Exercícios (2):

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1- a) $\lim_{x \to +\infty} f(x) = +\infty + 1 =$ $+\infty$

b) $\lim_{x \to -\infty} f(x) = -\infty + 1 =$ $-\infty$

2- a) $\lim_{x \to +\infty} f(x) = -\infty + 1 =$ $-\infty$

b) $\lim_{x \to -\infty} f(x) = +\infty + 1 =$ $+\infty$

3- a) $\lim (\infty^2 - 11) = +\infty^2 =$ $+\infty$

b) $\lim (4 - (-\infty^3)) = 4 + \infty =$ $+\infty$

c) $\lim 3^{+\infty} =$ $+\infty$

d) $\lim \sqrt{(\sqrt{10})^2 - 1} = \sqrt{10 - 1} = \sqrt{9} =$ 3

e) $\lim \left(\frac{2}{3}\right)^{+\infty} =$ 0

f) $\lim \sqrt{+\infty - 1} =$ $+\infty$

g) $\lim \left(\frac{2}{3}\right)^{0} =$ 1

h) $\lim \sqrt{\left(\frac{3}{2}\right)^2 - 1} =$ $\frac{\sqrt{5}}{2}$

4- a) $\lim \frac{1}{0-3} =$ $-\frac{1}{3}$

b) $\lim \frac{1}{2-3} =$ -1

c) $\lim \frac{1}{3-3} = \frac{1}{0} =$ $\boxed{+\infty \text{ e } -\infty}$

d) $\lim \frac{1}{-3-3} =$ $-\frac{1}{6}$

e) $\lim \frac{1}{\frac{7}{2}-3} = \frac{1}{\frac{7-6}{2}} = \frac{1}{\frac{1}{2}} =$

$\cancel{1} \cdot \frac{2}{\cancel{1}} = 2$

f) $\lim \frac{\frac{1}{x}}{\frac{x}{x}-\frac{3}{x}} = \frac{\frac{1}{x}}{1-\frac{3}{x}} =$

$\frac{0}{1-0} = 0$

5- a) $\lim \frac{1}{+\infty} = 0$

b) $\lim \frac{1}{-\infty} = 0$

c) $\lim \frac{1}{0^+} = +\infty$

d) $\lim \frac{1}{0^-} = -\infty$

6- a) $\lim \frac{\frac{2}{x}}{\frac{x}{x}} = \frac{0}{1} = 0$

b) $\lim \frac{\frac{2}{x}}{\frac{x}{x}} = \frac{0}{1} = 0$

c) $\lim \frac{1}{x} + \frac{\frac{1}{x}}{\frac{x}{x}} = 0 + \frac{0}{1} =$

$\frac{1}{\infty} = 1$

d) $\lim \frac{1}{x} + \frac{\frac{1}{x}}{\frac{x}{x}} = 0 + \frac{0}{1} =$

$\frac{1}{\infty} = 1$

e) $\lim \frac{2}{0} = +\infty$

f) $\lim \frac{1}{0^2} = +\infty$

g) $\lim \frac{1}{\sqrt[3]{0^2}} = \frac{1}{0} = +\infty$

7- a) $\lim x^3\left(2-3x^2\cdot\frac{x}{x}-3x\cdot\frac{x^2}{x}+2\right)=$

$x^3(2-3x^2\cdot 0-3x\cdot 0+2)=$

$x^3(0) = +\infty$

b) $\lim\limits_{x\to-\infty} 2x^3-3x^2-3x+2=$

$\lim 2(-\infty)^3=$

$-\infty$

c) $\lim\limits_{x\to 1/2} \frac{2}{8}-\frac{3}{4}-\frac{3}{2}+2=$

$\frac{2-6-12+2}{8}=$

2

8- a) $\lim\limits_{x\to-\infty} 3x^4-2x^3+x-1=$

$x^4\left(3-2x^3\cdot\frac{x}{x}+x\cdot\frac{x^2}{x}-1\right)=$

$x^4(3-2\cdot 0+x\cdot 0-1)=x(2)=$

$-\infty^4(2) = +\infty$

b) $\lim\limits_{x\to+\infty} 3x^4-2x^3+x-1=$

$x^4\left(3-2x^3\cdot\frac{x}{x}+x\cdot\frac{x^2}{x}-1\right)=$

$x^4(3-2\cdot 0+x\cdot 0-1)=$

$x^4(2) = +\infty^4(2) = +\infty$

9- a) $\lim\limits_{x \to -\infty} \dfrac{\dfrac{5\cancel{x^2}}{\cancel{x^2}} - \dfrac{\cancel{x}}{\cancel{x^2}} + \dfrac{1}{\cancel{x^2}}}{\dfrac{2\cancel{x^2}}{\cancel{x^2}} + \dfrac{\cancel{x}}{\cancel{x^2}} - \dfrac{1}{\cancel{x^2}}} = \dfrac{5-0+0}{2+0-0} = \dfrac{5}{2}$

