N-body simulation of galaxy merger

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Introduction-N-body simulation of galaxy merger

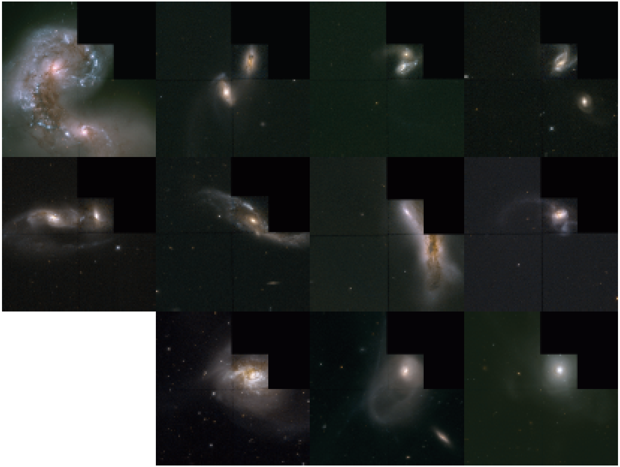

Figure 1: The Toomre sequence of merging galaxies seen at different phase.(source:https://www.cv.nrao.edu/ jhibbard/TSeqHST/)

Milky Way and Andromeda Merger

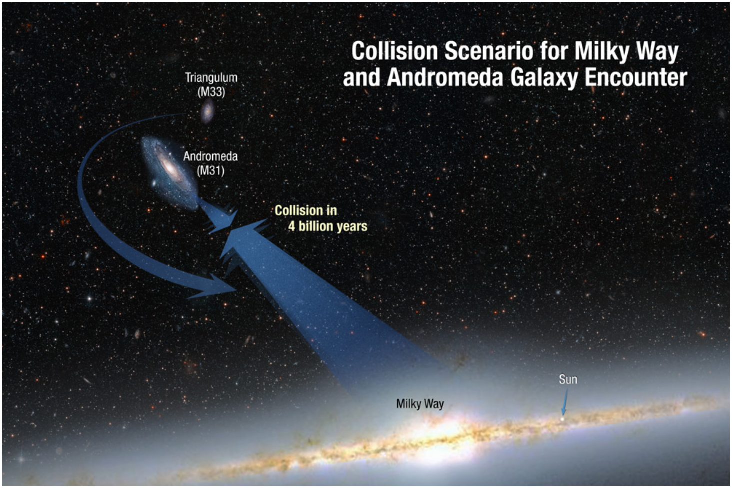

Figure 2: This illustration shows the collision paths of our Milky Way galaxy and the Andromeda galaxy.(Credit: NASA; ESA; A. Feild and R. van der Marel, STScI)

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Galaxy Model

The galaxy structure is shown in Fig. 20.

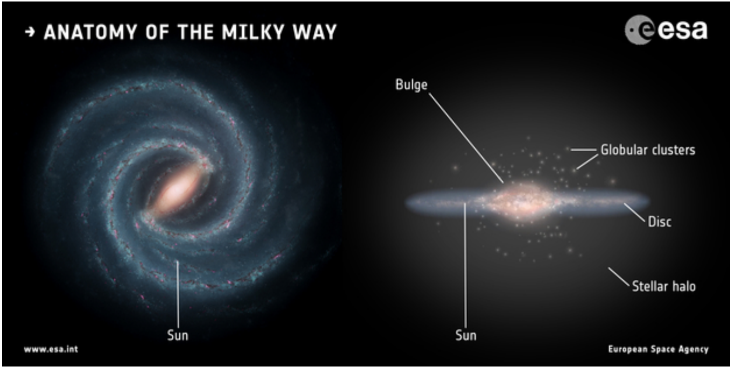


Figure 3: Milky Way anatomy

Galaxy Model–Stellar bulge

We use Hernquist's bulge potential[2]:

$$\Phi(r) = -\frac{GM_b}{r+a} \tag{1}$$

with a cumulative mass distribution of

$$M(r) = M_b \frac{r^2}{(r+a)^2} \tag{2}$$

Then, we assign a random number m to be enclosed mass between $(0, M_b)$, we get radius for the particles:

$$r = \frac{a}{(\frac{M_b}{m})^{\frac{1}{2}} - 1} \tag{3}$$

System of Units

/	Simulation value	Scaled value
Gravitational constant G	1	$6.67408 * 10^{-11} m^3 kg^{-1} s^{-2}$
Mass of bulge M_b	$\frac{1}{3}$	$1.8 * 10^{10} M_\odot$
Scale length of bulge a	0.1	0.35 kpc
Unit of velocity	1	262 km/s

