

Induction Log Forward Modeling: A Rigorous and Systematic Approach to Model Construction

W. D. Kennedy*

Abstract: In many instances the apparent resistivity response of the 6FF40 induction logging instrument is not a good estimate of the true formation resistivity. In some cases this is true in beds up to 200 feet thick. Traditional chart book corrections do not usually remedy this condition; however, when conditions warrant one-dimensional forward modeling can convert induction log apparent resistivity responses into more accurate estimates of formation resistivity. With the advent of commercially available, fast modeling codes packaged in easy-to-use interfaces, application of 1-d forward modeling has become a feasible interpretation option with the potential of substantially increasing reserve estimates. Unfortunately, the modeling of several hundred to several thousand feet of induction log response may seem a daunting task regardless of the speed of computer codes and the convenience of user interfaces, and the experience of the analyst. However, it has been discovered that, regardless of how complicated a log may appear, log responses can be catalogued into six easily recognized responses. The responses have been named for convenience: (1) the impulse response; (2) the step response; (3) the ramp response; (4) the whole space response; (5) the thin bed (blind frequency and anti-correlation) responses; (6) mixed responses. Responses not falling into these categories can be recognized and identified as two-dimensional responses, or in some cases erroneous responses due to, e.g., incorrectly set sonde errors. The six responses are founded in the tool physics, but they can also be used as practical rules of thumb; using the six responses to construct initial models and refine the subsequent results significantly reduces modeling time. Both field and theoretical examples of each type of response are illustrated. Recognition of the cataloged responses permits efficient forward modeling.

INTRODUCTION

Reserve estimates are based in part on log-derived measurements of formation resistivity. The uncorrected induction logging tool response has long been considered an accurate-enough estimator of true formation resistivity. That this is true in many cases is not in doubt; this report is not about those cases. Rather, this report focuses on the (perhaps equally) numerous cases where the uncorrected induction log response is an *in*accurate estimator of formation resistivity. The approach relies on duplicating the observed logging tool response in a numerical model of a geologic formation. To the degree that observed and modeled responses can be brought into agreement, the underlying numerical model can be accepted as an accurate representation of formation resistivity. Thus resistivities from the model can replace the "apparent" resistivity response observed on the log. This results in increased reserve estimates in many cases.

Numerical modeling of logging tool responses is a difficult business from the point of view both of mathematical analysis and computer programming. Few logging tool responses can be modeled efficiently and accurately enough to be used in day-to-day log analysis. Nonetheless, when conditions warrant an important petrophysical parameter which *can* be profitably modeled is the induction resistivity response. An exact solution to the mathematical boundary value problem of the electromagnetic fields of magnetic dipole sources in a plane-layered, conductive medium is possible, and appropriately rapid and exact methods for implementing the solution on digital computers have been developed (e.g., by Richard Hardman). Since the medium in this problem has physical properties (e.g., conductivity) that vary in only one direction (i.e., normal to the bedding planes), the method is called one-dimensional modeling. In many cases of practical interest the effects of the borehole and near-hole environment on the resistivity response are negligible. This will be the case when the borehole is not too large and varies little in radius, where the borehole fluid contributes little to the total signal, and where invasion of borehole fluids into the permeable formations is not deep or does not displace brine. Inspection of a typical induction log at first suggests that guessing a conductivity model to account for the observed induction log response would be a daunting task; however, it is found that to the extent that the one-dimensional earth approximation is valid, the induction log response can be entirely characterized in terms of six typical responses that I shall presently enumerate; conversely, if a one-dimensional response cannot be found the log must contain higher-dimensional responses, erroneous responses, or noise.

MODELING

The industry-standard induction logging sensor has been the so-called 6FF40 array from the early 1960s to the present. The induction tool was a significant improvement over conventional electrical surveys because it could be employed in oil-base mud systems where conventional logging was not possible, and because its response was symmetric and more easily interpreted than the asymmetric response of the 18" lateral array which it replaced. However, enthusiasm for the new tool and marketing hyperbole led to widely propagated (but exaggerated) claims for the 6FF40 response which have, unfortunately, become the conventional wisdom of many experienced log analysts and petrophysicists. Prominent in this conventional wisdom is the claim that the apparent resistivity response of the induction log can be used without correction as an adequate approximation for actual formation resistivities, summarized as

*Mobil Exploration and Producing Technical Center, Dallas, Texas.

$R_a = R_t$ (Dumanoir and others, 1957). But, as was recognized even at the time (Guyod, 1957), this simply is not generally true (Anderson 1986; Anderson and Barber, 1988). While few contemporary log analysts would extol $R_a = R_t$ for the induction log response, much of contemporaneous log analysis has been (and is) executed using uncorrected R_a as an accurate-enough estimator of R_t . In cases of high conductivity contrast (e.g., > 100:1) this will lead to very significant errors in calculated water saturations, and thus, reserve estimates. Before the late 1980s it mattered little from a practical standpoint whether one accepted (or was even aware of) the deficiencies of the induction log response because no means to make really meaningful corrections to the apparent resistivity response existed (chart book corrections are for ideal cases not often relevant to actual logs). The best that could be done (arguably), and the easiest, was to accept $R_a = R_t$. But with the advent of practical means of forward modeling the induction log response in the early 1990s, times have changed.

Familiarity with the idiosyncratic properties of the tool response is required for success in finding an appropriate model; for example, it is important to recognize that not every wiggle on the induction log response (i.e., the resistivity log) corresponds to a conductivity change in the earth, and conversely, a lack of wiggles does not necessarily guarantee that conductivity is homogeneous. As a further example, from the point of view of the instrument response, the signal-to-noise ratio is unfavorable when resistivities are high, and where possible an alternative resistivity survey (e.g., laterolog) would be used; however, when oil-base mud has been employed in the drilling program there is no alternative to induction logging for resistivity determination. In these situations, if R_t is high, a correct apparent resistivity response from the logging instrument depends critically on the correct determination of the difficult-to-measure parameter called sonde error. Incorrect sonde errors have a negligible effect on log responses in low resistivity formations, but are easy to recognize in high resistivity contrast formations by the rate of change of the apparent resistivity in passing from a conductive zone into a thick resistive zone. If the resistivity rises rapidly (within, say, 10 to 50 feet) from a low value to the induction-determined resistivity cutoff (i.e., 2000 Ω -m) the indication is that the sonde error is incorrect. Moreover, a technique formerly recommended by logging service companies for setting sonde-errors down hole when thick, very resistive formations are present can be shown not to give correct results in practice—formations are rarely either thick enough or resistive enough to provide a zero input signal.

RESPONSE FUNCTIONS

Induction logging tool response functions are based on the "geometrical" factor representation of the tool response. It is well known that Doll's (1949) original description of the tool response in terms of geometrical factors is a low-frequency, low-conductivity limiting

case of the actual induction tool response, and that when correctly described the "geometrical" factor contains non-geometrical terms (i.e., conductivity) as well (Thadani and Hall, 1982; Gianzero and Anderson, 1982; Moran 1982). Nevertheless, Doll's geometrical factor theory (GFT) is a convenient heuristic device. In a one-dimensional model where there is no conductivity variation in the radial direction, GFT suggests that the total response of the tool can be obtained as a sum of the individual contributions to the response of thin slices of formation taken parallel to bedding planes. This description is obtained from GFT by integrating the geometrical factor over the radial coordinate. The remaining function depends only on the axial coordinate and is usually referred to as the vertical response function; however, in one-dimensional models a spatial vertical response function is formally analogous to an impulse response for the tool; this nomenclature is used in the following sections.

