

Arps' Approximation Revisited and Revised

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ABSTRACT

Resistivity interpretations for conventional reservoirs depend upon an ability to estimate the resistivity of mud filtrate in flushed zones. An estimator, in use since 1953, was invented by Jan Jacob Arps. Arps worked out a method to use mud filtrate resistivity and its temperature as measured on the surface to estimate its resistivity at any other temperature, in particular the increased temperatures encountered at depth in wellbores. The method is known as Arps' approximation¹. It is also indispensable for extrapolating formation water resistivity from one depth and temperature to another depth at a different temperature.

Arps' approximation is based upon resistivity-temperature-salinity data as recorded in the *International Critical Tables of Numerical Data, Physics, Chemistry and Technology* (known as the ICT). There are 81 such data points extracted by Arps from the ICT. It is Arps stated goal to fully document his method; still, it is quite difficult to grasp his technique. However, the resulting Arps approximation is so simple and useful that it has not been subjected to further analysis in the intervening 67 years.

A re-examination of the basic ICT data has led to the formulation of a "first principle" relating the conductivity of a sodium chloride solution to its temperature as a function of salinity. The first principle takes the form of a simple differential equation from which the Arps approximation can be derived. Whereas Arps uses four pages and 2970 words written on 579 lines, this is done on five lines using the first principle, straight-forward step-by-step algebra, and five (or six) additional, but simple, equations without reference² to the ICT data.

A "reference temperature" arises in the derivation; this reference temperature is determined from the ICT data. In the Arps approximation the reference temperature is given as $T_0 = -6.7707$ °F. T_0 depends upon two arbitrary choices in Arps' analysis. Using Arps' choices the least squares fit of these data result in $T_0 = -6.7707$ °F.

However, this value might have been (slightly) different had Arps made other choices.

These discoveries give impetus to a re-examination of the evaluation of this constant, not only to validate Arps' estimate, but also to re-examine the choice of which data to include. For example, the ICT data is weighted toward low salinity (21 samples at 60, 100, 300 ppm) data and includes 10 samples at 32 °F; would these low-salinity, low-temperature samples unduly bias estimates at temperatures and salinities more representative of oil-field brines? Answering these questions leads to a collection of possibly more appropriate values for T_0 .

A final question is: does any of this re-analysis of the T_0 value have an impact on present and prior analysis using the Arps approximation? Fortunately, the answer to this question is "no"! Still, the new five-line derivation from a first principle and revaluation of T_0 improves our understanding of this venerable and ubiquitous approximation.

INTRODUCTION

Saturation equations in conventional hydrocarbon reservoirs rely upon the ratio of bulk formation resistivity to formation water resistivity. Thus, the first step in the formation evaluation of conventional hydrocarbon reservoirs is an estimation of formation water resistivity in the reservoir. Use of spontaneous potential and the Nernst equation was once the preferred method for doing this, but this method can require meticulous attention to this temperamental measurement (which in any event cannot be obtained offshore), so other methods are also in use including Pickett's plot and "inverse Archie". Another method relies on the constancy of the formation resistivity factor and reliable measurements of mud filtrate resistivity R_{mf} , flushed zone and virgin formation resistivity (R_{xo} , R_0) respectively. Then, writing $F = F$ in the form

$$\frac{R_0}{R_w} = \frac{R_{xo}}{R_{mf}} \quad \text{leads to} \quad R_w = \frac{R_{mf}}{R_{xo}} R_0. \quad (1)$$

Brine resistivities are functions of temperature. To use this relationship, it is necessary that all three resistivities on the right side be referred to the same temperature. This is, of course, assumed to be automatic for R_{xo} and

¹ See Appendix A: Grammar.

²Except for visual inspection of plots of these data.

R_0 since they are log responses at the same depth and time, and therefore, at the same temperature³. However, R_{mf} is known from a measurement made on the surface, typically at a much cooler temperature. Therefore, R_{mf} measured at surface temperature must be converted to the resistivity of the same mud-filtrate brine at formation temperature. In a “classical” by “hand and eye” interpretation a chart would be used to convert this brine resistivity from a resistivity at one temperature to its resistivity at another temperature, based upon the assumption that the brine salinity does not change as the mud filtrate circulates from the surface down to the depth of interest. This probably is the most robust assumption made in this calculation. For machine computation, a formula for making this conversion has long been preferred; this is the Arps (1953) approximation. It is based upon analysis of the data shown plotted in figure 1. The result is usually stated as

$$R_2 = R_1 \frac{T_1 + 6.77}{T_2 + 6.77} \quad (2)$$

when the temperatures are input as Fahrenheit degrees, R_1 and T_1 are mud-filtrate resistivity and temperature at the surface, T_2 is the estimated temperature at depth, and R_2 is the corresponding resistivity. Then taking R_2 as R_{mf} , the parameters on right side of the equation are all assigned values, and R_w is estimated using equation (1). In a “water leg” this formula could be applied at every depth giving a continuous estimate of R_w . However, if there are hydrocarbons present, then R_0 becomes perturbed by the amount of brine displaced by the hydrocarbons and is unknown. The nomenclature for bulk formation resistivity is then changed to R_t . However, R_w is still required at each depth in the reservoir where an interpretation is desired. For this Arps’ approximation is employed once again to project formation resistivity from a depth where it can be directly inferred to depths where it cannot be, identifying R_1 with R_0 at temperature T_1 , and projecting it to its corresponding resistivity $R_2 = R_t$ at T_2 . Of course, this use of Arps approximation is valid only if formation brine salinity is constant over the depth interval of interest.

³This is the implicit assumption made when using this relationship; however, it is not a rigorously true. The virgin formation where R_0 is estimated from a log response will (typically) be at a higher temperature than the flushed zone, which has been cooled by drilling mud. Thus, R_{vo} can be expected to have been sampled at a lower temperature than R_0 . This difference is seldom mentioned, let alone accounted for, in formation evaluation as routinely practiced.

The resulting estimate of R_w is then input into a saturation model; ultimately reserves are booked based upon this estimate. When the number of assumptions employed to estimate R_w is counted, it is astounding that we have any confidence at all in the estimate. But the fact is, we use it as if we do.

Arps published his approximation in 1953 in a very short⁴ paper (4 pages). As motivation he cites: “*Since some discrepancies were found in the charts ordinarily used for [interpretation], it was felt desirable to make a new study of this relationship⁵ based on the published measurement data [sic].*” My object here is to review and amplify Arps’ method, critique it, and offer a much simplified technique for arriving at Arps’ approximation⁶.

ARPS’ METHOD

Original Sources. Arps cites six sources. Three are from the then-contemporary literature, one is the *Handbook of Chemistry and Physics*, 32nd edition (1950), and the sinequa-non of his references, two citations of the *International Critical Tables of Numerical Data, Physics, Chemical and Technology* (ICT). The ICT were published in 7 volumes between 1926 and 1930. Arps citation of two of the volumes (**III**, **IV**) reflects his sources for density of NaCl solutions, and conductivity of NaCl solutions. The nature of such a collection of data is to reproduce data from original sources (e.g., journal articles). Thorough scholarship would require the checking of original sources as cited in the ICT volumes. I am foregoing thorough scholarship in favor of using the data that Arps selected for re-publication in his paper. These data will meet the needs of this article.

Molar conductivity. The usual unit of conductivity used in oil-field calculations is the Siemen/meter. The SI practice of honoring pioneers by assigning their names to basic quantities has the effect of obscuring the underlying units of mass, length, time, and charge (M, L, T, Q). The conductivity of a material is expressed in terms of the ratio of the current density \mathbf{J} (amperes/m²) induced to flow by an applied electric field \mathbf{E} (volts/m =

⁴It might be a better paper had it been a little longer. “Short writing makes for long reading” attributed to Professor Paul Samuelson cited in the preface to *Geophysical Signal Analysis*, Enders Robinson and Sven Treitel, 1980.

⁵That is, the salinity-resistivity-temperature transform.

⁶Some readers may wish to skip ahead to the section titled “A SIMPLER ROUTE TO EQUATION (6)” on page 5.

joule/coulomb/m). Or, conductivity units⁷ at this level are (amperes/volt)/m, or Siemens/m. Siemens are reciprocal Ohms. (See Appendix B: Units)

In electrochemistry it is most convenient to express the conductivity of a solution in terms of the conductivity attributable to a mole of solute per cubic meter of solution (mol/m^3 is equivalent to millimoles/liter, or in Arps now obsolete notation, milligram-moles/liter). This unit of conductivity is known as “molar conductivity”, denoted as $\Lambda_m = \sigma / C^8$, expressed as siemens/m per mole/ m^3 . In the limit of “infinite dilution” (i.e., zero concentration of solute) this conductivity is given the name “limiting molar conductivity”.