Table 1: system value

Galaxy Model–Stellar bulge

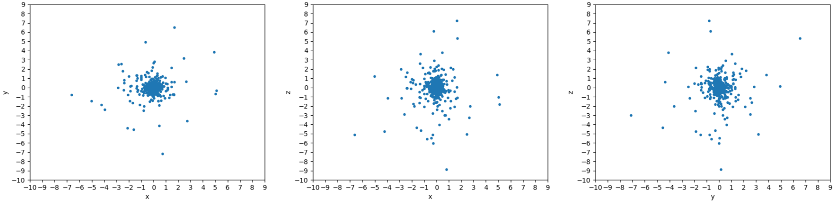


Figure 4: The xy, xz, yz plane's plot of bulge

Using enclosed mass and radius, we get circular velocity of the particles:

$$V_{cir} = \sqrt{\frac{Gm}{r}} \tag{4}$$

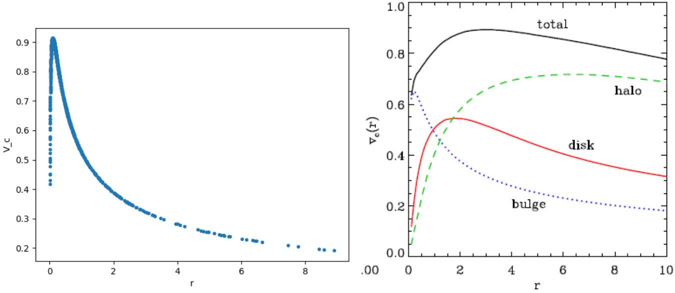


Figure 5: Rotation curve

Galaxy Model–Stellar disk

We use Kuzmin's thin disk potential[1]:

$$\Phi(r) = -\frac{GM_d}{\sqrt{r^2 + (b + |z|)^2}} \tag{5}$$

we set z = 0, so a cumulative mass distribution is

$$M(r) = \frac{M_d r^3}{(r^2 + b^2)^{\frac{2}{3}}} \tag{6}$$

Then, we assign a random number m to be enclosed mass between $(0, M_d)$, we get radius for the particles:

$$r = \frac{(\frac{m}{M_d})^{\frac{1}{3}} a}{\sqrt{1 - (\frac{m}{M})^{\frac{2}{3}}}} \tag{7}$$

System of Units

/	Simulation value	Scaled value
Gravitational constant G	1	$6.67408 * 10^{-11} m^3 kg^{-1} s^{-2}$
Mass of disk M_d	1	$5.6 * 10^{10} M_{\odot}$
Scale length of disk b	1	3.5 kpc
Unit of velocity	1	262 km/s

Table 2: system value

Galaxy Model–Stellar bulge

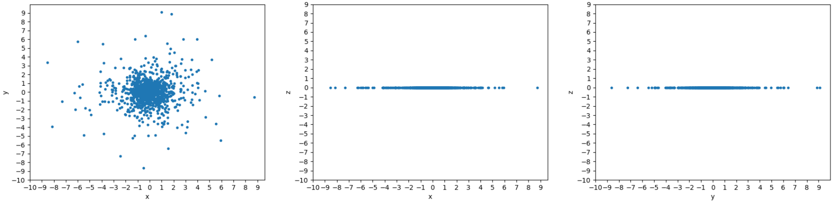


Figure 6: The xy, xz, yz plane's plot of disk

Using enclosed mass and radius, we get circular velocity of the particles:

