GEOPHYSICS, VOL. 59, NO. 6 (JUNE 1994); P. 918-927, 8 FIGS.

Electrical efficiency-A pore geometric theory for interpreting the electrical properties of reservoir rocks

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ABSTRACT

Knowledge of the way pore geometry affects the conductivity of rocks is important for the evaluation of sedimentary rocks for hydrocarbon potential. Virtually all formation evaluation from logs, however, uses the empirical Archie equation that has never been given a satisfactory physical basis. As an alternative, we have formulated a new model based on the efficiency with which water-bearing rocks conduct electric currents as determined by the conducting phase geometry. Electric current densities are not uniform throughout the conducting phase in the pore system of a rock. Numerical model studies show high current density in pore throats and low current density in nearly stagnant volumes in isolated parts of the pore system. The electrical efficiency of a rock is the ratio of the average power developed in all water-bearing parts of the pore system to the power developed in a straight tube with the same length and water volume as the rock. It is inherently independent of the bulk volume of water in the rock and determined only by the nonuniform current distribution caused by pore geometry and hydrocarbons in the pore system. Electrical efficiency can be calculated a priori given the geometrical distribution of the conducting phase. Empirical relationships between electrical efficiency and water content can be used to calculate hydrocarbon saturations from log data.

INTRODUCTION

Relating the hydrocarbon saturation of reservoir rocks to their bulk electrical conductivity is among the fundamental problems of log analysis. The correlation of high resistivity anomalies recorded on well logs with producing horizons was recognized about the time the first logs were recorded (Schlumberger et al., 1934). A quantitative empirical relationship between resistivity and hydrocarbon saturation was first achieved by Archie (1942), and is now known as Archie's "law." It remains the main tool for saturation estimates from resistivity measurements despite its known shortcomings (Guyod, 1944; Patnode and Wyllie, 1950; Wyllie and Rose, 1950; Winsauer et al., 1952; Keller, 1953; Poupon et al., 1954; Atkins and Smith, 1961; Carothers, 1968; Ransom, 1984; Focke and Munn, 1987; Givens, 1987). According to Archie's law $C_t = C_w S_w^n \phi^m$, where C_t is the bulk conductivity of the rock, C_w is the conductivity of the conducting phase (brine), ϕ is the porosity, S_w is the fractional water saturation, and m and n are empirically determined constant parameters. The physical meaning of C_t, C_w, ϕ, and S_w are obvious from their definitions; however, attempts to attribute physical significance to m and n have been unsatisfying. A new approach that we have called electrical efficiency theory (EET) explains the low-frequency bulk electrical properties of rocks in terms of the geometrical configuration of the brine phase that supports the flow of current. The electrical efficiency of a rock can be calculated from first principles if a complete description of the conducting phase geometry is known. For real rocks, such a description is intractable and parameters in EET must be empirically determined; however, EET provides physical insight into the significance of laboratory-determined parameters.

THE ELECTRICAL EFFICIENCY MODEL

A sample of reservoir rock can be considered as a volume containing a mixture of a conductive phase and one or more nonconductive phases. The conductive phase is primarily brine but might also have conductive mineral constituents; the nonconductive phases consist of rock-forming mineral grains such as calcite or quartz and any nonconductive fluids such as oil or gas that may be in the pore system. The

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conductive phase is distributed throughout the rock volume in a complicated network of pores and their interconnecting throats. We shall discuss a model applicable to the case where such brine- and hydrocarbon-filled networks give rise to homogeneous media with isotropic electrical properties; however, the model is equally applicable to inhomogeneous, anisotropic media by the replacement of the scalar conductivities with their tensor representations.

The development of our model uses the comparison of rocks to an idealized standard. The standard of comparison for the rock is imagined as a nonconducting material that has the same external dimensions as the rock sample (Figure 1). A straight cylindrical hole extends from end-to-end, oriented in the direction of the applied electric field. The volume of the cylindrical hole is equal to the volume of brine in the rock, and the hole is completely filled with the same brine as contained in the rock. This tubular geometric configuration maximizes the conductivity. The standard tube can be viewed as a model rock with the simplest possible pore system, a single linear capillary. The difference in electrical properties between the standard tube and the rock sample arise from the greater complexity of the distribution of brine in the rock.

The conductivity C_s of the standard tube, taken as a whole, is given by $C_s = C_w v_w$, where C_w is the bulk conductivity of the water in the tube and v_w is the fractional volume occupied by the tube. This volume fraction can be parameterized in terms of the porosity ϕ and water saturation S_w of the rock as $v_w = S_w \phi$, the water content of the rock sample; thus

$$C_s = C_w S_w \phi. \tag{1}$$

Similarly, the bulk conductivity of a rock sample devoid of conductive minerals (e.g., clays) is proportional to C,, S,, and ϕ; nonetheless, $C_t \neq C_s$ and the conductivity of the rock sample can be expressed as

$$C_t = C_w S_w \phi E, \tag{2}$$

where E is a proportionality factor appropriate for the rock sample in question. Since the geometrical distribution is the only difference in the conducting phase between the sample and the standard tube, E can be considered a geometrical parameter characteristic of the distribution of the conducting phase in the rock. The effects of tortuous conduction paths,

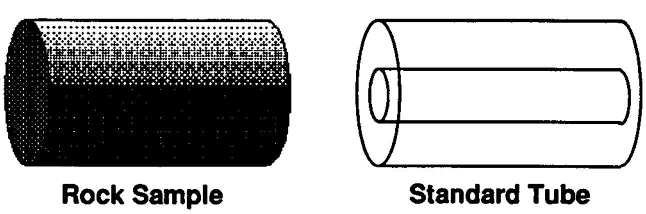

FIG. 1. A useful standard of comparison for the conductivity of a rock sample is obtained by reconfiguring the conducting phase into a tube. In this configuration the conducting phase has maximum conductance and the tube model has maximum conductivity. The ratio of (a) rock conductivity to (b) tube conductivity is defined as the electrical efficiency E of the rock.

pore-throat size distribution, pore connectivity, nonconducting fluid distribution, and other geometric characteristics that affect the conductivity of the rock are contained in E. If minerals that contribute to the rock conductivity are present (e.g., clays or pyrite), then their contribution is also contained in E, but only a brine conducting phase is considered here. Substitution of equation (1) into equation (2) gives $C_t = C_s E$ or

$$E = \frac{C_t}{C_s} = \frac{C_t}{C_w S_w \phi}. \tag{3}$$

From equation (3) it is clear that E is the ratio of the conductivity of the rock sample to the conductivity of the standard tube. Since C_t could be zero and since $C_t \leq C_s$, then $0 \leq E \leq 1$. E thus describes the efficiency with which brine distributed in the pore system of a rock sample conducts electric current compared to the maximum amount of current it could conduct in an optimum tubular geometric arrangement. The geometrical parameter E is therefore referred to as the *electrical efficiency* .

