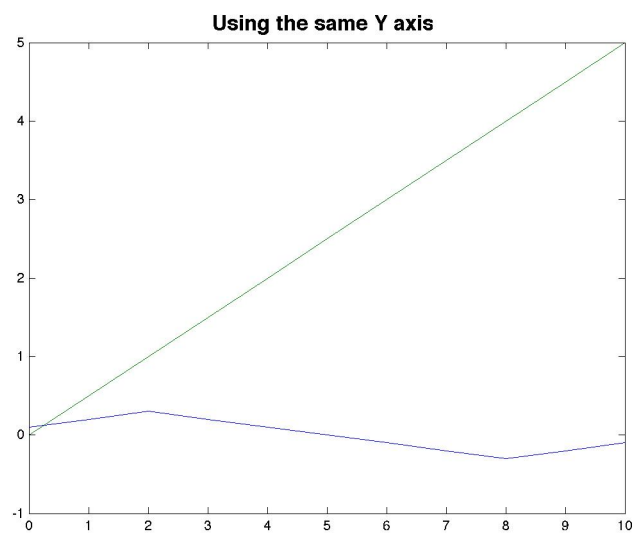


Figure 1: y_1 and y_2 are plotted against x using the same y axis.

```
x=linspace(0,10,11);
y1=[0.1 0.2 0.3 0.2 0.1 0.0 -0.1 -0.2 -0.3 -0.2 -0.1];
y2=[0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0];
figure
```

Answer:

```
plot(x,y1,x,y2);
% Also works: w=[y1;y2]; plot(x,w)

title('\fontsize{16}\bf Using the same Y axis');
```


2. (5 marks) Suppose we plot the following figure using the incomplete MatLab code below. Note the data plotted here is the same as in the question above!!!

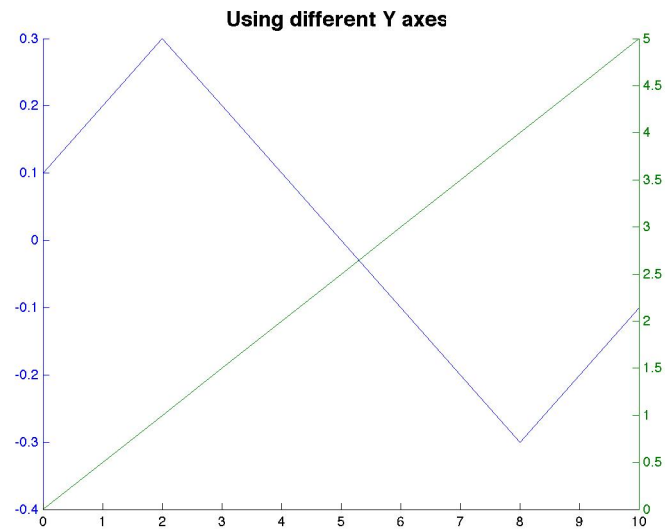


Figure 2: y_1 and y_2 are plotted against x using the different y axes.

```
x=linspace(0,10,11);
y1=[0.1 0.2 0.3 0.2 0.1 0.0 -0.1 -0.2 -0.3 -0.2 -0.1];
y2=[0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0];
figure
```

Answer:

```
plotyy(x,y1,x,y2);

title('\fontsize{16}\bf Using the different Y axes');
```

3. (5 marks) Suppose we plot the following figure using the incomplete MatLab code below.

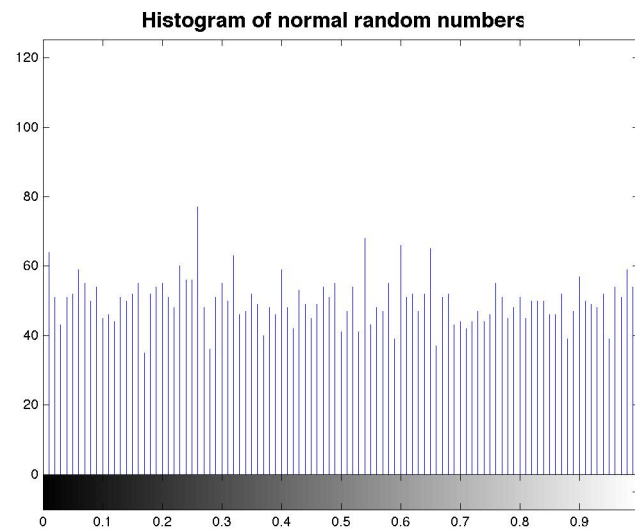


Figure 3: 5000 random normal values are histogram against 101 bins.

```
x=rand(5000,1,'double');  
figure
```