This is denoted as $\lim_{C \rightarrow 0} \Lambda_m \equiv \Lambda_m^0 \rightarrow \sigma / C$. Limiting molar conductivity is a function of temperature and has a unique value for each solvent and solute. For NaCl in water the value is 126.39 siemens/m / mol/m^3 at 77 degrees F. Arps Table 1 (See Appendix C) is a listing of molar conductivities for NaCl aqueous solutions at various concentrations ranging from 0.1 to 5,500 mol/m^3 , and for 7 temperatures ranging from 32 to 312.8 degrees Fahrenheit. There are 81 values in this table; these form the entire (published) basis for resistivity-temperature-concentration transforms used in log interpretation and formation evaluation. These data were first collected and

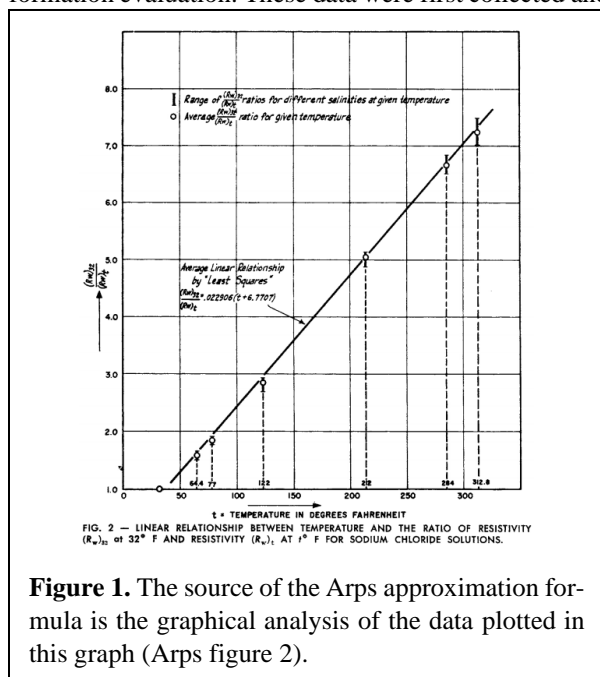


Figure 1. The source of the Arps approximation formula is the graphical analysis of the data plotted in this graph (Arps figure 2).

compiled in the late 1920s; Arps republished them in 1953. It is surprising (and appalling) that no new additional and confirmatory data have been added to this data set since Arps publication.

Errors in the ICT table. Arps was able to detect, by observing the patterns of numbers in the columns, that the three entries that he has marked with double asterisks belong in the column for 212 °F rather than their position in the 122 °F column, as published in the ICT table. Two errors, undetected by Arps, but also found by recognizing the deviation of the errors from the pattern established by the surrounding numbers, is that the 540.5 molar conductivity entry should have been entered as 450.5, and the entry 42.6 should have been entered as 46.2. These latter two errors in the Table 1 entries become apparent to a user only after significant interaction (i.e., plots of the data, etc.) with the ICT data set.

Arps Table 2, using data from volume **III** (1928) of the ICT produces the density (or specific gravity) of NaCl solutions of various concentrations at the 7 selected temperatures. This table delivers on the “density” part of Arps’ title: *The Effect of Temperature on the Density ... of Sodium Chloride Solutions*. Note The leftmost column is listed in terms of percent NaCl by weight (i.e., mass), but could have been listed as parts per million (i.e., $C[\text{ppm}] = 10,000 \times C[\%]$). Also note that the table is mostly filled by interpolation, extrapolation and estimation. The table contains 98 entries. The remainder of Arps paper is devoted to estimating the electrical resistivities of NaCl solutions using a subset of these estimates.

To proceed it is necessary to reformulate the data into “oil-field” units. This Arps begins in his Table 3 listing concentrations as functions of salinity (as expressed in ppm by mass) and temperature. Note that any gaps in this table have been filled by extrapolation and interpolation.

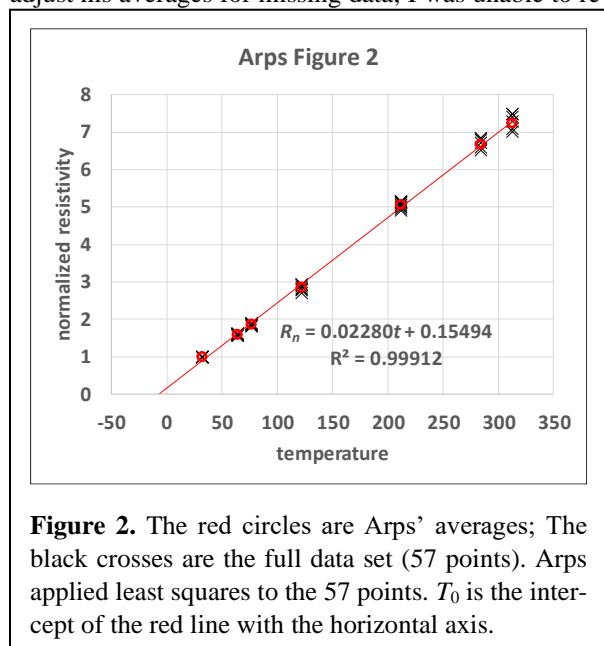
Arps’ Table 4 reproduces Table 1 with the addition of the concentrations at each molar concentration and temperature augmented by its concentration expressed in parts per million mass units. As with Table 1, there are 81 entries in the table. The concentrations expressed as salinities in ppm NaCl by mass are interpolated estimates.

⁷Even at this level, “amperes” and “volt” disguise the basic combinations of physical units which are amperes = $[Q/T]$ and volts = $[\text{energy}/\text{charge}] = [\text{joules}/\text{coulomb}] = [\text{ML}^2/\text{T}^2 / Q]$

⁸In electrochemistry conductivity is conventionally denoted as κ ; for this paper I shall employ the physics notation σ to mean the same thing.

Table 5 is the heart of the Arps paper, and Arps' approximation is based upon the data in this table. Note that the number of concentrations listed in the table is reduced to 10 (from 21) with concentrations less than 60 and more than 200,000 ppm NaCl ignored. Also note that *all* of the resistivity entries in the table are interpolations. Curiously, Arps has omitted 13 resistivity estimates from the table, perhaps because they require some extrapolation as well as interpolation (see Appendix E). The Table 5 contains data for 57 points. One additional data entry is produced for this table. The resistivity at each concentration is divided into the resistivity at that concentration at 32 °F. This is a kind of "normalization"; in conductivity terms it would be the conductivity of each solution divided by the conductivity of the same solution at 32 °F; viewed in this conductivity space, this kind of normalization seems more conventional. At any rate, the result is a collection of normalized resistivities at each of seven temperatures. Note that for each temperature the normalized resistivities are similar. This suggests that to obtain a single value at each temperature, one could just average the normalized resistivities at each temperature.

Arps gives the averages for each normalized resistivity in Table 5, in the bottom row⁹, and plots them as open circles in his figure 2. Arps has used some procedure to adjust his averages for missing data; I was unable to re-



⁹Quoting Arps: "The average ratios (corrected for the missing high salinity figures by the trends in $(R_w)_{32}$ and $(R_w)_{212}$) are shown at the bottom of this table."

produce his "averages" after trying various schemes, and so in the following I take these averages as given.

Fitting a Line to the Data. From this point forward, Arps applies the method of least squares to find the slope and intercept of the line that best fits his 57 data points (i.e., all 57 temperature - "normalized resistivity" pairs) in the "least squares sense". In following Arps, it is important to note that the trend line in figure 2 is *not* the result of fitting a line to the seven "averaged" normalized resistivities shown in figure 2. These data from Arps Table 5 are reproduced below.

