ELECTRICAL EFFICIENCY: A PORE GEOMETRIC MODEL
FOR THE ELECTRICAL PROPERTIES OF ROCKS

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ABSTRACT

An understanding of the way pore geometry affects the conductivity of rocks is important for proper interpretation of log and laboratory resistivity data when evaluating sedimentary rocks for hydrocarbon potential. The basis for virtually all formation evaluation from logs is the empirical Archie equation. This equation does not have a theoretical basis; rather, it is an *ad hoc* relationship derived from a graphical analysis of data from clean sandstones. No entirely satisfactory physical model based on pore geometry has been found which leads naturally to Archie's equation. We have formulated a new resistivity interpretation model based on the efficiency with which rocks conduct electric current and including an explicit parameter to account for pore geometry. The basis of the model is that electric current densities are not uniform throughout the conducting phase in the pore system of a rock. Numerical model studies show high current density in pore throats and low current density in nearly stagnant volumes in isolated parts of the pore system. The electrical efficiency is quantified by comparing the conductivity of water in a rock to the conductivity of a straight tube containing the same volume of water as the rock. The efficiency is inherently independent of the bulk volume of water in the rock, and is determined only by the non-uniform current distribution due to pore geometry and the distribution of hydrocarbons in the pore system. Moreover, the electrical efficiency can be calculated *a priori* given the geometrical distribution of the conducting phase. Variations in the distribution of water in a pore system due to diagenetic, depositional, or hydrocarbon emplacement processes result in varying electrical efficiency. Empirical relationships between electrical efficiency and water content can be used to calculate hydrocarbon saturations from log data independently of Archie's equation. The parameters in Archie's equation are explained using electrical efficiency theory.

INTRODUCTION

The fundamental problem of log analysis in wells drilled for oil and gas is the determination of the amount of hydrocarbon in the formations penetrated by the well. Quantitative interpretation of the resistivity of reservoir rocks R_t in terms of hydrocarbon saturation was first achieved by Archie (1942) using a synthesis of the resistivity index I and the resistivity formation factor F concepts: $R_t = FIR_w$, where R_w is the resistivity of the water in the formation. Porosity is related to F and water saturation is related to I through arbitrary power laws suggested by curve fitting; this is the origin of the two adjustable parameters in Archie's law, the so-called cementation and saturation exponents, m and n. Archie's method, with various modifications, remains virtually the only interpretational tool for resistivity measurements regardless of its empirical status and in spite of early and continuing recognition of instances where Archie's law fails (Patnode and Wyllie, 1950; Winsauer et al., 1952; Keller, 1953; Carothers, 1968). There have been many attempts to endow m and n with physical significance; however, since both elements of Archie's synthesis are empirical correlations, these efforts have not led to a physical model describing the electrical behavior of rocks in terms of their structures.

A new electrical efficiency theory (EET) describing the low-frequency resistivity of reservoir rocks has recently been formulated (Herrick and Kennedy, 1993). It focuses on the geometrical configuration of the conducting brine in the rock rather than on the electrical properties of the bulk rock. The EET relates the electric current carried by brine as it is distributed in the pore system to that of the same quantity of brine in its most conductive configuration, a straight tube. In contrast to Archie's law, EET can be derived from first principles and a geometrical description of the conducting brine. Archie's law is contained in the EET as a special

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case, but unlike Archie's law, the EET in *principle* requires no adjustable parameters. In *practice,* a complete geometrical description of the distribution of brine in the pore system of a rock is not feasible. Hence, like the Archie exponents, certain parameters in the EET must also be obtained from laboratory measurements. However, since the physical significance of the EET parameters is directly verifiable in numerical or scale models, EET offers, in addition to its quantitative aspects, a qualitative interpretation of conductivity in terms of pore geometry and brine distribution which Archie's law cannot provide.

THE ELECTRICAL EFFICIENCY MODEL

The conductivity C_t of a water-bearing rock composed of non-conducting minerals, and which may contain hydrocarbons displacing some of the original water from its intergranular pore-space, is a function of the conductivity of the conducting phase (usually brine) C_w, porosity ϕ, fractional volume occupied by the conducting phase S_w, and spatial distribution, or geometry, of the conducting phase; i.e., $C_t = f(C_w, \phi, S_w, \text{geometry})$. The effect of the geometry of the conducting phase on its electric current-carrying capability can be described by

$$C_t = C_w S_w \phi E, \qquad (1)$$

where E is a proportionality factor that accounts for the effects of the geometrical distribution of the brine. The product $C_w S_w \phi$ can be thought of as the conductivity of a straight brine-filled tube having the same brine content and external dimensions as the rock sample (Figure 1). This tube model will be referred to as the "equivalent tube" since it has the same dimensions and water content as the rock sample. The tubular configuration maximizes the conductance. Any geometric distribution other than a straight tube, such as that of brine in the pore system of a rock sample, has a lower conductance. The factor E relates the conductivity of a rock sample due to the brine distribution in the pore system to that of the same volume of brine in the equivalent tube,

$$E = \frac{C_t}{C_w S_w \phi}. \qquad (2)$$

From a modeling viewpoint, E is a geometrical factor by construction since none of the other factors

in equation (1) contains pore geometry, either explicitly or implicitly. Physically, E expresses the effect of the brine *distribution* in the pore system on the conductivity of the rock. It does not depend on the *amount* of brine ($S_w \phi$).

Since $C_t \le C_w S_w \phi$, E has values between zero and one and can be thought of as the *electrical efficiency* of the brine in the rock. The central concept of the EET is that E is a function of brine geometry alone. Consider, for example, the relationship between E and the fractional water content of the simple water-saturated rock models in Figure 2. The figure is a schematic representation of pore systems that vary in their complexity and in the amount of water that they contain. The rows contain models with identically complex pore systems, while the columns contain models of identical porosity. The first row across the top of Figure 2 contains models whose porosity is contained in varying numbers of identical straight tubes. The second model in the row has double the porosity of the first, and the third is three times as porous. However, since all of the tubes are straight, E is equal to one for each model, regardless of their porosities (i. e., their water content). The center row consists of models containing tortuous tubes; each has the same volume as the straight tube directly above it. The electrical efficiency is reduced for each model in the center row compared to the corresponding model in the top row, but it is the same for each model in the center row since the tubes in each model have the same geometry. Similarly, the bottom row of models containing tortuous tubes with pore-throat-like constrictions are less efficient conductors than the models in the upper two rows with the same porosities. These models illustrate that electrical efficiency can vary independently of the water content (the columns in Figure 2). Conversely, the water content can vary independently of E (the rows in Figure 2). The models shown employ tubes (of varying complexity) as "unit cells", but this particular geometry is not central to our argument; tubes were chosen as a convenient and easy-to-illustrate example. The pore-geometric unit cell can be arbitrarily complex without altering the conclusion. Electrical efficiency and water content are *inherently* independent.

The inherent independence of the electrical efficiency and the water content is useful since geometric effects can be considered separately from volumetric effects. For example, the relationship be-

tween the electrical efficiency and porosity for water saturated rocks can be used to classify them in terms of their electrical properties, or conversely, to use E and ϕ to determine the type of pore-system in a rock. Figure 3 illustrates relationships between E and ϕ for rocks with different kinds of pore systems. For example, tight carbonates with fractures parallel to the applied potential gradient have very high electrical efficiencies, approaching one in some cases, but have very low porosities, typically only a few percent. The other extreme is epitomized by tight, vuggy carbonates that can have very high porosities, but if the vugs are isolated, electrical connections are poor, and the electrical efficiency is low. Sandstones and carbonates having only intergranular pores typically have intermediate porosities and efficiencies. Shaly sands can appear to have unusually high electrical efficiencies since they exhibit surface conduction in addition to conduction due to brine in the pore-space.

In general, there is no relationship between electrical efficiency and the porosity of water-saturated rocks if comparisons are made between rocks that are genetically unrelated. On the other hand, if sandstone samples, for example, are taken from a formation in which different degrees of cementation have resulted in variations in porosity, then the process of cementation that caused the porosity variations may also produce consistent changes in the pore geometry as well. Cementation results in reduced porosity and increased complexity of the pore system. Hence, in spite of the inherent independence of E and ϕ, they can be coupled through diagenetic processes resulting in a correlation between electrical efficiency and porosity.

