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TUTORIAL

Introduction to Resistivity Principles for Formation Evaluation: A Tutorial Primer

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PROLOGUE

The standard model for relating bulk formation resistivity to porosity and water saturation was introduced to the petroleum industry in 1941; it remains the industry standard to this day. The model was discovered empirically by means of graphical analysis. Basically, G.E. Archie discovered that when the logarithm of formation resistivity factor was plotted against the logarithm of porosity the resulting trend could be fitted by a straight line. A similar relationship was discovered connecting the logarithms of resistivity index and water saturation. When these two power laws are combined into a single equation, it can be solved for water saturation (which is not observable from a borehole) in terms of bulk formation resistivity, interstitial brine resistivity, and porosity (all of which can be estimated from observations made in boreholes). This revolutionized log interpretation. There has always been a problem with the model in terms of its "explainability". That is, it cannot be derived in any straightforward way from accepted first principles of physics. It does not contradict any first principle, but neither does it seem to follow ineluctably from them. However, since the model works, most formation evaluators have memorized the relationships that follow from the model and simply "get used to them". That remains the situation to this day. However, there is a path around this obstacle to understanding formation resistivity at a fundamental level, and that way forward is to abandon the resistivity formulation in favor of its reciprocal, conductivity. It is surprising that such a seemingly trivial change could open a new vista into the relationships among formation electrical properties. A conductivity formulation permits the asking of questions about how a formation's conductivity *should* respond to changes not only in brine conductivity, but also in the fractional amount of brine in a formation, and its geometrical configuration. By answering these questions in an obvious way, and with some analysis of data taken in the laboratory, an intuitively obvious model explaining bulk formation conductivity emerges. The model is not the same as the Archie model. However, when certain parameters are taken to their limiting values, and the model is converted into resistivity space, Archie's power law model is revealed as an approximation to the limiting cases. Thus, from the conductivity formulation, an intuitive understanding of the Archie model emerges. Moreover, the conductivity model can be derived in at least three different ways, each yielding different insights into formation conductivity.

INTRODUCTION

This article is intended as a tutorial for novice formation evaluators. Our object is not to teach resistivity interpretation techniques, but to provide insight into the most commonly used model; namely, the Archie model for connecting the brine saturation of a porous rock to its bulk resistivity. Although our article is backed by the scholarship required of a *Petrophysics* article, the editor has granted us permission to withhold citations from the text (as in a textbook) in the interest of readability; we will supply a list of the references that we have used in the usual place following the end of the article. Also, we shall strive for clarity over brevity; if this means repeating ourselves, it is for pedagogical effect. (As has been said since Roman times, "Repetitio est mater studiorum", repetition is the mother of learning.) Our target audience is practitioners of formation evaluation newly introduced to the subject. Our goal is that a serious student having worked through this article should have a clear understanding of why rocks conduct the way that they conduct based upon intuitively obvious first principles, rather than a rote memorization. First principles can be stated verbally. An example from Isaac Newton would be "acceleration is proportional to applied force and inversely proportional to mass". However, to be useful in analysis, first principles must also be expressible as mathematics; e.g., $a = F/m$. A formula such as this is called a "model"; a successful understanding of the physical world will be based upon internalizing the first principle (which should be, more or less, obvious), not on memorizing a formula (which might be complicated). The mathematical level required for a basic understanding of the electric properties of rocks is at the level of secondary school algebra including the transformations of powers to roots (and vice versa), and the relationships

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between logarithms and exponents.

The literature, almost without exception, introduces the topic of formation resistivity with some variant of G.E. Archie's model for water saturation. That is, the topic is *introduced* by citing relationships, such as

$$S_w = \sqrt{\frac{R_w}{R_t \varphi^2}} \qquad \text{or} \qquad S_w = \sqrt[n]{\frac{aR_w}{R_t \varphi^m}}$$

with little or no preamble and little or no discussion of the provenance of the formulas. This is to put the end of Archie's own chain of reasoning at the beginning of an explanation. Archie himself eschews any discussion of physics or curve fitting, but he at least describes the results of his trend analysis of the data that led him to this formulation by beginning at the beginning. We shall follow Archie's example.

Archie's model is the oldest quantitative log interpretation technique; its literature begins in the 1940s. So it is expected that much has been written on resistivity interpretation since that time. As far as exposition is concerned two approaches are followed: (1) the equations of the Archie model are written down and justified as empirical discoveries without further explanation, or (2) the explanation begins with a cube or cylinder of conductive material and the Archie model is shown to be consistent with Ohm's law. Even the literature that attempts to put the Archie model on a theoretical footing restricts itself to the use of Ohm's law in terms of resistivity. The problem is that these methods do not explain the quirkiness of the Archie model. As we shall show in this article, the restriction of explanations to the "resistivity domain" prevents these efforts from achieving conceptual clarity. Suffice to say, consulting the "usual" sources of information on formation resistivity—porosity—water saturation provides only a single, and incomplete, point of view.

Archie's 1942 seminal paper contains only six equations. A tutorial purporting to elucidate the electrical behavior of rocks should begin with an introduction to the Archie model and these six equations, and we shall do so. However, we urge the student to download the original Archie paper from the SPE; its copyright has expired and it is available without charge at OnePetro. Archie's paper is often cited, but judging by claims attributed to Archie that he never made, seldom read. In the interest of good scholarship, if you should ever author an article that relies on Archie's work, you should review what he says before you cite him. If you rely on formation evaluation folklore, or even the citations of other scholars, for information, you are likely to misattribute something to Archie.

Archie's Model

Archie's model is based upon trends defined by resistivity observations that he recorded in two figures in his 1942 paper (see Fig. 1). These equations describe the trends suggested by Archie's own, and four additional, datasets. Archie discovered that for the rocks which he studied the ratio of a rock's bulk resistivity, R_0, to its interstitial brine resistivity, R_w, is a constant, F, an invariant property of the rock. That is, $R_0/R_w = F$; Archie expresses this as $R_0 = FR_w$ in Archie Eq. (1). Archie dubs F the "formation resistivity factor." The formation resistivity factor for different rock samples varies; Archie found by inspection of his plot of logarithm of F versus logarithm of porosity, φ, that the variation of F with porosity is described by the trend $F = 1/\varphi^m$ (Archie's Fig. 2 and Archie's Eq. (2)). Equating F in the definition of formation resistivity factor with F from trend analysis, Archie discovered a relationship between bulk rock resistivity and porosity, $R_0 = R_w \varphi^{-m}$ (Archie's Eq. (3)). (We note for our readers that Archie uses θ to denote porosity; we substitute the modern convention and use ϕ (introduced by Wyllie and Rose) in its script form φ. Another notational difference is Archie's use of R and S where the modern conventions are R_t and S_w.)

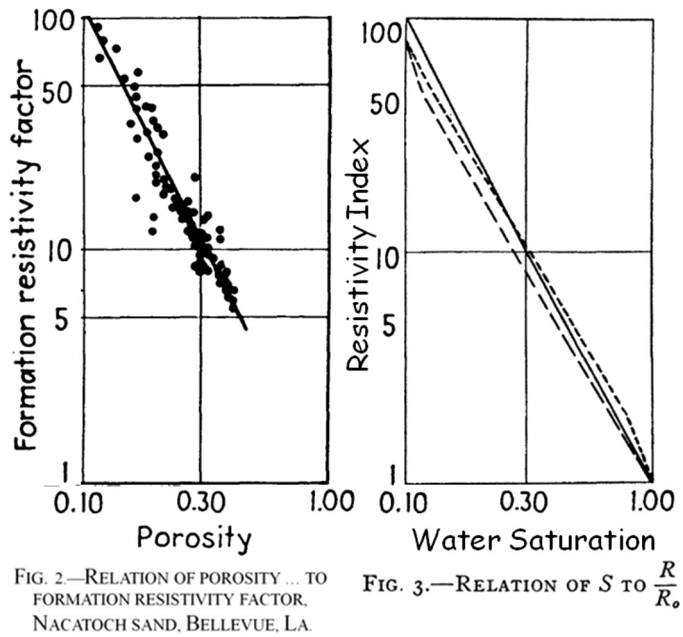

Fig. 1—Archie's figures defining the trends $F = 1/\varphi^m$ and $I = 1/S_w{}^n$ that are the basis of his model and much of modern formation evaluation (modified from Archie, 1942)

In addition, Archie analyzed four datasets available in literature published in the years 1936 through 1938. This is really an amazing contribution as the presentation of these data in the four papers is very different; at first glance they do not appear to be the same thing. Archie summarizes these data in a plot analogous to his formation resistivity factor-porosity plot and bases his next observation on the trend followed by the four datasets in this plot. Archie writes his fourth equation (which is really two equations), $S_w = (R_0/R_t)^{1/n}$ and $R_t = R_0 S_w^{-n}$ (Archie's Eq. (4)). The ratio R_t/R_0 is now known as the "resistivity index", although that term does not appear in Archie's paper; nor does an equation $I = R_t/R_0$ (now used as the definition of resistivity index), nor does $I = S_w^{-n}$, explicitly appear. Archie's contribution here is to discover that the resistivity index is predicted by a power law of water saturation. Finally, noting that $n = 2$ approximately, Archie writes two additional versions his of Eq. (4) explicitly using a square root, viz.,

$$S_w = \sqrt{\frac{R_0}{R_t}} \text{ (Archie's Eq. (5) and } S_w = \sqrt{\frac{FR_w}{R_t}} \text{ (Archie's Eq. (6)).}$$

That is the full explanation of Archie's contribution as he introduced it in 1941. It seems an obvious step given that he has written down Archie Eq. (4) as $R_t = R_0 S_w^{-n}$ to then substitute Archie Eq. (3) $R_0 = R_w \varphi^{-m}$ to produce

$$R_t = R_w \varphi^{-m} S_w^{-n} \text{,}$$

the combined equation probably most cited today as the Archie "law"; but, it is a step Archie himself did not take.

It is worth noting that Archie did not coin, and does not use, the misnamed "cementation exponent" nor does he introduce the factor *a,* (also poorly named) now called the "tortuosity" factor. These misnomers are later inventions of others. Finally, Archie relies entirely upon trend analysis for his discovery. Archie offers neither physical explanations, nor derivations from first principles. The questions raised in the minds of many who would study and use the Archie model, such as "why is it a power law?", and "why are the exponents approximately equal to 2?" are not even asked, let alone answered.

Archie's presentation to the AIME in Dallas was in the fall of 1941; by the time his paper appeared in 1942, the United States was at war and civilian research was largely moribund during the war years. When research resumed following the war, at least two publications attempted to explain the Archie law in terms of circuit theory principles framed in terms of resistance, but these were not really successful. The more sophisticated question of how to derive the Archie law from first principles was not even asked for 40 years following Archie's 1941 presentation.

Pabitra Sen was among the first, if not the first, to ask this question in the literature. His 1980 SPE conference paper, *The Dielectric and Conductivity Response of Sedimentary Rock*s, includes a section entitled "Derivation of Archie's law from First Principles." Although this indicates that, 39 years following Archie's introduction of resistivity interpretation, an interest in this topic was emerging among researchers, the "first principle" that Sen refers to is an "effective-medium theory" formula that many researchers would not elevate to a "first principle", and the "Archie law," which he derives has an exponent of 3/2; this is 1/2 too small to qualify as applying to most Archie reservoir rocks with exponents close to 2.

To make progress in intuitively understanding the physical basis of the Archie model the resistivity formulation of the problem must be abandoned in favor of a formulation based upon conductivity. This might sound like a trivial change in point of view, but it turns out to make this seemingly intractable problem as simple as it should be.