32	1
64.4	1.59078
77	1.85013
122	2.84238
212	5.05393
284	6.6735
312.8	7.2494

It is a trivial matter to drop these data into an Excel worksheet and fit the resulting plot with a linear trendline; however, these are not the data used in producing Arps' approximation, and the slope and intercept for the trend computed from these data are slightly different than those produced using all 57 data pairs. (see Appendix D)

Using all 57 data pairs, the result is a line expressing the normalized resistivity as a function of temperature; the least squares method has given the slope and intercept of the line. Arps' notation for temperature is t .

$$\frac{(R_w)_{32}}{(R_w)_t} = 0.022906 t + 0.155074 \quad (3)$$

Arps then factors the slope from the right side of the expression. The result is

$$\frac{(R_w)_{32}}{(R_w)_t} = 0.022906 (t + 6.7707), \quad (4)$$

these values being listed in the inset to Arps' figure 2. The constant 6.7707 is conventionally approximated by 6.77. Arps' final step is to ratio equation (4) at two different temperatures. Thus

$$\frac{(R_w)_{32}/(R_w)_{t_1}}{(R_w)_{32}/(R_w)_{t_2}} = \frac{0.022906(t_1 + 6.77)}{0.022906(t_2 + 6.77)} = \frac{(t_1 + 6.77)}{(t_2 + 6.77)} \quad (5)$$

which is Arps' formula:

$$(R_w)_{t_2} = (R_w)_{t_1} \frac{(t_1 + 6.77)}{(t_2 + 6.77)}. \quad (6)$$

This approximation is a triumph for data reduction, and a mark of Arps' genius. However, the method of derivation is tedious, requiring much effort to follow. A more transparent method leading to the same result would be an aid to understanding.

A SIMPLER ROUTE TO EQUATION (6)

Inspecting conductivity data plotted against temperature (figure 3) one will notice that the rate of change of conductivity with respect to a change in temperature depends mainly upon the salinity, or salt concentration, in the brine. This observation suggests that the rate of change of conductivity with respect to temperature for a given concentration depends upon a function of concentration alone; that is, it does not depend upon conductivity or temperature, at least not much.

Thus, am I prompted to speculate that

$$\frac{d\sigma}{dT} = f(C). \quad (7)$$

This is the most elementary kind of differential equation, solved almost trivially by the "separation of variables" method. Thus, multiplication of (7) by dT yields

$$d\sigma = f(C)dT \quad (8)$$

Integrating both side of (8) then yields

$$\sigma_1 = f(C) \int_{T_0}^{T_1} dT = f(C)(T_1 - T_0) \quad (9)$$

where the lower limit of temperature is a "reference" temperature, to be determined, and the upper limit is the temperature of a brine whose conductivity is σ_1 .

Applying the formula a second time with upper temperature limit T_2 , yielding conductivity σ_2 ; i.e.,

$$\sigma_2 = f(C) \int_{T_0}^{T_2} dT = f(C)(T_2 - T_0) \quad (10)$$

Forming the ratio of (9) to (10), the result is

$$\frac{\sigma_1}{\sigma_2} = \frac{f(C)(T_1 - T_0)}{f(C)(T_2 - T_0)} = \frac{T_1 - T_0}{T_2 - T_0}. \quad (11)$$

The function of temperature cancels from numerator and denominator. Summarizing, and solving for σ_1 gives

$$\sigma_1 = \sigma_2 \frac{T_1 - T_0}{T_2 - T_0}. \quad (12)$$

This is the Arps' formula in its conductivity guise. Finally, replacing conductivity with its reciprocal, resistivity, yields a formula with the same morphology as the Arps approximation:

$$R_2 = R_1 \frac{T_1 - T_0}{T_2 - T_0}. \quad (13)$$

The progression from equation (7) to equations (12) and (13) is certainly much easier to follow than Arps' arduous trek through 5 tables with many transformations, interpolations, and extrapolations.

For this method, it remains only to determine the value of the reference temperature, T_0 . The reference temperature T_0 is to be determined from observations of data using

$$T_0 = \frac{T_1 R_1 - T_2 R_2}{R_1 - R_2} \quad (14)$$

(i.e., equation (13) solved for T_0). Thus is the Arps approximation derived from a first principle, based upon observations, in a handful of equations.

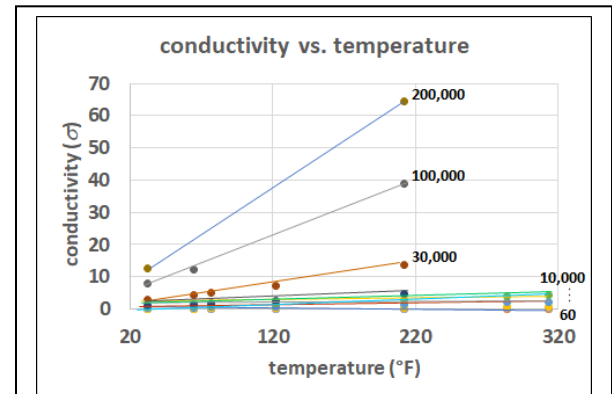


Figure 3. The ICT data with iso-concentration points approximately fit by straight lines in temperature-conductivity space. Salinity in ppm labels each iso-concentration line. Conductivity varies quasi-linearly with temperature for each concentration.

EVALUATING T_0 FROM DATA

If the iso-salinity lines on the ICT data plot were straight lines, then any two points on the same iso-salinity line would serve to evaluate T_0 . However, since the iso-salinity lines assumed by Arps' approximation are straight lines (represented on a log resistivity vs. log temperature plot), this procedure would fail using the ICT data. However, following Arps' 1953 publication, beginning in 1955, the Schlumberger salinity-resistivity-temperature chart Gen-6 plotted the Arps approximation rather than the ICT data itself. (This practice was reversed in the 1996 chart, when the iso-salinity lines became the curves honoring the ICT data.) Any service company (e.g., Schlumberger's Gen-6) chart that was plotted using the Arps approximation as iso-salinity curves will serve the purpose of T_0 evaluation.

Choose a temperature of 75 °F and arbitrarily pick a brine resistivity $R = 0.137 \Omega\text{-m}$, corresponding to an iso-salinity line of about 48,000 ppm NaCl. This avoids the slightly curved iso-salinity data that begins to appear at higher salinities. Following this iso-salinity line to a temperature of 185 °F gives a second temperature. The resistivity read at this temperature is, to two significant digits, $0.058 \Omega\text{-m}$. It will turn out that T_0 depends very sensitively upon the value of R_2 , and so one finds that, by trial and error, $R_2 = 0.058416 \Omega\text{-m}$ must be used in the formula for T_0 in order to produce $T_0 = -6.77$ °F.

ARPS "OOPSIES"

Arps' intention may have been that his contemporaries duplicate his work. In his words: "*as an aid to other workers in this field, an attempt has been made in this study to show every step in the calculations.*" I may be the first to actually follow in Arps' steps; certainly, I am the first to publish results on it. Having thoroughly vetted Arps' paper, including the numbers in most of his tables, I must point out several errors in the numbers in Arps' tables.

Numerical Oopsies. I have already mentioned errors that Arps discovered in the ICT data listed in his Table 1, in addition to two other more subtle errors that I discovered. I have not checked the numbers in Arps' Table 2; these I accept as published. Table 3 is produced using the density values in Table 2 and the formula

$$C_t = \frac{g_t \times ppm}{58.45} \quad (15)$$

(Arps equation (4)) where g_t is the density of the solution at temperature t (from Table 2), ppm is the salt concentration in parts per million NaCl by mass, and 58.45 is the molecular weight of NaCl; C_t is the salt concentration in mol/m^3 (mmol/liter). To check Arps the numbers in Table 2 are plugged into the equation and the numbers in numbers Table 3 will (or should) emerge. This expectation is verified for the temperature columns from 32 to 212 degrees, but not for the columns of 284 and 312.8 degrees, where the results in the table do not conform to the columns of Table 3 plugged into Arps' equation (4). The magnitude of the differences are shown for four cases below:

con. / temp	284	312.8	
10,000	158.392	157.040	Excel
	159.623	157.038	Arps
260,000	4903.747	4861.933	Excel
	4942.098	4862.059	Arps

The rows labeled "Excel" are the calculations based upon Arps' Table 2 and his equation (4); the rows labeled "Arps" are from Arps Table 3. I was unable to reproduce Arps numbers, leaving me without a guess as to the cause of the difference. However, the differences are small, in the 1% or less range.

Table 4 is the same 81 values listed in Table 1, but with the concentrations converted from mol/m^3 to ppm by mass, and the molar conductivities converted to resistivities in ohm-m . I accepted the numbers in this table as given.

Finally, the resistivity values in Table 5 were accepted as written, but I did check the "normalized" resistivity values; for these numbers (i.e., the final data) I found what appear to be three rounding errors (a difference of 1 in the least significant digit), and an apparent transcription error (an "8" written as "5") in the least significant digit. These minor errors would have no effect upon Arps' final estimates.

