A New Look at Electrical Conduction in Porous Media:
A Physical Description of Rock Conductivity

D. C. Herrick[1] and W. D. Kennedy[2]

Society of Petrophysicists and Well Log Analysts

ABSTRACT

The classical formulation of electrical conductivity in rocks, as described by G. E. Archie in 1942, has a strictly empirical origin. It is based upon the discovery of a power law relationship obeyed by the formation resistivity factor-porosity plot, and a similar power law connecting resistivity index to water saturation. These discoveries came from 1930s graphical analysis techniques resulting in an easily described linear trend on log-log graph paper. Although Archie's resulting famous equation is a good descriptor of the trends observed, no physics is inherent in the description. Consequently, the theory's parameters, m, n, and a, are a priori physically meaningless. Although it can be argued that these parameters must be functions of the distribution of the conducting phase, it is an ex post facto argument based on the observation that none of the other variables contain the effects of pore geometry.

There are many ways to determine and use trends observed in data to make predictions; the quality of the predictions can be comparable, so the choice of one method over another must be predicated upon a criterion other than accurate predictions. Archie's method is based upon decades-old custom and the enormous amount of historical data analyzed by his method. However, the ability of an analyst to make interpretations or predictions is impaired since Archie's parameters cannot be reliably interpreted in a meaningful physical sense.

The problem with the classical formulation is that it provides no physical intuition. However, it is not difficult to formulate a pore-geometric theory from first principles that not only makes correct predictions, but also provides a physical interpretation of

its adjustable parameters. It is clear that such a theory of conduction in rocks must be built upon the conductivity of, amount of, and geometrical distribution of, the conducting phase. The effect on conductivity of the brine conducting phase geometry can be characterized by comparing the current a rock with a given brine volume supports to the amount of current the same volume of brine configured in a tube would support. Making this comparison separates the electrical effects of pore geometry from those of pore volume and brine conductivity and is a statement of the conduction efficiency of brine in the rock.
.

INTRODUCTION

G. E. Archie (1942) invented the industry-standard model for describing the partitioning of pore fluids into conductive brine and nonconductive hydrocarbon phases. Archie's equation relates laboratory measurements of resistivity made on rock samples R_t, to measurements of brine resistivity R_w, porosity ϕ, and water saturation S_w made on rock samples. Although Archie did not explicitly synthesize "Archie's equation" (equation 1.a) in his paper, Archie observed a relationship between the formation porosity and the formation resistivity factor, which he defined as $F = R_0/R_w$, and a previously known relationship between the resistivity index I and water saturation (e.g., Jakosky and Hopper, 1937), yielding an equation that is descriptive of the resistivity of porous materials,

$$R_t = R_w\ \phi^{-m} S_w^{-n} \qquad (1.a)$$

(Archie, 1942). The model was originally framed in terms of resistivity, but theoretical discussions have long been reframed into terms of conductivity, which gives a numerically equivalent but analytically simpler form.

$$C_t = C_w\ \phi^m S_w^n. \qquad (1.b)$$

Archie (1942, 1950) was very conservative in his treatment of this model, and made no speculation as to how, or whether, the model could be related to physical first principles; for Archie the model was strictly an empirical means to organize trends in his

---

[1] Retired, formerly Baker Hughes, Inc.
[2] Baker Hughes, Inc.

data. The data Archie used, including those from the Nacatoch sandstone (figure 1), to develop this relationship is well described by equations (1).

Archie recognized immediately upon its discovery that his model could be applied using well log data with R_t estimated from a resistivity log, R_w estimated from R_0 in a water saturated zone or by "special means," and porosity ϕ estimated from a porosity log. In this use S_w can be estimated directly from suitable environmentally corrected well logs.

The formula in equation (1.b) prescribes a porosity-conductivity relationship given by $C_0 / C_w = \phi^m$ and a water saturation-conductivity relationship given by $C_t / C_0 = S_w^n$. Archie also implicitly assumed that the power law relationship between F and ϕ extrapolates to the origin ($C_0 / C_w = 1$ when $\phi = 1$).

Following the subsequent work of Winsauer and others (1952) the industry-standard Archie model became, and remains to this day,

$$C_t = a^{-1} C_w \ \phi^m S_w^n. \qquad (2)$$

where a is the intercept obtained when the observed relationship between F and ϕ does not extrapolate to the origin. This equation is well-suited to predicting
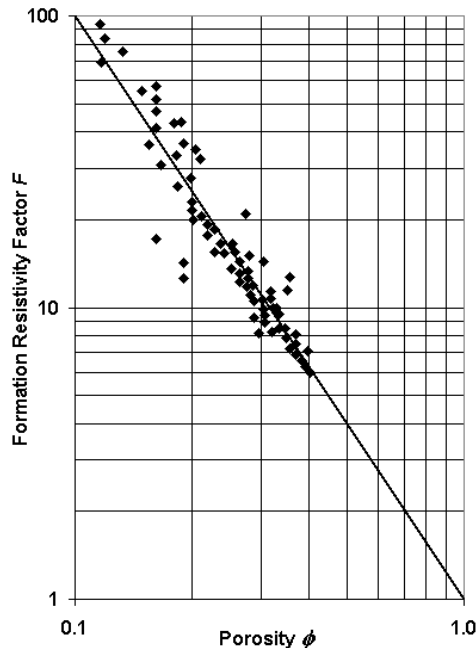

FIG. 1. Archie's Nacatoch sandstone resistivity-porosity data. The line corresponds to $F = \phi^{-2}$.

water saturation in a type of homogeneous, isotropic rock characterized by hydrophilic, approximately equidimensional, rock-forming mineral grains, comprising exclusively intergranular porosity with conduction due solely to brine in pores and on pore surfaces. Such rocks are similar to those studied by Archie, having conductivity well described by (2), and thus are called "Archie" rocks.

In modern interpretation practice, it is recognized that the parameters a, m, and n should be chosen based upon laboratory measurements made on the rock being described. In the evolution of the understanding of this relationship much effort has been expended in attempts to impute "physical significance" to these adjustable parameters. Since all conduction takes place in the brine, the physical significance of the amount of brine, ϕS_w, and the conductivity of the brine C_w, is clear. The bulk conductivity of a rock is also influenced by how the brine is geometrically distributed in a pore-system. Since C_w, ϕ, and S_w do not depend upon geometry, clearly the formation and brine geometries must reside in the exponents m and n, and in the factor a. Efforts to attribute physical significance to these factors has led to the commonly used names of "cementation" exponent for m (Guyod, 1944) and "tortuosity" factor for a (Schopper, 1966). However, any association of m with cementation or a with tortuosity is based strictly on correlative observations; there is no justification of these names that can be traced to any argument based a priori upon first principles.

In contemporary petrophysical interpretation, equation (2) is known as Archie's "law", and indeed in the form $C_0/C_w = \phi^m$ Archie's model has become widely entrenched in technical literatures, some of which are very far afield from petrophysics, as the rule that describes the conductivity of any brine-filled porous medium. As a purely descriptive predictor this is justified. However, use of the word "law" would seem to put Archie's model on an equal footing with Newton's "law of universal gravitation" or Maxwell's "laws of electromagnetism", and this is unwarranted. Archie's model is composited from two "power laws", but this is not the intended sense of the "law" in the phrase. In fact, Archie stated that his model was an approximation. It has never been, and cannot be, derived directly from first principles.

