

CONDUCTIVITY MODELS, PORE GEOMETRY, AND CONDUCTION MECHANISMS

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ABSTRACT

Rocks can be classified as "Archie" or "non-Archie" depending on the relationship between pore geometry, resistivity, and porosity. A mathematical model (such as Archie's equation) which appropriately describes the conduction of a specific rock depends on the conduction mechanism. The conduction mechanism, in turn, is determined by pore geometry as well as nongeometric factors such as wettability and the presence of conductive or surface conductive minerals. An understanding of the relationships between pore geometry, conduction mechanisms, and conductivity models is critical for proper formation evaluation from logs. The concept of electrical tortuosity can be used to understand conductivity models such as Archie's equation. However, the electrical tortuosity, as a factor relating pore geometry and conductivity, is ambiguous in that the pore geometry cannot be inferred from the tortuosity alone.

INTRODUCTION

Occasionally, facts which are "well known" or simply appear somewhere in the literature need to be pulled together and examined in a single context to gain some perspective and intuitive understanding on problems so common as to be taken for granted. For example, Archie's equation (Eq. (2)) is the basis for virtually all pay evaluation made from logs by the petroleum industry. It is frequently stated that Archie's porosity exponent (m), an important parameter in Archie's equation, is a function of electrical tortuosity. This is used as an explanation for the effect of pore geometry on electrical behavior. The actual relationship between " m " and electrical tortuosity, however, is very seldom, if ever, stated. The relationship between tortuosity, pore geometry, and electrical behavior is also not often described. The purpose of this paper is to look at the relationships between electrical tortuosity, pore geometry, and electrical behavior, not to provide an analytical solution to the problem (that has never been done!), but to provide some perspective for everyday use in routine thinking about these problems.

TORTUOSITY AND ARCHIE'S EXPONENTS

The formation resistivity factor (F), introduced by Sundberg (1932), was described by Archie's measurements on a set of sandstone samples (Archie, 1942). It is described by a linear relationship with negative slope and intercept at the origin ($S_w = 100\%$, $F = 1$) on a log-log formation factor-porosity plot (Figure 1a).^w The empirical equation describing this relationship is the well known Archie formation factor equation

$$F = \frac{1}{\phi^m} \quad (1a)$$

(All symbols and abbreviations used are listed in Table 2 at the end of the text.) Archie combined this observation with another, already well documented, exponential relationship between water saturation and resistivity,

$$S_w^{-n} = \frac{R_t}{R_o} \quad (1b)$$

(Figure 1b; Wyckoff and Botset (1936); Jakosky and Hepper (1937); Leverett (1938); Martin, et al. (1938)) to give the famous Archie equation for the calculation of water saturation from resistivity and porosity data

$$S_w^n = \frac{1}{\phi^m} \frac{R_w}{R_t} \quad (2)$$

This empirical equation is extremely important for pay evaluations made from logs. It is, therefore, important to have a good understanding of its physical meaning and limitations. Most of the terms in the equation are readily understood; however, the n and m exponents, having been empirically derived, had no initial theoretical or physical basis. They were simply proportionality constants relating resistivity, porosity, and water saturation. Archie and many others after him were, however, interested in interpreting n and m in terms of the physical characteristics of rocks and the distribution of the fluids contained in them. Guyod (1944) observed that the m exponent seemed to be related to the degree of cementation and so he labeled it the "cementation factor". This idea has subsequently been generalized to the concept of rock texture and pore geometry as the physical factors controlling the m exponent.

Attempts have been made to describe pore geometry in terms of the complexity of paths leading through the pore system, which control the conduction of both fluids and electricity. The concept of the "tortuosity" of the pore system is usually used to describe these paths.

Figure 2 represents a two-dimensional "rock" consisting of circular "grains". The top path through the rock is slightly affected by the grains. Comparing the path length to a straight path gives a tortuosity for the upper path of only 1.05. If the path through the "rock" is constrained to go half way around each grain, then the tortuosity becomes 1.4. The paths illustrated are "mechanical" paths, perhaps similar to a fluid flow path through a simple grain geometry. In general, for rocks with roughly equal sized grains and with at least moderately well connected pores, "mechanical" tortuosities of less than two may be expected.

Electrical tortuosity is usually thought of as the average path length over which conducting ions in solution must diffuse compared to the length of the sample. Electrical tortuosity can, therefore, be defined in terms of the apparent length of the conductive path through a rock. This is a

convenient, simple, and intuitive way to describe the complicated electrical conduction paths which exist in a natural rock sample.

An alternative way to think of electrical tortuosity has been described by Wyllie and Rose (1950), Winsauer, et al. (1952), and Hellander (1971). This definition of electrical tortuosity is probably the one most commonly in use. If the water in a water-saturated rock sample (Figure 3) is removed and placed in a capillary tube such that the water content of the tube is equal to the water content of the rock, and the end-to-end resistance of the tube and the rock are equal, then the electrical tortuosity is defined as the ratio of the length of the tube (l_e) to the length of the rock sample (l_c), $\tau_o = l_e/l_c$ (Figure 3). It can be shown that the electrical tortuosity of a water saturated rock τ_o is related to the formation factor F and the porosity ϕ (Wyllie and Rose (1950), Winsauer, et al. (1952), and Hellander (1971))

$$\tau_o^2 = F\phi \quad (3)$$

Eq. (3) can be rewritten as

$$\log F = -\log\phi + \log\tau_o^2 \quad (4)$$

Lines consisting of points having equal tortuosity can, therefore, be drawn on a formation factor plot (Figure 4). Eq. (4) indicates that the lines have -1 slope and an intercept of $\log\tau_o^2$. If four rock samples which obey Archie's equation and have porosities of 5%, 10%, 15%, and 20% and a common porosity exponent ($m = 2$) are plotted on the formation factor plot, then a single straight line connects them (Figure 5). The line has a slope of - m and passes through the origin ($\phi = 100\%$, $F = 1$). It is apparent from Figure 5 that m describes the change in electrical tortuosity with changes in porosity. By substituting Archie's formation factor equation (Eq. (1)) in Eq. (4), the m -tortuosity relationship can be described

$$m = 1 - \frac{\log \tau_o^2}{\log \phi} \quad (5)$$

The first term (1) in Eq. (5) reflects the formation factor - porosity relationship due only to the water content independent of geometric complexities. It is equivalent to a straight capillary tube model of the rock. The second term ($\log \tau_o^2 / \log \phi$) describes the tortuosity dependence on porosity, reflecting changes in the complexity of the conducting path with changes in porosity. Changes in complexity may result from changes in the length and size of pore connections and the average number of connections per pore as the result of changing porosity.

