Formation Resistivity Factor and Permeability Relationships in Rocks Characterized by Secondary Solution Porosity

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Abstract: Values of porosity exponent m obtained on a suite of rock samples having varying amounts of secondary porosity tend to be quite variable and unusually high. The variability complicates the prediction of electrical properties and calculated hydrocarbon saturations in these rocks. An understanding of how isolated void space in the form of vugs or moldic pores modifies the electrical behavior of the host rock can obviate problems with prediction and use of porosity exponents in formations with secondary porosity. Since the dominant pore-geometric factor controlling the flow of fluids and electric current in the pore space is the intergranular matrix of the host rock, addition of isolated voids affects the porosity in direct proportion to the void space added; however, the resistivity is much less affected. To correctly interpret the electrical (and, to a lesser extent, hydraulic) behavior of such rocks the two components of porosity must be separated. The Maxwell-Garnett equation can be used to describe F in terms of matrix porosity and a dilute concentration of uniform spherical vugs. This paper demonstrates that the Maxwell-Garnett relationship can be applied with a high degree of accuracy to numerical models which have a realistic concentration of vugs as well as real rocks. The separation of porosity components using the Maxwell-Garnett relationship is accomplished using core and log data. In favorable circumstances, the core data may not be required. The suggested separation of porosity components results in better estimates of water saturation and more reasonable porosity-permeability transforms than are usually obtained in vuggy rocks.

INTRODUCTION

For over fifty years, quantitative resistivity log interpretation in terms of hydrocarbon content has been based on Archie's well known empirical formula (Archie, 1942). Archie's law predicts the relationship between formation resistivity factor F and porosity ϕ by means of the power law $F = \phi^{-m}$ where m, the porosity exponent*, is an empirically determined constant corresponding to the slope of $F(\phi)$ plotted on log-log paper. F relates the bulk resistivity R_0 to the resistivity of the interstitial brine R_w. Thus knowledge of ϕ can be transformed into a prediction of R_0. If the actual formation resistivity R_t is greater than R_0, the increased resistivity is attributed to hydrocarbons having displaced conductive brine. Hence correct interpretation of hydrocarbon volume hinges on (1) the validity of the power law relating F to ϕ and (2) a reliable value for m. Unfortunately, Archie's method does not apply universally to petroleum reservoir rocks. Archie's observations were developed using clean sandstones from the Gulf coast. The pore system in these rocks consists only of intergranular porosity. Difficulties arise when Archie's equation is applied to the interpretation of sediments that are more complicated than the intergranular pore systems characteristic of clean sandstone.

The electrical properties of rocks can be classified according to whether or not they can be described by Archie's empirical law relating F and ϕ (Herrick, 1988). Rocks that exhibit Archie-law electrical behavior are described as "Archie" rocks; the remainder are "non-Archie" rocks. A reservoir rock's electrical behavior is influenced by the arrangement and conductivity of its constituent mineral grains and corresponding pores, as well as the distribution and conductivity of its pore fluids. If the pore geometry is characterized by a complex pore system that contains secondary, vuggy or moldic pores in addition to the intergranular pore system, then the electrical behavior is not readily described by Archie's equation.

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This article considers the case of bimodal pore systems consisting of vuggy or moldic pores in addition to the intergranular or intercrystalline pore system. The contribution of minerals such as certain clays or sulfides to the bulk conductivity is not considered. The electrical properties of this bimodal pore geometry are not readily described by Archie's equation. As a notational convenience, we shall use the term "vug" throughout the remainder of the paper to mean large pores, often formed by dissolution, which are significantly larger than the intergranular or intercrystalline pores of the host matrix. Vugs frequently result from the dissolution of a portion of the granular or crystalline rock matrix. Examples include vuggy and oomoldic limestones and dolomites, and sandstones with grain-sized moldic pores resulting from the preferential dissolution of certain constituents. The vugs or molds may be isolated from each other and electrically connected only through the intergranular pore system, or they may be physically connected to each other making more or less continuous channels through the rock. Several kinds of electrical behavior may result from the way in which vugs and molds are related to each other and to the intergranular pore system.

Regardless of its appropriateness, Archie's law is frequently applied to non-Archie rocks. Consider, for example, the F and ϕ data given in Figure 1a. The data are plotted in the conventional way. The scatter in the data

* This is sometimes referred to as the "cementation" exponent; our feeling is that this term nology manages to be simultaneously both too vague and too suggestive. The term "porosity exponent" is merely descriptive of the form of the terms in Archie's law and parallels the term "saturation exponent"

is treated as if it were due to experimental error, and a single best-fit line is drawn through the points giving an average value of $m = 2.7$. This method of determining an m value for use in Archie's equation is widely used in the core analysis service industry, but fails to account for the bimodal nature of the pore system. A second approach is the use of a separate porosity exponent for each rock sample (Figure 1b). Focke and Munn (1985) used this approach and correlated the m values of rock samples with porosity to allow prediction of m from porosity, assuming that the porosity variation was primarily due to differences in vuggy porosity. This procedure breaks down if there are variations in matrix porosity as well as vuggy porosity since a large range in F is possible for a single value of porosity.

As illustrated by the examples in Figure 1, the electrical behavior of non-Archie rocks is poorly understood. In the case of rocks having vuggy porosity this is partly due to the difficulty in varying the parameters of experimental models designed to study this behavior. Core samples, for example, do not provide data with systematically varying parameters. Laboratory models of rocks containing vugs present the would-be experimenter with difficulty in controlling the placement and size of the vugs. The problem can be examined theoretically using the Maxwell-Garnett (1904) canonical model; however, its applicability to real rocks (which violate some of the conditions imposed in deriving the canonical model) can be questioned. Consequently, numerical modeling seems to be an attractive option; unfortunately, numerical description of real rocks is intractable. Even if such description were feasible then numerical experiments would falter on the size of the resulting problem. The latter difficulties can be surmounted if we are prepared to accept certain (reasonable) limitations of our models wherein the results of small, simple numerical experiments are combined to simulate the results of more realistic, but intractable, modeling.