THEORETICAL BASIS FOR ELECTRICAL EFFICIENCY

The electrical efficiency in equation (3) is given in terms of the bulk conductivity C_t of a rock. This parameter provides a macroscopic description of the influence of the rock on average current as a result of an applied potential; however, no component of the rock/pore system has conductivity C_t, and the currents flowing at various places in the rock are not simply related to the average current. If E is to explicitly account for pore geometry, then it must be connected to the detailed distribution of electric current in the pore system, i.e., E should be expressible in terms of a microscopic description of the pore system. Such a description can be motivated by noting that current density varies from point-to-point in a pore system, and that the influence of pore geometry on current density arises from the interaction of the applied potential and the configuration of the conducting phase, thus determining the bulk properties of the rock. The current density can be modeled if the geometry of the conducting phase is known. Except at a source, the divergence of the current density must vanish by charge conservation, i.e., V l J = 0. Ohm's law is J = σ**E,** where **J** is current density, E is electric field, and $\sigma = \sigma(x, y, z)$, the spatial configuration of point conductivity. The electric field can in turn be expressed as the gradient of a scalar potential Φ: $E = -\nabla\Phi$. Together these relations give the differential equation satisfied by the potential in the rock,

$$\nabla \cdot \sigma \nabla \Phi = 0. \tag{4}$$

To a good approximation $\sigma = 0$ except in the conducting phase. Thus, through the potential distribution and Ohm's law, the geometry of the conducting phase (or pore geometry for brine-saturated rocks) is coupled to the current density J. Except in the case of some very simple geometries, no closed form expression linking the potential to the geometrical parameters of the medium is possible. However, for geometries representative of rock pore systems, numerical methods can be applied to solve the potential equation (4), and the numerical solutions contain pore geometry implicitly. For a given applied potential difference $v(= \nabla\Phi)$, once

J is determined, the current i can be computed. Conductance is obtained from i/u, and bulk conductivity C_t is obtained using conductance and sample dimensions. E can then be computed from equation (3).

While equation (3) focuses attention on the relationship of electrical efficiency to the bulk conductivity of a sample, it provides little insight into the connection between the bulk electrical properties and the electrical behavior of the conducting phase in the pore system itself. To shift the emphasis to the electrical behavior within the pores, the electrical efficiency of the medium can be expressed conveniently in terms of the power developed in the conducting phase. Consider the power developed when an electric current is induced to flow axially through a cylindrical rock sample occupying volume V. The total power P developed will be given by

$$P = \int_V \mathbf{J}(\mathbf{r}') \cdot \mathbf{E}(\mathbf{r}')\, dv', \tag{5}$$

where $\mathbf{r}'$ is the field point in V. The volume of integration is the entire sample; however, since $\mathbf{J} = 0$ outside the conducting phase, the integrand is nonzero only over the conducting phase in the pore space. For a given potential difference across such a sample, an upper bound on the power developed will be attained when the conductive phase is in its most efficient configuration, namely a tube parallel to **E.** For example, in a resistor $P = v^2/r$, so for a fixed voltage v, P is a maximum when resistance r is a minimum, i.e., when the conductor represented by 1/r is most efficient.

The power developed in a specific rock sample P_t can be expressed in terms of the maximum power P_s that could be developed if the conducting phase were in its most efficient configuration (a straight tube), i.e., $P_t = EP_s$, where P_t and P_s are computed using equation (5). In terms of power, electrical efficiency is defined as $E = P_t/P_s$. Thus, if **E** and **J** are known for any pore system, then the electrical efficiency can be calculated independently of equation (3). For a rock supporting a current density $\mathbf{J}_{rock}$ in response to an electric field $\mathbf{E}_{rock}$,

$$E = \frac{P_t}{P_s} = \frac{\int_{V_{rock}} \mathbf{E}_{rock} \cdot \mathbf{J}_{rock}\, dv'}{\int_{V_{tube}} \mathbf{E}_{tube} \cdot \mathbf{J}_{tube}\, dv'}. \tag{6}$$

The denominator of equation (6) represents the power in a standard tube carrying a current i in response to an applied voltage v. $|\mathbf{J}_{tube}| = i/a$ and $|\mathbf{E}_{tube}| = v/\ell$, where a is the cross-sectional area and ℓ the length of the conducting tube. In the standard tube corresponding to a given porosity, the scalar product is a constant; hence $\int_{V_{tube}} \mathbf{J}_{tube} \cdot \mathbf{E}_{tube}\, dv' = iv = P_s$. The volume integral is evaluated only where $\mathbf{J}_{tube} \neq 0$. The value of the numerator depends on the pore geometry since $\mathbf{J}_{rock} = 0$ except in the conductive phase in the pore system. In the conductive phase $\mathbf{J}_{rock}(x, y, z)$ follows the potential gradients established by the interaction of the applied potential and the configuration of conductive and nonconductive phases. For a specified water content each such configuration has a unique distribution of **J.** Further, substitution of $\mathbf{J}_{rock} = \sigma_w \mathbf{E}_{rock}$ and $\mathbf{J}_{tube} = \sigma_w \mathbf{E}_{tube}$ in equation (6) demonstrates that E is independent of σ_w.