Effect of Pore Geometry on E

The electrical efficiency of a water saturated rock sample is determined by its pore geometry. Unfortunately, the pore geometry of rocks is complicated. Except in very simple pore systems, E cannot be explicitly parameterized in terms of the geometry of the conductive paths. Nonetheless, assessment of the pore geometric parameters of simple systems is instructive and is indicative of the dominant parameters influencing E. Numerical studies of simple pore systems such as those illustrated in Figures 4 and 5 (Herrick and Kennedy, 1993) have shown that the pore-throat size, shape, and the current path tortuosity are important pore-geometric parameters. To evaluate the relative im-

portance of pore throats and pore bodies, a tube defined by the size and shape of the pore throats is constructed through each pore (heavy dashed lines, Figure 4). The electrical effect of the pore bodies can be determined by comparing the conductance of these pore-throat tubes to the total conductance that includes the contribution from the pore bodies:

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$$d = \frac{c_p}{c_{th}}, \qquad (3)$$

where c_p is the conductance of the entire pore system and c_{th} is the conductance of tubes defined by the pore throats. The minimum value of d is one since the minimum size of a pore is that of the throat-tube. Numerical studies of infinitely wide pores (Figure 5) have shown that the maximum value of d is about 3 (Herrick and Kennedy, 1993). For the simple two-dimensional pore system illustrated in Figure 4, the value of d is 1.5. For rocks with intergranular porosity, the value of d is expected to lie generally in the range from one to two.

The tortuosity τ of the current-flow path between pores is usually less than 2, and for rocks with equidimensional grains, the value of τ is less than 1.5 (Herrick, 1988). The conductance between pores in a rock can be expressed in terms of the throat-tubes adjusted for the width of the pore bodies and the length of the average current path, or inter-pore tortuosity:

$$c = c_{th} \frac{d}{\tau}. \qquad (4)$$

The d/τ ratio, however, is close to unity since both d and τ tend to have values close to 1.5. Thus electric conductance through pore systems in these models is governed almost entirely by the pore-throats. This strongly suggests that the value of E in clean rocks with intergranular porosity is determined primarily by the pore-throat sizes. Other pore-geometric parameters have only secondary effects on E.

Wyllie and Rose (1950), and Winsauer et al. (1952) introduced the idea of electric current streamline tortuosity. The concept was quantified by constructing a tube having the same volume of water as a rock sample of interest and the same end-to-end resistance. The ratio of the length of the tube to the length of the rock sample was called the tortuosity T. This definition is in common usage and results

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in the equivalence, $T^2 = F\phi$. Archie (1942) defined the formation resistivity factor as $F = R_0/R_w = C_w/C_0$, where R_0 and R_w are the resistivities of a water-saturated rock and of the water in the rock respectively. If the resistivity ratio F is substituted into equation (2), with $S_w = 1$ and $C_t = C_0$, then E expressed in terms of F is $E = 1/\phi F$. Hence, the Wyllie and Winsauer tortuosity factor $T^2 = 1/E$. Since E incorporates the electrical effects of all geometric variables excluding porosity, T^2 must include them as well. In fact, as shown above, the interpore streamline tortuosity is only a minor contributor to E; the most important geometric variable is the pore-throat size distribution. Accordingly, T^2 is not a measure of the electrical streamline tortuosity, as is generally believed.

Electrical Efficiency–Water Content Relationships

Although no inherent relationship exists between electrical efficiency (or pore geometry) and water content, the depositional, compactional and diagenetic processes that change porosity simultaneously affect pore geometry. As shown above, any process that simultaneously affects the pore-throat sizes and the pore volume can produce an E–ϕ correlation. Hence, similar rocks that experience similar diagenetic events, should exhibit some type of E–ϕ correlation.

To explore the nature of E–ϕ relationships electrical and porosity measurements were evaluated for several suites of rock samples. The relationship for three sandstones and one carbonate are shown in Figure 6. In each case, the relationship is linear,

$$E_0 = a_0\varphi + b_0, \qquad (5)$$

where a_0 and b_0 are the slope and intercept respectively, and the 0 subscript indicates the value of E when the rock sample is completely water saturated. Generally, clean rocks having intergranular porosity have a linear E_0–ϕ relationship, with electrical efficiency increasing with porosity. The scatter of individual sample points about the line is due to the variability in composition and local diagenetic environment that would be expected in any sedimentary rock column. The relationship describes the relative effect of compactional and diagenetic processes on the pores (ϕ) and pore throats (E). Although some functional relationship is thus

expected intuitively, the form of the function is not obtainable from the EET; however, the observed trends are linear for clay-free rocks with intergranular porosity.

In addition to changing the porosity from one sample to the next, brine content of a given sample can be varied by decreasing the water saturation with nonconductive hydrocarbons. Figure 7 shows the electrical efficiency-water content relationships for four samples each from four different areas. In each of the sixteen cases, a linear relationship is remarkably well determined. The small deviations from the linear relationships are likely to be experimental errors. The indication is that the electrical efficiency-water content relationship is linear for clean rocks having predominantly intergranular porosity and can be expressed as

$$E_t = a_t S_w\phi + b_t, \qquad (6)$$

in which the subscript t refers to rock samples with partial and varying water saturation and a_t and b_t are the slope and intercept of the linear E_t–$S_w\phi$ relationship. Note that both E_0 and E_t are both electrical efficiency E. The subscripts only indicate the mechanism whereby the water content of the rock is varied; i. e., E_0 varies with ϕ, E_t varies with S_w.

Water Saturation Equation

Since both E_0–ϕ and E_t–$S_w\phi$ are linear relationships for clean rocks with mainly intergranular porosity, a water saturation equation can be written which pertains to rocks having similar pore geometry. If equations (1), (5), and (6) are combined, a quadratic equation in water content results,

$$a_t(S_w\phi)^2 + \left[(a_0 - a_t)\phi + b_0\right](S_w\phi)$$
$$- C_t/C_w = 0, \qquad (7)$$

which is easily solved for S_w,

$$S_w = \frac{-B + \sqrt{B^2 + 4a_t C_t/C_w}}{2a_t\phi}, \qquad (8)$$

$$a_t S_w\varphi + b_t = a_0\varphi + b_0$$
$$\left. (a_t S_w\varphi + b_t)\right|_{S_w=1} = a_t\varphi + b_t = a_0\varphi + b_0$$
$$a_t\varphi + b_t = a_0\varphi + b_0$$
$$b_t = a_0\varphi + b_0 - a_t\varphi$$
$$b_t = (a_0 - a_t)\varphi + b_0$$

where $B = (a_0 - a_t)\phi + b_0$. Use of this equation to calculate water saturations from wireline porosity and resistivity log data requires three parameters a_0, b_0, and a_t which must be assumed or measured in the laboratory. a_0 and b_0 are obtained from routine resistivity measurements on water saturated rock samples (the same measurement required for deter-

mining the resistivity formation factor F). Routine laboratory measurements of resistivity measured as a function of water saturation (identical to resistivity index measurements) are required to determine a_t. Inspection of equation (7) shows that if the values of a_0, and a_t are both equal to one and b_0 is equal to zero, then the result is identical to Archie's equation with n and m both equal to two; a commonly made assumption in the absence of data.

For rocks with simple intergranular pore systems, the relationships between electrical efficiency and porosity or water saturation are observed to be linear and equation (8) can be used to calculate water saturations from wireline-log measurements. In general, however, the rocks may have more complicated pore systems, or may be shaly. The $E-S_w\phi$ relationship may no longer be linear (Figure 8). However, as long as there is *any* relationship between E and water content, water saturations can be calculated. A suitable arbitrary function may be used to model the $E-S_w\phi$ relationship and subsequently to determine water saturations.

$E_t-S_w\phi$ Relationships in Shaly Sandstones

A common type of nonlinear $E_t-S_w\phi$ relationship is observed in shaly sandstones. Figure 8 illustrates the $E_t-S_w\phi$ relationship for a sample from the Aux Vases Formation, Illinois. A water saturation equation based on the data in Figure 8 could be developed by fitting the data with an arbitrary function, for example, a polynomial curve. A more satisfying, and potentially more useful, water saturation equation can be based on a model incorporating the commonly assumed electrical behavior of shaly sands. To a first approximation, the conductivity of a shaly sand can be described as the volume-weighted sum of a bulk pore-fluid conductivity C_c unaffected by shale effects and a surface conductivity term C_{sh},

$$C = v_c C_c + v_{sh} C_{sh},\qquad (9)$$

where v_c and v_{sh} are the volume fractions of clean and shaly components (similar to Patnode and Wyllie, 1950; Waxman and Smits, 1968). By dividing equation (9) by $C_w S_w \phi$, the electrical efficiency of the sample is given by,

$$E_t = v_c \frac{C_c}{C_w S_w \phi} + v_{sh} \frac{C_{sh}}{C_w S_w \phi} .\qquad (10)$$

The first term of equation (10) is the fractional contribution to the electrical efficiency of the pore system exclusive of shale effects. Based on the experience summarized in Figures 7(a)-(d), a linear E_t-$S_w\phi$ relationship is assumed for a shale-free sandstone, hence equation (6) can substitute for the first term:

$$E_t = v_c \left(a_t S_w \phi + b_t\right) + v_{sh} \frac{C_{sh}}{C_w S_w \phi}.\qquad (11)$$

The relative contributions of the components are determined using equation (11) and the appropriate data. The fractional contribution from surface conductivity for the shaly sand in Figure 8 is determined to be 0.1 mho/m by this method.