As illustrated in figure 1, the one-dimensional induction log can be characterized completely in terms of six canonical responses*. The standard responses are: the homogeneous whole space response; the impulse response; the step response; the ramp response; the thin-bed responses; simple mixed responses. For the 6FF40 array these responses are non-linear generalizations of the corresponding functions in linear systems theory. Although formulas can be written only for the simplest cases, the six response concepts bring a measure of rigor to the method of searching for solutions to 1-d forward problems. These are described, with examples, below.

Whole Space Response

In a thick, conductive† bed the skin effect boosted apparent resistivity response equals (with negligible error) the resistivity of the space. However, the meaning of "thick" is a function of the conductivity of the bed, the shoulder beds, and the apparent dip. In conductive formations and low conductivity contrast resistive formations the whole-space response predominates; in many of these cases $R_a \approx R_t$. However, in some resistive beds $R_a \neq R_t$ even in beds thicker than 100 feet. From a theoretical viewpoint, only an infinitely thick bed (without a borehole!) yields $R_a = R_t$. Figure 1 suggests that the log response is flat in such beds.

* Of the six, five can be understood in terms of the remaining response—the impulse response (and its Fourier transform)—but this is a theoretical consideration not helpful in constructing conductivity models whereas the six canonical responses most definitely are.

† In the following there is a need to distinguish low resistivity beds from high resistivity beds. It is convenient to call low resistivity beds "conductive" beds and high resistivity beds "resistive" beds. The terms are relative; loosely speaking, a wiggle to the left on the log response is a conductive response, a wiggle to the right is a resistive response.

Impulse Response

A conductivity impulse is the limiting case of a thin, conductive bed immersed in a highly resistive medium as its thickness approaches zero and its conductivity approaches infinity in a manner that holds the conductivity-thickness product (or conductance) constant. The response of the 6FF40 antenna array as a function of distance from the impulse location is known as the impulse response. For linear systems (e.g., electronic amplifiers, geophones, etc.) the impulse response is a "constant" function; in particular, its shape and amplitude do not depend on the input signal; however, the spatial impulse response of the 6FF40 array is not a constant function. An important result in linear systems theory is that the input to a linear system can be obtained from its output and a knowledge of its impulse response. Since the output of such systems is mathematically described by the convolution of the input signal with the impulse response, obtaining the input from the output is termed *deconvolution*. The output of the 6FF40 antenna array can similarly be formally expressed as a convolution of the input signal (i.e., a conductivity profile) and the impulse response; however, the impulse response is, *ab initio*, an unknown function of the (also unknown) input signal. Thus the relatively simple methods of deconvolution developed in linear systems theory do not apply to the 6FF40 array response (i.e., induction logs).

The usual representation of the impulse response function of the 6FF40 induction logging array is illustrated in figure 2a. The impulse response function varies around zero in conductivity space—to study it in a logging situation it must be displayed on a log-linear plot in resistivity space (fig. 2b); however, zero conductivity cannot be represented on a logarithmic resistivity plot, so I introduce a small amount of conductivity (1 mS/m) in the space surrounding the conductivity impulse. Note in figure 2b the width of the resistivity response to a "very thin" impulse is about 6 feet at half-amplitude. Also note the horns, which logging naifs might mistake for resistive bedding; the width of the horns is about 1 foot at half-amplitude. I observe that resistivity anomalies corresponding to one-dimensional geological structure cannot be thinner than about 6 feet at half amplitude regardless of the thinness of the structure. Thinner apparent resistivity anomalies in wells with moderate relative dip are either associated with horns on the impulse response or higher-dimensional conductivity effects (Anderson, 1986). Figure 2c illustrates a corresponding case generated using a forward model; the response is to a 1 inch thick conductivity impulse with a resistivity of 1 Ω -m, which gives an apparent conductivity response approximately equal to the GFT impulse response. Cases for 0° and 60° relative dip are shown. Curiously, the apparent resistivity response is thinner in the case of the dipping configuration. Inspection of the responses in figure 2a-c reveals two side lobes on each side of the impulse, rather than one per side as usually illustrated. These additional side lobes are caused by the application of the (so-called) three point deconvolution operator to the raw array response.

Figure 2d* illustrates an observed conductivity impulse and corresponding model; the relative dip is about 12.5°. The impulse extends from 421 to 423 feet and has a resistivity of slightly less than 0.1 Ω -m. The apparent resistivity response and modeled response are in close agreement near the "bottom" (i.e., where the apparent resistivity is lowest) and the sides of the impulse response; moreover, an impulse having any other conductance than the one actually discovered will fail to allow the close fit of the curves a great distance both up-hole and downhole from the impulse location. On the other hand, any impulse with the same conductivity-thickness product will serve equally well to fit these features; however, the horns on the modeled response show a sensitive dependence on the width and conductivity of the impulse and the fit of observed horns to modeled horns often seems to distinguish the conductivity and thickness of an impulse separately. At the bottom of the impulse the modeled response shows a characteristic profile (resembling Batman; refer to figure 2a) whereas the observed response does not. It is often difficult to match the detailed shape of an impulse bottom; I speculate that this may be due to the finite coil sizes in the actual tool (the numerical model of the tool uses point dipoles as transmitters and receivers), or to minor asymmetries manufactured into the 6FF40 array but not reflected in its numerical representation. Although the conductivity anomaly in figure 2d qualifies as an impulse as far as the 6FF40 array is concerned, much more "impulsive" impulses are observed. The impulse in figure 2e (located at 996 feet) is less than 0.2 Ω -m in resistivity and less than 2 inches in width. This observed impulse response is only about 4 feet wide and this is consistent with the relative dip of this well being 60° (fig. 2c). The model of this particular impulse trades-off fit of the bottom of the resistivity anomaly and horns against a good fit in the resistive zone below the horn; i.e., the anomaly bottom and horn response can be better fit with a more conductive impulse but then the closeness of fit in the pay zones is lost. The observed discrepancy can be attributed to the presence of the infinitely resistive borehole. The net signal coming from the borehole region would be negative if the borehole had a non-zero conductivity, and would thus tend to diminish the observed signal; removal of conductive material near the borehole thus increases the apparent conductivity signal (since negative signal contributions are effectively being removed). This effect is greatest near the impulse location. The one-dimensional model cannot reproduce this effect near the impulse itself, but

* Figure 2d is the first of a number of response examples in actual wells. In obtaining models whose responses closely approximate the observed responses in actual wells, it has been found that given enough time to search the solution space, extremely good matches can usually be found. However, when we do choose to pursue perfection it is usually found that 90% of the modeling effort (and time) goes into eliminating the last 10% of the difference between the modeled and observed responses. Thus, although modeling efforts are usually terminated before the best possible model is obtained, they nevertheless have significant quantitative impact on the interpretation.

does produce correct results in the pay zone. Thus the lack of fit at the impulse location is consistent with what is known about the tool response for the model and suggests that the modeled impulse is correct. As before, the shape and amplitude of the horns are quite sensitive to the details of the model impulse.