It may not be obvious that the definitions of electrical efficiency given by equations (3) and (6) are equivalent. To show that $E = P_t/P_s$ is consistent with the definition in equation (3), consider the relationship of the axial bulk conductivity C of a cylindrical sample of length ℓ and area A to its resistance r: $C = (\ell/A)/r$. The power developed in a cylindrical resistance when a voltage v is applied between its ends is $P = v^2/r$. Combining these relations we have $P_t/P_s = (v^2/r_t)/(v^2/r_s) = C_t/C_s$ demonstrating that the definitions are equivalent for this example. Although the efficiency parameter is considered to be associated with the bulk properties of a rock, in the examination of current densities in various pore systems it will be noted that some volumes of the pore system conduct less current than other, similar volumes, i.e., some pores conduct more efficiently than others. To quantify these observations, a useful concept of *local* efficiency, or the efficiency at a point, ε, can be defined using a limiting process, viz.,

$$\lim_{\Delta V_i \to 0} \int_{\Delta V_{i,\, rock}} \mathbf{J} \cdot \mathbf{E}\, dv' = \varepsilon_i \lim_{\Delta V_j \to 0} \int_{\Delta V_{j,\, tube}} \mathbf{J} \cdot \mathbf{E}\, dv', \tag{7}$$

where $\Delta V_{i,\, rock}$ and $\Delta V_{j,\, tube}$ are the ith andjth incremental volumes in the rock and the standard tube, respectively, and are chosen to have the same shape. The ratio of the integral on the left to the integral on the right, presumed to remain finite in the limit, is defined as the local efficiency of the point surrounded by ΔV_i as ΔV_i vanishes. (Note, however, that the integral on the right does not depend on the index but only on the size of $\Delta V_j = \Delta V_i$; the value of this integral $= P_s \Delta V_i$.)

The introduction of local efficiency suggests that the global (or bulk) efficiency of a sample can be represented as the sum of its local efficiencies. This representation is developed from equation (7). If the limiting process is terminated at some finite volume small enough so that **J** is approximately constant, then the power developed in the ith such volume is $\Delta P_{t\ i} = \varepsilon_i \Delta P_{s.i}$. Note that $\Delta P_{s\ i}$ is the same constant for each AV in the standard tube and that $\varepsilon_i > 1$ is possible since the current densities in some pore throats might exceed the current density in the standard tube. Summing over N such volumes in the rock sample gives $P_t = \Delta P_{s,i} \sum_{i=1}^{N} \varepsilon_i$. Multiplying the right side by $1 = N/N$, and noting that $P_s = N\Delta P_{s\ i}$, gives $P_t = P_s \sum_{i=1}^{N} \varepsilon_i/N$. Comparison with equation (6) shows that

$$E = \frac{1}{N}\left(\sum_{i=1}^{N} \varepsilon_i\right). \tag{8}$$

Thus, the global electrical efficiency E is the average of the local efficiencies. Equation (8) provides a connection between the macroscopic bulk efficiency E, which is measured in the laboratory using the definition in equation (3), and its microscopic local counterpart ε which depends directly on current density in the individual pores of the system and is convenient for numerical modeling using equation (7).

EXAMPLES

Some simple examples are illuminating. Figure 2 illustrates two samples with the same external dimensions: a model with a single tortuous conducting tube and its equivalent standard tube. The conducting phase in the standard tube has a cross-sectional area A and length ℓ. The tortuous conducting tube in the model has length ℓ' related to the length of the conductor in the standard tube by a factor $\tau(\ell' = \tau\ell)$, while its area A' is decreased by the same factor ($A' = A/\tau$), maintaining the volume constant. Using equation (3). the ratio of conductances (c_{model}/c_s) is

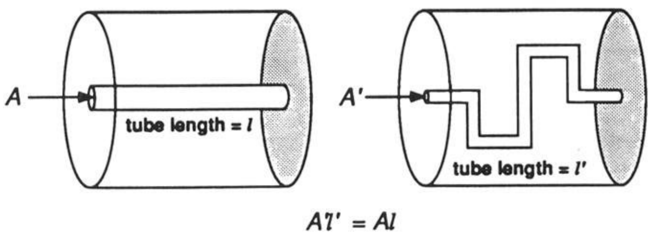

FIG. 2. The tortuous tube pore system is a poor representation of the complex pore geometries characteristic of rocks. It is, however, a model for which the electrical efficiency can be explicitly related to easily identified geometric parameters.

$$E = \frac{c_{model}}{c_s} = \frac{C_w A'/\ell'}{C_w A/\ell} = \frac{C_w \dfrac{A/\tau}{\ell\tau}}{C_w \dfrac{A}{\ell}} = \frac{\mathrm{I}}{\tau^2}. \tag{9}$$

The parameter τ, the streamline tortuosity, or ratio of the current streamline lengths between the model and the standard tube (ℓ'/ℓ), is a function only of the geometry of the tubes. Hence, the efficiency of this simple model is seen to depend solely and explicitly on the single geometrical parameter τ.

Figure 3a illustrates a 2-D tube of width A with an adjustable aperture, simulating a pore throat, that can impede the flow of electrical current to varying degrees as the aperture is adjusted from fully open to fully closed. The conductance of the tube with the aperture partially opened to width a (i.e., $c_{aperture}$) is greater than the conductance of a simple tube of the same width a (i.e., c_a) but less than the conductance of the equivalent tube (i.e., c_s) which does not have the aperture. Thus,

$$E = \frac{c_{aperture}}{c_s} \geq \frac{c_a}{c_s} = \frac{C_w a/\ell}{C_w A/\ell} = \frac{a}{A} \tag{10}$$

gives a lower bound for the electrical efficiency of this 2-D model (Figure 3b). The lower bound is clearly a geometrical limit determined by the size of the aperture. In this case the locus occupied by the curve in E - a/A space is a function of geometrical parameters such as the shape, thickness, and location of the aperture that cannot be parameterized readily in a formula. The specific curve for the model (Figure 3a), shown in Figure 3b, is obtained from a numerical simulation and is plotted as a function of the ratio a/A for unit length ℓ.