Other Oopsies. The source of Table 1 data is cited in the text as "The basic measurement data [sic] on the electrical conductivity of aqueous sodium chloride solutions, at different temperatures, as given in [the ICT] tables, are reproduced in Table 1." Yet the asterisk decorating the Table 1 label seems to contradict this assertion, citing "Ref. 2" as the data source. I think this has to be attributed to a typo since it is very clear that the data in Table 1 are from the ICT volume. There is another

possibility: a blemish on the paper at the top of the 64.4 °F column is probably a smudged asterisk. If that is true, then the column in Table 1 for 64.4 °F is footnoted that it has come from “Ref. 2”, and not the ICT source. This would seem to be a citation from the 1949 paper by Doll; i.e., “Ref. 2”. However, there is no such data in the Doll paper. Checking that Arps might be intending to refer to one of his other references, a check of the papers by Mounce and Rust (1945) and Wyllie (1949) does not turn up these data in those papers. Thus, it is unclear from the sources that Arps provides what the provenance of this data might be. I leave it as a final possibility that the data in the 64.4 °F column is taken from the *Handbook of Chemistry and Physics*, 32nd edition since this reference is not cited elsewhere in the text. These issues might be resolved by more thorough scholarship should it ever be profitable or necessary to resolve them.¹⁰

CONCLUSIONS

Will anything discovered in this investigation change future interpretation practice, or more importantly, past interpretations? The answer is, thankfully, no! All of the uncertainty in the approximation is reduced to the uncertainty in the parameter T_0 . Its exact value will depend upon arbitrary choices made in processing the ICT data. Arps choices included omitting some data at very low salinities from consideration, and his choice to “normalize” resistivities to their values at 32 °F. He could have omitted even more low salinity data than he did, and he might have normalized resistivity to a value in the mid-range, say 122 or 172.4 °F. Different choices would have resulted in different T_0 estimates. However, given that T_0 is small compared to temperatures normally used when mud filtrates are measured, or downhole temperatures, and given that T_0 varies little regardless of which (reasonable) assumptions are made, small variations in the T_0 value would translate to small variations in resistivity estimates of R_2 .

My *main* motivation for writing this article has been to introduce an alternative, and more easily understood, method to derive the Arps approximation. These are presented in equations (7) through (12) and (13). I certainly do not intend to diminish Arps’ contribution of his approximating formula, a mainstay of formation evaluation for nearly three quarters of a century. As I noted earlier, more scholarship might be done on this topic; I did not see the value in doing it merely for the sake of

completeness since, even if done, it would not result in any change to formation evaluation practice. However, as always, I welcome corrections, and additions, and other critiques to my scholarship.

ACKNOWLEDGEMENTS

I dedicate this article as a memorial to Ralph Payne and Willis Wright, my operators in Schlumberger’s district in West Sacramento, without whose unflagging support in the face of a months-long succession of “snake bit” jobs following my “breakout” 1973, I would never have stayed in this insane game of logging and formation evaluation.

NOMENCLATURE

Λ_m, Λ_m^0	molar conductivity; limiting molar conductivity.
$R_{mf}, R_{xo}, R_w, R_0, R_t$	resistivity of mud filtrate, flushed zone, formation brine, bulk formation resistivity.
$\sigma, \sigma_1, \sigma_2$	conductivity; conductivity of brine at temperature T_1 ; conductivity of brine at temperature T_2 .
T, T_0, T_1, T_2	T temperature; T_0 “reference” temperature; temperature of brine at temperature T_1 ; temperature of brine at temperature T_1 .
R, R_1, R_2	resistivity; resistivity of brine at temperature T_1 ; resistivity of brine at temperature T_2 .
C	concentration of NaCl in brine, or salinity
$f(C)$	function of concentration of NaCl in brine, or salinity

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¹⁰It occurred to me as I proofread this manuscript that Arps might have been referring to the ICT volume III, which he cites

after first citing volume IV. If so, correctly cited, this would be “Ref. 5”.

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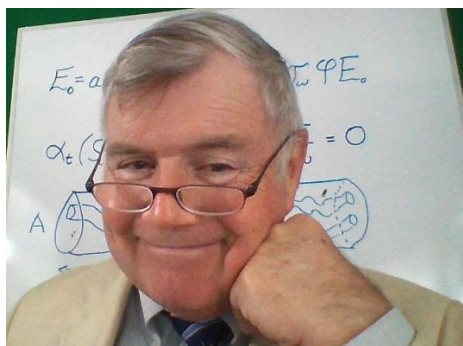
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ABOUT THE AUTHOR



David Kennedy began a career in the logging industry in 1973 following earning a B.S. in Physics at Georgia Tech. He entered the industry as a Schlumberger field engineer in California and Alaska, staying with Schlumberger for five years. Following that, Dave returned to school and earned Masters' degrees in Physics and Earth Sciences at the University of Texas at Dallas, with further studies at U. C. Berkeley. In his career he has worked in one or another capacity at Arco, Sohio Research and operations, Mobil Research and operations, ExxonMobil Research and operations, Baker-Hughes, PathFinder, and Southwestern Energy. David has been an SPWLA member for 40 years and served as V.P. Publications and Editor of *Petrophysics* from 1999-2002, V.P. Technology in 2009, and President (etc.) from 2014-2016. David is inventor or coinventor on 6 U.S. Patents, and has published about 45 articles as author or coauthor in conference proceedings and refereed journals. His major research interest has been formation conductivity interpretation and electromagnetic logging instrumentation. David has been an educator all his life,

having taught courses at the USMA at West Point, courses in computer science, physics, materials science, and electrical engineering at San Francisco Bay Area private and junior colleges. He was most recently a professor in the Petroleum Engineering Department at Texas A&M University instructing graduates and undergraduates in formation evaluation. Before all of that, David served as a Lieutenant of Infantry in Vietnam where he received both a Purple Heart and Bronze Star with V device.

APPENDIX A: Grammar

I learned the use of apostrophes to indicate possessive (or genitive) case from Mrs. Ratched¹¹ in 3rd or 4th grade. The usual rule is to add "'s" to the nominative case of the noun; e.g., "cow" becomes "cow's". But, what if the nominative case of the noun ends in "s"? Does the same rule apply? Should "Texas" become "Texas's"? Answer: No! The rule is to add an apostrophe following the final "s"; i.e., "Texas' " (not Texas's) is possessive for Texas. Applied to "Arps", this gives "Arps' approximation". On the other hand, when referring to the approximation as an entity using the definite article "the", the reference is "the Arps approximation" without any apostrophe. These are the English grammar rules that I learned from Mrs. Ratched, and that I apply in this article. Of course, being English grammar, that was just her opinion on usage. Other (self-appointed) grammar experts (like me) could have other opinions. The joy of speaking a living language is that the rules are constantly (if slowly) changing in real time! Don't you love English?

APPENDIX B: Units

Q (electric charge) coulomb q

M (mass) kilogram kg

L (displacement) meter m

T (time) second sec

Volume concentration: mole / m^3 or mmol/L

ampere (electric current) coulomb / sec, Q/T , i

volt (electric potential) joules/coulomb (energy per unit charge) v

E (electric field) volt / m = joules/coulomb (energy per unit charge) / m

¹¹The image of a strict grammarian, modeled on Nurse Ratched in Ken Kesey's 1962 novel, *One Flew Over the Cuckoo's Nest*,

portrayed by Louise Fletcher in the 1975 movie (winner of five Academy Awards).

J (current density) amperes / m^2 = coulomb/sec / m^2

j (energy) joules = $kg \cdot m^2/sec^2 = ML^2/T^2$

σ (conductivity) amp/ m^2 / volt/ m = amp/volt / m = siemen / m

R (resistivity) volt/ m / ampere/ m^2 = ohm - m

APPENDIX C: Arps' Tables

Arps' article is readily available from OnePetro, the SPE's online database. However, for convenience I reproduce his tables in this appendix.