Step Response

Near a conductivity step discontinuity the instrument response exhibits an easily recognizable character known as a step response. Mathematically speaking, the step response is the integral of the impulse response. When the impulse response is constant this integral is often easy to compute; however, in the case of the induction measurement, the impulse response varies with instrument location and is a priori unknown. The step response is obtained by solving the forward problem in the vicinity of the step. Like the impulse response, the response at the step contains various bumps and wiggles, or horns, which could naively be confused with conductive/resistive bedding.

Figure 3a illustrates an idealized step response at both 0° and 60° relative dip, while figure 3b illustrates an approximate step response observed in the field. Note that as intuitively expected, and unlike the impulse response, relative dip *broadens* the step response. The modeled horn in figure 3b is larger than the observed horn, probably due to the presence of the borehole, which cannot be included in a one-dimensional model. Conductive formation extends right through the tool location in the mathematical model, exaggerating the horn effect. The slow approach of the response to the high resistivity asymptote is termed shoulder effect; unrecognized shoulder effect can be the source of significant underestimation of reserves.

Ramp Response

A conductivity ramp is defined as a monotonic and continuous change in conductivity between two thick beds of constant conductivity. The simplest ramp is a linear increase in conductivity between its two constant-conductivity shoulder beds. (Since I have already mentioned that the step response is the integral of the ramp response, it should be mentioned that mathematically the ramp is the integral of the step.) The pure ramp response can be easily distinguished from the step response by a lack of "horns." In practice, a true ramp is not modeled since the analytical model does not admit the possibility of continuously varying conductivity; however ramps may be approximated as closely as desired by using a sequence of conductivity steps (i.e., conductive beds) approximating the ramp. Figure 4a illustrates an idealized ramp response for no dip and 60° relative dip; note that again the response in the presence of dip is broadened. Figure 4b shows a typical observed (and modeled) ramp response; note the absence of horns.

Periodic Conductivity Variation in Thin Beds

A thorough understanding of the two types of "thin-bed" responses requires a familiarity with the spectral representation of the impulse response. On the other hand, it is easy to imagine that for periodic* sequences of conductive and resistive beds, for periods smaller than the transmitter-receiver spacing of the antenna array, the instrument cannot respond to the conductivities of the individual beds but only to the average conductivity. The response in some cases cannot be distinguished from a thick bed of equivalent conductance. To distinguish between the cases, auxiliary information from cores or higher resolution logging instruments is required; the induction response itself may be undefinitive. More surprising and more difficult to visualize are cases where the apparent conductivity *does* respond to a sequence of thin beds, but 180° out of phase with the conductivity variation.

Thin bed response:

(1) Blind frequencies

If a periodic sequence of conductive/resistive thin beds is considered in the spectral representation, the spectrum is seen to be a discrete series of lines—in fact the magnitudes and wavenumbers of the lines are given by the Fourier series expansion of the periodic sequence. If the fundamental spatial frequency, or wavenumber, is k_0 , the spectrum will have lines at $k=0, k_0, 2k_0, 3k_0, \dots$. The line at $k=0$ gives the magnitude of the volume-weighted average, or "d.c.," component of conductivity. If k_0 happens to fall on the first zero in the transfer function of the 6FF40 array then the spectrum of the log will not contain the spectral component at k_0 (i.e., the tool will be "blind" to conductivity variation having a wavenumber of k_0), and the higher harmonic spectral components will be greatly attenuated because the transfer function approaches zero for higher values of k . If the periods are even thinner such that k_0 coincides with the second zero of the transfer function, then not only is the fundamental component removed from the log but the higher harmonics are shifted even further out the wavenumber axis where they are, for practical purposes, also eliminated by the rapidly diminishing transfer function. Thus the spectrum of the log of these thin beds is indistinguishable from the spectrum of a whole space; i.e., it is a single line at $k=0$ —representing the d.c. component. Thus the log of the sequence of thin beds with average conductivity σ is practically the same as the log of a homogeneous whole space with conductivity σ (which has *only* the component at d.c.). Moreover, any sequence of thin-enough beds with constant volume-weighted average conductivity, whether or not periodic, will have, except at $k=0$, nearly zero low

* It is important to distinguish between a bed and a period; the latter contains more than one bed. Thus periodic sequences have at least two beds per period and repeat themselves a number of times.

wavenumber components; the higher harmonics correspond to wavenumbers greatly attenuated by the transfer function. The resulting spectrum of the log has only the component at zero frequency corresponding to the volume-weighted average conductivity. Thus all thinly bedded sequences with the same d.c. component will give practically the same log response and cannot be uniquely modeled except by imposing constraints taken from other data sources such as cores, or logs having higher vertical resolution. For practical purposes the thin bed response at the blind frequencies turns out to be indistinguishable from the whole space response.

Figures 5a-c illustrate periodic sequences of thin beds and a single thick bed having nearly identical responses. This is a relatively common log response and figure 6 shows a typical example. In this example an otherwise thick sand is interrupted by many conductivity impulses. The locations and thicknesses of these impulses are chosen by inspection of higher vertical resolution logs (not shown), and the magnitude of each impulse is chosen to match the modeled response to the observed response. It has often been noted that induction responses are not uniquely invertible. Nonetheless, if the models are limited to one-dimensional step conductivity profiles then from a practical viewpoint, this type of thin-bed response seems to be the *only* case that ultimately calls for subjective judgment from the interpreter; i.e., this is the only response that seems to permit non-unique models. Further, if the conductivity and boundaries of the conductive beds are accurately known, then at least the average conductivity of the resistive beds can be accurately determined.

Thin bed response:

(2) Anticorrelation

A second type of thin-bed response is also possible—anticorrelation response. Periodic sequences of conductive/resistive beds with periods in the range of 4.72 beds per foot to 1.90 beds per foot (at 0° relative dip) have the curious property that in the periodic response apparent resistivity highs develop where true resistivities are low, and vice versa. This behavior is easy to understand in terms of the spectral representation of the tool response; unfortunately, space limitations prevent further discussion here (but see Anderson, 1988). Figure 7a illustrates the induction response to a periodic conductivity structure with a period of 4 feet per period (fpp) ($k_0 = 0.25$ periods per foot; ppf) logged at relative dips of 0° and 60°. At 0° the log responds to a conductivity high with a low apparent conductivity and to a conductivity low with a high apparent conductivity.

The anticorrelation response is not usually observed in sections containing as many periods as in figure 7a; usually only one or two periods are observed. Under such conditions the response may be difficult to recognize due to the transitions into and out of the anticorrelation response masking its presence. However, when beds are only a few feet in thickness, a trial model honoring the sense of conductivity variation suggested by

the observed response yielding negative results indicates an anticorrelation model should be tried. Figure 7b illustrates 2.5 periods of a sequence of beds with period 3 fpp for both 0° and 60°. Note that at this frequency the anticorrelation is well developed at both relative dips. The anticorrelation response is not rare, and figures 8a and 8b show a typical field example.

