

## An Odd Sum<sup>1</sup> (prob1)

### The Problem

Some numbers such as 40 can be written as the sum of consecutive integers. For example,

$$40 = 6 + 7 + 8 + 9 + 10.$$

Shiflett and Shultz<sup>1</sup> defined *odd-summing* natural numbers as “natural numbers that are the sum of two or more consecutive [positive] odd numbers.” Perfect squares (larger than one) are always odd-summable, while primes will not be. If a number is odd-summable, there may be more than one way to express it in that manner. For example,

$$40 = 19 + 21 = 7 + 9 + 11 + 13$$

For any odd summable number  $n$ , you are to enumerate all the sets of consecutive odd positive integers that sum to the given number  $n$ .

### Input

The first line of input will be the number of problems. The remaining lines will contain one number  $n$ ,  $1 \leq n \leq 10^9$  per line.

### Output

For each problem, output one line for each solution for a number in the input set. On the line, first have the original number, followed by a colon and a space. Then have the set of consecutive odd integers whose sum is  $n$ . Output the endpoints of each set as shown in the sample output.

When multiple solutions exist, output them sorted by the first term. If no solution exists for a given  $n$ , output a single line with “impossible” instead of the set.

### Sample Input

3  
7  
35  
400

### Sample Output (corresponding to sample input)

7: impossible  
35: [3, 11]  
400: [1, 39]  
400: [31, 49]  
400: [43, 57]  
400: [97, 103]  
400: [199, 201]

<sup>1</sup>Adapted from *An Odd Sum* by Ray C. Shiflett and Harris S. Shultz and published in *The Mathematics Teacher*, Vol. 95, No. 3 (March 2002), pp. 206-209