On the Quagmire of "Shaly Sand" Saturation Equations

D. C. Herrick[1] and W. D. Kennedy[2]

Society of Petrophysicists and Well Log Analysts

ABSTRACT

Calculating water saturation from resistivity and porosity logs in "shaly sands" has been problematic since Archie (1942) published his saturation equation for "clean" sands. Since then much effort has been expended trying to develop a generally applicable saturation equation for "shaly sands". Early confusion between "shale" and "clay" resulted in a whole class of saturation relationships, known as shaly sand equations, based on determining the volume of "shale" in sandstone (and other lithologies) using the log-derived properties of a separate thick reference shale. This methodology is popular since only log data are required. However, most of the material in sandstones treated as shale is actually clay of allogenic or, more commonly, authigenic origin. This clay usually has properties quite different from any neighboring shale bed. The failure of rocks to conform to the premises assumed results in saturation calculations of dubious value.

The shale-clay confusion was recognized by some scientists in the 1950s. Early efforts to account for excess conductivity attributed it to a volumetric component of conductive shale. In 1968 Waxman and Smits proposed a saturation model that advocated the use of cation exchange measurements to estimate conductivity due to cation exchange on clay in sandstones. The Waxman-Smits model eventually resulted in a new class of saturation equations, known as dispersed shale equations, based on modification of the Waxman-Smits model. These include the dual-water and Juhasz models. Unfortunately these new models have not relieved the confusion, but probably have compounded it.

Both classes of "shaly sand" saturation equations are in wide use. A review of the assumptions that underlie any of the equations in both classes shows that there are fatal flaws in all. It is clear that to date there is no "shaly sand" model that is defensible, or even self-consistent.

We hope that showing the explicit connections among the various shaly sand models, and pointing to the parts that cannot be logically justified, will spur the formulation of new, logically justified, self-consistent models relating water saturation to the distribution of conductive components that control the electrical properties of a reservoir rock. The development of a suitable equation based on fundamental physics is sorely needed to cope with the demands of the present and future evaluation of petroleum reserves. The essence of a true "shaly sand" equation is presented.

INTRODUCTION

Estimating the water saturation of porous rocks from wireline logs has been a valuable formation evaluation tool since Archie (1942) discovered a relationship between the resistivity, porosity and water saturation for rocks that are at least moderately homogeneous and isotropic and composed of more or less equidimensional grains. Archie's relationship does not accurately estimate the saturation of sediments with internal structure, such as thin beds, vugs and micro-porous grains.

Archie's equation also does not give correct saturation estimates when used for sediments referred to as "shaly sands" unless both n and m are allowed to vary and are calibrated with appropriate laboratory data. These sediments are typically clay-bearing with the clay distributed in the pore system or on grain surfaces of sandstone, or the clay may be a component of shale distributed within the sandstone. Early in the history of log analysis it was discovered that shales had distinctive log properties such as high radioactivity, high hydrogen index and low acoustic velocity, etc. Some sediments that were regarded as sandstones also shared these properties and were described as "shaly sands".

In the evolution of log analysis, some of the pioneers failed to distinguish shale and clay. As a result, the early literature is confusing. Much was written about "shaly sands", but with a considerable likelihood that sands containing clay, instead of shale, were under discussion. This confusion persists to this day. When "shaly sands" are referred to, it behooves the analyst to make certain exactly what is being discussed.

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[1] consultant/retired

[2] Baker Hughes Inteq, Inc., Houston, Texas.

The term "shale" refers to a particular kind of rock that is typically very fine-grained, indurated, laminated and fissile. Clays, on the other hand, are a class of minerals (not rocks) that tend to be very fine-grained and have a phyllosilicate crystal structure. There are many different clay minerals, each with different composition, structure and properties. Even individual clay minerals tend to have variable composition due to substitution of one element for another in the crystal structure. The term "clay" is also used as a size term describing grains of any mineralogy that are less than four micrometers in size. Clay mineral crystals and grains are typically clay-sized, hence the terminology.

Shales are often clay-rich with a typical clay content of 20 percent to 50 percent, averaging around 40 percent, the remainder being composed commonly of silt and clay-size quartz, rock fragments, volcanic material, carbonates and other minerals. Shales may also contain significant amounts of organic matter (kerogen) making some shales source rocks or even reservoirs for petroleum. The composition of any particular shale is dependent on provenance, weathering, sediment transportation, deposition and diagenesis. The electrical conductivity of shales in the subsurface depends on the porosity, brine chemistry, pore geometry and clay mineralogy.

ELECTRICAL PROPERTIES OF CLAY MINERALS

The electrical conductivity of clay-bearing rocks can be strongly affected by the surface chemistry of the clay minerals. Some clay minerals (e.g. mixed layer-illite, montmorillonite) contain so-called counterions; i.e., positively charged cations that are present between the negatively-charged aluminosilicate layers of the crystal structure. These counterions are held only by weak electrostatic forces (ionic bonds). When in contact with brine, the cations in the solution (usually Na^+) can exchange with the counterions of the clay. This is a continuous process due to the thermal agitation (Brownian motion) of water molecules and dissolved ions in the brine and of the counterions of the clay. When the clay-brine system is subjected to an electric field, the exchanging cations have a component of their motion in the direction of the field. This component constitutes an electric current that contributes to the total conductivity of the clay-brine system. The cation exchange process produces this contribution to the current, and thus the cation exchange current is restricted to the volume in very close proximity to the charged alumino-silicate surfaces of clay.

An additional source of electric current due to cation exchange results from the aluminosilicate sheet structure. Since the size of clay crystals is not infinite (in fact they are usually very small, commonly clay-size) the layers terminate with unsatisfied bonds at the crystal edges. Ions neutralizing the edge-charges are readily exchangeable with ions from brine and can contribute to electric currents due to ion exchange. These surface-ions are present in clay minerals that would not otherwise exhibit ion exchange, for example, kaolinite and pure illite. The volume next to charged clay surfaces and edges is referred to by chemists as the "double layer". The double layer contributes to the total conductivity in proportion to its volume, which can vary, and its conductivity, which also can vary. The properties of the double layer are discussed in detail in texts on surface chemistry.

If the conductivity of a brine-saturated sandstone sample (C_0) containing clay with cation exchange capability is measured and plotted as a function of saturating brine conductivity (C_w), then a positive intercept is observed on the C_0 axis when the linear portion of the relationship is extrapolated to zero brine conductivity (Figure 1). The intercept, C_s, is the contribution to the total conductivity due to cation exchange on clay mineral surfaces in the rock. This part of the total conductivity does not vary with changes in brine conductivity when the brine salinity is greater than about 15,000-20,000 ppm. When the brine salinity is in the range of 20,000 ppm or less, the C_0-C_w relationship curves downward to the origin. This curvature has been attributed to several phenomena, but is probably due to loss of exchangeable cations from the clay surface as the result of maintaining chemical equilibrium with the pore water.
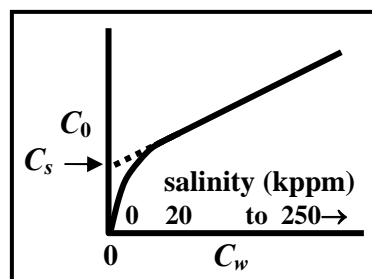

FIG. 1. Conductivity of a rock exhibiting surface conductivity as a function of brine conductivity.

