

# The Porosity-Water Saturation-Conductivity Relationship: An Alternative to Archie's Model<sup>1</sup>

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## ABSTRACT

The bulk volume fraction of conducting brine  $\beta$  in a reservoir rock is given by the product of porosity and water saturation,  $\beta = \varphi S_w$ , symmetric in  $\varphi$  and  $S_w$ . Bulk conductivity  $\sigma_t$  of a rock varies as this product varies. When  $\beta = 1$  bulk conductivity necessarily equals brine conductivity,  $\sigma_t = \sigma_w$ ; conversely, bulk conductivity typically vanishes at some  $\beta < 0$ . A mathematical description of the physical relationship underlying bulk rock conductivity and brine conductivity and the bulk volume fraction of brine must be constrained by these properties.

The classical three-parameter relationship connecting bulk formation conductivity to porosity and water saturation is an ad hoc formula comprising two separate power laws proposed by Archie in 1941 and modified by Winsauer et al. in 1952. The three adjustable parameters  $a$ ,  $m$ , and  $n$ , in the usual formulation of these laws are chosen, for particular data sets, by the method of least squares to minimize sums of squared residuals between observed and predicted data. The resulting theory depends upon the term  $\varphi^m S_w^n$ , breaking the symmetry inherent in the bulk volume brine fraction. While the laws have proved useful for predicting water saturations by interpolation within the bounds of observed data sets, they do not usually extrapolate to correct values at the limits of their domain of applicability, sug-

gesting that the Archie double-power law formulation cannot correctly represent the underlying physics.

The classical power laws devolve from an arbitrary selection of one particular class of fitting function from among several possible choices. It is possible to use the same porosity-resistivity data sets used by Archie and Winsauer to define an alternative three-parameter model. The proposed model's adjustable parameters are critical values of porosity, water saturation and conductivity and can be interpreted directly in terms of the percolation threshold of the medium. The proposed model simultaneously (1) treats the fractional brine volume  $\beta$  as the fundamental variable, (2) accommodates the boundary condition  $\sigma_t = 0$  at  $\beta = 0$  and satisfies  $\sigma_t = \sigma_w$  at  $\beta = 1$ , and (3) gives a smaller sum of squared residuals using the classical data sets than the classical models. Many additional insights into the conductivity-porosity-water saturation relationship follow from this new model.

The efficacy of the model is illustrated using Archie's and Winsauer's original data by better predicting conductivity from porosity, demonstrated by a smaller sum of squared residuals using those data, and using conductivity-porosity-water saturation data published by Hamada and others in 2002.

**Keywords:** Archie, water saturation, percolation theory, rock conductivity

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## INTRODUCTION

In situ estimates of bulk formation resistivity made on petroleum reservoirs by remote sensing instruments located in wellbores (i.e., resistivity logs) have been among the primary observations used in evaluating hydrocarbon volumes for more than a half-century, and resistivity estimates continue to remain an important element of formation evaluation. However, whereas the engineering of the data acquisition technology has kept pace with developments in new materials and new electronic methods, interpretation technology remains—perhaps arguably—unimproved for more than 60 years.

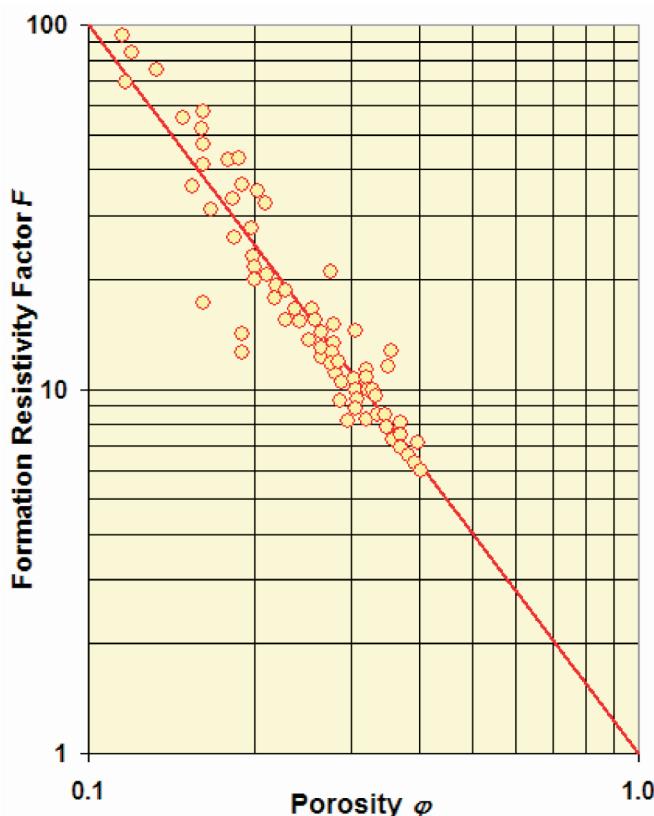
Empirical relationships among the bulk formation properties that determine the electrical resistivity of reservoir rocks were invented in the 1940s and 1950s. Originating as ad hoc models, these descriptions have never been understood, at least not from a classical-theoretic point of view. This is despite considerable effort to discover a simple pore-scale formulation of the problem that will scale up to the empirical models (See Jin et al., 2007, and the comprehensive reference list therein.) Conversely, this article aims to return to the seminal observations of the electrical behavior of reservoir rocks to investigate whether an entirely dif-

ferent macroscopic formulation of an empirical formation electrical properties model can lead to a better understanding of the empirical descriptions, new physical insights, and a new interpretation paradigm.

The industry-standard porosity–water saturation–resistivity interpretation method in “clean” (i.e., clay- or shale-free) clastic rocks, and some limestones, is based upon the analysis of the data sets used by Archie (1942) and Winsauer et al. (1952) (hereinafter referred to simply as “Winsauer”); the former resulting in a model known as Archie’s “law”, the latter giving rise to a minor generalization of Archie’s law known as the “Humble formula”.

The seminal data sets are simply ordered pairs of porosity–formation resistivity factor observations on core plugs arranged as two columns of numbers. Archie searched for a simple relationship relating the two columns. His search was constrained by the tractability of the representation chosen. There are many possible choices that Archie could have made, but one of the most tractable for the time, considering that in the 1940s every engineer would have had a desk drawer full of semi-log and log-log graph paper, was the plotting of the logarithm of porosity  $\varphi$  against the logarithm of the formation resistivity factor,  $F = R_0/R_w$ . Especially for Archie’s Nacatoch sandstone data, a linear relationship between the logarithm of  $\varphi$  and the logarithm of  $F$  (i.e., a power law) is strongly suggested (Figure 1). The suggested relationship is a power law expressed as  $F = \varphi^{-m}$  with  $m = 2$ . Note that this law correctly predicts that if  $\varphi = 1$  then  $F = 1$ . Archie’s plot has been retroactively interpreted as having physical significance, although Archie<sup>1</sup> himself never made this claim, and there has never been a convincing argument from any physical first principle that suggests a power law in this form is dictated by the laws of physics.

Application of Archie’s model to virtually every data set in the 1940s decade was assured, there being no alternative theory. Many instances must have been encountered where a rote application of Archie’s model did not fit the observations very closely. In fact, Archie in his seminal paper offered a second data set titled “gulf coast sandstones” for which the linear trend between  $\log\varphi$  and  $\log F$  was less pronounced, albeit still suggested. Winsauer and his coauthors revisited the question of predicting resistivity from porosity and suggested that if the requirement that the fitting line contain the point ( $\varphi = 1, F = 1$ ) were relaxed, the observations could be more closely fitted using a power law of the form  $F = a\varphi^{-m}$ ,  $a$  not necessarily equal to 1. Archie had noted that  $m = 2$ , and consequently for convenience he used a square root in his saturation equation. Based on a (very



**FIG. 1** Archie’s Nacatoch sandstone data.

<sup>1</sup>“It should be remembered that the equations given are not precise and represent only approximate relationships.” Archie, 1942

small) data set assembled from around the world (Figure 2), Winsauer proposed  $F = 0.62 \varphi^{-2.15}$ , which is famously known as the "Humble" formula. Because Winsauer's data set was assembled from a variety of widely geographically distributed sandstones, a certain (in hindsight, unwarranted) generality was attributed to the Humble formula, and it was used for a time with wild abandon in the industry. Although the parameter values of the Humble formula certainly do render the sum of the squared residuals for the Humble data set small<sup>1</sup>, after a time it was realized that best predictions in a given formation were based upon relations custom-developed for that formation; no generality for the parameter values  $a = 0.62$  and  $m = 2.15$  could be asserted. However, in the form  $F = a \varphi^{-m}$  with  $a$  and  $m$  custom-determined for each data set, the formulation of Winsauer (albeit under the double-misnomer "Archie's law") endures as today's industry standard method for representing the porosity-resistivity relation in clean sandstones. Note that if  $\varphi = 1$ , then  $F = a$ , which is physically correct only for the special case  $a = 1$ .

There are problems with the Archie-Winsauer (A-W) formulation of the porosity-resistivity function. If the data sets used to produce the correlations are viewed as having varying porosity due to the process of diagenetic cementation operating upon a rock homogeneous in its porosity at the time of deposition, say by preferentially reducing porosity at a greater rate in fine-grain components, then one expects conductivity to vanish with the porosity still greater than zero. In other words, the rock ceases to conduct electricity when the continuous conduction path through the rock is broken, and the path can be broken while isolated porosity remains in the rock. Conversely, based on extrapolation to zero porosity of the conductivity trend suggested by observations, some rocks would apparently continue to conduct at zero porosity (e.g., shaly sands).

The Archie-Winsauer model cannot honor these conditions. Also, clearly if  $\varphi = 1$  then  $F = 1$ , a physical fact that the Winsauer formulation does permit, but does not enforce. Thus, at the boundaries of the porosity domain, the A-W model fails. The failures of fit at the end-points of the porosity domain are summarized in Table 1.

A theory to account for conductivity over the whole porosity domain - including the limiting cases - would be more satisfying than a model that merely fits the observed data, at least if it managed to also fit the observations as well as the A-W model. Such a theory is possible.

## REEXAMINING THE SEMINAL DATA SETS

The form of the power laws in use for resistivity interpretation was basically imposed by the most readily available method in the 1940s to simplify the representation of relationships; i.e., graph paper that obviated the labor entailed in mathematical transformations of the data. Plotting porosity and formation resistivity factor data on bi-logarithmic graph paper was an arbitrary choice and is not well suited to every data set. (The principal disadvantages of the bi-logarithmic plot are illustrated in Appendix A.) There are many other choices of functions that would linearize the relationship among the data. For example, if the square root of  $1/F$  is plotted against linear porosity, the data falls on a linear trend. Moreover, there is no imperative to impose linearity if some other relationship is suggested by the data. Does the data conceal any useful trends that would suggest an alternative model that might in some way improve upon industry-standard methods?

Figure 3 plots the Archie Nacatoch sandstone data set as formation conductivity normalized by connate water conductivity (i.e.,  $\sigma_0/\sigma_w = 1/F$ ) on a linear scale, against poros-

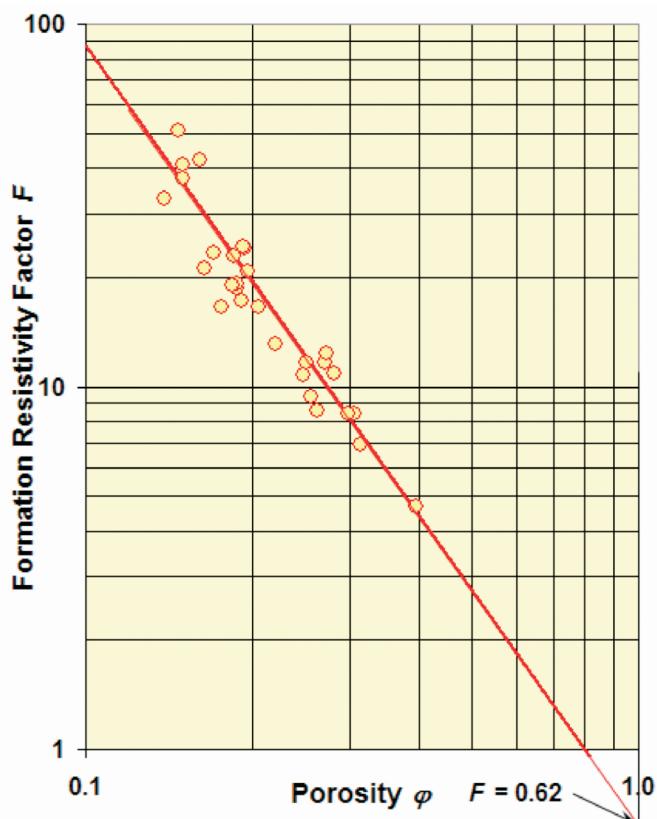


FIG. 2 The Winsauer et al. (1952) data set and the line representing the "Humble" formula,  $F = 0.62 \varphi^{-2.15}$ .

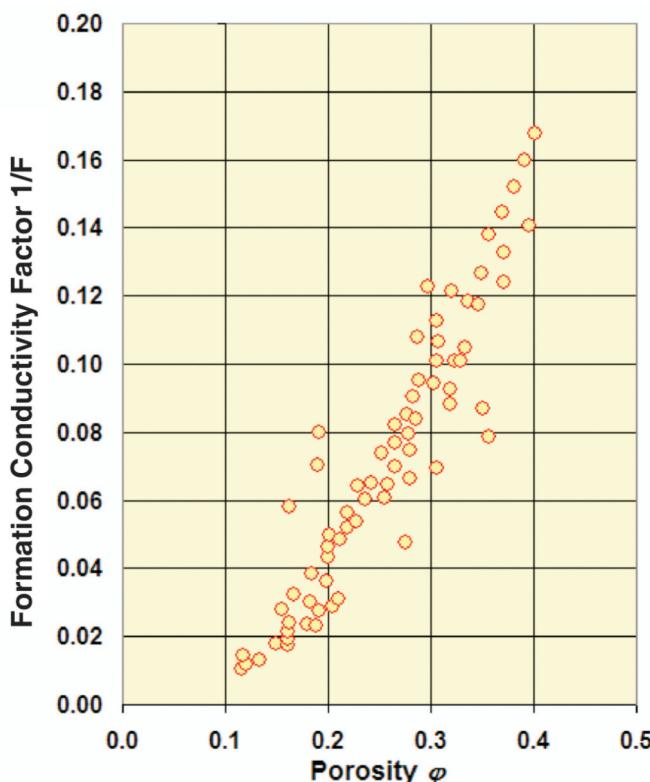
<sup>1</sup>The Humble formula parameters do not minimize the sum of the residuals for Winsauer's data. Perhaps his data was fit "by eye" rather than by mathematical least squares.

ity on a linear scale. There is a clear suggestion that the parent conductivity distribution underlying the data increases at an increasingly greater rate as porosity increases. There is also a hint that the underlying function will vanish at a porosity greater than zero. Looking at the data plotted on a scale that includes the 100% porosity end point (Figure 4) strengthens the suggestion of curvature.

### Porosity-conductivity analysis

Make the definition that the “fractional connected brine volume” is the fraction of porosity involved in conduction, represented as  $\beta_c = \varphi - \varphi_\delta$  where  $\varphi_\delta$  is the porosity remaining in the rock when the normalized conductivity vanishes. Notice that these data suggest that the rate of change of the normalized conductivity with respect to the fractional connected brine volume is approximately (possibly exactly) proportional to the fractional connected brine volume. I tentatively adopt the suggested direct proportionality relationship as a working hypothesis, and check whether the implications of the hypothesis are supported by observations on Archie's Nacatoch sandstone porosity-conductivity data.

