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Conductivity models for Archie rocks

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ABSTRACT

The petroleum industry's standard porosity-resistivity model (i.e., Archie's law), although it is fit for its purpose, remains poorly understood after seven decades of use. This results from the choice of the graphical display and trend formula used to analyze Archie's seminal porosity-resistivity data, taken in the Nacatoch sandstone, a petroliferous clastic formation in the Gulf of Mexico coastal area. Archie's model accurately predicts the conductivity-brine volume trend for this sandstone. Not all rocks follow the same porosity-resistivity trends observed in the Nacatoch sandstone, but those that do are defined as Archie rocks. Archie's Nacatoch sandstone data set has significant irreducible scatter, or noise. Data with significant scatter cannot be used to uniquely define a trend. Alternative graphical analyses of Archie's Nacatoch sandstone data indicates that Archie could have analyzed these data differently had it occurred to him to do so. A physics-based porosity-conductivity model, a "geometrical factor theory" (GFT), is preferred as an alternative to the Archie model because it has a physical interpretation. In this model, the bulk conductivity of an Archie rock is the product of three factors: brine conductivity, fractional brine volume, and an explicit geometrical factor. The model is offered in the form of a theorem, proved in three steps, to make our arguments as explicit and transparent as possible. The model is developed through its culmination as a saturation equation to illustrate that it is a complete theory for Archie rocks. The predictive power of the Archie model and GFT are similar, but unlike the adjustable parameters of the Archie model (m, n, and a), all of the parameters of GFT have a priori physical interpretations. Through a connection to site percolation theory, GFT has promise to connect porosity-conductivity interpretation to circuit theory first principles.

INTRODUCTION

Motivation

The so-called Archie law, relating the conductivity of a clay-free fluid-bearing rock to its porosity and brine volume, is an empirical formula describing a trend on bilogarithmic graph paper. Recent discussions (Freedman, 2009; Ransom, 2010) in a respected formation evaluation forum call attention to the important issue of whether most formation evaluationists understand the physics governing the relationship between water saturation and conductivity in Archie rocks. Paraphrasing one of Robert Freedman's remarks: "The fundamental problem with empirical equations is that reservoir rocks and fluids are too complex and variable to be accurately described by simple models; e.g., empirical equations a la Archie," Robert Ransom responds to Freedman in defense of Archie. Yet he observes, "after 67 years, many petrophysicists still do not understand Archie's relationships" and "at present, Archie's methodology is far from understood by the industry." One purpose of this paper is to obviate Freedman's concern, and also illuminate the physics of conductivity in Archie rocks, providing a remedy for Ransom's complaint. To understand conductivity in Archie rocks, it is necessary to begin by abandoning Archie's methodology as a *physical* explanation. We (Herrick and Kennedy, 1993, 1994, 2009) have published the basic tenets of the physics of conduction in Archie rocks in the past. However, we have continued to work on a pedagogy that will make our arguments as transparent as possible. This article, intended to serve both as a tutorial on conductivity in Archie rocks and a brief review of past attempts to explain it physically, is the most recent effort. It also connects conductivity in Archie rocks to the principles of site percolation theory and conduction in resistor networks.

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Outline

1) We define Archie rocks and list their characteristics.
2) Archie's data set has significant scatter, or noise. We explain the source of this noise and show that it is irreducible.
3) We use an analog problem to illustrate that data with significant noise cannot be used to uniquely define a trend.
4) This is followed by a similar analysis of Archie's Nacatoch sandstone data to show that Archie could have analyzed it differently had it occurred to him to do so.
5) We show that our industry's standard porosity-resistivity model (i.e., Archie's law) results from an arbitrary choice of the graphical display used to present Archie's seminal porosity-resistivity data set, taken in the Nacatoch sandstone. Archie's model works well for this sandstone and other rocks that share certain characteristics with the Nacatoch sandstone (e.g., an absence of surface condition).
6) We introduce and explain a physics-based porosity-conductivity model, which we have named geometrical factor theory, (GFT)[3]. The model is offered in the form of a theorem, proved in three steps to make our arguments as explicit and transparent as possible. The model is developed through its culmination as a saturation equation to illustrate that it is a complete theory for Archie rocks.
7) We compare GFT to the effective media approximations (EMA) that have been applied to petrophysics, showing that these EMA do not apply to Archie rocks. Finally, we show that the EMA based upon site-percolation theory and studied in 3D cubic lattices of conductors has the same formal structure as GFT. This correspondence suggests that GFT can ultimately be derived from the accepted tenets of circuit theory.

Figure 1. Scanning electron microscope image of a Fontainebleau sandstone rock sample (courtesy of Steve Bryant, University of Texas, Austin). The grain sizes range from 150 to 300 μm, but are well sorted. The absence of clay on the grain surfaces is important to the definition of an Archie rock. It is difficult to visualize what the pore space would look like if the grains were made invisible; however, it is a diaphanous network of brine that supports electric current.

Professor Paul Samuelson (Robinson and Treitel [1980], preface) has opined, "Short writing makes for long reading." In its role as a tutorial, this paper will eschew terse prose. A certain amount of redundancy can add to clarity. In its role as a review article, it will dwell on details that are well known to experienced professionals. We ask the forbearance of our more experienced colleagues in this matter.

Definition of an Archie rock

G. E. Archie invented (Archie, 1942) and coined the name for petrophysics (Archie, 1950). His articles describe the relationships among bulk resistivity, brine resistivity, porosity, and water saturation in Gulf of Mexico coast sandstones and other similar rocks. Archie's empirical model, in terms of conductivity,

$$\sigma_t = \sigma_w \varphi^m S_w^n / a,$$

adequately predicts water saturation S_w from measurements of porosity φ, bulk conductivity σ_t, and brine conductivity σ_w in such rocks. The model's predictions are optimized by a judicious choice of adjustable parameters in the form of exponents on porosity and water saturation, m and n, respectively. From an operational point of view, an Archie rock is any formation whose porosity-conductivity relationship is adequately described by Archie's model. What are the physical characteristics of these rocks? Figure 1 is a scanning electron micrograph of grains in an archetypical Archie rock. An Archie rock is water-wet and contains brine as the only conductive phase. It comprises nonconductive, approximately equidimensional grains with a simple unimodal intergranular pore system. Typically, these rocks are compacted and cemented. Over much of the porosity and water-saturation range, the three phases of nonconductive cemented rock-forming mineral grains, hydrocarbon, and conductive brine form continuous mutually interpenetrating connected networks.

In general terms of physical characteristics, rocks that are oil wet, or microporous, or contain moldic or fracture porosity, or conductive minerals (e.g., clay minerals, especially if distributed as a continuous phase) are not Archie rocks; their porosity-conductivity trends are not predicted by Archie's model.

Oil-wet rocks cannot be Archie rocks because for an oil-wet rock, the relationship connecting water saturation and resistivity index is disrupted. Archie rocks depend on grains coated by brine to conduct down to very low water saturations. When hydrocarbon displaces water on the grain surfaces, and the conducting phase is forced into the interiors of the pores and pore throats, the conducting phase disconnects at relatively high-water saturations, and the conductivity rapidly vanishes with increases in hydrocarbon saturation. This results in n values much greater than two, and a resistivity index-water-saturation relationship that is not a power law.

Background

Archie's law is entrenched in formation evaluation by 70 years of use. It is straightforward to apply and, being fit for its purpose and venerable, is not likely to ever be supplanted in practice.

[3]In our earliest work, we referred to GFT as "electrical efficiency" because it is the ratio of the actual conductivity of a rock to the conductivity of the brine in the pore space, removed and configured in its most conductive configuration, a tube. This ratio is the fraction of actual conductivity to the maximum possible conductivity for the amount of brine in the pore space, and thus, is the electrical efficiency of the brine in the pore space. The name change is to emphasize that this quantity is characteristic of the pore system in an Archie rock.

Nonetheless, how Archie's law, more accurately called a model, relates to the physics of conductivity, as our colleagues Freedman and Ransom have noted, remains poorly understood. Efforts to derive Archie's model from first principals, and interminable speculation over the meaning of its adjustable parameters, the so-called cementation exponent m and saturation exponent n (and since 1952, the so-called "tortuosity factor" a [Winsauer et al., 1952]), considering insights gained in the past few decades, ought to be brought to a close. Perhaps this can now be done. Why should something as seemingly simple as the bulk conductivity of a brine-wet Archie rock be so difficult to understand as to elude satisfactory explanation since 1942? In fact, such understanding is easily grasped, but to achieve it requires a reformulation of the problem.

Archie's industry-standard model relating the bulk conductivity of a rock σ_t to its brine conductivity σ_w, fractional brine volume φS_w, and geometrical distribution (related in some unspecified way to m, n, and a) of its conductive brine phase is $\sigma_t/\sigma_w = \varphi^m S_w^n/a$. The scalar quantities brine conductivity σ_w, porosity φ, and water saturation S_w are bulk properties of the brine and rock matrix and do not depend upon the geometrical arrangement of the pore space. On the other hand, the quantity σ_t does depend upon how the pore space is arranged; it is in fact a second-order tensor with three independent, direction-dependent components (Kennedy et al., 2001). Because neither σ_w, nor φ, nor S_w contain any geometrical information, in the Archie model, brine geometry information must necessarily reside in m, n, and a. The appearance of m and n as exponents on scalar factors make them unwieldy as tensors, and the tensor nature of the exponents has been largely ignored. On the other hand, historically, much effort in theoretical petrophysics has been focused on finding physical interpretations for these parameters.

Review of previous research

For three decades following Archie's discovery of an empirical model connecting conductivity, porosity, and water saturation, subsequent research focused mainly on characterizing its parameters in terms of rock properties (Guyod, 1944), refining its form (Winsauer et al., 1952), and extending it to accommodate rocks having a dispersed clay component (Wyllie, 1954; Waxman and Smits, 1968; Clavier et al., 1977, 1984). Indeed, similar studies on the relationship of exponents to grain shape, orientation, and texture (Montaron and Han, 2009) and the proper interpretation of the adjustable parameters (Glover, 2009) of the Archie model continue to this day. In clay-free rocks, the predictions of the Archie model are of such robustness that its appropriateness as a *physical* explanation has not been questioned.

From the late 1950s, physicists were developing the precepts of percolation theory (Broadbent and Hammersley, 1957), fractal geometry (Mandelbrot, 1977), and their applications in effective media approximations in pursuit of understanding certain emergent properties of condensed matter and other composite systems. Kirkpatrick (1973) applies percolation theory to 3D cubic arrays of resistors in computer simulations; Shankland and Waff (1974) validate Kirkpatrick's models in simple cubic arrays of physical resistors along with a discussion of how the results might be relevant to conductivity in rocks. Madden (1976) studies "embedded networks" (i.e., fractal or "self similar") and found that "... the universality of Archie's law appears to be more an accident of ... crack and pore width distribution functions than a fundamental property of porous media."

Prior to the 1980s, we are unable to find an attempt to derive Archie's law from first principles. Sen's (1980) SPE article includes a section titled "Derivation of Archie's law from first principles"; however, the derivation proceeds from Bruggeman's asymmetrical mixing rule (Hani, 1968). The Bruggeman mixing rule is derived for an emulsion of resistive spherical inclusions in a conducting matrix, and the power law that is derived assigns Archie's $m = 3/2$. Although this exponent value is in experimental agreement for the artificial medium of fused spherical glass beads, it is far from the $m \approx 2$ observed in conventional reservoir rocks.

Sen et al. (1981) continued work on self similar (i.e., fractal) networks to study the complex dielectric properties of shaly rocks. Sen et al. (1981) review the progress of applying percolation theory to transport in rocks through 1981; they conclude "... percolation theory ... needs to be extended ... before it can be applied to ... rocks." Sen et al. (1981) inaugurated further efforts to derive, or at least explain, Archie's law from fundamental principles. Representative articles in this genre are Balberg (1986), Roy and Tarafdar (1997), and Hunt (2004). Hunt (2005) provides an exhaustive review of attempts to blend percolation theory with fractal dimensionalities to predict the exponents of Archie-like power laws for porosity-conductivity relationships.

To date, none of these and similar attempts to connect Archie's law to first principles have met with complete and unqualified success. This lack of success suggested to us that it is possible that there is no fundamental significance to the Archie formula; it is a useful trend analysis tool and that is all. This idea is elaborated in our SPWLA *Transactions* article (Herrick and Kennedy, 2009), and will be set forth below in theorem form, together with arguments to support and illustrate the theorem. We further show that our result is consistent with the tenets of percolation models, and explains the nearness to two of the Archie porosity and saturation exponents.