It is important to note that the EET attributes all of the conductivity to the conductive fluid phase in the pore system. Hence, if there are other modes of conduction such as surface conduction on clays, or metallic conduction in sulfide minerals such as pyrite, the theory will predict unusually high electrical efficiency for the rock when the volume of conducting brine is small. This effect is typical of shaly sands and is illustrated in Figure 8. The electrical efficiency decreases with decreasing water content as expected in the shaly sand until about 6% water content where the efficiency begins to increase as the water content is further reduced. The water does not become a more efficient conductor as implied by the increasing electrical efficiency; the surface conductivity, which is assumed to be independent of water saturation, simply becomes the dominant conductive component. The first term of equation (11) describes the linear decrease in E_t with a reduction in water saturation due to the increasing complexity of the conducting phase geometry. At sufficiently low water content, the surface conductivity becomes proportionately more important so that the second term dominates. The surface conductivity in the second term is regarded as a constant in this model; however, the denominator decreases as water saturation is reduced. Hence, the second term has a hyperbolic relationship to E_t. The sum of the linear pore-geometric term and the hyperbolic surface-conductivity term gives the E_t-$S_w\phi$ relationship observed in Figure 8. Thus, the data have been fit to a function whose components correspond to the two conductivity components of the rock. The resolution of conductivity into these components provides a simple model for describing the rock's behavior in terms of the brine in the pore system

and conductive clay or other minerals on the grain surfaces.

ANALYSIS OF ARCHIE'S EQUATION USING ELECTRICAL EFFICIENCY

Formation Factor–Porosity Relationships

Perhaps the single most important contribution in petrophysics to formation evaluation is Archie's (1942) equation relating water saturation to resistivity and porosity,

$$S_w^n = \frac{R_w}{\phi^m R_t}. \qquad (12)$$

The equation does not have a theoretical basis, but is an ad hoc relationship derived from a graphical analysis of data from clean sandstones. No entirely satisfactory physical model has been found from which Archie's equation naturally follows.

Archie (1942) presented the data from which he devised the formation resistivity factor-porosity relationship only in graphical form. For the purposes of this paper, Archie's data were digitized for plotting and analysis. Archie's "consolidated sandstone cores of the Gulf Coast" and Nacatoch sand, Bellevue, Louisiana, sample sets are plotted in Figure 9. Regression equations, best-fit lines, and coefficients of determination (R^2) are also presented for each data set.

Archie (1942) did not discuss the basis of his choice of a log-log plot to represent the formation factor-porosity relationship. However, since his goal was an empirical correlation, the logarithmic relationship yielded a model which has been useful, if controversial, ever since. To appreciate the problem Archie was solving, the data are plotted on a linear scale in Figure 10. The nonlinear distribution is clear. It is also very suggestive of a hyperbolic relationship that can be linearized by taking the reciprocal of one of the variables. A conductivity formation factor f equal to the reciprocal of the resistivity formation factor ($f = 1/F$) can be used to this end. Archie's formation factor-porosity relationships for both sets of data are replotted in terms of conductivity in Figure 11. The use of the *conductivity* formation factor eliminates the necessity for using logarithmic scales to simplify the formation factor-porosity relationships. The coefficients of determination are given in Figures 9 and 11 for

both bi-logarithmic and linear plots respectively. The R^2 values for both the f–ϕ and log F–log ϕ relationships are essentially the same. Both relationships are, therefore, equally successful at establishing an empirical correlation on which to base a water saturation equation. Archie chose the log-log relationship, presumably because he was working with resistivity instead of conductivity, and possibly because a simple straight-line relationship was obtained with log-log paper. Archie's use of logarithms to straighten a hyperbolic relationship resulted in the porosity exponent m as the descriptive parameter. Had he converted resistivity to conductivity, Archie's equation would not have taken an exponential form, but could have been linear as suggested by Figure 11. If Archie had used this approach, much thought and discussion about the physical significance of m and its relationship to tortuosity and pore geometry would have been obviated.

An additional problem with establishing a relationship between either log F and log ϕ, or f and ϕ, is that f (or F) incorporates both E_0 and ϕ. Recall that Archie defined F as the R_0/R_w ratio. Substituting in equation (1) and setting S_w equal to one,

$$f = \frac{1}{F} = E_0\phi, \qquad (13)$$

for water saturated rocks. Hence, formation factor-porosity relationships have an inherent correlation since $f = E_0\phi$ is being related to ϕ. If the porosity dependence in the correlation is removed by dividing f by ϕ, then the resulting E_0–ϕ relationship describes changes in the electrical efficiency due only to pore geometry as a function of porosity. The relationships for both of Archie's data sets are shown in Figure 12. By removing the porosity dependence, the coefficient of determination (R^2) for the consolidated sandstone cores from the Gulf Coast is reduced from 0.74 to 0.30. Likewise, the coefficient of determination for the Nacatoch sandstone samples is reduced from 0.84 to 0.55. Almost all of the formation factor-porosity relationship observed by Archie, which he used as the basis for his well-known water saturation equation, is due to correlating porosity to a property proportional to porosity!

The lack of relationship between the electrical efficiency and porosity observed for the consolidated sandstone cores from the Gulf Coast is not surpris-

ing. The description suggests that these were samples selected from various formations and fields from the Gulf Coast area. As such, they would have had different depositional and diagenetic histories resulting in porosities and pore geometries that could not be expected to show a consistent relationship from one sample to the next. Therefore, no particular relationship should be expected between porosity and pore geometry, including the electrical properties determined by pore geometry.

There is, on the other hand, some evidence of a pore geometry-porosity relationship for the Nacatoch sandstone samples (Figure 12). Unlike the consolidated sandstone sample set, these samples were from a single formation in a localized area, perhaps from a single field. One would, therefore, expect more consistency in the depositional environment and state of diagenesis. The pore geometry-porosity relationship demonstrated in the electrical efficiency-porosity plot (Figure 12) reflects this consistency. In general, to be assured that a conductivity- (or resistivity-) porosity relationship exists which can serve as the basis for a water saturation equation such as Archie's, a relationship must exist between the electrical properties due to variations in pore geometry (the electrical efficiency) and porosity. Formation factor-porosity relationships that exist due only to the porosity dependence in the relationship can be misleading.

Porosity dependence also results in curved f–ϕ relationships. If Archie's Nacatoch Sandstone data or other data that have pronounced f–ϕ relationships are examined, then the relationship is observed to be curved (Figures 11 and 13). The source of the curvature is obvious if the linear relationship between E and ϕ expressed in equation (5) is substituted into equation (13) yielding the formation conductivity factor-porosity relationship:

$$f = a_0\phi^2 + b_0\phi$$ (14)

The resulting function (equation (14)) is quadratic in porosity and is parabolic in shape. This formation factor-porosity relationship is to be compared to Archie's relationship

$$f = \phi^m$$ (15)

Both equations can be regarded as curve-fitting equations used to fit experimentally obtained data. The single fitting-parameter used by Archie is the exponent m, while equation (14) uses two fitting parameters making it a more versatile parametric description. From inspection of equations (14) and (15) it is clear that when $a_0 = 1$ and $b_0 = 0$, $m = 2$. In general, however, $a_0 \neq 1$ and $b_0 \neq 0$; the single parameter m cannot describe the f–ϕ data as accurately as the two parameters, a_0 and b_0. Consequently, use of equation (15) to describe the f–ϕ relationship requires the introduction of fractional, and sometimes variable, values of m. Frequently a second fitting parameter a is introduced into Archie's law, suggestive that two parameters are demanded by the variability in the data.