A final word is to be said before leaving the discussion of Archie's model. Our intention here is not to disparage Archie or his model. His model is robust for the rocks where it is applicable and has been in use every day for over 75 years, including by your authors. Few artifacts or ideas from 1941 are still in use without modification today; Archie's law is a survivor and as a practical matter is unlikely to be supplanted any time soon, if ever. However, in order to really understand why it works, it is necessary to step out of the "resistivity domain" and into the "conductivity domain". We shall now take this step.

Conductivity in Brine-Saturated Archie Rocks

The paragraphs to follow will show that it is possible to predict the electrical behavior of a certain class of rocks known as "Archie rocks" from a set of (perhaps arguable) self-evident first principles. Due to its empirical nature, Archie's model best describes only these Archie rocks; so, although the log-derived fluid saturation model under this approximation is uncertain in many cases, it has been historically applied to almost every kind of rock, with mixed success. This method of evaluation has been universally accepted mainly due to a lack of serviceable alternatives. The attributes of Archie and non-Archie rocks are listed in Table 1.

Table 1—Effect of Pore Geometry on Electrical Rock Type

Rock Type	Geometrical Factors	Nongeometric Factors
Archie rocks	Unimodal pore systems Intergranular porosity	Water-wet No surface conduction No conductive minerals
Non-Archie rocks	Polymodal pore systems Moldic/vuggy porosity (oomoldoc porosity) Discontinuous microporosity (microporous chert) Continuous microporosity (authegenic clay coating)	Oil-wet Surface conduction (clay minerals) Conductive minerals (e.g., pyrite)

Archie formulated his analysis in terms of resistivity. This was a natural consequence of well logs being recorded in terms of resistivity, and measurements on core plugs also being reported in terms of resistivity. However, and unfortunately, the resistivity formulation of the electrical behavior of rocks does not lend itself to an intuitive grasp of the relationships involved.

The empirical nature of Archie's equation and any ambiguity associated with it can be avoided by using a relatively simple, straightforward and general approach to understanding the conductivity of rocks. It is founded upon an analysis of the geometrical configuration of the conducting brine in the rock. Based on this premise, the internal architecture of the rock can be explained by the introduction of an explicit geometrical factor, E_0.

Reformulated in terms of conductivity, the relationships become, if not trivial, at least possible to grasp with a minimum of mental effort. We begin by stating our premises. A brine-saturated Archie rock comprises two phases: nonconducting mineral grains and conducting brine with conductivity σ_w. By definition, for Archie rocks, all of the bulk rock conduction is due to the conducting brine in the void space between the mineral grains.

We now assert three principles regarding the bulk conductivity of an Archie rock. The conductivity model rests upon these principles.

The bulk conductivity, σ_0, of an Archie rock (or similar medium) is proportional to:

I. Conductivity of the brine σ_w, $\quad \sigma_0 \propto \sigma_w$

II. Amount (i.e., fractional volume) of brine (for constant pore geometry) φ, $\quad \sigma_0 \propto \varphi$

III. Geometry of brine as expressed by a function of pore geometry E_0, $\quad \sigma_0 \propto E_0$

These principles combine into a "triple-product theorem" which states:

$$\sigma_0 = \sigma_w \varphi E_0 \,. \tag{1}$$

This could be taken as a reasonable, almost self-evident, working hypothesis. However, we call this a "theorem" because it is capable of proof (or is falsifiable if the proof can be found to be in error). The proof is:

Proposition (I) is taken as axiomatically self-evident. If the conductivity of pore-filling brine in an Archie rock is doubled (or halved), the bulk conductivity of the rock is concomitantly doubled (or halved). The same would be true for any fractional change in brine conductivity producing the same fractional change in bulk rock conductivity.

Proposition (II) requires a demonstration of its meaning for clarification, especially for readers conditioned by prior exposure to the Archie model to regard bulk conductivity as being approximately proportional to φ^2. Consider the model core plug containing the single sinuous channel, as shown in the upper panel of Fig. 2. The core plug will exhibit a bulk conductivity. Next, consider the similar core plug in the lower panel which is identical to the plug in the upper panel except that the sinuous channel is duplicated. The dimensions and shape of the second sinuous channel are exactly the same as the dimensions and shape of the first. This is the meaning of "constant pore geometry." Obviously, the porosity and bulk conductivity of the plug in the lower panel are double the porosity and bulk conductivity of the plug in the upper panel. The reader might well object that these plugs are very "unrocklike" in their geometry. To answer this objection, the five-core-plug model of Fig. 3 is introduced, and explained in its caption. The conclusion with this more rocklike model is the same as with the sinuous channel model, and also validates Proposition (II).

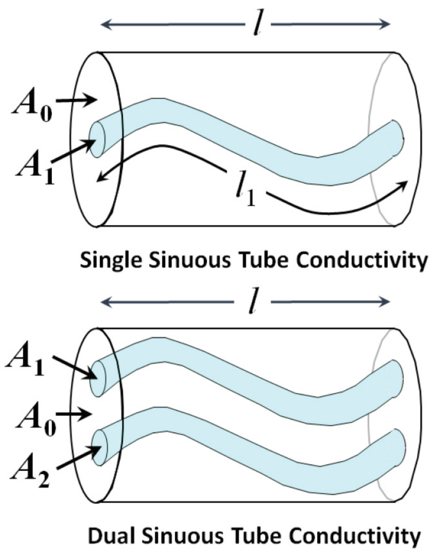

Fig. 2—In the upper panel a core plug is modeled as a cylinder with the plug outer dimensions area *A* and length *l*, and the void space, or porosity contained in a single sinuous tube of constant cross-sectional area A_1 and length l_1. In the lower panel the sinuous tube is duplicated, which doubles both the porosity and the bulk conductivity of the core plug while the pore geometry is held constant.

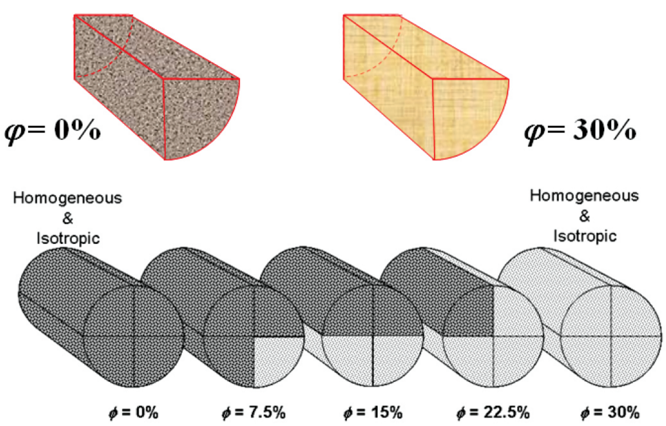

Fig. 3—Consider a model based upon two quarter-cylinders cut from two core plugs taken in Archie rocks, one with zero porosity and the other with 30% porosity. Next imagine that these two quarter-cylinders are replicated so that there are four of each kind. The pore systems within the quarter cylinders are the pore systems of the original rock. Now imagine putting these cylinders together in five configurations having composite porosities of 0, 7.5, 15, 22.5, and 30% porosity. Our claim is that the conductivities of these rocklike cylinders will vary in direct proportion to (i.e., to the first power of) the porosity.

Proposition (III) is also trivially self-evident if "unrocklike" geometrical distributions of the void-filling brine are permitted. The limiting cases are two: there is no end-to-end connection of the brine, in which case the bulk rock conductivity is zero; or, the brine occupies a cylindrical tube extending end-to-end through the core plug, in which case the bulk conductivity is a maximum (equal to $\sigma_{max} = \sigma_w\varphi$ as will be shown below). For distributions of brine in an (actual) Archie rock, a more detailed development will lead to a functional form for the geometrical factor E_0.

In the first place, the reader will note that bulk rock conductivity cannot be less than zero or more than the maximum conductivity permitted by the amount of brine available (i.e., $\sigma_{max} = \sigma_w\varphi$). This means that the range allowed for E_0 must be $0 \le E_0 \le 1$. Since a detailed description of a bulk rock sample containing tens of thousands to millions of pores and pore throats will be forever intractable, direct computation of E_0 except, perhaps, for very small samples, is impractical. However, note that all of the quantities in the triple-product formula except E_0 are measurable, and that when Eq. 1 is solved for E_0, it is computable by means of

$$E_0 = \frac{\sigma_0}{\sigma_w\varphi} = \frac{\sigma_0/\sigma_w}{\varphi} \quad . \tag{2}$$

So, E_0 is computable for any single real core plug. Since σ_0 can vary from 0 to $\sigma_{max} = \sigma_w\varphi$, the limiting conditions $0 \le E_0 \le 1$ are satisfied.

E_0 is the ratio of a rock's actual conductivity, σ_0, to its theoretical maximum conductivity if all of its brine were to be collected into a single through-going cylindrical tube, $\sigma_w\varphi$. This ratio thus measures the "electrical efficiency" of the brine phase of the rock and the notation for it is chosen partly in view of this fact.

From Eq. 2 an interesting observation follows. We have $\sigma_0/\sigma_w = \varphi E_0 = 1/F$ or, equivalently $F = 1/(\varphi E_0) = (1/\varphi)\cdot(1/E_0)$. This shows that a plot of F versus φ is equivalent to the product of two plots: a plot of $1/\sigma$ versus φ multiplied by a plot of $1/E_0$ versus φ. Plotting $1/\varphi$ versus φ on log-log paper will result in a straight line with slope −1. This amounts to a built-in correlation in *every* F versus φ plot. Obviously, there is no information regarding the rock in a $1/\varphi$ versus φ plot, so it is the remaining plot of E_0 versus φ that contains *all* of the pore geometric information. Since the slope of the product F versus φ is approximately −2, this suggests that the slope of $1/E_0$ versus φ will also be approximately −1, or $E_0 \approx \varphi$. In other words, to eliminate the influence of porosity from F, it must be multiplied by φ. That is to say, a plot of $\varphi F = 1/E_0$ versus φ would remove the influence of porosity from the plot, leaving only the influence of pore geometry. It is the built-in correlation of $1/\varphi$ versus φ, together with $1/E_0 \approx 1/\varphi$, that produces $m \approx 2$; i.e., about half of the slope value m (when $m \approx 2$) is due to correlating $1/\varphi$ with φ!

Since E_0 is a pore geometric factor, to proceed further it must be determined how E_0 varies with porosity. Since in real rocks (as opposed to models; e.g., the sinuous tube) pore geometry and porosity must vary concomitantly, we propose to *observe* how E_0 varies with porosity. Conductivity of the bulk rock should vary with cross-sectional area of the void space perpendicular to current flow direction. Expressed as a fraction of the total cross-sectional area of a rock sample, this is proportional to porosity. Given an ensemble of core plugs over a range of porosity, a correlation of the pore geometrical factor and porosity can be obtained. To illustrate this process, we apply the suggested procedure to Archie's Nacatoch sandstone data. The results are shown in Fig. 4.

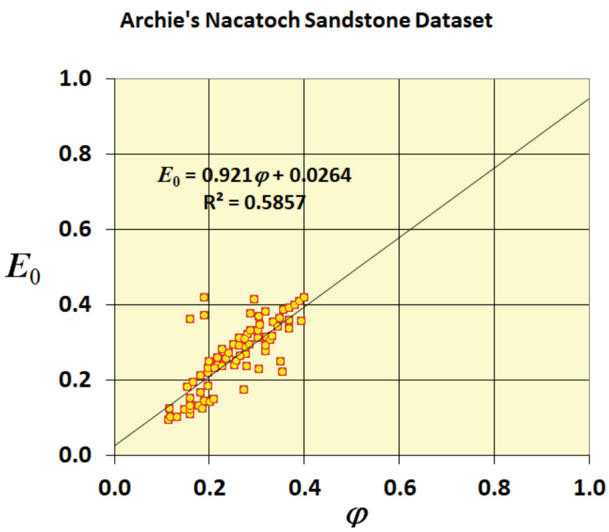

Fig. 4—The data in this figure is from Archie's Fig. (2) (see Fig. 1 in this tutorial) for the Nacatoch sandstone. It is converted to E_0 using Eq. 2; $E_0 = 1/(F\varphi)$. Note that the trend is linear, and that the slope is approximately 1 and the intercept approximately zero. Also note that the three points "northwest" of the main grouping of data are somehow different from the other data, probably taken in a different kind of rock, perhaps with fractures parallel to the direction of current flow. If these three points (out of 74) are omitted, the slope of trend is even closer to 1 (1.0336) and the intercept closer to zero (−0.0112) with a coefficient of determination $R^2 = 0.7633$.