Conductivity models: Brief history

The development of the important models describing (and predicting) the relationship of brine conductivity, matrix conductivity, and bulk conductivity to porosity and water saturation in conventional reservoir Archie rocks is summarized in brief by the sequence: "Archie precursors" (1936–1938); Archie's (1942) model; Winsauer et al.'s (1952) Humble formula. The four Archie precursors, viz., Wyckoff and Botset (1936), Jakosky and Hopper (1937), Leverett (1938), and Martin et al. (1938), are cited by Archie (1942) as the source of the resistivity index (RI) data found in his paper. Archie synthesizes $\mathrm{RI} = S_w^{-n}$ from the data and figures to be found in these four papers; he then defines a "formation resistivity factor," $F \equiv R_0/R_w = \sigma_w/\sigma_0$. Archie observed a relationship between F and porosity that he describes as $F = \varphi^{-m}$ and combines the formation resistivity factor and resistivity index to complete his model.

The modern notation for Archie's model for brine-filled rocks is $F \equiv \sigma_w/\sigma_0 = \varphi^{-m}$. The model proposed by Winsauer et al. (1952) (i.e., $F = a\varphi^{-m}$) amounts to the inclusion of a second adjustable parameter in the Archie model, making it a more flexible description of resistivity-porosity trends. Winsauer's assignment of values for the adjustable parameters is $m = 2.15$ and $a = 0.62$. Other investigators (Porter and Carothers, 1970; Carothers and Porter, 1971) emulate Winsauer's approach and study other collections of rock samples, each of which yield different values of a and m. As developed by Archie, the equation does not contain a as an explicit adjustable parameter. Presumably, this was due to the requirement that when porosity is 100%, $F = 1$. Archie's Nacatoch sandstone happened to satisfy this requirement almost perfectly. If the value of F is found to be other than one at 100% porosity, then the samples under investigation are either not "Archie rocks" or represent a mixture of rock types due to inappropriate sampling.

The values of the adjustable parameters picked by Winsauer et al. (1952) for the Humble equation reflected a straight-line fit (on bilogarithmic graph paper) to the formation resistivity factor–porosity plot of their particular data set. Unlike Archie's Nacatoch sandstone data from a single formation, Winsauer's samples were drawn from a few widely separated (in space, in time, and in provenance) locations and contained different kinds and combinations of cementing minerals. In modern practice (since the 1960s), a and m are taken as adjustable parameters, to be determined for each individual data set, ideally confined to a single formation. The formula $F = a\varphi^{-m}$ is almost always referred to as "Archie's law" in contemporary discussion, with both parameters adjustable, as needed, to fit particular data sets. The credit for Winsauer's et al. (1952) contribution has been lost in the modern formation evaluation lexicon.

TREND ANALYSIS

Physics and trend analysis

In electromagnetic field theory, Ohm's law takes the form of a constitutive relation between current density $\mathbf{J}$ and electric field $\mathbf{E}$ in which $\mathbf{J} = \sigma^* \mathbf{E}$, where the proportionality constant is the conductivity of the medium. The conductivity can be frequency- and dielectric-constant-dependent through $\sigma^* = \sigma + j\omega\varepsilon_0\kappa$, where κ is the dielectric constant, and σ is the zero-frequency conductivity, of the medium; ε_0 is the electric permittivity of free space, ω is angular frequency, and $j = (-1)^{1/2}$. This paper deals only with the low-frequency limit σ. It is clear how to apply this formula to a homogeneous and isotropic medium. In a medium comprising different immiscible materials in varying amounts and arrayed in varying arrangements, it is necessary to estimate the effective conductivity of the composite σ and also to predict changes in σ in response to changes in the amounts, conductivities, and arrangement of the components, or phases. Two methods of estimation are available: direct measurement and theoretical analysis. Each method has limitations. Trend analysis of direct measurements can establish formulas relating variations in conductivity to variations in the phases, but these, being empirical, do not satisfy the desire of physicists to understand the relationships at a fundamental level. On the other hand, physical analysis of a medium as geometrically complicated as the connected pore system of a rock defies direct analytical analysis; resort to simplified models and approximations is required. Early in the history of petrophysics, the industry settled on the particularly useful trend analysis formula that is known as Archie's law. For at least the past 35 years, efforts have been made to derive the empirical formula from first principles, but without significant success. We assert that this lack of success results from the absence of a physical basis for the industry-standard, but empirical, trend analysis model discovered by Archie.

Graph paper

Modern trend analysis relies on specialized computer programs (e.g., Microsoft Excel) running on the now ubiquitous personal computer. However, most modern engineering developments have roots reaching into the twentieth, and even the nineteenth centuries, and trend analysis is no exception. The graphs that now effortlessly appear on computer displays were created, until recently, by hand on standardized preprinted graph paper. The most common types of preprinted paper were linear, semilogarithmic (i.e., one axis scaled logarithmically), and so-called log-log (i.e., both axes scaled logarithmically) bilogarithmic paper. Prior to the universal availability of the personal computer, everyone working with experimental data kept a supply of preprinted graph paper in a desk drawer.

One of the first steps in the analysis of unfamiliar bivariate data was (and still is) to observe its graph plotted in various ways (e.g., on each of the commonly available preprinted graph papers) to identify any obvious trends. When data tend to fall on a straight line on any of the standardized plots, a functional form for the trend is known immediately. Predictions of the results of new observations can be made from the trends observed. Formulas based on fitting mathematical functions to such trends can be useful and reliable predictors, and deeper analysis is often unnecessary in engineering applications.

Irreducible scatter

Real data are imperfect. The cause is the imperfectability of measuring instruments and techniques on the one hand, and the variability of the physical world on the other. Each cause results in data that, when plotted, do not fall exactly upon a uniquely determined curve. Instruments and techniques have improved with time, reducing experimental error. However, variability in the physical world cannot be eliminated. The variability observed in porosity-conductivity data is partly due to sampling (e.g., the size and orientation of a core plug) and partly to natural variability in the rocks under investigation. Only a very small component of scatter is due to measurement error. In this view, the observed scatter in conductivity at constant porosity would follow a bell-shaped distribution were enough samples measured to see the distribution develop. In practice, there are rarely enough samples at the same porosity to construct a proper histogram and observe the bell-shape clearly; the location of the central tendency and the standard deviation can only be estimated from an undersampled distribution with an attending uncertainty, manifested as irreducible scatter in plots of the data. Thus, more than one function can be used to correlate the dependent variable to the independent variable with comparable efficacy (e.g., practically equal coefficients of determination, R^2). This lack of uniqueness in trend analysis presents a pitfall if one is attempting to relate a trend model to physical first principles.

To illustrate these points, an example is useful. The concept of a geometrical factor for pore geometry is formally developed in following sections; for this example, we call upon our reader's intuitive understanding of pore geometry. Figure 2 schematically depicts cross sections through several model pores having similar porosities, and geometries varying in orientation but not shape. The model pores are shown as cross sections through the centers of unit cells in a loose array of cubic packed spheres. A potential difference is applied from right to left across the pore. The conductance of the cell (i.e., the amount of current that is induced) depends on its orientation (one aspect of pore geometry) with respect to the applied potential. Cells of equal porosity are shown with their axes varying from parallel to the applied electric field to fully oblique (i.e., rotated 45°) with respect to the applied electric field; the geometric factor varies from a maximum to a minimum as the cell rotates.

Likewise, cells of equal geometrical factor are shown as their porosity varies from lowest, with axis parallel to the applied field, to highest, with axis most oblique to the applied field. Similarly, in Archie rocks, real pore systems having identical porosities can, and do, have distributions of pore geometries and conductivities. Likewise, identical pore geometries can, and do, have distributions of porosity. Thus, whatever the underlying relationship, or trend, of pore geometry to porosity may be, scatter of points around the trend will be observed. This scatter will be reflected in measured porosity-conductivity data; it is unavoidable and irreducible.

An analog problem

It is difficult to construct meaningful physical models having continuously variable porosity with concomitant changes in pore geometry and conductivity. Thus, to further illustrate the discussion of trend analysis in porosity-conductivity data, it is helpful to consider a more tractable analog model having a similar mathematical form. Consider the vertical displacement of a point mass falling freely in a constant gravitational field. Its displacement is described by

$$s = s_0 + v_0 t + g t^2/2, \tag{1}$$

where s is vertical displacement after a time t, s_0 and v_0 are initial conditions representing the displacement and velocity at $t = 0$, and g is the acceleration of gravity, equal to about 9.8 $\mathrm{m/s^2}$ taken to be approximately constant and independent of altitude for small displacements. With known values of s_0 and v_0, the displacement of the point mass can be calculated for convenient values of t. In this model, time will be a surrogate for porosity, which ranges from ~0.10 to ~0.40 in Archie rocks, so the values of time are chosen as $0.10 \leq t \leq 0.40$ seconds. Displacement, the surrogate for conductivity, is calculated at 0.01-s intervals; noise, or scatter (Appendix A), is added to the resulting displacements to simulate variation in conductivity in the analog model at constant porosity. Plots of displacement versus time are analogous to plots of conductivity versus porosity; plots of reciprocal displacement versus time are analogous to plots of resistivity versus porosity. Note that for $s_0 = 0$ and $v_0 = 0$, $s = (g/2)t^2$ is analogous to $1/F \equiv f = a^{-1}\varphi^2$; thus, for these parameter values, the analogy has a one-to-one correspondence with the Archie model of the porosity-conductivity relationship.

Figures 3 and 4 are plots of synthetic data, with noise added, modeled upon the freely falling point mass having an initial displacement of $s_0 = -0.0122$, and an initial velocity of $v_0 = -0.1463$; the remaining parameter is $g/2 = 4.9$. The reciprocal displacement data plotted in Figure 3a displays a curvature upward to the left and are fitted using a hyperbola; Figure 3b shows the plot is semilog with time on the linear axis and a trend that appears to be a straight line with the data fitted using an exponentially decaying function. In Figure 3c, the plot is also semilog with time on the logarithmic axis and is fit using the right limb of a Gaussian function. In Figure 3d, the plot is bilogarithmic with a linear trend suggested, fitted using a power law.

Figure 3e through 3h illustrates the reciprocals of same data plotted on the same graphs, except that the displacement s is plotted as a function of time. Note that the plot in Figure 3e graphs the fit of a second degree polynomial to the scattered data. The generating function is, of course, a second-degree polynomial, with initial conditions as shown on the plot. The fitting functions in Figure 3f–3h mirror those used in Figure 3b–3d to describe the trend. All of these functions are good predictors of the trends. These fitting functions use from two to four adjustable parameters. Only the three adjustable parameters in Figure 3e have direct physical interpretations because they directly correspond to the underlying function.

These plots serve to illustrate two points. First, when scattered data tend to cluster near a single line on one of the graphs, a trend function is thereby suggested; this is what makes the preprinted paper useful. Second, the representation of trends in the data can be accomplished with comparable efficacy using any number of functions. The functions illustrated in Figure 3 do not come close to exhausting the possibilities.

In Figure 3, the coefficients of determination vary from 0.84 to 0.94, with the highest correlation matched to the fitting equation that corresponds to the underlying model. However, the particular data set chosen for presentation here was chosen from among many data sets, generated at random for this study, precisely because the coefficients of the fitting polynomial more or less correspond to the parameters in the underlying model. This serves to make a connection for the reader between the underlying deterministic model and the data set with random noise added.

To illustrate this point, compare Figure 3e, with a fitting curve that corresponds to the underlying model, and Figure 3d, with a power-law-fitting curve similar to Archie's law. Figure 4 showcases a different model data set with coefficients of determination (i.e., R^2) so similar that one could not attribute much statistical significance to their small difference. It is easy to see why the power law might be preferred, especially if the data set itself is a table of time and reciprocal displacement; there would be little motivation for plotting time versus displacement. Note that the estimates of g,

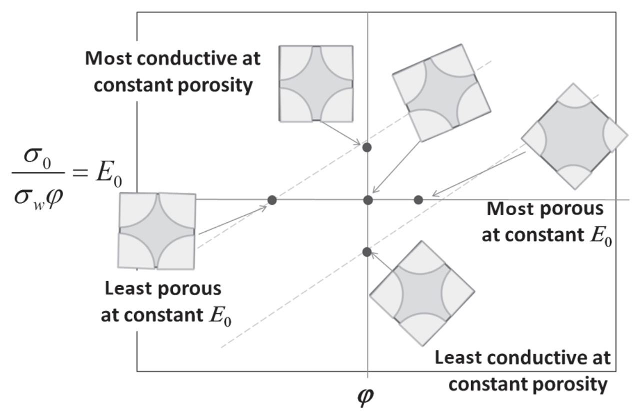

Figure 2. Illustrating sources of irreducible scatter in porosity-conductivity data. The sketch depicts variously oriented pores under the influence of an electric field applied from right to left. Three *identical* pores (vertical line of constant porosity) are shown in orientations having the maximum, the minimum, and an intermediate conductivity due to their orientations varying from fully parallel, to intermediate, to fully oblique, to the applied electric field. This illustrates that pores with the same porosity can have differing conductivity. Similarly, pores having the same pore geometry parameter E_0 can have differing porosity, depending upon whether their pore throat axes are oriented parallel to, or oblique to the applied electric field, as illustrated by the three pores of constant pore geometry. The result of this variability is that data are scattered over a region of the graph — no function is uniquely suggested by the cloud of points.