This paper reports on the methods and conclusions of our investigation of the impact of certain geometrical effects on the relation between F and ϕ in non-Archie rocks characterized by vuggy porosity. The applicability of the Maxwell-Garnett relationship is examined by comparison to rock-like two- and three-dimensional numerical models. The relationship holds even for high vuggy porosities, despite having been developed for "dilute" vug concentrations. Separation of total porosity into intergranular and vuggy porosity components permits interpretation of F in vuggy rocks as well as the development of more meaningful porosity-permeability transforms.

THEORY

A Canonical Model from Electromagnetic Theory for Vuggy Rocks

Rasmus and Kenyon (1985) based an oomoldic rock model on the Maxwell-Garnett (1904) relationship for the conductivity C, at low fequencies, of a "dilute" mixture of conductive spherical inclusions in a conductive host matrix:

$$C = C_i \left[\frac{1 + 2\phi_v \left(\frac{C_v - C_i}{C_v + 2C_i} \right)}{1 - \phi_v \left(\frac{C_v - C_i}{C_v - 2C_i} \right)} \right]. \qquad (1)$$

C_v is the conductivity of the spherical inclusions, ϕ_v is the volume fraction of inclusions, and C_i is the conductivity of the matrix consisting of nonconducting mineral grains with intergranular porosity. If the rock is water saturated, the conductivity of the inclusions (secondary pores) may be assumed to be that of bulk water C_w, while the matrix conductivity can be determined by an appropriate Archie equation. Thus, making the notational correspondances $C_w = C_v$ and $C_0 = C_i = \phi_i^m C_w$, and using (1), F for the model can be expressed as

$$F = \frac{1}{\phi_i^m} \left[\frac{1 - \phi_v \left(\frac{1 - \phi_i^m}{1 + 2\phi_i^m} \right)}{1 + 2\phi_v \left(\frac{1 - \phi_i^m}{1 + 2\phi_i^m} \right)} \right]. \qquad (2)$$

This theoretically obtained F applies to a host rock having matrix porosity ϕ_i with an Archie exponent m for the intergranular component, and a vug porosity component of ϕ_v. It can be shown that the total porosity of this model is given by $\phi = \phi_i + \phi_v - \phi_i\phi_v$; the $\phi_i\phi_v$ term in the equation corrects the total porosity for the effect of removing regions of host rock (including both mineral grains and brine) and replacing them with mineral-free, brine-filled vugs. Equation (2) holds for a "dilute" concentration of spherical vugs of equal radius distributed uniformly throughout the host rock. However, how well (indeed, whether) equation (2) approximates actual rocks with non-spherical vugs of different sizes, shapes and perhaps a large component of vug porosity is open to question. The answer must be sought by comparing the predictions of the Maxwell-Garnett model with those of carefully constructed, but cumbersome, numerical models.

Numerical Modeling the Electrical Behavior of Rocks

The distribution of electrical potential $V(x,y,z)$ due to a source of current I located at $\mathbf{r}'(x',y',z')$ in a medium of conductivity $\sigma(x,y,z)$* is governed by

$$\mathrm{div}(\sigma\,\mathrm{grad}V) = I(\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') \qquad (3)$$

where the delta function locates the source and $\mathbf{r}$ is a field point (Aiken, et al, 1973). Once the potentials have been determined for specified σ and I, the current densities $\mathbf{J}$ are determined from the field equation analog of Ohm's law: $\mathbf{J} = \sigma\mathbf{E}$ where the electric field $\mathbf{E} = -\mathrm{grad}\,V$. By integration of $\mathbf{J}$ over one of the boundary surfaces the total current I_t flowing in a model due to potentials imposed on two equipotential surfaces bounding the model can be determined. The bulk resistance of the model is obtained as the ratio of $\Delta V/I_t$, where ΔV is the potential difference between the equipotential surfaces bounding the model rock. Bulk resistivity is determined from sample geometry and bulk resistance. Knowing the bulk resistivity of a model together with the resistivity of the interstitial brine, F can be readily computed. Equation (3) can be solved using any convenient method. We employ finite differences.

Our models comprise distributions of conductivities representing grains, pores, vugs, and rock matricies, applied potentials on parallel model boundaries, and periodic boundary conditions on the remaining boundaries; the calculation solves (3) over a finite difference grid to give the potential distribution. Since the finite difference technique requires that the model be discretized at a scale significantly smaller than the mineral grains and pores, a three-dimensional finite difference model of even a small rock sample is not computationally feasible. Certain simplifying assumptions were required to reduce the size of our models. Our models were constructed from "unit cells" representative of the pore systems studied. These unit cells can be subsequently combined to simulate a rock, obviating the need for very large models. To minimize the computational burden, two-dimensional models were used to develop the basic results; subsequently our important conclusions were validated using three-dimensional models.

* The symbol σ for conductivity is used to conform with the standard electromagnetic notation for a "point" conductivity. The symbol C used earlier is the bulk conductivity as usually expressed using oil-industry notation.

MODELING CONSIDERATIONS

Computing F values of Numerical Models

F can be computed directly from the finite difference model for any specified pore (and grain) geometry. The finite difference model gives the potential distribution and potential gradients throughout the specified pore system. Since any electric currents which enter the unit cell must also exit the unit cell (there are no current sources or sinks in a rock), the total current flowing through the unit cell must be equal to the current entering the cell via the conductive pore throats. Once $V(x,y,z)$ is determined, current is calculated from Ohm's law using the resistance (determined from model resistivity and discretization interval) and the potential gradient across the first row of computational nodes in the pore throats through which currents enter the cell. The resistance for the whole unit cell is obtained by Ohm's law from the total current flowing through the cell and the applied potential across the cell; resistivity is then determined using the cell geometry. F is readily determined from the ratio of the unit cell resistivity to the resistivity of the water filling the pore space of the cell. F is thus clearly seen to be determined solely by pore-geometry.

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Modeling Isolated Vuggy Porosity

Two-dimensional models containing physically isolated but electrically connected vugs (through the intervening intergranular pore space), can serve as good approximations of corresponding three-dimensional vuggy rocks. The details of the intergranular pore system need not be considered to specify the bulk resistivity of the host rock. Models are constructed which contain vugs of a desired shape in a host rock of otherwise uniform electrical properties. The host rock is assigned properties of real rocks. For example, porosities in the range of 10 to 30 percent, and porosity exponents in the range of 1.8 to 2.1 are representative of most Archie rocks; parameters in these ranges are used to model the host rock matrix.