Answer:

```
imhist(x,101);  
  
title('\fontsize{16}\bf Histogram of normal random numbers');
```

4. (5 marks) Suppose we plot the following errorbar using the incomplete MatLab code below.

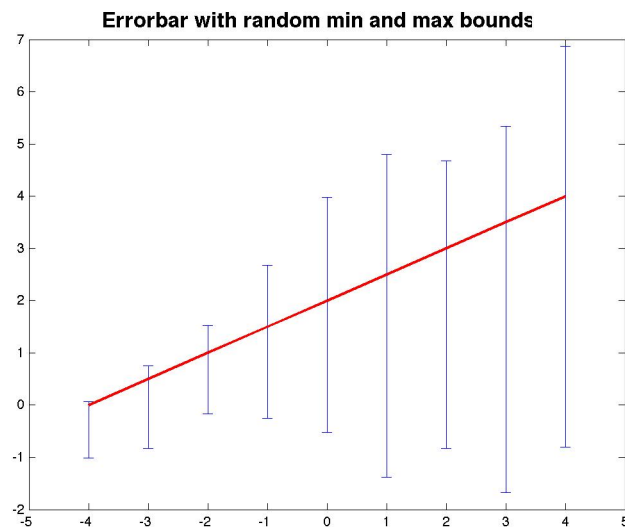


Figure 4: Errorbar with minimum and maximum bars.

```
x=linspace(-4,4,9);
y=[0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0];
U=y+abs(randn(1,9));
L=y-abs(randn(1,9));
figure
```

Answer:

```
errorbar(x,y,U,L)

hold on
% plot a straight red line through the line
% given by the data points x,y
plot(x,y,'linewidth',2.0,'color','red');
hold off
title('\fontsize{16}\bf Errorbar with random min and max bounds');
```

5. (5 marks) Suppose we plot the following quiver plot using the incomplete MatLab code below.

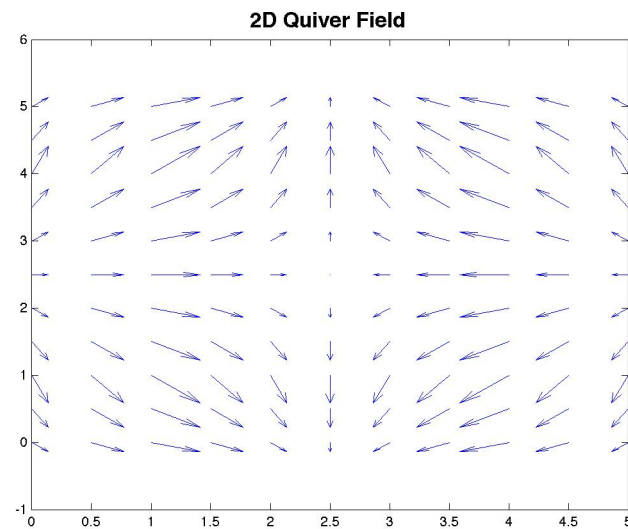


Figure 5: The u,v velocities at the meshgrid location of x and y values.

```
x=[0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0];
y=[0.0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0];
u=[0.1 0.2 0.3 0.2 0.1 0.0 -0.1 -0.2 -0.3 -0.2 -0.1];
v=[-0.1 -0.2 -0.3 -0.2 -0.1 0.0 0.1 0.2 0.3 0.2 0.1];
[X,Y]=meshgrid(x,y);
[U,V]=meshgrid(u,v);
figure
```

Answer:

```
quiver(X,Y,U,V);
```

```
title('\fontsize{16}\bf 2D Quiver Field');
```

(5) (25 marks) Consider the following 3D graphs and the incomplete MatLab code used to generate them. Fill in the single missing MatLab statement (after the `figure` statement) in each case.