These observations are summarized in the mathematical statement



**FIG. 3** Archie's Nacatoch sandstone data plotted as normalized linear conductivity versus linear porosity.

**TABLE 1** Observed boundary values of normalized conductivity and brine volume, and the corresponding values demanded by Archie's and Winsauer's models. Electrical efficiency theory (EET) discussed hereinafter, is an alternative theory that also does not accommodate the observed boundary conditions. The blue background represents observations; green background indicates agreement of theory with observations; yellow backgrounds indicate disagreement with observations. The observations are that normalized conductivity  $\sigma_0/\sigma_w$  does not vanish together with brine volume  $\varphi S_w = 0$ , and that  $\sigma_0/\sigma_w = 1$  when  $\beta = \varphi S_w = 1$ . This table records that only Archie's model at  $\varphi S_w = 1$  gives a correct prediction at one of the boundaries.

		boundary conditions	
		$\varphi S_w = 0$	$\varphi S_w = 1$
		observed	Archie
theory	$\sigma_t/\sigma_w$	$\neq 0$	0
Archie	$\varphi S_w = 0$	1	1
Winsauer / Humble	$\varphi S_w = 1$	0	$1/a$
EET	$\varphi S_w = 1$	0	$a_t + b_t \neq 1$

$$\frac{d(\sigma_0 / \sigma_w)}{d(\varphi - \varphi_\delta)} \propto (\varphi - \varphi_\delta) \quad (1)$$

where  $\varphi_\delta$  is a “percolation threshold” and where  $\varphi - \varphi_\delta$  is the amount of connected brine volume that supports conduction. Expressing (1) as an equality yields

$$\frac{d(\sigma_0 / \sigma_w)}{d(\varphi - \varphi_\delta)} = \alpha_0 (\varphi - \varphi_\delta) \quad (2)$$

where  $\alpha_0$  is a proportionality constant. Also note the boundary conditions that  $\sigma_0/\sigma_w = 1$  when  $\varphi = 1$ , and  $\sigma_0/\sigma_w = 0$  when  $\varphi = \varphi_\delta$ . Equation (2) is immediately integrable, the solution being

$$\frac{\sigma_0}{\sigma_w} = \frac{\alpha_0}{2} (\varphi - \varphi_\delta)^2 + \alpha_1 \quad (3)$$

with  $\alpha_1$  being a constant of integration. Evaluating the boundary conditions yields  $\alpha_1 = 0$  and  $\alpha_0 = 2/(1 - \varphi_\delta)^2$ , whence

$$\frac{\sigma_0}{\sigma_w} = \left( \frac{\varphi - \varphi_\theta}{1 - \varphi_\theta} \right)^2. \quad (4)$$

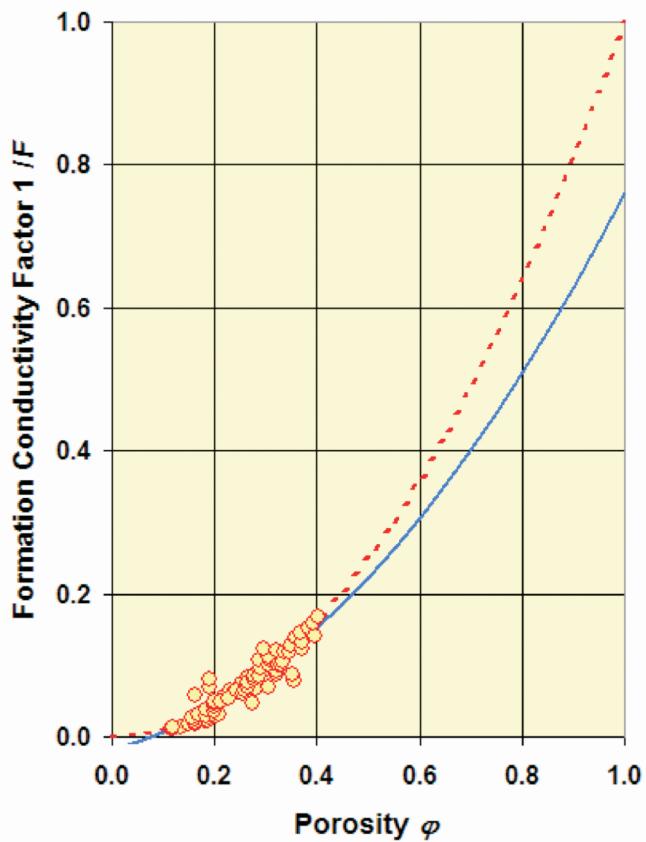
This result, having been derived from Archie's data, should apply only to rocks similar to those studied by Archie and Winsauer, known as "Archie" rocks. Archie rocks are characterized as comprising non-conductive, water-wet, convex, equidimensional mineral grains, with interconnected brine being the only conducting phase in the rock. The domain of the independent variable  $\varphi$  is  $0 \leq \varphi \leq 1$ . Rocks occupy only a subinterval of this domain. Archie rocks occupy the "Archie sub-domain," roughly  $0.10 \leq \varphi \leq .40$ . I also include the brine point  $\varphi = 1.0$  in the Archie sub-domain. (Appendix B is a discussion of the sedimentation process that argues the case for including  $\varphi = 1.0$  in the Archie sub-domain.)

$\varphi_\theta$  is not a true percolation threshold—the porosity value at which electrical conduction vanishes, approaching zero from the right on the porosity-normalized conductivity plot. Rather,  $\varphi_\theta$  is the projection to the porosity axis of the trend established by the porosity – normalized conductivity variation in samples representing the reservoir on the Archie-rock sub-domain of the porosity domain. In a mature, low-porosity Archie rock, as cementation progresses pore throats begin to occlude, and the conductivity at some porosity value begins to decrease more rapidly with porosity reduction than on the Archie-rock sub-domain, eventually reaching a true percolation threshold, different from the threshold predicted by a projection of the porosity–conductivity trend observed on the Archie sub-domain to the porosity axis. Consequently, I refer to  $\varphi_\theta$  as a "pseudo" percolation threshold (PPT). I refer to equation (4) as a pseudo-percolation threshold theory (PPTT). Experience suggests that core plugs used in analysis of Archie rocks are typically cut from the Archie sub-domain, with porosity values greater than, say, roughly, 0.10, and few—if any—core plugs are on the trend that ends at the true percolation threshold. When this condition is satisfied the PPT can be used to parametrically characterize the data space in terms of this "pseudo" rock property.

#### Connections of PPTT to A-W Theory<sup>1</sup>

Having analyzed Archie's data differently than Archie, and having reached a different result, it is interesting to find points at which the two analyses touch. In conductivity

notation, Archie's equation for the formation resistivity factor – porosity relationship is  $\sigma_0/\sigma_w = a^{-1} \varphi^m$  where Winsauer's  $a$  is included in the expression. Archie's model is seen to be a special case of equation (4) where  $\varphi_\theta = 0$ , implying  $a = 1, m = 2$ . The purpose of  $m$  and  $a$  as adjustable parameters is to allow sufficient freedom for the Archie-Winsauer function to be fit to observations with a reasonably small residual error. Nevertheless, the A-W function is constrained to pass through  $(\varphi, \sigma_0/\sigma_w) = (0,0)$  and is not constrained to contain  $(\varphi, \sigma_0/\sigma_w) = (1,1)$ . Thus, while A-W may be fit for the purpose of interpolating within the observations, extrapolated to the boundaries it cannot correctly predict the boundary values. Equation (4), on the other hand, has only a single adjustable parameter, but is capable of representing a percolation threshold,  $(\varphi, \sigma_0/\sigma_w) = (\varphi_\theta, 0)$  and always contains  $(\varphi, \sigma_0/\sigma_w) = (1,1)$ . In



**FIG. 4** Archie's data plotted on a scale to include 100 percent porosity. The blue curve is an unconstrained quadratic least squares fit to the data in this bi-linear space. Note that this curve intercepts the porosity axis at  $\varphi > 0$ . The dotted red curve plots  $1/F = \varphi^2$ , or Archie's law with  $m = 2$ . Archie's law honors  $F = 1$  at  $\varphi = 1$ , but forces the fit through the origin  $F = \infty$  at  $\varphi = 0$ , an unnatural constraint.

<sup>1</sup>Interestingly, Winsauer and his coauthors (1952) suggested a fitting equation with a pseudo-percolation threshold for the Humble data set:  $F = (\varphi - 0.06)^{-1.49}$ . However, they did not further develop the idea.

matching the boundary conditions (4) is successful, whereas A-W is not.

Figure 5 illustrates additional connections between  $m$  and  $\varphi_\vartheta$ . In particular, low porosity and variation in  $m$  are seen to be associated; for porosities greater than about 10 percent and percolation thresholds ranging from  $-0.05 \leq \varphi_\vartheta \leq 0.05$ ,  $m$  is almost constant and approximately equal to 2.  $m < 2$  is associated with negative percolation thresholds, suggestive of rocks with a conductive matrix component. For  $\varphi_\vartheta$  increasingly greater than 0,  $m$  is increasingly greater than 2. This suggests that well-cemented rocks have relatively greater percolation thresholds. At low porosity  $m$  varies significantly with porosity. While the very low porosity sub-domain is not in the range of PPTT normalized conductivity prediction, the need to introduce “variable  $m$ ” at low porosity is seen to be attributable to Archie’s law lacking a percolation threshold parameter. None of these correspondences contradicts any observations of  $m$  or percolation thresholds. However, a percolation threshold has a direct physical interpretation, whereas  $m$  is merely the slope of a line<sup>1</sup>, suggesting the former model incorporates more physics than the latter. The criterion for evaluating the relative merit of the PPTT versus the A-W functions *vis-a-vis* any particular data set will be the degree to which the sums of the squared residuals are minimized.

<sup>1</sup> $m$  is sometimes asserted to be a property of a single core plug. However,  $m$  is the slope of a line. Since a line requires two points for its definition, either  $m$  for a single point does not exist, or since  $m = (\log F - \log(1))/(\log \varphi - \log(1)) = \log F/\log \varphi$  then  $m$  is the slope of the line connecting the given point to the point  $(\varphi, F) = (1, 1)$ .

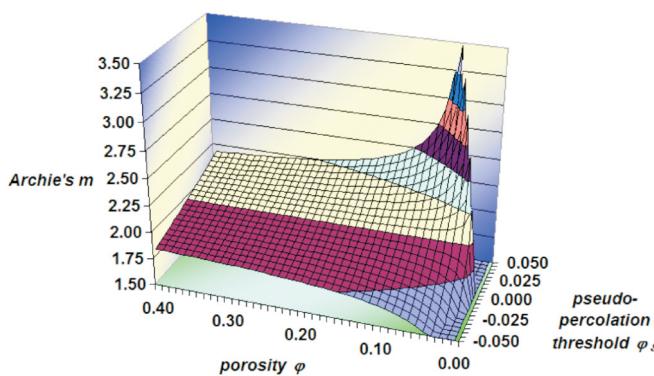


FIG. 5 The connection between Archie’s  $m$  and  $\varphi_\vartheta$ , the pseudo-percolation threshold parameter. This “ $m$  surface” is a plot of  $\sigma_0/\sigma_w = \varphi^m = ((\varphi - \varphi_\vartheta)/(1 - \varphi_\vartheta))^2$  solved for  $m$ .

## Connections of PPTT to EET

The normalized conductivity per unit length of a cylindrical brine-filled tube fashioned from insulating material (e.g., quartz) of outer radius  $R$  and inner radius  $r$  is given by  $\sigma_0/\sigma_w = \varphi$  where  $\varphi = r^2/R^2$  is the fractional cross-sectional area of brine having conductivity  $\sigma_w$ , and also the “porosity” of the tube. The cylindrical configuration of the brine phase gives it the maximum conductivity for a given volume of brine. Any redistribution of the brine phase into a more complicated geometrical configuration must reduce the conductivity. So, in general,  $\sigma_0/\sigma_w = \varphi E_0$  where  $E_0$  is a function of the geometrical configuration of the conducting phase, and where  $0 \leq E_0 \leq 1$  (Herrick and Kennedy; 1993, 1994). This is referred to as the “electrical efficiency theory” (EET).

Although  $\sigma_0/\sigma_w = \varphi E_0$  always holds true by definition,  $E_0$  can be determined by straightforward analysis only for a few simple cases. For rocks, the form of  $E_0$  is empirically determined using standard measurements of porosity and conductivity. When EET is applied to Archie rocks and  $E_0 (= \sigma_0/(\sigma_w \varphi))$  is plotted against  $\varphi$ , the resulting trend tends to be linear:  $E_0(\varphi) = a_0 \varphi + \beta_0$ . From empirical observations,  $a_0 \approx 1$  and  $\beta_0 \approx 0$ . In summary, for Archie rocks in the Archie sub-domain, according to EET

$$\frac{\sigma_0}{\sigma_w} = \varphi E_0 = a_0 \varphi^2 + b_0 \varphi. \quad (5.a)$$

Note that  $E_0 \approx \varphi$  as a consequence of  $a_0 \approx 1$ ,  $b_0 \approx 0$ .

PPTT begins directly with observations of  $\sigma_0/\sigma_w$  versus  $\varphi$ , leading to equation (4). Expansion of equation (4) leads to

$$\begin{aligned} \frac{\sigma_0}{\sigma_w} &= \frac{1}{(1 - \varphi_\vartheta)^2} \varphi^2 + \frac{-2\varphi_\vartheta}{(1 - \varphi_\vartheta)^2} \varphi + \frac{\varphi_\vartheta^2}{(1 - \varphi_\vartheta)^2} \\ &\equiv a_p \varphi^2 + b_p \varphi + c_p. \end{aligned} \quad (6.a)$$

Note that  $a_p + b_p + c_p \equiv 1$  and that for  $\varphi_\vartheta \approx 0$ ,  $a_p \gg b_p \gg c_p$ , and  $a_p \approx 1$ ,  $b_p \approx 0$ ,  $c_p \approx 0$ .  $c_p$  is so small as to be unobservable in practice. Further

$$\frac{\sigma_0}{\sigma_w} = \varphi \left( \varphi \left( \frac{1 - \varphi_\vartheta / \varphi}{1 - \varphi_\vartheta} \right)^2 \right) \equiv \varphi E_p. \quad (7)$$

Thus,  $E_p \approx \varphi$  explicitly. PPTT and EET applied to Archie rocks lead to the same result. The linearity of  $E_p$  versus  $\varphi$  and the magnitudes of  $a_p \approx 1$  and  $b_p \approx 0$  are analogous to the corresponding EET quantities. The EET and PPTT approaches are mutually consistent for Archie rocks

since both are based on the same observation, namely that the normalized conductivity approaches a quasi-quadratic relationship to water content. The difference is that EET begins with a statement of principle followed by inspection of the  $E_0$  and  $\varphi$  data to observe the form of  $E_0$  versus  $\varphi$ , whereas PPTT begins with an inspection of  $\sigma_0/\sigma_w$  versus  $\varphi$  data, in search of an underlying principle for Archie rocks. However, the convergence of the results strongly suggests that the underlying porosity-normalized conductivity relationship is quasi-quadratic, as indicated, and indeed, Archie's relationship with  $m = 2$  reduces to a pure quadratic. Since the form of PPTT, EET and A-W are each based on the same observations, it is not surprising that essentially the same limiting case (i.e.,  $\varphi_\theta \rightarrow 0, \Rightarrow a_0 \rightarrow a_p \rightarrow 1; b_0 \rightarrow b_p \rightarrow 0 \Rightarrow m \rightarrow 2, a \rightarrow 1$ ) is obtained regardless of the approach employed. In Appendix C it is shown that a slight reformulation of the premises of EET brings the two models into an exact formal conformance, differing only in point of view.

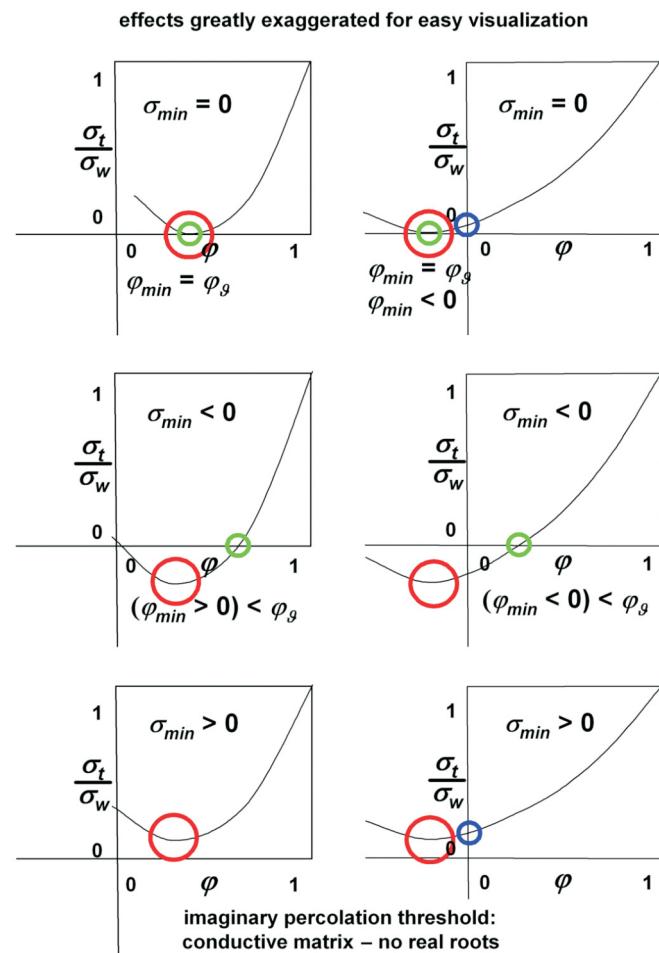
For values of  $\varphi$  typically observed in Archie rocks, the term  $c_p/\varphi$  would be unobservable in the presence of noise. For example, the smallest value of porosity in Archie's Nacatoch sandstone data set is approximately  $\varphi = 0.10$ . Using (4), the sum of the squared residuals using this data is minimized when  $\varphi_\theta = -0.002$ . Using this  $\varphi_\theta$  to compute the coefficients, then at 0.10 porosity  $a_p\varphi^2 = 0.00996$ ,  $b_p\varphi = 0.0003984$  and  $c_p/\varphi = 0.0000398$ . Thus  $E_p \approx a_p\varphi + b_p$ , the same relationship empirically found for EET. The value of  $c_p/\varphi$  is so small compared to  $a_p$  and  $b_p$  as to be unobservable in practice (0.4 percent of  $a_p\varphi^2$ ). The relative contribution of  $c_p/\varphi$  diminishes at higher porosities. Thus, while EET is in some sense conceptually more general than PPTT, the properties of the EET parameter  $E_0(\varphi)$  for Archie rocks can also be described in terms of a pseudo-percolation threshold.

Although explicitly developed using only Archie rocks, in clay-bearing sandstones the PPTT term  $c_p/\varphi$  becomes non-negligible, and can be considered as representing a parallel conductivity component that is independent of brine conductivity and fractional volume. This result was anticipated by the EET, which found for shaly sandstones that  $E_0 = a_0\varphi + b_0 + c_0/\varphi$  (Herrick and Kennedy, 1993). According to Herrick and Kennedy,  $c_0 = V_{sh}\sigma_{sh}/\sigma_w S_w$ , providing a pseudo-percolation threshold-free physical interpretation for the  $c_0/\varphi$  term. While PPTT does not purport to supply a physical explanation for shaly sandstone conductivity, it can nonetheless be useful for the description of the porosity-normalized conductivity relationship in shaly sandstones.