Resistivity (and Conductivity) Index-Water Saturation Relationships

The relationship between resistivity and water saturation which Archie used was taken from prior literature (Wycoff and Botset, 1936; Jakosky and Hopper, 1937; Martin et al., 1938; Leverett, 1939). This relationship was also empirical, relating the resistivity index I to water saturation:

$$I = \frac{R_t}{R_0} = S_w^{-n}.$$ (16)

The saturation exponent n is usually obtained from the slope of a log-log plot of laboratory derived I-S_w data Figure 14(a). As in the case of the formation resistivity factor-porosity relationship, a log-log plot was used by Archie to straighten the hyperbolic relationship between I and S_w Figure 14(b). A formation *conductivity* index i can be defined by analogy to the resistivity index in terms of a conductivity ratio,

$$i = \frac{C_t}{C_0} = I^{-1} = S_w^n.$$ (17)

The conductivity index is a function of the electrical efficiency and the water content as shown by substituting equations (2) and (13) into equation (17),

$$i = \frac{1}{E_0\phi}\left(S_w \phi E_t\right).$$ (18)

Recall that, for clean rocks with intergranular porosity, a linear relationship is observed between E_t and water content (equation (6)). Substitution of equation (6) into (18) results in a quadratic relation-

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ship between i and S_w as illustrated in Figure 14(c). The relationship is given by

$$i = \frac{1}{E_0 \phi}\left[a_t(S_w\phi)^2 + b_t(S_w\phi)\right]. \qquad (19)$$

The product $E_0\phi$ is a constant for a given rock sample, so it is clear that plotting i (or I) versus S_w results in an inherent correlation since S_w is being related to a function of S_w (equation(18)). The inherent correlation can be removed by using the electrical efficiency-water content relationship (equation (6)), and plotting E_t versus S_w (Figure 14(d)) which gives a linear relationship.

Both equation (19) and Archie's equation (16) are fitting equations for empirically relating conductivity (or resistivity) index data to water saturation. Equation (16) has only one fitting parameter (n), whereas equation (19) has two (a_t and b_t) making it a more versatile fitting equation. When $a_t = 1$ and $b_t = 0$, then equation (19) is identical to Archie's equation (16) with $n = m = 2$. In general, however, the pore and brine geometry will be such that these parameter values will not fit the measured data satisfactorily. When this occurs, the required values of a_t and b_t will be different from one and zero, respectively, and fractional values of n and m will be required in Archie's equation. Non-integer values of n and m, other than 2, simply result from using a two-parameter fitting equation to describe data that require at least three parameters for a proper fit. Equation (8) is a water-saturation equation with three parameters that is suitable for rocks with linear electrical efficiency-water content relationships of the type for which Archie devised his equation.

Comparison of S_w Predictions

Correlations between electrical efficiency and water content tend to be linear for clean rocks with only intergranular porosity (Figures 6 and 7). When both n and m are equal to two, the water saturations calculated from the electrical efficiency and Archie's equation agree for all values of ϕ and S_w. However, when n and m are not two, the calculated water saturations may not agree over the entire range of S_w and ϕ of interest. The relationship between electrical efficiency and Archie's equation can be obtained by substituting equation (2) into (12) resulting in

$$E = S_w^{n-1}\phi^{m-1} \qquad (20)$$

For water saturated rocks, $E_0 = \phi^{m-1}$. Figure 15a, for example, illustrates the E_0–ϕ curves predicted by Archie's equation for a range of m values from 1.7 to 2.3. The curves are nearly linear for porosities above ten percent, but below that, Archie's equation predicts a nonlinear electrical efficiency-water content relationship that is curved, passing through the origin. A comparison between the E_0–ϕ curve predicted by Archie's equation for $m = 1.7$ and the corresponding linear relationship $E_0 = 1.2\phi + 0.08$ is illustrated in Figure 15a (heavy dashed line). For porosities above 0.1 the two relationships agree closely, however, below $\phi = 0.1$, they diverge. The same nonlinear relationship exists between E_t and S_w for values of E_t predicted from Archie's equation for partial water saturation and different values of n. Figure 15b illustrates the difference in S_w values calculated from the linear E_t–$S_w\phi$ and nonlinear Archie relationships for $\phi = 0.2$ and $\phi = 0.05$, assuming $n = m = 1.7$. For porosity values above 0.1, the Archie relationship is approximately linear and the S_w difference is small. However, for porosity values less than 0.1, the S_w value calculated from Archie's equation can be significantly lower than that derived from the linear E_t–$S_w\phi$ relationship. In the example given (Figure 15b), the difference in calculated water saturation between the two models is a great as 13% (saturation units).

Provided that both m and n in Archie's equation and the coefficients in the linear E_t–$S_w\phi$ relationship are determined from core measurements in the porosity range of interest, either approach will give substantially the same calculated water saturations. If either water-saturation equation is applied beyond the porosity and water saturation range for which the calibration was made, the electrical efficiency and Archie derived water saturations may diverge. In particular, at low porosities the difference may be considerable. Since the Archie-derived E_t–$S_w\phi$ relationship is constrained to go through the origin, and since the evidence suggests that measured E_t–$S_w\phi$ relationships are linear, Archie's equation may yield erroneous calculated water saturations for low porosities and water saturations.

CONCLUSIONS

In summary, the electrical efficiency theory predicts the low-frequency bulk electrical properties of any specified geometrical configuration of conducting and non-conducting phases in reservoir rocks,

using $C_t = C_w S_w \phi E$. The theoretical predictions do not depend on the values of empirically determined parameters, but on a geometric factor E which ideally can be calculated from first principles and a geometrical description of the conductivity distribution in a rock. In real rocks where the numerical prediction of bulk electrical properties is intractable, observed relationships between efficiency and porosity allow the construction of water-saturation equations that are specific to each particular reservoir. In many cases the observed electrical efficiency is a linear relationship in S_w and ϕ, and the resulting water-saturation equation is quadratic in S_w. In these cases three adjustable parameters are required to describe the observed data. Archie's equation is contained as a special case of the electrical efficiency model. In the context of the efficiency theory the requirement for fractional saturation and cementation exponents can be partly understood as resulting from the under-parameterization inherent in Archie's law and partly from an incomplete model; i.e., it is a power law, but not quadratic in $S_w \phi$.

Although some interesting aspects, such as linear E_0–ϕ and E_t–$S_w \phi$ correlations, remain to be explained, the electrical efficiency model describes the electrical conductivity of rocks.

ACKNOWLEDGMENTS

We are grateful for the comments and advice given by our colleagues J. Hook, W. D. Lyle, W. R. Mills, J. A. Nieto and R. E. Maute. We thank Mobil Research and Development Corporation for permission to publish this work.

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**Rock Sample**   **Water-Volume Tube Model**
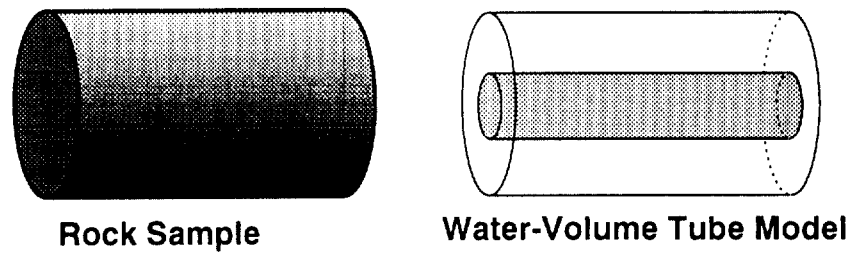
Figure 1. A useful standard of comparison for the conductivity of a rock sample is obtained by reconfiguring the conducting phase into a tube having the same external dimensions as the rock. In this configuration the conducting phase has maximum conductance and the tube model has maximum conductivity. The ratio of rock conductivity to tube conductivity is defined as the electrical efficiency E of the rock.
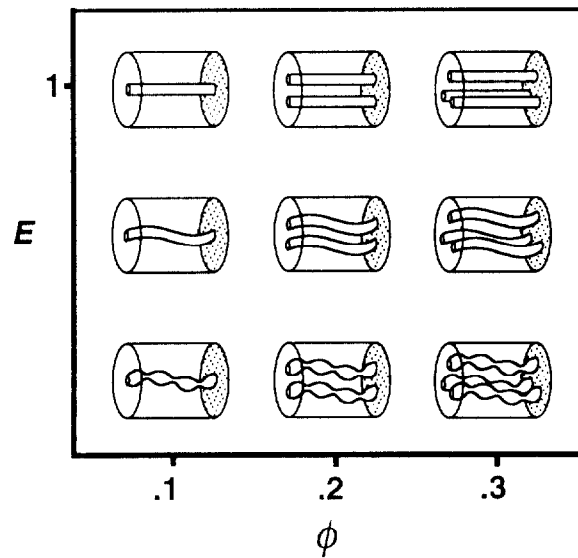