The electrical double layer associated with the surfaces of clay crystals consists mainly of cations in various states of hydration, depending on the salinity of the brine in contact with the clay. If the salinity is less than 15-to-20 thousand parts per million, the cation exchange process causes the surface to ionize to maintain a chemical equilibrium between the brine and the clay mineral surfaces (Figure 1). The result is unsatisfied negatively charged sites on the clay surface and fewer

exchangeable cations to carry an electric current. The positive end of water molecules is attracted to the unsatisfied negative charge. The surface ionization and attraction for water molecule dipoles causes the thickness of the double layer to increase. In general, when the pore water is relatively fresh, a decrease in C_w causes the thickness of the double layer to increase and the surface conductivity to decrease.

Pure illite contains only potassium ions in the inter-layer sites. The potassium ion (K^+) fits perfectly between the aluminosilicate layers comprising illite and does not hydrate readily. Thus, a K^+ ion occupies a potential energy minimum where it is unlikely to be displaced by collisions with water (or other) molecules; consequently, cation exchange of the interlayer K^+ ions is rare. However, illite usually exhibits a significant cation exchange capacity due to unsatisfied bonds and exchange sites on external surfaces, as well as to electric charge due to substitutions in the aluminosilicate structure. Illite may be a mixture of layers of illite and montmorillonite (so-called mixed-layer illite). The counterions in the inter-layers of montmorillonite are readily exchangeable and contribute to the cation exchange capability, and thus the surface conductivity, of illite.

SATURATION EQUATIONS

The problem of interpreting the conductivity of shaly and clay-bearing sandstones has existed since the beginning of quantitative log analysis, and has been the source of much work and voluminous publication. These sandstones are more conductive than they would otherwise be if the clay or shale could be removed. If this excess conductivity is not properly attributed to the clay or shale, hydrocarbon saturations will be underestimated using Archie's conventional resistivity-porosity-water saturation model for sandstone.

The conventional log-analytical model for a shaly-sand comprises shale-free sand and "sand-free" shale as end members. Between the two end members, three classes of so-called shaly sand are defined: structural, laminated, and dispersed. This is a log-analytical operational definition because "sand-free" shale is not a rigorous concept. As used in log analysis, the term "shaly sand" generally includes laminated thin sandstone and shale beds that cannot be resolved by a resistivity logging instrument. The term is also used for sandstones containing shale clasts and mixtures of sand and mud due to, for example, bioturbation or severe slumping. Unfortunately, the descriptor "shaly sand" is also used to include sandstones that contain clay of diagenetic origin or as grain coatings due to deflocculation during deposition. Clay in sandstone originating with these processes is not shale and must be treated separately.

To our knowledge there is no published saturation equation for sandstone containing shale clasts (often referred to as "structural shale"). Commonly, shale clasts in sandstone are a minor component and widely separated from each other. In this case, the shale clasts have little effect on conductivity since conduction occurs mainly through the pore system. Only in the unlikely event that the clasts are so numerous so as to be in contact with each other, forming a continuous conduction path, would the presence of shale clasts in a sandstone have a significant electrical effect, comparable to conduction through the pore system.

There are two types of shaly and clay-bearing sandstone conductivity analysis equations. The first type seeks to quantitatively account for the surface conductivity of clay minerals distributed in the pore space and on grain surfaces. These are categorized as cation-exchange saturation equations. The second type presumes that the sandstone's excess conductivity is due to shale present in sandstone and makes the conductivity correction based on an estimate of the volume of shale in the sandstone. Unfortunately these two sources of excess conductivity, cation exchange on clay mineral surfaces and conduction due to shale, have become confused; frequently shale conductivity estimates are misused to correct for conduction by cation exchange.

This article is not meant to be a comprehensive listing of saturation equations, as has been published by Worthington (1985), but instead evaluates commonly used cation exchange and shale conductivity equations employed for determining fluid saturations.

CATION EXCHANGE OR
SURFACE CONDUCTION EQUATIONS

Waxman-Smits Equation

The Waxman-Smits (W-S) saturation equation for clay-bearing sandstone (Waxman and Smits, 1968) was motivated by the recognition that cation exchange capacity measurements of core samples could be used to estimate and correct for the surface conductivity of clay minerals distributed on the grain surfaces and disseminated in the pore system of a sandstone. The equation is based on the following assumptions.

1. The form of the equation is assumed to be $C_t = C_p + C_s$, where C_p indicates conductivity due to brine in the bulk pore space and not including any conductivity due to cation exchange on clay. C_p is the conductivity that the rock would have if exchangeable cations on clay surfaces were hypothetically rendered immobile and non-conductive, but all other

aspects of pore geometry, composition, etc., remain unchanged. The contribution to the total conductivity from ion exchange on clay surfaces is added as a separate term C_s. This is a parallel conduction model.

There is evidence to support treating conductivity due to clay surfaces as a parallel additive component. Plots of the conductivity of a rock sample containing surface conductive clay, when saturated with brines of different salinity, show a constant offset when the brine salinity is above about 20 kppm (Figure 1). The offset is attributed to a constant surface conductivity component due to cation exchange on clay.

2. The conductivity due to cation exchange on clay surfaces C_s, is assumed to be proportional to the number of milli-equivalents of exchangeable clay counterions per unit volume of pore space, Q_v. The proportionality constant B is an experimentally determined factor used to convert Q_v to conductivity.

3. The third assumption, as stated by Waxman and Smits (1968) is "*We assume next that the electric current transported by the counterions associated with the clay travels along the same tortuous path as the current attributed to ions in the pore water*." The same conduction geometry implies that they also have the same formation conductivity factor $f*$ ($f* = 1/F*$ where $F*$ is the Archie formation resistivity factor of the "shaly" sand). The Waxman-Smits equation for water-saturated "shaly" sands can then be written as

$$C_0 = f* \, (C_w + BQ_v) ; \qquad (1)$$

and for rocks only partially brine saturated,

$$C_t = f* \, i* \, (C_w + BQ_v/S_w), \qquad (2)$$

where i^*, the conductivity index, is the reciprocal of the resistivity index ($i^* = 1/I^*$). Re-expressed in terms of Archie's equation, (2) becomes

$$C_t = S_w^{n*} \, \phi^{m*} \, (C_w + BQ_v/S_w). \qquad (3)$$

Note Waxman's and Smits' presumption that both clay counterions and pore water have the same conduction geometry continues to be expressed in (3), as it is in (1). The properties designated by "*" translate from $f*$ to $m*$ and from $i*$ to $n*$.

The premise that the same formation conductivity factor and conductivity index apply to both the pore-water and the clay counterions is asserted by Waxman and Smits (1968) without justification or explanation. The premise contradicts the concept of conduction by cation

exchange in the region of the double layer near the clay surface since this volume and geometry is different from the pore volume and its conduction geometry.

Conduction by cation exchange is restricted to a volume very close to clay surfaces. The mobility of a sodium ion in solution at room temperature is 5.2×10^{-8} m^2 sec^{-1} V^{-1} (i.e. velocity, m/sec, per unit electric field, volt/m). If a sodium ion is subjected to an alternating electric field with a square wave-form at a frequency of 20 kHz and an electric field of one volt per meter, then during one half of one cycle the sodium ion will be displaced only 0.052 Ångstrom units, roughly five percent of the diameter of the Na$^+$ ion. A displacement of this magnitude will not move an ion from a clay surface an appreciable distance into a pore. The electric fields in the rocks surrounding most logging tools are usually only in the millivolt per meter range, causing even smaller cation displacements. In addition, it is likely that the mobility of sodium ions that are part of the double layer on clay surfaces is less than when in solution in the pore-water. Hence the very small displacement of sodium ions is confined to the vicinity of the clay double layer and is controlled by the geometry of the clay surfaces.

