

ELE2038: Signals and Control

1 Problem 1

A system with a transfer function

$$G(s) = \frac{s^3 + s^2 - 5s - 1}{8s^5 + 38s^4 + 65s^3 + 50s^2 + 17s + 2}, \quad (1.1)$$

can be shown to be BIBO-stable through the use of Routh's criterion. Defining G as $G(s) = \frac{P(s)}{Q(s)}$, we can take Q as the characteristic polynomial and construct a Routh's tabulation using its coefficients as shown in Table 1. When tabulated, it can be seen that all values in the first column are positive, therefore, all roots of Q have a negative real part, meaning G is BIBO-stable. G can also be shown to be "sufficiently stable" if all of its poles are more than 0.2 from the imaginary axis. We can show this by constructing another Routh's tabulation with the coefficients of $Q(s - c)$ where c is the distance from the axis, in this case, 0.2. If all values in the first column are positive, the poles of $G(s)$ are more than 0.2 from the imaginary axis. Using the binomial theorem to expand $Q(s - c)$ and calculate the coefficients,

$$Q(s + 0.2) = 8(s + 0.2)^5 + 38(s + 0.2)^4 + 65(s + 0.2)^3 + 50(s + 0.2)^2 + 17(s + 0.2) + 2 \quad (1.2)$$

$$= 8s^5 + 30s^4 + 37.8s^3 + 19.48s^2 + 3.648s + 0.138 \quad (1.3)$$

Table 2 shows that this is the case, hence G is BIBO-stable with all poles more than 0.2 from the axis.

s^5	8	65	17
s^4	38	50	2
s^3	54.47	16.58	0
s^2	38.43	2	0
s^1	13.74	0	0
s^0	2	0	0

Table 1: Routh's tabulation of $Q(s)$.

s^5	8	37.80	3.648
s^4	30	19.48	0.138
s^3	32.61	3.611	0
s^2	16.16	0.138	0
s^1	3.332	0	0
s^0	0.138	0	0

Table 2: Routh's tabulation of $Q(s + 0.2)$.

2 Problem 2

References

- [1] P. Sopaskis, *Control Systems: An introduction*, 1.0.2-rc.6. Applied Mathematix Press, 2023.