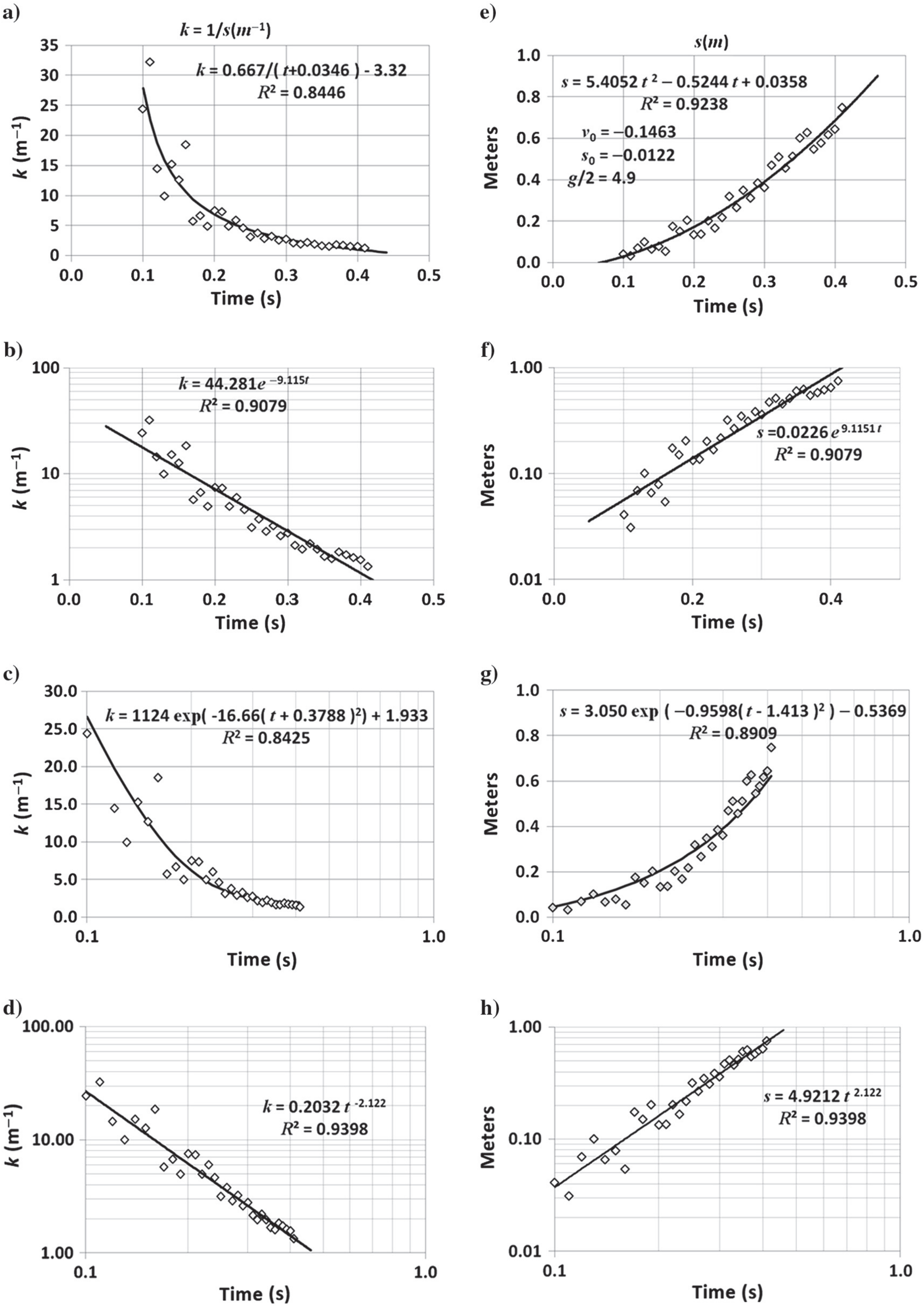

Figure 3. A data set based on the displacement of a freely falling body with time, with scatter added. The data are plotted in eight different ways, based on standard graph paper: linear, log-linear, and log-log, with the data taken as displacement (right-hand column of graphs), and the reciprocal of the data (left-hand column). Historically, a data set might be plotted in some, or all, of these ways in search of a trend. The lower-left graph (d) might seem to the eye to display the best-fitted trend, a power law, and it *is* a good fit to the data. However, the upper right (e) is the actual function (plus noise) used to generate the data. Note the values of the adjustable parameters are not particularly close to the true values of the initial conditions but at least are estimates of them. Compare those with the adjustable parameters in (d) and imagine how these numbers, analogs of a and m, might relate to s_0, v_0, and $g/2$.

v_0, and s_0 in the polynomial model are of very low quality. Based on our experience with these models, this is the usual situation with data containing significant scatter. This points to a pitfall: even when the underlying function is known exactly, estimates of these parameters from scatter-prone observations are highly uncertain.

These examples are intended to illustrate the folly (from a theorist's point of view) of settling upon *one* of the representations, and its adjustable parameters, as having physical significance without having some a priori notion, derived from a priori physical analysis, of the functional form to be expected.

Support from history

Archie's research relied upon resistivity index data published in four studies from 1936 to 1938. Wyckoff and Botset (1936) publish their resistivity index data as a plot of water saturation versus conductivity on a linear scale. Jakosky and Hopper (1937) publish their data in a table, and offer the tabulated data as a linear plot of resistivity versus water saturation, and a second plot of the data on semilog paper with resistivity shown as an approximately exponentially decaying function of water saturation. Martin et al. (1938) publish their resistivity index data as two curves plotted on semilog paper with water saturation scaled linearly and resistivity scaled logarithmically. Leverett's (1938) data are plotted as water saturation versus conductivity on a linear scale. It was left to Archie to synthesize all of these data into straight lines on a bilogarithmic plot. This illustrates that every kind of commonly used graph paper is featured in the evolution of Archie's model as attempts in a data-fitting exercise. The power law is the simplest fit to the data. On the other hand, the conductivity-water-saturation plots from Wyckoff and Botset (1936) and Leverett (1938) are well-defined parabolas, with only minor scatter in their data. We will argue that the underlying physical relationship is best represented by conductivity plotted as a function of water saturation on linearly scaled paper.

Archie's data

Archie (1942) presents two data sets. One set, in which Archie called "Gulf Coast sandstones," is a collection of core data from multifarious core samples from the onshore Gulf of Mexico region; the second set, the Nacatoch sandstone, features core samples from a single formation taken in a single field near Bellevue, Louisiana. The raw data available would have been a table of core plug porosity-resistivity pairs. (Archie shows only plots of his data — there are no tables in Archie's 1942 paper. The data used in this paper are hand-digitized from Archie's plots [Kennedy, 2007]). Archie chose to plot these data as resistivity versus porosity on bilogarithmic graphs; the trends that he observed are linear on those plots. The trend for the Gulf Coast sandstones, which are unrelated by provenance, is only loosely correlated; the trend for the Nacatoch sandstone from cores collected in a single oil field has a formation that is more tightly correlated, and suggestive of a straight line in the bilogarithmic space of the plot. However, when plotted as conductivity versus porosity on linear graph paper an equally strong (i.e., equal coefficients of determination) quadratic trend suggests itself. These plots are shown in Figures 5 and 6, analogous to Figures 3 and 4.

The curve-fitting in Figure 5 illustrates that the Nacatoch sandstone data set can be fit by a hyperbola (Figure 5a), a decaying exponential (5b), the right limb of a Gaussian bell (5c), and a power law (5d). Figure 5e–5h mirrors the fits in Figure 5a–5d in the reciprocal space. Figure 5e is important. It shows that the formation conductivity factor f can be fit with a quadratic polynomial in φ with the same degree of confidence as the power law in Figure 5d fits the resistivity representation. Thus, when making a choice between a power law and quadratic-fitting functions, we should bring to bear criteria beyond how close R^2 is to unity.

Note that the coefficient of the power law in Figure 5d is $0.9706 \approx 1.0$, and the exponent is $2.023 \approx 2.0$. Thus, Archie found $R_0/R_w \equiv F \approx \varphi^{-2}$. This is generalized by Archie to $F = \varphi^{-m}$ (and later, beginning with Winsauer et al.'s (1952) Humble formula $F = 0.62\varphi^{-2.15}$, finally evolving to $F = a\varphi^{-m}$, with parameters chosen to minimize residuals in each particular data set). A physical basis for this relationship, or at least the physical significance of the adjustable parameters appearing in the power law, have been sought — without notable success — since 1944.

In Figure 6, the polynomial and power-law fits are displayed in the same plots for easy comparison. The plots are convincing evidence that there is no a priori reason to choose the power law as a better representation than the quadratic. In fact, there are many equally effective fitting functions that can be applied to these data (see Appendix B). So, is there a reason why the quadratic should be preferred, at least for theoretical purposes? Indeed, there is.

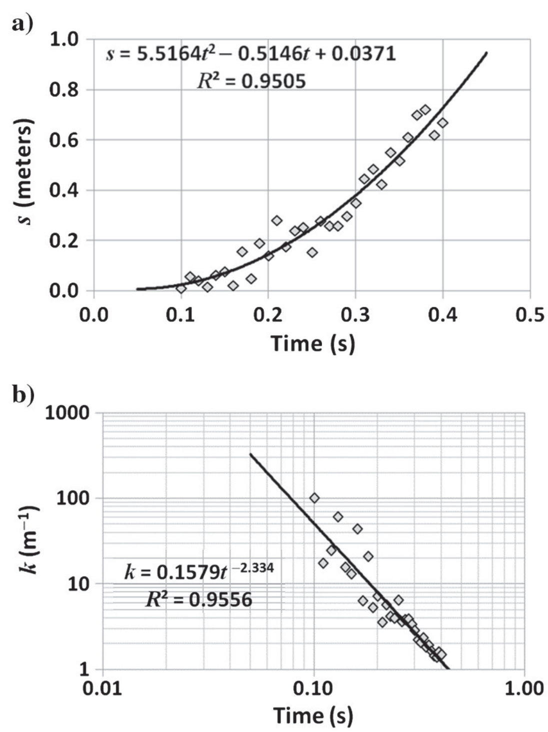

Figure 4. A second instance of the displacement versus time analog problem with random scatter added. The example is chosen because the coefficients of determination are comparable. This example, using the same initial conditions as in Figure 3, illustrates the difficulty in deducing underlying physics from raw data. The fitting coefficients in (a) would be hard to associate with the underlying actual values regardless that the fitting function is identical with the generating function. The trend in (b) is suggested at least equally, if not more, strongly. If there is difficultly in assigning physical significance to the adjustable parameters in (a), where they correspond to known physical quantities, it is folly to make the attempt in (b) where the fitting parameters (analogous to the Archie law a and m) have no a priori physical interpretation.