Concentration (C_t) (milligram mols per liter of solution)	TEMPERATURE IN DEGREES FAHRENHEIT t						
	32°	64.4°	77°	122°	212°	284°	312.8°
0.1	107.88
0.2	107.60
0.5	106.95	125.0	354.9	491	545
1	65.8	106.27	124.12	195.3	352.5
2	65.2	105.31	123.03	193.6	348.5	481	534
5	64.2	103.54	120.88	190.1	341.9
10	63.2	101.72	118.63	185.6	335.0	461	511
20	61.7	99.40	115.85	179.9	325.4
50	59.5	95.51	111.08	170.8	309.7
70	58.7	109.01	167.1	303.2
80	411	540.5
100	57.7	91.82	106.66	163.4	295.6	403.5	441.5
200	55.6	87.53	101.55	155.6	287.9
500	51.8	80.76	93.31	141.9	274.4
700	49.9	263.6
1,000	47.5	74.19	247.5
2,000	41.4	64.66	209.7**
3,000	36.5	56.4	184.1**
4,000	31.5	42.6	164.6**
5,000	26.6	44.7
5,500	24.6

*Data from Ref. 2.
 **Probably erroneous, should be for 212° F.

Table 1* — Electrical Conductivity Λ of Aqueous Sodium Chloride Solutions as a Function of Concentration C_t and Temperature t . (Expressed as 10^6 times the specific conductivity in $ohm^{-1}cm^{-1}$ per unit of concentration in milligram mols per liter of solution) [specific conductivity \equiv molar conductivity].

Note that 540.5 and 42.6 should probably be 450.5 and 46.2 respectively.

Percent NaCl by weight	TEMPERATURE IN DEGREES FAHRENHEIT t						
	32°	64.4°*	77°	122°	212°	284°**	312.8°†
1	1.00747	1.00569	1.00409	0.99482	0.9651	0.9258	0.9179
2	1.01509	1.01285	1.01112	1.00161	0.9719	0.9323	0.9244
4	1.03038	1.02728	1.02530	1.01531	0.9855	0.9455	0.9375
6	1.04575	1.04183	1.03963	1.02919	0.9994	0.9588	0.9507
8	1.06121	1.05653	1.05412	1.04326	1.0134	0.9723	0.9640
10	1.07677	1.07138	1.06879	1.05753	1.0276	0.9859	0.9775
12	1.09244	1.08642	1.08365	1.07202	1.0420	0.9997	0.9912
14	1.10824	1.10166	1.09872	1.08674	1.0565	1.0137	1.0051
16	1.12419	1.11708	1.11401	1.10170	1.0713	1.0278	1.0191
18	1.14031	1.13281	1.12954	1.11691	1.0864	1.0423	1.0334
20	1.15663	1.14874	1.14533	1.13238	1.1017	1.0569	1.0479
22	1.17318	1.16494	1.16140	1.14812	1.1172	1.0718	1.0627
24	1.18999	1.18143	1.17776	1.16414	1.1331	1.0870	1.0777
26	1.20709	1.19824	1.19443	1.18045	1.1492	1.1024	1.0930

*Interpolated between g_{50}° and g_{68}° .

**0.92085 x g_{68}° .

†0.91301 x g_{68}° .

Table 2 — Specific Gravity g_t or Density (g/cc) of Aqueous Sodium Chloride Solutions as a Function of Sodium Chloride Weight Percentages and Temperatures t .

(ppm) Parts per Million by Weight (mg NaCl per Kg solution)	TEMPERATURE IN DEGREES FAHRENHEIT t						
	32°	64.4°	77°	122°	212°	284°	312.8°
10,000	172.364	172.059	171.786	170.200	165.115	159.623	157.038
20,000	347.336	346.570	345.978	342.724	332.558	321.503	316.300
40,000	705.136	703.014	701.660	694.823	674.423	652.121	641.560
60,000	1,073.482	1,069.460	1,067.199	1,056.482	1,025.902	991.966	975.901
80,000	1,452.469	1,446.050	1,442.765	1,427.901	1,387.032	1,341.192	1,319.470
100,000	1,842.207	1,832.989	1,828.554	1,809.290	1,758.084	1,699.972	1,672.411
120,000	2,242.819	2,230.460	2,224.773	2,200.896	2,139.264	2,068.508	2,035.408
140,000	2,654.466	2,638.711	2,631.664	2,602.969	2,530.538	2,447.025	2,407.300
160,000	3,077.338	3,057.875	3,049.471	3,015.775	2,932.558	2,835.620	2,789.690
180,000	3,511.647	3,488.538	3,478.481	3,439.586	3,345.629	3,234.914	3,182.523
200,000	3,957.673	3,930.676	3,919.008	3,874.696	3,769.718	3,644.808	3,585.778
220,000	4,415.732	4,384.725	4,371.393	4,321.409	4,205.029	4,065.736	3,999.889
240,000	4,886.182	4,851.051	4,835.965	4,780.040	4,652.588	4,498.033	4,425.185
260,000	5,369.434	5,329.284	5,313.119	5,250.932	5,111.925	4,942.098	4,862.059

Table 3 — Concentration C_t of Aqueous Sodium Chloride Solutions in Milligram Mols per Liter of Solution as a Function of Sodium Chloride Concentration by Weight (ppm) and Temperature t .

Concentration (C_t) (milligram moles per liter of solution)	TEMPERATURE IN DEGREES FAHRENHEIT (t)													
	32°		64.4°		77°		122°		212°		284°		312.8°	
	ppm	Res.	ppm	Res.	ppm	Res.	ppm	Res.	ppm	Res.	ppm	Res.	ppm	Res.
0.1	—	—	5.812	926.96	—	—	—	—	—	—	—	—	—	—
0.2	—	—	11.624	464.88	—	—	—	—	—	—	—	—	—	—
0.5	—	—	29.060	187.00	29.106	180.00	—	—	30.282	56.353	31.324	40.733	31.839	36.697
1	58.017	151.97	58.120	94.100	58.212	80.56	58.754	51.203	60.564	28.392	—	—	—	—
2	116.03	76.687	116.24	47.479	116.42	40.64	117.51	25.826	121.13	14.347	125.30	10.395	127.36	9.363
5	290.08	31.152	290.60	19.316	291.06	16.545	293.77	10.521	302.82	5.849	—	—	—	—
10	580.17	15.822	581.20	9.831	582.12	8.430	587.54	5.388	605.64	2.985	626.48	2.1690	636.79	1.956
20	1,160.3	8.1037	1,162.4	5.030	1,164.2	4.316	1,175.1	2.779	1,211.3	1.536	—	—	—	—
50	2,900.8	3.361	2,906.0	2.0940	2,910.6	1.801	2,937.7	1.171	3,028.2	0.6458	—	—	—	—
70	4,061.2	2.434	—	—	4,074.8	1.310	4,112.8	0.8549	4,239.5	0.4711	—	—	—	—
80	—	—	—	—	—	—	—	—	—	—	5,011.8	0.30414	5,094.3	0.249
100	5,801.7	1.733	5,812.0	1.08908	5,821.2	0.9375	5,875.4	0.6119	6,056.4	0.3383	6,264.8	0.24783	6,367.9	0.226
200	11,579	0.8993	11,601	0.57123	11,620	0.4924	11,727	0.3213	12,083	0.1737	—	—	—	—
500	28,533	0.3861	28,609	0.24765	28,661	0.2143	28,934	0.1409	29,796	0.07288	—	—	—	—
700	39,712	0.2863	—	—	—	—	—	—	41,455	0.05419	—	—	—	—
1,000	56,010	0.2105	56,209	0.13479	—	—	—	—	58,526	0.0404	—	—	—	—
2,000	107,877	0.1208	108,403	0.07733	—	—	—	—	112,693	0.02384*	—	—	—	—
3,000	156,342	0.09132	157,238	0.05910	—	—	—	—	163,265	0.01810*	—	—	—	—
4,000	201,848	0.07936	203,054	0.05888	—	—	—	—	210,580	0.01519*	—	—	—	—
5,000	244,710	0.07519	—	—	246,876	0.04474	—	—	—	—	—	—	—	—
5,500	265,800	0.07391	—	—	—	—	—	—	—	—	—	—	—	—

* Conductivity data were reported in I.C.T. for 122° F.; should be 212° F.

Table 4 — Electrical Resistivity (R_w), of Aqueous Sodium Chloride Solutions in Ohm-meters for Various Temperatures t as a Function of Sodium Chloride Concentration C_t or as Parts per Million by Weight.