From the results in Fig. 4, E_0 is correlated to porosity by $E_0=a_0\varphi + b_0$. Note that $a_0 \approx 1$ and $b_0 \approx 0$. These approximate values for a_0 and b_0 hold not only for Archie's Nacatoch sandstone dataset, but are present in all of the Archie rock datasets observed to date.

Our plot indicates that E_0 approximately equals porosity. This is interesting since it establishes that, for example, for a rock specimen with 20% porosity only about 20% of the bulk porous medium conducts most of the electrical current, regardless that in an Archie rock the total porosity is interconnected. With increasing porosity transmissibility increases and tortuosity decreases making the rock an increasingly efficient conductor.

To summarize, we have the triple-product theorem $\sigma_0 = \sigma_w \varphi E_0$ and the observation that $E_0=a_0\varphi + b_0$. Combining these results yields $\sigma_0 = \sigma_w\varphi(a_0\varphi + b_0)=\sigma_w (a_0\varphi^2 + b_0\varphi) \approx \sigma_w\varphi^2$. For comparison, Archie's model in conductivity terms expresses this relationship as $\sigma_0 = \sigma_w\varphi^m$ with $m \approx 2$. Archie's model is based upon a trend analysis of resistivity-porosity data uninformed by any physics. We have shown that, expressed in conductivity terms and specifically accounting for a geometrical factor at the outset, a formula similar to the Archie model emerges from three proportionalities that bulk rock conductivity must obey, and the discovery by observation the relationship between pore geometry and porosity is linear in Archie rocks. With this new insight, it is readily seen why Archie's m must ≈ 2 (i.e., $a_0\varphi^2 + b_0\varphi \equiv \varphi^m$ where the single adjustable parameter m must account for the same variation found in the two parameters $a_0 \approx 1$ and $b_0 \approx 0$ with the exponent on the a_0 term being exactly 2). If that is considered a mystery solved, it introduces the new mystery of why $a_0 \approx 1$ and $b_0 \approx 0$. This question is further elaborated in the discussion section.

We have already mentioned that

$$E_0 = \frac{\sigma_0}{\sigma_w\varphi} = \frac{\sigma_0/\sigma_w}{\varphi} = \frac{1}{F\varphi}\ , \quad \text{or} \quad F = \frac{1}{E_0\varphi}\ . \tag{3}$$

This illustrates a deficiency of the Archie model in that the formation resistivity factor combines the effects of pore geometry and porosity. In sympathy with Archie and other pioneers working with a resistivity formulation for the electrical properties of rocks, $R_0 = FR_w$ seems to be a perfectly rational approach to the problem. However, in this formulation it is not obvious how F can be broken into a factor dependent upon porosity and another factor dependent on pore geometry. Indeed, nothing in the resistivity domain even prompts the question. However, formulated in the conductivity domain in terms of our three first principles, porosity and pore geometry are introduced as separate factors a priori, before any data is examined. Thus $\sigma_0 = \sigma_w\varphi E_0$ and it is noticed subsequently that a formation conductivity factor can be defined as $f = \varphi E_0 = 1/F$ in terms of reciprocal F. So, it is recognized that $\sigma_0 = f\sigma_w$ in the conductivity domain is the equivalent of $R_0 = FR_w$ in the resistivity domain. Viewed from the resistivity domain it is hard to see how the leap to an explicit pore geometry factor can be made, and indeed, it never has been.

As a final observation, writing the Archie model in terms of conductivity, $\sigma_0 = \sigma_w\varphi^m$, we know that the left side of the formula (i.e., bulk conductivity) depends upon pore geometry. However, on the right side neither σ_w nor φ depend upon pore geometry. By process of elimination, the pore geometry must therefore be contained in m. This has

$$a_t S_w \varphi + b_t = a_0 \varphi + b_0$$
$$\left(a_t S_w \varphi + b_t\right)\big|_{S_w=1} = a_t \varphi + b_t = a_0 \varphi + b_0$$
$$a_t \varphi + b_t = a_0 \varphi + b_0$$
$$b_t = a_0 \varphi + b_0 - a_t \varphi$$
$$b_t = \left(a_0 - a_t\right)\varphi + b_0$$

been known for a long time, but it is not quite the whole story. Factoring the Archie model into the triple product theorem terms, $\sigma_0 = \sigma_w \varphi^m = \sigma_w \varphi \varphi^{m-1} = \sigma_w \varphi E_0$. The term-by-term comparison sets the geometrical factor to $E_0 = \varphi^{m-1}$ suggesting that there is more to the geometrical factor in the Archie model than m alone can account for.

To summarize the main idea introduced to this point, the bulk conductivity of a brine-saturated Archie rock depends only on properties of the brine. Bulk rock conductivity is proportional to: (I) conductivity of the brine; (II) amount (or fractional volume) of the brine; (III) the geometrical configuration of the brine determined by pore geometry. Obviously, pore geometry must vary as porosity varies. It is observed that the postulated geometrical factor is correlated with porosity, and expressible as a linear function of porosity.

Conductivity in Hydrocarbon-Bearing Archie Rocks

Archie rocks are water-wet. When (nonconducting) hydrocarbon displaces brine in an Archie rock, it moves into the "centers" of the pores and pore throats. A film of conducting brine will remain on the mineral grain surfaces in the rock. The things that change with the emplacement of hydrocarbons are the amount of brine (which is reduced) and the geometry of the brine (which presumably becomes more complicated or, at least, is changed). The reduced amount of brine is accounted for using a coefficient, S_w, to express the reduction from the maximum amount of brine, φ, so $S_w\varphi$ gives the fraction of brine in the bulk rock. The change in brine geometry is accounted for by an additional geometrical factor, e_t. Thus, $\sigma_0 = \sigma_w \varphi E_0$ is modified to

$$\sigma_t = \sigma_w \left(S_w \varphi\right)\left(E_0 e_t\right) = \sigma_w \left(S_w \varphi\right) E_t \ , \tag{4}$$

where $E_t = E_0 e_t$ gives the geometrical factor for the partially hydrocarbon-saturated Archie rock. Note that this retains the form of, indeed is a more general form for, the triple-product theorem. No new proof or additional argumentation is required. The amount of brine is reduced and its geometry is changed, but that is all. In this case σ_t, σ_w, and $S_w\varphi$ are known from measurements. E_t is determined from

$$E_t = \frac{\sigma_t}{\sigma_w S_w \varphi} = \frac{\sigma_t / \sigma_w}{S_w \varphi} \ . \tag{5}$$

When E_t is determined for an ensemble of core plugs over a range of $S_w\varphi$, the trend suggested has always been observed to be linear, described by

$$E_t = \alpha_t S_w \varphi + \beta_t \ , \tag{6}$$

Substituting this observed trend into the triple-product theorem, $\sigma_t = \sigma_w (S_w\varphi) E_t$, yields

$$\alpha_t \left(S_w \varphi\right)^2 + \beta_t \left(S_w \varphi\right) - \sigma_t / \sigma_w = 0 \ ; \tag{7}$$

this beautiful quadratic equation in $S_w\varphi$ has flowed naturally from our three first principles. The solution for S_w is elementary:

$$S_w = \frac{-\beta_t + \sqrt{\beta_t^2 + 4\alpha_t \left(\sigma_t / \sigma_w\right)}}{2\alpha_t \varphi} \ . \tag{8}$$

In the case of Eq. 6, from observation it is not necessarily true that $\alpha_t \approx 1$ and $\beta_t \approx 0$. However, assuming that this is so, the equation reduces to

$$S_w = \frac{\sqrt{\left(\sigma_t / \sigma_w\right)}}{\varphi} = \sqrt{\frac{\sigma_t}{\sigma_w \varphi^2}} = \sqrt{\frac{R_w}{R_t \varphi^2}} \ , \tag{9}$$

which is the Archie model with $m = n = 2$. This article began with this same equation and the observation that the Archie model for the electrical behavior of rocks is often introduced using this equation, and the further observation that no physical explanation for the relationship is forthcoming in those presentations. Even when the Archie model is introduced as

$$\sigma_t = \sigma_w \varphi^m S_w^n \ , \tag{10}$$

there is no *physical* explanation as to why the conducting phase, $S_w\varphi$ should be broken into two factors and each given its own, potentially different, exponent. We can ask how, physically, would this make sense?

Of course, the actual explanation of the need for, and use of, two distinct exponents is that the porosity exponent, *m,* in the Archie model is determined in an experiment using a number of brine-saturated core plugs in one plot, while the saturation exponent, n, is determined in a different experiment (or a series of experiments, one for each core plug) where brine saturation is varied from $S_w = 1.0$ to values as low as possible in the experiment. The observation that for many of these determinations that frequently $m \approx n \approx 2$ is fairly strong evidence in favor of the triple-product theorem's correctness since it accounts for the observation in a natural way.

As you might imagine, there is a great deal of additional interesting and informative and perhaps controversial discussion that could follow on this topic. However, our object of providing a basic introduction to electrical properties of rocks has been met. This article is aimed at new students of formation evaluation. In their mind's eye, if the student can visualize that the bulk rock conductivity depends upon three properties of the interstitial brine, its conductivity, its amount, and its geometry (this much is pure logic) and also remember that it is observed that the geometrical factor is approximately equal to the fractional brine volume, then it is easy to write out

$$\sigma_t = \sigma_w \varphi S_w E_t \approx \sigma_w \left(\varphi S_w\right)^2 \;, \tag{11}$$

from which, by elementary algebra, distributing the exponent and substituting the Archie parameters for 2, then

$$R_t \approx R_w \left(\varphi S_w\right)^{-2} \approx R_w \varphi^{-m} S_w^{-n} \;. \tag{12}$$

recovering Archie's resistivity model. Then follow all of Archie's results; viz.:

$$S_w = \sqrt[n]{\frac{R_w}{R_t\, \varphi^m}} = \sqrt{\frac{F\, R_w}{R_t}} = \sqrt{\frac{R_0}{R_t}} \;. \tag{13}$$

We shall continue in the next section by showing that the results above are consistent with more conventional methods of derivation.