$$V_{cir} = \sqrt{\frac{Gm}{r}} \tag{8}$$

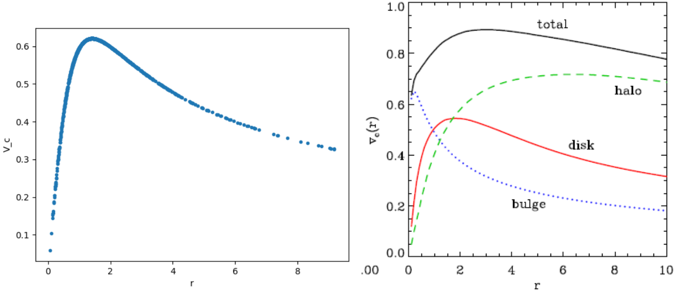


Figure 7: Rotation curve

Galaxy Model-Galaxy Set up

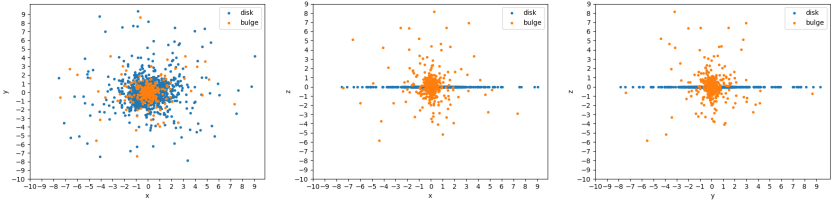


Figure 8: The xy, xz, yz plane's plot of galaxy

Direct N-body force calculations:

- Very accurate
- Used for precise results or when N is low
- Slow and bad scaling $O(N^2)$

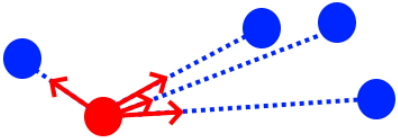


Figure 9: Direct force calculation

This works fine for small scale systems (Sun with planets) but will quickly require a supercomputer for larger systems (Galaxies).

Gravity Solver

Tree structure N-body force calculations:[3]

- Loses in accuracy by grouping particles together
- Used for large N if no suitable potential is available
- Usually faster than direct and has better scaling $O(NlogN)$

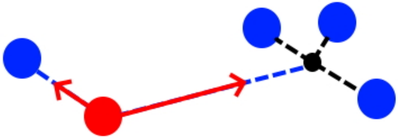
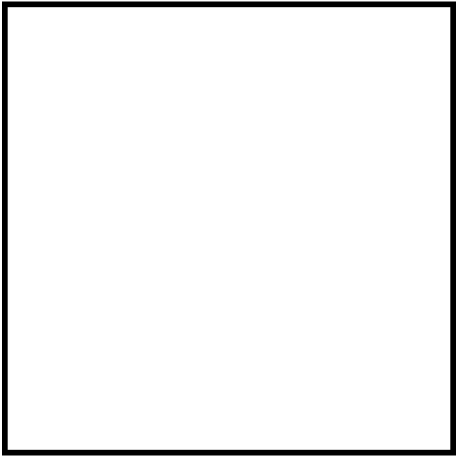


Figure 10: Force calculation through approximation

Clumping together far away particles for force calculations allows even your grandma's laptop to simulate our galaxy,

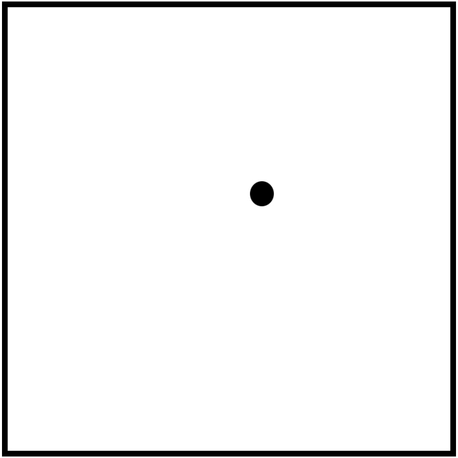
Subdividing the system

So how does this even work? Let's first make a box structure and then explain how we can use this.



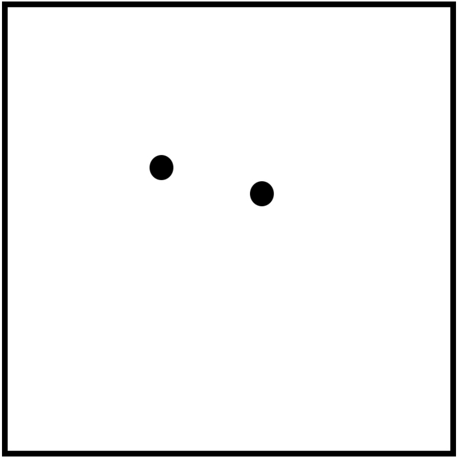
Subdividing the system

We insert a single particle to start.