The effect of conduction geometry on conductivity is most readily appreciated using the method of Herrick and Kennedy (1993, 1994, 2009). Rather than combining the effects of pore geometry and pore volume, as Archie's formation resistivity factor F does, the effects can be separated. The conductivity of a clay-free brine-saturated rock can be described by

$$C_0 = C_w \phi E_0, \qquad (4)$$

where E_0 accounts for the effects of the pore geometry of a porous medium on electrical conduction. (Note that $1/F = \phi E_0$.) The magnitude of the effects of pore volume (ϕ) and pore geometry (E_0) are similar, neither dominates. This equation describes the conductivity of brine-containing porous rocks as effectively as Archie's empirical equation, but has the advantage of having an explicit theoretical basis.

When hydrocarbon displaces brine, the bulk volume fraction of the remaining brine is $S_w \phi$. All conduction takes place in this volume. To account for the changes in volume and brine geometry, equation (4) is modified by including the additional factors S_w and e_t, respectively. The conductivity of partially brine-saturated and clay-free rocks is described by

$$C_t = C_w (S_w \phi)(e_t E_0), \qquad (5)$$

where e_t represents a reduction of E_0 that accounts for the change in brine geometry in the pore-system due to the introduction of non-conductive hydrocarbons, just as S_w represents a reduction of ϕ to account for the reduction in brine volume, also due to the introduction of hydrocarbons.

Equation (5), for Archie rocks, is extended, à la Waxman-Smits, to clay-bearing rocks assuming that surface conductance is in parallel with pore conductance. Thus

$$C_t = C_w(S_w\phi)(e_t^* E_0^*) + C_s , \qquad (6)$$

where C_s is the conductivity component *due to* cation exchange on clay and the "*" notation has the same meaning as used in the Waxman-Smits equation (equation (2)). Equation (6) accounts for different conduction geometry in the bulk pore system ($e_t E_0$) (Herrick and Kennedy, 2009) and a separate term having different geometry of conduction on and near clay surfaces capable of cation exchange. This distinction is not drawn in the Waxman-Smits equation (i.e., equation (3)).

The interpretation of the second term in the Waxman-Smits equation (3), BQ_v /S_w, is ambiguous. Waxman-Smits (1968) describe Q_v /S_w as "the effective concentration of exchange ions at $S_o > 0$", where S_o is the oil saturation. To clarify the interpretation, the Waxman-Smits equation (3) can be rewritten in terms of the conduction geometry parameters of (6),

$$C_t = (S_w\phi) (e_t^* E_0^*) (C_w + BQ_v/S_w). \qquad (7)$$

Multiplying by $S_w\phi$ as indicated by equation (7) gives

$$C_t = (e_t^* E_0^*) (C_w S_w \phi + BQ_v \phi) . \qquad (8)$$

The first factor of (8), ($e_t^* E_0^*$), accounts for the electrical effect of the geometry of the pore-water, independent of any cation exchange effects due to clay. The product of this geometric factor and the first term within the parentheses in equation (8) is the conductivity component due to the pore water, independent of cation exchange. One could equally well use equation (3) and arrive at the same results (equation 8a), although multiplying by $S_w\phi$ for interpretation is less obvious:

$$C_t = S_w^{n^*-1} \phi^{m^*-1} (C_w S_w \phi + BQ_v \phi) \qquad (8a)$$

The second term within parentheses in equations (8) and (8a) can now be interpreted. Keep in mind that porosity is defined as the ratio of the volume of void space in a rock sample to the total volume of the sample. Q_v is the number of milli-equivalents of exchangeable cations

per unit pore volume. The product $Q_v\phi$ has units of milli-equivalents of exchangeable cations per unit volume of rock. Multiplying Q_v by ϕ converts units of pore volume to units of rock volume. This is important since conductivity is defined as the conductance of a unit volume of rock. But note that the same geometric term, presumed to be *independent* of any cation exchange effects due to clay ($e_t^* E_0^*$ or $S_w^{n^*-1} \phi^{m^*-1}$), is applied to the conductivity term ($BQ_v \phi$) that is due to cation exchange on clay! This internal contradiction makes the efficacy of**,** and interpretation of, the Waxman-Smits equation problematic. Unfortunately, any saturation model derived from Waxman-Smits also inherits this contradiction.

Dual-Water Model

Clavier, Coates and Dumanoir (1977, 1984) recognized a shortcoming of the Waxman-Smits conductivity model, namely that the volume within which surface conduction occurs can be significant and must be accounted for. They proposed a "dual-water" conductivity model that partitions the total pore volume between the volume in and near the double layer in which surface conduction takes place (referred to hereinafter as the "double layer" to simplify discussion) and the remaining pore space, free of clay surface effects. The dual-water model is a modification of the Waxman-Smits' model and retains the self-contradictory premise that the conduction geometry of free pore water and the clay double layer is the same. In this context the term "free water" is meant to indicate all the water contained in the pore system not located in the clay double layer.

The dual-water model describes an effective water conductivity C_{we},

$$C_{we} = V_{fw}C_w + V_{dl}C_{dl} , \qquad (9)$$

where V_{fw} is the fractional volume of pore-water exclusive of the double layer, C_w is its conductivity, V_{dl} is the fractional volume of the double layer and C_{dl} is the conductivity of the double layer. $V_{dl} + V_{fw} = 1$ when $S_w = 1$. For partially brine saturated rocks, the volume fraction of brine is no longer unity but equal to the water saturation; i.e., $V_{dl} + V_{fw} = S_w$ when $S_w < 1$**.**

This volumetric relationship leads to the dual-water model saturation equation as given by Clavier, Coates and Dumanoir (1977, 1984),

$$C_t = f_0 S_w^n \left[\left(1 - \frac{\alpha v_Q Q_v}{S_w} \right) C_w + \frac{\beta Q_v}{S_w} \right], \qquad (10)$$

where f_0 is the formation conductivity factor ($f_0 = 1/F_0$). The "0" subscript distinguishes the dual-water model formation conductivity factor from Archie's f and Waxman-Smits f^*. The fraction of the pore volume occupied by the double layer per milli-equivalent of counter-ions is v_Q; α is a salinity-dependent double layer expansion factor needed at low salinity. When the pore water is less saline than about twenty thousand parts per million NaCl equivalent, cation exchange sites on clay surfaces tend to ionize leaving unsatisfied negatively charged sites on the surfaces. Water molecule dipoles are attracted to the charged sites and the double layer expands accordingly. Above this salinity range, no ionization or double layer expansion occurs and α is equal to one. For a given rock at constant brine salinity and temperature, the product $\alpha\, v_Q\, C_{dl}$ is constant, and in the second term in the sum in (10) is represented by $\beta = \alpha v_Q\, C_{dl}$.

LOG-BASED DUAL-WATER-TYPE SATURATION EQUATIONS

Shale properties

Shales typically exhibit gamma-ray radioactivity. This can be due to potassium-bearing minerals such as feldspar, mica and the clay mineral illite. The presence of clay minerals, however, does not assure radioactivity. Only certain clay minerals, e.g., illite and glauconite, are intrinsically radioactive, containing potassium. Others such as montmorillonite, kaolinite and chlorite, do not require potassium (or other radioactive elements) in their crystal structures, and these clay minerals are not radioactive.

The radioactivity of some shales is partly due to uranium or thorium, not necessarily associated with clay. Weathering and abrasion-resistant minerals such as zircon may be present; these may contain thorium and uranium. Organic matter, especially in marine shales, tends to concentrate uranium. In sandstones, neither uranium nor thorium give any information on clay that has resulted from diagenetic processes, since neither is a required structural element in clay crystals. The content of each of these radioactive components of shale can vary considerably, giving different shales different amounts and kinds of radioactivity. Thus, the gamma-ray activity of rocks does not invariably correlate with clay content or "shaliness."