We have previously introduced a theory of conduction in Archie rocks (Herrick and Kennedy, 1993, 1994). In the interim, this theory has been further refined and better understood, and now seems complete, the resulting insights having matured. In this article we shall review how to rigorously construct a

relationship connecting bulk conductivity to brine conductivity, brine volume, and brine geometry. We shall show that when the rigorous description is applied to practical problems, for example when computing water saturation, it generally becomes computationally intractable due to the complexity of the pore geometries of rocks, beyond which point use of empirical methods is required for quantitative application. We will argue that both the method we describe for application and Archie's model belong to a class of models (i.e., fitting equations) with many members, all of which are equally efficacious as predictors in their domain of conductivity-porosity-water saturation observations. Each model has its own unique, model-dependent, adjustable parameters, and usually these parameters will have no a priori physical significance. However, at least one equation, based on first principles, *can* be used as a fitting equation, and this equation does, by its construction, have physically interpretable adjustable parameters, albeit the values of these parameters will usually need to be determined by empirical observations made on rock samples since rock pore geometries are unknown a priori.

AN A PRIORI MODEL OF AN ARCHIE ROCK

We now define a conductivity-porosity-water saturation model for an Archie rock based upon first principles. For purposes of this definition, we restrict our model to rocks that satisfy two conditions:

1) all conduction is due to brine;

2) grain surfaces are brine-wet, so a continuous conductive path is always present regardless of the S_w value, $S_w > 0$.

Consequently we assert the hypothesis that bulk rock conductivity depends upon three and only three factors, viz.: (a) brine conductivity; (b) amount of brine; (c) geometrical distribution of brine.

Dependence on the conducting phase: C_w

The only conducting phase is water. If the water has no conductivity (e.g., deionized, fresh water) then the rock would have no conductivity. Formation water found in nature is conductive brine. Suppose the bulk conductivity of a rock is known by measurement; if the conductivity of the brine is doubled, all other physical properties remaining unchanged, the bulk conductivity of the rock will double. Thus, the conductivity of the rock must be directly proportional to brine conductivity with proportionality factor unity. This proposition we take as trivially self-evident.

Dependence on the quantity of brine: $S_w \phi$

Porosity Dependence. The amount of brine is controlled by porosity, and hydrocarbon saturation, if any. The simplest case is when hydrocarbons are not present, the pore volume is completely brine-filled, and $S_w = 1$. Obviously the conductivity increases as the brine volume increases, but the question is: what is the functional relationship?

A thought experiment is helpful in determining this (see figure 2.a). Consider a cylindrical core plug of cross sectional area A and radius R, $A = \pi R^2$, and length L having brine-filled porosity ϕ. Imagine that the pore volume is collected into a cylinder of radius r and area $a = \pi r^2$ coaxial with the core plug. This cylinder is filled with the brine. The rock-forming mineral grains (e.g., quartz) are used to form the wall of a pipe, or tube, around the brine-filled cylinder.
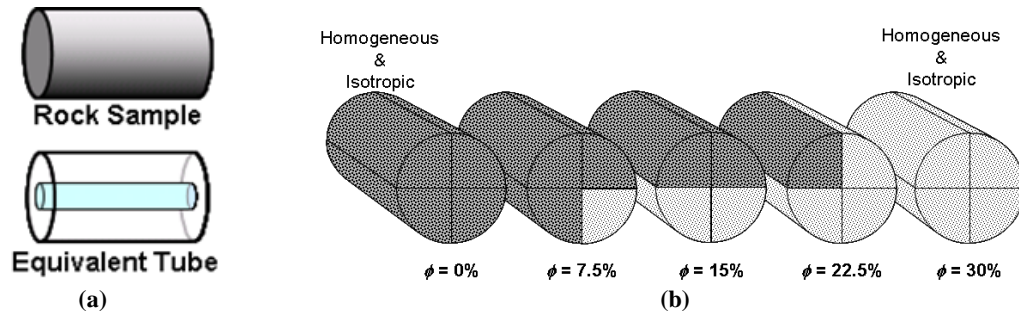

FIG. 2. (a) A rock with a distributed, brine-filled pore system, and the same material components separated into separate regions in a pipe, or tube. The wall of the tube is the non-conducting rock-forming mineral; the hole through the tube holds all of the original pore space and brine; **(b)** In real rocks, following sedimentation, changes in porosity are due to diagenesis (compaction, and cementation), and are always accompanied by changes in pore geometry. However, this figure illustrates a realizable thought experiment illustrating that pore geometry and bulk porosity are intrinsically independent variables.

The pipe has inner radius r and outer radius R. For this geometry, it is easy to show that the ratio of the areas a/A is equal to ϕ, the porosity. Also, for this geometry the bulk conductivity of the tube model is obviously proportional to the area of the conductor: a, and hence ϕ. If a is doubled, ϕ is also doubled and the brine content of the model is also doubled thereby increasing the conductivity by a factor of two. Thus for the arrangement of brine and mineral in the thought experiment, it is clear that conductivity is directly proportional to porosity, with a unity proportionality constant

Any redistribution of the brine in the same volume into a more complicated geometrical network will reduce the bulk conductivity of the sample, but for a fixed pore geometry the direct proportionality remains. To see this consider a second thought experiment. A method of varying pore volume without a concomitant varying of pore geometry is illustrated in figure 2.b. Quarter cylinders with the same external geometry are assembled in five configurations. The darker quarter cylinders have no porosity, and the lighter quarter cylinders have identical pore geometry and 30 percent porosity. In the thought experiment, the conductivity and porosity of 5 cases is illustrated. Clearly, the bulk conductivity and bulk porosity of these models are directly proportional to the conducting volume with same constant of proportionality, viz. one. The case is admittedly contrived, but it clearly illustrates that for *fixed* pore geometry of the conducting phase, the conductivity of the bulk rock (i.e., the full cylinder) must remain directly proportional to porosity with proportionality constant equal to one.

Water Saturation Dependence. In a brine-filled rock, the brine takes on the geometry of the pore space. When hydrocarbon displaces brine in water-wet rocks, brine is displaced from the central regions of the pores, leaving brine coating mineral grains, in micropores and in pendular regions at grain contacts. The fractional volume of the pore space occupied by the brine is the water saturation, S_w. The fractional volume of brine in the bulk rock when $S_w < 1$ is ϕS_w. As far as the bulk conductivity of a sample is concerned, brine is the sole conducting phase. The first of the thought experiments described above can be repeated with the hydrocarbon phase lumped with the rock-forming mineral phase as a non-conductor. The phases can be arranged as concentric cylinders. The area of the conducting phase is ϕS_w. Consider desaturating a sample from $S_w = 1$ to $S_w = 0.5$. The area of the conducting phase diminishes by a factor of $1/2$, and the conductivity also diminishes by the same factor, confirming the direct proportionality of bulk conductivity to bulk brine volume, or ϕS_w.

Dependence on pore geometry: E_0

It can be shown (Herrick and Kennedy, 1994) that for low frequencies the electric potential Φ in a source-free region satisfies

$$\nabla \cdot C_0 \nabla \Phi = 0 \ . \qquad (3)$$

Equation (3) expands to $\nabla C_0 \cdot \nabla \Phi + C_0 \cdot \nabla^2 \Phi = 0$, where Φ is electric potential. The first term is zero except on boundaries where $\nabla C_0 \neq 0$. Thus, except on grain boundaries, (3) reduces to the Laplace equation, $\nabla^2 \Phi = 0$. Numerical finite element and finite difference solutions to Laplace's equation and equation (3), together with appropriate boundary conditions, are well known and easily applied to small regions. The electric field $\mathbf{E}$ is the gradient of Φ, and Ohm's law is written $\mathbf{J} = - C_0 \nabla \Phi = C_0 \mathbf{E}$ where $\mathbf{J}$ is current density. Thus, given the applied potential difference and conductivity distribution $C_0(x,y,z)$, the current density at each point in a model is determined. $\mathbf{J}$ determines the direction of current flow, which is everywhere normal to the equipotential surfaces in the medium, and the total current determined from $I = \int \mathbf{J} \cdot d\mathbf{A}$ where $\mathbf{A}$ is the directed area normal to the direction of current flow. The integration is over any equipotential surface in the model; I, the total electric current, will be same for all.