Figure 2. Electrical efficiency E is a function of pore geometry, but is not inherently dependent on porosity.
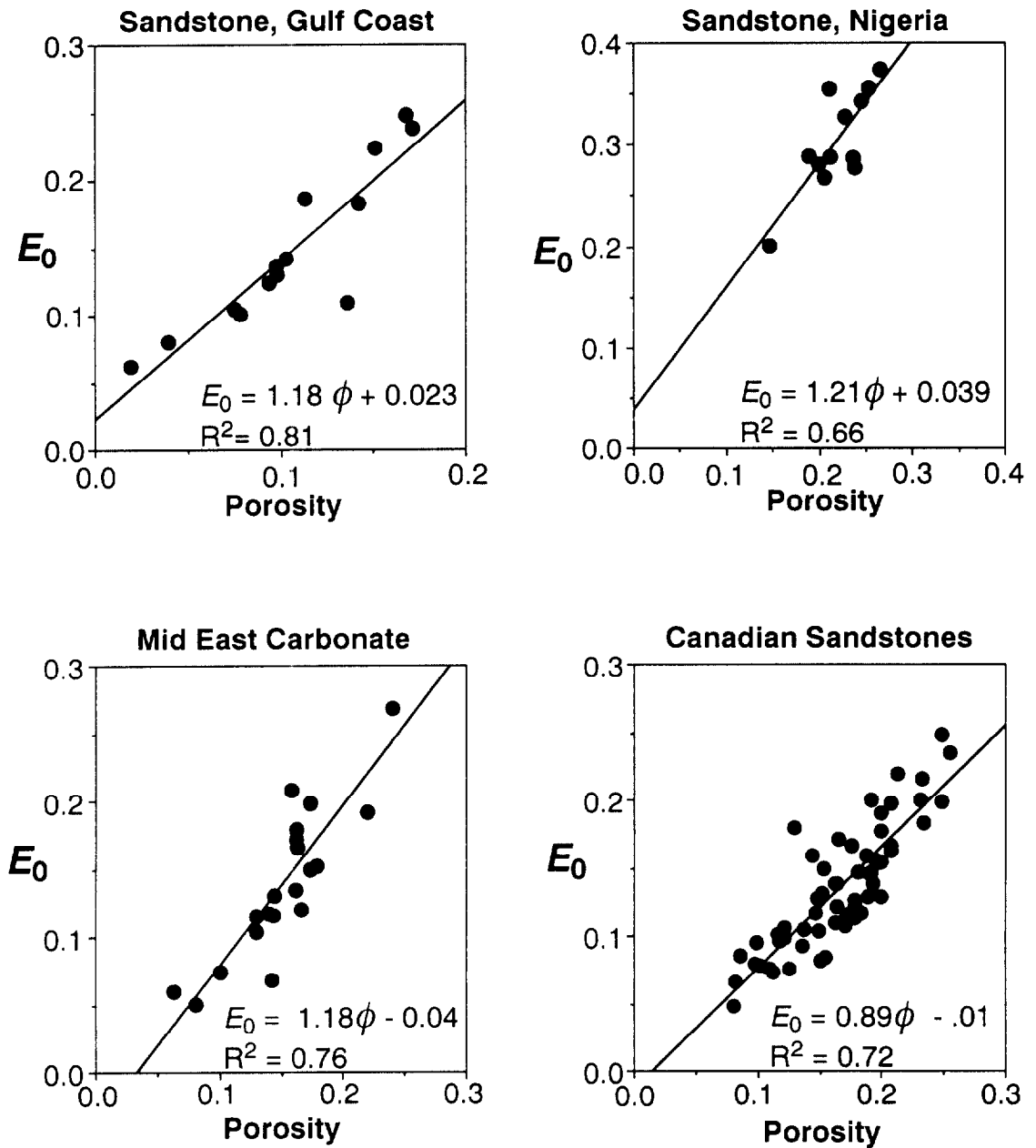
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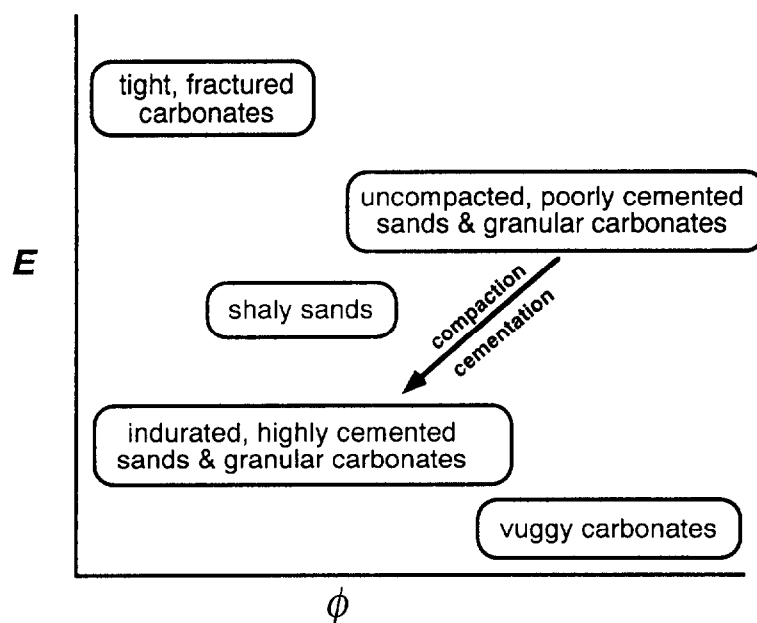
Figure 3. The electrical efficiency is determined solely by pore geometry and is not inherently related to porosity. E-ϕ relationships can exist, however, since changes in porosity due to depositional factors and diagenesis are usually accompanied by changes in pore geometry. As schematically illustrated, a given pore geometry can be used to determine E qualitatively; and conversely, a knowledge of E can be used to predict the pore geometry.
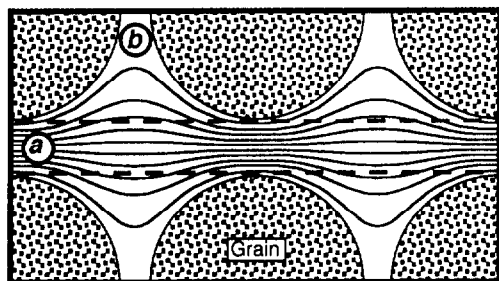
Figure 4. The electrical efficiency of a rock depends on the contribution of all conductive components of the pore space in the rock. Water in location (a) carries a relatively high current and makes a large contribution to the efficiency. Water at (b) carries little current and contributes little to the global efficiency of the rock.
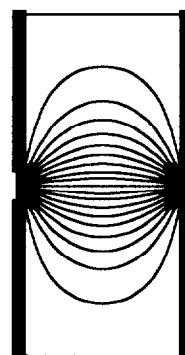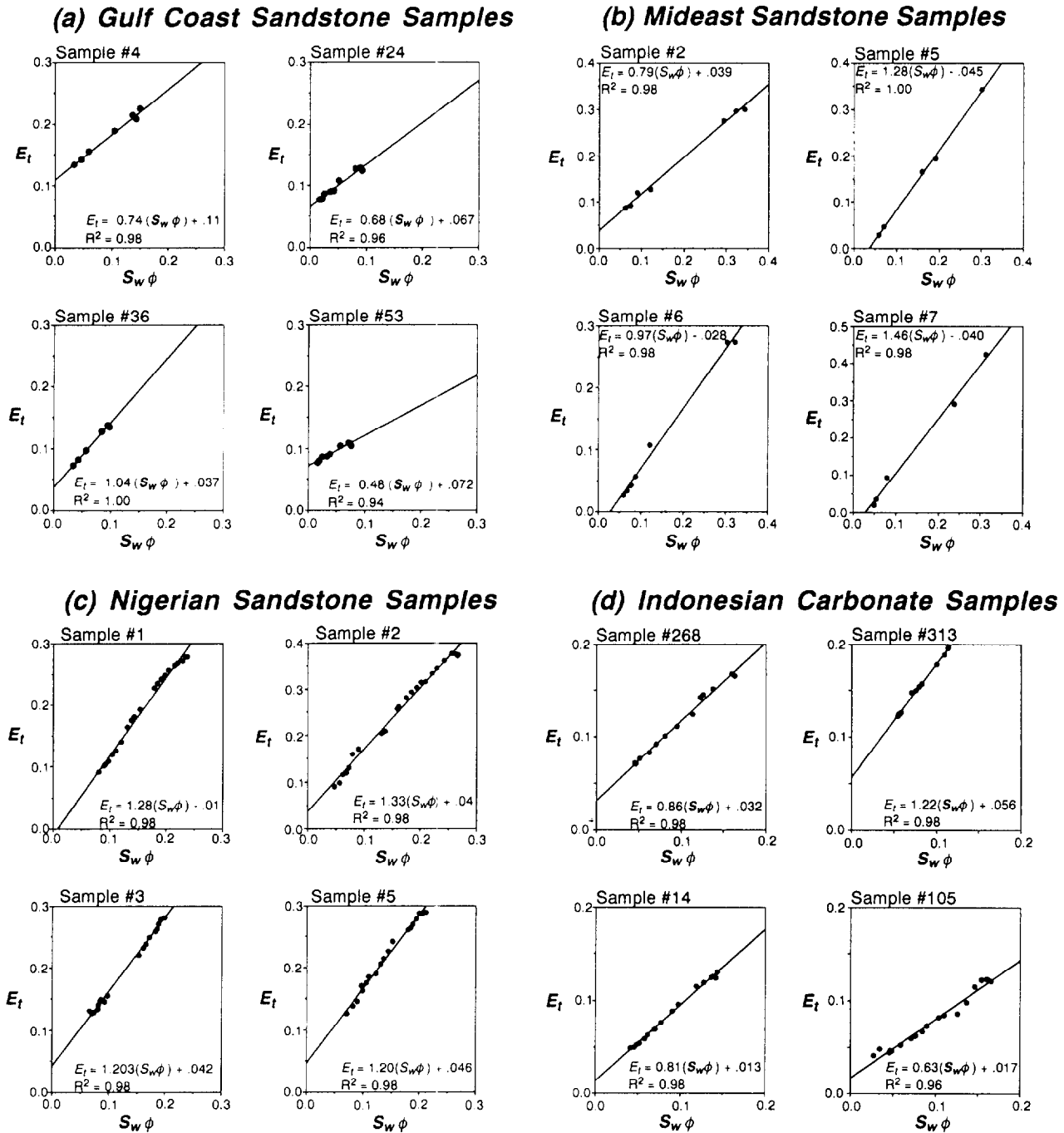