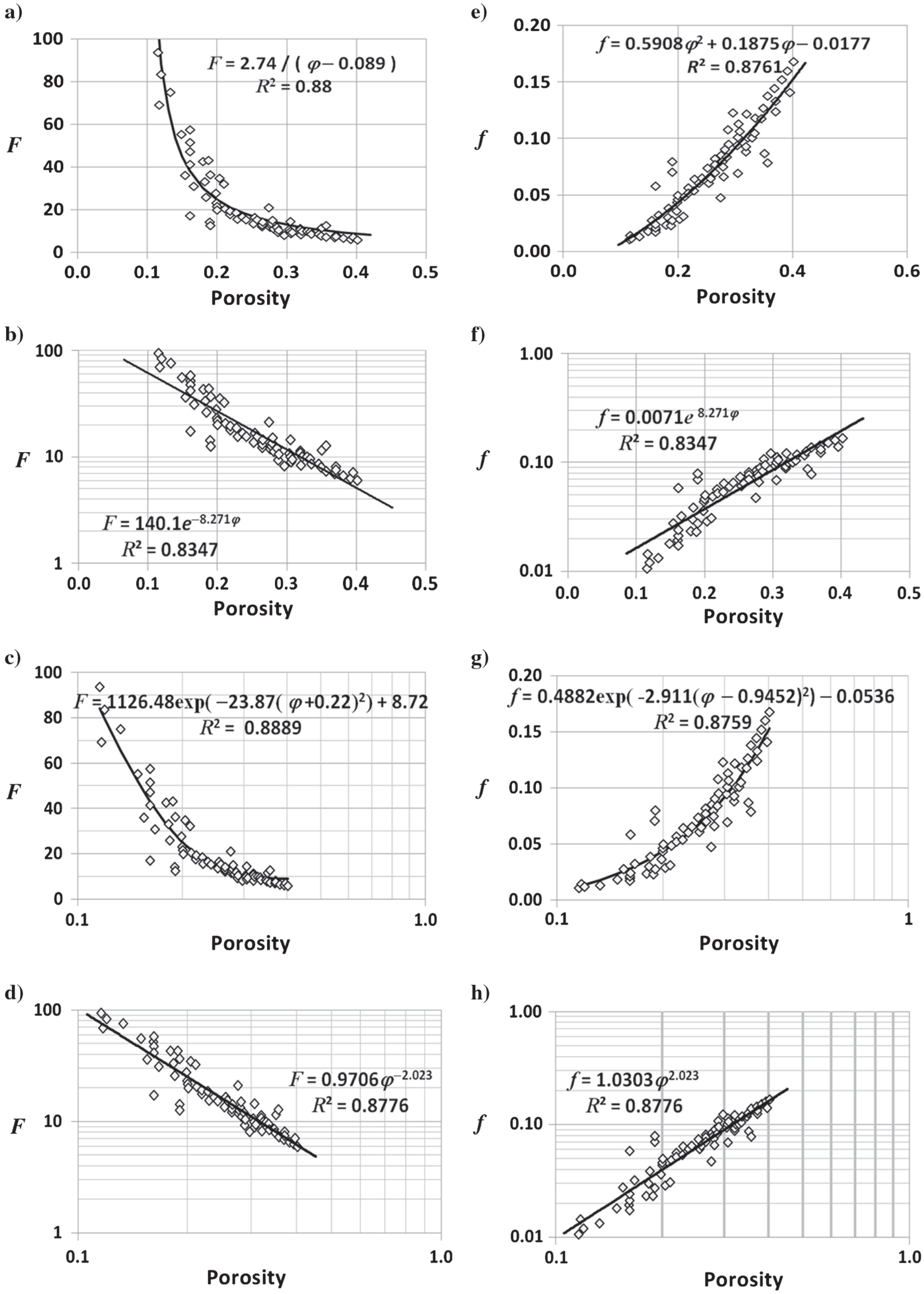

Figure 5. Archie's Nacatoch sandstone data plotted in various ways. Formation resistivity factor (F) data are plotted in the left column; formation conductivity factor data (f) are plotted in the right column. There are 72 porosity-conductivity pairs. Strong trends exist regardless of how the data are plotted. The choice of fitting function is arbitrary. Shown here are (a) a hyperbola, (b) a decaying exponential, (c) right slope of Gaussian bell, (d) a decreasing power law, (e) a quadratic, (f) increasing exponential, (g) left slope of Gaussian bell, and (h) an increasing power law. Archie chose (d) as his model. The geometrical factor model is plotted in (e). These cases hardly exhaust the possibilities, and several additional examples are shown in Appendix B.

CONDUCTIVITY IN ARCHIE ROCKS

A theory for conductivity in rocks can be developed from first principles. We present a theory in the form of a theorem, which we prove. This form is chosen to facilitate critiques from our colleagues and readers. In our model, we envision end-to-end conduction under the influence of an electric field applied parallel to the axis of a sample in the form of a core plug. Note that this common experimental set up has the geometry of a tube, with average current density and electric field parallel to the axis of the plug, and average equipotentials perpendicular to the axis. Of course, at a microscale, these quantities will vary in direction, but the tube geometry lends itself to tube models. The current in this model can be partitioned into an arbitrary number of nonintersecting (and noninteracting) approximately lenticular stream tubes conducting in parallel through the sample. This feature applies generally to any conductivity model in Archie rocks.

Theorem: The bulk conductivity of a brine-saturated Archie rock σ_0 is the product of three factors: brine conductivity σ_w, brine volume φ, and a factor E_0 describing the dependence of the bulk electrical conductivity of the rock on the spatial distribution of its interstitial brine phase.

Proof:

Factor 1.

***Axiom*: Brine conductivity σ_w.** Asserted as self-evident: the bulk conductivity of an Archie rock is directly proportional to the conductivity of its interstitial brine with constant of proportionality one; e.g., doubling the conductivity of brine will double the conductivity of the bulk rock, and so on.

Factor 2.

***Lemma*: Fractional brine volume φ.** In Archie rocks, bulk conductivity is proportional to fractional volume of interstitial brine.

Background: In an Archie rock, any change in the fractional volume of brine due to diagenesis or hydrocarbon emplacement is invariably accompanied by a concomitant change in brine geometry. This coupling of fractional brine volume and brine geometry contributes to confusion regarding the individual roles of these components. Given a means to vary fractional brine volume while holding brine geometry constant, and vice versa, these components can be studied separately. The decoupling is easily studied in thought experiments (Figure 7). In the absence of evidence to the contrary, the results of the thought experiment can be extended to Archie rocks.

Proof: Consider a cylindrical volume (to represent the volume of a core plug) containing a single through-going brine-filled channel. The direction and diameter of the channel can vary along its length, but the geometry and volume fraction of the channel in the cylinder is fixed for further use in the thought experiment. Figure 7a shows a simple case of a constant diameter channel having variable direction. The bulk conductivity of the cylinder is determined by the conductance of the channel and the dimensions of the cylinder. If the volume fraction is φ and the brine conductivity is σ_w, then the bulk conductivity would be described by $\sigma_{\mathrm{tube}} = \sigma_w \varphi_{\mathrm{tube}}$ if the channel and brine geometry were a through-going cylindrical tube. For the variable geometry tubes in Figure 7a and 7c equality no longer holds, but the bulk conductivity remains proportional, of course to the brine conductivity, but also to fractional brine volume. That is, for a single through-going channel

$$\sigma_1 \propto \sigma_w \varphi_{\mathrm{ch}}, \tag{2}$$

where φ_{ch} is the fractional volume of the channel. Consider next a similar cylinder containing two identical channels (Figure 7b and 7d). Obviously the fractional volume occupied by brine, and the current carrying capacity of the core plug, increases by a factor of two, and the pore geometry is the same for each channel. For the thought experiment, just as obviously, the bulk conductivity must also increase by a factor of two. Thus,

$$\sigma_2 \propto (\sigma_w \varphi_{\mathrm{ch}} + \sigma_w \varphi_{\mathrm{ch}}) = \sigma_w(\varphi_{\mathrm{ch}} + \varphi_{\mathrm{ch}}) = 2\sigma_w \varphi_{\mathrm{ch}}, \tag{3}$$

or, for two identical channels, $\sigma_2 \propto 2\sigma_w \varphi_{\mathrm{ch}}$. This follows from the two channels being electrically in parallel, and from the imposed condition that their geometry be the same. Similarly, addition of n such channels results in $\sigma_n \propto \sigma_w n \varphi_{\mathrm{ch}} = \sigma_w \varphi$, where the total porosity $\varphi = n\varphi_{\mathrm{ch}}$. Thus, recalling that φ is the fractional volume of the interstitial brine, and identifying σ_n with σ_0, the lemma is proved.

For the thought experiment, increasing the fractional volume of brine by incremental addition of channels increases the bulk conductivity by the same fractional increment. Other, arguably more realistic, thought experiments can be constructed that also confirm this deduction (see Figure 7e and its caption). Thus, it is demonstrated that for Archie rocks having fixed-pore geometry, but variable porosity, the bulk conductivity is proportional to the fractional volume of the conducting phase; i.e., porosity φ when $S_w = 1.0$, or φS_w when $S_w < 1.0$.

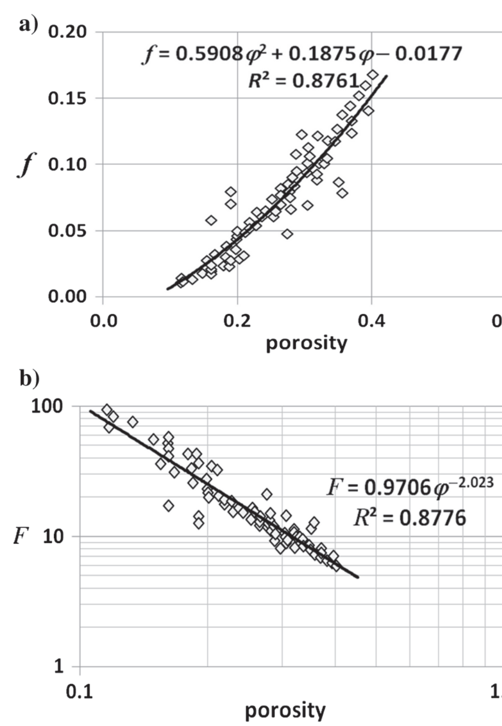

Figure 6. The upper-right (5e) and lower-left (5d) graphs from Figure 5. Insofar as possible, scales are chosen to be similar to those in Figure 4. Note the values of R^2 in (a) for the power law and polynomial, are almost identical. The graph in (b) shows that for Archie's Nacatoch sandstone data the coefficient is approximately 1.0 and the exponent approximately 2.0.

We claim that this proportionality self-evidently continues to hold (how could it not?) in real Archie rocks regardless that changes in pore volume are always accompanied by concomitant changes in pore geometry.

Factor 3.

***Definition*: Brine geometrical distribution.** If porosity exists as isolated inclusions, without an end-to-end connected channel, the bulk conductivity of a sample is identically zero regardless that $\varphi > 0$. If the porosity is distributed as cylindrical tubes with axes parallel to the applied electric field, then — as has been shown — $\sigma_0 = \sigma_w \varphi$ (see Figure 8). For any through-going connected pore network typical of Archie rocks, the bulk conductivity must be greater than zero and less than $\sigma_w \varphi$. This implies that there exists a multiplicative factor E_0 in which $0 \le E_0 \le 1$ that describes the proportionality of bulk conductivity to $\sigma_w \varphi$ for each particular pore network.

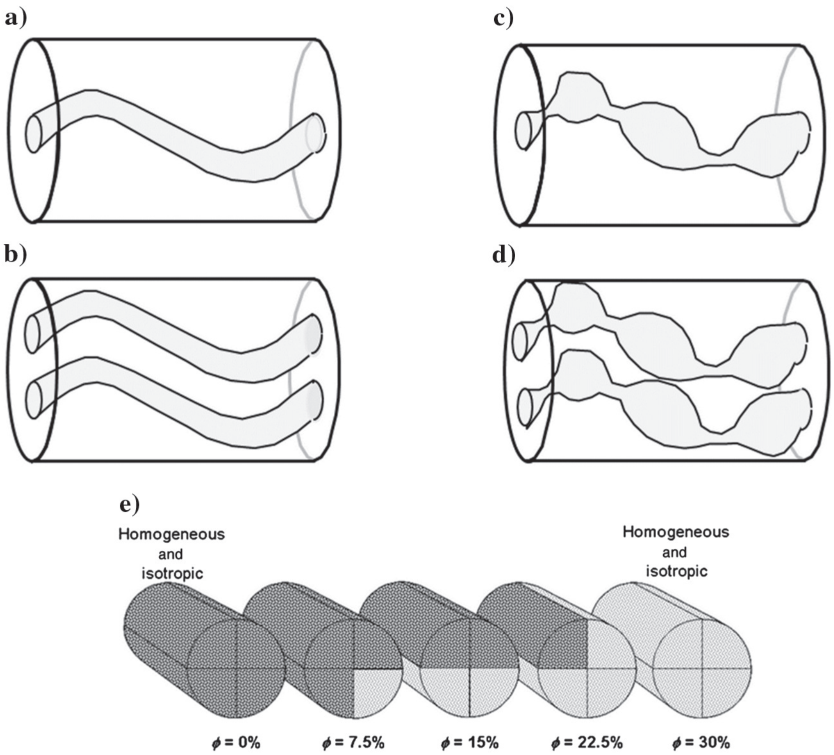

Figure 7. In real rocks, porosity cannot vary without a concomitant variation in pore geometry. This fact has hindered the recognition that, for constant pore geometry, conductivity is directly proportional to porosity. The three thought experiments illustrated in the figure and explained below show that, for fixed for geometry, conductivity is proportional to porosity. (a) A model rock comprising a single sinuous end-to-end channel containing all the porosity in the specimen. (b) A model rock similar in every way to the top model except that the sinuous channel is duplicated. Obviously, in a model like this, the conductivity will be proportional to the number of channels and hence to the porosity. (c and d) These models are similar to (a and b) except that the channel is more complicated, but the argument that the total conductivity is the sum of the conductivity of each channel, and therefore porosity, continues to hold. (e) A final thought experiment uses an arguably more general model rock. Consider two kinds of identical quarter cylinders that can be assembled into a model core plug. One of the quarter-cylinder segments has zero porosity; the other has 30% porosity; the pore system in each quarter cylinder is exactly the same and comprises an Archie pore system. Putting together the model quarter cylinders in the five assemblages shown progressively increases in porosity and conductivity, left to right, by the same fraction (i.e., $1/4$) of the final porosity and conductivity. The conclusion is that for fixed-pore geometry, conductivity is directly proportional to porosity, is thus established.

Thus, the relationship between bulk conductivity, brine conductivity, fractional brine volume, and brine distribution can be expressed as a product of three factors:

$$\sigma_0 = \sigma_w \varphi E_0. \tag{4}$$

Q.E.D.