Sodium Chloride Concentration (in ppm by weight)		TEMPERATURE IN DEGREES FAHRENHEIT						
		32°	64.4°	77°	122°	212°	284°	312.8°
60	$(R_w)_t$	147.0105	91.1872	78.1909	50.1520	28.6560	21.4719	19.6554
	$(R_w)_{32}/(R_w)_t$	1.	1.61215	1.88015	2.93130	5.13018	6.84665	7.47939
100	$(R_w)_t$	88.8074	55.0804	47.2224	30.2861	17.3268	12.9807	11.8824
	$(R_w)_{32}/(R_w)_t$	1.	1.61232	1.88062	2.93228	5.12544	6.84150	7.47386
300	$(R_w)_t$	30.1407	18.7255	16.0642	10.3099	5.9028	4.4425	4.0692
	$(R_w)_{32}/(R_w)_t$	1.	1.60961	1.87627	2.92347	5.10616	6.78462	7.40703
1,000	$(R_w)_t$	9.3543	5.81792	4.9988	3.24205	1.84581	1.39497	1.2844
	$(R_w)_{32}/(R_w)_t$	1.	1.60784	1.87131	2.88530	5.06786	6.70574	7.28301
3,000	$(R_w)_t$	3.25432	2.03200	1.75034	1.14824	.65155	.49395	.45834
	$(R_w)_{32}/(R_w)_t$	1.	1.60154	1.85925	2.83418	4.99473	6.58836	7.10023
6,000	$(R_w)_t$	1.67809	1.05717	.911432	.60004	.34124	.25784	.23948
	$(R_w)_{32}/(R_w)_t$	1.	1.58734	1.84116	2.79663	4.91762	6.50826	7.00722
10,000	$(R_w)_t$	1.03366	.656130	.56632	.37273	.208558
	$(R_w)_{32}/(R_w)_t$	1.	1.57539	1.82522	2.77321	4.95622
30,000	$(R_w)_t$.36897	.23729	.20548	.136328	.07243
	$(R_w)_{32}/(R_w)_t$	1.	1.55493	1.79565	2.70649	5.09416
100,000	$(R_w)_t$.12881	.08278902577
	$(R_w)_{32}/(R_w)_t$	1.	1.55588	4.99845
200,000	$(R_w)_t$.0797501549
	$(R_w)_{32}/(R_w)_t$	1.	5.14848
Avg.	$(R_w)_{32}/(R_w)_t$	1.00000	1.59078	1.85013	2.84238	5.05393	6.67350	7.24940

Table 5 — Electrical Resistivity (R_w), in Ohm-meters and Ratio $(R_w)_{32}/(R_w)_t$ of Aqueous Sodium Chloride Solutions for Various Temperatures t as a Function of Sodium Chloride Concentration C_t or as Parts per Million by Weight. The ellipses represent 13 data points that Arps has declined to interpolate/extrapolate.

APPENDIX D: Second Guessing Arps

There is a pitfall in decoding Arps' instructions. In conjunction with Arps' figure 2 (reproduced as figure 1 in this paper), it is easy to misread the following collection of passages [I have added italics to the tricky phrases]:

"The average ratios (*corrected for the missing high salinity figures by the trends in $(R_w)_{32}$ and $(R_w)_{212}$*) are shown *at the bottom of this table.*"

"... the averages are shown as circles ..."

"From inspection of the chart of Fig. 2 it will be observed that *the ratios $(R_w)_{32}/(R_w)_t$ very nearly seem to follow a linear pattern* with respect to temperature."

Arps' mention of his correction of the seven average ratios for missing data, his representation of the corrected averages with open circles and their seemingly being fit by a prominent trend line invites a hasty reader to assume that the line represents the best fit of these seven average data points. However, the best fit to these points gives a (slightly) different result ($T_0 = -6.7959$ vs. $T_0 = -6.7707$) than Arps' claims for his fit. Arps' intended meaning was that *all 57 of the ratios, not just the seven estimated averages*, be used in estimating the trend. Perhaps not all of Arps' readers would have been misled (as I was for a time). For those who have been, or would be, I have included this section.

APPENDIX E: Alternative Choices for T_0

Interpolating Missing Data. As I have already noted, the "scholarship" that I have done in researching this ar-

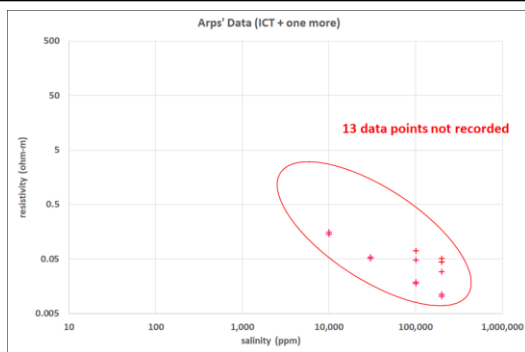


Figure E.1.a There are 13 lacunas in the data entries in Arps' Table 5. Their approximate locations in salinity-resistivity space are plotted here. It will be a challenge to fill these gaps as accurately as possible by interpolation and extrapolation.

ticle is incomplete, meaning that I did not examine the ICT volumes to check their original sources, nor attempt to discover the source of discrepancies in Arps' references. I did, however, examine reasonable alternatives to the choices that Arps made in analyzing the ICT data. As I have already noted, Arps chose only a subset of the 81 data points in the ICT volumes for his analysis, illustrating that he has "high graded" the data to be analyzed. This Appendix will present some alternative choices for exclusion of ICT data, and alternative methods of normalizing the data. These will lead to different (although not very different) values for T_0 .

Arps tells us *what* he did in minute (if sometimes unclear) detail. He tells us:

"... an attempt has been made in this study to show every step in the calculations."

and he has attempted to do so. However, he does not tell us *why* he made certain choices that he made. Maybe it did not occur to him that he was making choices (unwitting bias) from among various possibilities, and that is why he does not discuss his specific choices; he perhaps just assumed (without realizing it) that no other choices were possible. Unfortunately, it is too late to ask Arps.

In the following, I will not be "bashing" J. J. Arps, who worked with pencil and paper and an adding machine in the early 1950s. I will just be answering the question: are there other ways to process these data, and is $T_0 = -6.77$ the only, or best, value possible?

Extrapolating Missing Data. Arps' Table 5 does not include values for 13 pairs of temperature and resistivity points, representing the region of the ICT data listing at

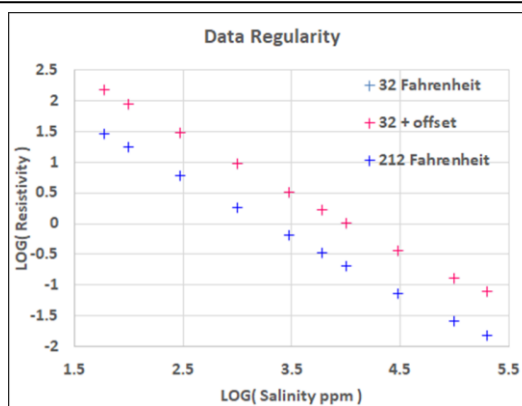


Figure E.1.b The trends at 32 and 212 °F.

high salinity and temperature where these data are also omitted (see Figure E.1.a). However, in estimating the averages of normalized resistivity, he simply offers “*corrected for the missing high salinity figures by the trends in $(R_w)_{32}$ and $(R_w)_{212}$* ”. I was unable to reproduce Arps’ estimated averages, so I cannot say what his method might have been. (I also note, as an aside, that it is curious that he did not fill these gaps in his Table 5 given that *all* of the numbers in the table have been interpolated to ppm from the values given in Tables 1 and 4 stated in mmol/L.) However, in order to proceed with my analysis, I needed to estimate the missing values.

Arps alludes to using “trends”. There are just two “complete” data columns in the Table 5 data; those for 32° and 212° F. Plotting the logarithm of resistivity against the logarithm of salinity (in ppm) for these data, it is ob-

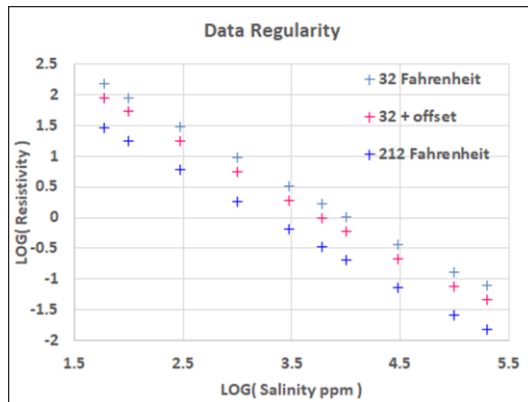


Figure E.1.c The trends at 32 and 212 °F, and also showing a “synthetic” trend, generated by adding a constant offset to the 32 degree trend.