Mixed Response:

(1) Impulse within step

A response frequently occurs, which although superficially resembling a step response cannot be duplicated using a simple conductivity step. This response is identified by its possession of a faster rate of resistivity change and more exaggerated horns than can be obtained with a step response. It is often signaled by a small region of lower apparent resistivity which occurs immediately adjacent to the most rapidly changing apparent resistivity. The features of this observed response can frequently be modeled by including an impulse of appropriate width and conductivity at the edge of the observed conductivity step. This impulse does not manifest itself by an impulse response on the log, but rather by steepening the rate of rise and/or amplifying the horns at what might otherwise appear to be a simple step change in conductivity. An example is shown in figure 9a.

Another manifestation is an apparent conductivity step with horns that cannot be closely matched by a simple step model. Although it should be recognized that horns may masquerade as conductivity variation and the modeling of many beds where one would suffice should be avoided, sometimes conductivity *does* vary exactly at the location of the horn. Usually these cases are suggested by difficulty in finding a suitable step profile; inspection of porosity and gamma ray logs will then usually suggest alternative bedding models whose conductivity can be empirically determined. Typical examples are illustrated in figure 9b.

Mixed Response:

(2) Impulse within ramp

A conductive impulse within a ramp produces a distinctive change in slope that can result in a temporarily reduced slope, or flattening, or even temporary reversal of the slope, of the ramp. A sigmoidal response observed in a ramp can almost always be modeled by inserting a properly formed impulse somewhere in the ramp. The location of the impulse and the location of the corresponding log response may differ by several feet. A typical case is illustrated in figure 9c.

OTHER MODELING CONSIDERATIONS

The basic responses described above are recognized by their shape on the induction log apparent resistivity response. It is important to realize that these basic response shapes have been modified by "signal processing" built into the instrument response. The basic instru-

ment response itself depends on the angle between the antenna axis and geological bedding planes (i.e., relative dip), and the influence of this signal processing sometimes profoundly modifies the basic instrument response at high relative dip. In addition, some observed (and repeatable) induction responses are not catalogued above and cannot be obtained in a one-dimensional model. In this section the effects of the standard enhancements to the induction log response, i.e., the three point deconvolution and skin effect are noted. The recognition of two-dimensional effects and sonde error are also discussed.

Three Point Deconvolution and Skin Effect Boost

The purpose of the three-point "deconvolution" is to make the induction response to a step change in conductivity appear more steplike. The effect is successful unless conductivity contrasts become large, say $> 20:1$; for large conductivity contrasts the three-point deconvolution greatly exaggerates the wiggles and bumps on the impulse and step responses; it fails to bring the overall response closer to the true resistivity and obscures the actual conductivity structure. Further, the three-point deconvolution process also introduces additional sidelobes into the instrument response, effectively adding to the confusion as to what is (or is not) legitimate geological bedding. The "deconvolved" log is subsequently "skin-effect boosted," meaning that the "deconvolved" apparent conductivity reads too low in conductive formations and needs to be "boosted" to a higher apparent conductivity. The intent is to keep the otherwise diminishing impulse response constant as the conductivity of a surrounding homogeneous whole space increases. However, since conductivity contrast is increased the effect is to further exaggerate horns already exaggerated (and introduced) by the deconvolution. Thus highly conductive but thin beds in otherwise highly resistive thick formations are apparently surrounded by pairs of even more resistive beds; however, these are not beds but exaggerated horns. The effect can be seen on the responses in figures 2, 9, and 10.

Dip effects

At low-to-moderate relative dips (out to 20° , say) the effects due to dip can be neglected (although this should be verified in each instance). The trade-off in neglecting dip is a faster computation and thus reduced modeling time and effort against a slightly degraded result. However, especially in cases where resistivity contrast is high, relative dip effects begin to play an increasingly important role in the shape of horns and other diagnostic features. Slight changes in dip can produce very significant changes in the appearance of various details on the log such as horns; thus it is important to have a good idea of the relative dip before attempting to model wells deviated over 30° . Possibly, models having incorrect relative dips can be found to produce induction log responses matching observed responses; however, experience has taught that increasing or decreasing the relative dip by 5° in a well with a nominal relative dip

of, say, 50° may simplify the model and greatly reduce the effort required to match modeled response to observed response. Figure 10 illustrates a case where the nominal relative dip was calculated from well records to be about 50° but where the model fit much better at 56.5° . Application of Occam's razor suggests that the easiest response to model might well be the one with the correct dip. Reconciliation of the dip suggested by the resistivity model and that computed by honoring other well records may then become enigmatic; alternatively, the reason for erroneous or approximate data values in the well records may be identified.

Two-dimensional effects

On unlucky occasions the one-dimensional approximation is inadequate due to significant higher-dimensional conductivity effects. The simplest higher-dimensional conductivity structure is two-dimensional axisymmetric. If deep invasion is indicated by the resistivity profile, 1-d modeling is contraindicated. In another case, Anderson (1986) has shown that conductive rings coaxial with the borehole in such models can account for apparent "noise" on the induction log response. Such noise is sometimes observed in hydrocarbon-bearing zones of wells drilled with oil-base mud (which is an emulsion of a highly conductive salt-saturated aqueous solution suspended in highly resistive diesel oil). On the other hand, this "noise" turns out to be absolutely repeatable, indicating that it is not noise at all (in the usual meaning of random, unrepeatable signals). As we have argued in discussing the impulse response, a one-dimensional response cannot be thinner than the instrument impulse response. If thinner responses are observed, they must therefore be attributed to other causes. A typical example is shown in figure 11. The smoothness of the 1-d model response serves to illustrate the magnitude of the 2-d noise effect. Addition of more detail to the one-dimensional model in an attempt to better fit the observed response fails.

Sonde error

In the section describing the 6FF40 the importance of correctly determined sonde error was discussed. Figure 11 also shows a case where modeling reveals that sonde error has been incorrectly set. In this case the observed apparent resistivity "pegs" at $2000 \Omega\text{-m}$. The model accordingly uses $2000 \Omega\text{-m}$ as a model end-point resistivity. What the figure reveals is that even if the true formation resistivity were $2000 \Omega\text{-m}$, the apparent resistivity response in the model would track the model response and could attain only just over $1000 \Omega\text{-m}$. Had the sonde error been correctly set, the observed apparent resistivity would have more closely followed the modeled response. The rate of rise of apparent conductivity from about $200 \Omega\text{-m}$ to $2000 \Omega\text{-m}$ within 15-to-20 feet is not a possible response for a correctly (sonde-error) compensated instrument; such rapid rates of increase inside thick resistive beds are diagnostic of sonde error problems.

CONCLUSION

The means for the modeling of the one-dimensional induction resistivity response has been available to petrophysicists for several years, but use has not become routine. Resistivity modeling has been demonstrated to be well worth the effort in terms of correctly estimating reserves; it should certainly be assessed as to its applicability in new wells. Moreover, investment of modeling effort in reassessing existing fields can yield significant dividends. The responses enumerated in this study should serve as a time-saving guide to future modelers.