#### Additional degree of freedom

PPTT as explained thus far is elegant, and consistent

with A-W and the behavior of EET. Unfortunately for elegance, in order to allow for best performance as an interpolating tool, the theory as expressed to this point is incomplete. The PPTT function as expressed in equation (4) is a parabola having its vertex tangent to the  $\sigma_0/\sigma_w = 0$  plane. This function cannot accommodate observations where the trend of the data demands that the fitting function is not tangent to the  $\varphi$  axis. Figure 6 illustrates some possible configurations. In order that the vertex of the parabola can be located above or below the  $\sigma_0/\sigma_w = 0$  axis another degree of



**FIG. 6** Generally data will suggest the vertex of the normalized conductivity parabola is not tangent to the porosity axis; another degree of freedom in the fitting equation must be introduced to account for this. The red circle locates the vertex of the parabola; the green circle locates the PPT. Note that if the vertex is above the abscissa there is no pseudo-percolation threshold. Some rocks with a conductive mineral component in parallel with the conductive brine, such as clay coating sand grains, will exhibit this behavior. The blue circles above the  $\varphi$  axis locate the excess conductivity for a slightly shaly rock. Archie's Nacatoch sandstone has this property.

freedom in the fitting function is required. With the introduction of this degree of freedom the pseudo-percolation threshold is no longer at the vertex of the parabola, and no longer appears in the conductivity-porosity formula. Equation (4) is modified to include an additional term, and the  $\varphi_\theta$  term is replaced by the porosity coordinate of the vertex, which is no longer the pseudo-percolation threshold. The coordinates of the vertex of the parabola are denoted by  $\varphi_{min}$  and  $\sigma_{min}$  respectively, and the data are assumed to satisfy

$$\frac{\frac{\sigma_0}{\sigma_w} - \frac{\sigma_{min}}{\sigma_w}}{1 - \frac{\sigma_{min}}{\sigma_w}} = \left( \frac{\varphi - \varphi_{min}}{1 - \varphi_{min}} \right)^2. \quad (8)$$

The adjustable parameters are the coordinates of the vertex of the parabola.  $\sigma_{min}/\sigma_w$  is the minimum normalized conductivity value attained by the function and  $\varphi_{min}$  is the porosity corresponding to this value. The left side remains normalized as  $\sigma_0/\sigma_w \rightarrow 1$  and reduces to  $\sigma_0/\sigma_w$  as  $\sigma_{min} \rightarrow 0$ . Although  $\varphi_{min}$  and  $\sigma_{min}$  do not have the direct physical significance of  $\varphi_\theta$ , the physical significance of the pseudo-percolation threshold is not lost; the conductivity trend defined by the observations still vanishes at the pseudo-percolation threshold. In that sense  $\varphi_\theta$  remains the key “observable” of PPTT; however, the PPT is not the curve fitting parameter. The intercept of the conductivity parabola with the porosity axis (i.e., the PPT,  $\varphi_\theta$ ), is given in terms of the vertex coordinates by

$$\varphi_\theta = \varphi_{min} + (1 - \varphi_{min}) \sqrt{\frac{-\sigma_{min} / \sigma_w}{(1 - \sigma_{min} / \sigma_w)}}, \quad (9)$$

$$\sigma_{min} < 0,$$

where  $\sigma_{min}$  and  $\varphi_{min}$  are selected to minimize the sum of the squared residuals using  $(\varphi, \sigma_0)$  observations and equation (8).

As shown in Figure 6, it is possible for the trend implied by the data to have a negative intercept ( $\varphi_\theta < 0$ ), or even completely fail to intercept the porosity axis. This could happen for a conductive rock matrix. If the corresponding  $\sigma_{min}$  is positive; there is no pseudo-percolation threshold. In such a case  $\varphi_\theta$  from (9) is a complex number with no obvious physical interpretation. However  $\varphi_{min}$  and  $\sigma_{min}$  are still useful for curve fitting.

Equation (8) is analogous to the A-W expression  $\sigma_0/\sigma_w = a^{-1}\varphi^m$ . Both equations require two adjustable parameters to best honor the observations. The equations become identical when  $\varphi_{min}$  and  $\sigma_{min}$  are both equal to zero, and  $m=2, a=1$ . A-W is simpler in appearance, but it cannot accommodate the existence of a pseudo-percolation threshold different from zero except by making  $m$  a poros-

ity-dependent variable  $m(\varphi)$ , the long-used expedient for extending A-W to low porosity<sup>1</sup>. But, A-W gives no prescription for the determination of  $m = m(\varphi)$ . Further, no a priori apparent physical significance can be assigned to  $m$  or  $a$ . Finally, for  $a \neq 1$ , the A-W formula fails to give  $F=1$  at  $\varphi = 1$ . Although A-W fails to honor the boundary conditions inherent in the conductivity – porosity relationship, the real comparison of efficacy is the minimization of the sum of the squared residuals between the observed data and the predictions of the two models. From the statistical point of view (and assuming the data contains no anomalous outliers), the method with the smallest sum of squared residuals is the better predictor of the relationship between the porosity and conductivity variables.

The expansion of equation (8), followed by comparison to EET, shows that inclusion of  $\sigma_{min}/\sigma_w$  merely multiplies  $a_p$ ,  $b_p$ , and  $c_p$ , by a factor of  $1 - \sigma_{min}/\sigma_w$  ( $\approx 1$ ) without changing the behaviors of the coefficients already described. However, the remaining  $\sigma_{min}/\sigma_w$  ( $\approx 0$ ) term provides a means to shift the vertex of the parabola slightly off the  $\varphi$  axis if necessary.

Although  $\sigma_{min}$  has been introduced as an ad hoc parameter in equation (8), in Appendix D equation (8) is shown to follow from boundary conditions consistent with the observed trend in the data, and thus is less ad hoc in character than it first appears.

### Comparing PPTT to A-W

Comparisons using Archie’s data are complicated by his not having published his data except as a postage-stamp-sized figure. Numerical comparisons thus begin with a digitization of this data (see Appendix H). Some of the data points in the figure are hard to visually resolve, and different analysts might not only find slightly different coordinates for the data points, but might even disagree as to the number of points. With this caveat in mind, I hand-digitized Archie’s data. I then, according to equation (8), computed the sums of the squared residuals, using Archie’s power law model, to be 0.015234 (with  $m=2, a=1$ ) and the sums of the squared residuals using PPTT to be 0.015020 (using  $\sigma_{min}/\sigma_w = 0.004, \varphi_{min} = 0.0075$ ). The difference is very small, but the small difference in the sums of the squared residuals is in favor of PPTT. Whereas Archie’s model demands that conductivity vanish at zero porosity, PPTT indicates a small residual conductivity ( $\sigma_0 \approx .004\sigma_w$ ) at zero porosity. Note that  $\sigma_{min} > 0$  implies that the pseudo-percolation threshold is complex; i.e., the curve does not quite intersect the porosity axis. This is consistent

<sup>1</sup>See 1972 Schlumberger Log Interpretation Charts, p. 10, chart Por-1 C-10.

with Archie's (1950; p. 947) description of the Nacatoch sandstone as shaly<sup>1</sup>.

Archie's data for the Nacatoch sandstone fits  $F = \varphi^{-2}$  almost perfectly in the least squares sense, although Archie never claims that  $m = 2$  for the Nacatoch data set — only that the range of  $m$  includes 2. Using my digitization of Archie's data, and fixing  $a = 1$ , the sum of the squared residuals is a minimum (0.015220) for  $m = 1.995$ , indistinguishable from  $m = 2$  using graphical methods to determine the slope. Note that this sum of squared residuals is still larger than the PPTT minimum sum of squared residuals.

The comparable sum of squared residuals for the Winsauer data and model (using  $m = 2.15$ ,  $a = 0.62$ ) is 0.003636 versus 0.003393 using PPTT with  $\sigma_{min}/\sigma_w = -0.015$  and  $\varphi_{min} = -0.091$ , corresponding to  $\varphi_\theta = 0.0416$ . Again, this difference, while small, is in favor of the PPTT model. Winsauer achieved the closeness of fit by allowing  $\sigma_0/\sigma_w$  to assume the value 1.61 at  $\varphi = 1.0$ , while PPTT honors  $\sigma_0/\sigma_w = 1$ . My result is obtained by not including the lowest and highest porosity points in the Winsauer data, rejecting them as outliers. Winsauer himself omitted the lowest porosity point (i.e., his Oil Creek, sample #3) as an outlier in his estimate of the Humble formula parameters. My observation is that the highest porosity point (Ottawa sand, which is not a rock at all, but unconsolidated, loose sand from the St. Peter sandstone) is also an anomalous outlier. Had it also been omitted from Winsauer's data (and using the normalized conductivity residuals in minimization), the Humble formula would have been  $F = 0.781 \varphi^{-1.955}$ , illustrating the sensitivity of the Humble formula parameters to this one point. I omit this point as well as omitting Oil Creek sample #3. Since Winsauer listed his data, one has the impression that a numerical regression was actually performed. However, I cannot find a method of regressing Winsauer's published data that yields the Humble formula parameters. Results vary depending upon whether the regression is done using logarithm of porosity and resistivity, linear porosity and resistivity, or linear porosity and conductivity, etc. Each method requires slightly different values of  $a$  and  $m$  to minimize the residual for that regression. Since Winsauer does not specify a method, perhaps his method, like Archie's, was graphical. In any event, Winsauer's specification of parameters to two and three significant figures cannot be interpreted as an expression of accuracy or precision in his result. Indeed, although the form of the Humble equation has become the

industry-standard, the actual parameter values are determined anew for each new set of data.

To summarize, the slight reduction in the sums of the squared residuals using PPTT, versus A-W, is less significant than the inclusion of proper boundary conditions by PPTT. However, to be taken seriously PPTT needs to predict the observations at least as well as classical A-W theory. The small sums of squared residuals, comparable to A-W sums of squared residuals, validates this requirement.

## CONDUCTIVITY-WATER SATURATION RELATIONSHIP

### The saturation exponent

A plot of resistivity index versus water saturation ( $I-S_w$ ) is similar in spirit and appearance to the formation resistivity factor versus porosity plot. The resistivity index is the ratio of core plug resistivity at arbitrary  $S_w$  to its resistivity at  $S_w = 1$ . The plot illustrates how resistivity increases as brine volume diminishes and so is similar to formation resistivity factor versus porosity, except that in the former brine volume is diminished by decreasing water saturation whereas in the latter it is diminished by decreasing porosity. There is no seminal data set comparable to Archie's<sup>2</sup> or Winsauer's for resistivity index – water saturation observations, but data from a recently published article (Hamada et al., 2002) is available to motivate a discussion. The practical significance of conductivity measurements made on reservoir rocks is their use in estimating water saturation given resistivity and porosity from logging instruments. Archie (1942) adopted from earlier workers the plotting of the logarithm of the water saturation,  $S_w$ , against the logarithm of resistivity index,  $I = R_i/R_0$ . On this plot the data suggest a relationship given by  $S_w = I^n$  (always written  $I = S_w^{-n}$  in the modern literature) where  $n$  ( $\approx 2$ ) is called the saturation exponent. The data for this plot originates from a single core plug which is assigned a porosity value consistent with its average porosity as determined by core analysis. The value of  $n$  assigned to this sample is the slope of a least-squares regression line fitted to the  $I-S_w$  data on a bi-logarithmic plot.

Characterization of a reservoir unit requires an  $n$  that is representative of the entire unit, and this requires that resistivity index - water saturation analysis be performed upon multiple samples from the same reservoir, both with various different samples having the same porosity, and also with

<sup>1</sup>"Friable sandstone, poorly sorted grains, shaly and calcareous; comparatively high porosity for permeability (Nacatoch formation, Bellevue, Louisiana)," Archie 1950.

<sup>2</sup>Archie cites studies by Leverett, Wyckoff and Botset, Jakosky and Hopper, and Martin, Murray and Gillingham as his sources for the  $I - S_w$  relationship.

samples displaying as much variation in porosity as the reservoir under study provides. The resulting data set yields multiple values of  $n$ ; one for each core plug in the sample (as illustrated in Figure 7). The problem then is to use this data in some fashion to characterize an entire reservoir. Further, there is no a priori reason to rule out the dependence of saturation exponents upon porosity or upon water saturation. (A water saturation-dependent  $n$  is analogous to a porosity-dependent  $m$ .) Attempts to honor these dependencies may occasionally be made in formation evaluation, but commonly the data is averaged to yield a single value of  $n$  to characterize an entire reservoir. There is no industry-standard method for calculating this average. The arithmetic average

$$\bar{n} = (1/N) \sum_{i=1}^N n_i$$

is invariably used; but why should the arithmetic be chosen over, say, the geometric  $\bar{n} = \sqrt[N]{\prod_{i=1}^N n_i}$  or harmonic  $1/\bar{n} = (1/N) \sum_{i=1}^N 1/n_i$  averages? In fact, since  $n$  is a mea-

sure of slope, it can easily be shown that the arithmetic average of multiple  $n$  values is not appropriate. For example consider the extreme case of averaging the slopes of a horizontal line (zero slope) and a vertical line (infinite slope), the result for the arithmetic average being infinite. On the other hand, the average of the angles associated with these slopes ( $0^\circ$  and  $90^\circ$  respectively) is  $45^\circ$ , a more representative result, corresponding to a slope of unity. This suggests the mean of  $N$  samples defined as

$$\bar{n} = \tan \left[ (1/N) \sum_{i=1}^N \tan^{-1}(n_i) \right] \quad (10)$$

provides an average most properly suited to the saturation exponent.

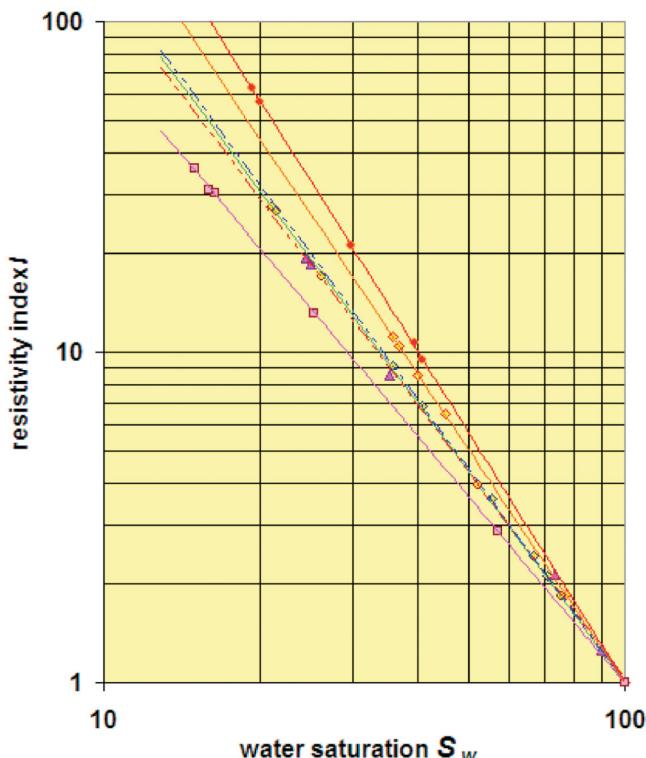
As discussed above, in conventional formation evaluation, the use of saturation exponent data derived from a collection of cores necessarily involves a choice of some method of averaging the data. But this prompts the question: Is it possible to globally analyze this data giving equal (or some statistically appropriate) weight to each data point rather than first analyzing the core plugs individually to extract saturation exponents, then averaging the saturation exponents?

### Water saturation-conductivity analysis

First note that the bi-logarithmic water saturation – resistivity index plot can be recast as a plot of water saturation versus “conductivity” index, all the scales being linear. Viewed in this space the conductivity index appears as a non-linear function of water saturation, very similar to the linear porosity–formation conductivity factor plot. The data shown in Figure 7 are replotted in Figure 8. Note that the abscissa in Figure 8 could be rescaled for each core plug by multiplying  $S_w$  by the porosity of the plug  $\varphi$ , scaling the abscissa as fractional brine volume. The result is a plot of conductivity index versus fractional brine volume for each individual core plug. If the ordinates are multiplied by the conductivity formation factor, the result is the normalized conductivity  $\sigma_t/\sigma_w$ . The resulting data, when plotted above the  $\varphi-S_w$  plane (Figure 9) is scattered above and below some smooth but a priori unknown normalized conductivity surface representing the data’s parent distribution in porosity–water saturation space. A representation of this surface can be developed in terms of the data and boundary conditions using a method similar to the development of equations (4) and (8).