Shales also have other properties causing characteristic log responses. Neutron instruments respond to the presence of hydrogen in clay minerals and organic matter. Clay grain density varies with composition. The densities of clay minerals in shales are often reported to be less, even much less, than that of quartz. This is due mainly to faulty analytical procedures. The actual density of most clay minerals is similar to or greater than that of quartz.

Due to the presence of clay minerals and organic matter, shales tend to be relatively compressible and exhibit low acoustic velocities (long interval transit times).

Sandstones may contain clay minerals. These minerals comprise either allogenic clays that have adhered to sand grain surfaces during deposition, or have been produced after deposition by authigenic chemical processes. Continual authigenic chemical changes in clays with burial are also common.

The properties of shales vary considerably, and sometimes dramatically, for a variety of reasons. In addition, shale and clay have different properties and different log responses. In petrophysics, shale and clay are distinct and different. The use of various logs, whether singly or in combination, cannot be relied upon to give information on surface conductivity (or "shaliness") unless a suitable correlation is independently established and verified.

Determining the electrical "shaliness" of sandstones based on the log response of a thick reference shale to estimate the volume of shale in the sandstone requires assuming that the log responses of shales deposited along with shaly or clay-bearing sandstones can be used to estimate the clay content and electrical properties of the sandstones. For validity, this assumption requires that the clay mineralogy, texture, log response and pore geometry of the shale are identical to those of the shale in the shaly sandstone. In order to proceed with log analysis, this assumption is universally accepted by formation evaluators; it is however, rarely, if ever, validated in practice.

V_{sh}-based "Modified Dual-Water Model"

The idea expressed in the dual-water model—that water associated with clay surfaces occupies a different volume and should have different properties than pore water, which is unaffected by clay effects—is sound. But an analyst must take care to define the point where one becomes the other. Defining this distinction has been a perennial problem in log analysis. Most methodologies based only on log data (e.g., Best , Gardner and Dumanoir, 1978), attempt to make this distinction using the properties of a shale bed associated with the target sandstones and thick enough to be characterized by logs.

In "dual-water" models based upon observed log responses, the estimation of the amount of water in which surface conduction occurs is based on an ad hoc algorithm. Being ad hoc, such an algorithm is not unique. The first such algorithm was offered by Best, Gardener, and Dumanoir (1978). The first step in their method is to estimate the "bound water saturation" S_{wb}, presumably the fractional amount of water associated with clay surfaces in which surface conduction occurs. This is accomplished by computing "shale indices" from gamma-ray, SP and neutron logs at each depth and using the minimum of the three "shale indices" in an empirical graphical transform to obtain S_{wb} (Figure 2). Moreover, at a single depth in a well, which indicator is the minimum of the three may well depend upon subjectively assigned adjustable parameters in the log response – to – V_{sh} transform.
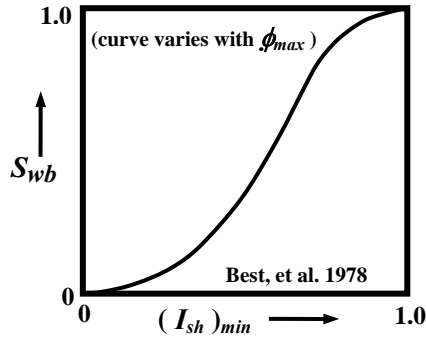

FIG. 2. Empirical graphical transform for converting a minimum shale index to S_{wb}.

Best et al. report the shape of their transform to vary as the result of several qualitative observations, including the behavior of S_{wb} at high gamma-ray values, that of the SP log in zones with low clay/shale content, and a flattening of the curve with increased maximum porosity of shale-free sandstone. No theoretical justification for the curve is given; hence the value of the "bound water saturation" needed to compute hydrocarbon pore volume using this approach is obtained by a transform with an unexplicated theoretical basis, and founded on an ad hoc choice of the minimum of three shale indices.

The value of S_{wb} obtained in this way is used in an equation similar to that of the dual-water model,

$$C_t = \left[\left(1 - \frac{S_{wb}}{S_w} \right) C_{wf} + \frac{S_{wb}}{S_w} C_{wb} \right] (S_w \phi)^2, \quad (11)$$

wherein the volume in which conduction occurs by cation exchange, $\alpha v_q Q_v$, is replaced by S_{wb} as determined from the transform shown in figure 2. C_{wf} and

C_{wb} are the conductivity of free and "bound" water, respectively.

Use of this log-based version of the dual-water model requires the following assumptions:

1. All the premises of the Waxman-Smits and dual-water model equations. These include the assumption in the Waxman-Smits and dual-water model equations that the conduction geometry (in this case it is presumed to be expressed by the Archie parameters f and i) is applied to *both* the pore volume in which surface conduction takes place and the water in the remaining pore volume;

2. The clay mineralogy and conduction geometry of associated sandstones and thick reference shale are identical and known;

3. S_{wb} from traditional V_{sh} indicators and the graphical transform gives an accurate estimate of the fractional volume in which surface conduction occurs;

4. The conductivity of water in a thick reference shale, calculated using Archie's equation by assuming a value for the porosity exponent m, is equal to the surface conductivity in a shale- or clay-bearing sandstone;

5. The Archie exponents n and m are both equal to two (although laboratory-derived values could be substituted, if available);

6. The conductivity of water deduced from the conductivity of a clean sandstone is the same as the water conductivity in the sandstone being analyzed.

Assumptions 1, 2, 3 and 4 are especially problematic. We have previously discussed the problems with assumption 1 in conjunction with the Waxman-Smits and dual-water model equations.

Assumption 2 requires estimating the volume of water in which surface conduction occurs by presuming that surface conduction takes place in shale disseminated in the pore system of the sandstone being analyzed. This is hardly axiomatic or self-evident. Shale-like material can be distributed in the pore system of a sandstone due to infiltration, bioturbation, deposition from turbidity currents and extreme deformation by slumping. But, there is no guarantee that this material has the same or even similar composition and electrical properties as a thick shale used as the shale reference.

In general, surface conductivity due to clay minerals of depositional or authigenic origin is more common than that due to disseminated shale. Estimating the volume and electrical properties of disseminated clay by using

similar properties in a reference shale has no rigorous basis and its success depends entirely upon coincidence. Log data alone provide little information on clay mineralogy, double layer volume and surface conductivity. V_{sh} determined by conventional means gives little information about the clay content of rocks. Thus we conclude that assumption 2 is likely to be invalid for most reservoirs.

The use of a thick shale as a reference for the properties of clay, or the shale-like contents of an associated sandstone, is further complicated by use of assumption 3 above. Even supposing the clay volume and mineralogy of the reference shale and the sandstone are the same, it is unlikely that their conduction geometry is identical. Shales are typically not 100 percent clay, and other shale constituents (commonly quartz) are instrumental in defining the pore geometry. The pore and surface conduction geometry of a shale determines its formation conductivity factor and Archie m values. These properties are not easily measured in the laboratory and generally have to be assigned by educated guessing. This amounts to assuming the conduction geometry of the reference shale. Making this assignment and then assuming that the conduction geometry of the reference shale is identical to the surface conduction geometry of material in a sandstone that has not been shown to be similar to the reference shale introduces uncalibrated uncertainty into the quantitative estimates made using the analysis.