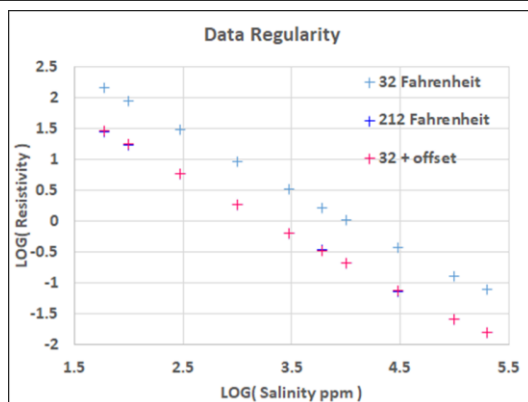


Figure E.1.d The trend at 212 °F is overlain by the synthetic trend with a 180 °F offset from 32 °F, illustrating its usefulness for interpolation (also used for extrapolation).

served that the slopes of the data appear to be nearly the same (see figure E.1.b). How nearly equal are they? To check this graphically, I added a “synthetic” data trend produced by adding a constant resistivity offset to the trend at 32° F. This trend is shown in figure E.1.c for an offset between 32 and 212 °F. Finally, figure E.1.d shows the synthetic trend almost exactly overlying the 212 °F observations when the offset is 180 °F, as can be seen by the red crosses overlying (and mostly hiding) the blue crosses. Assuming there is some error in the data listed both for 32 and for 212 °F, and assuming that the offset function is linear, then a linear offset from 32 °F is an adequate interpolating function between the two temperatures, and can also reasonably be expected to extrapolate to 284 and 312.8 °F.

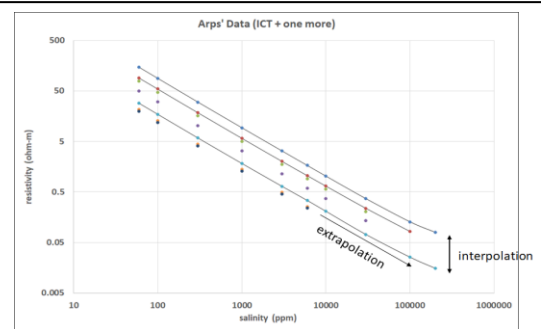


Figure E.1.e The trends at each of the seven temperatures can be extrapolated from the observations at lower resistivity and salinity to the missing data points at higher resistivity and salinity.

	32	64.4	77	122	212	284	312.8	
60	147.0105	91.1872	78.1909	50.152	28.656	21.4719	19.6554	
1	1.612184	1.880148	2.931299	5.130182	6.846646	7.479395		
100	88.8074	55.0804	47.2224	30.2861	17.3268	12.9807	11.8824	
1	1.612323	1.88062	2.932282	5.125436	6.841495	7.47386		
300	30.1407	18.7255	16.0642	10.3099	5.9028	4.4425	4.0692	
1	1.609607	1.876265	2.923472	5.10617	6.784626	7.407033		
1,000	9.3543	5.81792	4.9988	3.24205	1.84581	1.39497	1.2844	
1	1.607843	1.871309	2.885304	5.067856	6.705736	7.283012		
3,000	3.25432	2.032	1.75034	1.14824	0.65155	0.49395	0.45834	
1	1.601535	1.85925	2.834181	4.994736	6.588359	7.100231		
6,000	1.67809	1.05717	0.911432	0.60004	0.34124	0.25784	0.23948	
1	1.587342	1.841158	2.79663	4.917624	6.508261	7.007224		
10,000	1.03366	0.65613	0.56632	0.37273	0.208558	0.155426	0.144545	
1	1.575389	1.825222	2.773214	4.956223	6.650492	7.151132		
30,000	0.36897	0.23729	0.20548	0.136328	0.07243	0.054109	0.050515	
1	1.554933	1.795649	2.706487	5.09416	6.818965	7.304144		
100,000	0.12881	0.082789	0.071734	0.047595	0.02577	0.018884	0.017634	
1	1.555883	1.795655	2.70636	4.998448	6.821283	7.304602		
200,000	0.07975	0.050794	0.043934	0.028967	0.01549	0.011239	0.010455	
1	1.57007	1.815231	2.753108	5.148483	7.09557	7.62811		
1	1.58871	1.84405	2.82423	5.05393	6.76614	7.31387	Kennedy	
1	1.59078	1.85013	2.84238	5.05393	6.67350	7.24940	Arps	

Figure E.2 Arps’ Table 5 with the “data gaps” filled in with estimates, shown in color. Below the table I show the averaged “normalized” resistivity average estimates using the filled in table columns verses the values given by Arps in Table 5. Although similar, they are not the same.

There is another check on estimating the missing 13 data pairs. At each of the seven temperatures the data at low salinity provides a trend that can be extrapolated to the higher salinity points. Comparison of these extrapolations with the interpolated values is used to constrain the estimates to values compatible (to graphical accuracy) with both methods of estimation. (figure E.1.e)

With Table 5 now filled-in (figure E.2), the normalized resistivities can be calculated and the averages of the normalized resistivity computed. It is these averages that

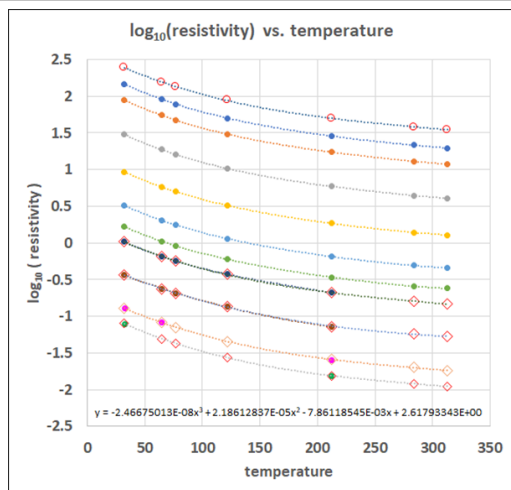


Figure E.3 A check on the quality of the interpolations. The plot is iso-concentrations from 60 to 200,000 ppm. The solid dots are observed data; the open diamonds are predictions of an extrapolating function (3rd order polynomial). Note the predictions enclose the observations where observations exist, and fall on or near the similarly-shaped trend lines defined by the observations. (See text for further explanation)

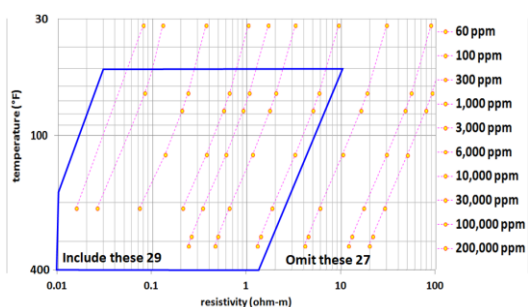


Figure E.4 Arps omitted 25 points from the ICT data that did not correspond to oil-field brines. He might well have excluded additional data.

are used in Arps' figure 2 plot. Although the averages are different, at the scale of the plot these differences are not visible.

Figure E.3 illustrates the regularity of these data when plotted as logarithm of resistivity versus temperature.

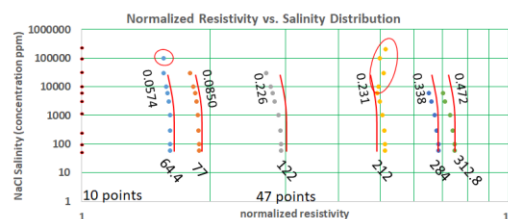


Figure E.5 Salinity versus normalized resistivity. The plot shows that only the data at 32 and 212 °F covers the entire salinity range. The data groups for each temperature show nearly vertical trends for all. The trend at normalized resistivity equal 1 is vertical since by definition all of these normalized resistivities are equal to 1. The trends are labeled at bottom with their respective temperatures, and at the top with the width of the distributions in normalized resistivity for that temperature. The shape of the trends is indicated by the offset red lines. The figure shows that the top four points at 212 degrees are off the trend, and the top one (or maybe two) points at 64.4 degrees are also off the trend. These anomalies are most apparent only in this plot.

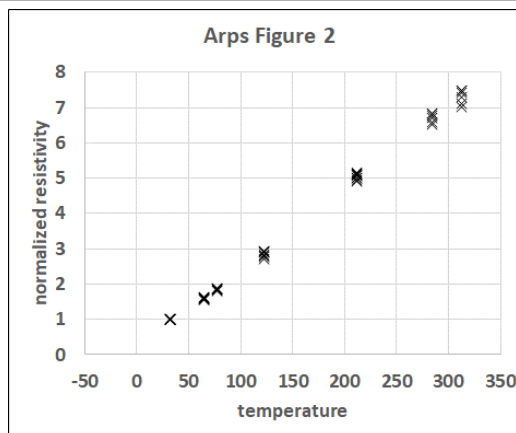


Figure E.6 Resistivity “normalized” to 32 °F versus temperature as Arps plotted it in his figure 2. Note the “spread” in the data for all temperature greater than 32 °F, where all of the normalized resistivities are equal to 1, and plot at the same point.