### Brine volume-conductivity analysis

Inspection of  $\sigma_t/\sigma_w$  plotted against  $\varphi S_w$  with  $\varphi$  held constant (the data for this plot is the same as required for a resistivity index - water saturation plot), with the introduction of



**FIG. 7** A conventional water saturation - resistivity index plot using data from Hamada et al. well B. The lines are fitted through the data acquired for a number of core plugs. Power law behavior is strongly suggested, but the suggested trends would probably curve upward if lower water saturations could be achieved in the desaturation experiment.

a variable  $\vartheta$  to denote the pseudo-percolation threshold for this case, suggests that the analog to equation (1) for  $S_w < 1$  is

$$\frac{d(\sigma_t / \sigma_w)}{d(\varphi S_w - \vartheta)} \propto (\varphi S_w - \vartheta). \quad (11)$$

$\vartheta$  is a generalization of  $\varphi_\vartheta$  to a curve in the  $\varphi-S_w$  plane, and  $\varphi S_w - \vartheta$  is a generalization of the connected fractional brine volume to the case where  $S_w < 1$ . As with equation (1), the rate of change of normalized conductivity with respect to connected brine volume seems to be proportional to the connected brine volume. In this proportionality it is the connected fractional volume of the conducting phase,  $\beta_c = \varphi S_w - \vartheta$ , rather than fractional brine volume  $\varphi S_w$  alone, that plays the main role in the rate of change of normalized conductivity. Writing equation (11) with a proportionality constant  $\alpha_2$

$$\frac{d(\sigma_t / \sigma_w)}{d(\varphi S_w - \vartheta)} = \alpha_2 (\varphi S_w - \vartheta) \quad (12)$$

subject to the boundary condition that  $\sigma_t/\sigma_w \rightarrow 1$  as  $\varphi S_w \rightarrow 1$ , and  $\sigma_t/\sigma_w \rightarrow 0$  as  $\varphi S_w \rightarrow \vartheta$ . The general solution is

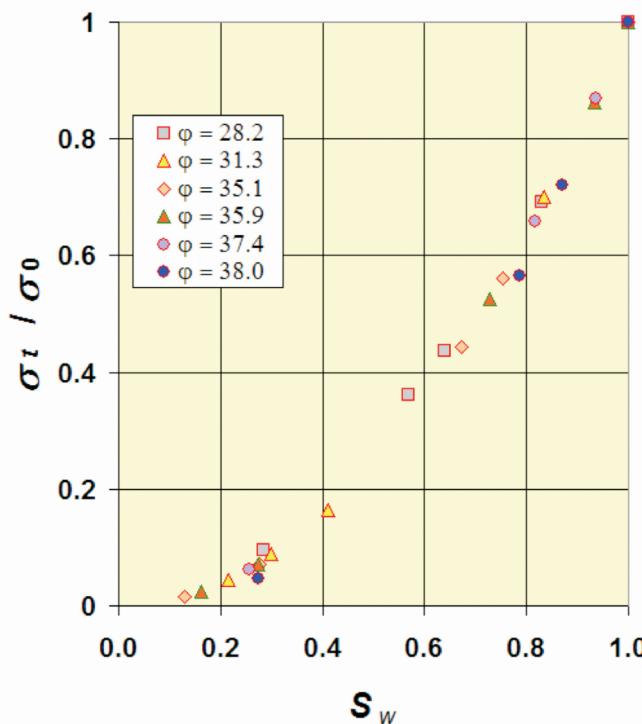


FIG. 8 Linear water-saturation – conductivity-index plots exhibit a resemblance to porosity-conductivity relationships. This is the same data as in Figure 7.

$$\frac{\sigma_t}{\sigma_w} = \frac{\alpha_2}{2} (\varphi S_w - \vartheta)^2 + \alpha_3. \quad (13)$$

The boundary condition  $\sigma_t/\sigma_w \rightarrow 0$  as  $\varphi S_w \rightarrow \vartheta$  requires that  $\alpha_3 = 0$ . The condition that  $\sigma_t/\sigma_w \rightarrow 1$  when  $\varphi S_w \rightarrow 1$  requires that  $\alpha_2 = 2/(1 - \vartheta_I)^2$  where  $\vartheta_I = \vartheta(\varphi S_w = 1)$ . Thus

$$\frac{\sigma_t}{\sigma_w} = \left( \frac{\varphi S_w - \vartheta}{1 - \vartheta_I} \right)^2. \quad (14.a)$$

This expression must satisfy certain conditions. By analogy with equation (4), the argument in the parenthesis of (14.a) is expected to be the difference between the total fractional brine volume surface  $\sigma_t/\sigma_w$ , and an isolated fractional brine volume surface  $\beta_i$ , and is expected to reduce to (4) when  $S_w = 1$ , or to an analogous form in  $S_w$  when  $\varphi = 1$ . The form of the fractional connected brine volume,  $\varphi S_w - \vartheta$ , is suggestive that  $\vartheta$  is more complicated than a simple constant, and the general expectation is that  $\vartheta = \vartheta(\varphi, S_w)$ . Discussion of the satisfaction of these requirements is deferred pending a brief digression introducing the unit conductivity cube.

#### The unit conductivity cube

Equation (14.a) expresses the normalized conductivity as a function of porosity and water saturation. Noting that both  $\varphi$  and  $S_w$  are constrained to the interval between 0 and

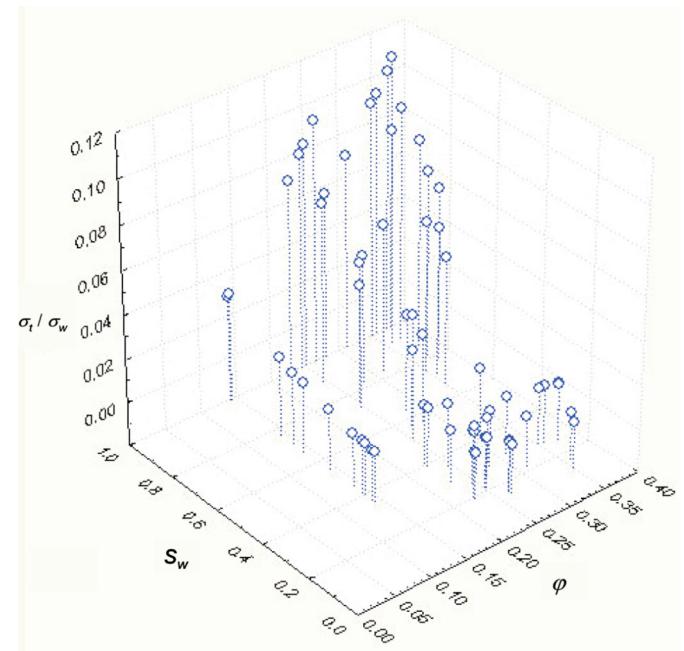


FIG. 9 Hamada's data presented as normalized conductivity above the  $\varphi-S_w$  plane. Each core plug generates a row of points having constant  $\varphi$ , variable  $S_w$  and  $\sigma_t/\sigma_w$ .

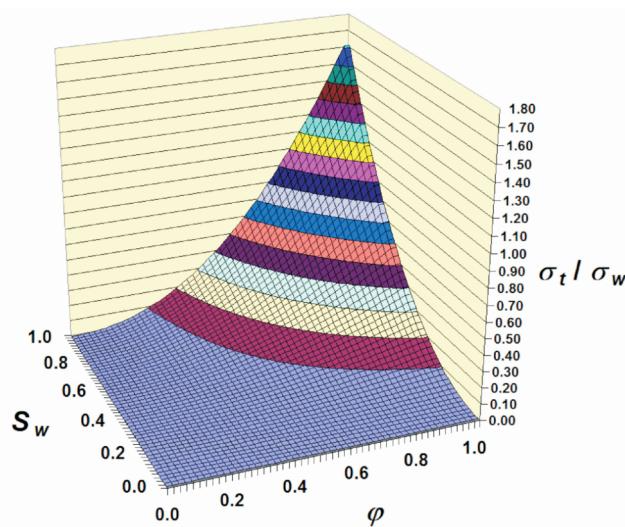
1, a convenient representation is to associate  $\varphi$  and  $S_w$  with orthogonal axes in a plane with  $\sigma_t/\sigma_w$  represented as a surface above this plane. Note also that  $0 \leq \sigma_t/\sigma_w \leq 1$ , so that the data space is a 3-dimensional unit cube. This cube contains, and renders visualizable, all the data in the porosity – water-saturation – normalized-conductivity experiment, including the boundary values. This is in contrast to the bi-logarithmic scales customarily used, which can display neither zero  $\varphi$  or  $S_w$ , nor infinite  $F$  or  $I$ , nor  $\sigma_t/\sigma_w = 0$ .

Consider, for example, the Humble formula  $\sigma_t/\sigma_w = \varphi^{2.15} S_w^2 / 0.62$  plotted above the  $\varphi$ - $S_w$  plane (Figure 10). According to the formula,  $\sigma_t/\sigma_w$  vanishes along the  $\varphi$  and  $S_w$  axes, and attains the value of  $1.0/0.62 = 1.612$  above the point called the brine pole,  $(\varphi, S_w) = (1, 1)$ . The values at the boundaries required by the Humble formula are not those observed: at  $(\varphi, S_w) = (1, 1)$ ,  $\sigma_t/\sigma_w$  should equal 1.0, and physically  $\sigma_t/\sigma_w$  does not vanish everywhere precisely on the  $\varphi$  and  $S_w$  axes. Conversely, when equation (14.a) is plotted in the unit conductivity cube  $\sigma_t/\sigma_w$  at  $S_w = 1$  always is equal to 1.0, and, in general, the normalized conductivity vanishes along a curved contour in the  $\varphi$ - $S_w$  plane rather than precisely on the axes. A surface defined by real data necessarily more closely resembles the pseudo-percolation threshold surface than the surface defined by the Humble formula. Conductivity measured upon real rocks should vanish along a curve on the  $\varphi$ - $S_w$  plane and include the brine pole  $(\varphi, S_w) = (1, 1)$  (Figure 11). Other physics-induced details of this surface are discussed in Appendix E.

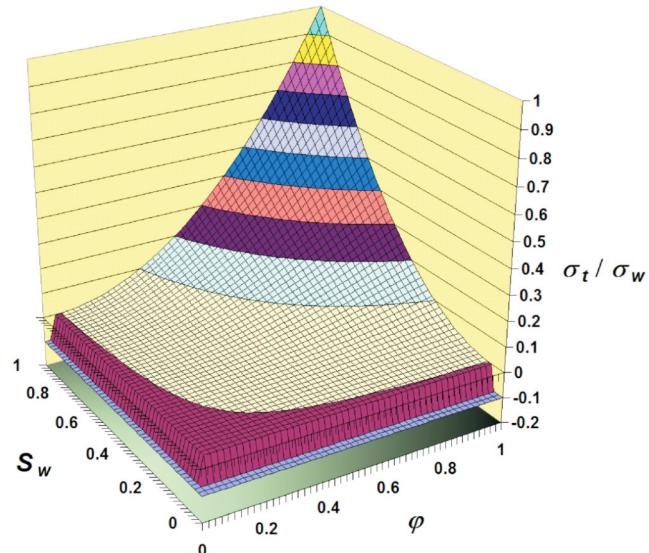
Archie rocks will have porosities limited to between about 0.10 and about 0.40, Hamada's rocks being an example (Figure 9). I have called this sub-domain of the porosity–water saturation space the “Archie sub-domain” (Figure 12). A close look at the Archie sub-domain reveals that all the resistivity data acquired on core plugs is accommodated in this plot. For example the data customarily plotted as logarithm of porosity against logarithm of formation resistivity factor for water-filled rocks plots near the red line in the  $S_w = 1$  plane as formation conductivity factor (or normalized conductivity) against porosity. The data from the water saturation – resistivity index experiment plots parallel to (the green) lines of constant porosity and variable water saturation. Conductivity vanishes along a contour in the  $\varphi$ - $S_w$  plane highlighted by the yellow line. According to equation (14.a) normalized conductivity is symmetric in  $\varphi$  and  $S_w$  (assuming  $\vartheta(\varphi, S_w)$  is also symmetric in  $\varphi$  and  $S_w$ ). A white line extending from the origin is the axis of  $\varphi$ - $S_w$  symmetry.

#### The interpretation of $\vartheta(\varphi, S_w)$

In equation (4), which refers to the  $S_w = 1$  plane,  $\varphi_\theta$  is a simple constant; however the situation is more complicated in the case that  $S_w < 1$ . A comparison of equations (1) and (11) on the one hand, and (4) and (14.a) on the other, suggests that the argument of the squared term in (14.a) should be identified with a connected brine volume,  $\beta_c$ . Consider



**FIG. 10** The surface in the unit conductivity cube defined by the Humble formula. This surface has unphysical properties; conductivity vanishes on the  $\varphi = 0$  and  $S_w = 0$  axes, and  $\sigma_t/\sigma_w \rightarrow 1.0/0.62 = 1.61$  as  $\varphi, S_w \rightarrow 1$ .



**FIG. 11** The normalized conductivity surface in the unit conductivity cube corresponding to the pseudo-percolation threshold theory, equation (14.a). Conductivity vanishes on the boundary contour between the cream-colored surface and the maroon surface, and  $\sigma_t/\sigma_w \rightarrow 1$  as  $\varphi, S_w \rightarrow 1$ .

the connected brine volume function in the  $S_w = 1$  plane to be extended parallel to the  $S_w$  axis. The result is a plane with contours parallel to the  $S_w$  axis, inclined upward toward  $\varphi = 1$ . This plane is described by

$$\beta_{c,\varphi} = \varphi - \frac{\varphi_\vartheta}{1-\varphi_\vartheta} (1-\varphi). \quad (15)$$

Also consider the analogous connected brine volume function in the  $\varphi = 1$  plane extended parallel to the  $\varphi$  axis. This plane is described by

$$\beta_{c,S_w} = S_w - \frac{S_\vartheta}{1-S_\vartheta} (1-S_w). \quad (16)$$

The product of these extended planes

$$\beta_c = \left( \varphi - \frac{\varphi_\vartheta}{1-\varphi_\vartheta} (1-\varphi) \right) \left( S_w - \frac{S_\vartheta}{1-S_\vartheta} (1-S_w) \right), \quad (17)$$

expanded and grouped as

$$\begin{aligned} \beta_c &= \varphi S_w - \\ &\left[ \frac{\varphi_\vartheta}{(1-\varphi_\vartheta)} (1-\varphi) S_w + \frac{S_\vartheta}{(1-S_\vartheta)} (1-S_w) \varphi \quad (18) \right. \\ &\left. - \frac{\varphi_\vartheta}{(1-\varphi_\vartheta)} \frac{S_\vartheta}{(1-S_\vartheta)} (1-\varphi)(1-S_w) \right] \end{aligned}$$

is in the form  $\beta - \beta_i$  if the term in the square brackets (i.e.,  $\beta_i$ ) is interpreted as the isolated fractional brine volume not participating in conduction. Rationalizing this expression gives

$$\beta_c = \frac{\varphi S_w - (\varphi_\vartheta S_w + \varphi_\vartheta S_\vartheta - \varphi_\vartheta S_\vartheta)}{1 - (\varphi_\vartheta + S_\vartheta - \varphi_\vartheta S_\vartheta)}. \quad (19)$$

Comparing this result to equation (14.a), note that making the definitions

$$\vartheta(\varphi, S_w) \equiv \varphi S_\vartheta + \varphi_\vartheta S_w - \varphi_\vartheta S_\vartheta. \quad (20.a)$$

and

$$\vartheta_I \equiv \varphi_\vartheta + S_\vartheta - \varphi_\vartheta S_\vartheta = \vartheta(1,1) \quad (20.b)$$

brings the argument of expression (14.a) and (19) into conformance. The factored form of (19) is

$$\beta_c = \frac{\varphi - \varphi_\vartheta}{1 - \varphi_\vartheta} \cdot \frac{S_w - S_\vartheta}{1 - S_\vartheta}. \quad (21)$$

This clearly reduces to the argument of equation (4) for  $S_w \rightarrow 1$ . Normalized conductivity in terms of connected

fractional brine volume  $\beta_c = \beta - \beta_i$  can be expressed in several forms. The most convenient form for the display of PPTT adjustable parameters is, in terms of normalized conductivity

$$\frac{\sigma_t}{\sigma_w} = \left( \frac{\varphi - \varphi_\vartheta}{1 - \varphi_\vartheta} \right)^2 \left( \frac{S_w - S_\vartheta}{1 - S_\vartheta} \right)^2. \quad (14.b)$$

### More Electrical Efficiency Theory connections

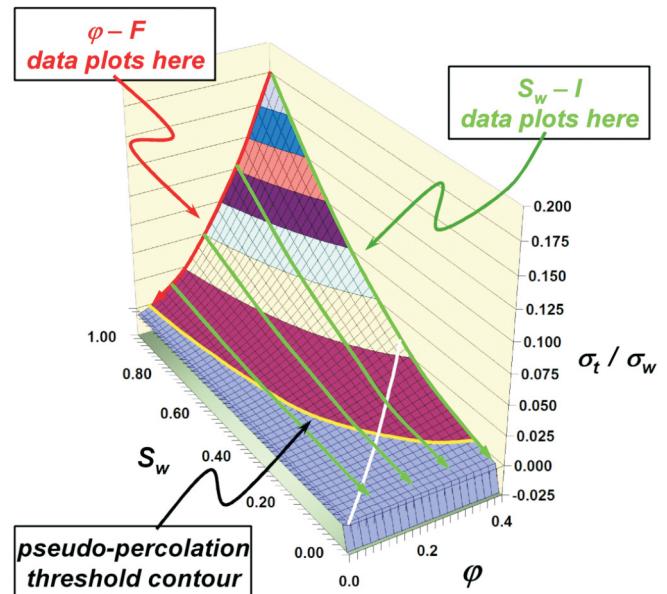
For  $S_w < 1$  the EET is expressed as (Herrick and Kennedy, 1994)

$$\sigma_t/\sigma_w = E_t \varphi S_w = a_t (\varphi S_w)^2 + b_t \varphi S_w. \quad (5.b)$$

Expanding equation (14.b) and making substitutions from equations (20.a) and (20.b) results in

$$\begin{aligned} \frac{\sigma_t}{\sigma_w} &= \frac{1}{(1-\vartheta_I)^2} (\varphi S_w)^2 + \frac{-2\vartheta}{(1-\vartheta_I)^2} \varphi S_w + \frac{\vartheta^2}{(1-\vartheta_I)^2} \quad (6.b) \\ &= a_p (\varphi S_w)^2 + b_p \varphi S_w + c_p. \end{aligned}$$

It is seen that the relationship of EET as a special case of PPTT remains valid for  $S_w < 1$ .