- (5 marks) Suppose we plot the following slice figure using the incomplete MatLab code below.

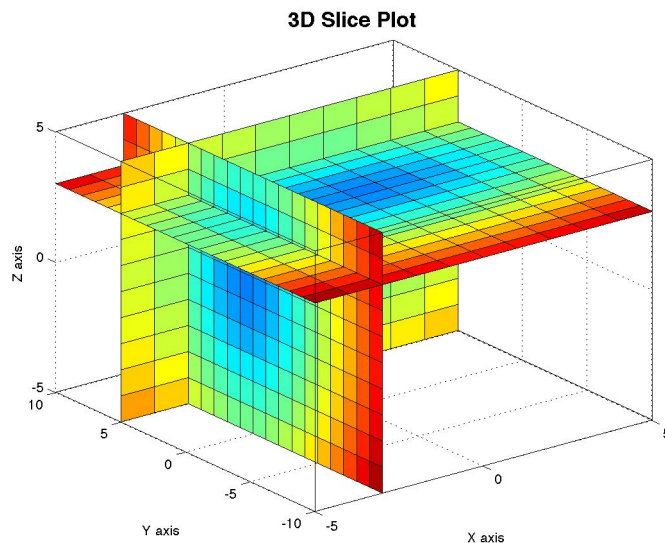


Figure 6: 3D slice image

```
x=linspace(-5,5,11);
y=-10:10;
z=-5:5;
[X,Y,Z]=meshgrid(x,y,z);
V=sqrt(X.^2+Y.^2+Z.^2);
figure
```

Answer:

```
slice(X,Y,Z,V,-3,5,3);

grid on; box on;
xlabel('X axis');
ylabel('Y axis');
zlabel('Z axis');
title('\fontsize{16}\bf 3D Slice Plot');
```

2. (5 marks) Suppose we plot the following helix figure using the incomplete MatLab code below.

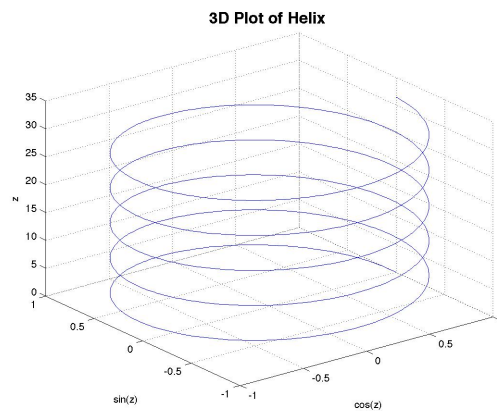


Figure 7: 3D helix image.

```
z=linspace(0,10*pi,1000);  
c=cos(z);  
s=sin(z);  
figure
```

Answer:

```
plot3(c,s,z)  
  
grid on  
xlabel('cos(z)')  
ylabel('sin(z)')  
zlabel('z')  
title('\fontsize{16}\bf 3D Plot of Helix');
```

3. (5 marks) Suppose we plot the following **ezsurf** figure using the incomplete MatLab code below.

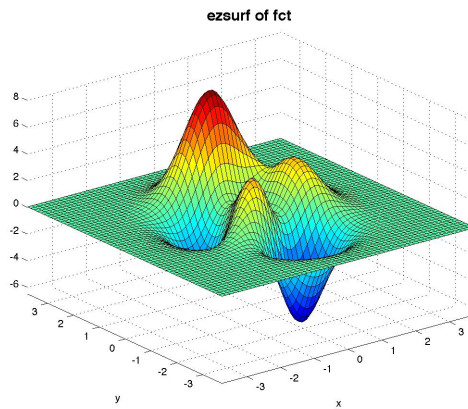


Figure 8: 3D ezsurf plot (of the peaks function!!!).

```
fct=['3*(1-x).^2.*exp(-(x.^2)-(y+1).^2)' ...
    '-10*(x/5-x.^3-y.^5).*exp(-x.^2-y.^2)' ...
    '-1/3*exp(-(x+1).^2-y.^2)'];
figure
```

Answer:

```
ezsurf(fct);

title('\fontsize{15}\bf ezsurf of fct');
```

4. (5 marks) Suppose we plot the following **ezcontour** figure using the incomplete MatLab code below.

