## Introduction

We begin by reviewing the flow of fluids and electricity. Only simple Darcy flow will be considered; the effects of turbulence or other complicated flow is beyond the scope of this discussion. A fluid which wets a cylindrical tube has a velocity flow profile which is parabolic in cross section parallel to the tube having zero velocity at the surface and the maximum speed in the center of the tube. The flow profile of a non-wetting fluid is piston-like with a uniform velocity across the tube. The flow profiles of fluids with intermediate wettability lie between the two.

Electricity behaves like a non-wetting fluid. Electrons oscillate very rapidly, moving randomly due to interactions with other electrons. When placed in an electromagnetic field, there is a tendency for the ensemble of electrons to drift slowly parallel to the direction of the field. This drift velocity is much slower than the frenetic movement of individual electrons. While the random speed of electrons is exceedingly high, their drift velocity in a copper wire is only about **40cm/sec** in a 1V/cm field.

## Fluid flow in porous rocks

The flow of water in a porous rock is impeded by the porosity and the internal geometry of the pore system which is measured as the permeability of a rock sample. The flow rate of water in a cylindrical rock sample is given by Darcy's Equation:

$$Q_r = k\frac{\nabla P}{\mu}\frac{A}{L} \tag{1}$$

where $Q_r$ is the flow rate, $k$ is the permeability, $\nabla P$ is the pressure gradient, $\mu$ is the viscosity of the fluid and $A$ and $L$ are the cross-sectional area and length of the rock sample. The flow of a wetting fluid through a tube is similar and given by the Hagen-Poiseuille equation:

$$Q_{tube} = \frac{a}{8\pi}\frac{\nabla P}{\mu}\frac{A}{L} \tag{2}$$

where $a$ is the cross-sectional area of the tube. The permeability of the tube is $a/8\pi$ and is due to the zero velocity of the fluid at the tube surface and the viscous interaction of the fluid molecules resulting in the parabolic flow profile.

Consider a thick-walled straight tube having the same pore volume and external dimensions as a cylindrical porous rock sample. The ratio of the flow through the rock $Q_r$ to the flow through the tube $Q_{tube}$ gives a measure of the efficiency $H$ with which the pore system of the rock transmits fluid compared to flow in the tube which has the highest flow rate.

$$H = \frac{Q_r}{Q_{tube}} \tag{3}$$

A cylindrical rock sample, like a core plug, with cross-sectional area of 9 cm$^2$, porosity of 30% and permeability to water of one Darcy, is a high quality rock by any standard. But the hydraulic efficiency of this rock is only $3 \times 10^{-7}$. Recall that one Darcy is $9.9\times10^{-9}$ cm$^2$. In general, rocks are very poor conductors of fluids.

## Flow of fluids and electricity in rocks

Let's consider the similarities and differences between the flow of electricity and a wetting fluid like water.

In figure 1 are two identical 2-D pores, both filled with identical brine. The pore on the left has an electrical potential across it from left to right. The right-hand pore has a pressure gradient across it from left to right. The distribution of the equipotential surfaces in both cases is identical (black curves). A potential gradient is a potential gradient, so what happens depends on the properties of what's in the gradient. In this case, the grains are both impervious to flow or electric current and the only place flow or conduction can occur is in the pore space. The two problems are the same. Well, no, they are NOT the same because you are talking about "boundary value problems" (as physicists call them) and boundary conditions are not the same. THAT is why the flow differs in the two cases.

Flow occurs due to the potential gradients. The left part of figure 2 shows current streamlines in red that contain the

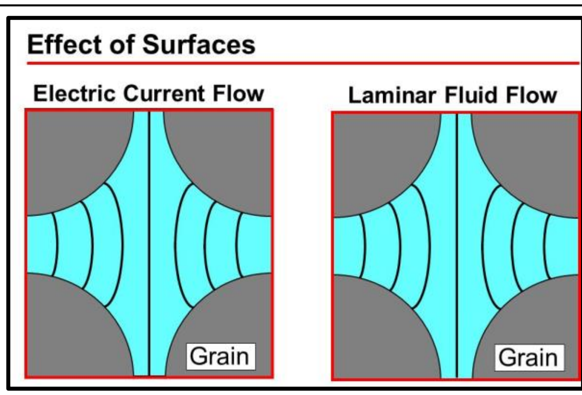


Figure 1: Two identical 2-D pores with equipotential surfaces shown in black curves.