**FIG. 12** The Archie sub-domain in the unit conductivity cube. Porosity-conductivity data plots near the red curve in the  $S_w = 1$  plane; water saturation-resistivity index data plots along lines parallel to the green lines. Normalized conductivity vanishes on the yellow contour, the pseudo-percolation threshold. The line of  $\varphi - S_w$  symmetry is shown by the white trace.

### Additional degree of freedom

As with equation (4), where the surface corresponding to equation (14) meets the  $\varphi$ - $S_w$  plane it is everywhere tangent to the plane. In order to accommodate other relationships of the normalized conductivity surface not tangent to the  $\varphi$ - $S_w$  plane that observations usually require, it is necessary to provide another degree of freedom. This degree of freedom is supplied exactly as in equation (8) to yield

$$\frac{\sigma_t - \sigma_{min}}{1 - \frac{\sigma_{min}}{\sigma_w}} = \left( \frac{\varphi S_w - \vartheta_{min}}{1 - \vartheta_I} \right)^2 \quad (22.a)$$

where  $\vartheta_{min}$  is the contour in the  $\varphi$ - $S_w$  plane where  $\sigma_t = \sigma_{min}$ . Equation (22.a) can also be displayed in factored form as

$$\frac{\sigma_t - \sigma_{min}}{1 - \frac{\sigma_{min}}{\sigma_w}} = \left( \frac{\varphi - \varphi_{min}}{1 - \varphi_{min}} \right)^2 \left( \frac{S_w - S_{min}}{1 - S_{min}} \right)^2 \quad (22.b)$$

from which the definitions  $\vartheta_{min} \equiv \varphi S_{min} + \varphi_{min} S_w - \varphi_{min} S_{min}$  and  $\vartheta_I \equiv \varphi_{min} + S_{min} - \varphi_{min} S_{min}$  follow.  $\varphi_{min}$  is the value assumed by  $\varphi$  in the  $S_w = 1$  plane on the  $\vartheta_{min}$  contour and  $S_{min}$  is the value assumed by  $S_w$  in the  $\varphi = 1$  plane on the  $\vartheta_{min}$  contour.  $\sigma_t = \sigma_{min}$  everywhere on this contour; i.e., the left side of equation (22) vanishes. In (22.b) all of the curve-fitting parameters appear not only explicitly, but also with each parameter uncoupled from the others in its own separate factor, making equation (22.b) the most convenient representation for their determination by numerical methods. It is also obvious that in the limit  $S_w \rightarrow 1$  that equation (22.b) reduces to equation (8) as is required.

The exponents arising in equations (4) and (22) being equal to 2 results from accepting a working hypothesis of direct proportionality between  $d(\sigma_0/\sigma_w)/d(\varphi - \varphi_\vartheta)$  and  $\varphi - \varphi_\vartheta$ ; similarly for  $d(\sigma_t/\sigma_w)/d(\varphi S_w - \vartheta)$  and  $\varphi S_w - \vartheta$ . However, inspection of the data does not rule out the possibility that, for example,  $d(\sigma_0/\sigma_w)/d(\varphi - \varphi_\vartheta) \propto (\varphi - \varphi_\vartheta)^{m-1}$  where  $m \approx 2$ . Adopting this premise, the analytical development would proceed exactly as discussed heretofore except that the result would differ slightly in exponent values from those displayed in equation (22.b). There is support from percolation theory for this possibility. It is discussed further in Appendix F.

### $\varphi$ - $S_w$ symmetry

The A-W theory, including water saturation, is usually stated as  $R_t = a \varphi^{-m} S_w^{-n} R_w$ , and has the normalized conductivity formulation  $\sigma_t/\sigma_w = \varphi^m S_w^n/a$ . These formulations

seem to give the porosity and water saturation factors unequal effect in influencing conductivity, weighting the factors asymmetrically according to any difference in  $m$  and  $n$  ( $\varphi^m S_w^n \neq S_w^m \varphi^n$ ). However, the bulk volume fraction of the conducting phase is a single number,  $\beta = \varphi S_w$ , where the factors in this product are formally symmetric ( $\varphi S_w = S_w \varphi$ ). Moreover, although the abscissa of the water saturation – resistivity index plot is conventionally presented scaled in water saturation from 0-to-1, if the labels on the abscissa scale are multiplied by porosity, the axis is converted to a fractional bulk volume of brine axis, the curve shape remaining unchanged. Recalling that Archie's equation is assembled from  $F = R_0/R_w = a\varphi^{-m}$  and  $I = R_t/R_0 = S_w^{-n}$  by elimination of  $R_0$ , forming the product  $IF$ , a plot of  $IF$  ( $= R_t/R_w$ ) against  $\varphi S_w$  is a graphical means of achieving a similar result. All resistivity data for the formation resistivity factor plot and resistivity index plot can be represented on this single plot (Herrick, 1988). Using these relabeled axes an Archie-like power law would have the form  $IF = R_t/R_w = \sigma_w/\sigma_t = (\varphi S_w)^{-n'}$ , where  $IF \equiv G$  is a generalized resistivity index and  $n'$  is the slope of the line that minimizes the sums of the squared residuals. This slope is a generalized saturation exponent,  $n'$ , based upon the entire data ensemble.

The Archie porosity exponent is recovered from

$$\lim_{S_w \rightarrow 1} n' \rightarrow m$$

where  $S_w \rightarrow 1$  means omitting data for  $S_w < 1$ .  $m$  in this limit has the same value as when it is determined using the conventional  $\log\varphi$ - $\log F$  plot. However, an explicit porosity exponent, separate from an explicit saturation exponent, is not required in using this method.

Plotting  $IF$  against  $\varphi S_w$  puts  $\varphi$  and  $S_w$  on a formally equal footing using Archie's conventional bi-logarithmic plot. The function  $G = (\varphi S_w)^{-n'}$  retains formal symmetry in the factors of fractional brine volume,  $\varphi$  and  $S_w$ , and reduces to Archie's equation with  $m = n$  for all values of  $n'$ , including  $n' = 2$ .) Moreover, by plotting the generalized resistivity index of an entire ensemble of core plugs against  $\varphi S_w$  (instead of merely  $S_w$ ) on the same plot, and fitting a single line to the resulting cluster of points, this technique avoids an explicit choice of an averaging method for the saturation exponent. The disadvantage, from the curve-fitting point of view, is that there is only the single adjustable parameter  $n'$  (or only two, if Winsauer's  $a$  is also used), and this will place a higher limit on the minimum sum of the squared residuals than the use of three adjustable parameters. Like the conventional Archie model, this formulation does not extrapolate correctly to the boundaries, nor is there any obvious physical significance for  $n'$ .

These considerations illustrate my assertion that

Archie's particular formulation of the formation conductivity function is not inevitable but is merely one among several alternative possible relationships, even in the context of the customary bi-logarithmic plots.

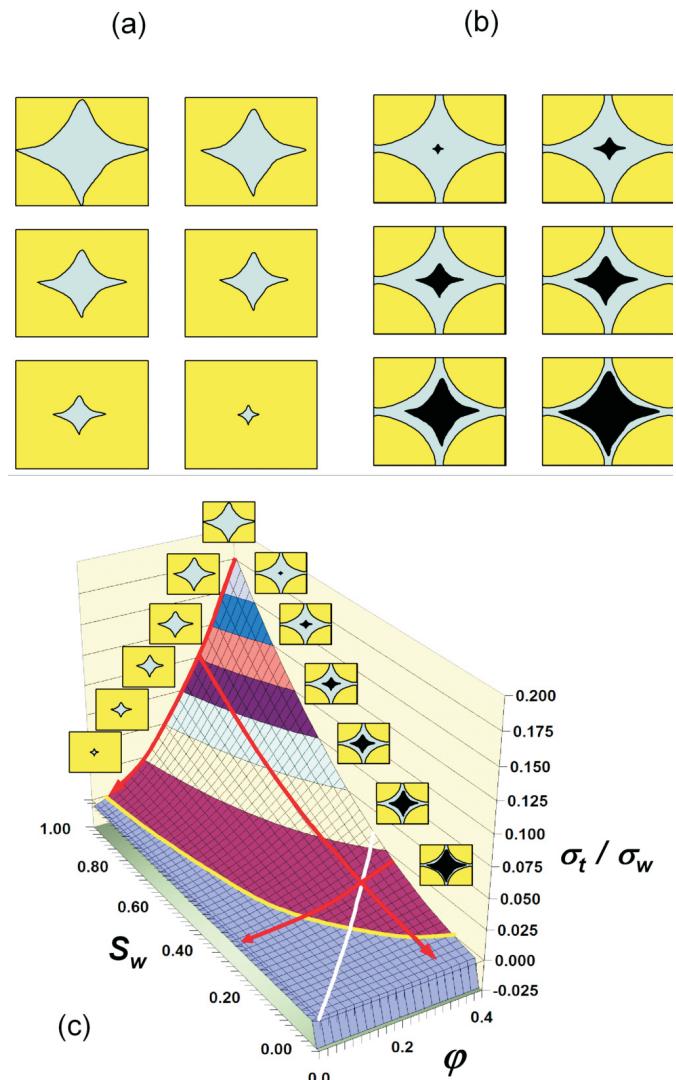
Symmetry in the normalized conductivity function is not unreasonable when normalized conductivity is viewed as a surface over the  $\varphi-S_w$  plane (Figure 12). The gradient in the  $\varphi$  direction ( $S_w$  coordinate held constant) is similar to the gradient in the  $S_w$  direction ( $\varphi$  coordinate held constant). In both cases the conductivity is diminished by similar reductions in the brine volume fraction, the difference being that in the former case the brine volume is reduced by reduction of the pore volume, the water saturation being constant, while in the latter case the brine volume is reduced by displacing brine with a non-conductive hydrocarbon phase, the porosity being constant. These brine configurations are illustrated in Figure 13. Whether the behavior of the normalized conductivity differs in these two orthogonal directions depends upon whether the geometry of the conducting phase in the directions of the respective gradients influences conductivity differently. This is a matter to be decided by observations on rocks; i.e., by experiment. The observations suggest that in many cases similar reductions in fractional brine volume lead to similar reductions in conductivity regardless of the mode of fractional brine volume reduction. This corresponds to the common observation in A-W theory that  $m \approx n$ , and suggests that bulk conductivity, at least in Archie rocks, is not especially sensitive to the detailed geometry of the conducting phase. This observation suggests that the minimum cross-sectional area of the brine phase normal to current flow controls the bulk conductivity of the medium, since the bulk conductivity is ultimately determined (and limited) by this cross-sectional area. Note that there is no role for tortuosity in this formulation. This is supportive of Herrick's and Kennedy's (1994) demonstration that tortuosity in Archie rocks represents, at most, a minor if not negligible perturbation of the primary conduction-limiting mechanism, the pore throat aperture area.

### A saturation equation

The modeled normalized conductivity - fractional brine volume relationship is

$$\frac{\sigma_t}{\sigma_w} = \frac{\sigma_{min}}{\sigma_w} + \left(1 - \frac{\varphi_{min}}{\varphi_w}\right) \left( \frac{\varphi S_w - \vartheta_{min}}{1 - \vartheta_I} \right)^2. \quad (23)$$

The usual interest in the conductivity-fractional brine volume relationship in formation evaluation is for the estimation of water saturation from conductivity and porosity as inferred from logging instrument responses. In this form equation (23) is



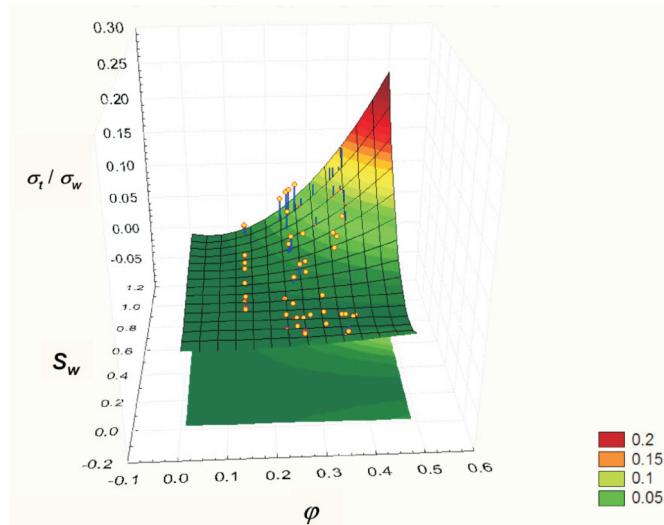
**FIG. 13** In real rocks changes in  $\varphi$  produce changes in brine geometry and volume (cartooned in the two left columns above, (a) that are different from the changes in brine geometry of the same volume produced by changes in  $S_w$  (cartooned in the two right columns, (b)). Fractional brine volume changes arising from reduction in porosity (along the line of constant  $S_w = 1$ ) obviously results in a significantly different configuration of brine than changes arising from reduction in water saturation (along lines of constant  $\varphi$ ). This difference might be expected to induce large differences in how conductivity changes with fractional brine volume according to the mode of brine volume reduction. However, it is observed that frequently conductivity is not particularly sensitive to which mode of brine reduction is operating. In other words, albeit surprising, the normalized conductivity surface ((c) above) seems to be nearly symmetric with respect to an exchange of  $\varphi$  and  $S_w$ . In A-W theory this near-symmetry is expressed as  $m \approx n$ . In PPTT theory the near-symmetry is a consequence of keeping the fractional brine volume of the conducting phase, the product of porosity and water saturation, as a single variable,  $\beta = \varphi S_w$ .

$$S_w = \frac{1}{\varphi} \left( \vartheta_{min} - (1 - \vartheta_I) \sqrt{\frac{\sigma_t / \sigma_w - \sigma_{min} / \sigma_w}{1 - \sigma_{min} / \sigma_w}} \right). \quad (24)$$

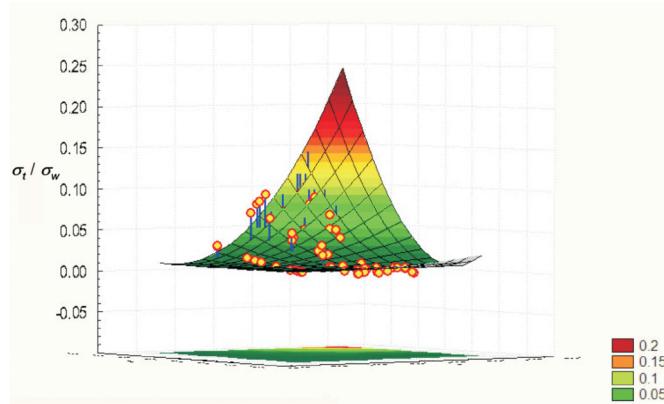
This saturation equation reduces to Archie's saturation equation evaluated with  $m = n = 2$  for the case where  $\sigma_{min}$ ,  $\varphi_{min}$ , and  $S_{min}$  all equal zero.

### Application to Hamada et al. data

The use of equation (24) is to estimate water saturation everywhere in the Archie sub-domain given a normalized



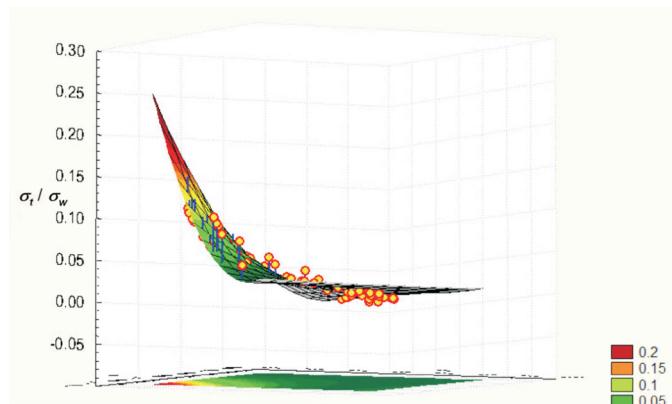
**FIG. 14** Hamada's porosity – water-saturation – normalized conductivity data for his well A and well B. Each line of constant-porosity points represents data taken from a single core plug.



**FIG. 15** The normalized conductivity data distributed astride the PPTT normalized conductivity surface that minimizes the sums of the squared residuals about the surface.

conductivity surface predicted from observations at discrete points in the sub-domain. The estimation is effected by finding adjustable parameters that minimize the sums of the squared residuals with respect to observed  $\sigma_t / \sigma_w$  and the predictions of equation (23); i.e., least squares. Although the PPTT model is nonlinear in its adjustable parameters,  $\sigma_t / \sigma_w$  is a smooth, well-behaved function of these same parameters. It is a simple matter to use any number of commercially available programs (e.g., Microsoft Excel's Solver) to discover values of the parameters that minimize the sums of the squared residuals between observed data and a PPTT normalized conductivity surface. Use of Hamada's data in the PPTT finds  $\sigma_{min} = 0.059$ ,  $\varphi_{min} = 0.056$ , and  $S_{min} = 0.232$ . Figure 14 is a view of this surface together with the data (shown as yellow dots) above and below the surface. Figure 15 shows the same surface with the  $\sigma_t / \sigma_w = 0$  plane nearly edge-on and illustrates how the fitted normalized conductivity surface divides the observations. The surface interpolates normalized conductivity given porosity and water saturation. Figure 16 is a similar view of the data with the surface viewed nearly edge-on.