Ohm's law applies at every point in the medium, thus determining the current density everywhere. The integration of the current density over any equipotential surface in the model gives the total current. The total current I and the applied potential difference $\Delta \Phi$ determine the conductance c ($= I / \Delta \Phi$) of the model, which is converted to bulk conductivity by $C_0 = c A / L$ where A is the cross sectional area and L the length of the model. For fixed conductivities of the constituents in the pore scale model, only the porosity and geometry vary. These control the total current, and thus the bulk conductivity, which for a given porosity, may be larger or smaller depending upon the geometrical arrangement of the grains.

Consider a volume of a conductive homogeneous and isotropic medium such as a cylinder of brine (figure 3.a). If an electric potential is applied across the brine then an electric field $\mathbf{E}$ results parallel to the cylinder's axis producing an electric field gradient in the brine that is uniform throughout, with equipotential surfaces that are planar and perpendicular to the field (figure 3.b). The current density $\mathbf{J}$ is also constant

throughout since both the electric field gradient and the conductivity are the same everywhere within the cylinder. If the cylindrical conductor is surrounded by nonconductive walls of finite thickness with the interface between the container and the brine parallel to the electric field in a tubular configuration, then the electric field and current density are also uniform throughout the brine (figure 3.c).

When the surfaces of a nonconductor are not everywhere parallel to an applied electric field, such as a quartz sandstone for instance, then the field within brine contained in the sandstone becomes distorted and non-uniform. Equipotential surfaces in the brine intersect nonconductive grains perpendicular to the brine-grain interface causing the surfaces to be curved within the conducting brine. The curved equipotential surfaces result in a non-uniform electrical field gradient which results in a current density that varies throughout the brine.

The electrical field gradient and the shape of the equipotential surfaces can be determined for any conductor-nonconductor configuration by discretizing the volume of interest and solving Laplace's equation at each point of the discretization, subject to appropriate boundary conditions, including potentials applied on the boundaries. The curved equipotential surfaces that result from the circular grains in a simple two dimensional "rock" are illustrated in figure

3.d. The electric field gradient is non-uniform resulting in a non-uniform current density distribution.

The streamlines along which electric currents flow are perpendicular to the equipotential surfaces. The streamlines pictured in figure 3.d enclose equal amounts of electric current between each pair. The streamlines are compressed in the constrictions, or pore-throats, between grains along the direction of the applied electric field and the current density is relatively high. The streamlines in the center of the pore between the constrictions are more separated indicating a lower current density. The separation between streamlines increases away from the center of the pore indicating that the current density becomes even less in the distal parts of the pore. The result of grain surfaces that are not parallel to the applied electric field is a non-uniform current density with high density in pore constrictions and along the predominant direction of current flow, and low current density in volumes remote from the main current path.

The current density in a thick-walled tube is uniform as described above when the tube walls are parallel to an applied electric potential. The tube walls do not distort the electric field in the tube since planar equipotential surfaces intersect the tube walls perpendicular to the tube surface. Under these circumstances the total current is maximum and the only impedance to current flow is the conductivity of the brine contained
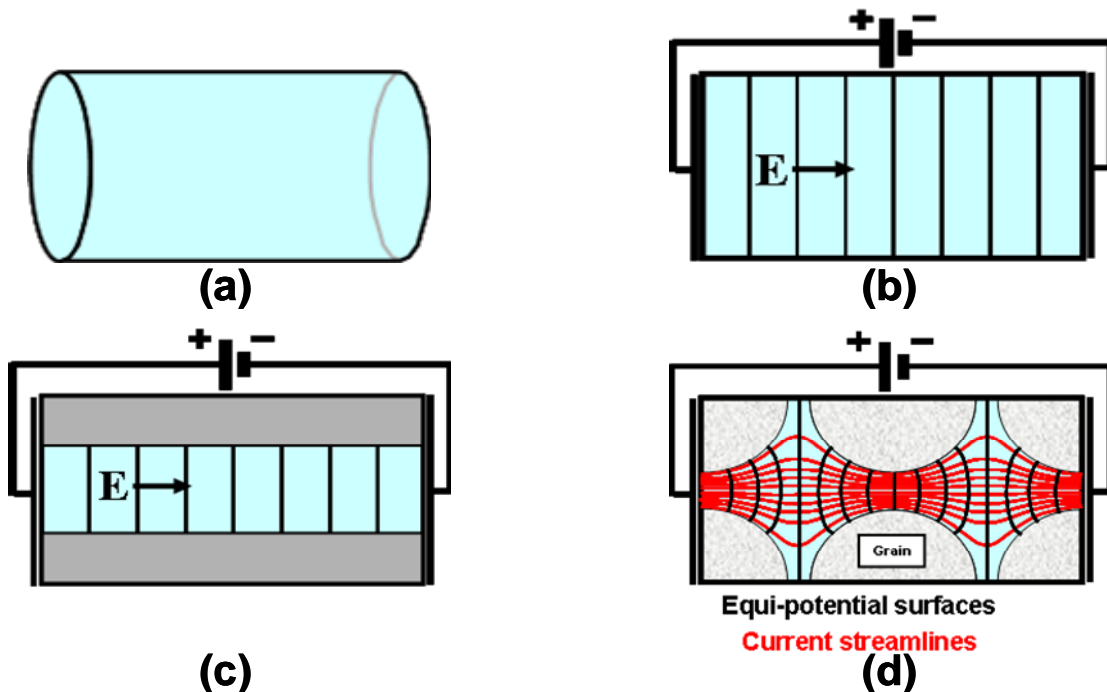

FIG 3. A thought experiment. (a) A thin-walled brine-filled tube. (b) Equipotential surfaces established in a brine-filled tube by a battery. (c) A thick-walled brine filled tube showing equipotential surfaces. (d) The wall material redistributed into a model pore system. See text for extensive discussion.

in the tube. Hence the conductivity of the tube, as a whole, is limited only by the brine conductivity and the brine content, or porosity of the tube. The conductivity of the tube is given by $C_0 = C_w \phi$, where C_0 is the conductivity of the tube, C_w is the conductivity of the brine and ϕ is the porosity.

When an electrical potential is placed across a simple two-dimensional "rock" model, the resulting electric field is distorted since the grain surfaces are generally not perpendicular to the applied potential (figure 3.d). The distorted field results in a non-uniform electric current density. The total current flowing through the rock model is less than that flowing in a tube of equal porosity. The diminished current is due to the smaller cross-sectional area of the constrictions, or pore throats and their concomitant higher resistance, and to the diminished current density in areas of the pore space located away from the region of main current flow. Most of the electric current is confined to pore constrictions along the electric field direction and in the center of pores between these constrictions. The current density can approach zero where pore surfaces are nearly perpendicular to the applied electric field direction or in pore volumes that are isolated by very small connections to the main current stream or even completely isolated with no connections preventing current flow through the volume.

The total current carried by a given volume of conducting brine in a container of a specified length can vary from a maximum if the container walls are parallel to the applied electric potential (a straight tube) to zero if the container has no end-to-end continuous conducting path. Any other container configuration, including the model "rock" (figure 3.d) will conduct an amount of current between the two extremes.

As an example, consider a porous cylindrical container whose pore volume has walls that define a tortuous tube of constant cross-sectional area (figure 4). The tortuosity of this tube can be quantified by $\tau = L_p / L$, where τ is the tortuosity, L_p is the conducting path length and L is the length of the container.
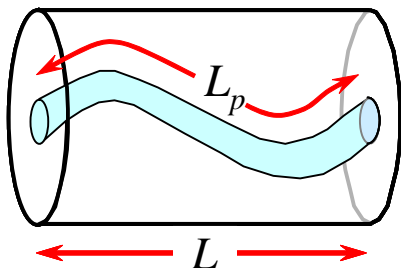

FIG. 4. A tortuous tube of constant cross-sectional area.