It is interesting to note that this relationship can also be deduced by the same process that Archie used to define his formation resistivity factor. Archie removed the dependence of rock conductivity on the interstitial brine conductivity by dividing it out, $F = R_0/R_w$, where F is a property of the rock, but it contains porosity and pore geometry. In terms of formation conductivity factor, this relation is expressed as $f = \sigma_0/\sigma_w$. The next obvious step, not taken by Archie, would have been to also divide out the effects of brine volume leaving only the effect of brine geometry, $E_0 = \sigma_0/\sigma_w\varphi$. In other words $F = (E_0\varphi)^{-1}$, which shows explicitly how the formation resistivity factor depends separately upon pore geometry and porosity.

DISCUSSION

Brine-saturated Archie rocks

Because E_0 is called a "geometrical" factor, ideally one would expect to see it parameterized in terms of geometrical quantities; e.g., pore dimensions such as length, volume, coordination numbers, etc. In fact, the geometrical factor E_0 can be directly derived in this way for some very simple model pore networks (Herrick and Kennedy, 1993 1994). No real pore system of interest (e.g., an oil reservoir comprising Archie rocks) can be described in this way. Direct calculation of E_0 by numerical modeling, while possible in principle, is intractable for real rock samples. Thus for any particular rock sample the geometrical factor is expressed in terms of macroscopically observable quantities σ_w, σ_0, and φ. Because σ_w, σ_0, and φ are all readily measurable,

$$E_0 = \frac{\sigma_0}{\sigma_w \varphi} \tag{5}$$

gives the dependence of a single sample on its brine (or pore) geometry. In spite of appearances in eqaution 5, E_0 for each single sample is by its construction independent of porosity and conductivity. Dividing σ_0 by σ_w cancels the linear dependence of the quotient on σ_w; dividing σ_0 by φ cancels the linear dependence of the quotient on φ; we have proved that the dependence of σ_0 on the factors σ_w and φ is linear; therefore, the quotient $\sigma_0/\varphi\sigma_w$ contains only information on the geometrical distribution of the brine phase. The single number for each sample E_0 characterizes the low-frequency electrical effects of the geometrical distribution of the conducting brine connected through an enormous number of pores and pore throats. It is analogous to temperature, a single number that represents the average behavior of an enormous number of independently moving particles.

Due to the nature of the geologic processes that produce and modify detrital sedimentary rocks, a change in porosity necessarily is accompanied by a change in pore geometry. In other words, $dE_0/d\varphi \neq 0$. But if not zero, then what? The simplest assumption is that $dE_0/d\varphi = a_0$, where a_0 is a constant > 0 (i.e., geometrical factor and conductivity increase with an increase in porosity). With this assumption, $E_0 = a_0\varphi + b_0$, where b_0 is an integration constant whose value is either inferred from boundary conditions or observed directly by linear regression on a data set. Because a_0 represents the part of E_0 that changes as porosity changes, and b_0 represents the part of E_0 that is independent of porosity, based upon experience with Archie rocks, b_0 would be expected to be small.

We can go further if we examine the behavior of brine-mineral mixtures falling outside the porosity range of Archie rocks by extrapolating the trend to maximum and minimum porosity values, namely, 1.0 and φ_ϑ, respectively. These are the boundary conditions alluded to above. The two boundary conditions are: when $\varphi = 1.0$, $E_0 = 1.0 = a_0 + b_0$, and, in general, $E_0 \to 0$ when $\varphi \to \varphi_\vartheta > 0$ for some small value of porosity, denoted by φ_ϑ, and can be thought of as a pseudopercolation threshold (which is further described subsequently). The percolation threshold is the value of porosity at which pore connectivity (or interaccessibility) is established throughout a volume and conduction commences; at the threshold $0 = a_0\varphi_\vartheta + b_0$. Then $\varphi_\vartheta = -b_0/a_0 = -b_0/(1 - b_0)$. Now φ_ϑ is close to zero because we know that rocks can continue to conduct to very low values of porosity (e.g., 1% or less), which leads to the expectation that $b_0 \approx 0$. Consequently, $a_0 \approx 1$. So, we have a priori expectations for the magnitudes of a_0 and b_0. Granted, these expectations are based upon the assumption $dE_0/d\varphi = a_0$, which is unjustified on the grounds of physical intuition, relying only upon Occam's razor for support. But in fact, these expectations for a_0 and b_0 are observed in actual data measured on Archie rocks. Figure 9 illustrates the concepts discussed above using Archie's Nacatoch sandstone data. Note the expectation that the slope be approximately one and the intercept approximately zero are well met.

We have established that $\sigma_0/\sigma_w = \varphi(a_0\varphi + b_0) = a_0\varphi(\varphi - \varphi_\vartheta)$. Imposing $E_0 \to 1$ as $\varphi \to 1$ requires $a_0 = 1/(1 - \varphi_\vartheta)$. Thus,

$$\frac{\sigma_0}{\sigma_w} = \varphi E_0 = \varphi \frac{\varphi - \varphi_\vartheta}{1 - \varphi_\vartheta}. \tag{6}$$

where φE_0 applies to individual samples; the rightmost term applies to the trend established by many samples.

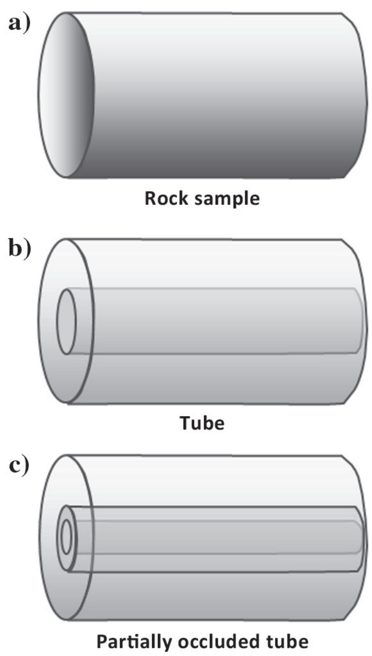

Figure 8. (a) A rock with a distributed, brine-filled pore system, and (b) the same material components separated into separate regions in a pipe, or tube. The wall of the tube is the nonconducting rock-forming mineral; the hole through the tube holds all of the original pore space and brine; the conductivity of the rock sample is σ_0; the conductivity of the tube is $\sigma_w\varphi > \sigma_0$, where φ is the area of the tube normal to current flow. (c) The conducting capacity of the actual rock can be modeled as if it were a tube with a smaller cross-sectional area, described by $E_0\varphi < \varphi$. In the figure, water participating in conduction is shown as the central cylinder; this water occupies the pore throats and the current paths through the pores connecting the throats. The water not participating in conduction is located between the inner and the intermediate cylinder (which encloses the same volume as shown in [b]); this represents pore volumes not close to the current carrying paths located between pore throats. The division is, of course, artificial in that current density would vary smoothly from zero to some maximum value throughout the pore system. To divide the brine into a phase that conducts and another phase that does not conduct, we imagine that all the current is constrained to be near the axes of the current carrying channels, and the current density is negligible elsewhere. The main teaching of the geometrical factor theory of conductivity in Archie rocks is that the bulk conductivity of the rock is proportional to the porosity, diminished by a factor that accounts for the percentage of cross-sectional area participating in conduction.

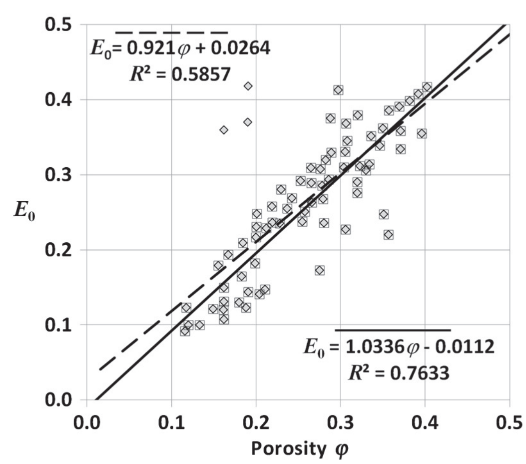

Figure 9. Pore geometrical factor E_0 plotted against porosity for Archie's Nacatoch sandstone data. The dashed trend line is for the complete data set; this trend shows a negative pseudopercolation threshold suggesting the formation is pseudo-Archie. The solid line omits the three outliers at the top of the trend giving the trend expected for an Archie rock.

When the pore geometry is a straight tube, $E_0 = 1$ and $\sigma_0/\sigma_w = \varphi$, where φ is the cross sectional area of the brine-filled tube, equal to the porosity. If the pore system is not a straight tube, but whose cross section varies along its length in a more rock-like fashion, then the electric current does not distribute itself uniformly, but concentrates in constrictions (e.g., pore throats) and in the pore volume directly between the constrictions. Pore volumes located remotely from the constrictions and connecting pore volumes or in dead-end volumes with no outlet may carry little if any current. When these conditions obtain as in rocks, then E_0 must be less than one for equations 5 or 6 to be satisfied. E_0 is, in fact, a direct measure of the efficiency of conduction of the water contained in a rock, whereas E_0 is an *explicitly* geometrical factor that can be thought of for convenience as the fraction of the pore volume contributing to conduction (Figure 8c).

The parameters of this theory of geometrical factors are: E_0, the geometrical factor, that can be viewed as the fractional cross-sectional area of the porosity participating in conduction (i.e., fractional area in pore throats that control where, and how much, electric current flows in any cross-section normal to the direction of current flow); a_0 is the rate of change of cross-sectional area participating in conduction with a change in porosity, b_0 is the fraction of cross-sectional area participating in conduction that is independent of porosity (a negative quantity in Archie rocks, meaning the effective cross-sectional area participating in conduction approaches zero before porosity approaches zero), and $\varphi_\vartheta (= -b_0/a_0)$ is an apparent (or pseudo-) percolation threshold, the critical porosity threshold at which, conceptually, conduction would abruptly commence, or cease.

From equation 6:

$$E_0 = \frac{\varphi - \varphi_\vartheta}{1 - \varphi_\vartheta} \approx \varphi. \tag{7}$$

The change in geometrical factor with changes in porosity thus correlated is quantified in terms of a percolation threshold parameter. Further, the parameter φ_ϑ is, at least in principle, observable by extrapolation of the trend established by linear regression in an observed data set to the $E_0 = 0$ axis. Note the fractional cross-sectional area of the brine phase normal to the applied electric field (i.e., E_0) is approximately equal to porosity for $\varphi \gg \varphi_\vartheta$, but diminishes as $\varphi \to \varphi_\vartheta$. Note that tortuosity does not enter into this model, and can be shown to have little influence on conductivity in Archie rocks (Herrick and Kennedy, 1993). Also, noting that φ_ϑ is a small number, $\varphi E_0 \approx \varphi^2$, or $\varphi E_0 = \varphi^m$ for some value of $m \approx 2$ if $\varphi_\vartheta \approx 0$. Thus, the reason $m \approx 2$ is rendered transparent in terms supplied by GFT.

Equation 6, and the resulting form for E_0 given by equation 7, is developed by use of boundary conditions outside the porosity range where Archie rocks reside. Thus, although equation 7 gives a theoretical insight as to why $E_0 \approx \varphi$ in terms of φ_ϑ, a_0, and b_0 determined through linear regression (i.e., minimum sum of squared residuals, or larger R^2) should be used in making predictions for the analysis of real data.

Although we assert that Archie rocks have no conduction mechanism other than brine, many rocks are quasi-Archie in that they possess a small residual conductivity at zero porosity. We denote this component as c_0, and introduce normalized conductivity for quasi-Archie rocks as

$$\frac{\sigma_0}{\sigma_w} = \varphi E_0 + c_0 = a_0\varphi^2 + b_0\varphi + c_0. \tag{8}$$

If c_0 is large, the rock departs from Archie behavior and enters the realm of non-Archie rocks; e.g., the "shaly" sandstones not discussed in this article. Setting equation 8 to zero, the solution of the resulting quadratic in φ is φ_ϑ. Essentially, the description in equation 8 is a tube with cross-sectional area φE_0 in parallel with a conductor having conductivity c_0.

Thus, in this view, conductivity in Archie rocks is reduced to simple form, and each coefficient (E_0, a_0, b_0, c_0) has a well-defined physical interpretation. There are no mysterious parameters, such as m, n, or a, featured in Archie's model, to investigate for physical significance.

Partially water-saturated Archie rocks

When interstitial brine in the pore system of an Archie rock is displaced by a nonconductive hydrocarbon phase, the resulting fractional brine volume is denoted φS_w. For a given pore system with fixed porosity (e.g., a core plug), φS_w behaves no differently than the fractional brine volume with $S_w = 1$. However, emplacement of hydrocarbons modifies the fractional volume and geometrical configuration of the brine. The effect of desaturation upon brine geometry is accounted for by inclusion of a second geometrical factor e_t. Consider the product of the geometrical factors $e_t E_0$. This product should reduce to E_0 in the limit $S_w = 1$, and reduce to zero in the limit $S_w = 0$. Thus, e_t must lie between limits

$$0 \le e_t \le 1. \tag{9}$$

Putting these results together gives

$$\sigma_t = \sigma_w(S_w\varphi)(e_t E_0), \tag{10}$$

where parentheses are used to group the terms specifying the fractional brine volume factors and the geometrical factors. With factors grouped in this way, the bulk conductivity of the model cylinder remains a triple product of brine conductivity, fractional brine volume, and a geometrical factor $e_t E_0 \equiv E_t$; i.e., $\sigma_t = \sigma_w \cdot \varphi S_w \cdot E_t$. Thus the triple product theorem remains directly applicable.