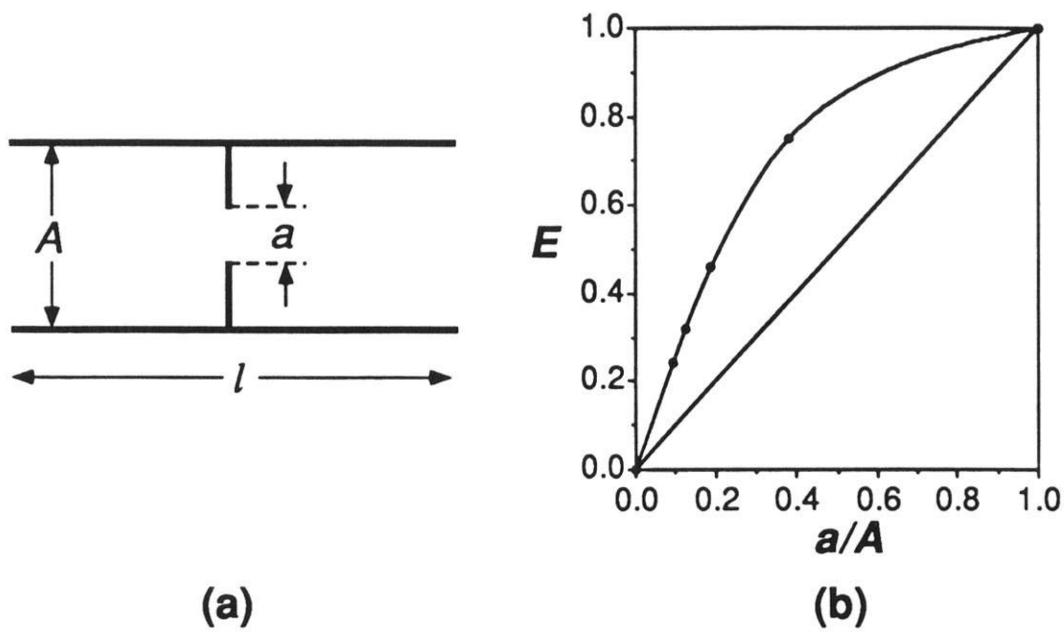

FIG. 3. Seemingly simple, the obstructed tube model of a pore is already too complex to permit the development of a closed-form formula for the electrical efficiency in terms of its geometrical parameters. However, a numerical simulation parameterizes E in terms of the geometrical variables a/A, for a given length e.

Note that for a 3-D tube, the cross-sectional area replaces width, and the lower bound becomes $E \geq (r/R)^2$ if r and R are the radii of the model and the standard tube, respectively.

The relationship between current density distribution and electrical efficiency is more realistically illustrated in a portion of a rectangular 2-D array of circular grains (Figure 4). The pore space between the grains is filled with conductive brine. An electric field is applied so that current flows horizontally through the model. The curved lines traversing the model from left to right are current streamlines whose locations were determined by numerically solving the field equations for the model geometry using equation (4). Each pair of contiguous streamlines encloses 10 percent of the current flowing through the model. From inspection of the streamlines, it is clear that water in the pore throats (location ***a***) and in the centers of the pores between throats, supports a relatively high current density, has a high local efficiency, and contributes significantly to the global electrical efficiency of the model. On the other hand, water between the grains (location ***b***) is electrically stagnant, carries little current, and decreases the electrical efficiency of the model. As a result, electrical measurements are insensitive to the composition of fluids in the electrically stagnant parts of the pore system. For example, if oil were to replace water in location ***b*** in Figure 4, a resistivity measurement would indicate that the model is almost completely water saturated. The local efficiencies required to calculate the global efficiency of even this simple model of a pore cannot be determined analytically; E has been determined by numerical methods to be $E = 0.59$.

Analysis of the pore geometry in Figure 4, as well as other simple geometries, indicates that local and global electrical efficiencies are determined by the degree to which the pore space in a rock is connected by pore throats. For example, rocks having large, poorly connected pores, like some oomoldic limestones, have low electrical efficiencies because of a large fraction of electrically stagnant pore space. Others, like clean, well-sorted and well-rounded sandstones, have less electrically stagnant pore volume and consequently have higher electrical efficiencies. In general, rocks with relatively uniform current densities in all parts of the conductive phase have high electrical efficiencies compared to rocks with large variations in current density that tend to have electrically stagnant volumes.

PARAMETRIC DESCRIPTION OF ROCKS

The alternative to modeling real rocks is to study simple pore models representative of specific pore geometries found in rocks using numerical modeling techniques, such as finite differences (used for our simulations) or diffusion simulation (Schwartz and Banavar, 1989).

The unique reconstruction of even a simple pore system from a parametric description alone is not possible; however, characteristic parameters can be identified that adequately describe its bulk electrical properties. A series of numerical experiments was conducted on models similar to those in Figures 4 and 5. The important parameters include pore-throat size, size and shape of pores relative to their throats, the length of current paths through a rock as determined by pore-throat sizes, and the paths by which pores are connected.

The electrical properties of a pore can be approximately represented by a tube having the length of the pore and cross-sectional area defined by the cross-sectional area of its pore throats, a "throat tube." This model can be refined by

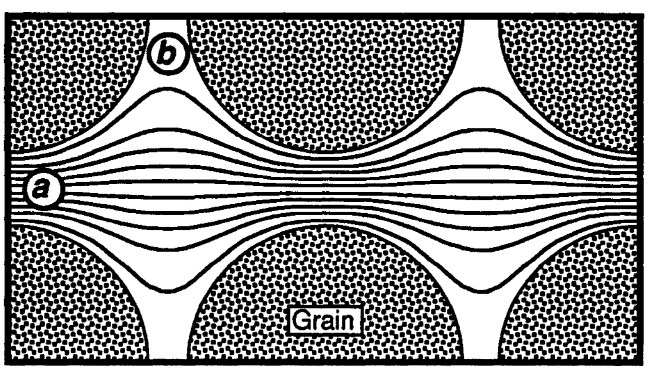

FIG. 4. The electrical efficiency of a rock depends on the contribution of all conductive components of the pore space in the rock to the overall electrical efficiency. Water in location ***a*** carries a relatively high current and makes a large contribution to the efficiency. Water at ***b*** carries little current and contributes little to the global efficiency of the rock.