F-ϕ Relationship for Single-Vug Unit Cells

Figure 2 shows two dimensional finite difference models of a uniform host rock having 10 per cent intergranular porosity ϕ_i and porosity exponent $m_i = 2$, along with circular and elliptical vugs imbedded in the same host. Current streamlines demonstrate the perturbing influence of the conductive vugs in the relatively resistive host rock. F values were calculated from the finite difference models, and are shown on an F-ϕ plot in Figure 2. F values for vugs with radii of 6, 8, 10, 12 and 14 finite difference grid units are shown as open squares on Figure 2 along with a least squares best-fit line drawn

through these points. If the porosity exponent m for a group of samples is regarded as the slope of a linear F-ϕ relationship, then m for these circular vug-bearing two-dimensional models is only 0.64. The intercept of the line corresponds to the factor a, a modification often made to Archie's equation (Winsauer, et al, 1952), which has a value of 8.3.

Qualitatively, low values of m for suites of vuggy rocks with a similar matrix are easily understood. Since the matrix of the examples in Figure 2 has only 10 per cent pore space, the addition of a vuggy pore has a direct impact on the cell porosity. The cell conductivity, on the other hand, is affected much less directly. Reference to Figure 2 illustrates that current paths in a vug-containing cell are generally longer than corresponding paths in a vug-free cell. Such longer current paths are more resistive than corresponding paths in vug-free cells if they do not intersect a vug, but may be either more or less resistive if a vug is intersected, depending on the relative lengths of the path segments inside and outside the vug. The net result is that the resistivity of a vug-containing cell decreases only slightly with each incremental increase in vug porosity. A change in vug radius, therefore changes the cell porosity proportionately, but F changes only to a small extent. Hence the low slope of the F-ϕ relationship for models and rocks with varying vuggy porosity, but constant matrix properties.

The shape of a vug also affects F. F values of models with elliptical vugs having about the same area as a circular vug with radius 10 grid units are displaced somewhat above and below the relationship defined by the circular vugs (Figure 2), depending on orientation. The main impact of secondary pores is on the porosity, and only secondarily on F, regardless of vug shape. Only a small change in slope results by substituting either of the elliptical vugs for a circular one. The effect of isolated conductive vugs in a resistive host rock is a F-ϕ relationship characterized by low slope and a large intercept.

F-ϕ Relationship for Multiple Random Vug Models

If the size of a cell containing a vug is made large enough relative to the vug size, then the electrical effects of the vug on the potential gradients and current streamlines can be substantially restricted to the interior of the cell. The electrical properties of cells contiguous to a vug-bearing cell can thereby be made independent of the vug. Hence other cells can be placed around a vug-bearing cell and the electrical properties of the aggregate can be calculated from a series/parallel resistor network using the resistances of the individual cells. This approach greatly simplifies the computation required for determining the F of a multicell, multiple-vug model. The resistance of each type of matrix and vug cell can be determined separately using the finite difference method. The finite difference representation of the entire problem is not required, being replaced with a smaller resistor network model. The electrical independance of contigous cells also approaches the "dilute" concentration of vugs required by the Maxwell-Garnett relationship.

The validity of treating combinations of matrix and vug-bearing cells as independent conductive elements is verified by comparing the results of a finite difference calculation made on three contiguous cells with the conductance of the three cells treated as separate conductive elements in parallel (Figure 3). The conductance of the two matrix cells and one vug-bearing cell (radius=14 units), when treated as a unit, differed by only 0.2 per cent from the sum of the conductances of the individual cells. If the ratio of the vug diameter to the unit cell dimension is on the order of 1:2 or less, then the conductance of a two-dimensional "rock" model consisting of many host rock and vug cells is determined from a simple resistor network calculation.

Large multiple-vug models can be constructed by randomly selecting the size of the vugs and vug-bearing cells (Figure 4). The resulting model is very similar to thin sections made from some oomoldic limestones. The amount of vug porosity can be varied by changing the number of vug-bearing cells to determine the effect of varying vuggy porosity on F. Square models made up of one hundred-cells (10x10) were constructed to evaluate the F-ϕ relationship for rocks with constant matrix pore geometry, but varying vuggy porosity. The fraction of vug-bearing cells was varied from 0 to 35 per cent. The F-ϕ relationship is given in Figure 5 for a matrix with $\phi_i = 10$ and $m_i = 2$. The result is similar to that seen previously in Figure 2. The slope of the F-ϕ relationship is low ($m = 0.32$) and the intercept is considerably above the origin ($a = 14.6$). The explanation is also similar. The porosity is greatly affected by the vug volume, while the resistance is only slightly reduced by the addition of vugs since it is controlled by the intervening resistive host rock.

Models similar to that shown in Figure 4 can be used to determine the porosity and F for varying matrix and vuggy porosity. The results are compared to those predicted by the Maxwell-Garnett relationship (Figure 6). The agreement is remarkable indicating that the requirement of the Maxwell-Garnett relationship for a dilute concentration of vugs is comparable to the electrical independence of neighboring cells in the 2-D model.

Similar numerical experiments were conducted for three dimensional models made of spherical vugs in cubic cells. Porosity was varied by changing the vug size. For

vug diameters less than about 70% of the cubic cell edge (vuggy porosity less than 20%), electrical independence between cells is maintained in three dimensions and again there is excellent agreement between the F values determined numerically for the models and for those predicted by the Maxwell-Garnett relationship (Figure 7). For vugs having vug-diameter/cell-edge ratios greater than 0.7, the F predicted by Maxwell-Garnett is slightly higher than that determined by the model. The equipotential surfaces in neighboring cells are perturbed by the presence of the vug when the vug diameter is greater than about 70% of the cell edge. The "dilute" vug concentration criterion needed for application of the Maxwell-Garnett relationship no longer holds, however the error resulting from use of the relationship is small, even at vug-diameter/cell-edge ratios as large as 0.9 (corresponding to a vuggy cell porosity of 40%). A ratio of 0.9 results in an underestimation of F by only 8% when using the Maxwell-Garnett relationship.