The resistivity response of partially water-saturated rocks can be described in terms of a "generalized formation factor" plot (Figure 6). The "formation factor" of a partially water-saturated rock is R_t/R_w . It is important to note that the formation factor plot describes the relationship between resistivity and the water content of a rock. For water-saturated rocks, the water content is equal to the porosity. For rocks only partially saturated with water, R_t/R_w is related to the product of the water saturation and the porosity ($S_w \phi$). This generalized formation factor plot has the advantage of displaying the laboratory data from both formation factor and resistivity index plots.

The electrical tortuosity can be defined for partially water-saturated rocks in an analogous way to the definition given earlier for water-saturated rocks:

$$\tau_t^2 = \frac{R_t}{R_w} (S_w \phi) \quad (6)$$

This equation can be used to define isotortuosity lines on the generalized formation factor plot for partially water-saturated rocks (Figure 6). Rocks which have a resistivity response described by Archie's equation (Eq. (2)) plot as a straight line on this diagram having a slope of $-n$, the Archie saturation exponent (Figure 7).

Like the cementation exponent, the saturation exponent (n) describes the change in electrical tortuosity with changes in water content. In the case of the cementation exponent, the variation in water content is due to changes in porosity. The saturation exponent describes the change in electrical tortuosity with changes in water saturation. The n -tortuosity relationship can be expressed mathematically by substituting Eqs. (2) and (3) into Eq. (6) and solving for " n ",

$$n = 1 + \frac{\log \tau_o^2 - \log \tau_t^2}{\log S_w} \quad (7)$$

The first term (1) reflects the resistivity- S_w relationship due only to the water content, independent of the complexities of the pore system as if all the water in the rock is present in straight capillary tubes. The second term $((\log \tau_o^2 - \log \tau_t^2) / \log S_w)$ describes the change in electrical tortuosity from that of the water-saturated state due to changes in the geometric complexity of the distribution of the conducting water in the pore system of the partially saturated rock.

PORE GEOMETRY, CONDUCTION MODELS, AND MECHANISMSArchie Rocks: Clean Sands

Rocks may be classified by their conductivity-porosity relationship as Archie or non-Archie rocks (Table 1). Archie rocks, those which obey Archie's equation, have at least three significant characteristics. First, they have a unimodal pore size distribution. Secondly, the relationship between the pore sizes and the pore aperture sizes varies in a regular way. Finally, conduction occurs by the diffusion of ions in the intergranular pore space in the rock in response to an electric field (Figure 8). Clean, well-sorted quartz sands are probably the best examples of Archie rocks. The electrical behavior of rocks with these characteristics is a function of changes in water content as described by Archie's equations. Consequently, they exhibit a linear trend which extrapolates to the origin of a generalized formation factor plot (Figure 6) with changes in either water saturation or porosity. Rocks with pore geometries other than those described above tend to show electrical behavior which is inconsistent with Archie's equations and can be considered as non-Archie rocks. Examples of non-Archie rocks, which will be discussed below, are mainly those with polymodal pore systems, such as shaly sands, rocks with moldic porosity, and rocks containing unconnected microporous grains.

Archie's equation can be written in terms of conductivity

$$C_t = (S_w^n \phi^m) C_w \quad (8)$$

where C_t is the conductivity of rock only partially saturated with water, and C_w is the conductivity of the water in the rock.

The coefficient $S_w^n \phi^m$ can be thought of as the water content of the rock corrected for electrical tortuosity. It can be further described as follows: If one considers a cylindrical rock sample having water content ($S_w \phi$) and obeying Archie's equation (Eq. (8)), then a capillary equivalent to this rock can be defined as a straight tube extending through an otherwise nonporous model "rock" having the same dimensions as the real rock (Figure 9). The capillary equivalent has water content $S_w \phi_e$, and the same end-to-end conductance (C) as the rock ($C_t(\text{rock}) = C_e^e$ (capillary equivalent)). If the same water composition is used in both the rock sample and the equivalent capillary tube ($C_w(\text{rock}) = C_w(\text{tube})$), then the water content of the equivalent capillary tube ($S_w \phi_e$) is equal to the water content corrected for electrical tortuosity of the pore system of the real rock:

$$S_w^n \phi^m (\text{rock}) = S_w \phi_e (\text{equivalent capillary}) \quad (9)$$

The term $S_w^n \phi^m$ can, therefore, be considered to be the "electrically effective water content" of the rock. It is the amount of water which seems to

be in the rock sample as indicated by the resistivity. It can also be thought of as the proportionality factor which must be used to correct C_w for the effect of the pore geometry of the rock to get C_t . Knowing the values of the Archie exponents n and m allows one to calculate the actual water content of a rock from the electrically effective water content.

Non-Archie Rocks: Shaly Sands

Shaly rocks can be characterized as those which do not obey Archie's equation and which contain clay minerals. They are often classified as having "dispersed", "layered" (or "laminated"), or "structural" shale (Figure 10). Accounting for the non-Archie behavior of shaly sands has been a continuing subject of research for over 40 years. Several conductivity equations have been proposed to account for the non-Archie electrical behavior of shaly sands. Interbedded sands and shales are described by the laminated shaly sand equation of Poupon, et al. (1954). The electrical effect of dispersed "shale" in shaly sands is accounted for by the Waxman and Smits model (1968). Another shaly sand equation has recently been proposed by Givens (1986) which is purported to "be applicable to a large number of different rock types," and be "independent of any specific conduction mechanism".

It is instructive to compare these models to emphasize their similarities and differences. Each equation can be written in terms of conductivity for ease of comparison:

1. Clean Sand: Archie $C_t = (S_w^n \phi^m) C_w$ (10)

2. Dispersed Shaly Sand: $C_t = (S_w^n \phi^m) (C_w + BQ_v/S_w)$ (11)
Waxman Smits

3. Layered Shaly Sand: $C_t = (S_w^n \phi^m) C_w (1 - V_{sh}) + C_{sh}(V_{sh})$ (12)
Poupon

4. Conductive Rock $C_t = (S_w^n \phi^m) C_w + C_r$ (13)
Matrix Model: Givens

It is important to realize that the electrically effective water content ($S_w^n \phi^m$), the Archie term which accounts for the effect of pore geometry, is identical in each of the four equations. In each case, this term refers to the tortuosity and formation factor of only the interconnected, intergranular macropore part of the pore system (the Archie part), which is independent of cation exchange or microporosity associated with clay minerals. In fact, the first part of each of the last three equations is identical to Archie's equation. The second term in each of the three shaly sand equations is meant to correct for the effect of "shale".

