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CS181-S18

Assignment #3
Due: 11:59pm March 23, 2018

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Homework 3: Max-Margin and SVM

Introduction

This homework assignment will have you work with max-margin methods and SVM classification. The aim of the assignment is (1) to further develop your geometrical intuition behind margin-based classification and decision boundaries, (2) to have you implement a basic Kernel-based classifier and get some experience in implementing a model/algorithm from an academic paper in the field, and (3) to have you reflect on the ethics lecture and to address the scenario discussed in class in more depth by considering the labor market dynamically.

There is a mathematical component and a programming component to this homework. Please submit your PDF and Python files to Canvas, and push all of your work to your GitHub repository. If a question requires you to make any plots, like Problem 3, please include those in the writeup.

Problem 1 (Fitting an SVM by hand, 7pts)

For this problem you will solve an SVM without the help of a computer, relying instead on principled rules and properties of these classifiers.

Consider a dataset with the following 7 data points each with $x \in \mathbb{R}$:

$$\{(x_i, y_i)\}_i = \{(-3, +1), (-2, +1), (-1, -1), (0, -1), (1, -1), (2, +1), (3, +1)\}$$

Consider mapping these points to 2 dimensions using the feature vector $\boldsymbol{\phi}(x) = (x, x^2)$. The hard margin classifier training problem is:

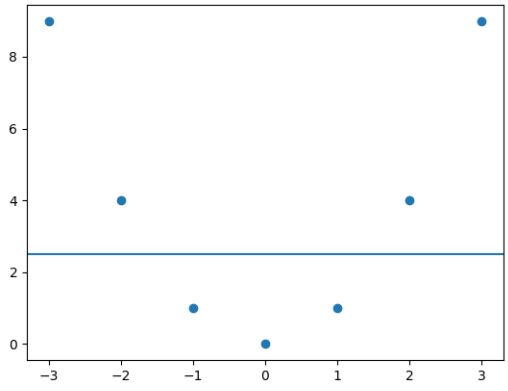
$$\begin{aligned} &\min_{\mathbf{w}, w_0} \|\mathbf{w}\|_2^2 \\ \text{s.t.} \quad &y_i(\mathbf{w}^\top \boldsymbol{\phi}(x_i) + w_0) \geq 1, \ \forall i \in \{1, \ldots, n\} \end{aligned} \tag{1}$$

The exercise has been broken down into a series of questions, each providing a part of the solution. Make sure to follow the logical structure of the exercise when composing your answer and to justify each step.

1. Plot the training data in $\mathbb{R}^2$ and draw the decision boundary of the max margin classifer.
2. What is the value of the margin achieved by the optimal decision boundary?
3. What is a vector that is orthogonal to the decision boundary?
4. Considering discriminant $h(\boldsymbol{\phi}(x); \mathbf{w}, w_0) = \mathbf{w}^\top \boldsymbol{\phi}(x) + w_0$, give an expression for *all possible* $(\mathbf{w}, w_0)$ that define the optimal decision boundary. Justify your answer.
5. Consider now the training problem (1). Using your answers so far, what particular solution to $\mathbf{w}$ will be optimal for this optimization problem?
6. Now solve for the corresponding value of w_0, using your general expression from part (4.) for the optimal decision boundary. Write down the discriminant function $h(\boldsymbol{\phi}(x); \mathbf{w}, w_0)$.
7. What are the support vectors of the classifier? Confirm that the solution in part (6.) makes the constraints in (1) binding for support vectors.

Solution

1.