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The Six Canonical Induction Tool Responses

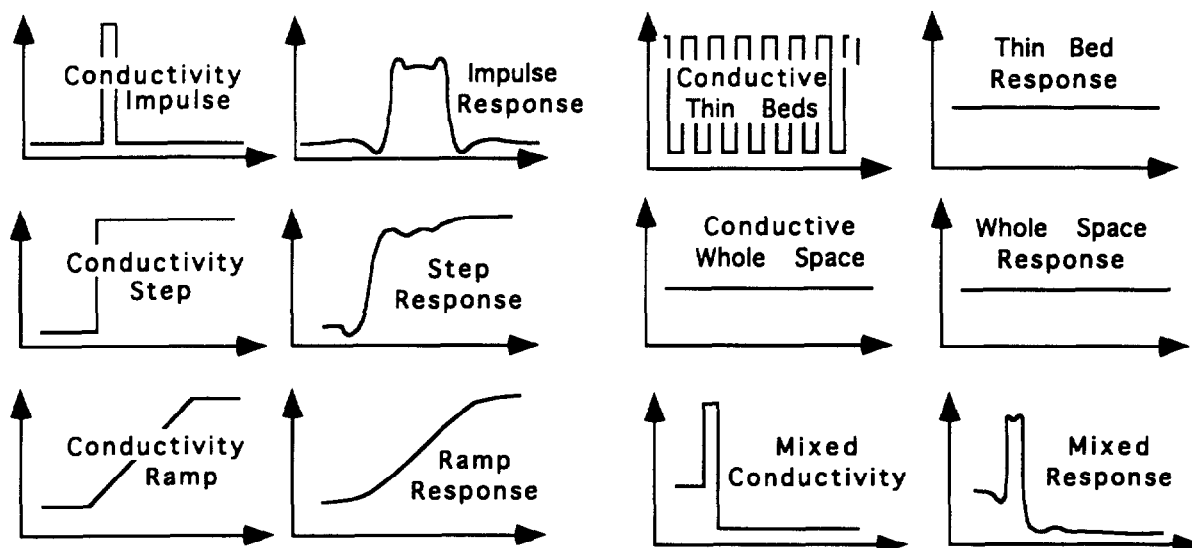


Figure 1. For the schematic tool response pairs pictured above, apparent conductivity response is on the y axis and depth on the x axis. Induction log responses can, in most cases, be recognized as consisting of combinations of these six canonical responses. The impulse response can be used to generate the other five responses and in that sense is the most fundamental of the set, but it is convenient in interpretation to recognize the others as separate responses. As is well known, the thin bed response is sometimes not distinguishable from a whole space response.

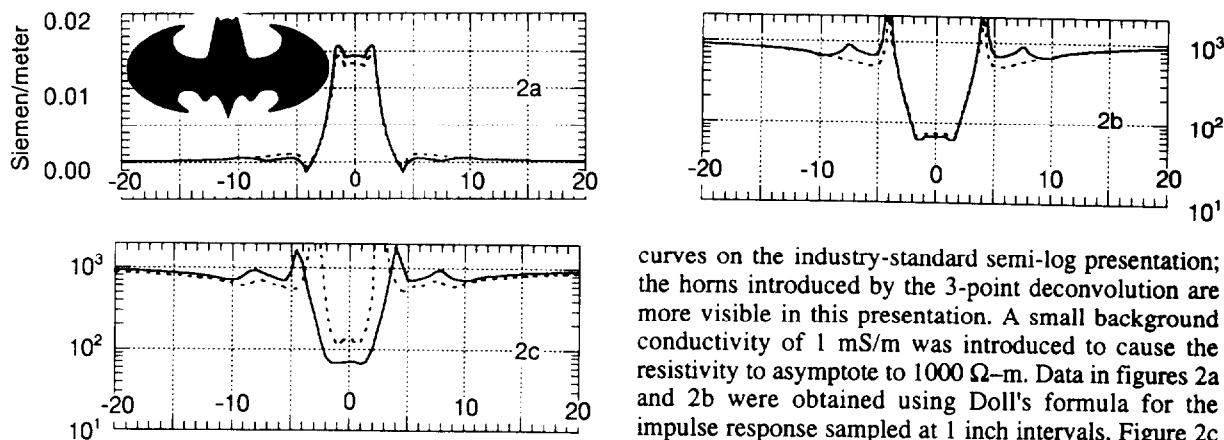


Figure 2. Figure 2a shows the well known vertical response function of the 6FF40 array as the dashed curve. The solid curve is the 6FF40 response with 3-point deconvolution applied. At the aspect ratio shown the impulse response profile resembles Batman. 3-point deconvolution introduces an additional set of horns. Figure 2b is the same data presented as resistivity

curves on the industry-standard semi-log presentation; the horns introduced by the 3-point deconvolution are more visible in this presentation. A small background conductivity of 1 mS/m was introduced to cause the resistivity to asymptote to 1000 Ω -m. Data in figures 2a and 2b were obtained using Doll's formula for the impulse response sampled at 1 inch intervals. Figure 2c is the response of a modeling program to a conductivity impulse. The solid curve corresponds to the solid curve in 2b except that the sampling interval is taken as 5 inches rather than 1 inch. The dashed curve in 2c is the response to the same impulse with the array tilted at 60° with respect to the impulse to simulate relative dip. It is well known that the impulse response will differ for a tilted array; curiously, however, this response is narrower than for a vertical tool.

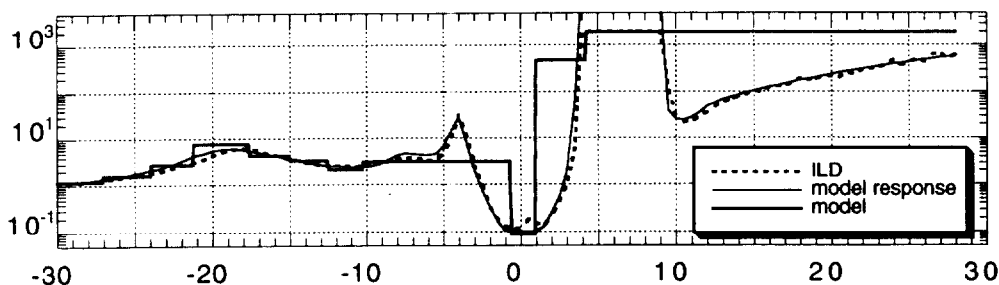


Figure 2d shows a typical conductivity impulse—in this case about 1.5 feet in width. The Batman profile is apparent but subdued on the observed log. The rounded "bottom" with lack of a Batman profile on the modeled response is typical of wide impulses. Note the horn to the right of (or downhole from) the impulse is very wide and resistive. This is characteristic of a high resistivity contrast between the impulse (.085 Ω -m) and the lower bed (1500 Ω -m). If the impulse width had been made narrower and the resistivity lower such that the conductivity-thickness product (or conductance) was held constant the Batman profile could have been developed on the model response. The shape of the log is affected by the impulse conductance for dozens of feet around the impulse. The details of horn shape and amplitude as well as the detail of the "bat" ears are very sensitive to the width of the impulse. Modeling of isolated impulses thus leads to very unique models. (relative dip = 12.5°.)