The second equation is the well known Waxman-Smiths equation (Waxman-Smiths, 1968), intended to correct calculated water saturations for the increase in conductivity from cation exchange on certain clay minerals, mainly smectite and illite. The term BQ_v gives the conductance per unit volume of pore space (C/ϕ) which is due to exchangeable cations. BQ_v/S_w is the conductance per unit water content ($C/S_w\phi$). The second term of the Waxman-Smiths' equation is, therefore, the electrically effective water content times the exchange ion conductance per unit water content or the exchange ion conductance of the partially water-saturated rock. The first term of the Waxman-Smiths' equation is simply Archie's equation describing the component of conductivity due to the intergranular pore space, independent of the exchange ions.

It has often been stated that the Waxman-Smiths' equation describes two parallel conductors: the intergranular pore space and the active clay surfaces. It does not. The second term of the equation ($(S_w^n \phi^m)(BQ_v/S_w)$) constrains the exchange ions to conduct "along the same tortuous path as the current attributed to the ions in the pore water" (Waxman and Smits, 1968). This is one of the basic assumptions made by Waxman and Smits in the development of their model, but it often seems to be overlooked. Furthermore, the conductance due to the exchange ions is variable, depending on the water saturation of the intergranular part of the pore system. The surface conductance is, therefore, not independent of the conductance of the intergranular pore system and the system does not behave like two independent parallel conductors. The Waxman-Smiths conduction model constrains the exchange ions on the clay minerals to be added to ions in the intergranular pore fluid by an ionization or ion exchange process. The conduction due to the exchange is added to that of the ions in solution in the intergranular pore water. Rather than two parallel conductors, the Waxman-Smiths' equation describes a system more like a small battery (exchangeable ions on clays) in series with a conductor (intergranular pore water). This model is also additive, but the two conductors (the battery and the resistor) are not parallel and do not conduct independently.

The Waxman-Smiths disseminated shale conductivity model has been modified by Clavier, et al. (1977) to account for the volume of the double layer, or "bound water" associated with the charged surfaces of the clay particles. The total pore volume was apportioned between that of the double layer and the intergranular or "free" (Archie-like) water. Otherwise, the "dual water model" of Clavier, et al. (1977) implies the same relationship between the ions in the intergranular pore water and the clay exchange ions. The use of the same formation factor to describe the electrical behavior of the exchange ions as well as that of the ions in the bulk pore water in the intergranular pore space is consistent between the two equations and implies a battery-resistor analog rather than independent parallel conductors.

The layered shale model of Poupon, et al. (1954) is a true parallel conductor model. It describes interbedded sands and shales with Archie's equation used for the sand beds. The total rock volume is apportioned between sand beds and shale beds using the volume of shale (V_{sh}) term. The conductivity of the intervening shale layers is not modeled in any

way, but the measured, or assumed, bulk conductivity of the shale layers is used with the assumption of 100% water saturation.

A fourth shaly conductivity model is the "conductive rock matrix model", or CRMM (Givens (1986)). The assumptions on which the model is based are: (1) "the total electrical conductance of many rocks can be represented by two conductors in parallel" - the intergranular pore space and the "matrix" and (2) "any conductance due to the matrix can be treated strictly as an electrical property independent of any specific conductive mechanism." The model is said to be "applicable to a large number of rock types."

This model also describes two independent parallel conductors: the intergranular pore space which obeys Archie's law ($S_w^n \phi^m C_w$) and the conductive "rock matrix" (C_r). Despite Givens' implication that this is a general equation describing low resistivity shaly sands, it is not equivalent to Waxman-Smiths' equation since the Waxman-Smiths' model is not a parallel model with independent conductors. The Waxman-Smiths' model and its implied conduction mechanism cannot be described by the CRMM.

Comparison of the CRMM with the laminated shale model demonstrates that the two models are very similar. The laminated model is based on the same parallel conductor assumption as Givens' model. Givens' model, however, does not account for any bulk volume or pore volume which may need to be attributed to the "matrix". The entire bulk and pore volumes are accounted for in the first (Archie) term of the equation, with no volume correction for the conductive "matrix". The CRMM is, therefore, not truly a new or general conductivity model, but will be considered equivalent to the laminated model.

Conduction Mechanisms

Rocks which obey Archie's equation conduct by ion diffusion in an electric field in intergranular pore water (Figure 8). There are no other conductive paths and the pore system can be modeled using the concept of electrical tortuosity.

The Waxman-Smiths' model describes rocks which do not obey Archie's equation. These shaly rocks have two sources of conduction which are both dependent on the water saturation and amount of intergranular pore space. Clay minerals with a significant cation exchange capacity and of authigenic origin are usually disseminated throughout the pore system on grain surfaces (Figure 11). The exchangeable cations associated with the clay minerals, according to the Waxman-Smiths' model, are ionized or exchanged into the intergranular pore space and add to the conducting ions present in solution in the intergranular pore space. The total conductivity is the sum of the conductivity of the diffusing ions from both sources, modified by the geometric complexity of the intergranular pore system.

The laminated shale model and Givens' conductive rock matrix model apply to rocks which have two parallel independent conducting media such as interlaminated sand and shale beds. According to Givens' model, rocks having a continuous, connected microporous pore network in addition to the

intergranular pore space conduct as if they are layered. Rocks whose grains are continuously coated with authigenic clay minerals, such as the illite-coated grains of the Jurassic Nugget Formation samples which Givens studied, may be an example. The authigenic clay crystals form a continuous microporous network on grain surfaces (Figure 12). The micropore network tends to remain water-saturated due to the high capillary force preventing the intrusion of hydrocarbons. The continuous micropore network is, therefore, unaffected by changes in the water saturation of the intergranular pore space, and functions as an independent conductor. The important aspect of Givens' work, then, was to recognize that rocks having a continuous connected micropore network in addition to the intergranular pore network conduct like two independent parallel conductors. A massive sandstone composed of grains which are clay coated can, therefore, behave as if it were laminated.

Givens' second assumption, that the CRMM is independent of conduction mechanism, is invalid. The conduction mechanism dictates the applicable model. A rock, for example, which conducts by a mechanism described by the Waxman-Smits' or Dual Water Model cannot be modeled by a parallel or laminated shaly sand model, such as the conductive rock matrix model.