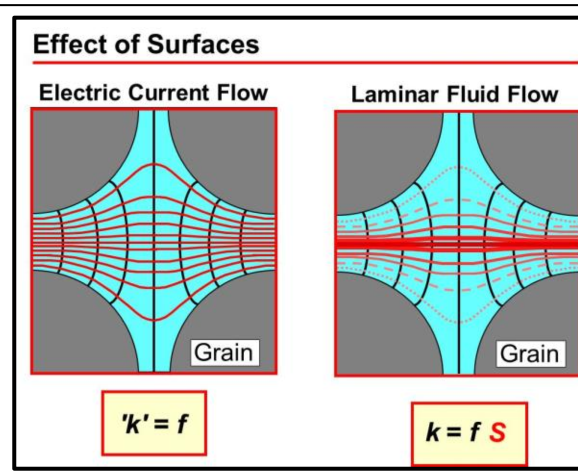


Figure 2. Streamlines in red showing the flow of electricity and water through identical pores.

same amount of current between each pair. Note that most of the current is constrained to the center of the pore with little current flowing in the upper and lower portions. In the right part of figure 2, the current density is depicted by red current streamlines that also vary in weight showing the amount of flow. Water flow is even more constrained to the center of the pore due to the parabolic flow profile in the pore throats. The "permeability" to electric current is the formation conductivity factor $f$. $f$ is the proportionality factor relating the conductivity of a rock sample $C_0$ to that of the water contained in the sample $C_w$, $C_0 = f\,C_w$. $f$ reflects the effect of pore volume and pore geometry on conduction. The permeability to wetting fluid flow is affected by the same volumetric and geometric factors as electric currents, but the effect of the parabolic flow profile $S$, especially in the pore throats, dominates fluid flow, $k = f\,S$. The effect of surfaces is the main reason that rocks are such poor conductors of wetting fluids like water.

**Modeling current flow**

The flow of electricity through a rock sample can be divided into a large number of individual stream tubes. The conduction through each tube is controlled mainly by the smallest constriction which limits flow. Hence, any stream tube such as that in figure 3 can be modeled with a straight tube with the cross-sectional area equal to that of the smallest limiting constriction $a$. The formation factor is then modeled by the sum of the areas of all limiting constrictions as a fraction of the cross-sectional area of the rock sample $A_{rock}$.

The pore space in a rock sample can be modeled with a bundle of $n$ stream tubes where $n$ is the number of all the stream tubes in the rock. $n$ times the average stream tube size $a$ is the total cross-sectional area of the stream tube model $a_t$ which provides an electrical flow model of the whole rock sample, as illustrated in figure 4.

Wetting fluid flow rate $Q$ through the model is given by

$$Q = \frac{a_t}{A}\frac{a}{8\pi}\frac{\nabla P}{\mu}\frac{A}{L} \qquad (4)$$

where $A$ is the cross-sectional area of the rock sample. Hence the permeability is given by

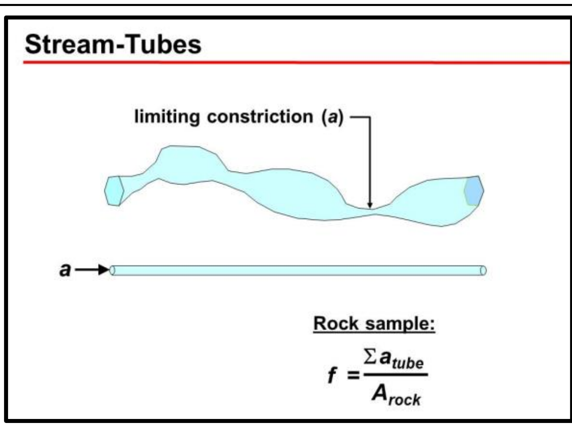

Figure 3. Model of a stream tube.

$$k = \frac{a_t}{A}\frac{a}{8\pi} \qquad (5)$$

and since $f = a_t/A$, then $S = a/8\pi$.

Recall that $S$ is the effect of the parabolic flow profile in the limiting constrictions and $a$ is the average size of the limiting constrictions.