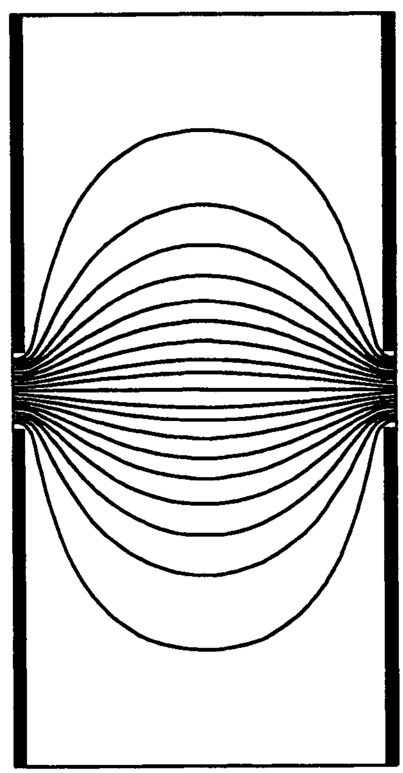

FIG. 5. A simple model is shown for the study of the influence of pore-throat size to pore-body size for a pore of unit length. As suggested by the figure, the throat is the controlling geometrical factor. Increases in width do not result in significant conductivity increases past a width/length ratio of 1.5.

accounting for the cross-sectional area of the pore that is generally larger than that of the throat tube. The larger cross-sectional area inside the pore is less resistive than the smaller pore throats. For this reason, pores conduct by a factor d more than the throat-tube, where d = pore conductance/throat-tube conductance. Limits on the value of d can be determined by numerical modeling of electric current flow in simple pores. The minimum value of d is one since the minimum size of a pore is that of the throat tube. The d value of the simple pore system in Figure 4 is 1.5.

The wide rectangular pore illustrated in Figure 5 is used to study the limiting effects of pore throats on current flow. Two parameters control the amount of current flowing through this model: the aspect ratio (width/length) and the pore-throat size/pore-width ratio. Finite-difference modeling has shown that if the pore-width/pore-throat size ratio is at least 5 and the aspect ratio of the pore is at least 1.5, then pore conductivity is a maximum. Increasing the width of the pore further will not result in any significant increase in current. The size of the pore throat is the limiting factor. Pores of the type depicted in Figure 5 are at most three times more conductive than the throat tube ($d = 3$) ; typically the value of d is between 1 and 2 in granular rocks.

The principal flux tubes that carry most of the electric current through these models and in rocks are not parallel to the applied potential, but are tortuous. The tortuosity τ of the current-flow path between pores is usually less than 2 (Herrick, 1988). In rocks with intergranular pore systems, tortuosities less than 1.5 are common. The conductance c between pores can be expressed in terms of the throat-tube conductance c_{th} adjusted for the width of the pore bodies d and the length of the average current path, or interpore tortuosity τ, or $c = (d/\tau)c_{th}$. The d/τ ratio, however, is close to unity since both d and τ tend to have values close to 1.5. Thus electrical conductance through pore systems in these models is governed almost entirely by the pore throats. This strongly suggests that the value of E in clean rocks with intergranular porosity is determined primarily by the pore-throat sizes. Other pore-geometric parameters have only secondary effects on E. The electrical properties of rocks with simple pore systems can, therefore, be estimated based on a knowledge of the area of the limiting pore throats alone. Etris et al. (1988 and 1989), for example, derived a statistical correlation between formation resistivity factor and pore throat area estimated from a combination of petrographic image analysis and capillary pressure measurements.

The concept of electric current streamline tortuosity was introduced by Wyllie and Rose (1950) and Winsauer et al. (1952). They considered the length of a fictitious tube having the same pore volume and end-to-end resistance as a rock sample. The ratio of the length of this tube to the length of the actual sample was defined as the tortuosity T. It is well known that this tortuosity can be expressed by $T^2 = F\phi$ where $F = C_w/C_0$ is the formation resistivity factor and C_w and C_0 are the conductivities of the brine and brine-saturated rock sample, respectively. Setting $S_w = 1$, $C_t = C_0$, and substitution of F into equation (2) results in $E = 1/F\phi$. Hence the Wyllie-Winsauer tortuosity factor is related to electrical efficiency by $E = 1/T^2$. Since E contains ***all*** pore-geometric effects, $1/T^2$ includes them as well. We have shown above that the interpore streamline tortuosity is a minor component of E; the dominant geometric property is the pore-throat size distribution. It follows that, contrary to the general view, T^2 is not a measure of streamline tortuosity.

E-ϕ RELATIONSHIPS

The central concept of the EET is that E is a function only of the geometric distribution of brine in the pore system of a rock and is not inherently affected by the brine content. This characteristic of E is implied by equation (2) in which the quantity of conducting brine (i.e., $S_w\phi$) and the effects of the brine distribution (i.e., E) are expressed as separate, independent variables. The independence of these variables is illustrated in Figure 6. Nine simple tube models are illustrated by three rows having different tube geometries and three columns with varying numbers of tubes, each of which has the same volume. In each row, the porosity is varied by changing the number of tubes; however, the geometry of the tubes is invariant. In each column the geometry of the tubes is varied resulting in variations in E, but the pore volume is held constant. By changing the number of tubes, or the geometry of tubes, E and water content can be varied independently. In principle, there is no inherent relationship between water content and electrical efficiency. Tubes with various geometries are used in Figure 6 for simplicity of illustration only and are not meant to imply that the pore systems in rocks are tubular. The conclusion, that the water content and electrical efficiency are inherently independent, is applicable to pore systems of any degree of complexity.