2. r = 1.5
3. $< 0, 1 >$ is orthogonal
4. $\forall i : h(\phi(x))y_i \geq 1$
   This is any $\vec{w}$ that satisfies the hard margin constraint is a linear separator

   So for our data:
   $-3w_1 + 9w_2 + w_0 \geq 1$
   $-2w_1 + 4w_2 + w_0 \geq 1$
   $w_1 - w_2 - w_0 \geq 1$
   $-w_0 \geq 1$
   $-w_1 - w_2 - w_0 \geq 1$
   $2w_1 + 4w_2 + w_0 \geq 1$
   $3w_1 + 9w_2 + w_0 \geq 1$

5. $\vec{w} - < 0, \frac{2}{3} >$
6. $w_0 = \frac{-5}{3}$
   Writing down $h(\phi(x))$:
   $h(\phi(x)) = \frac{2}{3}x^2 - \frac{5}{3}$
7. The positive classification support vector is $x = -2, +2$ and the negative classification support vector is $x = -1, 1$.

   $-2w_1 + 4w_2 + w_0 = 1$
   $w_1 - w_2 - w_0 = 1$
   $-w_1 - w_2 - w_0 = 1$
   $2w_1 + 4w_2 + w_0 = 1$

   So the constraints are binding for these points.

Problem 2 (Scaling up your SVM solver, 10pts (+opportunity for extra credit))

For this problem you will build a simple SVM classifier for a binary classification problem. We have provided you two files for experimentation: training *data.csv* and validation *val.csv*.

- First read the paper at http://www.jmlr.org/papers/volume6/bordes05a/bordes05a.pdf and implement the Kernel Perceptron algorithm and the Budget Kernel Perceptron algorithm. Aim to make the optimization as fast as possible. Implement this algorithm in *problem2.py*.

  [Hint: For this problem, efficiency will be an issue. Instead of directly implementing this algorithm using numpy matrices, you should utilize Python dictionaries to represent sparse matrices. This will be necessary to have the algorithm run in a reasonable amount of time.]

- Next experiment with the hyperparameters for each of these models. Try seeing if you can identify some patterns by changing β, N (the maximum number of support vectors), or the number of random training samples taken during the Randomized Search procedure (Section 4.3). Note the training time, training and validation accuracy, and number of support vectors for various setups.

- Lastly, compare the classification to the naive SVM imported from scikit-learn by reporting accuracy on the provided validation data. *For extra credit, implement the SMO algorithm and implement the LASVM process and do the same as above.*[a]

We are intentionally leaving this problem open-ended to allow for experimentation, and so we will be looking for your thought process and not a particular graph. Visualizations should be generated using the provided code. You can use the trivial $K(\mathbf{x}, \mathbf{x}') = \mathbf{x}^\top \mathbf{x}'$ kernel for this problem, though you are welcome to experiment with more interesting kernels too.

In addition, provide answers the following reading questions **in one or two sentences for each**.

1. In one short sentence, state the main purpose of the paper.
2. Describe each of the parameters in Eq. 1 in the paper
3. State, informally, one guarantee about the Kernel perceptron algorithm described in the paper.
4. What is the main way the budget kernel perceptron algorithm tries to improve on the perceptron algorithm?
5. (*if you did the extra credit*) In simple words, what is the theoretical guarantee of LASVM algorithm? How does it compare to its practical performance?

[a] Extra credit only makes a difference to your grade at the end of the semester if you are on a grade boundary.

Solution

Metric	Train time	Train accuracy	Val accuracy	Support Vectors
K	0s	.0	.0	1
$\beta = 0, N = 100$	2.3900s	0.9999	1.0	100
$\beta = -.1, N = 100$	0.0934s	0.5026	0.4976	100
$\beta = -.5, N = 100$	0.0970s	0.5026	0.4976	100
$\beta = .1, N = 100$	24.7758s	0.9390	0.9385	100
$\beta = .5, N = 100$	105.0660s	0.8209	0.8168	100
$\beta = 0, N = 25$	2.8009s	0.8427	0.8421	25
sklearn	7.76s	0.9999	1.0	2

1. To investigate how to weigh different datapoints during the SVM training to maximize the efficiency of the training for computing time.