The two- and three-dimensional numerical modeling experiments described above have shown that the Maxwell-Garnett relationship can be used to predict F values for vuggy rocks from matrix and vuggy porosity components with a high degree of accuracy, even when the vuggy porosities and vug sizes indicate that the requirement of a "dilute" concentration of vugs is not met. The Maxwell-Garnett relationship specifically applies to spherical vugs, but it will give a good approximation for rocks with roughly equidimensional vugs. Even if the vugs tend to be oblate or prolate, the prediction of F is not seriously affected (Figure 2). Hence, the Maxwell-Garnett relationship can be used as a viable predictor for F of most rocks composed of a porous conductive matrix containing vugs, molds or secondary solution porosity.

Application of the Maxwell-Garnett Vug Resistivity Model

Interpretation of F and ϕ data for vuggy rocks depends on resolving the total pore space into separate porosity components accounting respectively for host rock (or matrix) porosity and vug porosity. The storage capacity of the rock is greatly influenced by the vuggy porosity, while the electrical properties and the fluid flow properties are governed by the matrix porosity, particularly the pore throats of the intergranular pore system. The Maxwell-Garnett vug resistivity model can be visualized in a useful way by plotting curves of constant ϕ_i and of constant ϕ_v on an F-ϕ plot (Figure 8). The location of the resulting grid lines depends on the matrix porosity exponent m_i. Values of m_i for indurated rock matrices having exclusively intergranular porosity generally lie in a restricted range averaging about 2.0. Therefore, if the porosity exponent of a rock matrix is known, or assumed to be about 2, and if F and ϕ are known, the matrix and vuggy porosity can be separated numerically or graphically. For example, a rock having $F = 46$, and $\phi = 25$ per cent has about 12 per cent matrix porosity, and about 15 per cent vuggy porosity (Figure 8).

POROSITY-PERMEABILITY RELATIONSHIPS

The method described for separating the components of porosity can be used to determine the matrix porosity of a series of rocks, which in turn can be used to rank them in terms of permeability. Consider the porosity and F data plotted in Figure 1. As discussed in the introduction, a widely used method of determining a porosity exponent for the data is to treat the scatter in the data as if it were experimental error and attempt to fit a line (or perhaps some other functional form) to the data using least squares (Figure 1a). However, the scatter exhibited in the data for vuggy rocks is usually not due to experimental error—the data obtained are repeatable. Alternatively, the construction of a function relating m to porosity can be attempted (Focke and Munn, 1985), but the effort will fail if the porosity components vary independently and simultaneously giving a range of F values for a singe value of ϕ. Only by recognizing the bimodal distribution of porosity in these rocks can the difficulty be resolved.

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The Maxwell-Garnett vuggy rock resistivity model allows a better porosity interpretation than either of the previous approaches. The model separates total porosity into matrix and vuggy porosity components. The matrix porosity exponent can either be known from core measurements, or assumed with a reasonable degree of confidence since the value of m_i is usually close to two, typical of an intergranular pore system. The value of F as a function of total porosity, then depends on the amount of vug pore space added to the intergranular porosity (Figure 9a). Using the graphical representation of the Maxwell-Garnett vug resistivity model (Figure 9b), the pore space of each example can be apportioned between matrix and vugs. Note that the matrix porosity of the two examples having the highest F values is about the same, while the matrix porosity of the example with the lowest F value is about twice that of the other two examples.

While there is rarely a good porosity-permeability relationship for vuggy rocks, there is often a well-determined relationship for rocks having only clean intergranular pores. Since the pore space in rocks with isolated vugs is connected only through the intergranular pore system of the matrix, the permeability k of this rock-type should correlate with the matrix porosity estimated by the Maxwell-Garnett vuggy rock resistivity model. Example "b", Figure 9b, should, therefore, have

the highest permeability, while examples "a" and "c" would be expected to have similar permeabilities, but lower than example "b". Note that the permeability estimated for these examples does not correlate with the total porosity, but only with the matrix porosity. Permeability prediction in a relative sense can be made using the Maxwell-Garnett vug resistivity model, as long as the pore geometry of the real rock is consistent with the model. It is not necessary to know the actual value of the porosity exponent of the matrix, it only needs to remain relatively constant.

Data from the Khuff reservoir, North Field, Qatar, described by Focke and Munn (1985), can be used as an example of permeability estimation. Their data are plotted in Figure 10. Two types of pore systems were identified: carbonate rocks with primarily intergranular porosity, and rocks with predominantly vuggy (or oomoldic) porosity. Three permeability classes were identified: $k < 0.1$ md; $0.1\text{md} < k < 1\text{md}$; and $k > 1$ md. If it is assumed that the m_i values of all the rock samples are similar, then the permeability can be related to the matrix porosity estimated from the Maxwell-Garnett vug resistivity model. Inspection of Figure 10 shows that, while the permeability groups overlap considerably, the center of the lowest permeability group ($k < .01$md, circle symbol) corresponds to the lowest matrix porosity, the middle permeability group (.01md < k < 1md, plus symbol) relates to an intermediate value of matrix porosity, and the center of the highest permeability group (k > 1md, triangle symbol) is consistent with the highest matrix porosity. The considerable scatter in each group is due to the broad permeability range defined by each group, and the lack of petrographic information on the degree of isolation of the vugs and on the nature of the matrix pore geometry, particularly the role of microfractures which could dramatically influence such low permeabilities and affect m_i. The influence of the matrix pore geometry on the permeability of these rocks is still strongly suggested.

A second example of the potential of the Maxwell-Garnett vug resistivity model for permeability prediction is given in Figure 11 for oomoldic carbonates from another Mideast field. Porosity, permeability and F were measured on core samples. The data are given in Figure 11a. The correlation between porosity and the logarithm of permeability is poor, as it is for most oomoldic and vuggy carbonates, and is not statistically significant at the 95 per cent confidence level (Figure 11b). On the other hand, the correlation between the matrix porosity, as estimated from the Maxwell-Garnett vug resistivity model (Figure 11c), and the permeability is statistically significant at the 95 per cent confidence level (Figure 11d). If porosity and resistivity log data are available from water saturated or flushed zones for similar oomoldic parts of the formation so that F can be obtained, then the Maxwell-Garnett vug resistivity model can be used to estimate permeability in a relative sense from commonly available log data. A quantitative estimate of the permeability can be made using the correlation equation shown in Figure 11d.