Triple-Product Theorem from Ohm's Law and Circuit Theory

In its role as a tutorial targeted at novice formation evaluators it will be interesting to visit some topics of perennial interest in petrophysics and formation evaluation. Let us begin with a derivation of the triple-product formula from Ohm's law. In our derivation using Ohm's law, in deference to the customary notation used in electrical engineering, we shall let R denote resistance and ρ denote resistivity. Ohm formulated his law in terms of electrical circuits. His voltage sources were bimetal junction thermocouples of known potential; he measured current with a galvanometer. His circuit elements were "standard" wires of fixed material and diameter whose resistance varied with the length of the wire. He announced his result in a formula relating current to the ratio of voltage and resistance; i.e., $I = V/R$ in modern notation. A more modern formulation is $R = V/I$ which indicates that the current through a circuit element known as a resistor adjusts in reaction to the applied voltage so that the ratio remains a constant, R, dependent on the electrical properties and geometry of the material. This relationship is typically expressed as $R = \rho l/A$ where ρ is the resistivity of the material formed in a cylinder or prism of length l and cross-sectional area A. Ohm published in 1827 (100 years before the first well log was acquired). However, to determine ρ for a material one first constructs the cylinder of known geometry and then measures V and I with a voltmeter and an ammeter from which follows $\rho = V/I \times A/l$. The point is that use of resistance and resistivity is an accident of history, the result of an arbitrary choice, and is in no way fundamental. In an alternative universe Ohm might have written

$$\frac{I}{V} = g = \sigma \frac{A}{l} \;, \tag{14}$$

where g is the electrical engineer's notation for conductance and σ is the physicist's notation for conductivity. Referring to Fig. 5, for the core plug having volume $\nu = Al$ containing the single sinuous tube pore of cross-sectional area A_1 and length l_1, the measured voltage drop and current would be exactly the same for both cases; only the dimensions of the conductor vary. So, equating the conductance of the bulk medium (i.e., the core plug) and the sinuous channel, $g_0 = g_1$ and substituting the conductivity expressions, then

$$\sigma_0 \frac{A}{l} = \sigma_w \frac{A_1}{l_1} \;, \tag{15}$$

and this is solved for the equivalent conductivity of the core plug as

$$\sigma_0 = \frac{A_1/A}{l_1/l} \sigma_w \;. \tag{16}$$

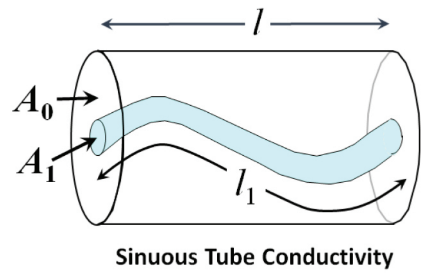

Fig. 5—The triple-product model can be derived from Ohm's law by equating the conductance of the sinuous channel to the conductance of the model plug, substituting for conductivity and geometry, and solving for the model plug equivalent conductivity and grouping terms into brine conductivity, fractional brine volume, and geometric factors.

The quantity in the denominator is recognized as the definition of tortuosity, τ. In a rock-model where the void space is collected into a single through-going cylinder, the porosity is expressible as $\varphi = A_1/A$ where A_1 is the area of the through-going cylinder, or $A_1 = \varphi A$. In actual rocks the area exposed in cross section is not equal to the area of the cross section of the through-going cylinder, but is proportional to it since the exposed void space in a random cross section of an Archie rock will be less than the area of the through-going cylinder. This is expressed as $A_1 = \alpha\varphi A$ where $0 \le \alpha \le 1$. We call α transmissibility. Thus

$$\sigma_0 = \frac{\alpha\varphi A/A}{\tau}\sigma_w \ , \tag{17}$$

which upon cancelation of the A terms, and grouping the other terms becomes

$$\sigma_0 = \left(\frac{\alpha}{\tau}\right)\varphi\sigma_w \ . \tag{18}$$

Note that this expression has the form of the triple-product theorem if the geometrical factor is defined as

$$E_0 = \frac{\alpha}{\tau} \ . \tag{19}$$

Since E_0 is by definition a geometrical factor, and since α/τ is a function of ratios of areas and lengths, it seems the requirements of the triple-product theorem are met. Thus

$$\sigma_0 = \left(\frac{\alpha}{\tau}\right)\varphi\sigma_w = E_0\varphi\sigma_w \ , \tag{20}$$

or reordering the factors on the right side, $\sigma_0 = \sigma_w\varphi E_0$. The triple-product theorem is thus consistent with (and can be derived using) Ohm's law from circuit theory.

The form $E_0 = \alpha/\tau$ invites some discussion. The factor $1/\tau = 1/\langle l_1/l \rangle$, where $\langle l_1/l \rangle$ is the average tortuosity of the medium, by its definition is ≤ 1. The parameter α describes the "transmissibility" of the pore system to electrical current, and is also a number ≤ 1. Thus, E_0 is ≤ 1. However, we also have from observation that $E_0 = a_0\varphi + b_0$. Thus

$$E_0 = \left(\frac{\alpha}{\tau}\right) = a_0\varphi + b_0 \ . \tag{21}$$

In most formulations all of the reduction in conductivity (increase in resistivity) due to the presence of mineral grains in the brine has been attributed to the tortuosity term; the transmissibility term has been ignored *even in publications where its existence is explicitly acknowledged* (e.g., Wyllie and Rose,1950; Winsauer et al., 1952). Putting all of the reduction in conductance in the tortuosity term tends to give it a misleadingly large value. However, there is ample evidence that tortuosity in Archie rocks should not be, and in fact is not, a large number but in the neighborhood of 2 or less. Thus if

$$\alpha = \left(a_0\varphi + b_0\right)\tau \ . \tag{22}$$

then the transmissibility is on the order of $\tau\varphi$.

To conclude this discussion, we make good on our promise to show that $\sigma_{max} = \sigma_w\varphi$. We noted in our derivation of the triple-product theorem from Ohm's law that $\sigma_0 = (A_1/A/l_1/l)\sigma_w$ where A_1 is the cross-sectional area of a sinuous tube of length l_1 embedded in a cylinder of cross-sectional area A and length l. The denominator represents tortuosity, a number $\tau > 1$; however, in the limit $l_1 \rightarrow l$, that is, as the sinuous tube becomes straight, then $\tau \rightarrow 1$ and the right side approaches $A_1/A \equiv \varphi \ge (A_1/A)/(l_1/l)$. Thus, $\sigma_0 \rightarrow \varphi\sigma_w$ is the maximum value that the bulk conductivity can assume.

Connections to Percolation Theory

Solid-state physicists commenced the study of conductivity of alloys and mixtures of materials at the atomic level in support of the development of transistors and integrated circuits in the 1950s. In mixtures of equidimensional conductive and nonconductive spheres they found there is a definite ratio of conducting to nonconducting spheres at which conduction commences. They dubbed the value of this ratio the "percolation threshold." For spheres, conduction begins when the ratio of conducting spheres to total spheres is, roughly, 1:3. In the 1970s geophysicists studying the conductivity of the whole earth applied percolation theory to geophysical systems, first publishing the formula

$$\frac{\sigma_0}{\sigma_w} = \left(\frac{\varphi - \varphi_\vartheta}{1 - \varphi_\vartheta}\right)^r \ , \tag{23}$$

where $r \approx 2$. (We have modified the notation from the original article to conform with our notation.) The parameter φ_ϑ is the percolation threshold parameter. By definition $\sigma_0/\sigma_w = 0$ for $\varphi < \varphi_\vartheta$; in other words, the bulk conductivity of the rock is zero when the porosity is less than a critical porosity, or percolation threshold. Percolation thresholds in rocks are much lower, approaching zero, than in sphere mixtures and other models studied by physicists, (e.g., cubic resistor lattices). We observe that Eq. 23 is not formally derived from first principles, but rather an empirical model consistent with the data presented in the source article and its references.

Combining Eq. 1 and $E_0 = a_0\varphi + b_0$ gives

$\sigma_0/\sigma_w = \varphi E_0 = \varphi(a_0\varphi + b_0)$. We now apply boundary conditions to this formula. The boundaries are the percolation threshold where bulk conductivity is zero, and 100% porosity where bulk conductivity equals σ_w and $\sigma_0/\sigma_w = 1$. We call these limits the left and right boundaries, respectively. Begin by factoring a_0 from the parenthesis:

$$\frac{\sigma_0}{\sigma_w} = \varphi\left(a_0\varphi + b_0\right) = a_0\varphi\left(\varphi + \frac{b_0}{a_0}\right). \tag{24}$$

Noting that at the left boundary $\sigma_0/\sigma_w = 0$ when $\varphi = -b_0/a_0$, then $b_0/a_0 = \varphi_\vartheta$ by definition. At the right boundary

$$\lim_{\varphi\to 1}\frac{\sigma_0}{\sigma_w} = \lim_{\varphi\to 1} a_0\varphi\left(\varphi - \varphi_\vartheta\right) = a_0\left(1 - \varphi_\vartheta\right) = 1, \tag{25}$$

So

$$a_0 = \frac{1}{1 - \varphi_\vartheta}. \tag{26}$$

Thus

$$\frac{\sigma_0}{\sigma_w} = \varphi\left(\frac{\varphi - \varphi_\vartheta}{1 - \varphi_\vartheta}\right) = \varphi E_0. \tag{27}$$

In this form, the geometrical-factor theory and triple-product theorem are seen to be consistent with percolation theory, and indeed E_0 is seen to be definable in terms of a percolation threshold. However, comparison of Eqs. 23 and 27 reveals a difference in that Eq. 23 has (approximately) two factors $(\varphi - \varphi_\vartheta)/(1 - \varphi_\vartheta)$ whereas Eq. 27 has only one. Since we are interested in derivations from first principles, this difference leads us to re-examine our list of three first principles. We notice that the second principle is not quite correct: it should be stated in terms of $\varphi - \varphi_\vartheta$, that is:

The bulk conductivity of an Archie rock (or similar medium), σ_0, is proportional to:

II. amount of connected and conducting brine (for constant pore geometry) $\varphi_c = (\varphi - \varphi_\vartheta)$; i.e., $\sigma_0 \propto \varphi_c$.

This would also change the computation of the geometrical factor to $E_c = (\sigma_0/\sigma_w) / (\varphi - \varphi_\vartheta)$.

We shall not pursue the implications of this correction to our geometrical-factor theory further in this venue. Suffice to say that any practical consequences would be minimal due to the small magnitude of the percolation threshold parameter. Practical use of the geometrical factor theory requires plotting observations to determine a_0 and b_0. Practical limits of porosity for conventional reservoir rocks are roughly from to 10 to 40% porosity. Extrapolation from this range to minimum (e.g., zero) conductivity on the left and maximum conductivity on the right requires imposition of a functional form for the fit that may not be capable of fitting observations as well as if no boundary conditions are imposed. The paper by Kennedy (2016) is devoted to this issue.

Stepping Beyond Archie Rocks: Parallel Conduction Model

Possibly the first use of a conductivity formulation for describing the electrical properties of rocks is made in the paper *Electrical Conductivities in Oil-Bearing Shaly Sands* by M. Waxman and L.J.M. Smits (1968). They propose to model a "shaly sand" as two resistors (with resistances r_1 and r_2) in parallel. In terms of resistance, the equivalent resistance r_0 of the two resistors would be

$$\frac{1}{r_0} = \frac{1}{r_1} + \frac{1}{r_2}, \tag{28}$$

or

$$r_0 = \frac{r_1 r_2}{r_1 + r_2}. \tag{29}$$

Expressed in term of conductance where $g = r^{-1}$ the corresponding relationship is

$$g_0 = g_1 + g_2. \tag{30}$$

As you can see, Eq. 30 is considerably simpler than Eq. 29. We saw in Eq. 14 that conductances and conductivities are connected by $g = \sigma(A/l)$. Making this substitution

$$\sigma_0\frac{A}{l} = \sigma_1\frac{A_1}{l_1} + \sigma_2\frac{A_2}{l_2}. \tag{31}$$

This system of conductors is illustrated in Fig. 6. A brief digression into the notation of the Waxman-Smits article will be helpful for our purposes here and for those readers who may read the Waxman-Smits paper. In the notation used in Waxman-Smits, Eq. 30 for conductance is rendered $C_{rock} = C_c + C_{el}$ (W-S Eq. (1)) where the c subscript denotes clay conductance and the el subscript denotes electrolyte conductance. The solution of Eq. 31 for σ_0 is written as $C_0 = xC_e + yC_w$ (W-S Eq. (2)) where x and y are called "appropriate geometric constants." The e subscript denotes cation exchange conductivity and the w subscript denotes brine conductivity. (Note that in the Waxman-Smits notation both conductance and conductivity are (confusingly) denoted by C; more conventional notation uses the separate symbols g and σ, respectively, for these different quantities.) However, Waxman and Smits do not provide a derivation of the constants x and y. They just assert that there are such constants. We shall now show that a form for these constants is readily derived in the parallel conduction model.