Herrick and Kennedy (1994) determined that the conductivity of a rock sample $C_0$ is the product of only three factors, the water conductivity $C_w$, the volumetric property porosity $\varphi$ and a geometric factor that accounted for all pore geometric effects independent of volumetric effects, $E_0$

$$C_0 = C_w \varphi E_0 \qquad (6)$$

Since the formation conductivity factor $f$ is defined as $C_0/C_w$,

$$f = \varphi E_0 = \frac{a_t}{A} \qquad (7)$$

Combining (8) and (5) gives permeability in terms of the electrical properties and the average limiting constriction size,

$$k = \varphi E_0 \frac{a}{8\pi}\ . \qquad (8)$$

Since the formation resistivity factor $F$=1/$f$,

$$a = 8\pi k F\ . \qquad (9)$$

This relationship allows the bundle of capillaries model for both fluid flow and electrical conductivity to be tested. Mercury injection capillary pressure measurements can be made to determine an average limiting constriction size. The limiting constriction size can also be calculated from permeability and formation resistivity measurements and the two compared.

Mercury injection, permeability and formation factor data were obtained for a wide variety of rocks including clean and shaly sands, carbonates and the worst-case scenario, an oomoldic dolomite. The limiting constriction sizes were determined by the two methods and compared in figure 5. The

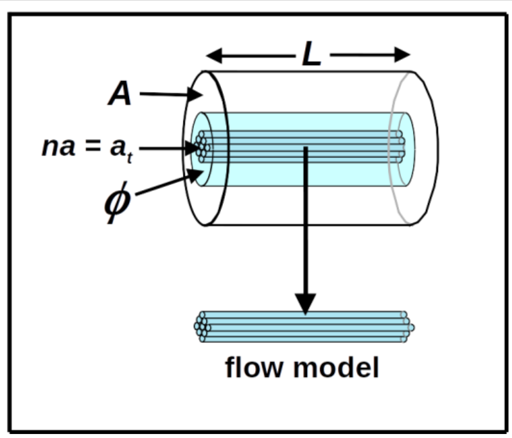

Figure 4. Stream tube model for the electrical conductance of a rock.

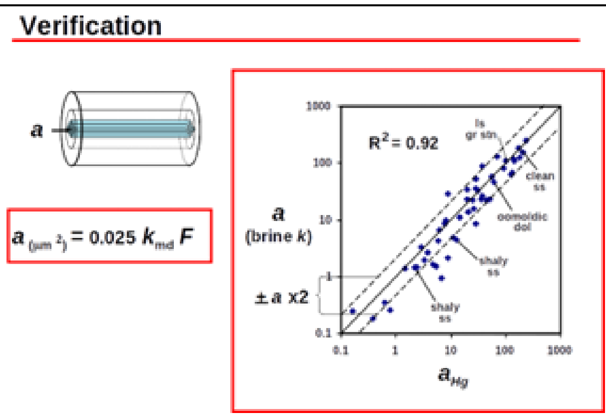


Figure 5. Verification of the bundle of capillaries tube model for electrical conductivity and permeability.

factor of 0.025 is due to units used and the value $8\pi$. It is not arbitrary. The strong correlation, $R^2 = 92\%$, verifies that the two values of the limiting constrictions for each rock sample are indeed the same verifying the bundle of capillary tubes model for both rock conductivity and permeability. The

strong agreement is particularly remarkable given that the two limiting constriction measurements are not the same. In the case of mercury injection, a sample is surrounded by mercury and injected from all sides while the conductivity measurement is directional along the axis of a core plug. Additional components contributing to the scatter are experimental errors, differences in connectivity, constrictivity, tortuosity and pore bodies, etc

Note that shaly sands tend to fall below the least-squares line. This reflects the additional conductivity due to clay surfaces not affected by the limiting constrictions. Most samples agree within a factor of two. This is remarkable since permeability correlations are often only within an order of magnitude.

**Conclusions**

The limiting constriction size is the most important pore geometric parameter. The effects of tortuosity, connectivity and pore bodies, etc. are minimal. Pore systems conduct both electricity and fluids as if they were a bundle of tubes whose size is determined by the limiting constriction sizes. The critical difference between electrical and fluid conductivity is fluid viscosity and surface wetting in the case of wetting fluid flow resulting in zero velocity at grain surfaces resulting in a parabolic flow profile.

**Reference**

Herrick, D. C. and W. D. Kennedy, 1994, Electrical efficiency-A pore geometric theory for interpreting the electrical properties of reservoir rocks, *Geophysics*, vol.59, No. 6, pp. 918-927.