MOLDIC POROSITY

Moldic pores result from the dissolution of individual grains or particles forming pores which are grain-sized. As normally used, the term "mold" may refer to pores of any size resulting from the dissolution of fossils, oolites, pisolites, etc., which may be much larger than intergranular pores in a finer grained matrix surrounding and connecting the molds. Microporous grains are partially dissolved or altered matrix grains which contribute to the bulk porosity and tend to remain water saturated due to the large capillary forces associated with the micropore structure. In this paper, the term "mold" is restricted to matrix-grain sized pores. Thus, the pore geometry of surrounding matrix pore space must be considered in modeling the effects of moldic porosity on the electrical properties of rocks.

Modeling the effects of moldic and microporous-grain porosity numerically is more demanding than modeling the effects of vugs since the entire pore structure must be included in the model. Evaluation of large three dimensional grain structures with our resources is prohibitive. We have modeled two dimensional hexagonal grain packs with moldic porosity and microporous grains. Unfortunately, to have adequate discretization of the pore throats to avoid discretization artifacts, requires relatively large pore throats such as depicted in Figure 12. The resulting two-dimensional porosity is unrealistically high, however, the electrical effects of additional moldic porosity or microporous grains behave qualitatively as predicted by the Maxwell-Garnett relationship.

A hexagonal unit cell of circular grains is shown in Figure 12a. The shading patterns illustrate the potential distribution. Current streamlines are depicted by solid curves between grains. If the central grain is removed, a moldic pore results as shown in Figure 12b. The current distribution and flux was determined for both cells by the finite difference method from which F values were calculated. The F values are plotted as a function of porosity in Figure 12c. The m value associated with the change in F with ϕ ($m = 1.2$) is greater than that observed for vugs (Figure 2), however it is still less than that due to changes in intergranular porosity alone.

A strategy similar to that used for vugs can be used to evaluate the electrical effect of varying moldic porosity in large (100 cell) numerical models. The influence of a moldic pore or microporous grain on neighboring cells

having only intergranular pores can be determined by modeling three cells separately, then as a single three-cell unit (Figure 13) and comparing the results. Only a small difference exists between the sum of the currents of two cells with only intergranular porosity plus one cell with a moldic pore and the three cell aggregate model (Figure 13b). The difference is attributed to the discretization requirements which have exaggerated the size of the pore throats relative to the size of the moldic pores in two dimensions. Hence, it is possible to treat large multicell two-dimensional moldic porosity models as simple series/parallel resistor networks in the same way multicell vug models were treated.

The F-ϕ relationship for varying moldic porosity can be determined using large multicell network models. F and ϕ were calculated for a one hundred cell model having only intergranular porosity (Figure 14a), and for models of the same size containing from 5 per cent to 35 per cent cells with moldic pores (Figure 14b). The F-ϕ relationship is given in Figure 14c. The increase in porosity due to the addition of 35 moldic pores is small relative to the high porosity of the two-dimensional hexagonal grain distribution (Figure 14a). The slope of the relationship between porosity and F, given in Figure 14c (m =1.2), is the same as that for individual unit cells (Figure 12c).

The explanation of the low value of m for 2-dimensional moldic rock models is similar to that for vuggy rock models. The formation of a moldic pore increases the porosity and the conductance within the pore, however, the resistance of the entire model is governed mainly by the pore throats which are unaffected by the porosity increase.

CONCLUSIONS

The current distribution and formation resistivity factor-porosity relationship of a specified pore system can be calculated solely from geometric considerations using a finite difference solution of Laplace's equation and a digital model specifying the conductivity and distribution of conducting and nonconducting phases. Models containing vugs which are isolated from each other, but electrically connected through the intervening intergranular pore space of a porous matrix, can serve as good approximations of corresponding vuggy rocks. To a good approximation, the Maxwell-Garnett vug resistivity model can be used to apportion the total porosity between vuggy porosity and matrix porosity for real rocks. Despite being developed for a dilute concentration of vugs, the Maxwell-Garnett vug resistivity model gives a good prediction of F, even for vuggy porosities as high as 40%. Since the intergranular porosity of the matrix controls the flow of fluids through the rock, an estimate of the matrix porosity from the Maxwell-Garnett vug resistivity model can be used to estimate the permeability of vuggy rock samples.

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W. David Kennedy's research interests are centered on the application of electromagnetic theory to problems arising in petroleum exploration and production. His academic training was obtained at Georgia Tech, the University of Texas at Dallas, and the University of California at Berkeley; his degrees are in Physics and Geophysics. He has been a researcher with Mobil Oil Corp. since 1988.

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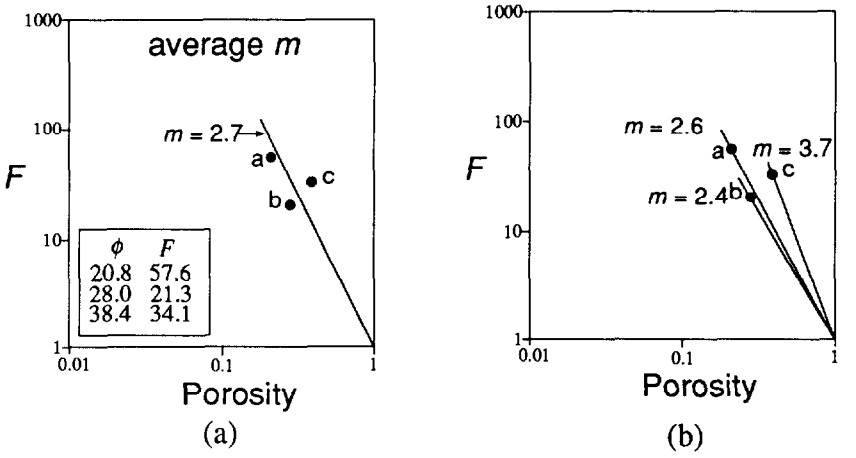


Figure 1. For non-Archie rocks, F-ϕ relationships are nonlinear (in log-log space) and R_0 cannot be predicted from a knowledge of ϕ. (a) an average m is determined with scatter in the data treated as if it were due to experimental error; (b) m values are determined for each sample seeking an m-ϕ relationship.