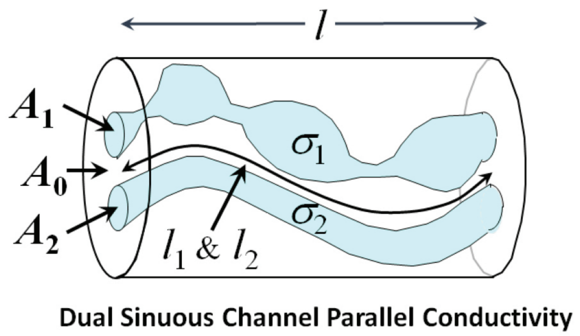

Fig. 6—A cartoon representation of the parallel conduction model where the channels have both different geometries and different conductivities.

Performing the algebra to isolate the bulk conductivity on the left side of the equation in Eq. 31, then

$$\sigma_0 = \frac{A_1/A}{l_1/l}\sigma_1 + \frac{A_2/A}{l_2/l}\sigma_2 \,, \tag{32}$$

and noting that the fractional cross-sectional areas are not equal to, but less than, the porosities of the individual channels (i.e., $A_i = \alpha_i \varphi_i A$ where $0 \le \alpha_i \le 1$) by the same arguments as used to justify Eq. 17 above,

$$\sigma_0 = \frac{\alpha_1 \varphi_1 A/A}{\tau_1}\sigma_1 + \frac{\alpha_2 \varphi_2 A/A}{\tau_2}\sigma_2 \,, \tag{33}$$

where we note that the total porosity of the plug is sum of the individual channel porosities, so that $\varphi_2 = \varphi - \varphi_1$. Then, canceling the area ratios and grouping the terms, gives

$$\sigma_0 = \left(\frac{\alpha_1}{\tau_1}\right)\varphi_1 \sigma_1 + \left(\frac{\alpha_2}{\tau_2}\right)\left(\varphi - \varphi_1\right)\sigma_2 \,, \tag{34}$$

and noting that the ratios contain only geometrical terms, then $E_i = \alpha_i/\tau_i$ and

$$\sigma_0 = E_1 \varphi_1 \sigma_1 + E_2\left(\varphi - \varphi_1\right)\sigma_2 \,, \tag{35}$$

or in Waxman and Smits terms

$$\sigma_0 = x\sigma_1 + y\sigma_2 \,, \tag{36}$$

where the "appropriate geometrical factors" of W-S are identified with $x = (\alpha_1/\tau_1)\varphi_1$ and $y = (\alpha_2/\tau_2)(\varphi - \varphi_1)$.

Although we shall not follow them there, Waxman and Smits then assert "We assume next that the electric current transported by the counterions associated with the clay travels along the same tortuous path as the current attributed to the ions in the pore water," and by this assumption they set $x = y = (F^*)^{-1}$, where by F^* they mean "the shaly sand formation resistivity factor"; presumably this means

$$F^* = \frac{C_e + C_w}{C_0} = \frac{R_0}{R_w}\frac{R_e + R_w}{R_e} = F\frac{R_e + R_w}{R_e} = \frac{1}{\varphi^m}\frac{R_e + R_w}{R_e} \,. \tag{37}$$

$R_e = 1/C_e$ denotes exchange cation resistivity; this could vary from a low value up to infinity (e.g., if the clay volume is zero, although a problem with the Waxman-Smits model is that it lacks an explicit volumetric partitioning of porosity into an interstitial brine volume and a cation exchange, or clay, volume). In the limit of zero clay volume $F^* = F$; otherwise $F^* > F$. According to their assumptions, the Waxman-Smits model will be valid when $(\alpha_e/\tau_e)\varphi_e = (\alpha_w/\tau_w)(\varphi - \varphi_e) = (F^*)^{-1}$ is satisfied. In the case of $(\alpha_e/\tau_e) = (\alpha_w/\tau_w)$ or $E_e = E_w$ (i.e., "the same tortuous path") then $\varphi_e = \varphi_w$ and the porosities are implicitly equally apportioned between the conducting phases. Otherwise (i.e., if $x \neq y$), then $E_w/E_e = \varphi_e/\varphi_w$ or $\varphi_e = E_w/E_e\varphi_w$ meaning if $E_e > E_w$, then $\varphi_e < \varphi_w$ which might seem to be the case for an authigenic clay coating grains in a predominately brine-filled void space. The Waxman-Smits requirement that $x = y = 1/F^*$ places severe constraints upon the rocks where it can be applied with accuracy.

CONCLUSIONS

Discussion

The cornerstone of the Archie model is the definition of the formation resistivity factor, $F = R_0/R_w$. This answers the question of how bulk rock resistivity depends upon a particular rock and its interstitial brine resistivity, $R_0 = FR_w$; the model connects brine resistivity to bulk rock resistivity through a rock-dependent factor but is silent as to how brine volume and brine geometry separately influence R_0. We must resort to the laboratory to discover the connection. However, even the observation that $F = \varphi^{-m}$ from the lab does not explicitly break down into a volume fraction and a geometrical factor.

In contrast, the geometrical factor representation focuses on how the bulk rock conductivity is directly proportional to the three properties of the conducting phase: conductivity, fractional volume, and geometry; i.e., $\sigma_0 = \sigma_w \varphi E_0$, our triple-product theorem. Comparison to the Archie model expressed in conductivity terms, $\sigma_0 = \sigma_w / F$ reveals that $1/F = \varphi E_0$ which is obviously a formation *conductivity* factor; i.e., $\sigma_0 = f\sigma_w$. Then f, and thus $1/F$, is seen to be the product of the two factors, porosity and geometric factor. It is true that we must still retire to the laboratory to discover a correlation between E_0 and φ; however, E_0 is an explicit geometrical factor directly correlated to porosity.

The correlation of the geometrical factor to porosity is $E_0 = a_0\varphi + b_0$. In contrast, it is not clear how to partition φ^m into a volume fraction and a geometrical factor. It would be hard to guess that the partition is $\varphi \cdot \varphi^{m-1}$, and indeed, the problem was never solved in this way, but always by putting the entire contribution from geometry into m.

We have shown that the triple-product theorem is consistent with Ohm's law by deriving the theorem from the law. This formulation also leads to the expression of E_0 as the ratio of transmissibility to tortuosity. In the appendix we offer a third derivation of the triple-product theorem in terms of ionic conductivity. The triple-product theorem also gives explicit form to the geometrical factors used in the parallel conductivity models employed for shaly sand interpretation.

One of the mysterious aspects of the Archie model is the closeness of m and n to 2. In terms of the Archie model derived from empirical observations this is just accepted as what the data dictate. At first glance the triple-product theorem seems to resolve this mystery, since σ_0 and σ_t are quadratic in φ and $S_w\varphi$, respectively, with dominant terms having exponents exactly equal to 2. However, for the triple-product theorem the empirical step is to determine how E_0 and E_t depend upon φ and $S_w\varphi$, respectively, using crossplots similar to Archie's use of crossplots to determine m and n. The resulting correlations $E_0 = a_0\varphi + b_0$ and $E_t = a_t S_w \varphi + b_t$, where a_0 and $a_t \approx 1$, and b_0 and $b_t \approx 0$, have served only to shift the mystery from the Archie parameters to the geometrical factor theory parameters. The observations restrict the values of the a and b parameters to approximately 1 and 0, respectively, leading to $E_0 \approx \varphi$ and $E_t \approx S_w\varphi$; substitution of these equivalencies into the triple-product theorem formulas reduces them to the Archie model with $m = n = 2$. Our analysis provides an answer to the question of the Archie exponents being nearly equal to 2, but it does so by shifting the question to why the geometrical factors E_0 and E_t should be so nearly equal to the brine volumes (i.e.; φ and $S_w\varphi$) in Archie rocks.

As a final observation as to the relationships $E_0 = a_0\varphi + b_0$ and $E_0 = \alpha/\tau \approx \varphi$ and why the electrical efficiency of the Archie rock should approximately equal the porosity, we can speculate that since the cross-sectional area normal to current flow is one of the main controllers of how much current is passed, to the degree that effective tortuosity approaches 1, then transmissivity approaches porosity in value. This may be evidence of our claim that tortuosity in fact is a number approaching 1 for Archie rocks.

The development above does not take into account formation conductivity (or resistivity) anisotropy. Real rocks are almost invariably anisotropic, and the triple-product theorem is readily extensible to anisotropic rocks. Archie's model can also be extended to anisotropic rocks. We will develop anisotropy in the next article of this tutorial.

Concluding Thought

Our purpose in this article has been to introduce and illuminate the physical processes responsible for conduction in rocks. We have framed the problem in three distinct ways: a derivation from three physical first principles; a derivation from Ohm's law in conductivity terms; a derivation from brine conductivity in terms of ion concentration (see Appendix). All three methods lead to the triple-product theorem formula, which is then trivially transformed to the Archie model in certain limiting cases. Since the physical first principles are very nearly self-evident, and lead to formulas that produce the Archie model directly, there is every reason to embrace them. Empiricism is still required to determine the geometrical dependence, but in the triple-product theorem formulation the dependence on geometry is explicit, not being shoved into parameters m and n because there is nowhere else to put the dependence. This fulfils our mission.

We have often paused to wonder why formation evaluation has resisted progress when compared to other 20th century technical developments. Heisenberg (1925) and Schrodinger (1926) published the foundations of quantum mechanics just prior to 1927, the year that the first well log was acquired in France. In the intervening 90 years quantum mechanics, invented to explain atomic physics, has been applied secondly to nuclear physics and then to the physics of protons and neutrons as particles comprised of still smaller particles, quarks. Neil Armstrong set foot on the moon just 65 years following the first powered flight. Given these examples (and there are many others) one may well wonder why in the 78 years since Archie announced his empirical model corresponding progress has not been made in formation evaluation theory. Is formation evaluation more difficult than physics' "standard model" or the engineering challenges of space flight? Perhaps the answer lies in that in physics and the engineering of flight thousands of scientists and engineers in hundreds of universities and government-supported agencies and companies were engaged in competitions to decipher the workings of matter, whether of atoms or airfoils. Conversely, in formation evaluation only a handful of thoughtful people, distributed over several competing commercial organizations and distributed over seven decades in time, were dedicated to thinking about conductivity in reservoir rocks. Archie's model works so well in so many cases managers had little incentive to allocate resources to what seemed to them to be a non-problem. Moreover, management goals tended (and still tend) to change annually, so only a very few lucky scientists (e.g., Wyllie at Gulf, and Archie at Shell) were able to devote years-long effort to the problem.

The thought processes of researchers were so entrenched in the resistivity formulation that even when they were trying, they were not able to overcome the barrier posed by the resistivity formulation of the problem, namely that the formation resistivity factor is the product of (reciprocal) porosity and a (reciprocal) geometrical factor. Further, even though the role of brine cross-sectional area was recognized in print by some of the pioneers (Wyllie and Rose, 1950; Winsauer et al., 1952), their models focused wholly on tortuosity to the exclusion of transmissibility. To the best of our knowledge, Dr. David Herrick, at the Amoco research laboratory in the 1980s, was the first to completely rethink the problem without reference to prior models. His thinking was the source of the ideas embodied in the geometrical factor theory, and were independently confirmed by the work of Professor C.F. Berg in 2012.

It is our hope that this exposition will inspire its readers to "think out of the box" in terms of resistivity interpretation. Although the subject here is mostly confined to conventional Archie rocks, the shaly sand problem might be profitably revisited. The application of unconventional thinking to unconventional resources is surely the way to progress in the future. We hope that this article will trigger a renewed and vigorous, even if contentious, discussion of this topic. It will only be through the disputation of ideas, new and old, that will lead to progress and, perhaps, consensus among formation evaluators as to the form a new model will take.