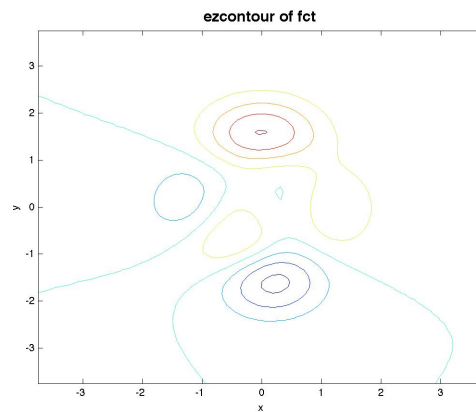


Figure 9: 3D ezcontour plot (of the peaks function!!!).

```
fct=['3*(1-x).^2.*exp(-(x.^2)-(y+1).^2)' ...
    '-10*(x/5-x.^3-y.^5).*exp(-x.^2-y.^2)' ...
    '-1/3*exp(-(x+1).^2-y.^2)'];
figure
```

Answer:

```
ezcontour(fct);
```

```
title('\fontsize{15}\bf ezcontour of fct');
```


5. (5 marks) Suppose we plot the following sinc (jinc) function figure using the incomplete MatLab code below.

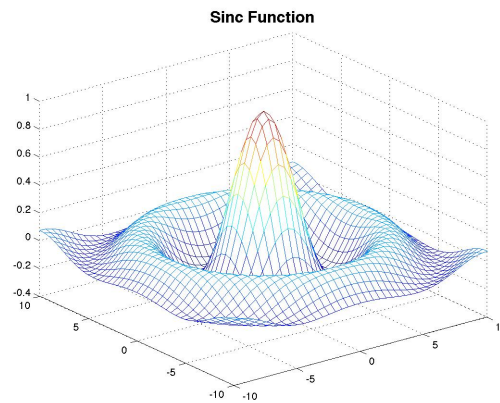


Figure 10: 3D Sinc Function.

```
[X,Y]=meshgrid(-10:0.5:10);  
R=sqrt(X.^2+Y.^2)+eps;  
Z=sin(R)./R;  
figure
```

Answer:

```
mesh(X,Y,Z)
```

```
title('\fontsize{16}\bf Sinc Function');
```

(6) (20 marks) This questions is also on 2D and 3D plotting and the use of Handle Graphics to change the appearance of graphs.

1. (2 marks) What does the **rotate3D** button in the toolbar let you do to a 3D image?

Answer:

Rotate the 3D graph in 3D using mouse motions

2. (2 marks) Consider the plot of a 2D graph using `plot(x,y)`, where `x` and `y` are appropriately set vectors. How would you change the linewidth of the graph to be 2.0?

Answer:

```
plot(x,y,'linewidth',2.0); or set(gca,'linewidth',2.0);
```

3. (2 marks) Consider the plot of a 2D graph using `plot(x,y)`, where `x` and `y` are appropriately set vectors. How would you change the colour of the graph to be red?

Answer:

```
plot(x,y,'color','red'); or set(gca,'color',[1 0 0]);
```

4. (4 marks) Suppose you have successfully written 4 MatLab functions, `G1(X,Y,Z)`, `G2(X,Y,Z)`, `G3(X,Y,Z)` and `G4(X,Y,Z)` to plot 4 3D graphs. `X`, `Y` and `Z` have been correctly computed elsewhere. How would you plot these 4 graphs as a single plot using **subplot**? Do not label the subplots with titles.