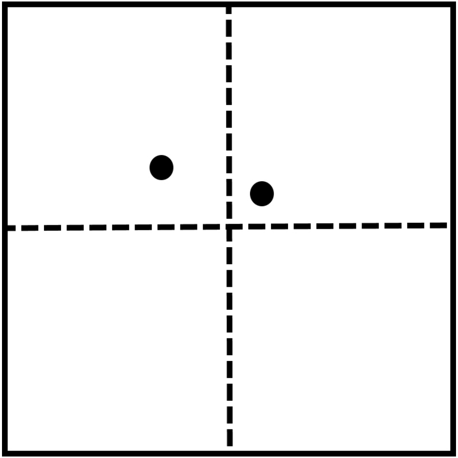
Subdividing the system

Then we add a second particle.



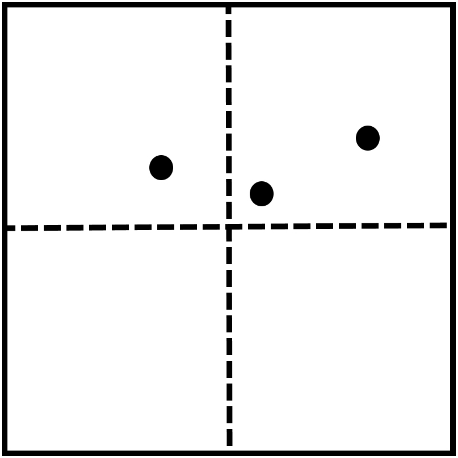
Subdividing the system

We want each particle in it's own box so we split it in four.



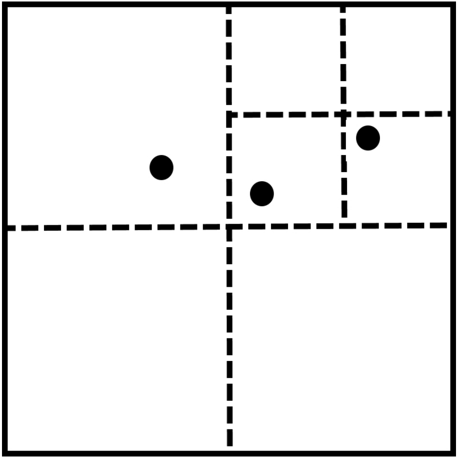
Subdividing the system

We continue adding particles.



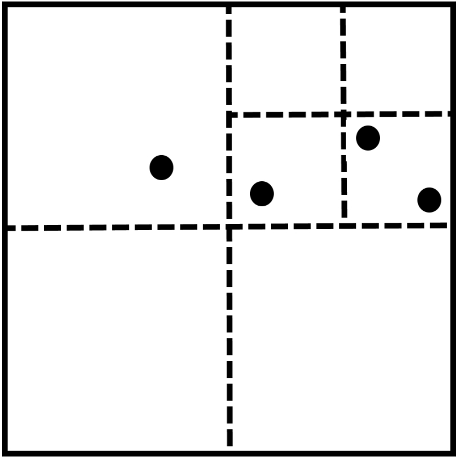
Subdividing the system

Our subbox now has 2 particles so we split only that box in four again.



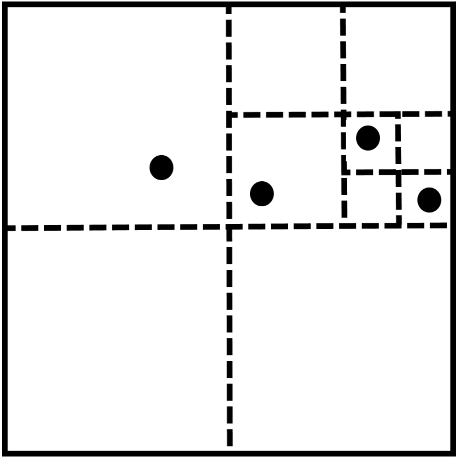
Subdividing the system

And then this process can be repeated for all particles.



Subdividing the system

And we will end with each particle in it's own box.