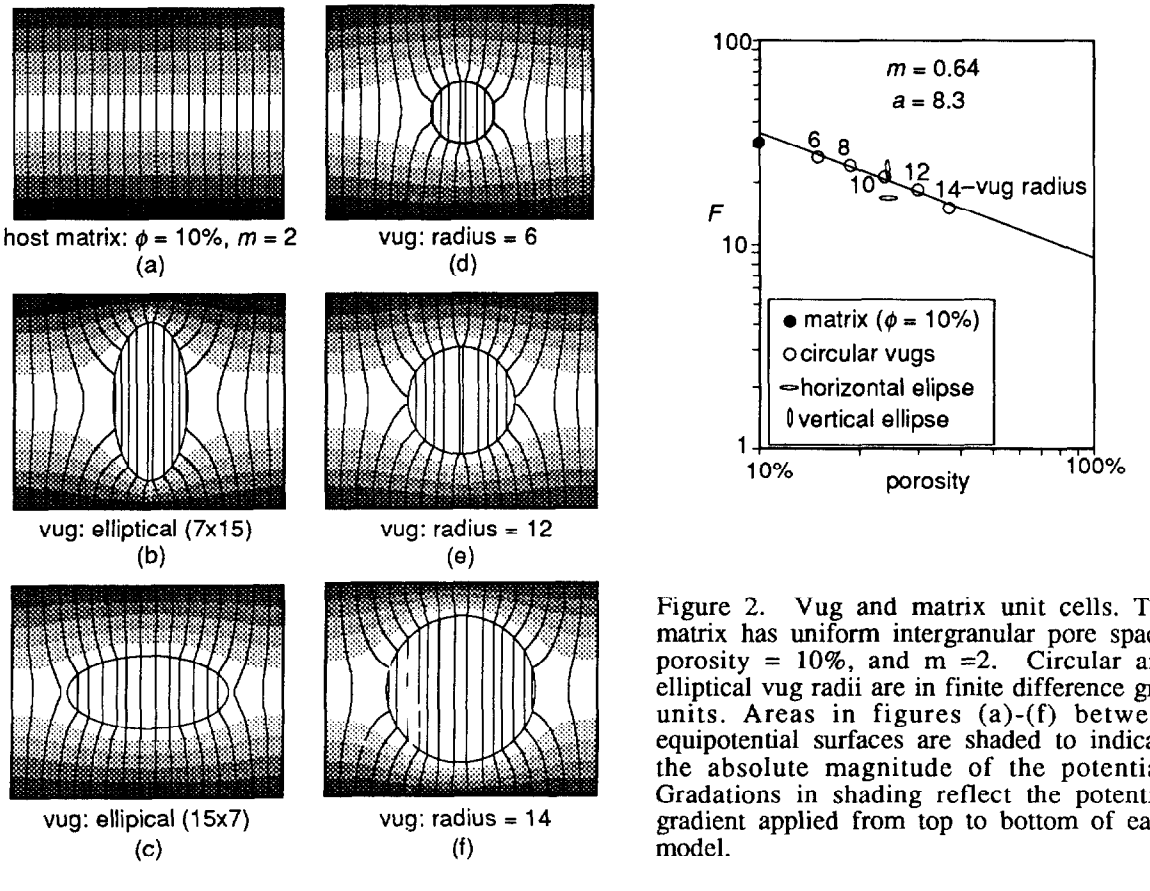


Figure 2. Vug and matrix unit cells. The matrix has uniform intergranular pore space, porosity = 10%, and m =2. Circular and elliptical vug radii are in finite difference grid units. Areas in figures (a)-(f) between equipotential surfaces are shaded to indicate the absolute magnitude of the potential. Gradations in shading reflect the potential gradient applied from top to bottom of each model.

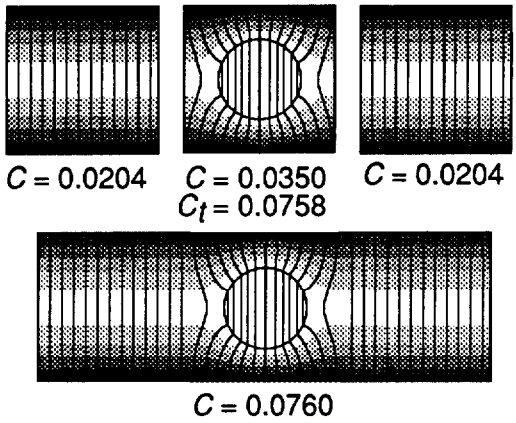


Figure 3. Comparison of the sum of the conductance of two matrix cells and one vug-bearing cell with the conductance of the combination.

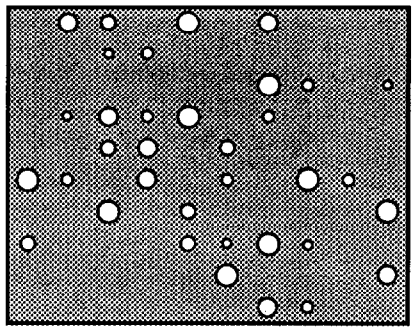


Figure 4. 2-D vuggy rock model with 35% vug-bearing cells

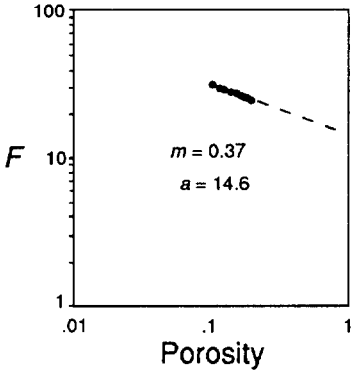


Figure 5. F-ϕ relationship for varying porosity, $\phi = 10\%$, $m = 2$.

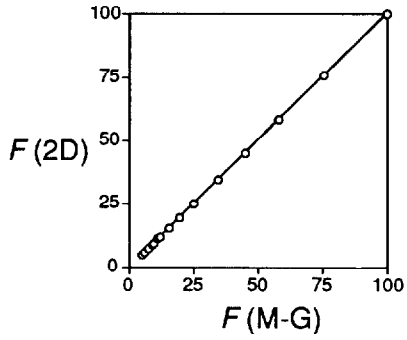


Figure 6. Comparison between F values determined by 2-D modeling and calculated from the Maxwell-Garnett relationship for models with $\phi_i = 10\%$, 20% and 30% and with ϕ_v varying from 0% to 40% in 5% increments.