The difficulty with log-based versions of the "dual-water model" is that they attempt to overcome an absence of ground truth, as determined from measurements taken on recovered core samples, by making an assumed equivalence between the clay/shale-like contents of a sandstone and a thick reference shale. To be valid, an exact equivalence in composition and pore geometry is required. Such an equivalence is unlikely, rendering assumption 4 equally unlikely. Having assumed such equivalence, shale properties are substituted for clay properties in a dual-water equation that assumes that the conduction geometry of the pore volume in which surface conduction occurs is identical to that of the remainder of the pore system. The questionable validity of this assumption introduces an unquantifiable degree of uncertainty into fluid saturation estimates made using the dual-water conductivity model with log-derived estimates of its adjustable parameters.

The Juhasz "Dual-Water Model"

Another modification of the Waxman-Smits saturation equation was proposed by Juhasz (1981), also with the goal of using well log data as a replacement for laboratory data and correlations between lab and log data.

Recall that Q_v in W-S theory is a concentration of exchangeable counterions in milliequivalents per unit porosity. Juhasz based his approach on a "normalized Q_v" Q_{vn},

$$Q_{vn} = \frac{Q_{vss}}{Q_{vsh}} \qquad (12)$$

where Q_{vss} is Q_v of the sandstone of interest and Q_{vsh} is the Q_v of an associated thick reference shale from the same sedimentary sequence as the sandstone.

Recognizing that "disseminated shale" in sandstones is in reality usually clay, Juhasz expressed Q_{vss} in terms of the volume of clay present in the sandstone V_{clss},

$$Q_{vss} = \frac{V_{clss}\,\rho_{cl}\,CEC_{cl}}{\phi}, \qquad (13)$$

where ρ_{cl} is the grain density of the dry clay, CEC_{cl} is the cation exchange capacity of the clay and ϕ is the porosity of the sandstone containing the clay. Q_{vsh} is also described by (13) using V_{clsh}, the volume of clay in the shale. Juhasz assumed that the clay in the sandstone is identical to that in the shale with the same density and CEC. Equation (12) can be rewritten in terms of clay volumes

$$Q_{vn} = \frac{Q_{vss}}{Q_{vsh}} = \frac{V_{clss}\,\rho_{cl}\,CEC_{cl}}{V_{clsh}\,\rho_{cl}\,CEC_{cl}} \cdot \frac{\phi_{sh}}{\phi_{ss}}$$

$$= \frac{V_{clss}}{V_{clsh}} \cdot \frac{\phi_{sh}}{\phi_{ss}} \qquad (14)$$

The neutron log responds to the hydrogen index of clay and is often used to estimate the volume of clay present in a sandstone or shale,

$$V_{cl} = \frac{\phi_n - \phi}{HI_{cl}}, \qquad (15)$$

where ϕ_n is the neutron porosity of the shale or sandstone and ϕ is the true porosity of the sandstone or shale, often estimated from a density or NMR log. The hydrogen index of the clay HI_{cl} is characteristic of the type of clay believed to be present in both shale and sandstone. Q_{vn} can be then be rewritten in terms of porosity from neutron and density logs (or any other accurate porosity log replacing the density log),

$$Q_{vn} = \left(\frac{\phi_{nss} - \phi_{ss}}{\phi_{nsh} - \phi_{sh}} \right) \cdot \frac{\phi_{sh}}{\phi_{ss}}. \qquad (16)$$

where ϕ_{nss} is the indicated neutron porosity response from a well log, and ϕ_{ss}, ϕ_{sh}, and ϕ_{nsh} are the end-member responses of the density and neutron logs in shale-free sand and "sand-free" shale, respectively. The ratio $(\phi_{nss} - \phi_{ss})/(\phi_{nsh} - \phi_{sh})$ is a conventional V_{sh} estimate, however in this case the meaning of V_{sh} is unconventional in that it refers only to the clay content and not the shale as a whole,

$$V_{shJ} \equiv \left(\frac{\phi_{nss} - \phi_{ss}}{\phi_{nsh} - \phi_{sh}}\right) = \frac{V_{clss}}{V_{clsh}}, \qquad (17)$$

distinguished with the "J" subscript to differentiate this V_{shJ} parameter from the conventional meaning of V_{sh}. The subscripts "$clss$" and "$clsh$" refer to clay in the sandstone and in shale, respectively. Following Juhasz, the V_{sh} nomenclature is retained and Q_{vn} in terms of V_{shJ} becomes

$$Q_{vn} = V_{shJ} \frac{\phi_{sh}}{\phi_{ss}}. \qquad (18)$$

Using (12), the Waxman-Smits clay conductivity term BQ_v becomes (where $Q_{vss} = Q_v$)

$$BQ_v = BQ_{vn} Q_{vsh} = B\left(V_{shJ} \frac{\phi_{sh}}{\phi}\right)Q_{vsh}. \qquad (19)$$

According to W-S theory, the apparent water conductivity of a shale C_{wash} comprises the sum of two terms: the formation water conductivity C_w and the conductivity of the shale due to clay cation exchange BQ_{vsh}.

$$C_{wash} = C_w + BQ_{vsh}. \qquad (20)$$

Using Archie's equation to express the conductivity of a shale,

$$C_{sh} = \phi_{sh}^{m*} C_{wash}. \qquad (21)$$

Equation (21) substituted in (20) gives an expression for the surface conductivity due to clay in shale,

$$BQ_{vsh} = C_{sh} \phi_{sh}^{-m*} - C_w. \qquad (22)$$

Using the definition of Q_{vn} (12) gives the following expression for the Waxman-Smits clay conductivity term BQ_v,

$$\begin{aligned} BQ_v &= Q_{vn}\left(C_{sh} \phi_{sh}^{-m*} - C_w\right) \\ &= \frac{V_{shJ} \phi_{sh}}{\phi}\left(C_{sh} \phi_{sh}^{-m*} - C_w\right). \end{aligned} \qquad (23)$$

Substitution of (23) for BQ_v in the Waxman-Smits saturation equation (3) gives the Juhasz saturation equation.

$$C_t = \left[\left(1 - \frac{V_{shJ} \phi_{sh}}{\phi_{ss} S_w}\right)C_w + \frac{V_{shJ} \phi_{sh}}{\phi_{ss} S_w}\phi_{sh}^{-m*}C_{sh}\right] S_w^{n*} \phi_{ss}^{m*}. \qquad (24)$$

The variables in this saturation equation, with the exception of $n*$, can all be estimated from log data using various cross-plotting techniques described by Juhasz (1981). The value of $n*$ can only be obtained from laboratory data, or assigned by educated guess.

Use of the Juhasz saturation equation requires the following assumptions:

1. The same assumptions made by Waxman and Smits (1968) and by Clavier, Coates and Dumanoir (1977, 1984), especially the assumption that the sandstone and shale have the same conduction geometry including the same values of the Waxman-Smits porosity exponent $m*$;

2. The clay mineralogy and conduction geometry of associated sandstones and thick reference shale are identical and known;

3. All equation parameters, except $n*$ can be determined from logs. $n*$ must be assumed or measured in the laboratory;

4. The neutron log in conjunction with another log or logs that give an accurate measure of porosity can be used to obtain a valid clay volume;

5. Q_{vn} is comparable to $\alpha v_q Q_v$ in the dual-water model and S_{wb} in the Best et al. (1978) equation and gives an accurate estimate of the volume of surface conduction in sandstone;

6. $(C_{sh} \phi_{sh}^{-m*} - C_w)$ gives the surface conductivity due to clay in shale;

7. The ratio of Q_v of the sandstone of interest to that of the associated reference shale is equal to the ratio of volumes of clay in each, normalized to their respective porosities.

The main problems with the Juhasz (1981) clay-bearing sand conductivity model arise from assumptions 1 and 2. The premise of Waxman and Smits, that the sand-

stone and shale have the same conduction geometry, is unlikely to be true for rocks. Further, at present there is no reliable method for accurately determining or verifying clay mineralogy from log data alone.