The critical differences between conductivity models can be illustrated using a "generalized formation factor plot" (Figure 13). The shape of lines on this log-log plot is governed by the mathematical form of each of the conductivity models. For example, the form of Archie's, Waxman-Smits', and Givens' models can be generalized in terms of $\log(C_t)$ as a function of $\log(S_w)$:

$$\text{Archie: } \log(C_t) = -\log(aS_w^n) \quad (14)$$

$$\text{Waxman-Smits: } \log(C_t) = -\log(bS_w^n + cS_w^{n-1}) \quad (15)$$

$$\text{Givens: } \log(C_t) = -\log(dS_w^n + e) \quad (16)$$

where a, b, c, d, and e are constants. The Archie equation (Eq. (14)) is a simple exponential in S_w and therefore produces a straight line on a $\log(C_t) - \log(S_w)$ plot. Eq. (15), however, is a polynomial in S_w and, therefore, results in a curve. Givens' CRMM (essentially a layered model) is a single exponential in S_w ; however, a constant (e) is added, producing a higher degree of curvature than the polynomial of Eq. (15), which lacks an added constant. Figure 13 illustrates the curve shape for the three shaly sand models for a hypothetical rock having 20% porosity, $n = 1.5$, $m = 2$, surface or matrix conductivity = 0.2 mho-m and partially saturated with 0.1 ohm-m water. It should be noted that due to the form of the equations, a rock that conducts by a Waxman-Smits (Eq. (15)) mechanism can be modeled only approximately with a Givens-type equation (Eq. (16)). The "matrix" or shale layer conductivity which would be predicted by modeling

a rock having disseminated shale with a layered shale model would not reflect the true shale contribution to the total conductivity.

A confusing issue in Givens paper is that he used illite-bearing rocks from the "Anschutz" Nugget Formation. Illite has the capacity to suppress the formation resistivity by two mechanisms. It can form a continuous micropore network, as indicated by his SEM pictures. Illite also has significant cation exchange capacity, so that it can conduct by a Waxman-Smits' mechanism as well. To what degree each mechanism was involved in the observed resistivity suppression is unknown. It may be possible to separate the two effects by making a resistivity index measurement with a wide range of saturation values, using first a low salinity water to emphasize cation exchange conduction, and then a high salinity water to minimize the effect of cation exchange and emphasize the micropore network conduction.

Non-Archie shaly sand resistivity-porosity data is sometimes fit with a "modified" Archie equation

$$F = \frac{a}{\phi^m} \quad (17)$$

Over a limited porosity range, or with some scatter in measured resistivity-porosity data, either disseminated or layered sands may appear to give straight lines on a formation factor plot. The lines are characterized by a low slope (equivalent to $m < 2$) and a positive intercept ($\log a$). However, a value of "a" other than one is simply an artifact of attempting to make non-Archie rocks fit a pseudo-Archie model which has no theoretical basis.

Combined Conductivity Model

Shaly sands can conduct in the intergranular pore space (Archie), by cation exchange (Waxman-Smits) and by parallel conduction in a micropore network (Givens' layered model). A single general model which can be used to describe rocks which conduct by any or all of these mechanisms is a combination of the three (Patchett and Herrick (1982))

$$C_t = (S_w^n \phi^m) \left(C_w + \frac{BQ_v}{S_w} \right) (1 - V_{sh}) + C_{sh} (V_{sh}) \quad (18)$$

Note that the shale conductivity term, C_{sh} , can apply to either shale laminae or mineral coatings that form a continuous microporous pore network. In either case, the volume must be apportioned with the V_{sh} term. If the parameters m , n , ϕ , and BQ_v are known for the "shale" phase, then the C_{sh} term could be replaced by an appropriate descriptive equation such as Archie's or Waxman-Smits' equations. The combination model can be written in a statistical form:

$$C_t = a S_w^n + b S_w^{n-1} + c \quad (19)$$

where a, b, c, and n are parameters determined by a statistical fit of laboratory data.

Non-Archie Rocks: Other Pore Geometries

Non-Archie electrical behavior is exhibited by rocks other than shaly sands. Focke and Munn (1987), for example, have recently described the problems of interpreting resistivity measurements in rocks with oomoldic porosity. Rocks which have undergone partial dissolution during diagenesis often have secondary moldic porosity due to the complete dissolution of some of the more soluble grains. For example, feldspar dissolution in sandstones can produce appreciable moldic porosity. Dissolution of shell fragments, oolites, or other soluble grains in carbonates can also result in a high proportion of moldic porosity. The pore geometry which results from the complete dissolution of grains is characterized by relatively large grain-sized pores which tend to be connected only through the intervening intergranular pores and pore connections. The result is a rock with significantly increased porosity, but relatively unchanged permeability and resistivity. Vuggy carbonates with large, poorly connected vugs also exhibit similar pore geometry, permeability, and resistivity. Hydrocarbon saturation of the large secondary pores is controlled by the size, number, and wettability of pore throats connecting the pores to the remainder of the rock. If hydrocarbons are able to overcome the capillary forces of the pore throats to enter the large secondary pores, then the pores tend to become saturated with hydrocarbons.

A formation well known for oomoldic porosity is the Smackover formation in Texas and Louisiana. Figure 15 shows photomicrographs of thin sections of two Smackover samples. Photograph A is of a dolomitized mudstone having porosity of 19%, formation factor of 37 and electrical tortuosity of 2.7. Photograph B is a wackestone originally consisting of oolites in lime mud. The mud has been dolomitized, and the oolites have been dissolved, resulting in an oomoldic dolomite with porosity of 32%, formation factor of 43, and electrical tortuosity of 3.7. The dolomitized mud of photograph A is very similar to the dolomite matrix of photograph B. The effect of oolite dissolution is to increase porosity, while having relatively little effect on resistivity. A result is that the electrical tortuosity increases dramatically from 2.7 to 3.7 and the cementation exponent m increases from 2.2 to 3.3 (Figure 16).

A change in m with porosity is, of course, characteristic of non-Archie rocks. Archie's formation factor equation is, therefore, not a suitable model for rocks with isolated secondary pores such as oomoldic carbonates. Rasmus (1986) and Rasmus and Kenyon (1985) have modeled the electrical behavior of rocks with moldic porosity using theory developed by Sen, et al. (1981) for spherical conductive inclusions in a conductive host medium (Figure 17). This geometry is most similar to oomoldic rocks, but Rasmus recommends the model for any rocks with moldic porosity since "it has been found that the current responds mainly to the intergranular porosity so the shape or placement of the secondary porosity is not impor-

tant." The application of the model to rocks with partial hydrocarbon saturation, however, is difficult since assumptions are required about the wettability and water saturation of the moldic pores.