The solid colored dots represent the ICT data; the open diamonds represent the interpolated/extrapolated predictions. Where measurements exist the locations of the data are very well predicted; this suggests that predictions where there is no data will be adequate representations of the missing data.

Excluding Irrelevant Data. The ICT data set comprises 81 points ranging in concentration from 0.1 mmol/L to 5500 mmol/L and in temperature from 32 °F to 312.8 °F.

These concentrations correspond to a salinity range of 5.8 ppm to 265,800 ppm. Arps omits salinities of less than 60 ppm and more than 200,000 ppm from his analysis; he retains all of the temperatures. The justification for omitting low salinity would be that such low salinities are not encountered in practice; conversely, high salinities up to saturated solutions are indeed encountered in formation brines. However, leaving them out of the analysis focuses attention on the almost linear parts of the iso-concentration curves. Thus, Arps analysis includes only 57 data points compared to the ICT's 81 data points.

You might well ask why include the data at 32 °F since such low temperatures would never be encountered in the subsurface, and even on a very cold day at the surface a mud sample caught at the shale-shaker will be warm or hot, and measured in a mud-logger's shack or logging unit would not cool down to less than comfortable temperatures. Likewise, salinities of 60, 100, and 300 ppm are essentially no more saline than water from a kitchen tap¹². If the ICT samples at 32 °F and salinities of less than 1000 ppm are excluded, then the number of Arps interpolated data points remaining is just 29 with 27 of Arps' 57 data points omitted. You can see that observations in the real world would seldom or never fall into the range of the omitted data, and excluding them from the analysis might well improve the quality of the estimates made in the actual range of most wellsite data. (figure E.4)

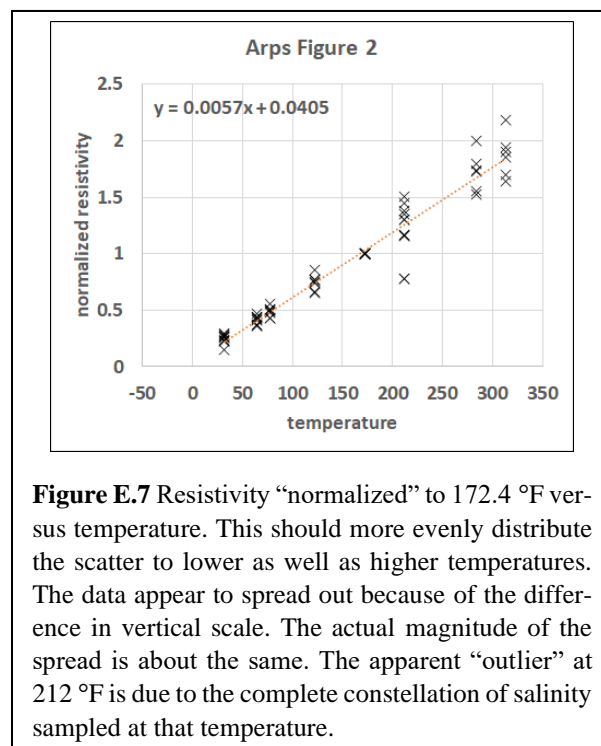
Using these 29 data points, the estimate for $T_0 = -4.2744$.

(RE-)NORMALIZING THE RESISTIVITY DATA

Arps method depends upon a plot of averaged "normalized" resistivity data versus temperature. "Normalized"¹³

is my word for the quotients of brine resistivity at a given salinity and across a range of temperatures divided into the brine resistivity at 32 °F. Arps calls these quotients "ratios". The normalized data at each temperature tends to be about the same. As Arps puts it: "*these ratios are reasonably constant for a given temperature throughout the entire salinity range studied*". (figure E.5) This is what makes Arps method work. Since Arps uses 32 °F as his reference temperature, this has the effect of making all of the normalized resistivity values at 32 °F equal to 1 (see figure E.6). We could well ask whether a temperature at the limit of the temperature domain for this data is the best choice to use. Not only is 32 °F at the left limit of the range, it is also out of the temperature range that would normally be found in field data.

Figure E.7 plots the resistivity data normalized to the mid-temperature of the range, 172.4 °F. Note the reduced range on the vertical axis due to the increased value of the normalizing temperatures at 172.4 °F, and the reduction in the spread of the range of the remaining normalized temperatures. Dividing the intercept by the slope in this plot gives $T_0 = -6.8801$.



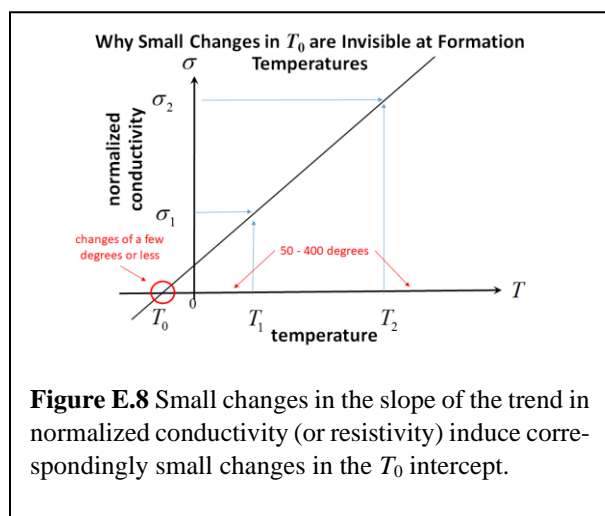
¹²The upper limit on potable water is about 1000 ppm, with fresh water supply limited to 500 ppm; tap water in the United States is typically 100 ppm or less. https://www.engineeringtoolbox.com/water-salinity-d_1251.html

¹³The usual convention for "normalizing" a parameter is to divide it by some reference value. Arps is dividing his data into the reference value, rather than by the reference value. You may note that in conductivity terms, Arps' method conforms to the usual definition of normalized conductivity.

DOES IT MATTER?

Yes, but not much! T_0 is the intercept on the horizontal axis of the line defined by the trend of the average normalized resistivity values versus temperature fit by the least squares method. The temperatures of interest will (at least mostly) lie between 50 and 400 °F; the slope of the line in this range will tend to be almost the same regardless of whether or not low salinity points are omitted from the calculation of the normalized resistivity. Since the locus of the remaining average normalized salinity does not change much, the intercept will not change much either, as shown in figure E.8.

The effect on interpretation is illustrated in figure E.9. Consider a brine having resistivity of 10 Ω -m at 50 °F. Using Arps' approximation ($T_0 = -6.7707$) the resistivity at the opposite temperature limit of the chart is estimated at 1.40 Ω -m at 400 °F; using $T_0 = -4.2744$ °F the resistivity estimate is 1.35 Ω -m, a difference of 0.05 Ω -m, hardly visible when using the Gen-6 chart. At the opposite end of the chart, the minimum resistivity is 0.06 Ω -m at 50 °F; the maximum resistivities using the two values of T_0 are 0.008374 Ω -m using $T_0 = -6.7707$ and 0.008055 Ω -m using $T_0 = -4.2744$. The difference between these values is hardly visible on the chart.



The effect of renormalization to 172.4 °F is to estimate $T_0 = -6.8801$ °F. The differences in the approximation using this reference temperature would be truly negligible and hardly visible on a Gen-6 chart, or not at all.

TWO RECOMMENDATIONS

The Arps approximation is, well, an approximation, and not an exact relationship. $T_0 = -6.77$ may not be the optimal value (if there is one) but small changes in T_0 will not much affect the results of the approximation formula. So, should there be an effort to change T_0 to a “more correct” and justifiable value? My recommendation has to be “no” not only because the difference that this would make in interpretations would be minuscule, but also (based upon my experience) it is doubtful whether petrophysicists could ever reach any consensus regarding a “Most Correct” estimate of T_0 .

Perhaps we could agree upon two improvements in practice going forward: (1) formation evaluation educators ought to inform students of the choices that Arps made in determining T_0 , and (2) log analysis software ought to provide a choice of T_0 values including user-defined values.