ACKNOWLEDGEMENTS

The fundamental ideas developed in this article originated in the fertile brain of Dr. David Herrick. Recently and independently, our colleague Professor Carl Fredrik Berg developed the identical concept with a rigor acceptable to the physics community. This article would not have been attempted without the invitation and encouragement of Professor Carlos Torres-Verdin.

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Sen has a section entitled "Derivation of Archie's Law from First Principles." However, a more accurate title for the section might have been "Derivation of a Resistivity Power Law from an Effective Medium Approximation."

Wyllie, M.R.J., and Rose, W.D., 1950, Some Theoretical Considerations Related to the Quantitative Interpretations of the Physical Characteristics of Reservoir Rock from Electric Log Data, Paper SPE-950105-G, *Journal of Petroleum Technology*, **2**(4), 105-118. DOI: 10.2118/950105-G.

This paper contains the first attempt that we know of to derive the Archie model from theoretical principles. Interestingly, all of the necessary principles are indeed discussed, in particular the role of cross-sectional area normal to the direction of current flow, and its proportionality to porosity. Wyllie and Rose then proceed to *not* use this fact in their analysis and thereby miss the opportunity to have discovered the geometrical factor theory and triple product theorem.

Wyllie, M.R.J., 1952, Role of Clay in Well Log Interpretation, *Clays and Clay Minerals*, **1**(1), 282–305. DOI: 10.1346/CCMN.1952.0010125.

We include this paper as one of interest for the shaly sand problem because it indicates pretty clearly that Wyllie understood all of the issues 16 years before the Waxman-Smits publication appeared. Perhaps arguably, his analysis is even more cogent when pore geometrical aspects are concerned. For example, Wyllie writes $C_{wa} = C_f + C_w = C_f + C_c/F$. followed by "It may be noted that in Eq. 6 C_s/F' may be substituted for C_f." In other words, $C_{wa} = C_s/F' + C_c/F$. "Here C_s is the actual conductivity of the conductive solids and F' their formation factor." This is equivalent to Waxman's and Smit's Eq. 3 but with different formation factors (i.e., pore geometries) for the two conductive phases. We can speculate that Wyllie carried this no further as there is no obvious way to separately determine F and F'. Still, it would have been profitable to have opened a discussion of the issue.

Winsauer W.O., Shearin, H.M., Masson P.H., and Williams, M., 1952, Resistivity of Brine-Saturated Sands in Relation to Pore Geometry, *AAPG Bulletin,* **36**(2), 253–277.

This paper makes extensive reference to the role of conductor cross-sectional area in discussion. It then fails to use it in its analysis. It is also the source of the "Humble formula"; i.e., $F = 0.62 / \varphi^{2.15}$.

Waxman, M.H., and Smits L.J.M., 1968, Electrical Conductivities in Oil-Bearing Sands, Paper SPE-1863-A, SPE *Journal*, **8**(2), 107–122. DOI: 10.2118-1863-A.

This paper is, perhaps arguably, the seminal paper for resistivity log interpretation in clay-bearing rocks. Its longevity is remarkable considering its derivation contains very questionable assumptions regarding pore geometry and its lack of volume-weighting for its parallel conductive components.

Clavier, C, Coates, G., Dumanoir, J., 1984, Theoretical and Experimental Bases for the Dual-Water Model for Interpretation of Shaly Sands, Paper SPE-6859, *SPE Journal*, **24**(2), 153–168. DOI: 10.2118/6859-PA.

This paper initially appeared in 1977 as a conference paper and in a quite different version as a peer reviewed paper. As far as its physics and electrochemistry is concerned, it is basically the same as in the Waxman-Smits paper; however, these authors correct the Waxman-Smits failure to partition the pore volume into interstitial brine and exchange cation brine. Unfortunately, the Waxman-Smits assumption that the geometry of the pores and the surfaces is the same is retained in the dual-water model.

Shankland, T.J., and Waff, H.S., 1974, Conductivity in Fluid-Bearing Rocks, *Journal of Geophysical Research*, **79**(32), 4863–4868. DOI: 10.1029/JB079i032p04863.

Although the formula was never adopted into petrophysical use, these authors were first to apply percolation theory percepts to conduction in rocks. Perhaps their work went unnoticed in formation evaluation because they were geophysicists publishing in the *Journal of Geophysical Research* rather than petroleum engineers or geologists publishing is SPE or AAPG venues.

Geometrical Factor Theory Papers

The exposition of the Geometrical Factor Theory, first published in 1993 and 1994, evolved over 18 years as its inventors realized its implications, eventually culminating in its formulation as the Triple-Product Theorem. Prof. C.F. Berg, a physicist then working at Statoil, independently and with mathematical rigor, reinvented the GFT in 2012.

Berg, C.F., 2012, Re-Examining Archie's law: Conductance Description by Tortuosity and Constriction, *Physical Review E*, **86**(4)., DOI:10.1103/PhysRevE.86.046314.

Herrick, D.C. and Kennedy, W.D., 1993, Electrical Efficiency: A Pore Geometric Model for the Electrical Properties of Rocks, Paper HH, *Transactions*, *SPWLA 34th Annual Logging Symposium*, Calgary, Canada, 13–16 June.

Herrick, D.C. and Kennedy, W.D., 1994, Electrical Efficiency—A Pore Geometric Theory for Interpreting the Electrical Properties of Reservoir Rocks, *Geophysics*, **59**(6), 918–927. DOI: 10.1190/1.1443651.

Herrick, D.C. and Kennedy, W.D., 2009, A New Look at Electrical Conduction in Porous Media: A Physical Description of Rock Conductivity, Paper BB, *Transactions*, *SPWLA 50th Annual Logging Symposium*, The Woodlands, Texas, 21–24 June.

Kennedy, W.D., Herrick, D.C., 2012, Conductivity Models for Archie Rocks, *Geophysics*, **77**(3), WA109-WA128. DOI: 10.1190/geo2011-0297.1.

Kennedy, W. D., 2016, Conducting Connected Porosity: A Concept for Unifying Resistivity-Porosity Models, Paper U, *Transactions, SPWLA 57th Annual Logging Symposium*, Reykjavik, Iceland, 25–29 June.

Service Company Literature

Schlumberger, 1972*, Log Interpretation*: *Volume* I – *Principles*.

Notes: Chapter 1. *Formation Factor and Porosity*. Here we find the first mention of formation resistivity factor. There is no discussion of causality, but the article begins with "It has been established experimentally that the resistivity of a clean formation … is proportional to the resistivity of the brine with which it is fully saturated." A misattribution to Archie follows, "Archie proposed the formula $F = a/\phi^m$ where m is the *cementation factor*." In fact, Archie (1942) (and the reference is cited) proposed only $F = 1/\phi^m$, without any use or mention of a, and Archie does not refer to a "cementation factor". This latter coinage is from H. Guyod writing in 1944. The earliest reference that we have found to a is in an article by Owens, 1952.

*Water Saturation***.** In this section Archie's Eqs. 4 and 5 are

listed. The only justification is that "Archie determined experimentally …" Actually, Archie deduced his Eqs. 4 from the literature.

Invasion, Vertical Saturation Gradients and Vertical Fluid Migration. There is a nice sketch of invasion profiles provided on page 3. The accompanying discussion is in very general terms. Some of its claims would be hard to follow for a person without a reservoir engineer's background in fractional fluid flow.

Anisotropic Formations. Anisotropy is briefly mentioned but attributed mainly to shale and mudcake. Laminated shale-sand is mentioned as being anisotropic without elaboration.

Chapter 14**.** *Resistivity Interpretation*.

In spite of the title of this chapter, it is mainly about which log should be used in which circumstances, and which charts (in a separate Chart Book volume) should be employed in interpretation. Step resistivity profiles are mentioned but their limitations are not discussed except for the case of a resistivity annulus.

Chapter 15. *Determination of Saturation* (Clean Formations).

The Archie Formula. The section begins "All water saturation determinations from resistivity logs in clean formations with homogenous intergranular porosity are based on Archie's formula or variations thereof." Once again, there is no discussion of any formation resistivity (or conductivity) physics, but merely a recitation of formulas and the logs that should be obtained to have proper parameters to plug into the formulas.

Chapter 16. *Shaly Formations*

Introduction. The "Thomas-Stieber" model with its Laminated-Dispersed-Structural shale distribution is presented in picture form. The publication of Principles – Volume 1 proceeds the Thomas-Stieber publication, so we assume that Thomas and Stieber drew on the pictures in this volume in the production of their analytical model.

Laminated Sand-Shale Simplified Model. This presents the parallel conduction model for laminated sand and shale formations as a formula. It is correct but there is no discussion of how it arises from the physics of the medium.

Dispersed Shale Simplified Model, Shaliness and Cation Exchange & Total Shale Model. Again, there is no discussion of conduction mechanisms in these sections, only formulas. The only "surviving" technique mentioned in these articles is the Waxman-Smits model. We have addressed the limitations of the Waxman-Smits parallel conduction model in our main article on parallel conduction.

Schlumberger, 1989, *Log Interpretation Principles/Applications*.

Notes: This is the most recent Schlumberger offering, now 30 years old. Although the organization of the content has changed and been added to, much of it remains identical to the 1972 *Log Interpretation Volume I – Principles*. Additions have been made to the shaly sand resistivity/conductivity interpretation discussion. For example, "There are many formulas that relate resistivity to water saturation in shaly sands. Most are generally of the form:

$$\frac{1}{R_t} = \frac{S_w^2\left(1-V_x\right)}{FR_w} + \frac{CV_x}{R_x} .$$

This is a parallel conductivity model where V_x is a fractional volume (or a term depending upon fractional volume) of a conductive phase identified with the x subscript, and C is a term "related to water saturation." Translated into conductivity terms gives

$$C_t = \left(1-V_x\right)\frac{S_w^2}{F}C_w + V_x C\, C_x ,$$

and substituting Waxman-Smits parameter notation, $C_x = BQ_v$, $S_w^{\,2}/F = 1/G^*$, and $C = F^*/\,S_w^{\,n+1^*} = 1/S_w$ then

$$C_t = \frac{1}{G^*}\left[\left(1-V_x\right)C_w + V_x\frac{BQ_v}{S_w}\right] = \frac{S_w^{n^*}}{F^*}\left[\left(1-V_x\right)C_w + V_x\frac{BQ_v}{S_w}\right] ,$$

which is the Waxman-Smits formula except that it includes volume weights for the two conductive phases.

There are several other service company publications equivalent to the Schlumberger *Principles* volumes, including from Gerhart Owens International, Welex, Halliburton, Atlas (in several forms: Dresser Atlas, Western Atlas, Baker Hughes, etc.). None of these publications depart from a conventional discussion of the development of formation resistivity, and we do not review them separately for this summary.

Books: Petrophysical Theory and Tool Physics

Doveton, John H., 2014, *Principles of Mathematical Petrophysics*, International Association for Mathematical Geology Studies in Mathematical Geology **9**, Oxford University Press. ISBN: 978-0199978045.

Doveton's Chapter 1 is a review of formation resistivity as it developed historically, beginning with a formula from J.C. Maxwell's *Treatise on Electricity and Magnetism*. He does point out the distinction between the empirically-based resistivity formulations and physics-based conductivity formulations on the very first page of his book. In the remainder of the chapter the resistivity models are introduced more or less in chronological order of their publications. This book, and its Chapter 1 provide an excellent place for novice formation evaluators to begin their continuing education on formation resistivity methods.

Ellis, D.E., Singer, J.M., 2007, *Well Logging for Earth Scientists*, 2nd Edition, Springer. ISBN: 978-1402037382.