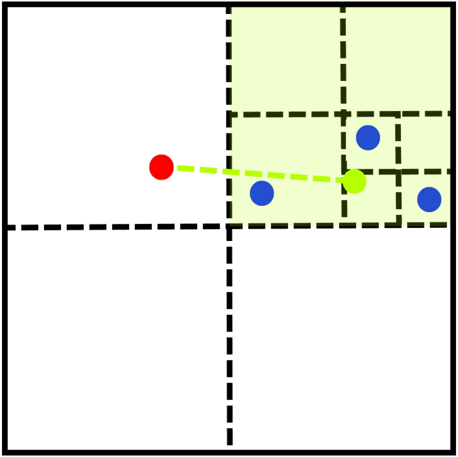
Calculating the force

We have divided the system but now the forces have to be calculated. To do this the following process will be repeated for every particle:

- Calculate distance from particle to center of mass of the box
- Calculate $\alpha = \frac{\textit{Length Box}}{\textit{Distance}}$ and check if $\alpha < \theta =$ angle criterion
- If $\alpha < \theta$ calculate the force with this box's Center of Mass
- If $\alpha >= \theta$ repeat this process for the subboxes

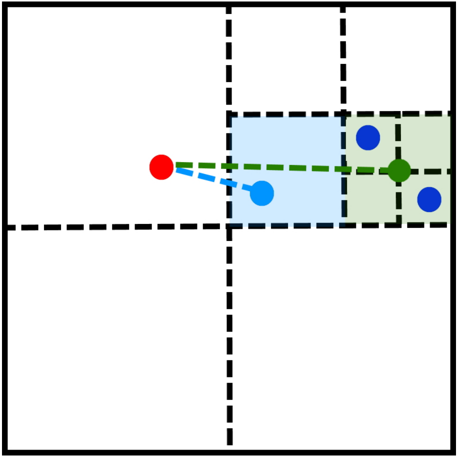
Calculating the force

For example in our previous system calculating the force for the red particle. But if this does not fulfill the angle criterion we go deeper.



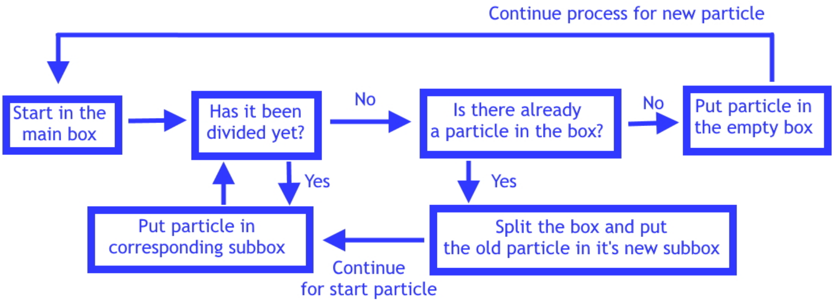
Calculating the force

Going deeper we only calculate for non empty boxes and we end up with 2 force calculations.



Code implementation

In our implementation we have a particle set which we update and nested arrays of references which form our tree-structure.



Code implementation

The actual gravity solving is done through a leap-frog integration scheme for simplicity. Other notable functions of the code include:

- Updating the center of masses in the tree-structure
- Dampening the force for galactic stability $R = \sqrt{r^2 + \epsilon^2}$
- Tracking amount of force calculations per particle

A quick and easy way to test a gravity solver is with a well known system.

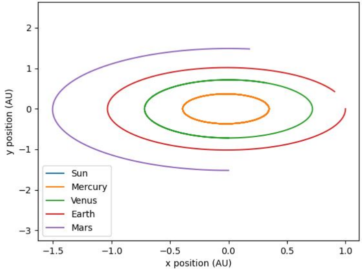


Figure 11: Planetary trajectories over 350 days

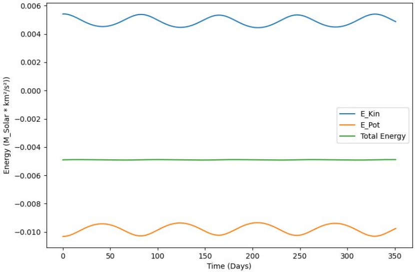


Figure 12: Energy plot solar system

But looks can be deceiving, energy is not perfectly conserved in the simulation!

Verification with solar system

But is it actually using the tree code?

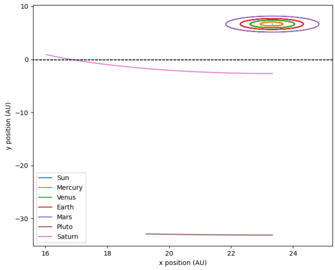


Figure 13: Celestial body trajectories over 1500 days

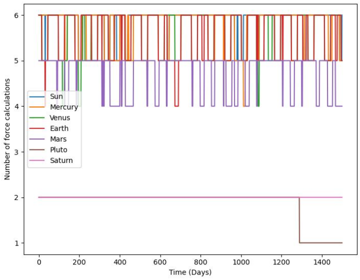


Figure 14: Amount of force calculations performed per particle during the simulation

Bodies far away from the heavier sun use much less calculations and there is even some speed up for nearby planets.