Observing e_t properties

Plotting the collection of points (S_w, RI) generated in a typical drainage experiment on a single core sample does not define a mere trend, but invariably (for Archie rocks), these points lie almost perfectly on a straight line on bilogarithmic graph paper (Figure 10a), small departures from the line are attributed to experimental uncertainties. There is typically little scatter in these data because it is performed on a single core sample; it also has a built in correlation to S_w. For the sample being measured, E_0 is a fixed property of the rock; the change in brine geometry concomitant with reduction in brine volume is accounted for using the saturation geometrical factor coefficient, $e_t = \sigma_t/\sigma_0 S_w$, which has the built-in correlation to S_w divided out. The example in Figure 10b is typical. The relationship is expressed as

$$e_t = a_t S_w + b_t. \tag{11}$$

There is a boundary condition at $S_w = 1$, where $e_t = 1$. Thus, $1 = a_t + b_t$, and

$$e_t = a_t S_w (1 - a_t), \tag{12}$$

which determines e_t in terms of a single adjustable parameter. Of course, each plug will have its own value of a_t. Also, analysis of a_t and b_t at the boundaries yields $e_t = (S_w - S_\vartheta)/(1 - S_\vartheta)$, where S_ϑ is value of water saturation where the brine phase disconnects. For brine-wet Archie rocks, this should be a very small quantity indeed. Thus, $e_t \approx S_w$.

Saturation equation

Given estimates of E_0 and e_t, water saturation can be estimated. Consider

$$\frac{\sigma_t}{\sigma_w} = (S_w \varphi)(e_t E_0). \tag{13}$$

Using equation 13 and substituting equation 12 for e_t, it follows that

$$S_w \left(S_w + \frac{1 - a_t}{a_t} \right) - \frac{1}{a_t} \frac{\sigma_t}{\sigma_0} = 0, \tag{14}$$

from which[4]

$$S_w = \frac{1}{2} \left(\frac{a_t - 1}{a_t} + \sqrt{\left(\frac{a_t - 1}{a_t} \right)^2 + \frac{4}{a_t} \frac{\sigma_t}{\sigma_0}} \right). \tag{15}$$

Water saturation depends upon three parameters: a_t, a_0, and b_0, which are empirically determined from a population of samples taken from a particular reservoir. The dependence upon a_t, determined as an average over a suite of samples, is explicit in equation 15; dependence upon a_0 and b_0 is implicit through $\sigma_0 = \sigma_w \varphi (a_0 \varphi + b_0)$, where a_0 and b_0 are the slope and intercept of the least-squares regression on the same suite of samples.

When equation 15 is applied to continuous log data, a_0 and b_0 have been predetermined by observing the trend in resistivity-porosity pairs measured on core samples from the formation being logged (same data as used in a φ-F plot to determine m and a), or else are estimated from experience. In equation 15, a_t is an average of a_t values taken over the same ensemble of samples (same data as used in an S_w-RI plot to determine n), or is estimated from experience. (Taking $a_t = 1$, $a_0 = 1$, and $b_0 = 0$ corresponds to taking $m = n = 2$ using Archie's model.)

EFFECTIVE MEDIA THEORIES

Mixing rules

Effective media approximations found their way into the theoretical petrophysics discussion in the early 1980s (Sen, 1980; Sen et al., 1981). Theoretical petrophysicists were curious because the mixing rules, some in the form of power laws, are approximations derived directly from physical first principles. Archie's law is a power law long in search of a first principle. Moreover, the problem of relating the bulk properties of an Archie rock to the

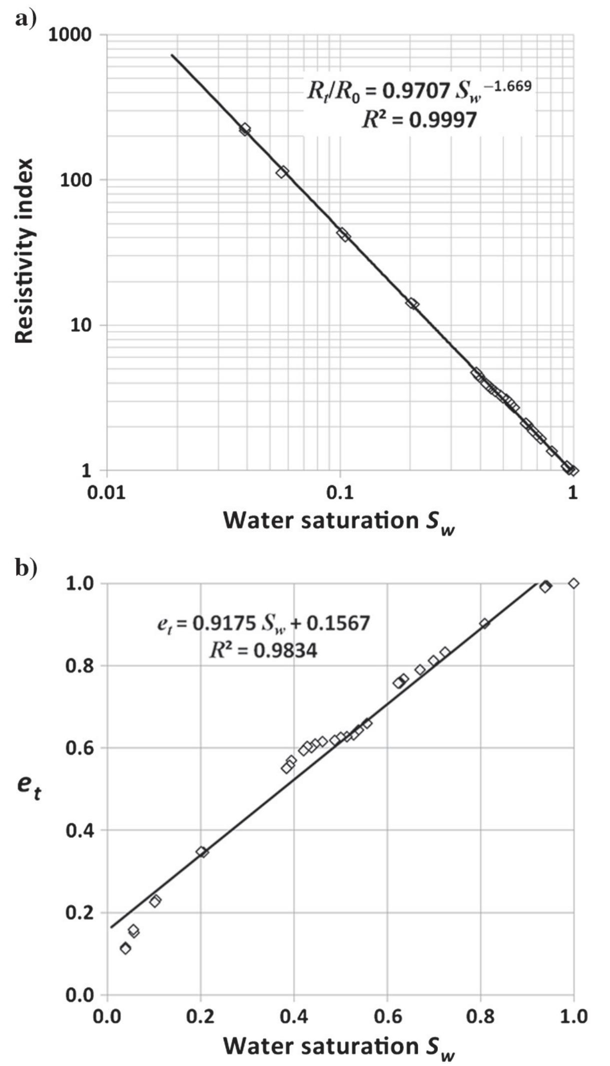

Figure 10. Resistivity index data from Lasswell et al. (2005) typifies resistivity index–water-saturation data. (a) When data are taken on a single-core plug as water saturation is varied continuously and plotted as resistivity index versus water saturation, the data points fall on a very well-defined line. (b) If e_t has the dependence on S_w divided out; then $e_t = \sigma_t/\sigma_0\, S_w$. The remaining correlation is less pronounced, the scatter probably being mostly due to experimental technique; e.g., too little equilibration time at some of the data points.

[4]We have already opined that Archie's model is unlikely to be replaced in practice given its fitness for its purpose. Comparison of equation 15, with the corresponding relationship in Archie's model, $S_w = ((\sigma_t/\sigma_w)\varphi^{-m})^{\frac{1}{n}}$, confirms that the Archie relationship is satisfyingly compact.

properties and arrangement of its conducting and nonconducting phases can be viewed as an attempt to find a correct mixing rule, although the Archie model is not usually framed in these terms. The initial hope that effective media theory would provide a theoretical and practical underpinning for the Archie model has not been realized in practice. However, the "effective medium" concept has also been applied to model resistor networks studied in percolation theory. These models *do* hold promise to provide a first-principles corroboration for our theory of conductivity in Archie rocks.

Background

Most material media are inhomogeneous. Physical theories take their simplest forms in homogeneous, isotropic media. Mathematical physics principles allow for the extension of physical theories to a class of inhomogeneous media comprising regions of homogeneous material separated by highly symmetric boundaries that conform to isosurfaces (e.g., equipotentials) in various coordinate systems (e.g., in spherical coordinates, these are surfaces of constant radius, surfaces of constant colatitude, and surfaces of constant azimuth). Within each homogeneous region, field quantities are related through simple constitutive relationships; at boundaries various boundary conditions hold (e.g., components of electric fields tangent to a boundary are continuous across the boundary). By these means, closed-form formulas for various fields can be derived. These are called boundary value problems. For cases of less, or no symmetry, numerical methods can be applied to compute estimates of field quantities, but closed-form formulas cannot be written. A numerical description of any realistic medium, such as an oil reservoir, is not possible in a practical sense. So, in the end, both analytical and numerical methods are intractable for the problem of modeling Archie rock conductivity. This motivates the search for approximations.

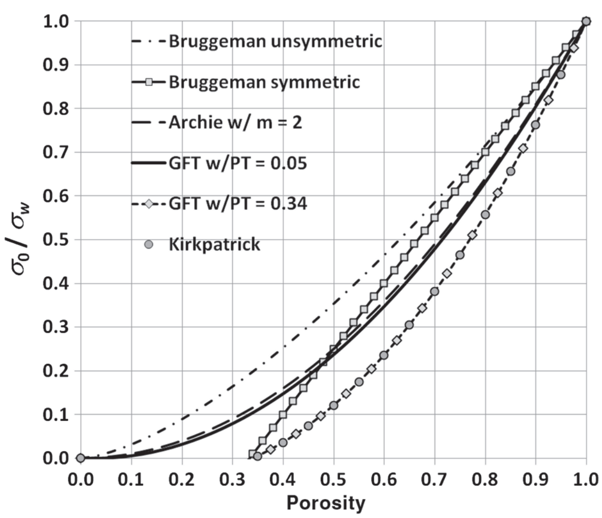

Figure 11. Effective medium approximations. This chart follows Landauer in its comparison of the effective medium formulas attributed to Bruggeman. It is modified to allow other comparisons. The axes are relabeled in petrophysically meaningful terms. The Bruggeman formulas are developed for spherical inclusions in a homogeneous background medium. The background component is taken to have zero conductivity. Bruggeman's unsymmetric formula reduces to an Archie-like law with $m = 3/2$, the dot-dash curve. Bruggeman's symmetric formula is linear (solid line marked with squares) with a percolation threshold at $\varphi_\vartheta = 1/3$. The GFT (marked by diamonds) and Kirkpatrick's formula (marked by circles) overlay exactly for $\varphi_\vartheta = 0.34$. Archie's law with $m = 2$ (long dashes) is shown for comparison. The curve labeled "GFT w/PT = 0.05" is the geometrical factor theory with a pseudopercolation threshold of 0.05. The figure clearly shows that for Archie rocks, the Bruggeman effective medium theories do not approximate observed conductivities. GFT (and Kirkpatrick's EMA) *do* approximate conductivities observed in rocks for realistic choices of the pseudopercolation threshold. Note the closeness of the solid GFT curve and the dashed Archie curve. For some choices of parameters, both theories degenerate to $\sigma_0/\sigma_w = \varphi^2$. Further, for $m \neq 2$, some value of a_0 and b_0 will generate almost the same curve. The predictive power of the two models is the same. The difference is that the parameters of GFT have a priori physical interpretations, whereas the Archie's m parameter does not.

For mixtures of immiscible components such as emulsions, as just noted, direct methods are impossible, or at least intractable, to apply. When the heterogeneous mixture is homogeneous at a larger scale, field quantities at the larger scale can be estimated by treating the medium as if it comprised a single material having its own unique properties. Ohm's law is still written $\mathbf{J} = \sigma \mathbf{E}$, but $\mathbf{J}$ and $\mathbf{E}$ are volume averages, and σ will be a composite quantity. The composite is called an "effective" medium. Although the properties of such a medium might be easy to measure, the problem of predicting the properties of an effective medium is not trivial, given the arrangement in space, fractional volumes and properties of the constituents.

The literature of mixing rules and effective media is venerable and voluminous, but the application to porosity-conductivity relationships in petrophysics is only 30 years old. Our discussion is guided by Landauer's (1978) review article. Landauer discusses two approximations: Bruggeman's (1935) "unsymmetrical" and "symmetrical" effective medium theories (EMT). The unsymmetrical form is

$$\left(\frac{\sigma_0 - \sigma_r}{\sigma_w - \sigma_r}\right)\left(\frac{\sigma_w}{\sigma_0}\right)^{\frac{1}{3}} = \varphi, \qquad (16)$$

where σ_r is the conductivity of the mineral grains. This is called unsymmetrical because interchanging the variables σ_r and σ_w, and replacing φ by $1 - \varphi$ does not leave the formula invariant. Although the equation has found use in shaly sand models in which σ_r accounts for a conductive matrix, in Archie rocks $\sigma_r \to 0$, and the equation reduces to the Archie power-law form $\sigma_0/\sigma_w = \varphi^{3/2}$. Sen et al. (1980) claim this is a derivation of Archie's law from first principles. However, the Bruggeman equation is designed for spherical inclusions; the $3/2$ exponent is indeed observed for spherical glass beads embedded in brine, and is close to similar observations made in sand packs. But the value of the exponent for Archie rocks is much closer to $4/2 = 2$. Also, it would be a matter of taste as to whether a mixing rule is a first principle. The graph of this unsymmetrical Bruggeman curve for a nonconductive matrix (i.e., closest approximation of an Archie rock using this formula) is illustrated, among several others, in Figure 11. The Bruggeman unsymmetrical form was adopted into formation evaluation, and survives to this day as an option for the analysis of shaly sands (Berg, 2007).