This book has three sections on resistivity interpretation Chapter 3 (Basic Resistivity and Spontaneous Potential), Chapter 4 (Empiricism: The Cornerstone of Interpretation), and Chapter 23 (Saturation and Permeability Estimation). Equation (23.1) is $S_w^{\,n} = R_w/(R_t\varphi^{-m})$. These authors mention the triple-product theorem (Eq, 4.12) in their discussion in the section "Alternative Models." This book should be on the shelf of every scientist planning to contribute, or contributing, to formation evaluation.

Hearst, J.R., Nelson, P.H., and Paillett, F.L., 2000, *Well Logging for Physical Properties*, 2nd Edition, John Wiley & Sons, Ltd. ISBN: 978-0471963059.

This book attempts to relate all of well-log interpretation, insofar as possible, to the basic underlying physical principles. It also has three sections that discuss resistivity and its interpretation. First is a brief mention in the section on Mixing Rules in the first chapter. Its Chapter 5 (Electrical and Magnetic Methods) provides (in 100 pages) a thorough discussion of hardware and interpretation physics. A subsection titled "Theoretical Understanding of Archie's Equation" (p. 75) provides a discussion based upon the same work that supports this paper. The authors finally return to resistivity interpretation in Chapter 14 (Saturation).

Books: Petrophysical Practice

Asquith, G., and Krygowski, D., 2004, *Basic Well Log Analysis*, 2nd Edition, AAPG Methods in Exploration Series, **16**. ISBN: 978-0891816676.

This book is very much oriented as to the "how to" aspects of formation evaluation using logs. Archie's model relating formation resistivity, brine resistivity, porosity, and water saturation using parameters m and n is introduced beginning on page 3. There is discussion of the model's formula but discussion of the physical basis of the model is not present except for an explanation of the relationship between resistivity and resistance. The author's claim that "G.E. Archie … presented a paper … which set forth the concepts used as a basis for modern quantitative log interpretation" overstates the scope of the Archie model. Archie puts forth no "concepts"; only formulas. Asquith and Krygowski also commit the common error of attributing the a in $F = a/\phi^m$ to Archie. There is some further discussion in Chapter 5 (Resistivity) which is primarily devoted to logging instrument hardware concepts, and Chapter 7 (Log Interpretation) but there is no discussion as to how the formulas relate to first principles.

Bassiouni, Z., 1994, *Theory, Measurement, and Interpretation of Well Logs*, SPE Textbook Series **4**. ISBN: 978-1555630560.

Bassiouni begins his book with a chapter on *The Electrical Resistivity of Rocks*. Section 1.4 is devoted to a derivation of the formation resistivity factor. The technique used is to equate the resistance of a core plug to the resistance of the brine in the core plug, both based upon the dimensions and resistivity of the plug. The argument is that to keep the volume of brine $V = \varphi AL$ constant in view of the longer-than-L lengths of the conductive paths, then the cross-sectional area must be modified accordingly; i.e., $V = \varphi AL(L_e/L_e) = \varphi A(L/L_e)\ L_e = (\varphi A/\tau)L_e$. As Bassiouni puts it: "The cross-sectional area of the equivalent water volume, A_c, has to be $\varphi A(L/L_e)$." But this is not so. If the final formula is to reflect the effect not only of tortuosity, but also the effect of reduced cross-sectional area then we can take $V = \alpha\varphi A(L/\alpha) = \alpha\varphi AL_e'$ as a more, or at least equally, valid parameterization of the brine volume. Then using this parameterization leads to $F = (\tau/\alpha)/\varphi = 1/(E_0\varphi)$ rather than Bassiouni's result $F = \tau^2/\varphi$ which uses two factors of tortuosity and none for transmissibility of the cross-sectional area.

Bateman, Richard M., 2012, *Openhole Log Analysis and Formation Interpretation*, 2nd Edition, Society of Petroleum Engineers. ISBN: 978-1613991565.

Unfortunately, much of the story of the historical development of formation evaluation is not established fact, but based upon peoples' notions of "how it had to have been" which in many cases is not "how it happened". This is so common that Barbara Anderson (Scientist at Schlumberger-Doll Research, Editor of *Petrophysics* (2004–2005), and President of SPWLA (1994–1995) coined the term "folklore" to describe it. Bateman's book devotes Chapter 6, section 6 to the development of the Archie model. In the lead-up to a key result Bateman states: "Eventually, Archie found that laboratory measured values of F could be related to the rock porosity by an equation of the form $F = a/\varphi^m$." Well, actually Archie never employs a in any of his four publications. Later in the text (Chapter 26, page 462) Bateman expands: "where a is a constant (the *Archie constant*) …" In fact, a is usually termed the "tortuosity factor", and as just mentioned, Archie never uses any form other than $F = 1/\varphi^m$. In section 6.6.3 Bateman reports "Archie's experiments showed that the saturation of a core could be related to its resistivity." We do not doubt that Archie made (or paid for) many such measurements during his long career; however, the relationship he reported in his 1941 presentation to the AIME and subsequent 1942 publication (i.e., $S_w = (R_t/R_0)^{1/n}$, and Archie's Eqs. (4), (5) and (6)) are all based upon data to be found in four papers cited by Archie from the literature of the late 1930s; Archie reports no experiments that he made to validate this relationship. Otherwise, the Bateman presentation is conventional and does not attempt any derivations for the Archie model.

Dewan, J.T., 1983, *Essentials of Modern Open-Hole Log Interpretation*, PennWell Publishing Company. ISBN: 978-0878142330.

Dewan's treatment of resistivity uses the artifice of brine- and sand-filed cubes to motivate a discussion of formation resistivity factor. However, his subsequent introduction of the Archie model is to just write down the relationships and claim they are justified by "general principles" which are not elaborated.

Helander, D.P., 1983, *Fundamentals of Formation Evaluation*, OGCI Publications. ISBN: 978-0930972028.

Helander attempts a rigorous justification of the Archie model on his pages 63 and 64. His final result is $F = \tau^2/\varphi$. where τ is tortuosity defined in the conventional way as L_e/L. This would, of course, mean that $\varphi \approx 1/\tau^2$, a remarkable result. This would certainly seem to warrant some discussion, but there is none. However, this result is sometimes cited in the literature, and Helander's derivation of the result is a good source for understanding the result. Helander does not number his equations, but the third equation in the left column of page 64 is the source of this result. It is an equation for the volume of brine in a core plug, $V_{cw} = A_cL\varphi$, where A_c is the cross-sectional area of the plug, L the length, and φ the porosity. He then multiplies this quantity by $1 = L_e/L_e$ and identifies $A_c(L/L_e)\varphi$ as the cross-sectional area of an equivalent plug of the same volume with length L_e. He then uses the cross-sectional area so defined in his expression for the equivalent resistance of the brine in the core. This leads with a little algebra to his final result. However, what the factor $L/L_e < 1$ does is lead to

a reduction in effective cross-sectional area (which is needed for the derivation); the same result could have achieved with the introduction of a coefficient $\alpha < 1$ on the porosity; i.e., $\alpha\varphi$. With this substitution (i.e., $\alpha/\alpha = 1$ rather than $L_e/L_e = 1$) Helander's result would have been $F = (\tau/\alpha)/\varphi = 1/E_0\varphi$.

Hilchie, D.W., 1978, *Applied Openhole Log Interpretation for Geologists and Petroleum Engineers,* self-published by D.W. Hilchie.

Introduction to resistivity and saturation analysis by use of the Archie model. There is no attempt to justify the model.

Hilchie, D.W., 1989, *Advanced Well Log Interpretation,* self-published by D.W. Hilchie.

Introduction to resistivity and saturation analysis by use of the Archie model. There is no attempt to justify the model. Although the title has changed from "*Applied*" to "*Advanced*", the treatment of Archie's resistivity model remains unchanged.

Peters, E.J., 2012, *Advanced Petrophysics*, v. 1, Geology, Porosity, Absolute Permeability, Heterogeneity, and Geostatiscis, Live Oak Book Company,

In spite of its title, the treatment of the Archie model in this book is completely conventional.

Rider, M., and Kennedy, M., 2011, *The Geological Interpretation of Well Logs*, 3rd Edition, self-published by Rider-French Consulting, Ltd. ISBN: 978-0954190682.

Although this book devotes its chapter 5 to Resistivity Logs, only 1/2 of one column on page 105 is devoted to repeating the Archie model equations, presented with water saturation on the left side. There is no connection made to physics, or even to trendline fitting.

Tittman, J., 1986, *Geophysical Well Logging*, excerpted from Methods in Experimental Physics, **24**: Geophysics, Academic Press. ISBN: 01206913900.

Tittman's book is mainly concerned with logging hardware, and is a good resource for logging instruments up to its date of publication. It's treatment of resistivity theory is conventional and cursory.

Tiab, D., and Donaldson, E.C., 1996, *Petrophysics: Theory and Practice of Measuring Reservoir Rocks and Fluid Transport Properties*, Student Edition, Gulf Professional Publishing.

This book is a comprehensive compendium on (as its title promises) petrophysics. It is not a "how-to" book on log analysis, and its treatment of formation resistivity is purely conventional. It does, however, include a discussion on "Theoretical Formula for F_R" based upon a 1976 paper by C.P. Rosales, "Generalization of the Maxwell Equation Formation Resistivity Factors," Paper SPE-5502-PA, published in JPT. We have not had time to review the reference for our present article, but suffice to say under most conditions the Rosales formula reduces to $F = a/\varphi^m$ which is not a surprise.

Wyllie, M.R J., 1963, *The Fundamentals of Well Log Interpretation*, 3rd Edition, Academic Press.

Wyllie's book may have been the first book published (in English, at least) on log interpretation. Wyllie was a research scientist at Gulf Oil Corporation. In 1963 he had been researching formation evaluation for over a decade. On page 2 of his book he opines: "In many ways it unfortunate that conductivity was not selected in place of resistivity as the standard term in the early days of electric logging. … Conductivity logs, although identical in general form to resistivity logs, would somewhat simplify the equations now used in log interpretation. It is probably too late to upset the entire terminology of logging, but it is still sometimes easier to speak and think in terms of conductivities instead of resistivities. … The word conductivity implying as it does the ability to conduct electric current, is a particularly convenient term when the mechanism of current flow in rocks is considered." Wyllie's derivation of formation resistivity factor F parallels the discussion in his 1950 paper with W.D. Rose. He states in that text "Any real rock … has a conducting area A perpendicular to the direction in which a resistance measurement is made which is *proportional* to the rock porosity, φ." [emphasis added] However, he does not use this proportionality in his derivation. For rock resistance he writes $r_{\text{Rock}} = RL_e/\varphi$ (where L_e is the length of the current path through a sample of length L) rather than $r_{\text{Rock}} = RL_e/\alpha\varphi$ (which includes a proportionality factor α and forms the ratio of this quantity to the bulk resistance of the saturating brine, $r_{\text{Solution}} = RL$, his result is $F = (L_e/L)/\varphi$, where L_e/L is by definition tortuosity τ. Had he included the porosity proportionality constant, his result would have been $F = (\tau/\alpha)/\varphi$, or in our notation $F = 1/E_0\varphi$. Wyllie's next equation is a statement of Archie's definition of formation resistivity factor, $FR_w = R_0$. Had Wyllie and Rose included the proportionality of cross-sectional area to porosity in their definition of F, they would have invented geometrical factor theory, $R_w/\varphi E_0 = R_0$, in 1950, or Wyllie could have done so in his book a decade later. Unfortunately, those opportunities were lost. From this point (page 13) in the text, and beyond, Wyllie is conventional in his explanations.

Internet Sources

One might expect oilfield professional societies to be the authoritative sources for this kind of material; however, in practice they are of limited use, not very different from the print sources cited above. Some are cited below.