The conductivity of this container with a tortuous-tube pore space is given by $C_0 = C_w \phi (1/\tau)$. The factor $1/\tau$ corrects the tube conductivity $C_w \phi$ for the extra conducting path length specified by τ.

If the cross-sectional area of the tortuous tube is not constant, but varies making constrictions and expanded sections along its length (figure 5), then an additional correction must be made to account for the effect of the constrictions and variable tube shape. In general, an analytical correction for this geometry is not available, but it could be evaluated numerically for a given geometry using Laplace's equation, as above, or even experimentally. In any case, the additional geometric correction can be designated in general as a variable in the conductivity equation for a container with pore space comprising a tortuous tube with constrictions: $C_0 = C_w \phi (1/\tau) X$, where X is a geometric factor that accounts for the effect of the constrictions and expanded volumes.

The electrical effect of tortuosity $1/\tau$ can be combined with that of constrictions X and any other geometric complications one wishes to consider to form a single geometric factor $E_0 (= X/\tau)$ accounting for all geometric effects on conductivity. E_0 can equally well be used to describe the electrical effects of the geometry of the pore space in the model rock (e.g., figures 3.d and 5) or any real rock whose conductivity is solely due to conductive brine in the pore volume. The conductivity of any container with arbitrary pore geometry, including rocks, that conducts only through brine in the pore system can be described with

$$C_0 = C_w \phi E_0 \, . \qquad (6)$$

Solving (6) for E_0,

$$E_0 = \frac{C_0}{C_w \phi}, \qquad (7)$$

the numerator is the conductivity of the container with its complicated pore geometry and the denomi-
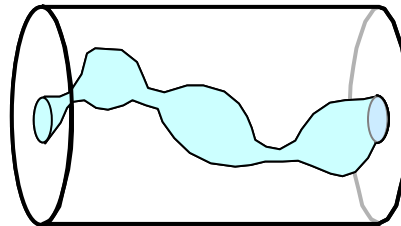

FIG. 5. A tortuous tube of variable cross-sectional area.

nator is the conductivity the container would have if there were no geometric complications; i.e. a tube. This equivalent tube exhibits the maximum conductivity possible for a given size container and porosity. E_0 expresses how well brine in the container conducts compared to how well it could conduct if the brine geometry were a tube and the conductivity maximized. Hence, E_0 expresses the efficiency with which water in the container conducts electricity. The value of E_0 is one if the container is a tube and zero if the container has pore space that is entirely isolated and cannot conduct.

In principle, E_0 can be evaluated numerically for any container or rock sample if the pore geometry is sufficiently well characterized. In practice, this is seldom, if ever, practical since in any small volume of rock there are tens of thousands of pores, each one connected to many others. Moreover, the details of electric current circulation in the pore system will depend on the direction of the applied electric field. Therefore, in practice, E_0 must be evaluated experimentally by measuring the porosity and conductivity of the container using brine with known C_w.

If both sides of (6) are divided by C_w, then $C_0 / C_w = \phi E_0$ results. The right side of this expression obviously does not depend on brine conductivity. The ratio on the left side must likewise be independent of brine conductivity, depending solely on properties of the rock's pore system.

The ratio C_0/C_w is clearly the reciprocal of Archie's formation resistivity factor, $F = R_0/R_w$ and can be designated the formation *conductivity* factor f and $f = C_0/C_w = \phi E_0$. The right side makes it clear that f depends both on the amount of brine, represented by ϕ, and the geometrical distribution of brine, represented by E_0. Thus, the formation conductivity (or resistivity) factor is compounded from two, more fundamental, variables, ϕ and E_0, a fact that goes unremarked upon in conventional interpretation theory. As a consequence, f is not uniquely related to porosity since a single value of f can result from various combinations of ϕ and E_0. Obviously, the formation conductivity factor changes as porosity varies. However, the quotient f / ϕ is independent of porosity, hence the factor $E_0 = f / \phi$ is independent of both porosity and brine conductivity. By its construction, the parameter E_0 depends only upon, and quantifies, the electrical effect of the distribution of the conducting phase; i.e., the brine geometry.

Dependence on the brine geometry: $e_t E_0$

To this point our discussion has been confined to brine-saturated rocks. Rocks of central interest are hydrocarbon-bearing. Hydrocarbons, whether liquid or gaseous, are nonconducting. In Archie rocks, brine remains the only conducting phase, but the amount of brine is reduced, some brine having been displaced by hydrocarbon. This reduction in the conducting phase is expressed as a fraction of the total pore space in a rock. The fractional volume of pore space occupied by brine S_w varies from a small number close to zero, to unity when the pore space is totally brine-filled. The fractional volume of brine in the bulk rock is then $S_w \phi$. Thus, S_w can be considered a coefficient applied to the porosity that expresses the reduction of the amount of brine in the pore system. The amount of brine cannot be reduced without a concomitant change in the geometry of the conducting phase. This change is quantified by a coefficient, e_t, which is applied to the basic pore-geometrical factor E_0. The geometrical factor for partially brine-saturated rocks is represented by $e_t E_0$. Thus

$$C_t = C_w (S_w \phi) (e_t E_0) \qquad (8)$$

where parentheses are used to group the factors that quantify the electrical effect of the amount of brine, and the distribution of brine.

The resulting bulk conductivity can be viewed as the result of two separate and sequential geological processes. The ϕ and E_0 factors are determined by the geologic processes that make rocks; i.e., deposition and digenesis, including compaction, cementation, and dissolution/recrystalization. The subsequent emplacement of hydrocarbons into the pore system is governed by the height of the hydrocarbon column and the capillary properties and pore-throat size distribution of reservoir rocks, with the concomitant displacement of brine, changing the saturation S_w and geometric distribution e_t of the remaining brine. Equation (8) and algebra yields

$$e_t = \frac{C_t}{C_w S_w \phi E_0} = \frac{C_t}{C_0 S_w} = \frac{i}{S_w} \qquad (9)$$

and, from (9) it is seen that e_t is the slope of the line in the graph of $i = e_t S_w$ where i is the formation conductivity index (reciprocal of the formation resistivity index).

The theoretical developments to this point can be summarized succinctly as $C_t = f i C_w$, where $f = \phi E_0$ and $i = e_t S_w$ with E_0 and e_t having been constructed explicitly as geometrical factors.

PRACTICAL APPLICATION

Determination of E_0

The geologic processes that produce clastic sedimentary rocks, including deposition, compaction and diagenesis, are continuous. These processes change porosity and pore geometry simultaneously causing a correlation between the geometric factor E_0 and the volumetric factor ϕ. With diagenesis, for example, porosity is commonly reduced, its pore geometry changes and its geometric factor and efficiency as a conductor also diminishes. Likewise, the displacement of conductive brine by non-conducting hydrocarbons is also a continuous process creating a link between the brine saturation S_w and the electrical properties due to the geometry of the remaining brine, $e_t E_0$. While the existence of these correlations is obvious, it is not at all clear what functional form the correlations should take. Due to the intractability of a geometrical description, it is impossible to establish formulas relating E_0 and e_t to the complicated geometry of conducting brine in a porous rock; they must be determined experimentally.