Answer:

```
subplot(2,2,1);  
G1(X,Y,Z);  
subplot(2,2,2);  
G2(X,Y,Z);  
subplot(2,2,3);  
G3(X,Y,Z);  
subplot(2,2,4);  
G4(X,Y,Z);
```

5. (4 marks) Suppose you wish to visually compare two 3D plots. The 1st plot has x , y and z ranges of $[x1min, x1max, y1min, y1max, z1min, z1max]$ while the 2nd plot has x , y and z ranges of $[x2min, x2max, y2min, y2max, z2min, z2max]$. What **axis** statement would you use to ensure both plots have the same x , y and z ranges.

Answer:

```
axis([min(x1min,x2min) max(x1max,x2max) ...  
      min(y1min,y2min) max(y1max,y2max) ...  
      min(z1min,z2min) max(z1max,z2max)]);  
% will axis auto also work? Freezes the axis for subsequent plots
```

6. (3 marks) One way to perform an animation is to plot the changing figures on top of each other without a **hold on**. However, if the figures are plotted too fast we need to slow down the rate at which figures are drawn on top of each other. How can we do this?

Answer:

Use a pause command between each figure plotting, for example, `pause(0.1)` pauses 0.1 seconds.

7. (3 marks) If, on the other hand, suppose the drawing of each figure is too slow (even without a pause) to produce a realistic animation. What can we do?

Answer:

Draw each image and save it as a movie frame. At the end, we can display the movie at a preset frame rate to get a smooth realistic animation.

(7) (15 marks) This questions is about doing computations on arrays with some **nans** in them. Give any computational results as fractions. Consider the 1D row array:

```
x=[4 5 9 nan 6 3 4 nan 2 8 1];
```

1. (1.5 marks) What is the value of `isnan(x)`?

Answer:

```
0 0 0 1 0 0 0 1 0 0 0
```

2. (1.5 marks) What is the value of `~isnan(x)`?

Answer:

```
1 1 1 0 1 1 1 0 1 1 1
```

3. (2 marks) What is the value of `mean(isnan(x))`:

Answer:

```
0.1818 (2/11)
```

4. (2 marks) What is the value of `mean(~isnan(x))`:

Answer:

```
0.8182 (9/11)
```

5. (4 marks) What is the value of `mean(x(isnan(x)))`

Answer:

```
nan
```

6. (4 marks) What is the value of `mean(x(~isnan(x)))`

Answer:

```
4.6667 (42/9=(4+5+9+6+3+4+2+8+1)/9)
```

(8) (30 marks) The question concerns the Matlab symbolic arithmetic toolbox.

1. (5 marks) Consider a function $f(x, y, z) = e^x \sin(y) \cos(z)$. Write a MatLab code segment to symbolically evaluate this integral:

$$\int \int \int f(x, y, z) dx dy dz.$$

Answer:

```
syms f x y z
f=exp(x)*sin(y)*cos(z)
int(int(int(f,z),y),x)
% value is -exp(x)*cos(y)*sin(z)
```

2. (5 marks) Write MatLab code to numerically evaluate this integral:

$$\int_{-2}^2 \int_{-3}^3 \int_{-4}^4 f(x, y, z) dx dy dz.$$

Answer:

```
syms f x y z
f=exp(x)*sin(y)*cos(z)
% This statement will work without eval
eval(int(int(int(f,x,-4,4),y,-3,3),z,-2,2))
% value is 0
```

3. (5 marks) Consider a function $f(x, y, z) = e^x \sin(y) \cos(y)$. Write a MatLab code segment to symbolically evaluate the derivative:

$$\frac{\partial^3}{\partial x \partial y \partial z} f(x, y, z)$$

Answer:

```
syms f x y z
f=exp(x)*sin(y)*cos(z)
diff(diff(diff(f,z),y),x)
% value is -exp(x)*cos(y)*sin(z)
```

4. (5 marks) Write MatLab code to numerically evaluate this derivative:

$$\frac{\partial^3}{\partial x \partial y \partial z} f(x, y, z)$$

at $x = 1$, $y = 2$ and $z = 3$.

Answer:

```
syms f x y z
f=exp(x)*sin(y)*cos(z)
x=1;
y=2;
z=3;
eval(diff(diff(diff(f,z),y),x))
% value is -2.4470
```

5. (10 marks) Write a MatLab code segment to symbolically solve the 3×3 linear system of equations:

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} r \\ s \\ t \end{bmatrix}.$$

which can be written more concisely as $AX = B$.