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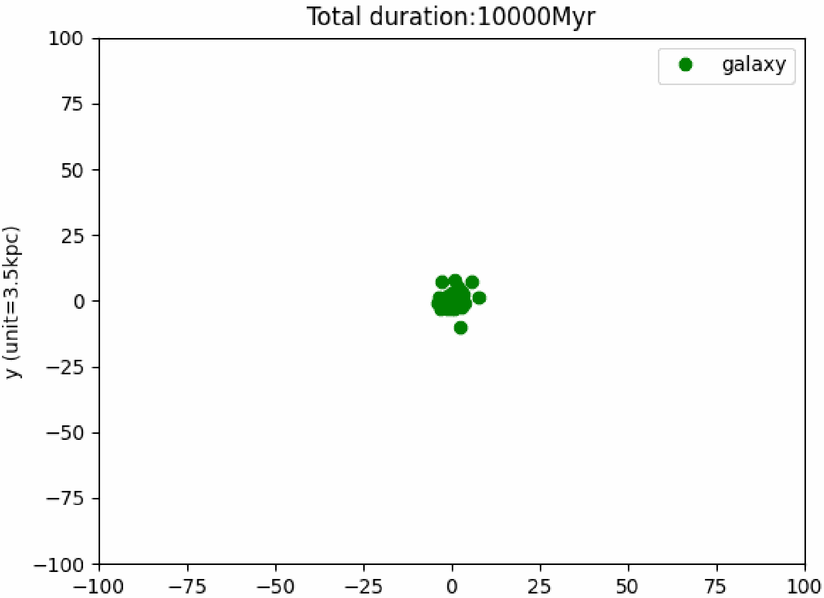
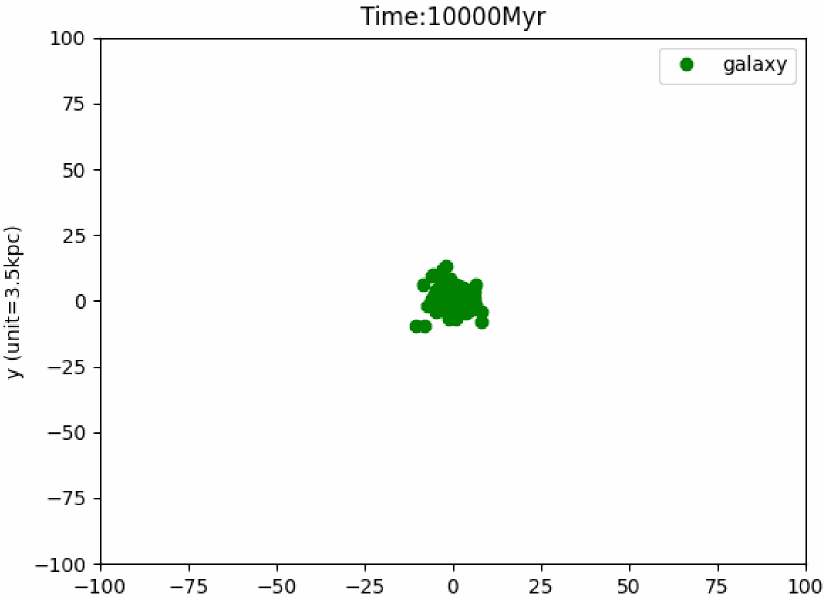
Test the stability of one single galaxy