These real data illustrate that often the available data are restricted to a narrow sub-domain on the porosity axis. In the case of the Hamada et al. data set  $0.15 \leq \varphi \leq 0.35$ . If the porosity sub-domain is narrow, when there is a large amount of scatter in such data, the data may not constrain the adjustable parameters to physically reasonable limits. Under such circumstances, similarly to Winsauer's  $a$  parameter (a number which should always be 1.0, but isn't), the parameters of the pseudo-percolation threshold theory may violate the expectation that they be small parameters, close to zero. The more restricted the porosity sub-domain,



**FIG. 16** A view showing the PPTT normalized conductivity surface edge-on suggests that a robust fitting scheme might be preferred over standard least squares for some data sets. The effect of robust estimation is to better fit the majority of the data by minimizing the influence of outliers.

the higher the porosity, and the noisier the data, the greater the possibility will be for physically unmeaningful values of  $\varphi_b$ , and relatively large values of  $(\varphi_{min}, S_{min}, \sigma_{min})$  to be observed. However, the resulting normalized conductivity surface will still be a good interpolator/predictor in the  $\varphi-S_w$  sub-domain defined by the observations and cannot, by its construction, give silly results at the boundaries.

Robust estimators are techniques for reducing the influence of data "outliers" on fitted functions. The scatter of the data points seen in Figures 15 and 16 suggest that robust estimators might be superior to the Excel Solver method for finding suitable adjustable parameters for Hamada's data set. However, application of robust estimation techniques to Hamada's data is beyond the scope of this study.

## INSIGHTS

Reviewing Archie's (and Winsauer's) and the PPTT (and ETT) models, can anything new be said about the conductivity of reservoir rocks? These models can be tied together with the introduction of the concept of an ideal Archie rock, characterized by the conductivity relationship

$$\sigma' = (\varphi' S'_w)^2. \quad (25)$$

Equation (25) allows a unified frame where the relations of all the methods can be compared. Contemplation of the adjustable parameters in equation (22.b) suggests that real Archie rocks differ by only small perturbations from this ideal Archie rock. If the parameters of the generalized Archie model are perturbed only slightly the model can be brought into close accord with Archie's observations and the empirical models derived from these data. The model perturbations can be constructed to honor boundary conditions, observed and inferred, satisfied by the physical conductivity-fractional brine volume relationship. These are

$$\sigma' = \frac{\sigma_t - \sigma_{min}}{\sigma_w - \sigma_{min}} = \frac{\frac{\sigma_t}{\sigma_w} - \frac{\sigma_{min}}{\sigma_w}}{1 - \frac{\sigma_{min}}{\sigma_w}} \quad (26.a)$$

$$\varphi' = \frac{\varphi - \varphi_{min}}{1 - \varphi_{min}} \quad (26.b)$$

$$S'_w = \frac{S_w - S_{min}}{1 - S_{min}} \quad (26.c)$$

and can be viewed as transformations from an ideal Archie space common to all conductivity-porosity-water saturation relationships into the physical space where any particular problem is observed. The correction terms are small pertur-

bations in each of the variables. The adjustable parameter values should correlate with the pore geometry of various rock textures. In the case of  $\sigma_{min} = 0$  the percolation threshold parameters become  $\varphi_{min} = \varphi_b$  and  $S_{min} = S_b$ , and lie in the  $\sigma/\sigma_w = 0$  plane. In this model the brine point in the ideal Archie space and in the unit conductivity cube are always co-located. The two spaces differ by the unit conductivity cube's origin being perturbed just slightly, displacing it from the origin in the ideal Archie space. Viewed from the ideal Archie space, the unit conductivity cube in real porosity-water-saturation-conductivity space behaves like a slightly distorted image of the ideal space, being compressed or stretched parallel to its edges into the unit conductivity cube. Substituting (26.a)-to-(26.c) into (25) leads to equation (22.b), an equation perfectly symmetrical in porosity and water saturation, the symmetry being broken slightly in the observations because  $\varphi_{min}$  and  $S_{min}$  are determined by different physical processes and so should not be, in general, equal.

## CONCLUSION

This research has been inspired by a hope of discovering whether the Archie-Winsauer power law connecting conductivity to fractional porosity and fractional brine volume could be shown to follow from physical principles, and if so, how so? Full success would have been an enunciation of an unequivocal physical principle from which a porosity-water-saturation-conductivity relationship would follow ineluctably. Be there such a principle, it is: the rate of change of normalized conductivity with respect to a change in connected fractional brine volume is proportional to the connected fractional brine volume. This statement is based upon observations of normalized conductivity of Archie rocks on the Archie sub-domain and its boundaries on the  $\varphi-S_w$  plane. No generalization to other rocks is claimed. Further remarks are offered in Appendix G.

At the least, I have shown that the Archie-Winsauer model is valid over at most only a portion of the fractional porosity and fractional brine volume range, and therefore cannot qualify as a physical theory itself, nor can it be shown to derive, by formal analysis, from physical principles. A-W theory merely minimizes the squared residuals between observations and an arbitrarily selected function, logarithm of resistivity proportional to logarithm of porosity. Thus while A-W is fit for the purpose of interpolating within the observations, the A-W power function cannot be used to extrapolate outside the data since it has unphysical (and therefore incorrect) behavior at the boundaries of the  $\varphi-S_w$  plane. These findings will not surprise thoughtful formation evolutionists.

I have also shown that for conventional special core

analysis data, that it is possible to formulate the porosity – water-saturation – normalized-conductivity relationship problem to honor boundary conditions while simultaneously fitting observations at least as well as the conventional methods. Although the pseudo-percolation threshold is merely quasi-physical, it puts adjustability into the fitting function where that adjustability can be used to most closely mimic the natural behavior of conductivity at low porosity and water saturation. Thus, the PPTT model provides insight into why various ad hoc extensions of Archie's law at low porosity and for shaly sands are required. Moreover, while both Archie-Winsauer and pseudo-percolation threshold theory are three-parameter models, the former requires the portioning of data into artificial categories ( $S_w = 1$  for determination of  $a$  and  $m$ ,  $S_w \leq 1$  for determination of  $n$ ) followed by the use of ad hoc procedures such as arithmetic averaging of  $ns$ ; the latter determines all of the fitting parameters simultaneously using all of the data simultaneously, giving all of the data equal influence, seemingly the most "democratic" method to use the data. Further, PPTT treats the fractional brine volume  $\varphi S_w$  as an indivisible unit, the conducting phase, rather than separating its components and treating them separately; e.g.,  $\varphi^n S_w^m$ . Finally, the pseudo-percolation threshold is at least notionally related to a property of the rock fabric—the PTT, whereas  $a$ ,  $m$ , and  $n$  are merely ad hoc model parameters with no a priori physical interpretations implied. PPTT can be used without modification for slightly shaly sands, such as Archie's Nacatoch and Hamada's A and B sands.

Not every feature of the normalized conductivity surface and its limitations are discussed above. There is, for example, a large sub-domain on the  $\varphi$ - $S_w$  plane where the normalized conductivity vanishes because, as discussed in Appendix E, non-conductive hydrocarbons fluids form the continuous phase of an emulsion. However, Archie rocks do not belong to this sub-domain. So, keeping in mind Archie's cautionary admonition quoted in the footnote on the second page of this article, PPTT remains valid on the Archie sub-domain and at the boundaries where normalized conductivity takes on its extreme values: zero and one.

Although this search for a first principle for the conductivity-porosity-water saturation relationship in Archie rocks has not succeeded in finding that principle, it has illustrated that curve- and surface-fitting functions with physically interpretable adjustable parameters can be found. For the industry's seminal data sets the PPTT model interpolates the data at least as efficaciously as the classical Archie-Winsauer model. This provides formation evaluationists with a practical alternative, with physically interpretable adjustable parameters, to the classical method for water saturation estimation.

## NOMENCLATURE

$\alpha_0, \alpha_2$	Proportionality constants
$\alpha_1, \alpha_3$	Integration constants
$\varphi$	(phi) porosity
$\varphi_\theta$	Pseudo-percolation threshold
$\varphi'$	Idealized porosity
$\varphi_{min}$	Porosity at $\sigma_{min}$
$\vartheta$	(theta) pseudo-percolation threshold contour
$\varphi - \varphi_\theta$	Connected fractional brine volume (CFBV)
$S_w$	Water saturation
$S_\theta$	Water saturation analog of porosity PPT
$S'_w$	Idealized water saturation
$S_{min}$	Water saturation at $\sigma_{min}$
$\beta = \varphi S_w$	Fractional brine volume = $\beta_c + \beta_i$
$\beta_i$	Isolated fractional brine volume
$\beta_c$	Connected fractional brine volume
$(\varphi - \varphi_\theta)/(1 - \varphi_\theta)$	Normalized CFBV
$R_0$	Resistivity if $S_w = 1$
$R_t$	Resistivity if $S_w < 1$
$R_w$	Brine resistivity
$V_{sh}$	Shale volume
$\sigma_0$	Conductivity if $S_w = 1$
$\sigma_t$	Conductivity if $S_w < 1$
$\sigma_w$	Brine conductivity
$\sigma_{min}$	Minimum conductivity parameter
$\sigma_{sh}$	Shale conductivity
$\sigma'$	Idealized conductivity
$\sigma_0/\sigma_w$	Normalized conductivity if $S_w = 1$
$\sigma_t/\sigma_w$	Normalized conductivity if $S_w < 1$
$a$	Winsauer's fudge factor
$m$	Archie's porosity exponent
$n$	Archie's saturation exponent
$n'$	Generalized saturation exponent
$\bar{n}$	Average saturation exponent
$F$	Formation resistivity factor = $R_0/R_w$
$f$	Formation conductivity factor = $1/F$
$I$	Resistivity index = $R_0/R_t$
$G$	Generalized resistivity index = $FI$
$E_0$	EET geometrical factor at $S_w = 1$
$E_t$	EET geometrical factor at $S_w < 1$
$E_p$	PPTT analog of $E_0$ and $E_t$
$a_0, a_t$	EET adjustable constants
$b_0, b_t$	EET adjustable constants
$a_p$	PPTT analog of $a_0$ and $a_t$
$b_p$	PPTT analog of $b_0$ and $b_t$
$c_p$	new EET constant suggested by PPTT

## ACKNOWLEDGMENTS

In a sense, this work has been in progress since 1977

when I first became curious as to the underlying physical basis for Archie's model. Over such a period of time interacting with colleagues, insight eventually happens. This study has been especially influenced, and would not have been possible without mentoring, encouragement and criticism from David Herrick. Most recently, Matt Honapour asked the question that started the thoughts about an optimum way to predict water saturation using a copious amount of special core analysis data. Arthur Thompson introduced me to the formal theory of the percolation threshold as elucidated in *Introduction to Percolation Theory* by D. Stauffer and A. Aharony. Mark Bowers, James Dixon, Quinn Passey, Keith Sullivan, David Hammock, and David Herrick all contributed valuable suggestions for improving the manuscript. Dr. Shin-Ju Ye and Dr. Suyu Ye found errors in my transcription of Hamada's data that, for a time, stalled progress. After seeing my presentation of these results at the 2006 SPWLA Symposium, Dr. Aleksandr Mousatov kindly introduced me to Allen Hunt's useful work, *Percolation Theory for Flow in Porous Media*. The comments of the anonymous reviewers, my associate editor Wim Looyestijn, and the guidance of the editor were all very helpful. Finally, I offer my thanks to ExxonMobil management both for making it possible to hatch these ideas, and for permission to publish them.

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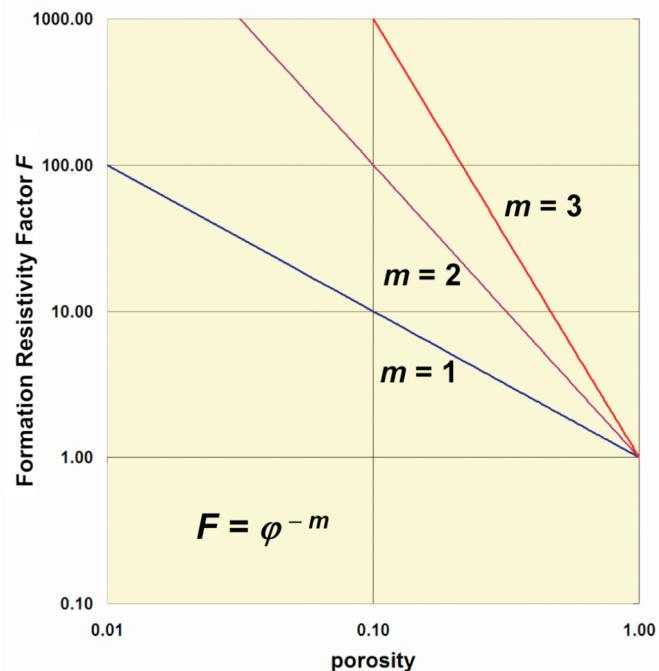
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## APPENDIX A

### Disadvantage of the bi-logarithmic plot

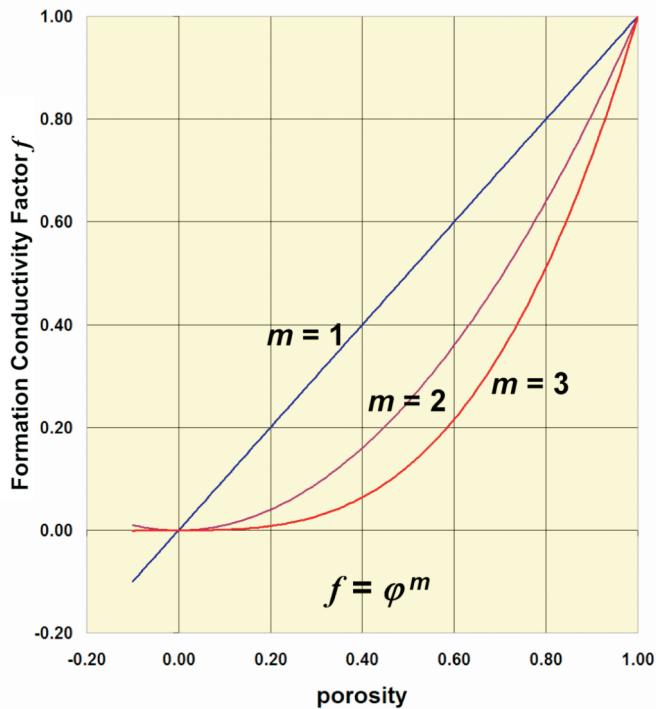
The representation of the conductivity – porosity relationship as a power law in resistivity and porosity deceives intuition. For example, consider the case of three lines in the logarithm of formation resistivity factor versus logarithm of porosity space, originating at the brine point (i.e.,  $(\phi, F) = (1, 1)$ ) and extending to the upper left boundaries of the plot toward lower porosity and higher resistivity values. Now consider whether these functions intersect at any points other than the point on the graph shown in Figure A.1. Intuition strongly compels the mind to reject the possibility that these lines, projecting as they do toward infinitesimal porosity and infinite resistivity, ever meet again. Indeed, for any finite extension of the graphical axes, the lines continue to diverge, leading to the illusion that the three functions intersect only at the single point shown on the graph.

On the other hand, if the same relationships are plotted as linear conductivity factor ( $f = 1/F$ ) versus linear formation porosity, the two points of intersection of the three func-



**FIG. A.1** Power law behavior. Three seemingly divergent lines, inviting the conclusion that the functions represented by the lines intersect only at the single point shown. However, this is not correct.

tions are readily apparent (Figure A.2). Thus it is realized that the bi-logarithmic porosity-resistivity plot will be unsuitable for the representation of high resistivity values occurring at low porosity values. Consider the usual case (for low porosity rocks) where resistivity rises at an increasingly greater rate as porosity diminishes. Physically, this represents a change in the mode of conductivity reduction; i.e., the occlusion of pore throats with progressive cementation. Eventually the continuous brine volume that supports electrical conduction is interrupted and the conduction ceases. Generally there will be porosity remaining in the rock when conduction ceases, but once formation brine becomes unconnected, porosity reduction by cementation is greatly reduced, or ceases, and porosity thereafter is essentially constant as long as the physical conditions in the rock (e.g., temperature and pressure) also remain constant. The resistivity will approach infinity along some constant-porosity asymptote; thus the graph of logarithm of formation resistivity factor versus logarithm of porosity must curve upward in this kind of rock. Another case is observed for rocks with a conductive matrix component.

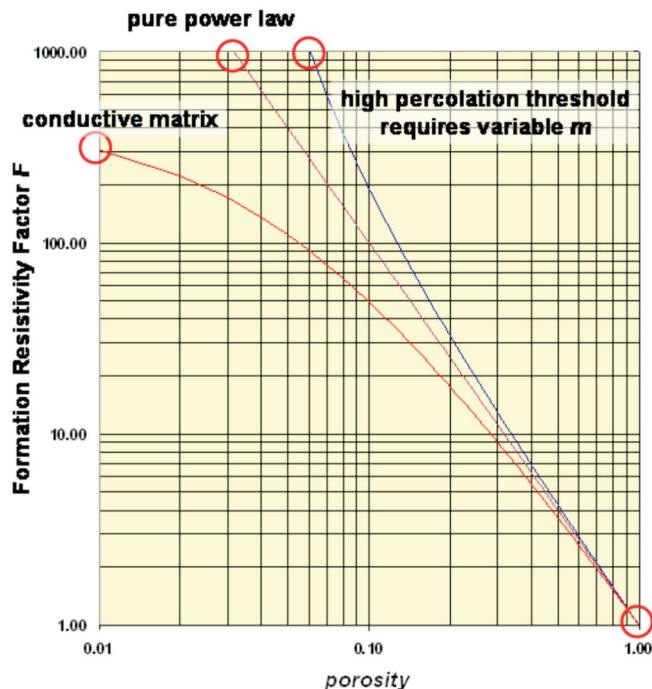


**FIG. A.2** Porosity - formation conductivity factor plot. The formation conductivity factor  $f = 1/F = \varphi^m$ . These are same functions as plotted in Figure A.1, but in this case plotted as linear formation conductivity factor versus linear porosity. The two points of intersection are obvious in this representation. The advantage of this representation for observing conductivity near zero porosity is also obvious.