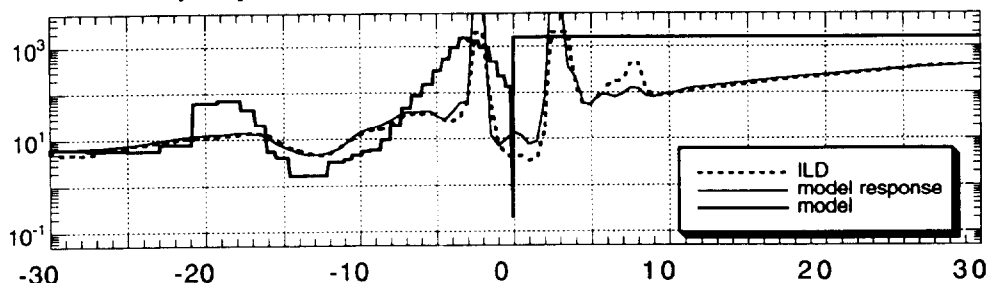


Figure 2e shows a very "impulsive" impulse. Note that every wiggle on the observed log has a corresponding wiggle in the model response. In particular the effect of the 3-point deconvolution is quite apparent downhole from the impulse, and the Batman profile is well developed. Matching the modeled and observed responses at the impulse location spoils the good agreement over the remainder of the log. The failure of the observed and modeled responses to match better at the impulse location seems consistent with a non-conductive borehole fluid. The negative geometrical factor in the borehole would increase the apparent resistivity response observed in the field if the mud had a non-zero conductivity. (relative dip = 60°.)

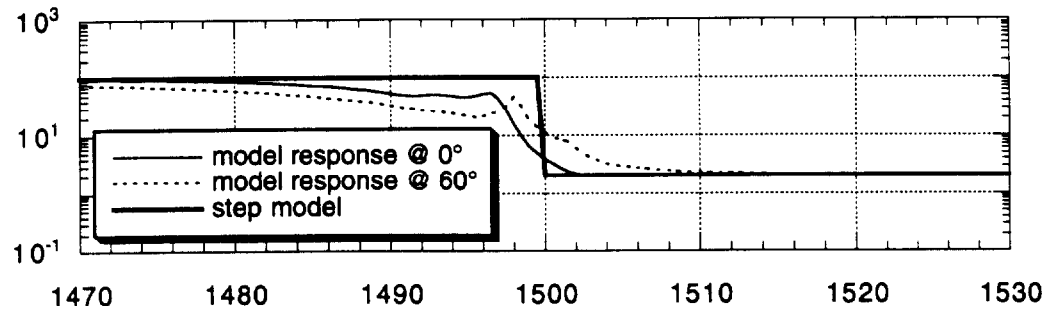


Figure 3a. Ideal step response at relative dips of 0 and 60 degrees.

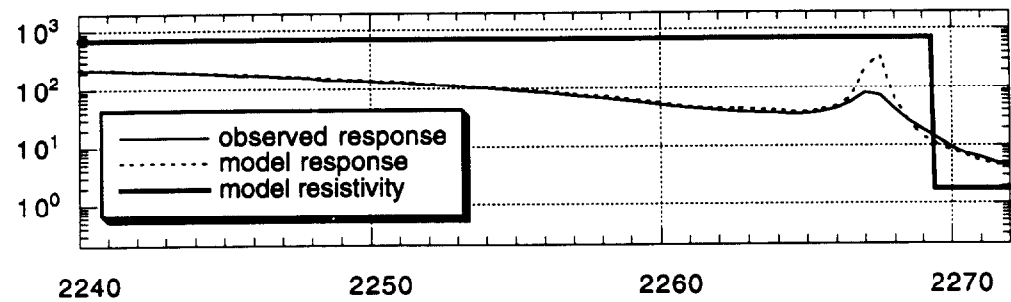


Figure 3b. An observed and modeled step response. The reduced amplitude of the horn in the observed response is probably due to the presence of the borehole (relative dip = 60°).

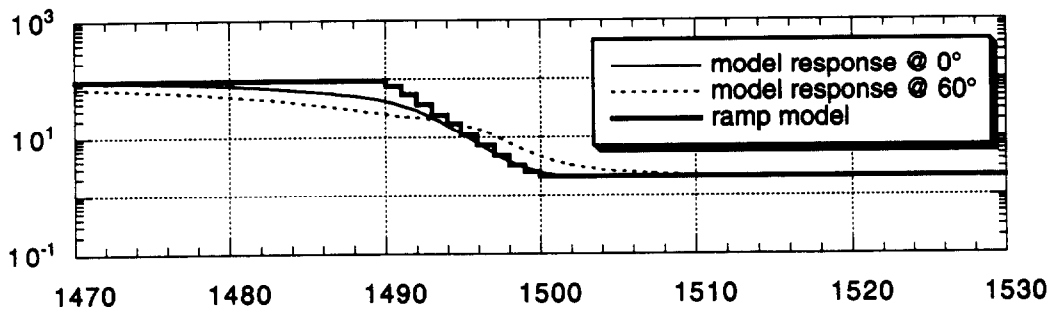


Figure 4a. Ideal ramp response at relative dips of 0 and 60 degrees.

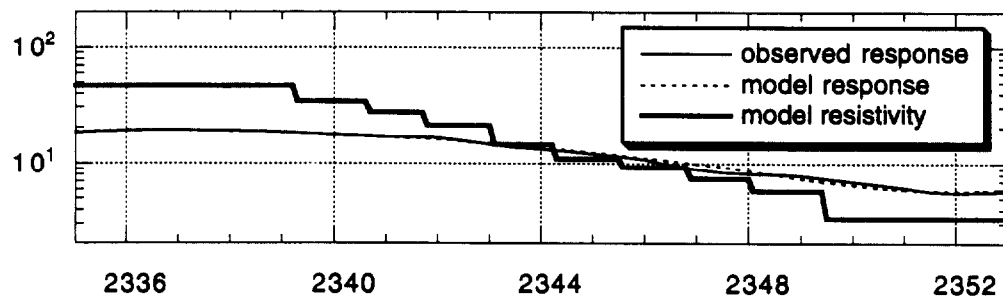


Figure 4b. An observed and modeled ramp response. Note the absence of horns (relative dip = 60°).

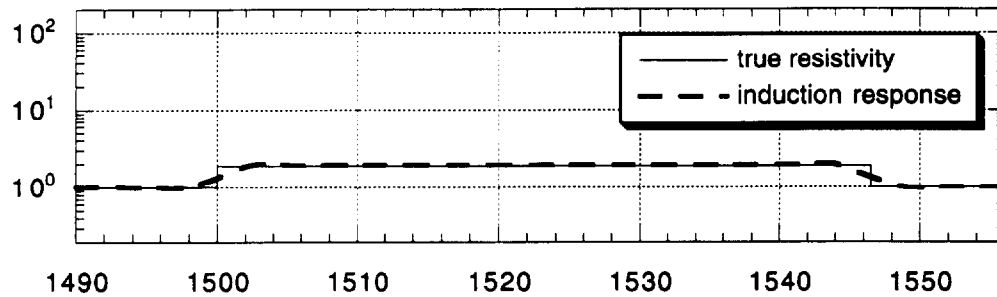


Figure 5a. An illustration of a "thick bed" response" in a 2 Ω -m bed about 46 feet thick.