The symmetrical EMT form of Bruggeman's theory is

$$\varphi \frac{\sigma_w - \sigma_0}{\sigma_w + 2\sigma_0} + (1-\varphi)\frac{\sigma_r - \sigma_0}{\sigma_r + 2\sigma_0} = 0. \qquad (17)$$

This equation is called symmetric because it is invariant to an exchange of the variable pairs σ_r and σ_w, φ and $1-\varphi$. The equation is quadratic in the effective conductivity of the medium σ_0. The solution for the effective medium conductivity is (Landauer, 1978)

$$\sigma_0 = \frac{1}{4}\left(\gamma + \sqrt{\gamma^2 + 8\sigma_r\sigma_w}\right), \qquad (18)$$

where

$$\gamma = (3(1-\varphi) - 1)\sigma_r + (3\varphi - 1)\sigma_w. \qquad (19)$$

If $\sigma_r \to 0$, then $\sigma_0/\sigma_w = 3\varphi/2 - 1/2$, and this form is seen to possess a percolation threshold; i.e., $\sigma_0/\sigma_w \to 0 \Rightarrow 0 = 3\varphi/2 - 1/2$, and $\varphi = 1/3$ is the porosity at which conductivity would commence (see Figure 11). This is actually correct for spherical conductive inclusions in an insulating host, but obviously does not describe a rock that remains conductive to quite low-porosity values. Neither the symmetrical nor unsymmetrical EMT theories of Bruggeman shed any light on conduction in Archie rocks.

Percolation theory

When applied to conduction in cubic 3D networks (called lattices) of conductors (i.e., resistors), percolation theory is an example of an effective medium theory in the sense that it relates the bulk properties of the network to the conductance and spatial distribution of conductors in the network. Kirkpatrick (1973) develops formulas for the bulk conductivity of lattices as a function of the number of conductors in the lattice as the conductors are removed one by one from random locations (called bond models), or as nodes are removed one by one (each node removing six conductors attached to the node, called site models). The models predict quite accurately the fractions of bonds or sites that must be present for conduction to begin. These fractions are called "percolation" thresholds, meaning no current can pass through the network until the threshold fraction is attained. For further increases in the fractional number of bonds connected, or sites occupied, current increases as predicted by these theories.

It has long been recognized that some form of percolation theory should be applicable to the flow of electrical current in rocks. Unfortunately, pioneering percolation theorists were not focused on making a connection to formation evaluation; earth scientists interested in the method have been unable (to date) to translate the theoretical results into readily applied explanations and formulas.

Five elements of this problem are discussed below:

1) Connectivity and conduction in Archie rocks

Electrical current can circulate and flow only in a network of *connected* pores. In a two phase system comprising brine and rock-forming mineral grains sharing a volume, when the fractional volume of the brine is zero, electrical current does not flow; when the fractional volume is one, current flow is maximized. Between these two limits there must exist a fractional volume for the brine phase that represents the singular point where a connected volume of brine (just) extends throughout the volume which is otherwise entirely occupied by nonconducting minerals and disconnected brine. Any less brine and the conducting path, or circuit, opens and the conductivity vanishes; for any greater volume of brine a through-going path is established and thereafter conductivity increases with porosity. Taking a sandstone deposited subaqueously in a brine (e.g., a turbidite) as an example of a rock, you can imagine the bed-load transport as a slurry, followed by sedimentation of sand grains, followed by burial, compaction and cementation, dissolution and recrystalization, and ending with the complete occlusion of the pore system. The conductivity of this system is reduced monotonically until it vanishes at some small value of porosity with the occlusion of enough pores and pore throats.

The physical pore network is a very complicated system; fortunately, a detailed knowledge of the pore system is not required. A simpler model captures the essence of conductivity in this system.

2) Percolation versus pseudopercolation threshold

In Archie rocks, there is an additional important detail in the porosity-conductivity trend. Although conductivity diminishes monotonically and continuously with reduction in porosity, there are two qualitatively different mechanisms contributing to conductivity reduction. The conductivity reduction that begins following sedimentation is primarily due to reduction in pore volume in response to compaction and cementation, with little reduction in the connectivity. In other words, the conductivity-limiting pore throats are reduced in size but not in number. The φ-E_0 trend is linear. However, at some point in porosity reduction, typically less than 10 pu, pore throats begin to occlude. This results in an increase in the rate of conductivity reduction. Thus, the slope of the porosity-conductivity trend becomes steeper at lower porosity, establishing a second trend. This trend will terminate at the true percolation threshold marking the point at which the final through-going conducting channel disconnects, and conductivity becomes zero. However, many, perhaps most, conventional reservoirs have porosity distributions that do not descend to this low-porosity region. The porosity-conductivity trend established over this higher porosity range will not extrapolate to the true percolation threshold at zero conductivity, but to wherever the extrapolated trend takes it. This extrapolated intercept is called a pseudopercolation threshold. It has no a priori physical significance, although it may prove useful as a correlative parameter.

A pseudopercolation threshold should be positive (i.e., $\varphi_\vartheta > 0$) for typical Archie rocks. A negative value of the pseudopercolation threshold would mean that some conductivity remains in the rocks at zero porosity, suggesting that in such a rock, there is a conducting mechanism in addition to the interstitial brine, and the rock is not a true Archie rock as we define Archie rocks. Note that in Figure 9, the pseudopercolation threshold is negative. This could indicate a small amount of surface conduction due to clay in the Nacatoch sandstone, ironically suggesting that this seminal formation for resistivity interpretation is a quasi-Archie rock. On the other hand, if the three outliers at the top of the data set are omitted from the trend, the remaining points conform to the trend style expected in an Archie rock.

3) Conduction in a simple cubic lattice of conductors

Consider a regular cubic lattice of points connected together by conductors (i.e., resistors by another name). In two dimensions, each node has four nearest neighbors spaced equidistantly, one at each point of the compass; in three dimensions, each node has six near neighbors in the grid. Imagine connecting each point in the lattice to its nearest neighbors using identical conductors. This is the lattice having maximum conductivity. Now imagine removing conductors, one at a time, from randomly chosen locations in the lattice, and recording the bulk conductivity of the lattice. The process is continued until conduction abruptly ceases with the removal of a single conductor in the last through-going conducting path. A similar, but different, process would be to choose a lattice point (or node) at random and remove all the conductors (six in a 3D cubic lattice) that meet at the point. These two procedures, called bond- and site-percolation models, respectively, have been modeled numerically and analytically (Kirkpatrick, 1973) and physically (Shankland and Waff, 1974).

4) Equivalence of conduction in Archie rocks and conductor lattices

Any network of conductive pores and their connecting pore throats could, in principle, be modeled by a conductor network, except that the coordination number (number of nearest neighbors) would, in general, not be six but would correspond to the number of pore throats connecting the pore to the network, and the conductors could have different conductance values. There is nothing in the derivation of the formulas that could not (in principle) be extended to higher coordination numbers, nor is it necessary that all the conductors have the same values. The cubic lattices are invoked because algebraic analysis and computer simulations on such lattices are relatively easy. The restriction on conductor values (i.e., a binary selection one member of which is zero conductance) makes for tractable analysis and compact formulas for the bulk resistivity of the network; in principle, this restriction is not required. Conductor values could be drawn from a distribution; qualitatively the behavior of the network would be the same although quantitatively the numerical values would be expected to differ. Given these observations, it is but a small leap of faith to apply the formulas developed on the regular lattice to the less regular networks found in rocks.

5) Impact of percolation theory in the petrophysics community

Percolation theory applied to conductivity in rocks has failed to have significant practical impact among formation evaluators. Its literature is arcane, the notation unfamiliar, the mathematics adumbrated, and the discussion dwells upon theoretical values of certain parameters which differ from corresponding values observed in rocks. It appears to be directly applicable to its archetypical models, a lattice of conductors and other regular arrays of particles such as crystals. But its direct relevance to formation evaluation is unclear. This is true even for articles invoking percolation theory in earth and soil science journals. What is available is a hodgepodge of formulas with no guidance as to how to apply them to rocks.

Geometrical factors and the site percolation model

In this section, we show that there is a formal isomorphism between our geometrical factor model of electrical conduction in rocks and site percolation on resistor networks. In our minds, the coincidence of these formalisms points to an underlying connection of the theories, and provides a simple connection of conductivity in Archie rocks to the first principles of electrical conduction in heterogeneous networks.

Recently, Montaron's (2008) equation 4 decoded the earlier literature (Kirkpatrick, 1973). Equation 20 is for site percolation in Montaron's notation, where p is the probability that a site chosen at random would be occupied and a is a "critical" probability, i.e., a percolation threshold. The conductivity of the network is

$$\frac{\sigma_0}{\sigma_w} = p\,\frac{p-a}{1-a}, \tag{20}$$

which is formally identical to equation 6:

$$\frac{\sigma_0}{\sigma_w} = \varphi\,\frac{\varphi-\varphi_\vartheta}{1-\varphi_\vartheta}.$$

The isomorphism of 20 and 6 is suggestive of a connection between GFT and site-percolation models. Montaron derives equation 20 from Kirkpatrick's equation 6.20 for site percolation, which reads,

$$G(x) = P^{(s)}(x)D(x)/D(1), \tag{21}$$

where x is the probability that a site will be connected to the conducting network (same as p in Montaron's notation, and corresponding to porosity φ), $G(x)$ is the normalized conductivity (σ_0/σ_w in formation evaluation notation), $P^{(s)}(x) = x$ is the fraction of accessible space in a volume (also φ in formation evaluation notation), and $D(x)/D(1)$ is a "normalized spin stiffness coefficient." Citing Izyumov (1966), Kirkpatrick expands this coefficient to

$$D(x)/D(1) = 1 - 1.52(1-x) + \mathcal{O}[(1-x)^2], \tag{22}$$

where $\mathcal{O}[(1-x)^2]$ represents higher-order terms of small magnitude, neglected in the further analysis. Kirkpatrick puts this together (in his equation 6.22) as

$$G(x) = x - 1.52\,x(1-x) + \mathcal{O}[(1-x)^2]. \tag{23}$$

Equation 23 is graphed in Figure 11. In terms of formation evaluation, equation 23 would be

$$\sigma_0/\sigma_w = \varphi - q\varphi(1-\varphi), \tag{24}$$

where $q = 1.52$. It can be shown that

$$\varphi - q\varphi(1-\varphi) = \varphi\left(\frac{\varphi - \frac{q-1}{q}}{1-\frac{q-1}{q}}\right) = \varphi\left(\frac{\varphi-\varphi_\vartheta}{1-\varphi_\vartheta}\right), \tag{25}$$

where $\varphi_\vartheta = (q-1)/q = 0.342$, which is close to the theoretical value for the 3D cubic lattice resistor array that Kirkpatrick analyzes. In terms of the coordination number z, in a conductor lattice (Hunt, 2005; cf. equation 1.45, p. 25),

$$\varphi\left(\frac{\varphi-\varphi_\vartheta}{1-\varphi_\vartheta}\right) \sim p\left(\frac{p-2/z}{1-2/z}\right), \tag{26}$$

where p is the probability that a site will be occupied in the cubic lattice, and where φ_ϑ corresponds in a cubic lattice to $2/z$, where (for $q = 1.52$) $z = 5.85 \approx 6$; i.e., $\varphi_\vartheta = 0.342 \approx 1/3$.

These results are echoed in Sahimi (1993) and in Keffer (1996), who recast Kirkpatrick's $D(x)/D(1)$ as a diffusivity ratio in a cubic crystal lattice using a lattice Green's function; this is further reviewed in Hunt's (2005) text. The element missing from the literature to date is a *transparent* explication of equations 21 and 22.

The percolation threshold for site percolation in the simple cubic network of resistors, with a coordination number of six, is much higher than is observed in Archie rocks. This would be because the coordination number of a pore in a rock can be higher than six. Equally important is the way in which the conductive phase is removed from the network; for site percolation, six resistors simultaneously change their conductivity to zero, modeling an instantaneous vanishing of porosity. This is not how porosity in rocks is reduced, which is a gradual process of porosity reduction by cementation; thus conductivity persists to much lower values of porosity. In the resistor network, the corresponding process would be a gradual and uncorrelated decrease in conductivity in connected sextuplets of variable resistors in the network, without the sudden vanishing of sextuplets of connected resistors. This would tend to move the percolation threshold to lower porosity values. For real rocks exhibiting percolation thresholds, the coordination number computed from $\varphi_\vartheta = 2/z$ is much higher than is physically possible. The explanation would have to be that the analogy of the resistor network to the pore system cannot be successfully pushed to low porosity in every respect. The interpretation of z as a coordination number, and $2/z$ as a percolation threshold, breaks down.