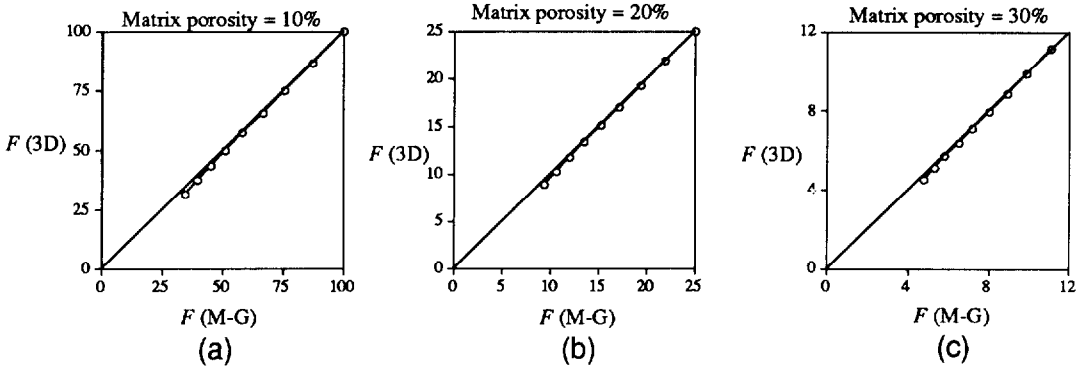


Figure 7. Comparison between F values determined by 3-D modeling and calculated from the Maxwell-Garnett relationship. (a) $\phi_i = 10\%$, (b) $\phi_i = 20\%$ and (c) $\phi_i = 30\%$. ϕ_v varys from 0% to 40% in 5% increments in each figure.

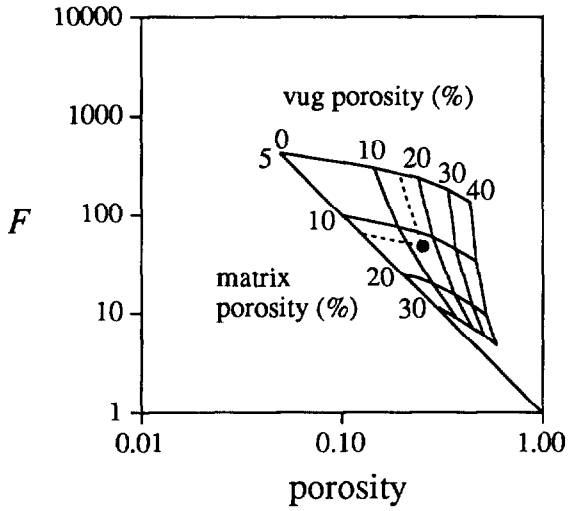


Figure 8. Graphical representation of the Maxwell-Garnett relationship for $m_i = 2.0$. Example of the division of total porosity into matrix and vuggy porosity: for $F = 46.2$, $\phi = 25\%$, $\phi_i = 12\%$, $\phi_v = 15\%$.

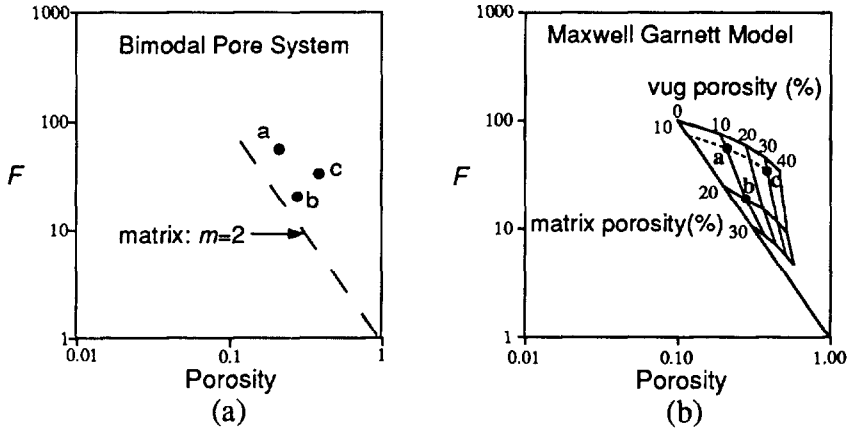


Figure 9. Interpretation of the F-ϕ relationship for rocks with a bimodal vug-matrix pore system.

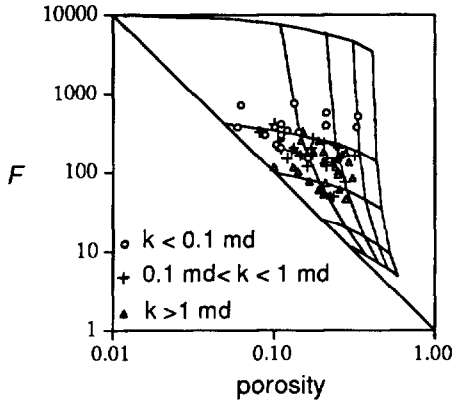
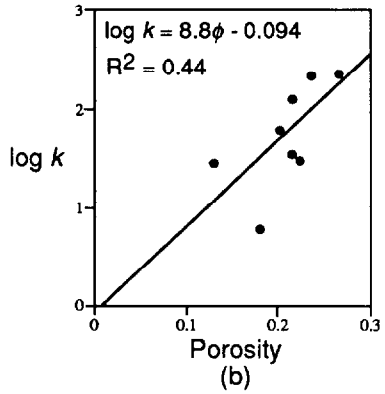


Figure 10. Relationship of matrix porosity to permeability for the Khuff Formation, North Field, Qatar.
(data from Focke and Munn, 1985)