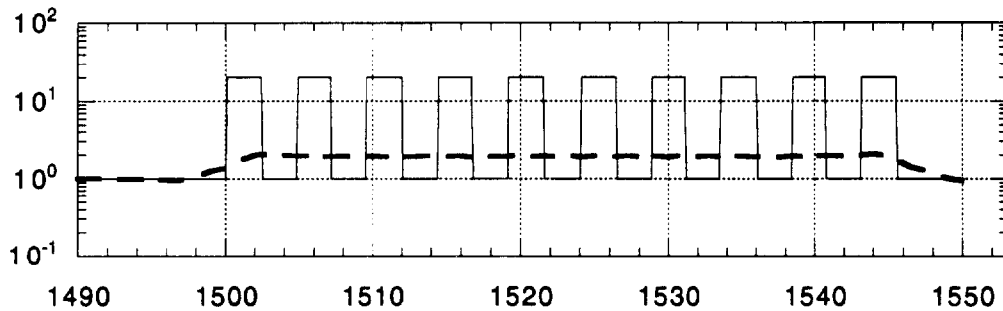


Figure 5b. A periodic sequence of beds with period at the lowest "blind" frequency (≈ 4.72 feet per period) has a response practically indistinguishable from the thick bed in figure 5a. Note the resistive beds have a resistivity of 20 Ω -m.

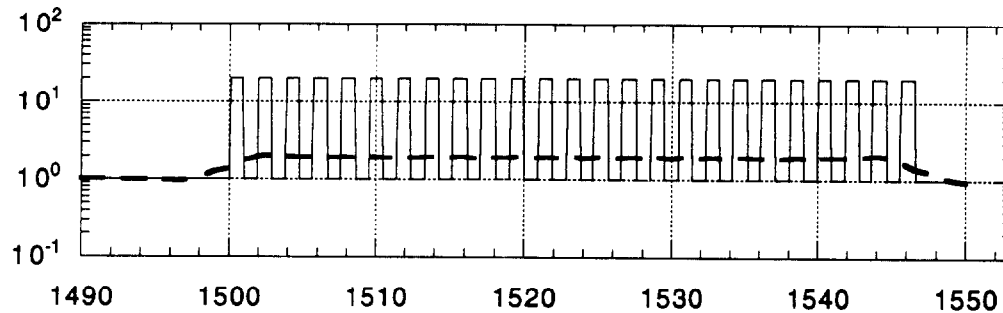


Figure 5c. A periodic sequence of beds with period at the second "blind" frequency (≈ 1.90 feet per period); again the response is practically indistinguishable from the thick bed in figure 5a. Resistivity contrast is the same as in 5b.

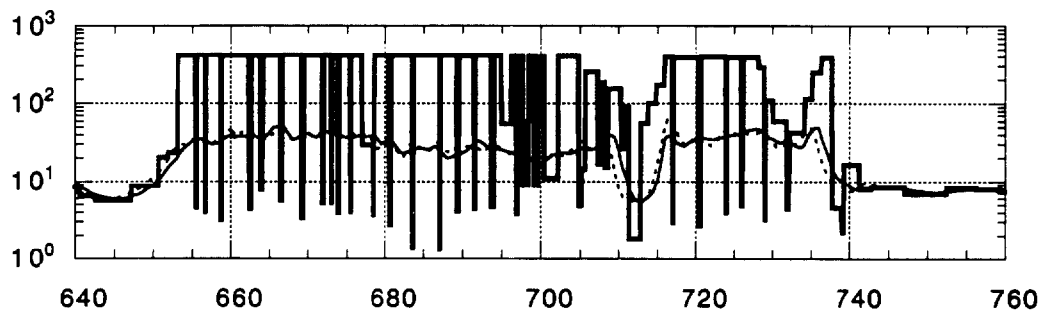


Figure 6. An example from the field. The choice of bed boundaries and resistivities is suggested by logs (not shown) having higher vertical resolution than the ILD (for the boundaries) and known sand/shale resistivity contrasts in pay sands from thicker beds in the same depositional environment. (relative dip = 60°)

Resistivity Model Construction

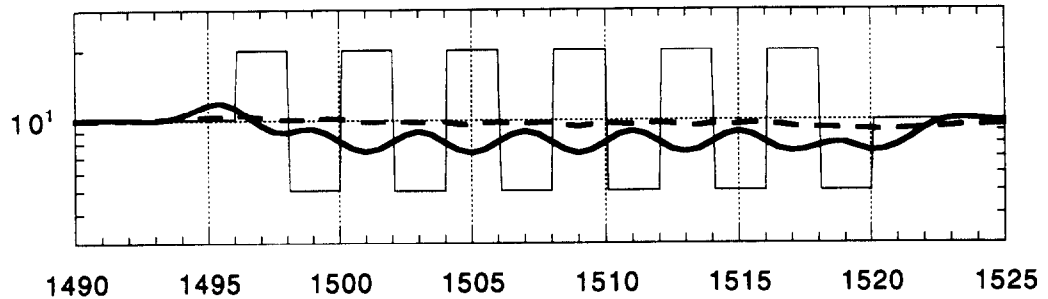


Figure 7a. Ideal responses to a thick periodic sequence of beds having period 4 feet exhibits anticorrelation response at 0° and illustrating that a blind frequency response occurs in the same sequence at 60° . An unlikely to be encountered case with six periods are shown.

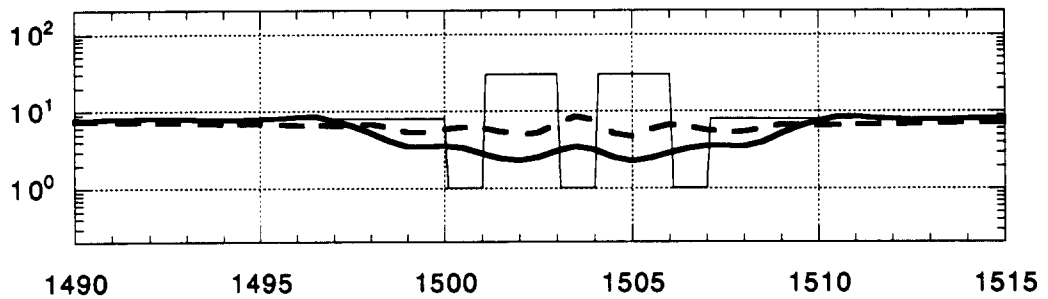


Figure 7b. Ideals responses in the more likely to be encountered case of a periodic sequence having 2.5 periods. The period in this case is about 3 feet and the response anticorrelates both at 0° and 60° .