Figure 5. A simple model is shown to illustrate the influence of pore-throat size and pore-body size. As suggested by the figure, the throat is the controlling geometrical factor; increases in width do not result in significant conductivity increases past a width/length ratio of 1.5.

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Sandstone, Gulf Coast

$$E_0 = 1.18 \, \phi + 0.023$$
$$R^2 = 0.81$$

Sandstone, Nigeria

$$E_0 = 1.21 \phi + 0.039$$
$$R^2 = 0.66$$

Mid East Carbonate

$$E_0 = 1.18\phi - 0.04$$
$$R^2 = 0.76$$

Canadian Sandstones
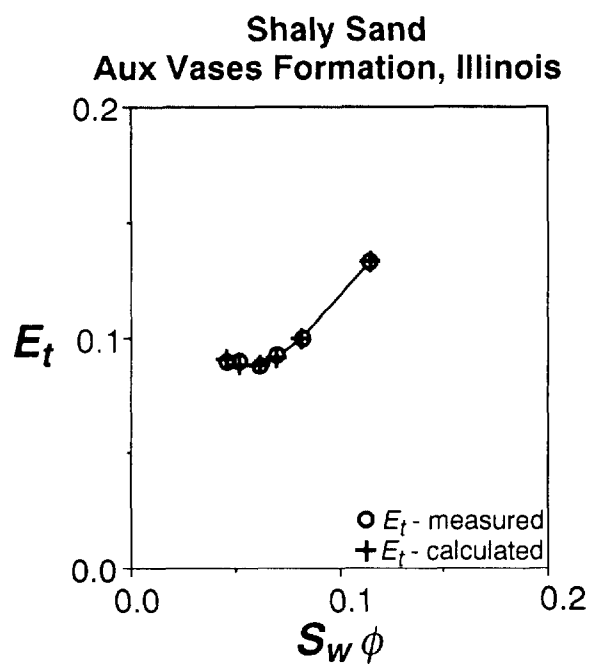
$$E_0 = 0.89\phi - .01$$
$$R^2 = 0.72$$

Figure 6. Electrical efficiency-porosity relationships for three sandstones and one carbonate sample. The relationship tends to be linear, unless there has been more than one porosity-changing process.

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(a) Gulf Coast Sandstone Samples

(b) Mideast Sandstone Samples

(c) Nigerian Sandstone Samples

(d) Indonesian Carbonate Samples

Figure 7. Four samples from each of four fields showing a linear decrease in electrical efficiency with decreasing water saturation. This behavior is typical of other samples from the same fields.

**Shaly Sand
Aux Vases Formation, Illinois**

E_t

0.2

0.1

0.0

$\bigcirc$ E_t - measured
$+$ E_t - calculated

0.0   0.1   0.2

$S_W \phi$

Figure 8. The E_t-$S_W\phi$ relationship for shaly sands has a characteristic hook-shape. It can be separated into conductivity components using the combined linear decrease in E_t with S_W for the intergranular pore-space and the hyperbolic relationship due to the microporosity and surface conductivity of clay.
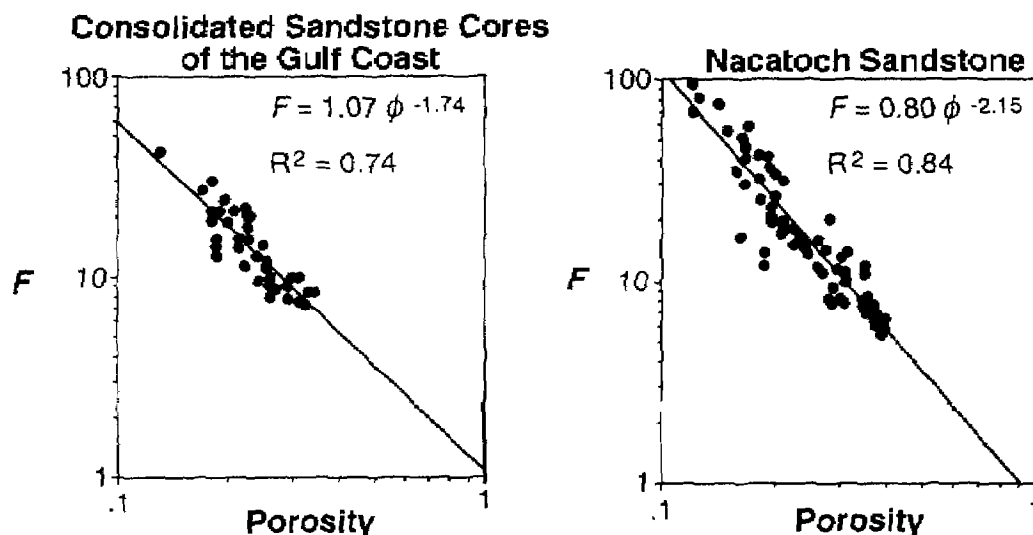
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Consolidated Sandstone Cores of the Gulf Coast