This kind of rock will appear to continue to conduct as the fractional brine volume diminishes toward zero. That is, porosity decreases more rapidly than resistivity increases. The result is a limiting value of formation resistivity factor as porosity approaches zero. The formation resistivity factor – porosity curve bends down parallel to the porosity axis in these rocks.

Pure power law behavior is often observed to be violated, and for these cases to preserve Archie's power law form, Archie's model is modified by allowing the exponent  $m$  to vary as a function of porosity. Thus, although the power-law form is preserved, the function is no longer a power law (i.e., the exponent is not a constant). These cases are illustrated in Figure A.3.

All porosity values measured on Archie rocks are closer to the zero-porosity left side of the porosity axis than to the all-porosity right side of the porosity axis. It is self-evident that studying the decrease toward zero of the conductivity



**FIG. A.3** Power law broken. A pure power law is a straight line on a bi-logarithmic plot. At low porosity, the power law character of the formation resistivity factor versus porosity relationship is violated. Conventionally, this is accounted for by introducing a variable exponent (e.g., "variable  $m$ " models) allowing the power law form of the equation to be preserved although the resulting function is no longer, speaking strictly, a power law. A conductive matrix deflects the resistivity trend downward. A high percolation threshold deflects it upward.

formation factor (i.e.,  $1/F$ ) as porosity approaches zero is clearly facilitated when zero porosity and zero conductivity are represented on the graph employed for the study; e.g., as in Figure A.2.

## APPENDIX B

### Sedimentation and diagenesis

In this study I have found it convenient to regard the porosity-conductivity formation factor as a continuous function with continuous derivatives over the entire porosity domain; i.e., over  $0 \leq \varphi \leq 1.0$ . It is obvious that this proposition is not valid. For example, consider the deposition of the rock by sedimentation from a slurry. Imagine the mineral phase of the slurry to begin at zero fractional volume and continuously increase in fractional volume until each mineral grain is in contact with other mineral grains, the grains eventually forming a continuous phase. Call this condition the precipitation event. I claim that between the precipitation event and 100 percent brine the bulk conductivity of the slurry is a continuous, smooth function of the fractional volume of the brine phase. That the conductivity is continuous is self-evident. I claim that the derivative is also continuous up to the precipitation event because there is no mechanism that would interrupt the smooth decline of conductivity as the mineral fraction displaces the brine fraction.

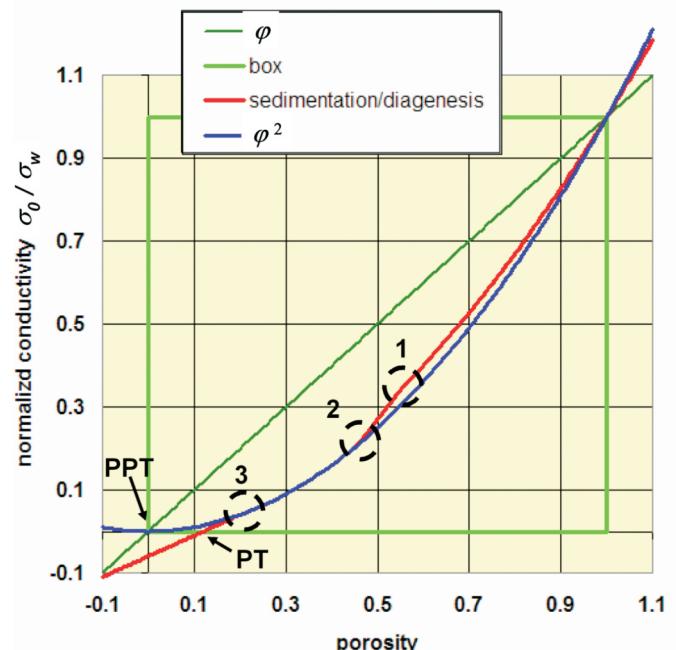
However, once the mineral fraction forms a continuous phase, deposition is finished; burial commences; compaction begins. The conductivity is expected to decrease at a more rapid rate during compaction than in deposition, as brine is squeezed out of the pore spaces between the mineral grains and the grains rearrange toward a minimum potential energy packing. Thus, there will be a break in the slope of the conductivity-porosity function to mark the boundary between the end of deposition and the beginning of compaction. This boundary should be near a brine fractional volume of 0.48, the porosity of cubic-packed spheres, a first approximation to the mineral grains in an Archie rock. This break in slope, or cusp, is shown in the circle labeled 1 in Figure B.1.

Compaction will proceed until the grains are locked in place by overburden pressure and incipient cementation and cannot further shift mechanically to respond to increasing local stresses. Conductivity decreases continuously and smoothly throughout the compaction process, but along a different trajectory than conductivity reduction in the slurry and the deposition process.

Once all grains are mechanically locked, further reduction in porosity occurs due primarily to cementation. The rate at which conductivity decreases with porosity during the cementation process will be less than during compac-

tion, so another break in the slope of the conductivity-porosity function is expected at this change in porosity reduction mechanisms. In the case of well-sorted sandstone, this cusp should occur prior to the rock's reaching 0.26 porosity, corresponding to hexagonal close-packed spheres. The exact location of this boundary is likely to be fuzzy because incipient cementation might begin well before compaction has ceased, for example, or initial porosity might vary due to the effects of sorting. The right edge of the Archie domain seems to coincide closely to the porosity at which spheres would be orthorhombically packed, about  $\varphi = 0.40$ , suggesting that for Archie rocks, compaction has already ceased near this boundary. This cusp is indicated in the circle labeled 2.

Once compaction has ceased and cementation has commenced the conductivity will decrease smoothly with reduction in porosity over 20-to-30 porosity units, until cementation has reduced the pore throats to a degree such that, with further cementation, pore throats begin to choke



**FIG. B.1** The porosity domain and normalized conductivity range are shown in the green box. During the processes of sedimentation and diagenesis conductivity will vary smoothly with porosity except at three cusps where the rate of conductivity reduction with respect to fractional volume of brine reduction changes. These cusps occur at (1) the onset of compaction, (2) the onset of cementation, and (3) the onset of non-Archie conductivity reduction. The changes in slope are subtle, and are exaggerated in this figure for ease of visibility. In the text I argue a hypothetical reversing of this process is perfectly smooth (blue curve).

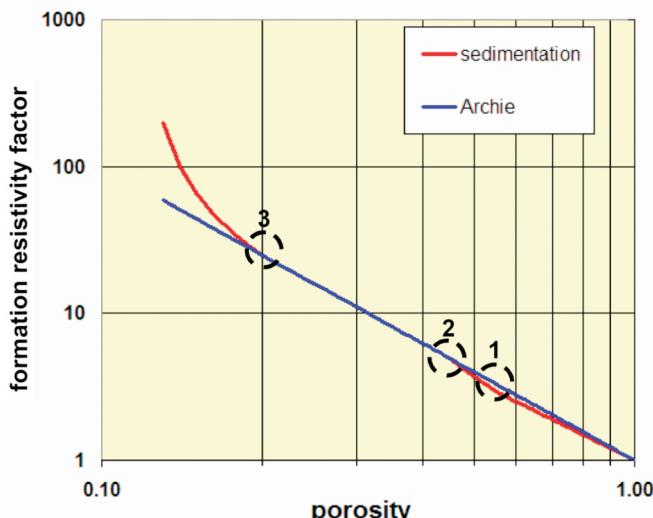
with cement and occlude, disconnecting conducting paths in the rocks. When this point is reached, probably at porosities less than 0.10, the rate of conductivity reduction increases with further porosity reduction. Thus, there is a final cusp on the porosity-conductivity curve, shown in the circle labeled 3 in Figure B.1. If the cementation continues, conduction will finally cease, all conductive paths having been disconnected by cement. However, the rock will still contain isolated porosity. The porosity value at the cessation of conduction is the true percolation threshold. Projection of the trend established in the Archie sub-domain (between cusps 2 and 3) beyond cusp 3 to the porosity axis gives a “pseudo-percolation threshold” which is convenient as a curve-fitting parameter.

This scenario envisions complicated conductivity-porosity behavior over the history of the rock. However, for the purpose of predicting conductivity from observations of porosity, only the conductivity variation in the Archie sub-domain, where many of the rocks encountered in hydrocarbon exploration reside, is of interest.

In this article I have proposed using a continuous function with continuous derivative determined by observations on rocks, and also containing the conductivity point at unit

fractional brine volume. This may appear unphysical. However, it is not an unreasonable assertion, which I illustrate with a thought experiment: beginning with a rock at the cusp marking the boundary between the final cementing phase in the rock’s evolution and the Archie sub-domain (cusp 3), and imagine running the diagenesis process in reverse. This begins by removing cement, thus increasing porosity and conductivity. When the boundary cusp between the Archie sub-domain and the “compaction sub-domain” is reached (i.e., cusp 2), imagine that the mineral grains are “magically” held in place, and further reduction in mineral fraction volume occurs by the dissolution of the mineral grains in situ, with the surface of each grain uniformly yielding up material, dissolving in the brine without any change in shape or orientation. This process continues until finally only brine is present. I claim that, in this imagined process, the conductivity and its derivative with respect to porosity are continuous. The conductivity is self-evidently continuous. The derivative is continuous because in the thought experiment there is no change in process that would cause the rate of conductivity increase with mineral volume decrease to change. Thus there would be no cusps, and the conductivity curve would be smooth.

Figure B.2 illustrates the cusps as they would appear on the usual formation resistivity factor - porosity bi-logarithmic plot. All of the parameter choices used in the production of these curves are exaggerated in order to make the effects more easily visualizable in the plot. Nevertheless, the cusps are hard to see. In a real rock the breaks in slope would be even more subtle. Thus, though the normalized conductivity - porosity curve is not rigorously smooth during the depositional process, it is very nearly smooth at the scales normally chosen to view the data.



**FIG. B.2** This is the same data as in Figure B.1, except that it is plotted on the conventional bi-logarithmic plot. Of course the two cusps at the lower left would be absent in real data. However, for low porosity data the resistivity factor is frequently observed to curve toward the top of the graph, illustrating that the low porosity behavior of the conductivity does not obey a power law, and that the bi-logarithmic graph is not a good tool for observing conductivity variation at low porosity. The point of departure from Archie behavior illustrated in this plot ( $\approx 0.20$ ) is “unphysical”, but is chosen to make the variation visible on the bilinear plot of Figure B.1. For most rocks this cusp would occur at approximately 0.10 or less (e.g., Archie’s Nacatoch Sandstone).

## APPENDIX C

### EET and PTTT

The electrical efficiency theory (Herrick and Kennedy, 1993, 1994) holds that when water saturation is equal to 1, the normalized conductivity of an Archie rock can be represented as

$$\frac{\sigma_0}{\sigma_w} = \varphi E_0 .$$

$E_0$  is determined as a function of  $\varphi$  by plotting  $\sigma_0/(\sigma_w \varphi) = E_0$  against the fractional brine volume. For Archie rocks the data will invariably suggest that, approximately

$$E_0 = a_0 \varphi + b_0 .$$

Combine  $E_0$  with the first equation, then (the equation numbering is carried over from the text article)

$$\frac{\sigma_0}{\sigma_w} = \varphi E_0 = \varphi(a_0\varphi + b_0) = a_0\varphi^2 + b_0\varphi, \quad (5.a)$$

This is EET for brine-filled Archie rocks, quadratic in  $\varphi$  with no constant term, or rather with an “invisible” constant term,  $c_0$ , representing normalized conductivity at  $\varphi = 0$ , equal to zero in this formulation. In the main text it was shown that this result is compatible with the pseudo-percolation threshold theory.

In fact, viewed only slightly differently, it becomes apparent that ETT and PPTT are not merely compatible, they are identical. To see this, let all assumptions of usual EET hold except that for this case take conductivity proportional to the amount of *connected* fractional brine volume,  $\varphi - \varphi_\vartheta$  rather than merely  $\varphi$ . Then

$$\frac{\sigma_0}{\sigma_w} = (\varphi - \varphi_\vartheta)E_0. \quad (C.1)$$

In this case plot the same laboratory data  $\sigma_0/(\sigma_w(\varphi - \varphi_\vartheta)) = E_0$  against connected fractional brine volume,  $\varphi - \varphi_\vartheta$ . Note that for the brine-filled tube that  $\varphi_\vartheta = 0$ , but for rocks generally,  $\varphi_\vartheta \neq 0$ . Then observe from plotting the data that, approximately,

$$E_0 = a'(\varphi - \varphi_\vartheta) + b'. \quad (C.2)$$

Combine this  $E_0$  with the equation (C.1), then

$$\frac{\sigma_0}{\sigma_w} = (\varphi - \varphi_\vartheta)(a'(\varphi - \varphi_\vartheta) + b'). \quad (C.3)$$

Or, after multiplying out the terms,

$$\frac{\sigma_0}{\sigma_w} = a'\varphi^2 + (b' - 2a'\varphi_\vartheta)\varphi + (a'\varphi_\vartheta^2 - b'\varphi_\vartheta), \quad (C.4)$$

quadratic in  $\varphi$ . If the coefficients of the porosity powers are the same as those of PPTT, then EET as formulated in terms of the connected fractional brine volume must be the same theory, arrived at by a different route. To resolve the question, first associate the coefficients of the porosity powers in (C.4) with the EET coefficients  $a_0$  and  $b_0$ , and include a term  $c_0$  to account for the possibility of a term independent of  $\varphi$ . Then

$$\frac{\sigma_0}{\sigma_w} = a_0\varphi^2 + b_0\varphi + c_0 \quad (C.5)$$

and ask what are  $a_0$ ,  $b_0$ , and  $c_0$  in terms of  $\varphi_\vartheta$ ?

It is already obvious by comparison of (C.4) and (C.5) that the coefficient of the  $\varphi^2$  term  $a' = a_0$ . To evaluate the

coefficients of the  $\varphi^1 = \varphi$  and  $\varphi^0 = 1$  terms from boundary conditions, make the following observations using equation (C.5):

$$\left. \frac{\sigma_0}{\sigma_w} \right|_{\varphi=1.0} = 1 = a_0 + b_0 + c_0 \quad (C.6)$$

and

$$\left. \frac{\sigma_0}{\sigma_w} \right|_{\varphi=\varphi_\vartheta} = 0 = a_0\varphi_\vartheta^2 + b_0\varphi_\vartheta + c_0. \quad (C.7)$$

Now, introduce the condition that

$$\left. \frac{d\left(\frac{\sigma_0}{\sigma_w}\right)}{d\varphi} \right|_{\begin{array}{l} \frac{\sigma_0}{\sigma_w}=0 \\ \varphi=\varphi_\vartheta \end{array}} = 0. \quad (C.8)$$

Then

$$a_0 + b_0 + c_0 = 1, \quad (C.9)$$

$$a_0\varphi_\vartheta^2 + b_0\varphi_\vartheta + c_0 = 0, \quad (C.10)$$

$$2a_0\varphi_\vartheta + b_0 = 0, \quad (C.11)$$

i.e.,

$$\begin{bmatrix} 1 & 1 & 1 \\ \varphi_\vartheta^2 & \varphi_\vartheta & 1 \\ 2\varphi_\vartheta & 1 & 0 \end{bmatrix} \begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (C.12)$$

is a system of three equations linear in the three unknowns  $a_0$ ,  $b_0$ ,  $c_0$ . The solution is

$$\begin{bmatrix} a_0 \\ b_0 \\ c_0 \end{bmatrix} = \frac{1}{(1-\varphi_\vartheta)^2} \begin{bmatrix} 1 \\ -2\varphi_\vartheta \\ \varphi_\vartheta^2 \end{bmatrix}. \quad (C.13)$$

Then from equations (C.4), (C.5), and (C.13)

$$a_0 = \frac{1}{(1-\varphi_\vartheta)^2} = a', \quad (C.14)$$

and

$$b_0 = b' - 2a'\varphi_\vartheta = \frac{-2\varphi_\vartheta}{(1-\varphi_\vartheta)^2}. \quad (C.15)$$

Thus

$$b' = b_0 + 2a'\varphi_{\vartheta} = \frac{-2\varphi_{\vartheta}}{(1-\varphi_{\vartheta})^2} + \frac{-2\varphi_{\vartheta}}{(1-\varphi_{\vartheta})^2} = 0. \quad (\text{C.16})$$

$$c' \equiv a'\varphi_{\vartheta}^2 - b'\varphi_{\vartheta} = a'\varphi_{\vartheta}^2, \quad (\text{C.17})$$

and

$$c' \equiv \frac{\varphi_{\vartheta}^2}{(1-\varphi_{\vartheta})^2} - b' = \frac{\varphi_{\vartheta}^2}{(1-\varphi_{\vartheta})^2} = c_0. \quad (\text{C.18})$$

So the foregoing considerations have shown that the EET coefficients in equation (C.4) and the corresponding coefficients of PPTT are precisely equivalent under the assumptions that: (1) it is the connected fractional brine volume, and not the porosity alone, that gives rise to conductivity behavior (i.e.,  $E_0$  is a function of  $\varphi - \varphi_{\vartheta}$  rather than  $\varphi$  alone); (2)  $\sigma_0 = \sigma_w$  when  $\varphi = 1.0$ ; and (3) the derivative  $d(\sigma_t/\sigma_w)/d(\varphi - \varphi_{\vartheta}) = 0$  when the normalized conductivity is zero.

This comparison makes clear some assumptions not explicitly honored in the original EET formulation; namely the assumptions numbered (1) and (3) in the preceding paragraph. The condition invoked on the derivative, equation (C.8), is implicitly imposed in EET (with  $\varphi_{\vartheta} = 0$ ) since conductivity and porosity vanish together.