Comparison of clay-bearing sandstone models

Archie:
$$C_t = C_w f i$$

Waxman-Smits:
$$C_t = \left[C_w + \frac{BQ_v}{S_w} \right] f * i *$$

Dual-Water Model:
$$C_t = \left[\left(1 - \frac{\alpha v_q Q_v}{S_w} \right) C_w + \frac{\beta Q_v}{S_w} \right] f_0 i$$

Best, et al (1978):
$$C_t = \left[\left(1 - \frac{S_{wb}}{S_w} \right) C_{wf} + \frac{S_{wb}}{S_w} C_{wb} \right] (\phi S_w)^2$$

Juhasz:
$$C_t = \left[\left(1 - \frac{V_{shJ} \phi_{sh}}{\phi_{ss} S_w} \right) C_w + \frac{V_{shJ} \phi_{sh}}{\phi_{ss} S_w} \phi_{sh}^{-m*} C_{sh} \right] f * i *$$

Archie's equation is meant for use in clay-free sandstones, however it can be used equally well for clay-bearing rocks. Cation exchange causes f and particularly i to vary with porosity and saturation. Use of Archie's equation for interpretation of clay-bearing sandstones requires laboratory measurement of f and i in the porosity and saturation ranges of interest.

Archie's, Waxman-Smits' and the dual-water model equations have different formation conductivity factors and conductivity indices. Relationships between formation conductivity factors are listed below,

Archie:
$$f = \frac{C_0}{C_w}. \qquad (25)$$

Waxman-Smits:
$$f^* = \frac{f}{1 + BQ_v / C_w}. \qquad (26)$$

Dual-Water Model:
$$f_0 = \frac{f^*}{1 - \alpha v_q Q_v} = \frac{f /(1 + BQ_v / C_w)}{(1 - \alpha v_q Q_v)}. \qquad (27)$$

and the relationship between i and i^* is

$$i^* = i \left[\frac{f / f^*}{1 + \frac{BQ_v}{C_w S_w}} \right]. \qquad (28)$$

i^* is used in both Waxman-Smits and dual-water model equations.

The V_{sh}–based "modified dual-water model" equation uses ϕ^2 and S_w^2 for the Archie f and i parameters, and the Juhasz equation uses the Waxman-Smits parameters f^* and i^*.

Surface Conductivity Equations Summarized

The Waxman-Smits equation purports to account for the surface conductivity of clay minerals that complicate interpretation in clay-bearing sandstones, but W-S employs a flawed premise regarding clay spatial distribution and pore geometry and omits entirely the volumetric dependence. To get the volume of water unaffected by surface conductivity, the dual-water model introduces a pore volume correction to account for the volume associated with clay surfaces in which conduction by cation exchange occurs. The Juhasz equation attempts the same thing but uses data almost exclusively from well logs. The Best, et al. (1978) equation is an oversimplified dual-water-type approach. Each of these conductivity equations is founded upon implicit and/or explicit assumptions, some of which are difficult to justify. The premise that all these models share is the Waxman-Smits assumption that the same formation conductivity factor and conductivity index apply not only to the bulk pore water, but also to surface conduction on clays as well. This premise is prima facie false since it requires that each conduction mechanism occurs in the same volume with the same conduction geometry; i.e., all conducting phases have the same formation resistivity factor.

OTHER LOG-BASED V_{sh} SATURATION EQUATIONS

Poupon equation for inter-bedded sediments

Poupon, Loy, and Tixier (1954) published the first widely-used shaly sand equation based on volumetric weighting. Their equation specifically applies to interpreting the resistivity of interbedded shales and sands that are not completely resolved by the resistivity logging instrument used. In the 1950's wells tended to be vertical and perpendicular to more or less horizontally bedded sediments. Given this geometry, the beds re-

spond as a parallel resistor network. The current carried by each bed is determined by its conductivity and thickness. Each bed is assumed to have constant thickness within the volume of the measurement so that the thickness of each bed is also equivalent to its volume. The total conductivity measured is then the volume-weighted average of the conductivities of all the beds within the measurement volume; i.e.,

$$C_t = V_1C_1 + V_2C_2 + V_3C_3 + ..., \qquad (29)$$

where the numbered subscripts refer to individual beds. If the sand and shale beds within the measurement volume have characteristic conductivities C_{sd} and C_{sh}, then the total conductivity can be apportioned by the amounts of shale (V_{sh}) and sand ($V_{sd} = 1 - V_{sh}$),

$$C_t = V_{sh}C_{sh} + \left(1 - V_{sh}\right)C_{sd}. \qquad (30)$$

If shale volume and conductivities are known, there are no approximations in this equation. If the sandstone is hydrocarbon-bearing, then the sandstone conductivity can be expressed in terms of Archie's equation (for example),

$$C_t = V_{sh}C_{sh} + \left(1 - V_{sh}\right)C_w\phi^m S_w^n. \qquad (31)$$

Equation (31) is commonly referred to as the "Poupon equation" used for determining saturations in thin-bedded sands and shales.

Simandoux saturation equation

Simandoux (1963) studied "homogeneous mixtures of sorted sand and natural clay in various proportions." The clay and sand were mixed in a $CaCl_2$ solution to prevent ion exchange. No details of the methodology were given.

Despite the use of $CaCl_2$, Simandoux observed a contribution to the conductivity due to the clay that was independent of the "shape factor" of the porous medium (i.e., Archie's F) and depended only on the amount of clay in the mixture. This component of the observed conductivity conducted in parallel with the solution saturating the porous medium. Simandoux stated that "it is quite natural to assume that this term [the conduction component] represents directly the surface conductivity of the clay particles, which is added in parallel to the conductivity of the water in the pores." Based on this assumption he wrote a conductivity equation for the sand-shale-brine mixture,

$$C_t = C_t^* + [clay]C_{cl}, \qquad (32)$$

where C_t^* is the conductivity of the mixture exclusive of any clay contribution, "[clay]" is the clay concentration and C_{cl} is the conductivity of the clay component.

He also wrote

$$C_t^* = AS_w^2 + BS_w \qquad (33)$$

without detailed explanation. Substituting equation (33) in (32) gives

$$C_t = AS_w^2 + BS_w + [clay]C_{cl}, \qquad (34)$$

which is the equation Simandoux actually published.

Since 1963 equation (31) has evolved considerably, and is now written as

$$C_t = C_w f\, S_w^2 + V_{sh}C_{sh}. \qquad (35)$$

In spite of the differences between (31) or (34), and (35), equation (35) has become known as the "Simandoux equation."

Simandoux's equation (34), if applied to rocks, and equation (35) share common failings. The first terms are not volume weighted, and the second, clay-effect, terms are not corrected for pore-geometric effects (i.e., there is no formation conductivity factor). The lack of correction makes the clay-term too large, artificially reducing the water saturation one would calculate. Relying on the modified Simandoux equation for formation evaluation or reserve estimates results in over-estimating hydrocarbon reserves. This feature, while giving incorrect hydrocarbon-in-place results, makes the Simandoux equation very popular for equity determination.

Poupon-Leveaux or "Indonesian" Equation

The Poupon-Leveaux (1968) or "Indonesian equation" was developed to cope with evaluation problems for certain clay-rich formations in Indonesia. These formations were characterized by high clay content (40-to-90 percent) and high R_w (less than 40 kppm NaCl equivalent). Poupon and Leveaux conducted an empirical computer study of "several R_t - S_w equations" and obtained "the best overall results" with

$$\sqrt{C_t} = S_w^{n/2}\left(\phi^{m/2}\sqrt{C_w/a} + V_{cl}^{1-Vcl/2}\sqrt{C_{cl}}\right), \qquad (36)$$

where a is a fitting parameter with no physical interpretation. No theoretical basis for (36) is offered, however it does reduce to Archie's equation if $V_{cl} = 0$. No information is given about how to obtain either V_{cl} or C_{cl}.

