

The University of Western Ontario

Computer Science 2035b

Solutions for Midterm Examination - Friday, February 27th, 2014

Surname	
Given Name	
Student Number	

This exam consists of 4 questions (8 pages including this page) worth a total of 100%. It is an open book exam, course notes and any MatLab book(s) are allowed. All answers are to be written in this booklet. Scrap work may be done on the back of each page; this will not be marked. No laptops or cell phones are allowed. The exam is 50 minutes long and comprises 20% of your final mark. Please print your full name and student number in the space provided below before you start this exam.

(1) 40%	
(2) 10%	
(3) 30%	
(4) 20%	
Total	

Professor: John Barron

(40%) Consider the following MatLab matrices A, B and C:

```
A= [19  4  5  6  8;
    30 22  7 12 13;
    28  2  3  5  9;
     8  9 10  5  4;
    19  4  6 24 9];
B=[2 6; 4 9];
C=[1; 2];
```

1. (4%) Using the original A above, if $A(2:4,2:4)=\text{eye}(3)$ what is the value of A?

```
A = 19      4      5      6      8
     30      1      0      0     13
     28      0      1      0      9
       8      0      0      1      4
     19      4      6     24      9
```

2. (4%) Using the original A above, if $A(2:4,2:4)=A(2:4,2:4)'$ what is the value of A?

```
A = 19      4      5      6      8
     30     22      2      9     13
     28      7      3     10      9
       8     12      5      5      4
     19      4      6     24      9
```

3. (4%) Using the original array **A**, what is the value of `reshape(A(2:3,2:4),3,2)`?
For partial marks, say what `A(2:3,2:4)` is.

`A(2:3,2:4)` is

22 7 12

2 3 5

`reshape(A(2:3,2:4),3,2)` is

22 3

2 12

7 5

[Take the elements of `A(2:3,2:4)` column by column.]

4. (4%) What is `B*B`:

`B*B`= 28 66

44 105

5. (4%) What is `B.*B`:

`B.*B`= 4 36

16 81

6. (4%) What if the value of `[B C]`?

`[B C]`= 2 6 1 using `C = 1`

4 9 2 2

7. (4%) What if the value of `[B; C']`?

$$\begin{array}{cc} 2 & 6 \\ 4 & 9 \end{array} \quad \text{using } C' = \begin{array}{cc} 1 & 2 \\ 1 & 2 \end{array}$$

8. (4%) Consider a 2 element column vector \mathbf{s} . How would you solve the system of equations $\mathbf{B} \cdot \mathbf{s} = \mathbf{C}$?

$\mathbf{s} = \mathbf{B} \backslash \mathbf{C}$; giving $\mathbf{s} = [0.5; 0.0]$ for $\mathbf{B} = \begin{bmatrix} 2 & 6 \\ 4 & 9 \end{bmatrix}$; and $\mathbf{C} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$;

Note $0.5 \cdot 2.0 + 0.0 \cdot 6 = 1$ and $0.5 \cdot 4 + 0.0 \cdot 9 = 2$, which are the the two rows of \mathbf{C} !!!

9. (4%) What happens when we execute $\mathbf{A} \cdot \mathbf{B}$?

$\mathbf{A} \cdot \mathbf{B}$

Inner matrix dimensions must agree.

10. (4%) What happens when we execute $\mathbf{A} . \cdot \mathbf{B}$?

$\mathbf{A} . \cdot \mathbf{B}$

Matrix dimensions must agree.

(2) (10%) This is the Lab question.

1. (3%) Consider the following expression: $1/2+3^2+4/5*6/7$. Parenthesize this expression according to the precedence of the operators. You do not need to compute its value. Parenthesized expression:

$1/2+3^2+4/5*6/7$ has value 10.1857

$((1/2)+(3^2))+(((4/5)*6)/7))$ has value 10.1857

2. (7%) Draw the graph plotted by:

```
x=[-1:0.1:1];  
plot(x,abs(x));  
title('absolute value');  
print abs.jpg -djpeg
```

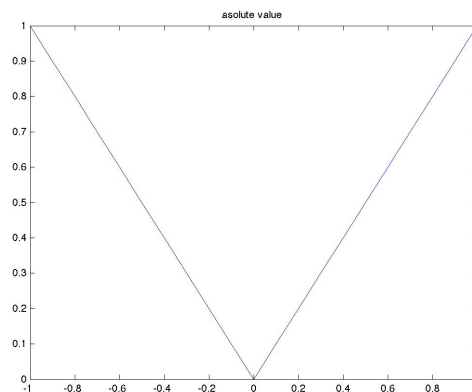


Figure 1: Absolute value graph between -1 and +1.