Finally EET informs PPTT regarding the proportionality of  $d(\sigma_0/\sigma_w)/d(\varphi - \varphi_{\vartheta})$  to  $\varphi - \varphi_{\vartheta}$  and  $d(\sigma_t/\sigma_w)/d(\varphi S_w - \vartheta)$  to  $\varphi S_w - \vartheta$ , adopted in equations (2) and (12) as the working hypothesis. When reciprocal resistivity data are plotted against connected fractional brine volume to determine the dependence of the electrical efficiency function upon that volume, the relationship in Archie rocks is invariably linear—strikingly so for single core plug reciprocal resistivity index data versus  $S_w - S_{\vartheta}$ . Thus resort to this representation is strong evidence that the exponent is indeed exactly equal to 2.

## APPENDIX D

### Equation (22.a) from a boundary condition

The left side of equation (22.a) was obtained by appeal to intuition. However, with the insight gained from experience with this model, the same equation can be shown to follow from a physically reasonable boundary condition. Beginning with the observations

$$\frac{d(\sigma_t / \sigma_w)}{d(\varphi S_w - \vartheta)} \propto (\varphi S_w - \vartheta) \quad (\text{11})$$

and proceeding to an assignment of functional form

$$\frac{d(\sigma_t / \sigma_w)}{d(\varphi S_w - \vartheta)} = \alpha_2 (\varphi S_w - \vartheta). \quad (\text{12})$$

Then

$$\int d\left(\frac{\sigma_t}{\sigma_w}\right) = \alpha_0 \int (\varphi S_w - \vartheta) d(\varphi S_w - \vartheta). \quad (\text{D.1})$$

The elementary integration leads to

$$\frac{\sigma_t}{\sigma_w} = \frac{\alpha_2}{2} (\varphi S_w - \vartheta)^2 + \alpha_3 \quad (\text{13})$$

where the notation for the equations and constants used earlier in the text is retained. The boundary conditions are that the normalized conductivity equal unity at porosity equal 1.0, i.e.,

$$\left. \frac{\sigma_t}{\sigma_w} \right|_{\sigma_t = \sigma_w} = 1 = \frac{\alpha_2}{2} (1 - \vartheta_1)^2 + \alpha_3, \quad (\text{D.2})$$

and, if at the opposite of the end of the porosity domain, pre-science developed from experience suggests that the vertex of the normalized conductivity parabola may not fall precisely on the porosity axis, and assign the coordinates of the vertex to be  $(\varphi, \sigma) = (\varphi_{min}, \sigma_{min})$ , then at the low-porosity end of the porosity domain

$$\left. \frac{\sigma_{min}}{\sigma_w} \right|_{\sigma_t = \sigma_{min}} = \frac{\alpha_2}{2} (\sigma_{min} S_{min} - \vartheta_{min})^2 + \alpha_3 \quad (\text{D.3})$$

where

$$\vartheta_{min} = \varphi_{min} S_{min}. \quad (\text{D.4})$$

Therefore

$$\alpha_3 = \frac{\sigma_{min}}{\sigma_w}. \quad (\text{D.5})$$

Substituting  $\alpha_3$  into equation (13) gives

$$\frac{\sigma_t}{\sigma_w} = \frac{\alpha_2}{2} (\varphi S_w - \vartheta_{min})^2 + \frac{\sigma_{min}}{\sigma_w} \quad (\text{D.6})$$

or

$$\frac{\sigma_t}{\sigma_w} - \frac{\sigma_{min}}{\sigma_w} = \frac{\alpha_2}{2} (\varphi S_w - \vartheta_{min})^2. \quad (\text{D.7})$$

Then, easily, evaluating equation (D.7) at  $\varphi S_w = 1$  ( $\Rightarrow \sigma_t = \sigma_w$ ) for  $\alpha_2$

$$\alpha_2|_{\varphi S_w=1} = \frac{2\left(1 - \frac{\sigma_{min}}{\sigma_w}\right)}{(1 - \vartheta_I)^2} \quad (D.8)$$

where  $\vartheta_{min}$  ( $\varphi = 1, S_w = 1$ ) =  $\vartheta_I$ , and substituting for  $\alpha_2$  in (D.7) and slightly rearranging

$$\frac{\frac{\sigma_t}{\sigma_w} - \frac{\sigma_{min}}{\sigma_w}}{1 - \frac{\sigma_{min}}{\sigma_w}} = \frac{(\varphi S_w - \vartheta_{min})^2}{(1 - \vartheta_I)} \quad (22.a)$$

the same expression equation (22.a) that was asserted by appeal to intuition in the text.

## APPENDIX E

### Details of the normalized conductivity surface

The normalized conductivity surface over the  $\varphi$ - $S_w$  plane represents a physical relationship only over portions of the plane where brine forms a continuous conducting phase. In the Archie sub-domain mineral grains, hydrocarbon fluids, and brine comprise interpenetrating, continuous phases. Near and outside the sub-domain boundaries the phases begin to disconnect. For example, as the  $S_w = 1$  plane is approached the hydrocarbon phase disconnects and vanishes; as  $\varphi \rightarrow 0$  porosity becomes discontinuous, disconnecting the brine phase; as  $S_w \rightarrow 0$  the brine phase disconnects. At the corner of the normalized conductivity cube where  $\varphi = 1.0, S_w = 0$ , clearly hydrocarbon is the only phase present. Surrounding this point are emulsions and slurries having a continuous hydrocarbon phase.

In any region of the  $\varphi$ - $S_w$  plane where brine is discontinuous, the electrical conductivity obviously vanishes. So the proper domain of the normalized conductivity surface is that region of the  $\varphi$ - $S_w$  plane capable of supporting conduction characterized physically by brine being a continuous phase. This truer representation of the unit conductivity cube is shown in Figure E.1.

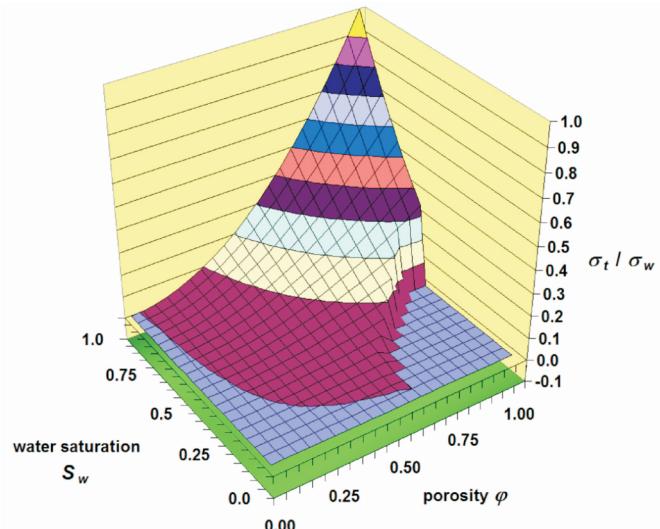
## APPENDIX F

### Possible generalizations of PPTT

The development of the pseudo-percolation threshold model from an inspection of the seminal data sets began with accepting a relationship suggested by Archie's observations, namely the direct proportionality of the rate of change of normalized conductivity with respect to the connected fractional brine volume, with a similar relationship holding when  $S_w < 1$ , leading to

$$\frac{\frac{\sigma_t}{\sigma_w} - \frac{\sigma_{min}}{\sigma_w}}{1 - \frac{\sigma_{min}}{\sigma_w}} = \left( \frac{\varphi - \varphi_{min}}{1 - \varphi_{min}} \right)^2 \left( \frac{S_w - S_{min}}{1 - S_{min}} \right)^2. \quad (22.b)$$

However, the acceptance of direct proportionality is notionally based upon the principle of Occam's razor. The data could as easily support a conjecture that the rate of change of normalized conductivity with respect to the connected fractional brine volume is proportional to a fractional power,  $m - 1$ , not quite equal to one if  $m \approx 2$ , of the connected fractional brine volume. This would lead to a power law form with  $m$  exponents not exactly equal to, but close to, two. The exponents are determined by the minimization of some objective function such as the sum of squared residuals. If desired, a "Winsauer" parameter  $1/\alpha$  can also be included, to yield



**FIG. E.1** For earlier convenience, an important property of the normalized conductivity surface was not discussed. This is that the conductivity is, of course, zero wherever brine is a discontinuous phase and the mineral phase is non-conducting (i.e., everywhere in an Archie rock). The vanishing of the normalized conductivity along the pseudo-percolation threshold contour on the  $\varphi$ - $S_w$  plane has already been described at length. This graphic illustrates that region of the surface near zero water saturation and unit porosity physically occupied by mixtures of oil and brine (emulsions) or oil and mineral grains (slurries) where brine forms a discontinuous phase, shown notionally as a bite taken from the lower right corner of the surface. The Archie sub-domain does not intersect this bite, and the existence of the bite has no implication for PPTT interpretations.

$$\frac{\sigma_t - \sigma_{min}}{\sigma_w - \sigma_w} = \frac{1}{a} \left( \frac{\varphi - \varphi_{min}}{1 - \varphi_{min}} \right)^m \left( \frac{S_w - S_{min}}{1 - S_{min}} \right)^n. \quad (22.c)$$

This is an Archie-like quasi-power law that recognizes the existence of pseudo-percolation thresholds. There are six adjustable parameters; a model of this form will be better able to minimize an objective function than a three parameter model. Inclusion of the Winsauer factor removes the constraint that  $\sigma_t/\sigma_w = 1$  at  $(\varphi, S_w) = (1, 1)$  if relaxing that restriction provides a better fit of the data to the surface represented by equation (22.c). This is a full generalization of the classical A-W model to include pseduo-percolation threshold parameters. From the point of view of curve fitting (22.c) seems to be on equal, or even better, footing with other fitting models. However, I am not comfortable with the ad hoc introduction of variable exponents, sans physical interpretation, for the sole purpose of increasing the number of adjustable parameters.

Percolation theory predicts for certain, more-or-less rock-like, three-dimensional conductivity models that the  $m$  exponent is equal to  $m = 1.88$ , and further posits that the value of this exponent is “universal”, applying to a whole class of three-dimensional conduction problems in porous media governed by percolation theory, including electrical conduction in Archie rocks (Hunt, 2005). The tenants of percolation theory being both relatively new and thoroughly arcane, this finding is not widely known in the formation evaluation community. Moreover, the claim to “universality” might be overly ambitious, the  $m = 1.88$  result being merely suggestive rather than definitive. Real rocks exhibit a distribution of  $m$  exponents with mean value in the neighborhood of 2. This is no surprise. However, the other element predicted by percolation theory is that there will be a percolation threshold. It is this latter element that has been omitted from classical formation evaluation.

## APPENDIX G

### Whimsey

Many physical principles are capable of parsimonious expression. On the other hand, historically many overly pithy principle statements, in earlier times generally accepted as reasonable and seemingly self-evident, have proved to be incorrect; e.g., the earth is flat, the orbits of the planets are circles, heavy objects fall faster than light objects, etc. This latter assertion by Aristotle can be mathematically expressed as

$$v - v_0 \propto \text{mass} \quad (G.1)$$

where  $v$  is speed and  $v_0$  is initial speed. Galileo Galilei, in testing this proposition, found instead that the velocity of falling objects is independent of mass, but depends upon the time spent in falling according to

$$v - v_0 \propto t - t_0 \quad (G.2)$$

where  $t$  is time and  $t_0$  is an initial time. Average speed is the change in displacement,  $s$ , during a specified time interval; i.e.,  $v = \Delta s / \Delta t$ . Thus Galileo found that

$$\frac{\Delta(s - s_0)}{\Delta(t - t_0)} \propto (t - t_0) \quad (G.3)$$

or, in words, a change in an increment of displacement  $\Delta(s - s_0)$  during a time increment  $\Delta(t - t_0)$  is proportional to the time elapsed since  $t = t_0$ . This proportionality has the same form as equation (1), and leads to the formula

$$s - s_0 = \frac{g}{2} (t - t_0)^2. \quad (G.4)$$

where  $g$  is the (constant) acceleration of gravity. Equation (G.3) might be construed as a “kinematic” first principle, but Galileo’s kinematical principles can be easily derived from Newton’s dynamical principles (i.e., the three laws of motion and universal gravitation). Consequently Galileo’s kinematical rules are not taught in school as “first” principles; before Newton’s work, they justifiably might have been. The formulation of equation (G.3) is based on many observations; likewise the formulation of (1) and (11) is based upon observation. The analogy of (1) and (11) to (G.3) is close. As Newton’s laws of motion and gravitation were (for a time) a hidden foundation for Galileo’s kinematical principles, surely a more fundamental principle than equations (1) or (11) must exist. Formation evaluation will remain as much art as science until such a principle be found.

## APPENDIX H

### Archie’s Nacatoch sandstone porosity – formation resistivity factor data

Winsauer and others (1952) published a tabulation of the data that was used in the estimation of the Humble formula parameters. Archie’s (1942) data is published only as a postage stamp-sized figure. In order to make quantitative comparisons, numerical values are required. After checking with several Shell colleagues as to whether Archie’s original data is extant in Shell’s archives, it seems that the original tabulation of this data is lost. To proceed, I digitized the data by first xerographically enlarging the postage stamp to

a size that permitted standard-sized rulers to be used, and accounting as best I could for xerographic distortion by digitizing between the subdivisions of the enlarged graph grid. The result is listed in Table H.1.

I numbered the points roughly in ascending order of porosity. The samples with indices 50, 55, 59, and 62 are highlighted to indicate that due to the crowding of the points in the original graph, there is some uncertainty in the location of these points, or even whether distinct data actually exist at these coordinates.

## ABOUT THE AUTHOR

**David Kennedy**'s research interests have been centered upon the measurement and interpretation of formation resistivity in hydrocarbon-bearing reservoirs. This has led to a description of conductive formations in terms of the tensor components of conductivity, their interpretation in terms of water saturation in the reservoir, their measurement using existing and next-generation instrumentation, and the numerical modeling of instrument responses in isotropic and anisotropic media. David has six patents and is author or co-author of a number of articles in the geophysical and well-logging literature. His career began in 1973 as a Schlumberger field engineer working in the western United States and Alaska. David left Schlumberger to pursue graduate studies beginning in 1978. He has acquired MS degrees in Physics and Earth Sciences from the University of Texas at Dallas, with further graduate studies at University of California at Berkeley. These academic pursuits served David's interest in acquiring, understanding, and interpreting borehole resistivity measurements. While studying at U.C. Berkeley David taught courses in computer science, materials science, physics, and electrical engineering to undergraduates at local collages in the San Francisco Bay area. David has also worked briefly in operations for ARCO, and in research for Standard Oil Company of Ohio (Sohio), and for Lockheed Missiles and Space Systems. In 1988 David began a 13 year career at Mobil Oil Corporation, working both in research and operations services. Kennedy has served ExxonMobil since the year 2000 merger of Exxon and Mobil as a research associate at Upstream Research Company, and a petrophysicist and formation evaluationist in the Exploration Company. David has been the industry-champion for the use of resistivity modeling as a routine element of log interpretation. David served the well logging industry as editor of the well-logging and petrophysics journal, *Petrophysics* from 1998 to 2002. During this same period he served as Vice President for Publications of the Society of Professional Well Log Analysts. Throughout his career David has taken an active interest in promoting international scholarship, and has collaborated on research with scholars from China, England, France, Korea, Russia, Turkey, South America, and the middle east. In 1999 David spent a week at the East China Petroleum University attending the 1999 International Workshop on Well Logging, exchanging views on new methods in petrophysics. He is an Honorary Professor of Engineering at the Xi'an Petroleum Institute in Xi'an, China. David is the proud father of a West Point plebe. He resides near Magnolia, Texas with his darling daughter and his gracious and lovely mate.

Archie's Published Resistivity-Porosity Data		
Index	<i>g</i>	<i>F</i>
1	0.116	93.722
2	0.120	83.567
3	0.133	75.128
4	0.117	69.265
5	0.149	55.330
6	0.162	57.618
7	0.161	51.647
8	0.162	47.148
9	0.162	41.366
10	0.156	36.027
11	0.180	42.788
12	0.188	43.089
13	0.167	30.956
14	0.191	36.406
15	0.183	33.221
16	0.204	34.841
17	0.184	25.926
18	0.210	32.195
19	0.198	27.663
20	0.162	17.188
21	0.200	23.050
22	0.200	21.631
23	0.201	20.063
24	0.219	19.257
25	0.219	17.718
26	0.212	20.596
27	0.190	14.202
28	0.191	12.546
29	0.229	15.539
30	0.237	16.561
31	0.275	21.040
32	0.228	18.612
33	0.253	13.569
34	0.266	12.206
35	0.279	13.369
36	0.282	11.078
37	0.280	15.095
38	0.306	14.408
39	0.304	10.623
40	0.319	11.345
41	0.357	12.734
42	0.351	11.498
43	0.319	10.781
44	0.242	15.345
45	0.254	16.510
46	0.258	15.461
47	0.266	14.309
48	0.265	13.022
49	0.278	12.573
50	0.276	11.754
51	0.286	11.914
52	0.289	10.497
53	0.288	9.262
54	0.305	9.905
55	0.308	9.396
56	0.306	8.863
57	0.297	8.155
58	0.320	8.229
59	0.323	9.928
60	0.330	9.917
61	0.334	9.532
62	0.336	8.464
63	0.346	8.507
64	0.360	7.898
65	0.357	7.256
66	0.371	8.072
67	0.371	7.522
68	0.369	8.922
69	0.382	6.572
70	0.396	7.105
71	0.392	8.258
72	0.402	5.960

**TABLE H.1** Digitization of porosity – formation resistivity factor data.