Being geometrical factors, inherently independent of brine content, E_0 or e_t in principle can be written down in terms of dimensional parameters (e.g., lengths, widths, heights, angles, etc.). Unfortunately, it is possible to derive E_0 only for a limited number of
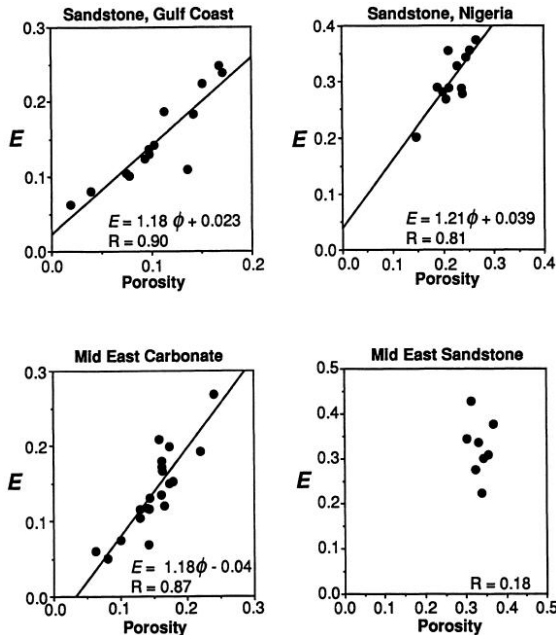

FIG. 6. Plots of the geometrical factor, E_0, against porosity for rock samples from four reservoirs. The scatter in the data are typical of resistivity porosity data. Where there are trends, they are linear.

very simple models (Herrick and Kennedy, 1993, 1994).

Only simple and small pore systems can be characterized by numerical modeling. On the other hand, real pore systems are much too complicated and extensive to characterize in formulas that honor the geometry of each pore. An idea of the complexity can be appreciated by considering a cubical sample bound by edges, each having 100 mineral grains. The total number of grains is $100^3 = 10^6$, and the number of pores and pore throats is comparable. The number of pores increases with the cube of the length of an edge. This renders pore-scale modeling of realistic cases practically intractable.

To establish useful ϕ–E_0 correlations, one must choose samples from a single formation that are likely to have experienced similar geologic histories, but to different degrees, usually manifested as a range in porosity. Measurement of the conductivity along with porosity allows establishment of the existence and degree of correlation. A lack of correlation indicates that samples have had different geologic histories and that results of two or more different processes are represented. If a good correlation exists, it indicates that only a single geologic process resulted in the variation in porosity and pore geometry (e.g. quartz cementation). Unfortunately there is very little published documentation that relates geologic processes to their pore volumetric and geometric results. One such study is that of Beard and Wyle (1973) who documented changes in porosity as the result of changes in sorting.

Measurements of ϕ and E_0 have established that they are linearly related (figure 6)

$$E_0(\phi) = a_0\ \phi + b_0 . \qquad (10)$$

This is consistent with Archie's observation that the porosity exponent m typically has a value of about two. Since $f = E_0\phi = \phi^m$ and m is often about two, E_0 and ϕ should be approximately equal. This is the observed relationship. If m is exactly equal to two, then the correlation between E_0 and ϕ is linear with a slope a_0 of one and intercept $b_0 = 0$ at the origin.

The laboratory desaturation process is performed on individual samples and is continuous. An excellent correlation between S_w and e_t is therefore expected. In practice this is achieved (figure 7). Figure 7 is expressed in terms of E_t, the geometric factor of a partially saturated rock sample. E_t is simply the product $e_t E_0$ and E_0 is a constant for each sample). The ob-

served relationship is $e_t = a_t S_w + b_t$. Evaluated at $S_w = 1$, this relationship demands that $b_t = 1 - a_t$. Thus

$$e_t = a_t S_w + (1 - a_t) \tag{11}$$

has only a single adjustable parameter.

Since the conductivity index $i = S_w^n = e_t S_w$, and since the value of the saturation exponent n is typically about two as Archie (1942) reported, then e_t is ex-

pected to have a value similar to that of S_w. This is indeed the case since the e_t–S_w relationship is typically observed to be linear with a slope of about one.

The relationship between Archie's equation and the pore geometric theory of conductivity is clear since they both describe the same phenomenon. Archie's equation, however, is entirely empirical while the geometric theory is based on the physics of conduction.
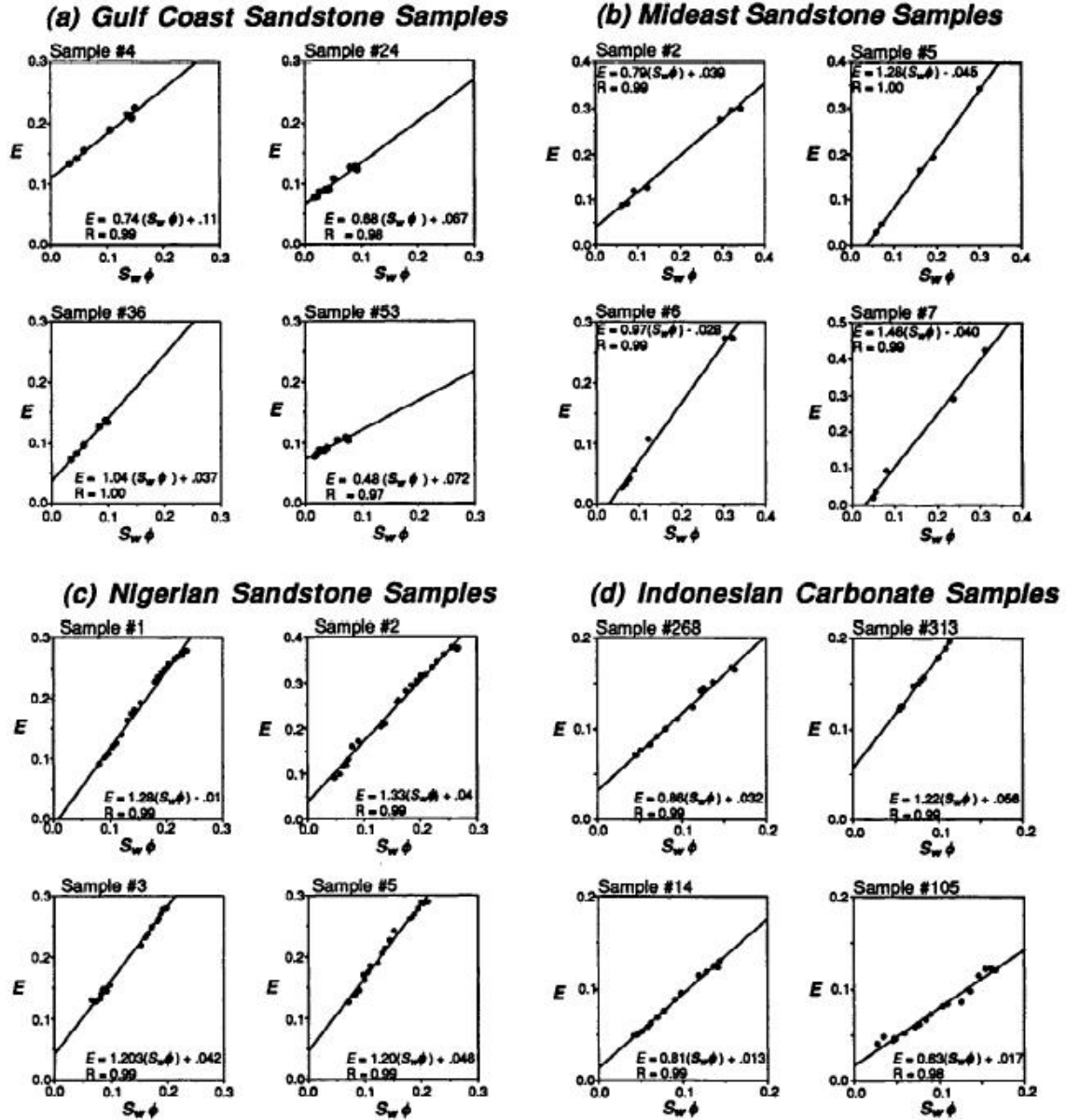

FIG. 7. Plots of the geometrical factor, $E_t = e_t E_0$, against brine volume ϕS_w for sixteen rock samples. The brine geometry is changing only in response to the displacement of brine by a nonconducting phase. That the trends are linear is obvious.