$$F = 1.07 \, \phi^{-1.74}$$

$$R^2 = 0.74$$

Nacatoch Sandstone

$$F = 0.80 \, \phi^{-2.15}$$

$$R^2 = 0.84$$

Figure 9. The F-ϕ relationships used by Archie (1942) as the basis for his well known equation for calculating water saturations from log-derived resistivity and porosity data.
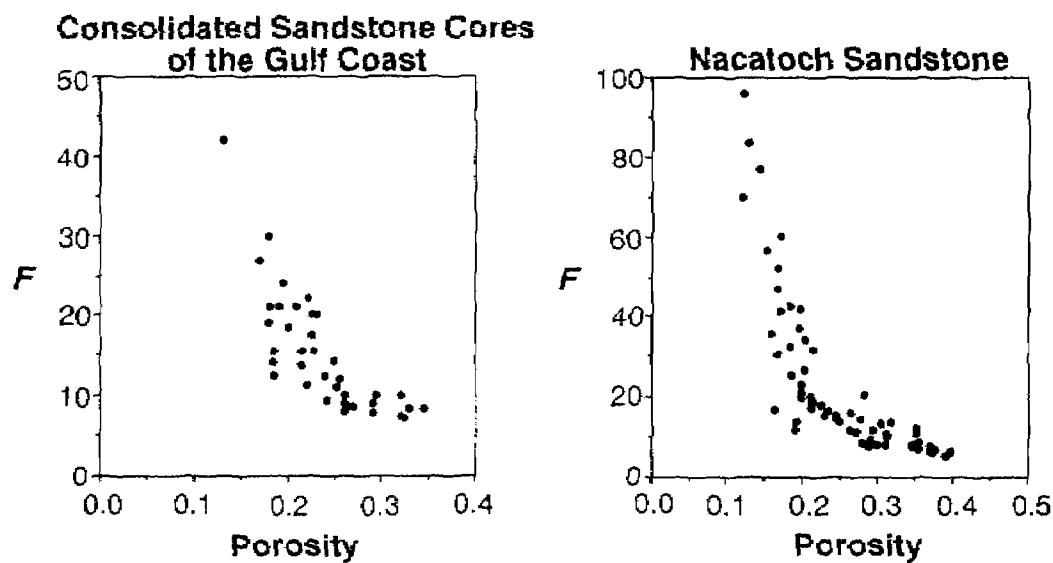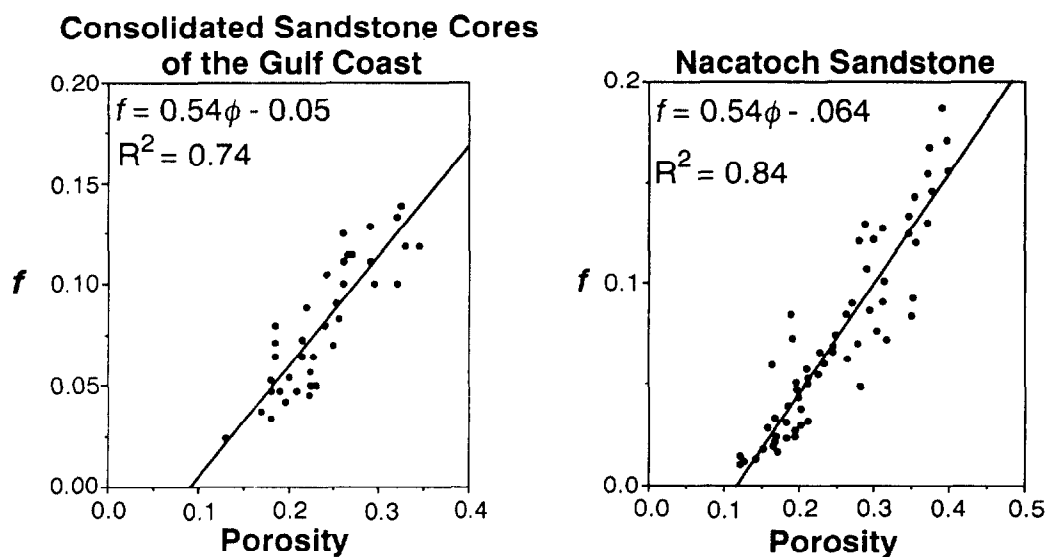
Consolidated Sandstone Cores of the Gulf Coast

Nacatoch Sandstone

Figure 10. Archie's F-ϕ relationships show pronounced curvature when plotted with linear scales. The curves suggest hyperbolas.

Consolidated Sandstone Cores of the Gulf Coast

$f = 0.54\phi - 0.05$

$R^2 = 0.74$

Nacatoch Sandstone

$f = 0.54\phi - .064$

$R^2 = 0.84$

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Figure 11. The hyperbolic relationship suggested in Figure 10 can be made linear by taking the reciprocal of the formation resistivity factor and converting it to a formation conductivity factor. Note the slight curvature which remains in the Nacatoch Sandstone data.
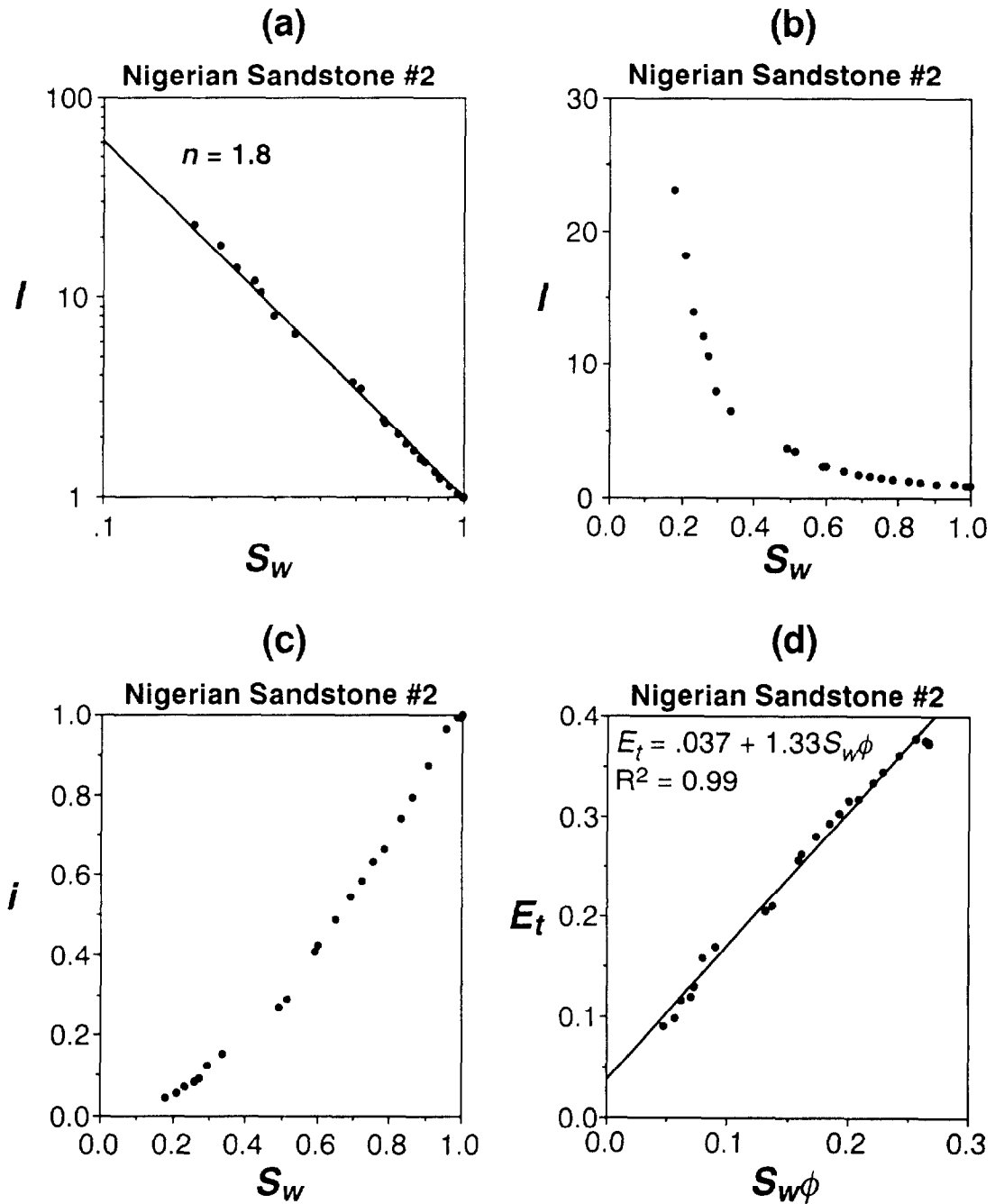
Consolidated Sandstone Cores of the Gulf Coast

$R^2 = 0.30$

Nacatoch Sandstone

$R^2 = 0.55$
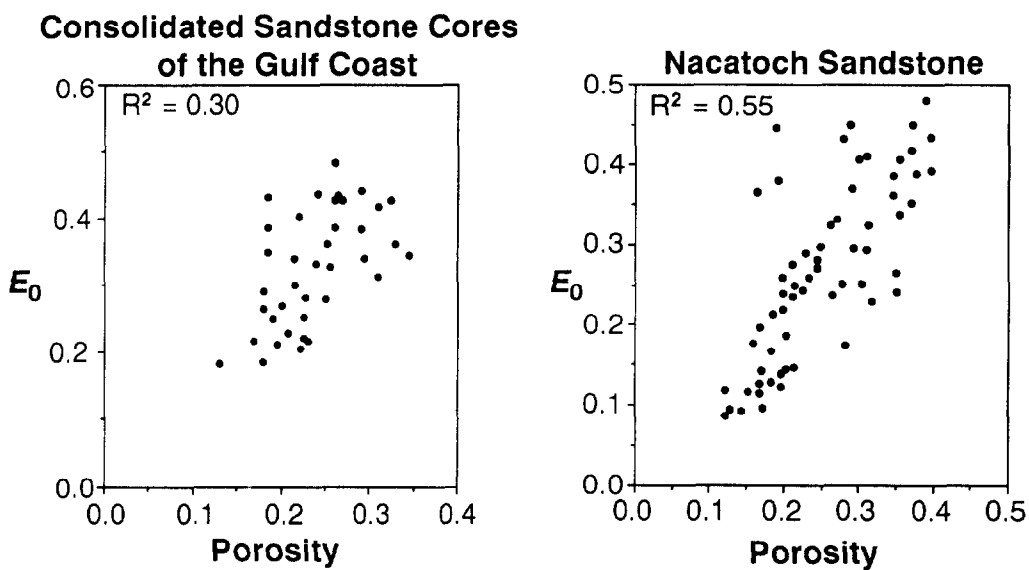
Figure 12. If the inherent porosity dependence in the F-ϕ relationship in Archie's data is removed by converting the formation factor to electrical efficiency, most of the correlation is also removed. There is essentially no pore geometry-porosity relationship between the Consolidated Sandstone samples and only a slight, but significant, relationship between the Nacatoch Sandstone samples.