Another type of pore geometry results from only partial dissolution of grains, resulting in a rock with spatially isolated conductive grains (Figure 18). Partial dissolution typically results in grains which are microporous. For example, microporous chert grains (Figure 19), fossil fragments, or oolites resulting from partial dissolution are common. The diagenetic alteration of feldspar to kaolinite can also result in spatially isolated microporosity in the kaolinite. Microporous grains are conductive since they tend to stay saturated with water due to the high differential pressure necessary to displace water from very small pores. "Structural shale" (Figure 10) is also included in this pore geometry. Cation exchange on clay minerals such as smectite or illite in the shale clasts may also contribute to the conductivity of the grains over and above that due to water in the micropores of the shale.

Swanson (1985) has modeled the electrical behavior of rocks containing spatially isolated conductive grains. He suggested that a series electrical circuit would provide a suitable model, since the electric current which passes through each conductive grain must have also passed through the intergranular pore space of the intervening matrix between conductive grains (Figure 20). In one respect, this pore geometry is simpler to model than moldic porosity, since the microporous, conductive grains tend to be unaffected by changes in water saturation of the intergranular pore space due to large capillary forces. The volume and conductivity of the conductive grains must be known to apportion the total pore space and conductivity between the conductive grains and the intergranular pore space. It is possible to measure the volume of microporosity using high pressure mercury injection capillary pressure measurements in the laboratory (Swanson, 1985), but it is more difficult to apportion the volume and conductivity of structural shale clasts. Apportioning the pore space is even more difficult (if not impossible) if wireline logs are the only source of data.

It may also be possible to model spatially isolated conductive grains with Rasmus' moldic porosity model by substituting the conductivity of the porous grains for that of water in moldic pores, with the viewpoint that completely dissolved grains are just a special case of partial dissolution. Whether Swanson's or Rasmus' model is used, the result is non-Archie electrical behavior since, compared to an Archie rock, large changes in amounts of conductive pore space are accompanied by relatively small changes in conductivity. For example, Figure 21 describes the changes in resistivity (formation factor) accompanying changes in different types of pore space. If a rock with 10% intergranular pore space and formation factor of 40 is assumed, then other rock samples with the same intergranular pore geometry, but with higher porosity, will tend to have a porosity-formation factor relationship described by Archie's equation, and will plot on the Archie line in Figure 21. If, on the other hand, the pore space had been increased by the addition of moldic pores without affecting the intergranular pore space of the remaining rock matrix, then the porosity-formation factor relationship is described by Rasmus' model. The pore

space is increased with only a small accompanying decrease in formation factor. Swanson's model describes the addition of pore space by only partial dissolution or alteration of certain minerals to form microporous conductive grains, which are connected by the unchanged intergranular pore space. Partial dissolution or grain alteration results in a formation factor-porosity relationship intermediate between those due to the Archie and Rasmus pore geometries.

TORTUOSITY AND PORE GEOMETRY

Of the various pore geometries discussed, only the intergranular geometry which has a formation factor-porosity relationship described by Archie's equation can be considered unimodal. Each of the other geometries is at least bimodal in that a second pore structure is superimposed on the intergranular pore geometry. Combinations of pore structures can result in polymodal pore systems with complicated formation factor-porosity relationships (Table 1).

Some general statements can be made about the relationship of electrical tortuosity and pore geometry, since the electrical tortuosity is simply the product of formation factor and porosity. For Archie rocks having at least moderate porosity ($>10\%$), tortuosities tend to have low to intermediate values, with tortuosity increasing with decreasing porosity (Figures 5, 14, and 22). Rocks having varying amounts of moldic porosity have the opposite relationship. The electrical tortuosity tends to increase with increasing amounts of moldic porosity (Figure 22). High values of electrical tortuosity accompanying high porosities are invariably associated with large spatially isolated moldic pores.

According to the formation factor-porosity relationship predicted by Swanson's pore model, rocks with relatively high tortuosities which change little with porosity may have varying amounts of microporous grains (Figure 22). Layered and dispersed shaly sands tend to have reduced resistivity, especially at low porosity or low water saturation. A decrease in porosity or water saturation, therefore, can result in a reduction in electrical tortuosity (Figure 14).

Factors other than pore geometry can affect the electrical tortuosity. Some nongeometric factors to be considered are listed in Table 1. For a given water saturation, the fluid distribution in a rock may vary considerably in response to changes in wettability. Changes in fluid distribution can change the conductivity, and therefore, change the electrical tortuosity. In shaly rocks, the higher the cation exchange capacity, the more conductive the rock becomes, reducing the electrical tortuosity. The presence of other conductive minerals such as pyrite can also decrease the electrical tortuosity.

It is clear from the discussion above that the electrical tortuosity gives some indications about pore geometry, and for extreme values of tortuosity, the pore geometry may be fairly well determined (e.g., moldic porosity, fractures). It is equally clear, however, that in general the electrical tortuosity is ambiguous. Knowing the electrical tortuosity does not establish the pore geometry. For example, a low porosity rock

with only intergranular porosity which obeys Archie's equation can have a rather high value of electrical tortuosity (Eq. (3)). The conduction paths can be represented by a sinuous capillary wrapped around the grains composing the rock (Figure 2, Figure 23). However, another higher porosity rock having appreciable moldic porosity, but a tighter less conductive matrix, can have exactly the same electrical tortuosity as the lower porosity Archie rock. Moldic pores can also be modeled with a modified capillary. If a straight capillary is enlarged into bubbles at intervals along its length, then the porosity is greatly increased (straight capillary with "vugs", Figure 23). The conductivity, and therefore the formation factor, is mainly controlled by the remaining small radius capillary connecting the bubbles. In a geometric sense, this straight bubbly capillary model has a mechanical tortuosity equal to one. The electrical tortuosity, however, can be high since it is the product of F and ϕ (Eq. (3)), and can even be numerically equal to that of the highly sinuous capillary of the low porosity Archie rock. For this example and many others, more information is obtainable about the pore geometry of the capillary models, or more importantly, of rocks, from the formation factor-porosity relationship. The distribution of data on a formation factor plot is generally more informative of pore geometry than the electrical tortuosity. In fact, it should be clear that the electrical tortuosity is a very poor way to describe the electrical behavior of rocks as a function of pore geometry.