In general, there is no relationship between electrical efficiency and the porosity of water-saturated rocks if comparisons are made between rocks that are genetically unrelated. On the other hand, if sandstone samples, for example, are taken from a formation in which different degrees of

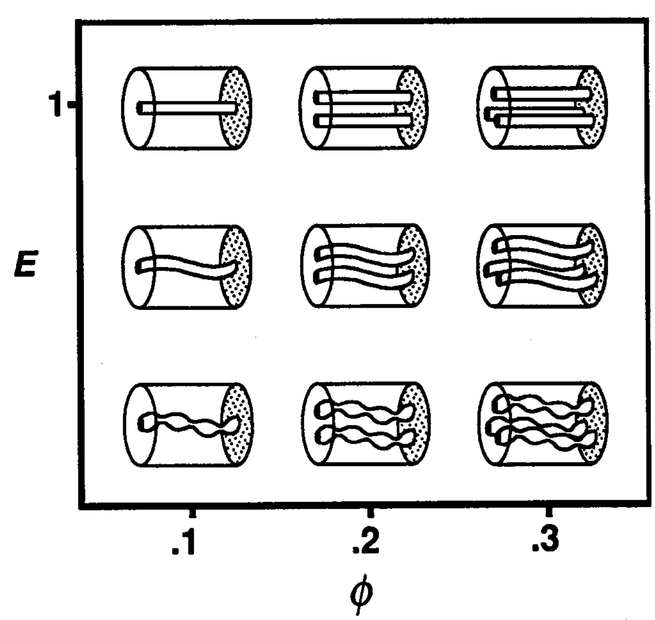

FIG. 6. Electrical efficiency is a function of pore geometry, but is not inherently dependent on porosity.

cementation have resulted in variations in porosity, then the process of cementation that caused the porosity variations may also produce consistent changes in the pore geometry as well. Cementation results in reduced porosity and increased complexity of the pore system such as reduced pore-throat sizes and increased amounts of electrically stagnant volumes. Hence, in spite of the inherent independence of E and ϕ, in any specified rock they are coupled through diagenetic processes. This results in a correlation between electrical efficiency and porosity.

Relationships between E and ϕ as a result of diagenesis or other geologic processes can be easily sought by applying equation (2) using conductivity and porosity measurements on suitable suites of rock samples. Examples of observed relationships are given in Figure 7 for samples from three sandstones and one carbonate. With the exception of the mid-East sandstone samples, an approximately linear relationship is observed between E and ϕ. This can be expressed as

$$E_0 = a_0\phi + b_0, \tag{11}$$

where the subscript 0 denotes complete water saturation. Experimental error is unlikely to account for the scatter about the regression lines in Figure 7. The variability is attributed to minor differences in depositional environment

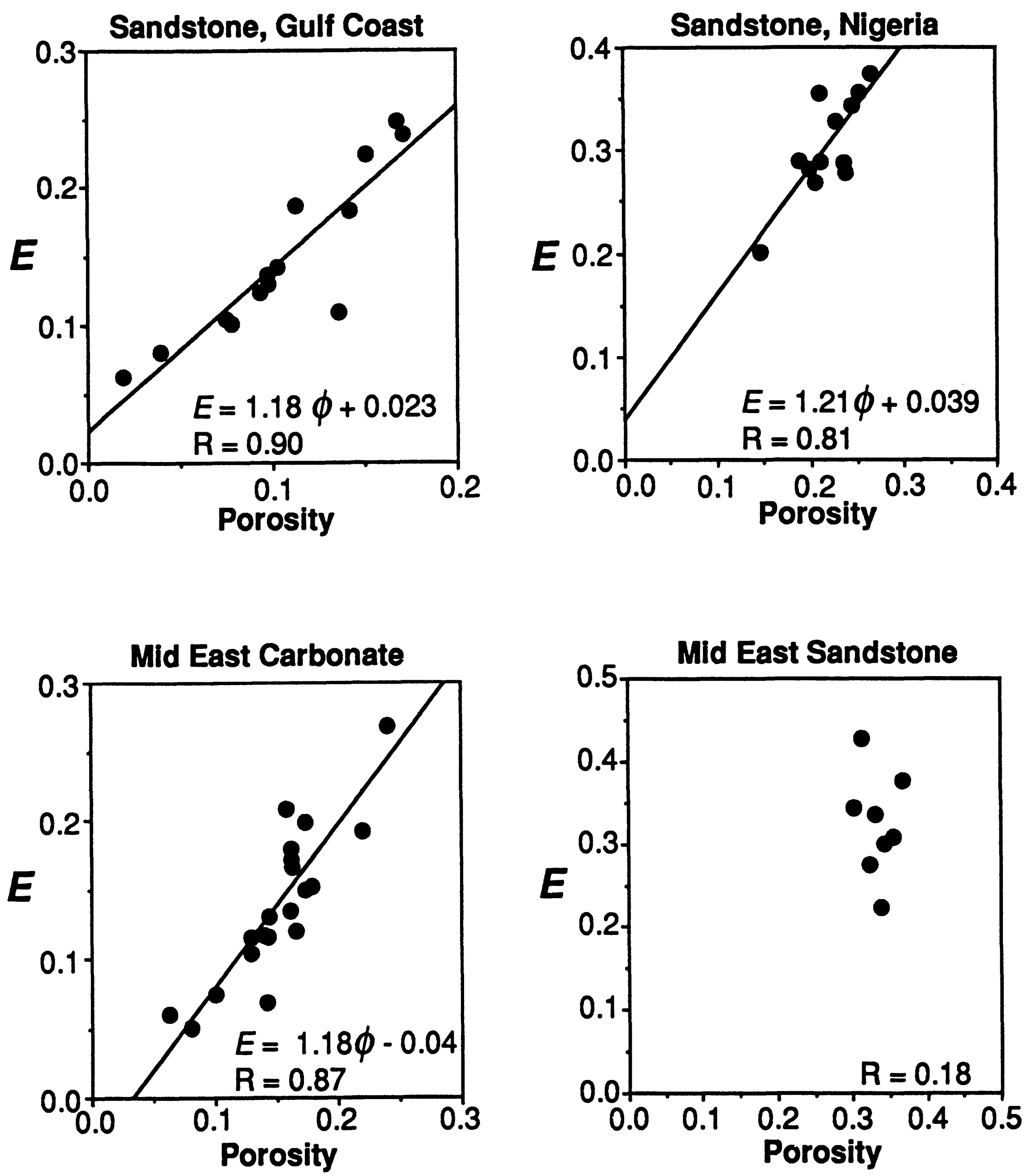

FIG. 7. Electrical efficiency-porosity relationships for three sandstones and one carbonate sample are shown. The relationship tends to be linear unless there is more than one porosity-changing mechanism.