Test the stability of one single galaxy



Merger of two galaxies

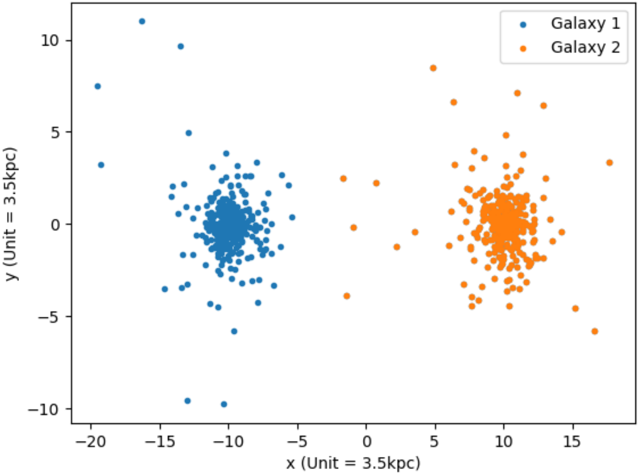
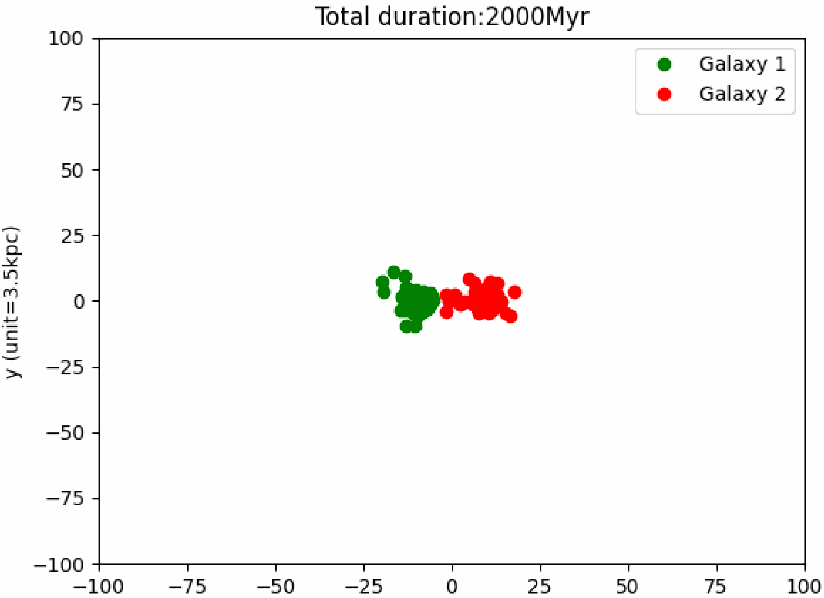