The petrology and mineralogy of the Indonesian sediments that Poupon and Leveaux studied are not discussed. Although "clay" is referred to throughout their paper, neither the clay mineralogy nor clay properties is mentioned.

Since little detail is provided in Poupon's and Leveaux's rock description, their reported clay volumes cannot be interpreted without resorting to speculation.

Homogeneous sediments that fit the description of 40-to-90 percent clay are typically shale. Consider that the maximum sandstone porosity is about 30 percent. In a homogeneous rock, if percentages of clay were 40 percent or higher, as they are reported to be, the clay would be supporting sand grains. Thus the reported range of 40-to-90 percent clay might conceivably refer to sediments with sand or carbonate grains suspended in a clay matrix. However, such sediments would not have been economically interesting hydrocarbon reservoirs since Indonesia in the late 1960s had an abundance of high quality, easily producible hydrocarbon reservoirs.

Assuming Poupon and Leveaux were analyzing economically plausible reservoirs with a "clay" content of this magnitude, it is likely that Poupon and Leveaux were actually referring to shale instead of clay. To have an economically productive reservoir, it is most likely that the sediments that they were discussing were actually thin inter-bedded sands and shales. If this was the case, then there was no need to introduce the "Indonesian equation" since Poupon had already published the parallel conductor model for inter-bedded sand and shale in 1954 (Poupon, Loy, and Tixier, 1954).

The "Indonesian equation" is an ad hoc empirical development without any theoretical basis. It was devised to give 100 percent brine saturation in clay/shale-bearing water sands and default to Archie's equation in clean sands. Since it was developed in the late 1960's, it may have been more due to an opportunity to experiment with computers, which were then new in the industry, than to solve a possibly complex formation evaluation problem.

If equation (33) is squared, three terms result:

$$C_t = C_w \phi^m S_w^n \qquad \text{(Term 1)}$$

$$+ C_{cl} V_{cl}^{2-Vcl} S_w^n \qquad \text{(Term 2)} \qquad (37)$$

$$+ 2 S_w^n \sqrt{V_{cl}^{2-Vcl} C_{cl} \phi^m C_w} \qquad \text{(Term 3)}$$

Term 1 is Archie's equation for clean sands. Term 2 seems to be a similar equation for the "clay" component with a formation conductivity factor term equal to V_{cl}^{2-Vcl}. No justification was presented for this, and it seems to have been a rather arbitrary addition. Term 2 of the equation also contains a saturation term S_w^n. The brine saturation, as given, is that of the whole rock, not that of the "clay," although it is present in the "clay" term. One would typically assume that the brine saturation of the "clay" (or shale) is 100 percent not requiring S_w in Term 2.

The form of equation (34) is similar to a type of fitting equation used when looking for structure in a two dimensional surface. The first two terms of the fitting equation account for variation in the two variables being described and the third term (the square-root of the product of the first two) describes the interaction of the variables. According to Poupon and Leveaux (1968), H. G. Doll had suggested in the 1940's that "there should be such a term in the conductivity equation to account for the cross-linkage [interaction] of the two networks [Archie and "clay" terms]." Equation (34) seems to have been an attempt to introduce such a cross-linkage. No justification other than Doll's opinion was given for including Term 3. To our knowledge, no evidence has been presented since the Indonesian equation was published that supports the need for such an interaction term.

The arbitrary nature of the Indonesian equation along with the "clay"-shale confusion, the nebulous shale formation factor along with the inclusion of S_w^n in the shale term (Term 2) and the presence of the interaction term make it essentially impossible to interpret the results of its use for calculating water saturation, unless V_{cl} is zero. Despite this, some analysts occasionally obtain "reasonable" results using the Indonesian equation. This simply underscores the oil-field adage that "anything will work somewhere."

A comprehensive "shaly sand" saturation equation

A comprehensive "shaly sand" equation must account for conductivity due to thin shale interbeds as well as conductivity due to clay contained in sandstone. Poupon's equation (equation (31)) correctly describes the conductivity of sand-shale interbeds and can serve as the starting point for a rigorous description of parallel conduction models comprising shale layers laminated with clay-bearing sandstones.

The conductivity of sandstone containing clay must include both the effects of conduction on clay surfaces by cation exchange and conduction within the pore space. These two sources of conduction must be con-

sidered separately since each has a separate conduction volume and geometry. The volume of the double layer on clay surfaces in which conduction by cation exchange occurs ϕ_{dl} and the volume of the inter-granular pore space ϕ_p constitute the entire pore volume ϕ such that $\phi_{dl} + \phi_p = \phi$. The electrical effects of the conduction geometry are embodied in the electrical efficiencies (Herrick and Kennedy; 1993, 1994, 2009) of each conducting volume. The conduction equation describing the conductivity of a "shaly sand" is the sum of the components:

$$C_t = V_{sh}C_{sh} + (1-V_{sh})(C_w S_{wp}\phi_p e_{tp}E_{0p} + C_{dl}\phi_{dl}E_{dl}) \quad (38)$$

If the conduction volumes and geometries are combined in terms of the formation conductivity factor and the conductivity index, (38) becomes

$$C_t = V_{sh}C_{sh} + (1-V_{sh})(C_w f_p i_p + C_{dl}f_{dl}) \quad (39)$$

The formation conductivity factor and conductivity index of the clay double layer are difficult to determine, experimentally or otherwise, and since $C_s = C_{dl}\,f_{dl}$, equation (39) is better expressed as

$$C_t = V_{sh}C_{sh} + (1-V_{sh})(C_w f_p i_p + C_s). \quad (40)$$

C_s can be evaluated in the laboratory with a C_0-C_w measurement (Figure 1). It might also be estimated from log data if a water saturated sandstone section is present, the connate brine conductivity is known, and the mud filtrate has a significantly different conductivity than the formation water and completely saturates the invaded zone. Then values of C_0 and C_w for both the flushed zone and the formation are known and a two-point C_0-C_w plot can be constructed. The C_0 intercept at $C_w = 0$ is an estimate of C_s as depicted in Figure 1.

Hydrocarbons will displace brine from larger pores in response to capillary effects. However, micropores tend to remain brine-saturated if the micropore surfaces are hydrophilic, as would typically be the case. If a continuous micropore system is present (e.g., chlorite grain coatings), then an additional conduction volume is present in addition to the macropore system and the double layer on clay surfaces. The assumption can reasonably be made that a continuous micropore system would conduct in parallel with the other conducting components and can be included in the conductivity equation,

$$C_t = V_{sh}C_{sh} + (1-V_{sh})(C_w f_p i_p + C_s + C_{mp}), \quad (41)$$

where the subscript "mp" signifies the continuous micropore system which has its own conduction geometry

so that $C_{mp} = f_{mp}\,C_w = E_{0mp}\,\phi_{mp}\,C_w$. Or equation (41) can also be expressed using an Archie model,

$$C_t = V_{sh}C_{sh} + (1-V_{sh})(C_w \phi_p^{m_p} S_w^{n_p} + C_s + C_w \phi_{mp}^{m_{mp}}). \quad (42)$$

Equation (42) can be considered a generalized "shaly sand" equation, including conduction due to unresolved shale interbeds, also including clay-bearing sandstone beds with conduction due to surface conductivity on clay, and due to a continuous clay and/or other micropore system.