$$a_t S_w \varphi + b_t = a_0 \varphi + b_0$$
$$\left(a_t S_w \varphi + b_t\right)\Big|_{S_w=1} = a_t \varphi + b_t = a_0 \varphi + b_0$$
$$a_t \varphi + b_t = a_0 \varphi + b_0$$
$$b_t = a_0 \varphi + b_0 - a_t \varphi$$
$$b_t = \left(a_0 - a_t\right)\varphi + b_0$$

and subsequent diagenetic history that can be expected in plugs taken along the length of a core. The lack of an E-ϕ relationship for the mid-East sandstone samples (Figure 7) illustrates an important point. The eight samples all have similar porosities; however, the electrical efficiency varies from about 0.2 to over 0.4, indicating large differences in pore geometry. This is suggestive of a significantly changing depositional and/or diagenetic environment during the time that the sampled rocks were deposited and buried. The E-ϕ relationship, in general, can be used as an indication of pore geometric relationships between rock samples as a result of the effects of depositional and diagenetic processes. A linear relationship between E and ϕ is characteristic of clay-free rocks having only intergranular porosity. The cause of the linearity has yet to be determined, although a relationship between the cross-sectional area of the pore throats and the pore volume is indicated.

E-S, RELATIONSHIPS

In addition to depositional and diagenetic processes, displacement of conductive brine by hydrocarbons also changes the electrical efficiency. Generally, if a rock is water wet, then the introduction of a hydrocarbon phase results in displacement of water from the larger and better connected pores. The remaining water is left in the smaller or more poorly connected pores and on grain surfaces. Hence the conductivity is reduced, not only because less water is contained in the rock, but also because the conducting geometry of the remaining water is complicated by the hydrocarbons. The increase in tortuosity and development of electrically stagnant volumes of water because of the presence of hydrocarbons tend to decrease the electrical efficiency of the remaining water in the rock. These arguments suggest a monotonic correlation between E and $S_w\phi$ should exist.

Figure 8 illustrates the observed decrease in electrical efficiency as water is displaced from four samples from each formation given in Figure 7. Inspection of the figure for each sample indicates that the electrical efficiency-water content relationship is also linear:

$$E_t = a_t S_w \phi + b_t, \tag{12}$$

where the subscript t indicates partial water saturation. The correlation coefficients for linear regressions computed with these data are statistically significant at greater than the 97 percent level of confidence in each case. The small scatter of points about the line is likely to be experimental error. The remarkable linearity of these trends is, like the E-ϕ correlation, not fully understood. Regardless of the cause of the observed linearity, it can be fruitfully exploited.

WATER-SATURATION EQUATION

A primary objective of formation evaluation using wireline log data is the estimation of water and hydrocarbon saturations. For clean, clay-free rocks that tend to exhibit linear electrical efficiency-porosity and electrical efficiency-water content relationships, a water-saturation equation can easily be derived. Combining equations (11) and (12), with S_w = 100 percent, and $E_t = E_0$, b_t can be evaluated as a function of porosity in terms of the other parameters:

$$b_t = (a_0 - a_t)\phi + b_0. \tag{13}$$

E_t is therefore given by

$$E_t = [a_t(S_w - 1) + a_0]\phi + b_0. \tag{14}$$

Equating E_t from equations (3) and (12) yields a quadratic equation in water content $S_w\phi$.

$$a_t(S_w\phi)^2 + b_t(S_w\phi) - C_t/C_w = 0. \tag{15}$$

Solved for S_w this gives a water-saturation equation that is dependent on the three parameters of the electrical efficiency relationships:

$$S_w = \frac{-b_t + \sqrt{b_t^2 + 4a_t C_t/C_w}}{2a_t\phi}. \tag{16}$$

The parameters a_0, a_t, and b_0 are obtained from conventional laboratory electrical measurements on core samples; however, the data are analyzed in terms of E_0-ϕ and E_t-$S_w\phi$ relationships. The relationship between measured conductivities (or resistivities) and E_0 or E_t is given in equation (2). When $a_0 = a_t = 1$ and $b_0 = 0$, equation (15) reduces to Archie's (1942) equation with $m = n = 2$; however, equation (15) is more general than Archie's equation and can be used to calculate water saturations from log data with at least comparable, or often better, accuracy.

The use of electrical efficiency concepts to obtain water saturations from log data has several advantages. E_0-ϕ and E_t-$S_w\phi$ relationships do not have inherent correlations, as do the resistivity relationships that Archie (1942) used to obtain his equation for determining water saturation (Herrick and Kennedy, 1993). The electrical efficiency relationships are observed to be linear for many rocks, resulting in the simple S_w equation (16). If the E_0-ϕ and' E_t-$S_w\phi$ relationships are not linear, as in shaly sands, for example, then an appropriate nonlinear equation can be fit to the data and a water-saturation equation can be determined. If the electrical efficiency E_t is observed to be a nonlinear function of ϕ and S_w, then equation (14) can be expressed as $E_t = f(S_w, \phi)$. Substituting this expression in equation (2) gives

$$S_w\phi f(S_w, \phi) - C_t/C_w = 0. \tag{17}$$

The $f(S_w, \phi)$ is obtained from laboratory-derived E_0-ϕ and E_t-$S_w\phi$ relationships and in general can be nonlinear. The expression $f(S_w, \phi)$ may be represented either parametrically (i.e., by curve fitting) or graphically to obtain an S_w-C_t relationship that fits the observed data as closely as desired.