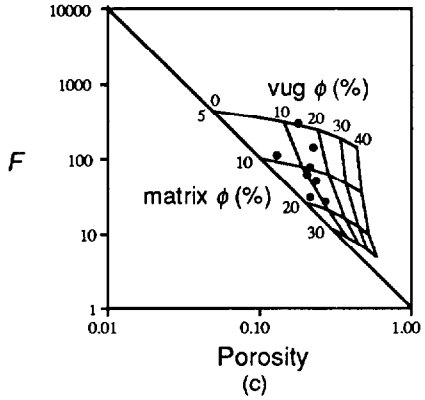
Mid-East Oomoldic Carbonates

Sample No.	ϕ (%)	k (md)	F	ϕ_i(%)	ϕ_v(%)
1	12.9	28	105	9.1	4.1
2	20.1	60	58	12.0	9.2
3	21.4	35	74	10.3	12.4
4	22.2	30	132	7.2	16.2
5	17.8	6	279	5.1	13.4
6	23.5	222	49	12.7	12.4
7	21.4	130	30	17.5	4.7
8	26.5	228	26	18.0	10.4

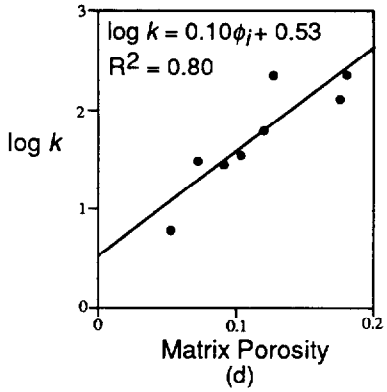
(a)



(b)

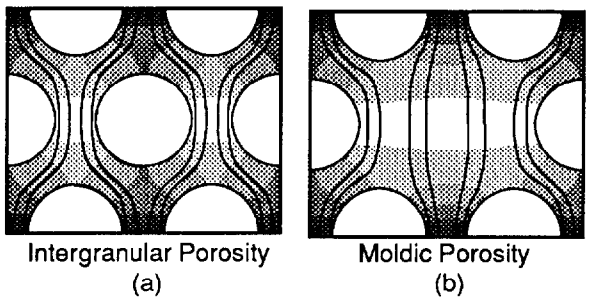


(c)

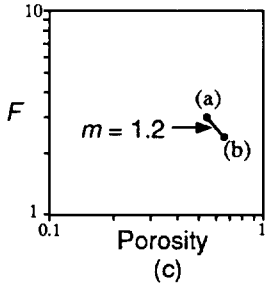


(d)

Figure 11. Application of the Maxwell-Garnett relationship to permeability prediction in oomoldic carbonates from the Mid-East.



(a) (b)



(c)

Figure 12. Potential and current distribution in unit cells with intergranular (a) and moldic (b) porosity and their F-ϕ relationship.

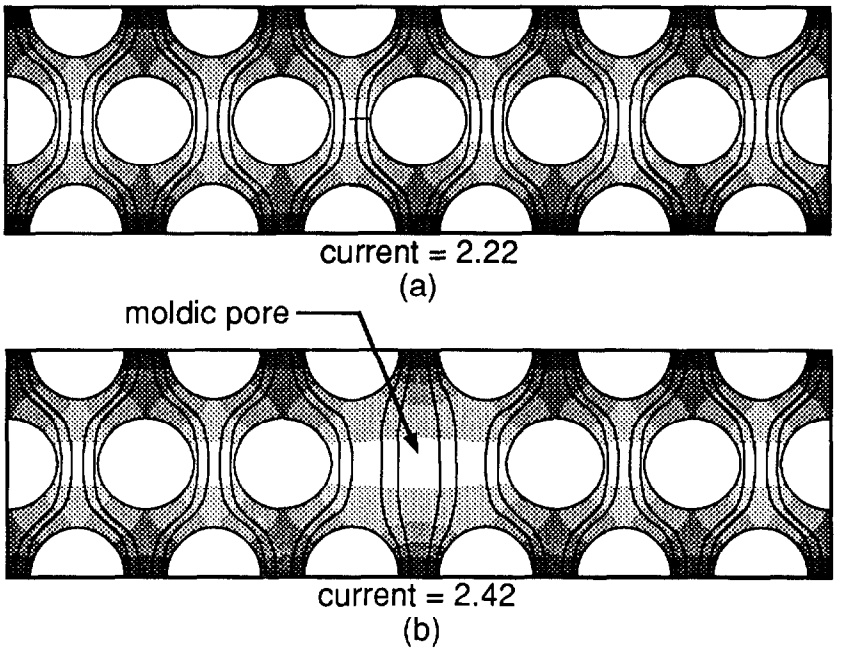


Figure 13. Comparison of the current streamlines in intergranular (a) and moldic (b) multiple-cell finite-difference models.

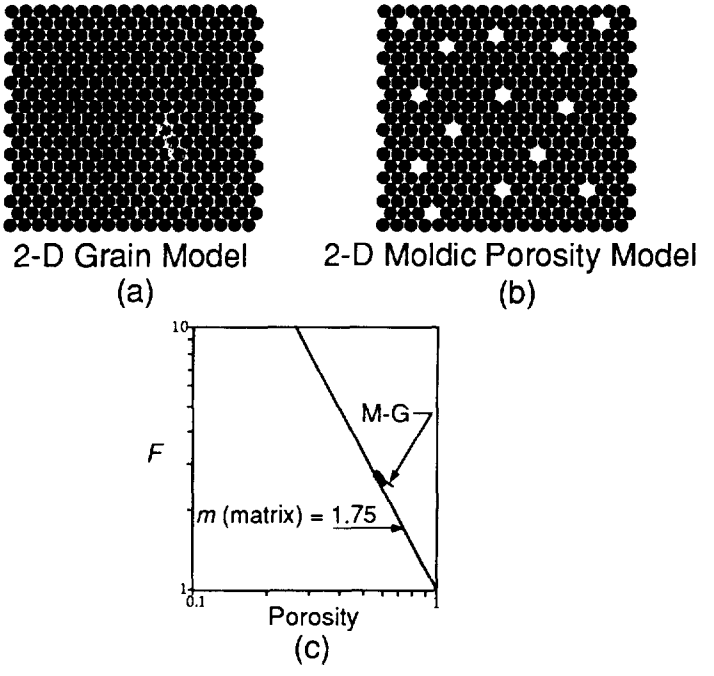


Figure 14. F-ϕ relationship for 100 cell moldic porosity models. (a) intergranular porosity; (b) model having 35% cells with moldic pores; (c) F-ϕ relationship for varying moldic porosity showing the Maxwell-Garnett (M-G) prediction.