SATURATION EQUATION

With a_0, b_0, and a_t determined experimentally, and recalling

$$C_t = C_w (S_w \phi) (e_t E_0) \qquad (8)$$

equation (8) is easily rearranged to

$$\frac{C_t}{C_0} = S_w \, e_t \, . \qquad (12)$$

Substituting for e_t, equation (12) is written as

$$S_w\left(a_t S_w + (1-a_t)\right) - \frac{C_t}{C_0} = 0 \qquad (13)$$

which is a quadratic equation for S_w, having the solution

$$S_w = \frac{a_t - 1 + \sqrt{(1-a_t)^2 + 4a_t (C_t/C_0)}}{2a_t} \qquad (14)$$

where $C_0 = C_w \, \phi \, (a_0\phi + b_0)$. Thus, given a_0, b_0, and a_t observed in the laboratory, and an estimate of C_w, well logs for $C_t \, (= 1/R_t)$ and ϕ can be converted into a computed log for S_w.

The conductivity representation of rock electrical properties is algebraically simpler than the resistivity representation since reciprocal powers are not required. Indeed, Archie's equation is more simply expressed in conductivity representation. The pore geometric theory is neither better nor worse than Archie's model at predicting S_w from ϕ and C_t. It is not in their roles as water saturation predictors that the most fruitful comparisons are made, but in the interpretability of their adjustable parameters.

It is interesting to compare the form of the two models:

$$C_t = a^{-1} C_w \, \phi^m \, S_w^{\,n}; \qquad (2)$$

$$C_t = C_w \, S_w \, \phi \, (a_0 \, \phi + b_0) \, (a_t \, S_w + (1 - a_t)). \qquad (15)$$

From a statistical point of view, there is no difference in the efficacy of these models at predicting S_w given the same laboratory and log data; i.e., their correlation coefficients (i.e., R^2) are almost equal. The obvious morphological difference is that Archie's model is capable of more parsimonious expression than the pore geometric theory. The most obvious

$$a_t S_w \varphi + b_t = a_0 \varphi + b_0$$

$$\left(a_t S_w \varphi + b_t\right)\Big|_{S_w=1} = a_t \varphi + b_t = a_0 \varphi + b_0$$

$$a_t \varphi + b_t = a_0 \varphi + b_0$$

$$b_t = a_0 \varphi + b_0 - a_t \varphi$$

$$b_t = \left(a_0 - a_t\right)\varphi + b_0$$

similarity is that both models have three adjustable parameters: a, m, and n, versus a_0, b_0 and a_t. This latter observation strongly suggests that *whatever* model is used, three adjustable parameters are the minimum number required to best fit the observations. The most important difference is the interpretation of the adjustable parameters. Despite much effort, no one has succeeded in finding any physical significance to a, m, and n other than by a posteriori correlation.

Conversely, $E_0 = a_0 \, \phi + b_0$ and $e_t = a_t \, S_w + (1 - a_t)$ have a priori physical significance at two levels. In the first place, the designed-in physical interpretation of E_0 and e_t compares the efficacy of the conducting phase to its most conductive configuration, and in that sense E_0 and e_t are direct measures of the complexity of the pore network without, and with, hydrocarbons emplaced, respectively. At a more subtle level, the slope of e_t vs. S_w is an indicator of the rate that at which brine geometry becomes complicated with hydrocarbon emplacement, and the slope of E_0 vs. ϕ is a fossil recording of diagenetic processes that have produced the present day rock from the original sediment, hinting at the impact of diagenetic processes on pore geometry. No doubt experience with a_0, b_0, and a_t would produce much discussion regarding their best interpretation in various geologic settings, but the starting point for such discussion would have more significance than a purely arbitrary curve-fitting parameter.

FITTING EQUATIONS

The oil and gas industry is very conservative. Revolutionary technologies typically require 10 years to replace the technology which they supplant (Robert Truman, 1995, *personal communication*). However, new technologies that meet an existing and unfilled need are embraced much more rapidly. Demand for well logging after its introduction was nearly exponential (Schlumberger 1934) as it met an unfulfilled need, viz., a visual and permanent record of formations that could be successfully correlated with productive intervals, and used as a guide for well completions. This led to the collection of voluminous data generating an incentive to discover quantitative relationships between formation resistivity as observed on logs and hydrocarbon producibility and volume. Likewise, Archie's model filled a void. The universal and nearly immediate acceptance of Archie's pioneering study reflects that it was timely, and filled a pressing need for a serviceable method for using log data. Archie uncovered a relationship so efficacious at predicting hydrocarbon saturations from resistivity logs that it was never analyzed critically. Indeed, it

has been raised to the status of a natural "law" by many authors.

Endowment of Archie's model with intrinsic but un-discovered physical significance led to a cottage industry attempting to divine the physical significance of the adjustable parameters in Archie's model (and Archie's model as modified by Winsauer et al., 1953). Thus, m has been become ubiquitously known as the "cementation" exponent (Guyod, 1944), and a characterized as a "tortuosity" factor (Schopper[1], 1966), although not many contemporary formation evaluators can articulate what m has to do with cementation, or a with tortuosity. In fact, Archie himself never endowed the "trends" that he observed with any significance beyond useful correlations (Archie, 1942, 1950).

Archie first presented his results in late 1941. He was collecting and analyzing his data in the late 1930s. His analysis tools included graph paper of these types: linear, semi-log, and log-log. The relationship of voltage to current was invariably expressed as Ohms law in the form $V = IR$. Archie's data both from the laboratory and from logs were expressed in terms of resistivity, and he used his resistivity data as it was delivered to him – without any application of mathematical transformations. The trend of formation resistivity plotted against formation porosity on log-log paper is unquestionably linear. But, such tidy relationships are not unique. There are numerous alternative ways to view (or plot) the data that have equally compelling trends that make equally efficacious predictions.

Figure 8a illustrates Archie's Nacatoch sandstone data plotted in the conventional log-log representation. Even in this representation the points can be fit with a straight line in more than one way. A commonly used spreadsheet program has a trendline utility providing five common functions[2]. Using this utility in the log-log space gives the blue line, $R^2 = 0.88$. If the logarithm of F is plotted against the logarithm of ϕ in linear space, and the trendline used to fit a line to the data, with the result plotted in log-log space, the red line is the result with $R^2 = 0.90$. Each line minimizes the sum of squared residuals in its own domain. The difference in correlation coefficients may appear to favor the latter procedure over the former, but actually the difference is not significant. For example, if various data points are omitted from the sample population before the curve fitting, not only would the correlation coefficients change,

the relationship between correlation coefficients magnitudes might reverse. And in any event, it is clear by inspection that both lines are good predictors of the trend in the data. This example uses the seminal data set and classical presentation to underscore the arbitrariness associated with trend prediction from data. There are yet other ways to represent the trend with equally efficacious predicting power.

Figure 8.b plots the Nacatoch sandstone data in linear porosity-resistivity space. The data can be imagined to scatter about the limb of a hyperboloid of the form $F = a \,/\, (\phi - \phi_0)^m$ with $m \approx 2$. This function fits the data with a correlation coefficient $R^2 = 0.90$, essentially the same predicting power as the log-log representation.