Answer:

```
syms f a b c d e f g h i x y z r s t A B X
A=[a b c; d e f; g h i];
B=[r; s; t];
X=A\B
%X=
% (b*f*t-c*e*t-b*i*s+c*h*s+e*i*r-f*h*r)/(a*e*i-a*f*h-b*d*i+b*f*g+c*d*h-c*e*g)
% -(a*f*t-c*d*t-a*i*s+c*g*s+d*i*r-f*g*r)/(a*e*i-a*f*h-b*d*i+b*f*g+c*d*h-c*e*g)
% (a*e*t-b*d*t-a*h*s+b*g*s+d*h*r-e*g*r)/(a*e*i-a*f*h-b*d*i+b*f*g+c*d*h-c*e*g)
```

(9) (30 marks) Consider the 25 x and y values plotted in the graph below. The following

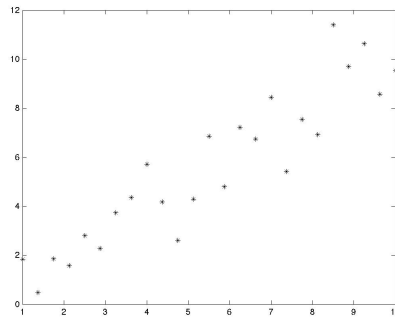


Figure 11: Scatter plot of some x and y data.

MatLab code segment produced this plot:

```
x=linspace(1,10,25);  
y=linspace(1,10,25)+randn(1,25,'double');  
plot(x,y,'*k');
```

Answer the following questions:

1. (10 marks) Fill in the incomplete MatLab code continued after the above MatLab code segment that produces the following line fit (1st order polynomial fit):

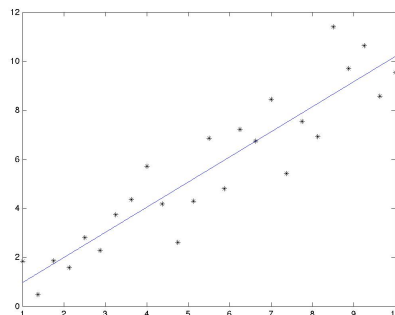


Figure 12: 1st polynomial fit of the x and y data.

```
hold on
xp=linspace(1,10,250);
```

Answer:

```
p1=polyfit(x,y,1);
plot(xp,polyval(p1,xp));
```

2. (10 marks) What are the two problems most commonly associated with high order polynomial fits to datasets?

Answer:

(1) High condition number of vandermonde matrix indicate numerical instability

(2) While the polynomial may fit the data well, in between the data points, the polynomial may vary wildly.

3. (5 marks) What is the difference between an **inlier** and an **outlier**?

Answer:

An outlier obviously does not belong to the dataset while an inlier obviously does. The outlier falls outside the trend of the data while the inlier falls within the trend of the dataset.

4. (5 marks) What effect can an outlier have on the fit of a line (or, indeed, the fit of any polynomial) to the data?

Answer:

An outlier can drastically change the line fit to the data (i.e. both its slope and y intercept). It has huge influence on the fit beyond what an inlier has.

(10) (10 marks) Consider edge detection as done by MatLab in Lecture 19 and Lab 11.

1. (4 marks) What is an advantage of using a white background with black edgels over a black background with white edgels?

Answer:

(1) A black background will use significantly more toner when printing!

(2) It is maybe easier to see black edges on a white background than vice versa (a subjective judgement).

2. (6 marks) Consider the colour edge detection done for Lab 11 (the last lab).

- (a) (2 marks) What does a red edgel mean?

Answer:

There is an edgel in the red image plane but not in the green or blue planes at that location.

- (b) (2 marks) What does a yellow edgel mean (yellow is made by combining red and green light in equal amounts)?

Answer:

There is an edgel in the red and green image planes but not in the blue plane at that location.

- (c) (2 marks) What does a white edgel mean?

Answer:

There is an edgel in all three image planes at that location.