It is clear, however, that the bulk conductivity of a lattice depends on the same three factors as an Archie rock. If $g(p)$ is the conductance of a lattice with sites occupied with probability p, and $g(1)$ is the conductance of the lattice with all the sites occupied, then

$$g(p) = g(1) \cdot p \cdot E, \tag{27}$$

where p is the probability that a site will be occupied in a connected network of nodes, and E is the fraction of conductance of the network compared with the conductance of the network if all the conductors were arranged so that there would be no unoccupied sites in the connected conducting paths (i.e., a "straight-through" in the lattice network). Clearly, this is the same triple product as postulated for the Archie rock, and E is formally identical to E_0, and it should come as no surprise that they are governed by the same equation.

CONCLUSION

Comparison of Archie's ad hoc model, which did not make explicit provision for the inclusion of the electrical effects of brine geometry, and our alternative model, which includes brine geometric effects as an a priori element, suggests that one difficulty in attaining an intuitive understanding of conductivity in rocks is the suitability of the Archie model to the problem. The imputing of physics to this model has been an assumption based on its success in predicting the brine volume-conductivity trend always observed in Archie rocks. In this paper, we have shown that there are many physics-free functional forms that predict the trend in scatter-prone Archie data equally well. The form chosen by Archie is an artifact of history due to his use of resistivity instead of conductivity, and to the difficulties of curve-fitting (particularly nonlinear curve-fitting) with the resources available in the late 1930s. A procedure in common use at that time was to plot data on linear, log-linear, and bilogarithmic paper in search of a trend which could be easily fit by using a straight edge. It is highly likely that this same procedure was used by Archie to obtain his relationship between formation resistivity factor and porosity. This approach results in a simple power law.

Archie himself, in his seminal paper, did not ascribe any deep significance to his model. He modestly admonished in his 1942 paper:

> It should be remembered that the equations given are not precise and represent only approximate relationships. It is believed, however, that under favorable conditions, their application falls within useful limits of accuracy.

History has substantiated Archie's belief; unfortunately, his cautionary warning is forgotten.

Summary of important points

Trend analysis, graph paper, and Archie's law

We have seen that Archie's law is a trend analysis based upon the technology of preprinted graph paper. Given the irreducible scatter in brine volume-conductivity data, Archie's law makes adequate predictions of water saturation. There is no physics to be found in Archie's m and n, or the Humble formula's a; they are mere curve-fitting parameters. However, because none of the quantities σ_w, φ, or S_w contain information on brine geometry, then m, n, and a *must* contain all information on brine geometry, albeit not in readily interpretable form.

A geometrical factor theorem for electrical conductivity in Archie rocks

The bulk conductivity of Archie rocks has been shown to be proportional to the product of three properties of its interstitial conductive brine, namely, its conductivity σ_w, its fractional volume φS_w, and a geometrical factor, $E_t = e_t E_0$. The geometrical factor is interpreted as the fraction of the porosity's cross-sectional area participating in conduction. This relationship is quite straightforward, and does not rely on empirical power laws for its description. The empirical constants that arise in $E_0 = a_0\varphi + b_0$, $e_t = a_t S_w + b_t$, and $\sigma_0/\sigma_w = \varphi E_0 + c_0$ have interpretations: E_0 is the fractional cross-sectional area of the porosity participating in conduction; a_0 is the rate of change of the fractional cross-sectional area of constrictions (e.g., pore throats) controlling conduction with respect to porosity and is determined by the geologic processes that change porosity; b_0 is a part of the geometrical factor that is independent of porosity, usually small; c_0 is "excess" conductivity that does not depend upon brine in the pore system; and, in principle, $c_0 = 0$ identically in Archie rocks. The ratio $\varphi_\vartheta = -b_0/a_0$ has an interpretation as a (pseudo-) percolation threshold. The Archie power-law form (i.e., $m \to 2$) is approached asymptotically in the limits $a_0 \to 1$, $b_0 \to 0$; or equivalently, $\varphi_\vartheta \to 0$. Similarly, $e_t E_0 \approx a_t a_0 S_w \varphi \leq S_w \varphi$ suggests that $e_t E_0$ gives the fractional cross-sectional area of brine volume participating in conduction, and $a_t a_0$ gives the rate of change of this fractional area with respect to $S_w \varphi$ variation. In conductivity terms, Archie's law and GFT are almost equivalent: $\sigma_0/\sigma_w = a_0\varphi^2 + b_0\varphi + c_0 \approx a\varphi^m$. The difference is most apparent at low porosity where the Archie conductivity vanishes with porosity, whereas GFT conductivity can vanish at a pseudopercolation threshold, not necessarily equal to, and usually expected to be greater than, zero. It is also clear from this correspondence why the value of the Archie exponent is approximately equal to two. The correspondence is exact when the pseudopercolation threshold $\varphi_\vartheta = 0$.

The formal equivalence of the geometrical factor and site percolation models

Analytically, the geometrical factor theory described above is formally identical to the site percolation model in a cubic lattice of conductors. This observation suggests a close correspondence, if not the identity, of the two conductivity problems. If this is true, the basis of geometrical factor theory in physical theory is not only self-evident from the postulates of the theory, but an identical formalism is also independently supported by models of site-percolation theory in 3D conductor networks, suggesting that GFT follows from the same first principles as site percolation.

Closing remarks

To recall our introductory remark, Archie's law "is straightforward to use, and being fit for its purpose, is not likely to ever be supplanted in practice by another rule." Given the uncertainty in brine volume-conductivity trends attendant upon the irreducible scatter in porosity-conductivity data, its predictions are as serviceable as any other fitting function that might be used. However, we hope it is now clear that there cannot be any real success in a search for the direct physical significance of the fitting parameters m, n, and a. They have none. This limits the interpretability of conductivity measurements, using the Archie model, to predicting conductivity from a porosity-conductivity trend given that the values of the adjustable parameters are known a priori. Neither is there any need for conductivity models that attempt to account for excess conduction in reservoir rocks (i.e., shaly sand models) to reduce to the Archie power law in the limit of vanishing secondary conduction.

We have thought about the problem of conductivity in Archie rocks since our careers began in the 1970s. Like everyone else learning the petrophysical craft, we were introduced to the topic through the work of Archie and his successors. Although we were working in the fourth decade following Archie's first publication, no one had been able to provide a transparent elucidation of the origin of the power law from physical principles. Certainly no one had suggested that there might be a more productive way to formulate the problem. Since its first use in petrophysics, there had not been any critical analysis of the Archie model.

According to the tenets set forth by Sir Karl Popper, no physical theory can be proven to be true. On the other hand, every physical theory can be proven false (or falsified) by a single incorrect prediction. In this article, our model is set forth in a way that invites easy falsification: bulk conductivity of an Archie rock σ_t is the product of three factors: brine conductivity σ_w, fractional brine volume φS_w, and the geometrical distribution of the brine captured in a single parameter E_t. The geometric parameter changes as brine volume changes in a certain way that can be anticipated a priori. Those anticipations have been generally validated by, and are calibrated by, measurements made in the laboratory. Each data set will have its own, unique adjustable parameters. All of the parameters in this model have a priori physical interpretations. That is the purpose of our model of conductivity in Archie rocks.

Our intent is not "Archie-bashing," as criticism of Archie's model has been termed. We have the utmost respect for Archie and his accomplishments. It is clear to us from reading his work that he did not regard his equation as being the ultimate and definitive answer to resistivity interpretation. This view can be attributed to his successors. We hope that our arguments serve to initiate a new look at the old problem of conductivity in Archie rocks. We are unable to find a case to falsify any of the tenets in our model in its restricted domain of Archie rocks. We invite and welcome analysis and critiques.

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APPENDIX A

MODELING IRREDUCIBLE SCATTER

Irreducible scatter is simulated using the formula $N * (\mathrm{RAND}() - 0.5)$ where the Microsoft Excel RAND function returns a pseudorandom number in the interval 0.0–1.0. The magnitude of the scale factor N is chosen to provide an amount of scatter in the resulting data set that is visually similar to the scatter observed in Archie's Nacatoch sandstone data set and was varied as the occasion required.

APPENDIX B

NONUNIQUENESS OF FITTING FUNCTIONS

Figure 5 illustrates that the same data can be graphed in many different ways, and that these different presentations suggest different fitting functions. In this appendix, we show that, indeed, there are many different fitting functions that can be applied to the *same* graph. Figure B-1 contains four graphs of resistivity versus porosity plotted on linear scales and four graphs of conductivity versus porosity, also plotted on linear scales. Each graph contains a different function used to fit the data, each function as efficacious as the others based upon their nearly equal figures of merit; i.e., the coefficients of determination R^2. This serves to emphasize the point that Archie's law is but a member of a class of ad hoc fitting functions that have no special relationship to the physics of conductivity in rocks.

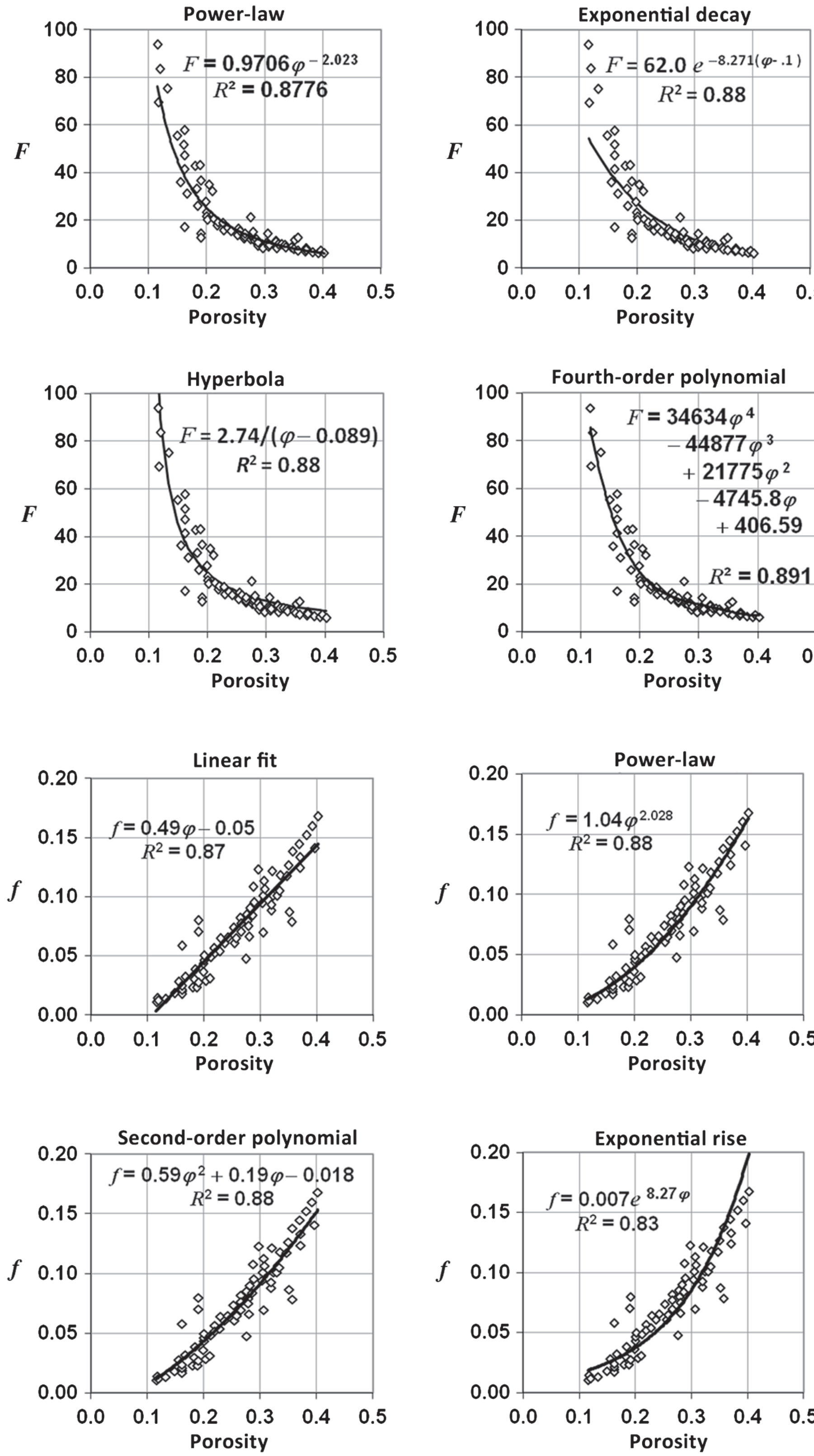

Figure B-1. Figures 3 and 5 illustrate that graph paper scaled in three ways can be used to search for simple trends in data. This figure illustrates that data plotted on the same scales can be fit equally well with many different functions. Shown here are four linear F-φ plots, and four linear f-φ plots, all fit equally well by various functions. These functions hardly exhaust the possibilities. Only one of the functions is Archie's law. This makes the powerful point that imputing physical meaning to adjustable parameters in curve-fitting functions does not make sense unless the fitting function has the physics built into it in its design.

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