Rather than pore geometry, the electrical tortuosity may be descriptive of the relationship between the water content of a rock and its electrical behavior. The square of the electrical tortuosity is equal to the ratio of the water content of a rock to the electrically effective water content (Eq. (20), Figure 9).

$$\tau_t^2 = \frac{S_w \phi}{S_w^n \phi^m} \quad (20)$$

It is the ratio of the water content of a rock to the amount of water which seems to be in the rock. This alternative definition of electrical tortuosity tells little about pore geometry, but the ratio of real to apparent water content is an intuitively useful quantity.

In a descriptive sense, the modified Archie formation factor equation (Eq. (17)) can be used for data from non-Archie rocks. In this sense, the parameters "a" and "m" relating to the intercept and the slope have no physical or theoretical significance, but are simply statistically derived. For most non-Archie rocks, "m" will be less than two and may even take a value of less than one. The intercept parameter "a" will always have values greater than one for non-Archie rocks. For rocks having the same type of pore geometry, "m" and "a" should be indicative of systematic changes in that geometry. Careful calibration should allow predictions of pore geometry from "a" and "m" and conversely. An empirical modified Archie equation should always be accompanied by the standard

error of the estimate of formation factor from porosity, and the porosity range over which the equation is valid must always be stated.

CONCLUSIONS

1. The exponents in Archie's equation, "n" and "m", can be explicitly related to the electrical tortuosity (Eqs. (5) and (7)).
2. Rocks can be classified as Archie rocks and non-Archie rocks. Archie rocks have linear formation factor-porosity and resistivity index-water saturation relationships. The intercept of the formation factor-porosity relationship for Archie rocks is the origin (porosity = 100%, formation factor = 1). Rocks exhibiting any other electrical behavior are non-Archie rocks.
3. Intergranular pore space which is connected and free of micropore space due to clay or other secondary minerals is typical of the pore geometry of Archie rocks.
4. Non-Archie rocks can be classified according to pore geometry, since the electrical behavior is largely controlled by pore geometry. Five common non-Archie pore geometries are disseminated, laminated, and structural shaly rocks; rocks having large moldic pores or vugs; and those containing microporous or conductive grains.
5. Givens' conductive rock matrix model (CRMM) is equivalent to the old laminated shale model; however, it does not correct for the volume of pore space associated with the "shale".
6. The laminated shale model can be used to describe rocks which have continuous connected pore space in addition to intergranular pore space. Rocks with continuous microporous clay coatings on grain surfaces may be an example, and will conduct as though clean layers are interlayered with shale laminae.
7. The three types of shaly sand electrical behavior can be combined into a single shaly sand model (Eq. (18)).
8. Electrical tortuosity can sometimes be related to pore geometry; however, the relationship is often ambiguous. Rather than pore geometry, the electrical tortuosity may be descriptive of the relationship between the water content of a rock and its electrical behavior. Measured data on a formation factor-porosity plot or a modified Archie equation gives a less ambiguous relationship of electrical behavior to pore geometry.

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TABLE 1: EFFECT OF PORE GEOMETRY ON ELECTRICAL ROCK TYPE		
Rock Type	Geometric Factors	Nongeometric Factors
Archie Rocks	<u>Unimodal Pore Systems</u> - intergranular porosity	Water wet No surface conduction No conductive minerals
Non-Archie Rocks	<u>Polymodal Pore Systems</u> - moldic/vuggy porosity (moldic carbonates) - discontinuous microporosity (structural shale) (microporous chert) - continuous microporosity (authigenic clay coatings)	Oil wet Surface conduction (clay minerals) Conduction minerals (pyrite)

TABLE 2: SYMBOLS AND ABBREVIATIONS

m	porosity exponent in Archie's formation factor equation
ϕ	porosity
F	formation factor = R_o/R_w
R_o	resistivity of a rock saturated with water (Ωm)
R_w	resistivity of aqueous solution (Ωm)
S_w	water saturation
R_t	resistivity of rock with $S_w \leq 100\%$ (Ωm)
n	"saturation exponent" in Archie's equation ($S_w^n = FR_w/R_t$)
τ_o	electrical tortuosity of a water saturated rock
τ_t	electrical tortuosity of a rock with $S_w \leq 100\%$
C_t	$C_t = 1/R_t$ - conductivity of rock with $S_w \leq 100\%$
C_w	conductivity of aqueous solution ($C_w = 1/R_w$)
B	equivalent conductance of exchangeable ions associated with clay surface

Q_v	concentration of exchangeable cations associated with clay surfaces per unit pore volume
V_{sh}	bulk volume of "shale", includes volume of minerals and associated pore space (usually micropore space)
C_{sh}	conductivity of a shale layer or bed
C_r	conductivity of Given's rock "matrix"
S_{we}	water saturation of an equivalent capillary tube having the same resistance as a rock sample

D

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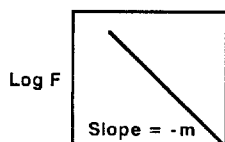
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D

LABORATORY MEASUREMENTS

Formation Factor

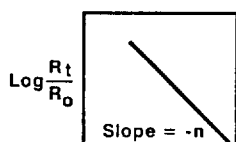


Log ϕ

$$F = \frac{1}{\phi^m}$$

Figure 1a

Resistivity Index



Log S_w

$$S_w^{-n} = \frac{R_t}{R_o}$$

Figure 1b

MECHANICAL TORTUOSITY

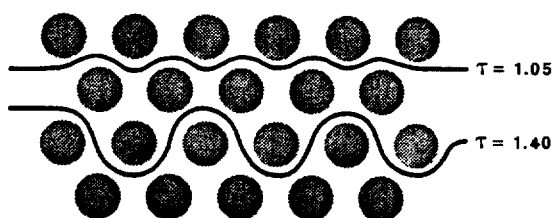
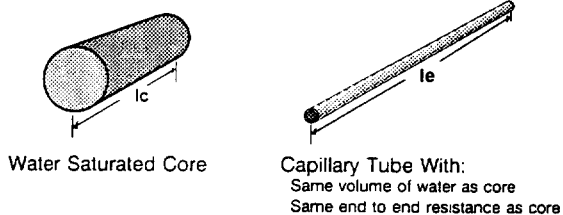