(3) (30%) This question is loosely related to Assignment 2.

1. Consider the evaluation a polynomial $p(x) = x^4 + x^3 + x^2 + x$ for x being a n component column vector $\mathbf{x} = \text{linspace}(0, 1, n)$.

(a) (10%) Give the straightforward vectorization of this polynomial:

$$\mathbf{p} = \mathbf{x}.^4 + \mathbf{x}.^3 + \mathbf{x}.^2 + \mathbf{x}$$

(b) (10%) Give the vectorization of this polynomial that does the least number of multiplications. One solution:

$$\begin{aligned} \mathbf{x2} &= \mathbf{x} .* \mathbf{x}; \\ \mathbf{x3} &= \mathbf{x2} .* \mathbf{x}; \\ \mathbf{x4} &= \mathbf{x3} .* \mathbf{x}; \\ \mathbf{p} &= \mathbf{x4} + \mathbf{x3} + \mathbf{x2} + \mathbf{x} \end{aligned}$$

The straightforward solution (a) does 6 element by element multiplications. The optimized solution does 3 element by element multiplications. Another solution (that I did not expect) uses Horner's rule:

$$\mathbf{p} = \mathbf{x}.^4 + \mathbf{x}.^3 + \mathbf{x}.^2 + \mathbf{x}$$

can be re-written as:

$$\begin{aligned} &= \mathbf{x} .* (\mathbf{x}.^3 + \mathbf{x}.^2 + \mathbf{x} + 1) \\ &= \mathbf{x} .* (\mathbf{x} .* (\mathbf{x}.^2 + \mathbf{x} + 1) + 1) \\ &= \mathbf{x} .* (\mathbf{x} .* (\mathbf{x} .* (\mathbf{x} + 1) + 1) + 1) \end{aligned}$$

This also requires 3 element by elements multiplications.

2. (10%) Consider the following serialized loop that computes the above polynomial:

```
for i=1:n
    x(i)=(i-1)/(n-1); % numbers between 0 and 1
    p(i)=x(i)^4+x(i)^3+x(i)^2+x(i);
end
```

Rewrite the original loop so that MatLab's JIT compiler can compile the loop.

Just pre-allocate the space for **x** and **p** using **zeros**. For efficiency, we could precompute $x2(i)=x(i)^2$, use it to compute $x3(i)=x(i)^3$ as $x3(i)=x2(i)*x(i)$ and compute $x4(i)=x(i)^4$ as $x4(i)=x3(i)*x(i)$.

```
x=zeros(1,n,'double');
p=zeros(1,n,'double');
```

(4) (20%) Consider the following matrix $Q = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$;

(a) (5%) What does $L = Q > 5$ print?

$L = Q > 5$

```

0      0      0
0      0      1
1      1      1

```

(b) (5%) What does $C = \text{find}(Q > 5)$ print? (Hint: `find` returns the coordinates of $Q > 5$ as if Q were reshaped to be a column vector). Note that $L(:)'$ as a 1D row vector is $0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 0 \ 1 \ 1$. That is we take the columns of L from left to right and stack them. The 1's are at the 3rd, 6th, 8th and 9th positions of $L(:)$.

$C = \text{find}(Q > 5)$

```

3
6
8
9

```

Note that $Q = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

converted into a 1D vector column by column as:

$Q(:)' = 1 \ 4 \ 7 \ 2 \ 5 \ 8 \ 3 \ 6 \ 9$

Then $Q(C)$ prints

```

7 --- 3rd element
8 --- 6th element

```


6 --- 8th element

9 --- 9th element

(c) (5%) What does `[m,n]=size(Q)` print?

`[n,m]=size(Q)`

`n = 3`

`m = 3`

(d) (5%) What does `sum(L(:))` compute?

`L=Q>5`

0 0 0

0 0 1

1 1 1

So `L'(:)` has values 0 0 1 0 0 1 0 1 1 and the `sum(L(:))=4`.