The ratio between a voltage (applied or induced) and current (induced or applied) quantifies a key electrical property of a rock. If the ratio is expressed as voltage/current it is called resistance and the associated material property is called resistivity. Resistivity was chosen as the classical unit, probably because Ohm and Kirchhoff worked in resistivity space ($R = V/I$), leading to circuit theory being taught to
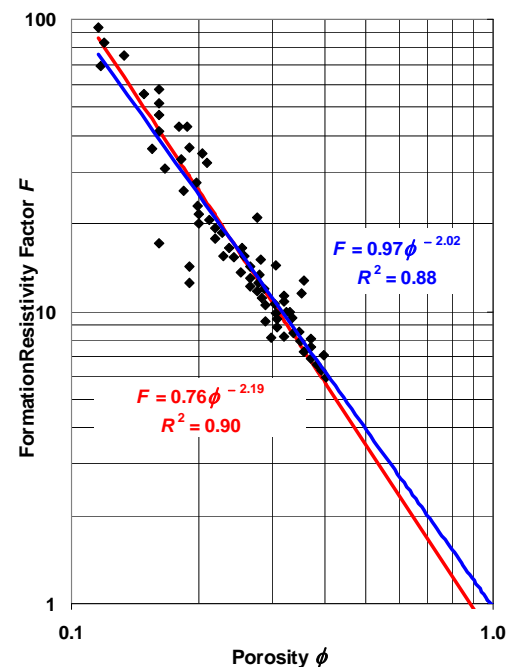

FIG. 8.a. There are many ways to characterize the relationship between F and ϕ even in seemingly the same space. The blue curve represents a least squares trend fitted by Excel's trendline tool with the choice of the power law function. The red line was obtained by least squares fitting a trend to the logarithms of the data.

---

[1]See the Appendix for history of provenance of these terms.
[2] If the Nacatoch data set is transformed into conductivity space, each of the five functions will fit the data with $R^2 > 0.8$.

engineers in terms of resistivity, and the subsequent invention of surface electrical prospecting being represented in terms of apparent resistivities, which in turn was carried over into well logging, which was invented by surface electrical prospectors. Consequently, the data supplied to the formation evaluation pioneers came to them in the form of resistivity and apparent resistivity. However, it happens that the electromagnetic field theory representation of Ohm's law that relates current density $\mathbf{J}$ to electric field intensity $\mathbf{E}$ ($\mathbf{J} = C_0\mathbf{E}$ in our notation) is more convenient for most theoretical purposes, circuit theory being the main exception.

If Archie's Nacatoch sandstone data is converted to apparent conductivity, quite a number of adequate fitting functions can be found. Figure 8.c plots the data in linear porosity – linear formation conductivity factor space, and uses the so-called "pseudo-percolation threshold theory" (Kennedy, 2007) as the fitting function. The function and values of the adjustable parameters that minimize the sum of squared residuals is shown on the figure. The correlation coefficient, $R^2 = 0.87$, is comparable to correlation coefficients in other panels of figure 8.

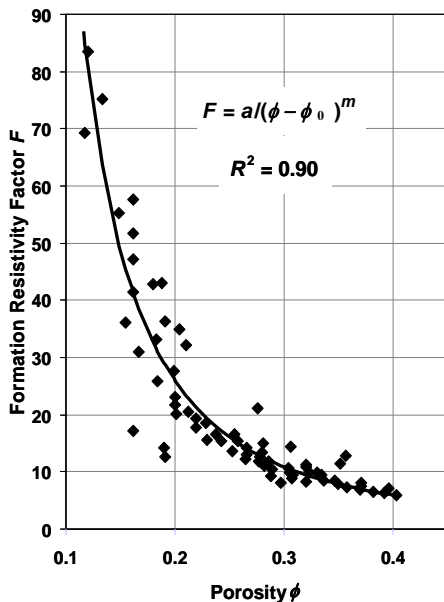Figure 8.d fits the Nacatoch sandstone linear conductivity – linear porosity representation to a quadratic

polynomial. The resulting correlation coefficient is $R^2 = 0.88$, again comparable to other correlation coefficients found for different fitting functions applied to this data. Many other fitting functions with equally adequate correlation coefficients are possible. The adjustable parameters of such functions will not in general have any physical interpretation, other than they must somehow contain the pore geometry of the models, as it does not appear in the other model parameters, viz. C_0, ϕ, S_w.

What do we learn from the observation that there are many equally efficacious means of predicting formation conductivity (or resistivity) factor from porosity, and many equally efficacious means of predicting conductivity (or resistivity) index from water saturation? We would make three points: (1) there is nothing special about Archie's model, and therefore further effort to understand it in terms of first principles would be effort better spent in other directions, and (2) given that there are many predictive correlations that can be drawn among the data, in some sense the simplest model with physically interpretable adjustable parameters is the "best" choice, and (3) such a model built upon a first-principles foundation with a priori physically interpretable adjustable parameters is relatively easy to construct.
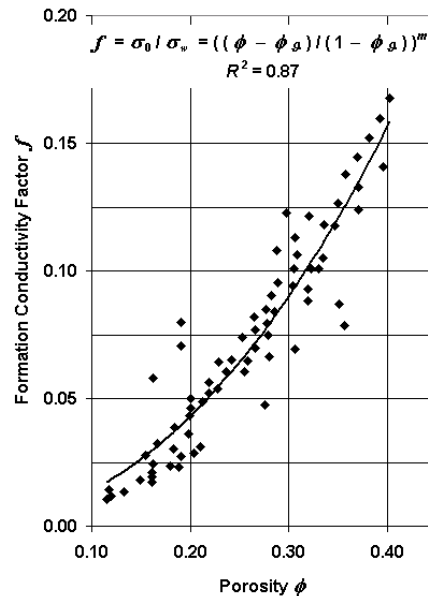

FIG. 8.b. Archie's data fitted using a hyperbola and three adjustable parameters, as shown by the equation on the chart. Note that the coefficient of determination, R^2, is similar to the R^2 obtained for the logarithmic fittings shown in figure 8.a. The resulting equation is $F = 0.86/(\phi - 0.12)^{2.04}$.

FIG. 8.c. Archie's data transformed into conductivity space, and fitted using a function from pseudo-percolation threshold theory with three adjustable parameters, $f = ((\phi - 0.047) / (1-0.047))^{2.18}$ (Kennedy, 2007). Note that R^2 is in the same range as for the other fitting functions.

SUMMARY AND CONCLUDING REMARKS

Like many researches before us, we have puzzled over Archie's equation, the industry-standard working relationship among resistivity, porosity, and water saturation. Those in our, and following, generations inherited the method in use from the pioneers of the techniques. The pioneers were practical people primarily concerned with using well logs for the identification and quantification of reserves. Their empirical solution to that problem served them well, and has continued to serve well to this day. However, Archie in particular made no hypothesis regarding why the particular rule that he had selected to quantify the relationship worked so well. Indeed, Archie cautions against attributing too much significance to the correlations that he discovered and exploited.

In spite of Archie's cautionings, many who followed the pioneers have attempted to endow the Archie method with physical significance, even referring to it as if it were a physical "law" of nature. And it has been exported to the wider world of science under

FIG. 8.d. Archie's data transformed into conductivity space, and fitted with a quadratic polynomial,

$$f = 0.59(\phi - 0.135)^2 + 0.35(\phi - 0.135) + 0.0185.$$

Note that R^2 is in the same range as the other fitting functions. There are four adjustable parameters in this function.

that banner. Like most of today's practitioners, our own teachers accepted the Archie method for daily

this banner without much concern for what physical principles might underlie the method.

To us, a more lucid explanation seemed possible, and to our satisfaction, we have found one, as reviewed in this article. In developing a relationship, insofar as is possible, between bulk rock conductivity, brine conductivity, porosity, water saturation, and brine geometry we were led to appreciate Archie's method for what it is, a curve fitting technique. This appreciation led to the discovery that *many* methods of curve fitting as efficacious as Archie's method could have been applied to Archie's original data. In this sense the particular method that we attribute to Archie is an accident of history. This being true, there is no particular significance (other than historical) to Archie's method

Still, there is a difference between the pore-geometric theory described herein and other curve-fitting schemes. The adjustable parameters of Archie's method have been associated with brine geometry by a posteriori realization that the brine geometry is not included in C_w, ϕ, or S_w, and so must be found in a, m, and n. On the other hand, the pore-geometric theory is assigned variables to explicitly account for brine geometry a priori. Like a, m, and n, the fitting parameters relating E_0 and ϕ and e_t and S_w (a_0, b_0, and a_t) are also determined by curve fitting; however, the connection of the latter to brine geometry is explicitly clear.