Figure 2

ELECTRICAL TORTUOSITY

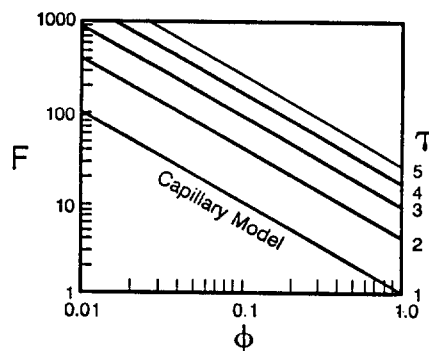


- Tortuosity = l_e / l_c
- $F = R_o / R_w = \left(\frac{l_e}{l_c} \right)^2 \frac{1}{\phi}$
- or,

$$\tau^2 = F\phi$$

Figure 3

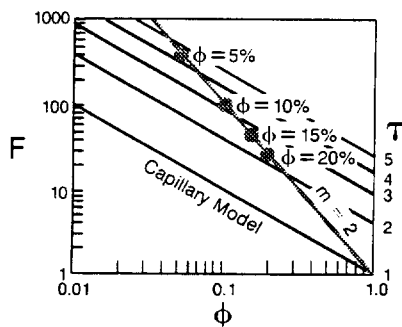
ELECTRICAL TORTUOSITY



$$\log F = -\log \phi + \log \tau^2$$

Figure 4

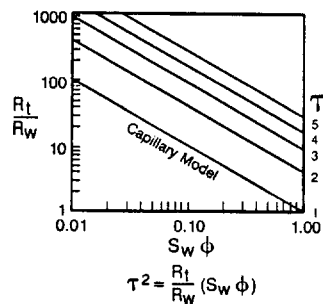
ELECTRICAL TORTUOSITY AND CEMENTATION EXPONENT



$$\log F = -\log \phi + \log \tau^2$$

Figure 5

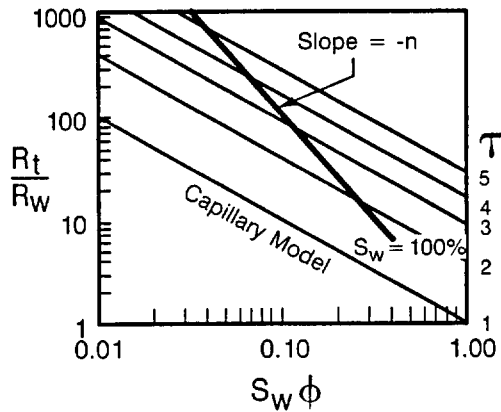
GENERALIZED "FORMATION FACTOR" PLOT



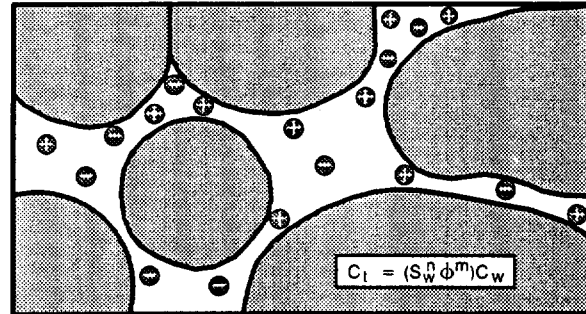
$$\tau^2 = \frac{R_t}{R_w} (S_w \phi)$$

Figure 6

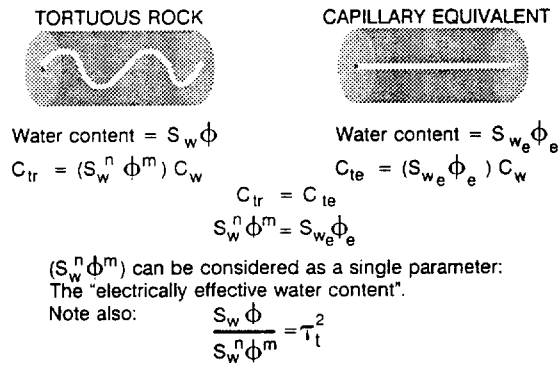
GENERALIZED "FORMATION FACTOR" PLOT



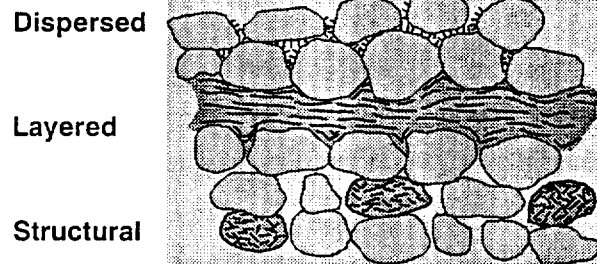
CONDUCTION MECHANISMS ARCHIE



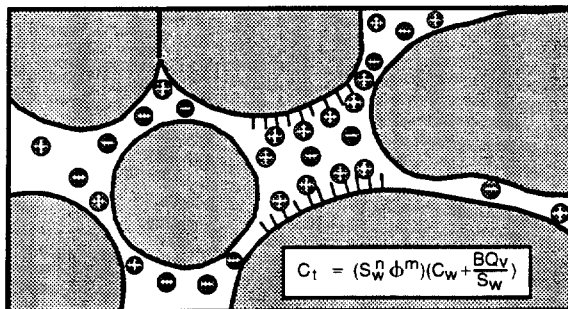
"ELECTRICALLY EFFECTIVE WATER CONTENT"



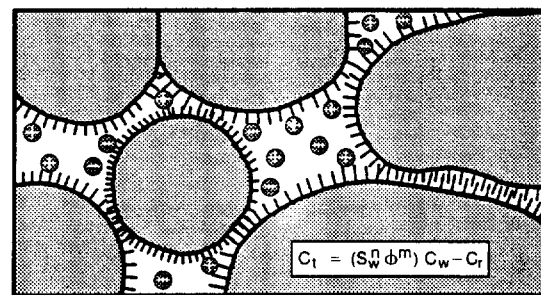
CLAY IN SHALY SAND



CONDUCTION MECHANISMS WAXMAN-SMITS



CONDUCTION MECHANISMS GIVENS



EFFECT OF SURFACE CONDUCTIVITY

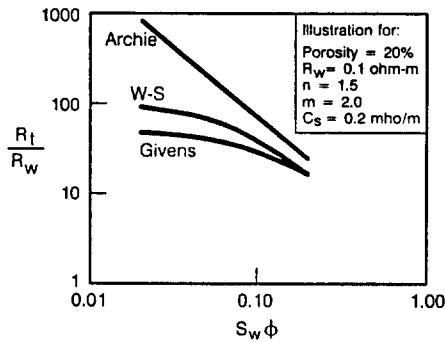


Figure 13

EFFECT OF SURFACE CONDUCTIVITY

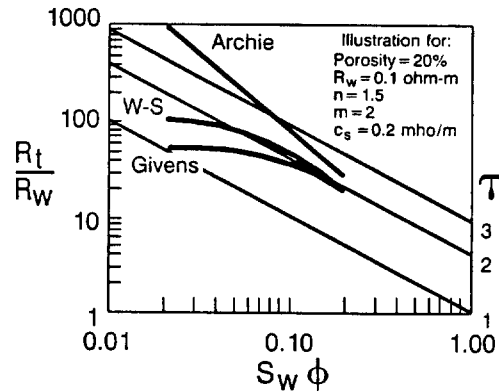


Figure 14 :