Wikipedia (https://en.wikipedia.org/wiki/Archie%27s_law). There is no derivation to be found here, but the model is given in its conductivity form as $C_t = C_w\varphi^m S_w{}^n/a$

AAPG Wiki (https://wiki.aapg.org/Archie_equation). This reference gives the Archie model solved for S_w (i.e., $S_w^n = R_w/R_t\varphi^m$)

and with no explanation of the physics. This is the type example of why resistivity in rocks is so poorly understood by so many who use logs.

S*PE Petrowiki* (https://petrowiki.org/Water_saturation_determination). There is a very thorough discussion of water saturation determination, but as far as the use of the Archie

model is concerned, the only equation is $S_w = (R_0/R_t)^{1/n}$. with a separate equation for R_0, $F = R_0/R_w$.

APPENDIX: TRIPLE-PRODUCT THEOREM FROM ELECTROLYTE CONDUCTIVITY

In the main text of this article the triple-product theorem was developed from a statement of three self-evident first principles and also by use of Ohm's law. In the former case the physical property of conductivity is implicitly assumed to exist; in the latter case conductivity is defined by ratios of electrical and geometrical properties; viz., I/V and l/A respectively. Neither of these methods considers the actual mechanism of conduction; i.e., mobile charges. We offer a final discussion of charges in motion that also leads to the triple-product theorem.

A fundamental relationship for transport is $Q_{vol} = A\mathbf{v}$ where Q_{vol} is the vector volumetric flow rate with units of m^3/sec, A is the area normal to flow direction, and $\mathbf{v}$ is the speed of the flow. If there is something in the volume, say particles, then the particle flow rate would be $Q_{part}=nA\mathbf{v}$ where n is particle density in particles/m^3. If the particles are charged (i.e., ions), then the charge flow rate will be $Q_{charge}=nqA\mathbf{v}$ where q is the charge per ion. Note that the unit is charge per second. Since this is electric current, let us change to the conventional notation for electric current, $\boldsymbol{I}$, and $\boldsymbol{I}=nq\mathbf{v}A$ where $\boldsymbol{I}$ is the current vector in particles (or ions) per second, n is the density of particles (ions/volume), q is the charge per ion, $|\mathbf{v}|$ is the speed of the ions, and A is the cross-sectional area of the flow. Dividing by A, the cross-sectional area, converts the left side to current density; $\mathbf{J}=nq\mathbf{v}$, where $\mathbf{J}$ is current density, is the fundamental equation relating current density to amount of charge passing through an area A in one second. The units are included in square brackets below to illustrate how they combine.

$$\begin{aligned}\mathbf{J} &= n\left[\frac{\text{particles}}{\text{m}^3}\right]q\left[\frac{\text{charge}}{\text{particle}}\right]\left[\frac{\text{m}}{\text{sec}}\right] \\ &= nq\mathbf{v}\left[\frac{\text{charge/sec}}{\text{m}^2}\right] \equiv \text{current denisty}\end{aligned} \tag{A.1}$$

You could well imagine the current density through a brine-filled cylinder (Fig. A.1) given by $\mathbf{J}_w=n_wq\mathbf{v}_w$ where the subscript w is the convention used to denote brine properties. The same brine suffusing a similar, but sand-filled, cylinder (Fig. A.2) will exhibit different properties. In particular, the amount of charge in each unit volume would be reduced to a fraction of the charge in the brine, the fraction being equal to the volume ratio of the brine in the sand-filled cylinder to the brine volume in the brine-filled cylinder; in other words, its porosity. In the sand-filled cylinder $\mathbf{J}_0=n_0q\mathbf{v}_0$ where the 0 subscript denotes the bulk properties of the sand-filled cylinder. In general, $\mathbf{J}_0$ is less than $\mathbf{J}_w$ because $n_0 < n_w$ by the brine volume ratio, and $\mathbf{v}_0 < \mathbf{v}_w$ since the end-to-end speed of the ions is reduced due to mobility restrictions placed upon the ions by the sand-grains acting as obstacles to flow, reducing the cross-sectional area (transmissibility) and increasing the streamline lengths (tortuosity).

Fig. A.1—A brine-filled cylinder.

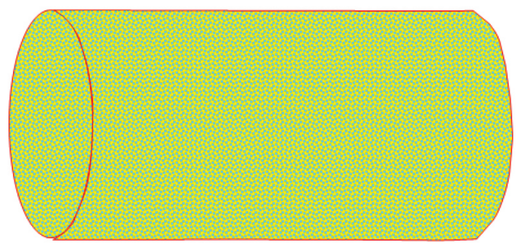


Fig. A.2—A sand-filled cylinder suffused with the same brine.

The general forms of the equations are not useful unless the speed of the ions is known. This may not be a convenient formula when the speed would be difficult to measure. Since the transport is induced by an applied electric field, it is convenient to convert the fundamental equation to a form including electric field strength. To do this the speed vector is multiplied by 1 in the form of applied electric field divided by applied electric field magnitude; i.e., for the brine

$$\mathbf{v}_w = |\mathbf{v}_w|\frac{\mathbf{E}}{|\mathbf{E}|} = \frac{|\mathbf{v}_w|}{|\mathbf{E}|}\mathbf{E} \equiv \mu_w\mathbf{E} \ , \tag{A.2}$$

and for the bulk rock volume

$$\boldsymbol{v}_0 = |\boldsymbol{v}_0|\frac{\boldsymbol{E}}{|\boldsymbol{E}|} = \frac{|\boldsymbol{v}_0|}{|\boldsymbol{E}|}\boldsymbol{E} \equiv \mu_0\boldsymbol{E} \ , \tag{A.3}$$

where the ratios $|\mathbf{v}_w|/|\mathbf{E}| = \mu_w$ and $|\mathbf{v}_0|/|\mathbf{E}| = \mu_0$ are called "mobilities". Their units are (speed per unit of $\mathbf{E}$ field strength) or ((m/sec) /(volt/m)). Values for mobilities are specific to each different kind of brine and must be measured in a laboratory, but once known, can be applied to any known solution.

So now we have $\mathbf{J}_w = n_w q\mu_w \mathbf{E}$ and $\mathbf{J}_0 = n_0 q\mu_0 \mathbf{E}$; a term by term comparison with Ohm's law shows that $\mathbf{J}_0 = n_0 q\mu_0 \mathbf{E} = \sigma_0 \mathbf{E}$ and $\mathbf{J}_w = n_w q\mu_w \mathbf{E} = \sigma_w \mathbf{E}$. Form the ratio of these quantities as

$$\frac{\mathbf{J}_0}{\mathbf{J}_w} = \frac{n_0 q \mathbf{v}_0}{n_w q \mathbf{v}_w} = \frac{n_0\left[\text{particles/volume fraction}\right]}{n_w\left[\text{particles/volume}\right]} \times \frac{q\left[\text{charge/particle}\right]}{q\left[\text{charge/particle}\right]} \times \frac{\mu_0\left[(\text{m/sec})/(\text{volt/m})\right]}{\mu_w\left[(\text{m/sec})/(\text{volt/m})\right]} \times \frac{\mathbf{E}}{\mathbf{E}} \tag{A.4}$$

The factors of q and $\mathbf{E}$ are the same in numerator and denominator; thus they cancel and

$$\frac{\sigma_0}{\sigma_w} = \frac{n_0}{n_w} \times \frac{q}{q} \times \frac{\mu_0}{\mu_w} = \varphi \cdot 1 \cdot E_0 \ , \tag{A.5}$$

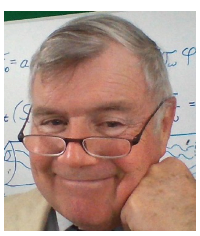
or, to put it succinctly

$$\frac{\sigma_0}{\sigma_w} = \varphi E_0 \ , \tag{A.6}$$

where porosity is identified with the ion density ratios in the two cylinders and where E_0 (not to be confused with electric field $\mathbf{E}$) is identified with the mobility ratio. Basically, it is the ratio of the time it would take for an ion to traverse the sand-filled cylinder with its complicated pore space to the time it would take the same ion to traverse the unobstructed brine-filled cylinder. Thus, the triple-product theorem, $\sigma_0 = \sigma_w \varphi E_0$, is recovered from a comparison of charge flow rates. The Archie model would follow experimentally by comparing E_0 for a number of sand-filled cylinders of differing porosity.

This discussion is simplified for heuristic purposes; an actual electrolytic solution would have ions of both positive and negative charge, each type with its own concentration, charge and mobility. Inclusion of these details would complicate the argument without altering the conclusion.

ABOUT THE AUTHORS



David Kennedy began a career in the logging industry in 1973 following earning a BS in Physics at Georgia Tech. He entered the industry as a Schlumberger field engineer in California and Alaska, staying with Schlumberger for five years. Following that, Dave returned to school and earned Masters' degrees in Physics and Earth Sciences at the University of Texas at Dallas, with further studies at UC Berkeley. In his career he has worked in one or another capacity at Arco, Sohio Research and operations, Mobil Research and operations, ExxonMobil Research and operations, Baker-Hughes, PathFinder, and Southwestern Energy. David has been an SPWLA member for 40 years and served as V.P. Publications and Editor of *Petrophysics* from 1999–2002, V.P. Technology in 2009, and President (etc.) from 2014–2016. David is inventor or coinventor on 6 U.S. Patents, and has published about 45 articles as author or coauthor in conference proceedings and refereed journals. His major research interest has been formation conductivity interpretation and electromagnetic logging instrumentation. David has been an educator all his life, having taught courses at the USMA at West Point, courses in computer science, physics, materials science, and electrical engineering at San Francisco Bay Area private and junior colleges. He is now a professor in the Petroleum Engineering Department at Texas A&M University instructing graduates and undergraduates in formation evaluation. Before all of that, David served as a Lieutenant of Infantry in Vietnam where he received both a Purple Heart and Bronze Star with V device.

Fredy R. García R., is an Exploration Petrophysicist Professional, grade I at Ecopetrol S.A., the largest petroleum extraction company in Colombia. In this position, Mr. García has had experience in the Geosciences and Exploration Department with openhole logging interpretation for all exploratory basins in Colombia and the Caribbean offshore, including the Gulf of México, Venezuela and the North Sea (United Kingdom). In addition, he has acquired expertise in problematic reservoirs: clean sands with heavy oil/fresh water, turbidites, tight gas sands, marls and shaly laminated sandstones.

Mr. García earned his BSc, in Petroleum Engineering at the America University Foundation in his native town of Bogotá, Colombia. He also holds a distinction degree for the MSc, in Petrophysics and Formation Evaluation at Aberdeen University, Scotland.

His professional experience has fluctuated between the academic and the technical world. In the first realm, he has been a TA for the course "Formation Evaluation and Well Logging Interpretation" at the National University of Colombia, a teacher for the course "Production Engineering and Open Hole Logging Interpretation" at the America University Foundation, a course developer in Basic and Intermediate Petrophysical Engineering and Oral Presenter and a visiting geoscientist for the AAPG. As part of his technical expertise, he has worked as a Development Petrophysicist in Gran Tierra Energy, a Reservoir Engineer and Petrophysicist in Vetra Exploration and Production, as a Well Completions Engineer at Schlumberger and a Production Engineer at Perenco.

An active member of the SPWLA and the AAPG, Mr. García has won the Imperial Barrel Award Competition of the AAPG with the exposition "Hydrocarbon Prospects in Cooper Eromanga Basin – Australia PEL Asset 106," as well as a professional recognition from Ecopetrol based on the petrophysical interpretation of oilfield discoveries in offshore basins of the Colombian northern coastline. He has also given oral presentations at SPWLA conventions in the United States and Colombia.

Like Brian May, BSc and ARCS in Physics of the Imperial College and lead guitar of the English rock group Queen, Mr. García also holds a special place in his activities for the study of the stars and music, being a member of the Colombian Association of Amateur Astronomers and a proficient drummer for different bands in Colombia and Scotland.