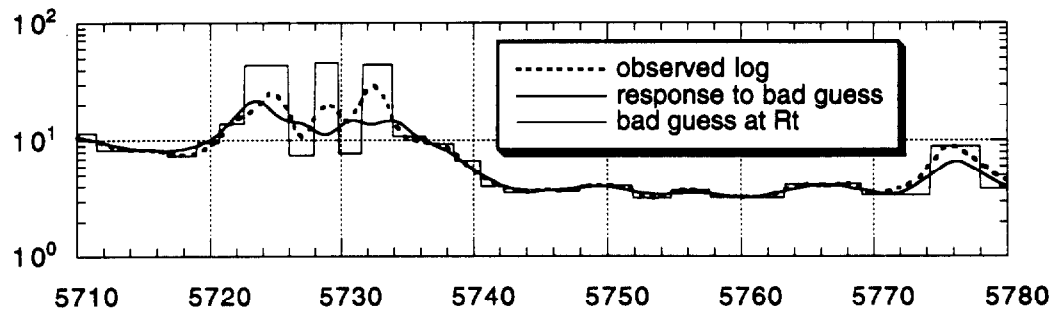


Figure 8a. The observed log might initially suggest the model shown with model resistivity highs and lows corresponding to apparent resistivity highs and lows. The resulting model response anticorrelates with the observed response.

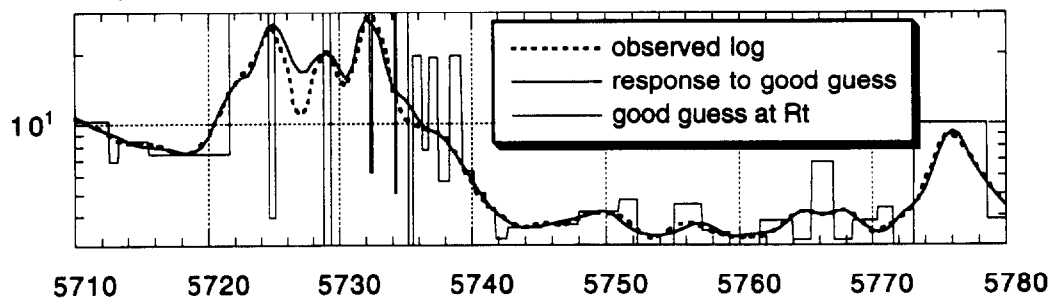


Figure 8b. A model of the same observed log honoring anticorrelations between 5720 and 5735 feet and at 5766 feet. The anomaly amplitude at 5727 could not be matched in a 1-dimensional model response, but the anticorrelation of the model resistivity and apparent resistivity response is clear. The anticorrelation response is not rare. (The resistivity scale is chosen to make the anticorrelations as visible as possible. The impulses at 5729 and 5735.5 bottom at $2 \Omega\text{-m}$ while the resistive zones between 5721 and 5736 are all set at about $500 \Omega\text{-m}$.)

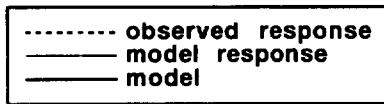
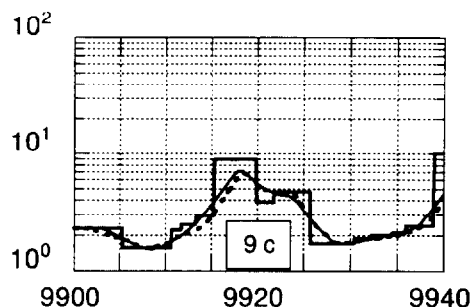
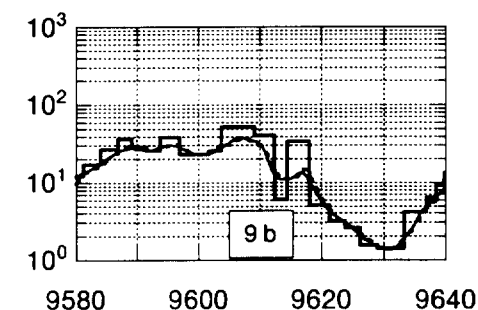
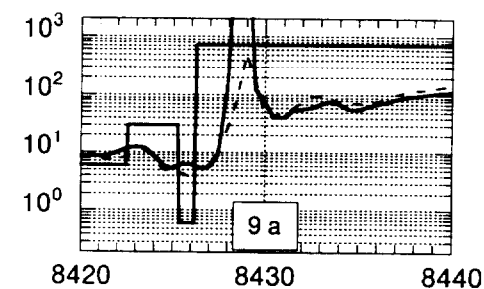


Figure 9. (a) "Impulse-within-a-step" type of mixed response. (b) A false horn generated by an impulse-within-a-step. The sigmoidal shape is diagnostic. (c) An impulse-within-a-ramp. The sigmoid character of the curve at 9920, although flattened, is still suggested. (relative dip = 12.5° in all cases)

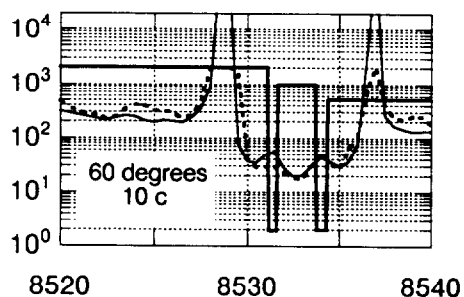
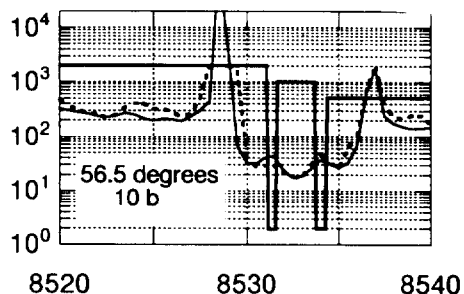
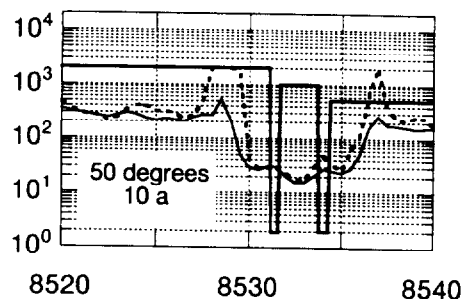


Figure 10. The effect of inaccurately specified relative dip is illustrated. Well records indicated a relative dip of 50° for this anomaly; however using a numerical search applied between 50° (10a) and 60° (10c) the dip which permitted the best fit, especially of the horns, was found to be 56.5° (10b). Choice of this dip permitted the specification of a simpler model for this well than would otherwise have been possible.

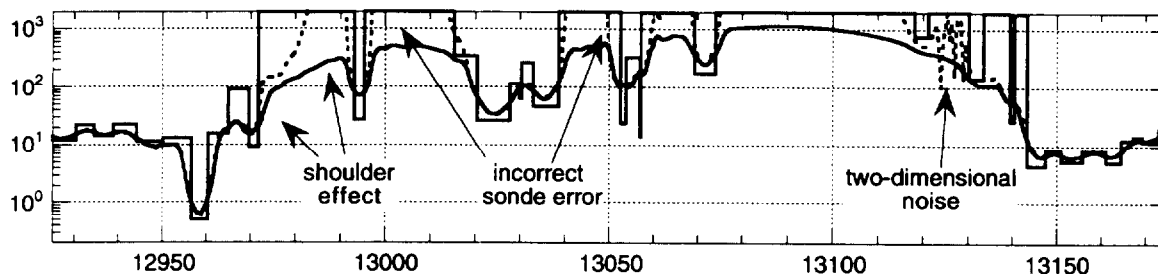


Figure 11. Both two-dimensional noise and an incorrect sonde error can be recognized on this log. (relative dip = 35°)