2. w is a dot product to project the basis features of x.
   ϕ is a basis transformation on the features of x.
   b is a shift to make the hyperplane. The slope is defined by the orthogonal vector separating the classes.
3. After a finite number of mistakes, it converges.
4. In order to achieve larger margins, the algorithm removes the furthest support vectors from the linear separator from S.

Problem 3 (Ethics Assignment, 10pts)

Recall our class activity:

> Hiring at Abercrombie and Fitch. Abercrombie and Fitch have hired a new computer science team to design an algorithm to predict the success of various job applicants to sales positions at Abercrombie and Fitch. As you go through the data and design the algorithm, you notice that African-American sales representatives have significantly fewer average sales than white sales representatives. The algorithm's output recommends hiring far fewer African-Americans than white applicants, when the percentage of applications from people of various races are adjusted for.

In class, we thought about the problem *statically*: given historical data, such as data about sales performance, who should Abercrombie and Fitch hire right now?

In this follow-up assignment, I want you to think about consumer behavior and firm hiring practice dynamically. Looking at features of the labor market dynamically allows you more, or different, degrees of freedom in your model. For example, in class, you probably took consumer preference about the race of their sales representative as given. What would happen if you allowed consumer preference to vary (say, on the basis of changing racial demographics in the sales force)?

Heres the new case:

> The US Secretary of Labor has heard about your team's success with Abercrombie and Fitch and comes to you with a request. The Department of Labor wants to reduce disparate impact discrimination in hiring. They want you to come up with a model of fair hiring practices in the labor market that will reduce disparate impact while also producing good outcomes for companies.

Write two or three paragraphs that address the following:

- What are the relevant socially good outcomes, for both workers and companies?
- What are some properties of your algorithm that might produce those socially good results?
  - Think about constraints that you might build in, such as the fairness constraints that we discussed in class, or how you might specify the prediction task that we are asking the machine to optimize.
- Are there tradeoffs that your algorithm has to balance? [optional]
- Are there any features of data collection, algorithm implementation, or the social world that make you wary of using machine learning in this case? [optional]

We expect that:

- You focus on one or two points of discussion for each question.
  - For example, for question 2, pick a single fairness criterion. item Depth over breadth here!
- You provide reasons in support of your answers (i.e., explain why you chose your answer).
  - For example, for the first question, you might choose the socially good outcome of increased profit for companies, and give reasons why profit is the right social goal.
- You are clear and concise - stick to plain, unadorned language.
- You do not do any outside research.
- You demonstrate a thoughtful engagement with the questions.

Solution

I think the main relevant outcomes that should be focused on that maximze the overall social good are the overall diversity of the workforce in almost all fields, increase overall profit/market activity, and improve consumer happiness/satisfaction while participating in the market. I believe the reality of these goals is that some of these outcomes may lower before it is possible to raise them all, so a model that perhaps has some stronger regularization at the beginning may be appropriate to prevent one of the outcomes from overwhelming the other outcomes.

For example, if historical data shows that customer happiness and overall spending is higher when there are more white sales representatives, but an ultimate goal is to increase the diversity in the workforce, relative to applications or the population, then it's likely necessary to prevent the model from devaluing the diversity outcome at the beginning, with the overall theory being that as diversity becomes normalized, the consumer preference on race of the sales representative becomes either a smaller factor for consumers overall.

I don't think that we should be wary of using machine learning except understanding that the models we create may very well be incorrect, or may not adequately incorporate the social outcomes that we want to improve. Especially when using human responses as input, we have to be careful to understand that human perception itself is flawed, even if ultimately that is our basis for judgement of the success of the algorithm. I don't believe that a more "human" argument without the use of computers is purely better in this case, and that a well designed model can, at the very least, show us options with some precision that we may not have considered. It would be silly to suggest that there were no flaws in making the model in the first place that may not have accounted for certain, complicated social goals, so ultimately a combination of machine learning, human interpretation and discussion would produce perhaps the best plan moving forward.

Calibration [1pt]

Approximately how long did this homework take you to complete?