SMACKOVER FORMATION

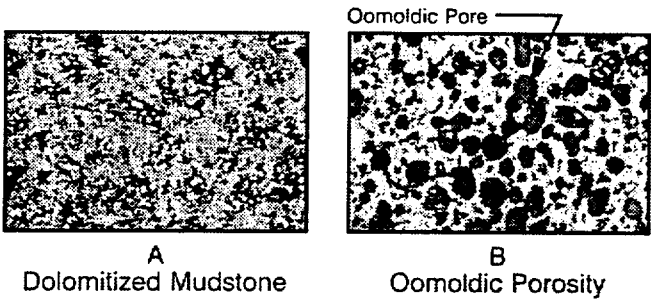


Figure 15

FORMATION FACTOR PLOT SMACKOVER

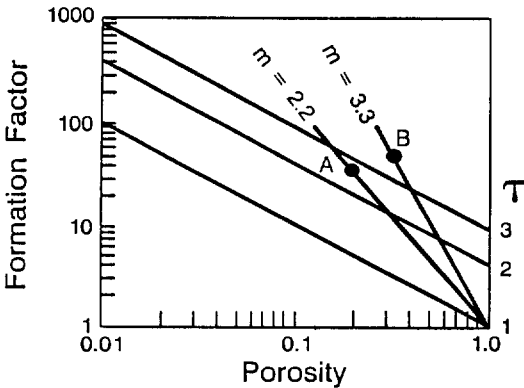


Figure 16

MOLDIC POROSITY: RASMUS MODEL

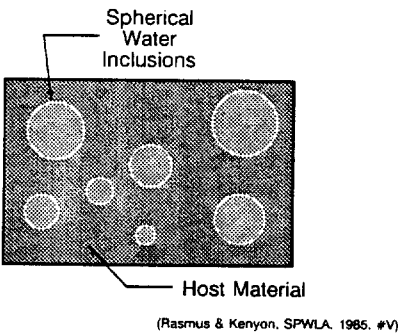


Figure 17

CONDUCTION MECHANISMS STRUCTURAL SHALE

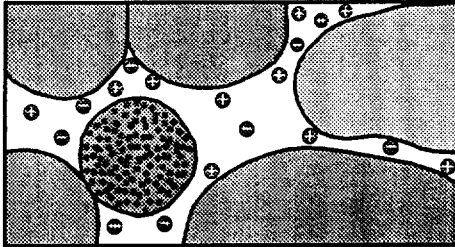


Figure 18

MICROPOROUS CHERT GRAINS

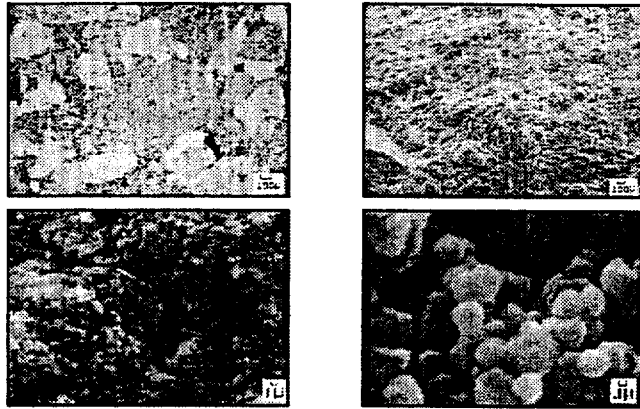


Figure 19

ISOLATED MICROPOROSITY SWANSON'S MODEL

Series Resistor Model:

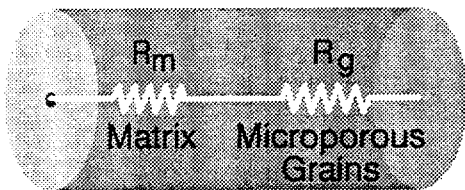


Figure 20

MOLDIC POROSITY MODELS

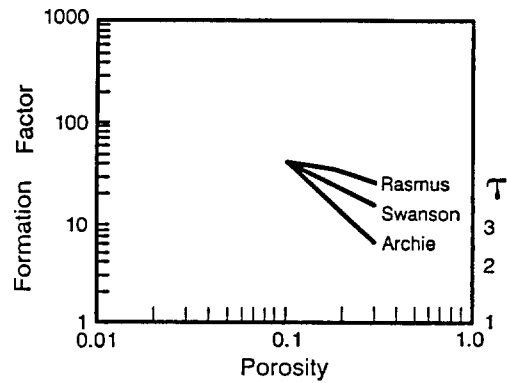


Figure 21

MOLDIC POROSITY MODELS

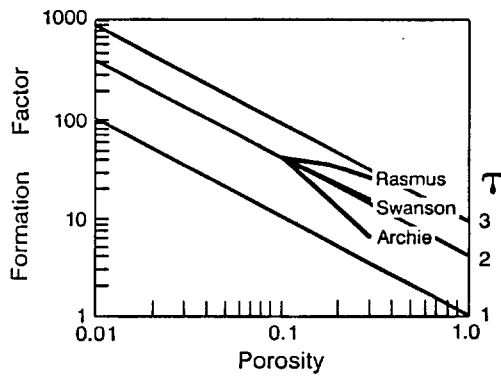


Figure 22

PROBLEM WITH TORTUOSITY:

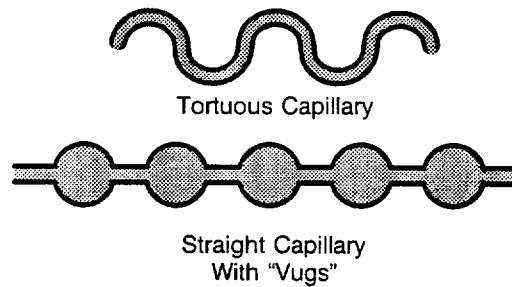


Figure 23

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