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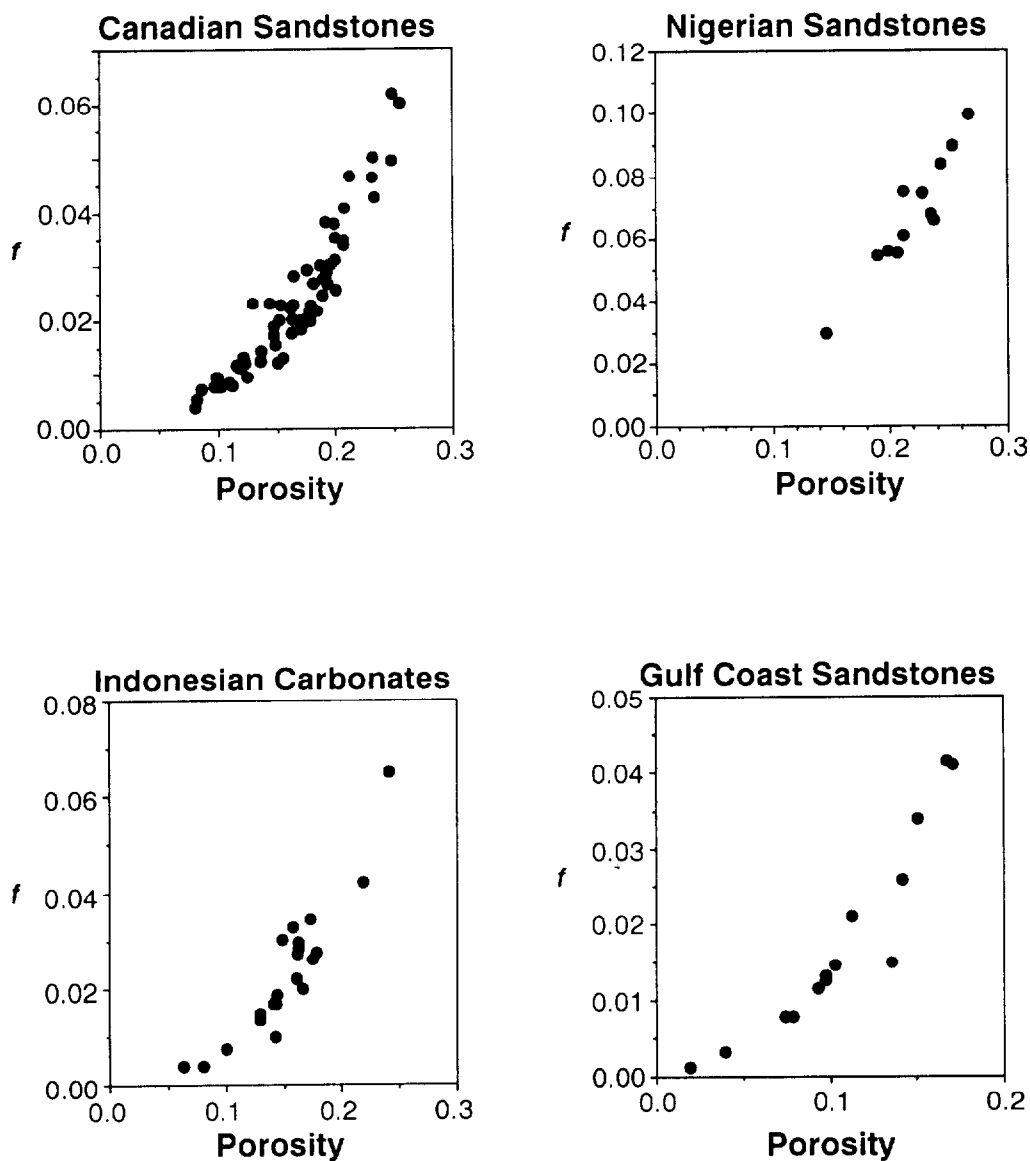
Figure 13. The formation conductivity factor-porosity relationship for other suites of sandstone samples is well defined with a slight concave-upward curvature. The curve shape is parabolic since the formation factor is a function of the product of ϕ and E, and E is also linearly related to ϕ in these rocks.

(a)

Nigerian Sandstone #2

$n = 1.8$

I

S_W

(b)

Nigerian Sandstone #2

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I

S_W

(c)

Nigerian Sandstone #2

i

S_W

(d)

Nigerian Sandstone #2

$E_t = .037 + 1.33 S_W\phi$
$R^2 = 0.99$

E_t

$S_W\phi$

Figure 14. The electrical efficiency-water content relationship is observed to be linear for many rocks (**d**), most notably clastics. The conductivity (or resistivity) index is a function of the product of E and water content, expressed as S_W or $S_W\phi$. The i-S_W relationship is therefore parabolic (**c**), and the I-S_W relation is hyperbolic (**b**). When the hyperbola in (**b**) is plotted with bi-logarithmic axes, a straight line results (**a**) which is the basis for Archie's law (1942).

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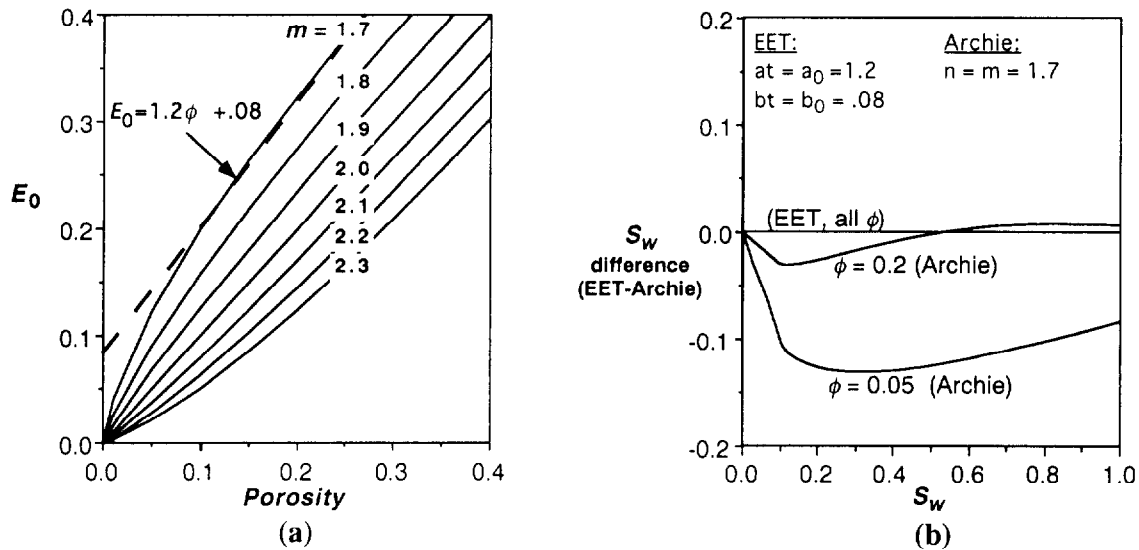
Figure 15. (a) Archie's equation predicts curves in E-$S_w\phi$ space rather than the linear relationships which are observed for clean rocks with intergranular porosity. The E_0-ϕ curves for m=1.7-2.3 are illustrated. The dashed line is the straight E_0-ϕ relationship which approximates Archie's equation with m=1.7 in the porosity range above 0.1. Below 0.1 porosity units, the curvature in Archie's equation results in deviation from the E_0-ϕ curve. (b) Generally, water saturations calculated from Archie's equation and electrical efficiency agree for S_w>0.5 and ϕ>0.1. Differences are observed if S_w<0.5 and the differences can be very significant if ϕ<0.05.