Empirical equations like Archie's are useful for, but limited to, making calculations. Descriptive equations that have a basis in physics can also be used for calculations, but are far more powerful since they give the practitioner the additional ability to understand the results of calculation in terms of pore geometry as well as making predictions.

This research has successfully uncovered the relationship of the industry-standard resistivity interpretation method to data modeling, the concomitant lack of necessity for any underlying physical significance for its adjustable parameters, and its efficaciousness at predicting water saturation from its input data. It has also produced a model equally efficacious as a predictor, but built from a priori physical principles, giving a priori physical significance to its adjustable parameters.
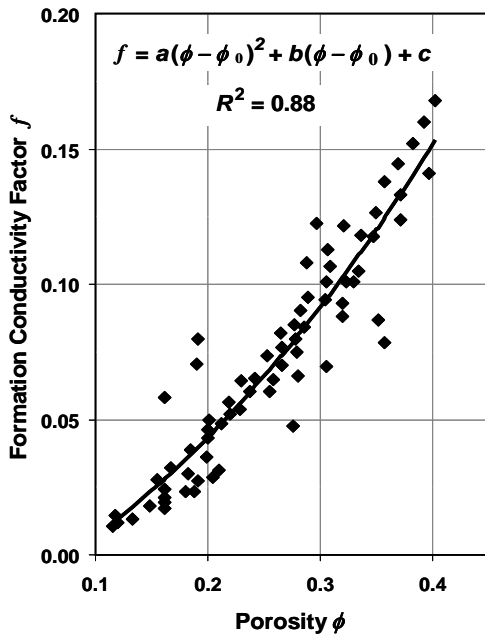
REFERENCES

Archie, G. E., 1942, The electrical resistivity log as an aid in determining some reservoir characteristics, *Transactions*, AIME, vol. 146.

Archie, G. E., 1950, Introduction to Petrophysics of Reservoir Rocks, *Bulletin of the AAPG*, vol. 34, no. 5, 943-961.

Beard, D.C., and Wyle, P.K., 1973, Influence of texture on porosity and permeability of unconsolidated sand, *AAPG Bulletin*; February 1973; Vol. 57, No. 2, pp. 349-369.

Guyod, Hubert, 1952, *Electric Well Logging Fundamentals*, Part 12, Fundamental Data for the Interpretation of Electric Logs, Well Instrument Developing Co., p. 76. (Probable original source is *Oil Weekly*, Fundamental Data for the Interpretation of Electric Logs, Oct. 30, 1944.)

Herrick, D. C. and W. D. Kennedy, 1993, Electrical efficiency: a pore geometric model for the electrical properties of rocks, *Transactions*, SPWLA 34th Annual Logging Symposium June 13-16, paper HH.

Herrick, D. C. and W. D. Kennedy, 1994, Electrical efficiency-A pore geometric theory for interpreting the electrical properties of reservoir rocks, *Geophysics*, vol.59, No. 6, pp. 918-927.

Herrick, D. C. and W. D. Kennedy, 2009, A New Look at Electrical Conduction in Porous Media: A Physical Description of Rock Conductivity, *Transactions*, SPWLA 50th Annual Logging Symposium June 21-24, this volume.

Kennedy, W.D., 2007, The Porosity-Water Saturation-Conductivity Relationship: An Alternative to Archie's Model, *Petrophysics*, Vol. 48, No. 5, pp. 335-361 .

Jakosky, J.J., Hopper, R. H., 1937, The Effect of Moisture on the Direct Current Resistivities of Oil Sands and Rocks, *Geophysics*, Vol. 2, No. 1, pp. 33-54.

Schlumberger, C., Schlumberger, M. Leonardon, E. G., 1934, Electrical Coring; a Method of Determining Bottom-hole Data by Electrical Measurements, Trans. AIME, vol. 110, p. 237-72.

Schopper, Jurgen, 1966, A Theoretical Investigation on the Formation Factor/Permeability/Porosity Relationship using a Network Model, *Geophysical Prospecting*, vol. 14, pp. 301-341.

Winsauer W. O., H. M. Shearin, P. H. Masson and M. Williams, 1952, Resistivity of Brine-Saturated Sands in Relation to Pore Geometry, *AAPG Bulletin* 36 (1952): 253-277.

Wyllie, M. R. J., Gardner, G. H. F., 1958, The Generalized Kozeny-Carman Equation, its Application to Problems of Multiphase Flow in Porous Media, *World Oil*, (Mar), 121 + (1958) (Apr), 210.

APPENDIX

A portion of Schopper's doctoral thesis appeared as an article in *Geophysical Prospecting* in 1966. This article appears to be the origin of the "tortuosity" coinage for the *a* in Winsauer and others (1952) formula, but Schopper does not apply the term to Winsauer's *a*. There are several errors in Schopper's citations, including the attribution of the term "cementation" exponent to Archie (in fact this coinage is due to Guyod, 1944). Following shortly after his misattribution of the *cementation exponent* coinage, Schopper appears to have coined the term "electrical tortuosity factor" to refer to a term X in the expression $F = X/\phi$, with $X \approx 1/\phi$, attributed to Wyllie and Gardner (1958). (Note that $X = 1/E_0$ in terms of GFT explicated herein.) Schopper applied the term "electrical tortuosity" to X. Subsequent authors apparently misidentified X with the Winsauer *a* factor, which Schopper never mentions. Such misidentification could not have been made if Schopper's equations (5), (6), and (13)-through-(16) had been properly understood and compared to Winsauer et al (1952). Schopper himself misidentifies the original source as Wyllie's book, which makes no mention of $F = X/\phi$, and cites Wyllie and Gardner (1958) as the source of (13)-(16). Even allowing that Schopper would have translated Wyllie's and Gardner's notation to his own notation, it is a stretch to find a one-to-one correspondence of Schopper's (13) to anything in Wyllie

and Gardner, let alone all of Schopper's (13)-(16). It is probable that Wyllie did make such observations, given his prolific record as an author and coauthor on this topic. However, (13)-(16) in Schopper appear to be misattributed to Wyllie and Gardner. The result of all this compounded comedy of lackadaisical scholarship is Winsauer's *a* is now universally known as the "tortuosity factor" more by reason of poor scholarship, rather than any demonstrated relation to tortuosity.

BIOGRAPHIES

David C. Herrick, is currently semi-retired and singularly available for consulting. He was trained in chemistry and geochemistry at Indiana University (B.S.) and Penn State (Ph.D.). Dave has conducted research, training and technical service during his thirty years in the petroleum industry for Conoco, Amoco, Mobil and Baker Hughes in the areas of geochemistry, petrology and petrophysics. He has recently been Senior Petrophysical Advisor and Chief Petrophysicist in the Houston Technology Center of Baker Hughes Inc. Dave has given many presentations and schools on petrophysics, resistivity interpretation, the meaning of porosity terminology and the impact of pore geometry on permeability and conductivity. He has been author or coauthor on about 20 publications and is the inventor on two patents.

W. David Kennedy is a Senior Research Advisor in Baker Hughes Inteq's Strategic Technology and Advanced Research group in the BHI Houston Technology Center. Dave trained in physics at Georgia Tech and U. T. Dallas, and geoscience at U. T. Dallas, with further training at U. C. Berkeley; and was employed serially by Schlumberger, Arco, Lockheed, Mobil, and ExxonMobil, and most recently at Baker Hughes Inteq. David stayed with ExxonMobil for 20 years to prove to his mother that he could hold a job. Having convinced his mother, David retired from ExxonMobil in 2007. He is coinventor and inventor on six patents and coauthor or author 30 or so publications, and is not quite done yet. Along the way, Dave has edited the SPWLA journal, *Petrophysics*, for four years, and served the SPWLA as VP Publications during that time.