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EEECS
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Assignment H4

ELE2038: Signals and Control

1 Problem 1

A system with a transfer function

$$G(s) = \frac{s^3 + s^2 - 5s - 1}{8s^5 + 38s^4 + 65s^3 + 50s^2 + 17s + 2}, \tag{1.1}$$

can be shown to be BIBO-stable through the use of Routh's criterion. Defining G as $G(s) = \frac{P(s)}{Q(s)}$, we can take Q as the characteristic polynomial and construct a Routh's tabulation using its coefficients as shown in Table 1. When tabulated, it can be seen that all values in the first column are positive, therefore, all roots of Q have a negative real part, meaning G is BIBO-stable. G can also be shown to be "sufficiently stable" if all of its poles are more than 0.2 from the imaginary axis. We can show this by constructing another Routh's tabulation with the coefficients of $Q(s - c)$ where c is the distance from the axis, in this case, 0.2. If all values in the first column are positive, the poles of $G(s)$ are more than 0.2 from the imaginary axis. Using the binomial theorem to expand $Q(s - c)$ and calculate the coefficients,

$$Q(s + 0.2) = 8(s + 0.2)^5 + 38(s + 0.2)^4 + 65(s + 0.2)^3 + 50(s + 0.2)^2 + 17(s + 0.2) + 2 \tag{1.2}$$

$$= 8s^5 + 30s^4 + 37.8s^3 + 19.48s^2 + 3.648s + 0.138 \tag{1.3}$$

Table 2 shows that this is the case, hence G is BIBO-stable with all poles more than 0.2 from the axis.

s^5	8	65	17
s^4	38	50	2
s^3	54.47	16.58	0
s^2	38.43	2	0
s^1	13.74	0	0
s^0	2	0	0

Table 1: Routh's tabulation of $Q(s)$.

s^5	8	37.80	3.648
s^4	30	19.48	0.138
s^3	32.61	3.611	0
s^2	16.16	0.138	0
s^1	3.332	0	0
s^0	0.138	0	0

Table 2: Routh's tablulation of $Q(s + 0.2)$.

2 Problem 2

To tune a PID controller for the system

$$G(s) = \frac{0.5s + 1}{(s + 1)^3(0.1s + 1)}, \tag{2.1}$$

various methods can be used.

2.1 Part (i)

First we will use the first Ziegler-Nichols method. We begin by finding the inflection point of the system's step response and plotting a tangent to the curve at that point. The plot of the step response and tangent are shown below in Figure 1. We can then find the values required to calculate the tuning parameters

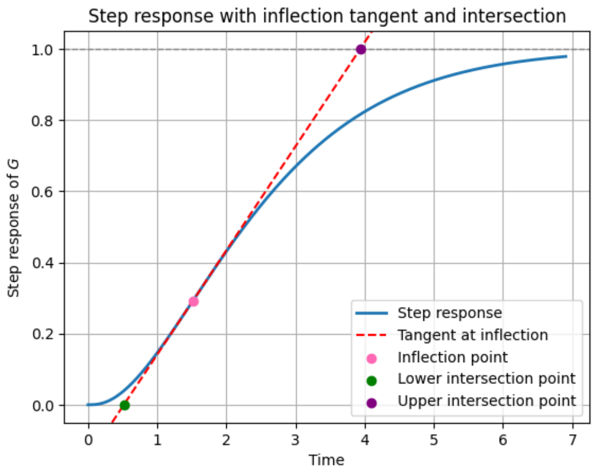

Figure 1: A plot of the step response of G and a tangent to the curve at the inflection point.

given the two intersection points marked in Figure 1. The resulting PID parameters are stated in Table 3 below. Applying these parameters to the system in Equation (2.1) produces the step response shown

K_c	τ_I	τ_D
7.927	1.035	0.259

Table 3: Ziegler-Nichols tuning parameters.

in Figure 2.

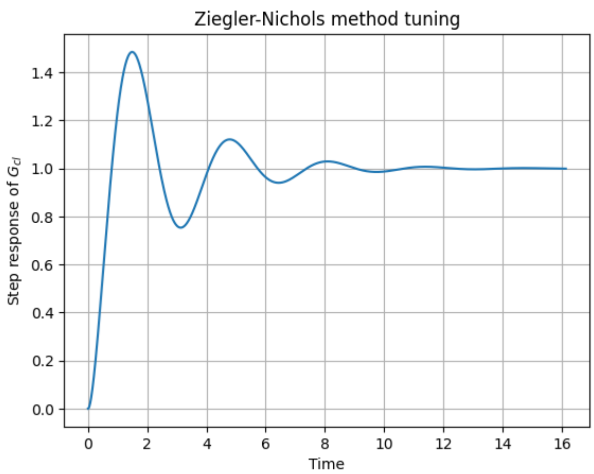

Figure 2: A plot of the step response of G_{cl} tuned using the first Ziegler-Nichols method.

2.2 Part (ii)

Second, we will use the Ziegler-Nichols ultimate sensitivity method. We start by finding the ultimate gain through experimentation, then we find the ultimate period. From these values we can calculate the PID controller parameters which are stated in Table ??. Applying these parameters to the system in Equation

K_p	τ_I	τ_D
14.24	0.462	0.116

Table 4: Ziegler-Nichols ultimate sensitivity method tuning parameters.

(2.1) produces the step response shown in Figure 3.

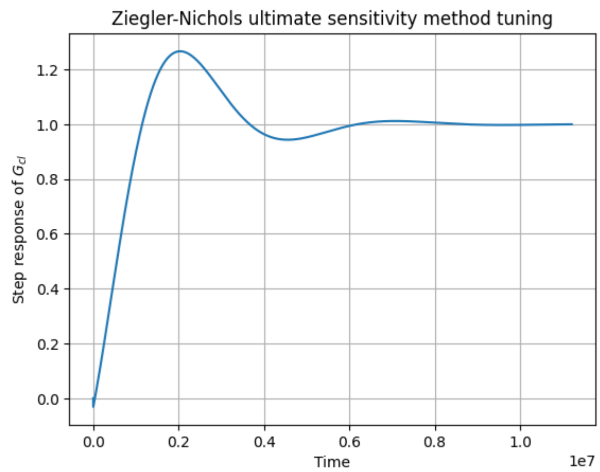

Figure 3: A plot of the step response of G_{cl} tuned using the Ziegler-Nichols ultimate sensitivity method.

3 Problem 3

References

[1] P. Sopaskis, *Control Systems: An introduction*, 1.0.2-rc.6. Applied Mathematix Press, 2023.