Another approach to a "shaly sand" saturation equation that may be simpler than experimentally evaluating all the parameters of equation (42) is a purely empirical approach that is still based on conduction physics. We know that brine content is related to resistivity in a reciprocal sense. To simplify interpretation it is better to establish relationships using conductivity thereby avoiding the necessity of working in log-log space. Then, in the laboratory, measure conductivity as a function of brine content and fit the results with the simplest equation that suitably minimizes the deviations between observations and predictions over the brine content range of interest. The result is a saturation equation that is suitable to that same rock-type throughout a reservoir. Other relationships must be determined for other rock-types. Relying on published saturation equations is unnecessary and can result in inaccurate estimates of hydrocarbon pore volume.

We also know that the formation conductivity factor is the product of volumetric and geometric variables, porosity and electrical efficiency. The formation factor is therefore not uniquely related to either. Many different values of porosity and efficiency can give the same formation factor. To improve interpretative capability we recommend separating the formation factor into its volumetric and geometric components.

SUMMARY AND CONCLUDING REMARKS

The term "shaly sand" is appropriate for sandstones with thin shale inter-beds or sandstones containing shale clasts. Other uses of the term are frequently misleading or wrong. It is commonly used to refer to sandstones containing clay of authigenic origin which is not "shale" by any definition. "Shale" refers to a particular type of rock that is typically fine-grained and fissile. Clay, on the other hand, refers to a class of phyllosilicate minerals that typically have a very small crystal or grain size. Clay minerals are a common detrital component of shales, averaging about 40 percent.

"Shaly sands" are often categorized as possessing "structural shale," "laminated shale," or "dispersed

shale." There are no published saturation equations for sandstones containing "structural shale," i.e. sandstones with shale clasts. Laminated shale is described by Poupon's equation (Poupon, Loy and Tixier, 1954) as the volume-weighted average of the inter-bedded shale and sandstone conductivities. Sandstones with dispersed shale, in the sense that sandstone contains true shale similar in all respects to that in associated shale beds, are likely to be rare. Examples might be debris flows or highly bioturbated sand/shale.

There are a variety of saturation equations purported to be applicable to sandstones that contain dispersed shale. These equations rely on estimating the volume of shale in sandstone using log responses and assuming that the properties of shale in the sandstone are identical to those of an associated shale bed thick enough to provide unambiguous log data. Most sandstones described as containing dispersed shale do not, in fact, contain shale, but have authigenic clay as grain coatings or in the pore space. The properties of an associated shale bed give no information about the properties of authigenic clay in clay-bearing sandstones. In addition, dispersed shale saturation equations such as the Poupon's and Leveaux's (1968), or the Indonesian equation, or that attributed to Simandoux (1963), are empirical having no theoretical basis. The quality of the assumptions required to use these dispersed shale saturation equations make them unsupportable from a scientific point of view.

Saturation equations that describe the conductivity of sandstones containing authigenic clay such as those of Waxman and Smits (1968), Clavier, et al. (1977) and Juhasz (1981) have some theoretical background, but are also based on the flawed premise that the same conduction geometry, as embodied in the formation resistivity factor, pertains to conduction both by brine in the pore system and by cation exchange on clay surfaces. The volumes in which these two conduction mechanisms occur are mutually exclusive, one being in the pore and the other on clay surfaces. The geometries of the clay surface and pore space are quite different. The premise that surface conduction and conduction in the pore space have the same formation resistivity factor (or electrical efficiency) is simply false.

Care must be taken when using published saturation equations for "shaly sands" since, with the exception of Poupon's (1954) equation for inter-bedded shale and sand, all are based on flawed assumptions.

Shaly sand and dispersed shale equations arose to meet a pressing need; however, there has been little in the way of technical critiques of the various methods. Consequently, a generation of formation evaluators has used these models based on faith in prior authority, and the need for a practical interpretation method. There is no doubt that these models will continue to be used unless and until more rational alternatives become available.

History, as reviewed in this article, teaches that it is not easy to characterize the conductivity components of a "dispersed-clay", or a "shaly-sand", system. What can be said about the conditions that a *proper* parallel conduction model must satisfy? The following requirements must be met: (1) each rock-type included in the analysis (e.g. thin beds of sandstone and shale) must be volume-weighted, and the weights must sum to unity; (2) each porosity component of each rock-type must allow for separate pore geometrical factors because the porosity components occupy different regions of space and can be expected to have independent—if perhaps complementary—geometry; (3) the total pore space of each rock-type must be apportioned among its porosity components, and must sum to the total porosity; (4) finally, water saturation must be apportioned among the porosity components, and the sum of water content and hydrocarbon content must equal porosity.

Any model that resolves bulk rock conductivity into a pure linear combination of conductivity factors is defective in some way unless all the conditions listed above can be met. This does not preclude models that are not of the purely volume-weighted parallel conduction type. However, other types of models will have to satisfy their own sets of restrictions, and from a scientific point of view these should be identified in advance of using the model, so that a model's strengths and deficiencies will be fully known from the beginning.

The purpose of this article has been to analyze and critique the most commonly used "shaly sand" saturation equations. On the other hand, perhaps the most valuable lesson to be learned is to have a healthy skepticism regarding popularly used equations and procedures. We should instead seek to discover and carefully evaluate their underlying assumptions, rejecting those without rigorous scientific justification.

We think it is likely that we petrophysicists in general have overlooked the most important lesson to be learned from Archie's saturation equation, and that is how he achieved it. Archie arrived at an empirically determined equation that suitably described the relationship he was looking for without preconception of what it should be; he obtained a satisfactory fitting equation using the curve fitting techniques that were readily available in his time (i.e., log-log graph paper). Times have changed. The universal availability of powerful electronic computers permits us to easily and rou-

tinely apply sophisticated curve-fitting technology, but our methodology should be Archie's: find a suitable relationship and use it. This is a purely empirical approach exemplified by the method Archie used. This approach, however, can be improved on by basing the fitting equation on established principles of physics, thereby endowing our empirically determined adjustable parameters with unambiguous physical meaning. The "best" fitting equation is one that suitably minimizes the errors in its predictions, is the simplest if there is more than one to choose from, and is based on physics.

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BIOGRAPHIES

David C. Herrick, is currently semi-retired and consulting. He was trained in chemistry and geochemistry at Indiana University (B.S.) and Penn State (Ph.D.). Dave has conducted research, training and technical service during his thirty years in the petroleum industry for Conoco, Amoco, Mobil and Baker Hughes in the areas of geochemistry, petrology and petrophysics. He has recently been Senior Petrophysical Advisor and Chief Petrophysicist in the Houston Technology Center of Baker Hughes Inc. Dave has given over fifty presentations and schools on petrophysics, resistivity interpretation, the meaning of porosity terminology and the impact of pore geometry on permeability and conductivity.

W. David Kennedy is a Senior Research Advisor in Baker Hughes Inteq's Strategic Technology and Advanced Research group in the BHI Houston Technology Center. Dave trained in physics at Georgia Tech and U. T. Dallas, and geoscience at U. T. Dallas, with fur-

ther training at U. C. Berkeley; and was employed serially by Schlumberger, Arco, Lockheed, Mobil, and ExxonMobil, and most recently at Baker Hughes Inteq. David stayed with ExxonMobil for 20 years to prove to his mother that he could hold a job. Having convinced his mother, David retired from ExxonMobil in 2007. He is coinventor and inventor on six patents and coauthor or author 30 or so publications, and is not quite done yet. Along the way, Dave has edited the SPWLA journal, *Petrophysics*, for four years, and served the SPWLA as VP Publications during that time.