Figure 15: Galaxy initial set up

Merger of two galaxies



Merger of two galaxies

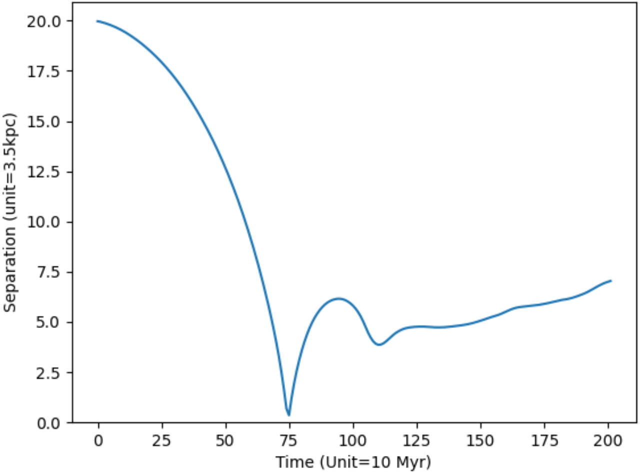


Figure 16: Separation between two galaxies

Milky Way and Andromeda merger

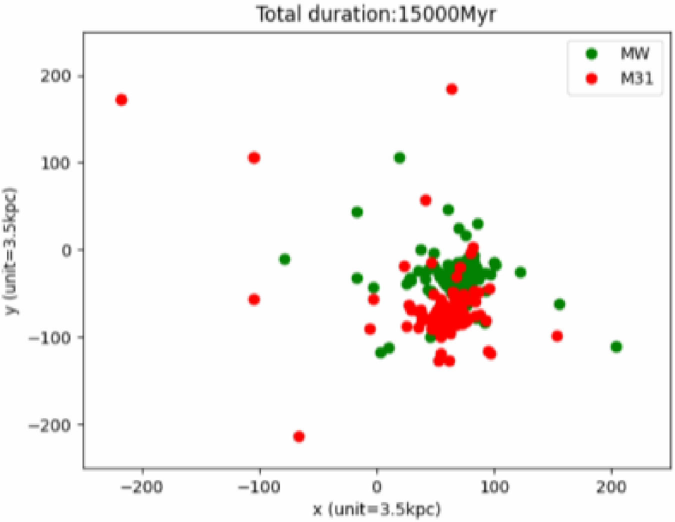


Figure 17: merger process

Milky Way and Andromeda merger

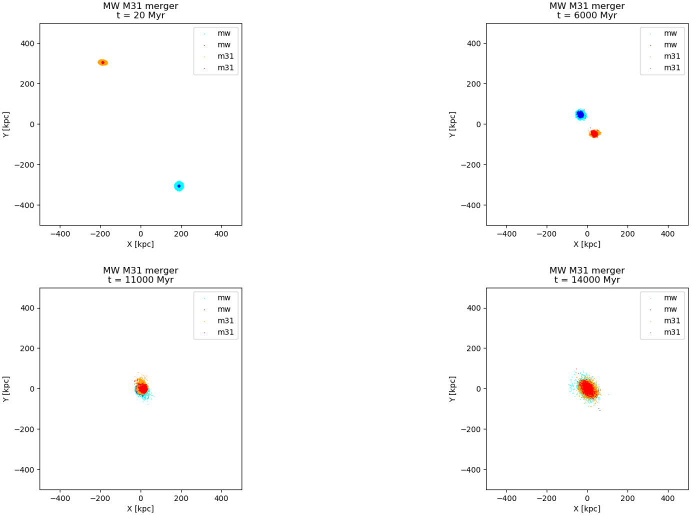


Figure 18: MW and M31 merger process.
Source:https://github.com/boson112358/sma-group-e-project

Milky Way and Andromeda merger

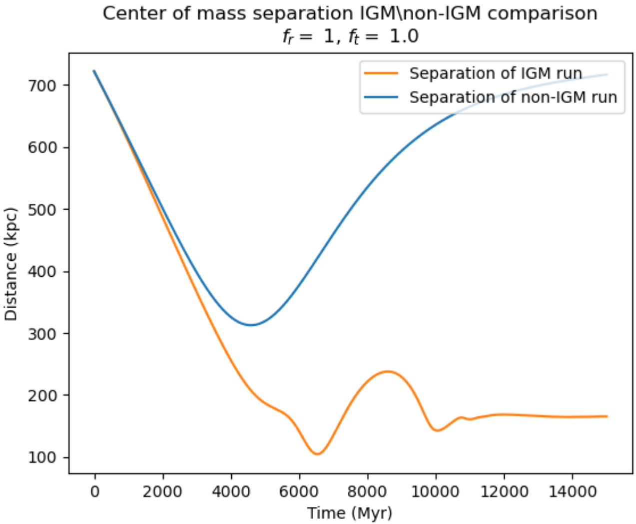


Figure 19: Separation between MW and M31

Milky Way and Andromeda merger

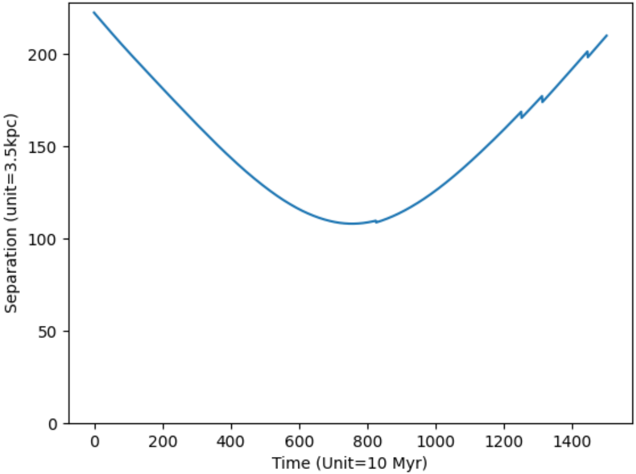


Figure 20: Separation between MW and M31

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Summary

Future development

- Add halo into galaxy model to get a more detailed galaxy model.
- Take merger angle into account
- Increase the simulation particle number
- Intergalactic medium and central supermassive black holes can be considered in the future work
- Change the boundary condition and the plotting issue it creates

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Reference

[1] Miyamoto, M., Nagai, R. (1975). Three-dimensional models for the distribution of mass in galaxies. Publications of the Astronomical Society of Japan, 27, 533-543.

[2] Hernquist, L. (1990). An analytical model for spherical galaxies and bulges. The Astrophysical Journal, 356, 359-364.

[3] J. E. Barnes and P. Hut. A hierarchical O(n-log-n) force calculation algorithm.Nature, 324:446, 1986.

QUESTIONS?