CONCLUSIONS

Our studies have been motivated by a desire to understand the effects of pore geometry on rock conductivity. The electrical efficiency theory predicts the low-frequency bulk electrical properties of any specified geometrical configuration of conducting and nonconducting phases in reservoir rocks, using $C_t = C_w S_w \phi E$. The theoretical predictions do not depend on the values of empirically determined parameters but on a geometric factor E which in principle can be calculated from a detailed geometrical description of the conductivity distribution in a rock. This contrasts with Archie's law $C_t = C_w \phi^m S_w^n$, where the relationship of m

and n to conducting phase geometry has never been quantified. However, Archie's equation is contained in electrical efficiency theory as a special case. Although as a pore-geometric factor, the electrical efficiency E does not intrinsically depend on water content $S_w\phi$, these parameters are coupled in real rocks through processes that increase the complexity of a pore system as porosity decreases. In many cases when the rocks are not shaly and are dominated by intergranular pore space, the observed electrical efficiency has a linear relationship to variations in both S_w and ϕ. This linear dependence of E on $S_w\phi$ would be observed in detailed numerical models were it practical to construct them; however, the linear relationship was first discovered in field data rather than in numerical models. Although this relationship can be usefully exploited in saturation determinations, it remains a goal to understand the observed linear dependence theoretically and to thereby predict its parameters $\boldsymbol{a}$ and $\boldsymbol{b}$ given a description of the pore geometry, and vice versa.

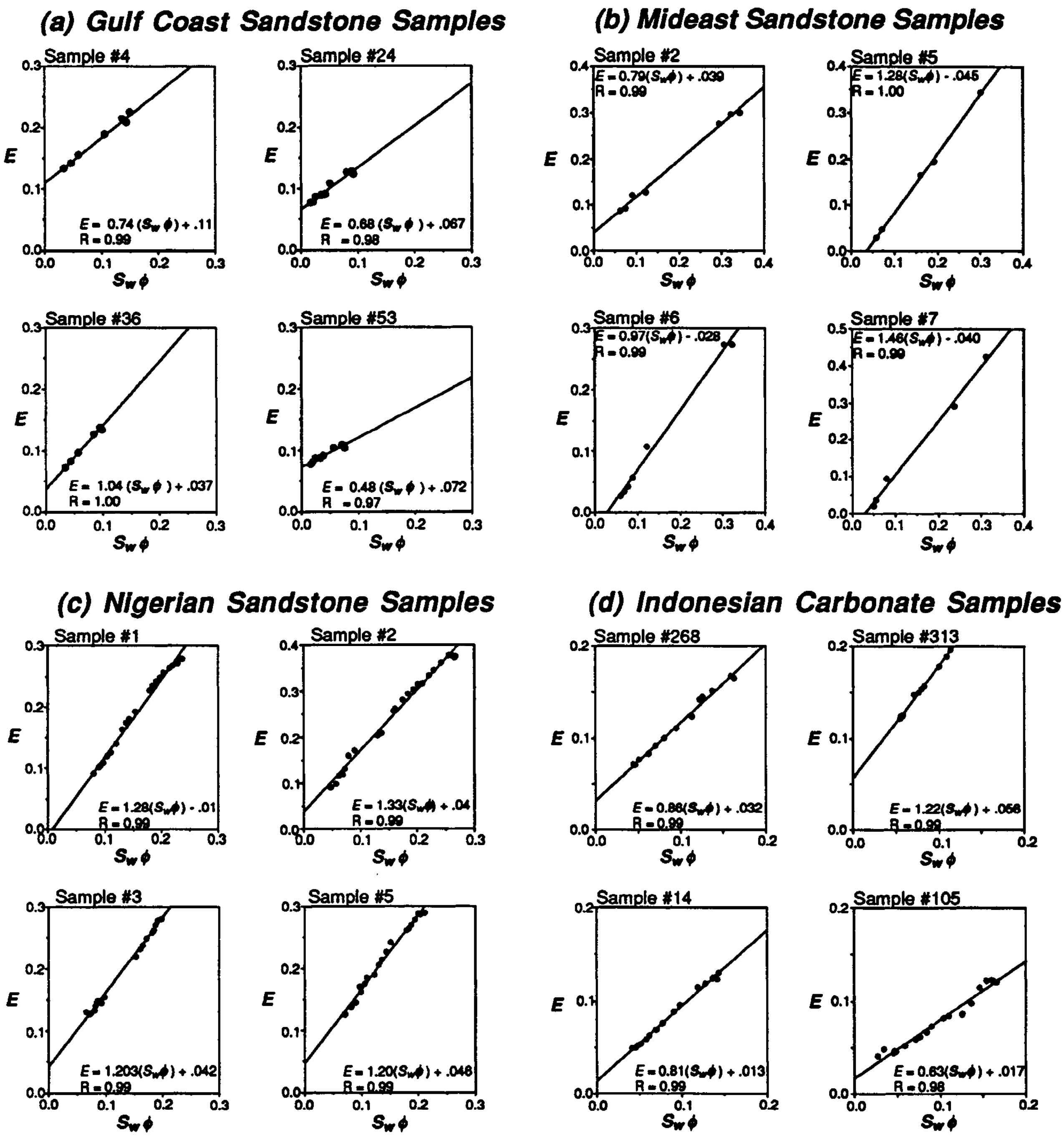

FIG. 8(a)-(d). Four samples from each of four fields showing a linear decrease in electrical efficiency with decreasing water saturation are shown. This behavior is typical of other samples from the same fields.

In the practical application of electrical efficiency theory to rocks, where pore geometry is unknown and the numerical prediction of bulk electrical properties is intractable, observed relationships between electrical efficiency and water content allow the construction of a water-saturation equation, quadratic in S_w, which is specific to each reservoir.

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