b) $\lim\limits_{x \to +\infty} \dfrac{\dfrac{\cancel{x^3}}{\cancel{x^3}} + \dfrac{\cancel{x}}{\cancel{x^3}} + \dfrac{1}{\cancel{x^3}}}{\dfrac{2\cancel{x^3}}{\cancel{x^3}} - \dfrac{\cancel{x}}{\cancel{x^3}}} = \dfrac{1+0+0}{2-0} = \dfrac{1}{2}$

c) $\lim\limits_{x \to +\infty} \dfrac{\dfrac{x}{x} - \dfrac{2}{x}}{\dfrac{x}{x} - \dfrac{1}{x}} = \dfrac{1-0}{1-0} = 1$

d) $\lim\limits_{x \to -\infty} \dfrac{\dfrac{3\cancel{x^3}}{\cancel{x^3}} - \dfrac{x}{x^3}}{\dfrac{x^2}{x^3} + \dfrac{1}{\cancel{x^3}}} = \dfrac{3 - \dfrac{1}{x^2}}{\dfrac{1}{x} + 0} = \dfrac{-\infty}{+\infty} = -\infty$

e) $\lim\limits_{x \to +\infty} \dfrac{\dfrac{x^4}{x^4} + \dfrac{x^3}{x^4} + \dfrac{2\cancel{x}}{\cancel{x^4}}}{\dfrac{x^3}{x^4} + \dfrac{x}{x^4} - \dfrac{1}{\cancel{x^4}}} = \dfrac{1 + \dfrac{1}{x} + 0}{\dfrac{1}{x} + \dfrac{1}{x^3} - 0} = \dfrac{+\infty}{+\infty} = +\infty$

f) $\lim\limits_{x \to -\infty} \dfrac{x^2}{x^3} = \dfrac{1}{x} = \dfrac{1}{-\infty} = 0$

g) $\lim\limits_{x \to +\infty} \dfrac{-1-x}{x^3+2} = \dfrac{-x}{x^3} = \dfrac{1}{x^2} = \dfrac{-1}{+\infty} = 0$

h) $\frac{1}{x-2} = \frac{1}{x} = \frac{1}{+\infty} = 0$

i) $\frac{1}{x^2} = \frac{1}{-\infty^2} = \frac{1}{+\infty} = 0$

j) $\frac{1}{-x} = \frac{1}{-\infty} =$

K) $\frac{2}{x^2-1} = \frac{2}{x^2} = \frac{2}{-\infty^2} = \frac{2}{+\infty} = 0$

L) $\sqrt{\frac{x+1}{x-1}} = \sqrt{\frac{x}{x}} = \sqrt{1} = 1$

M) $\sqrt[3]{\frac{\cancel{x^2}}{\cancel{x^2}}} = \sqrt[3]{-1} = -1$

N) $\frac{-x^2}{3x} = \frac{-(+\infty)}{3(+\infty)} = -\infty$

O) $\sqrt{\frac{x^5}{x^2}} = \sqrt{x^3} = +\infty$

2ª Parte: $1+\frac{1}{7x} = 1+\frac{1}{t}$: $7x = t$, $x = \frac{t}{7}$

$1 - \frac{3}{x} = 1 + \frac{1}{t}$

$-3t = x$

1- a) $\lim_{x \to +\infty} \left(1+\frac{1}{7x}\right)^x =$

$\left(1 + \frac{1}{7\frac{t}{7}}\right)^{\frac{t}{7}} = \left[\left(1+\frac{1}{t}\right)^t\right]^{\frac{1}{7}} = e^{\frac{1}{7}}$

b) $\lim_{x \to +\infty} \left(1+\frac{3}{x}\right)^x =$

$\left(1 - \frac{\cancel{3}}{-3t}\right)^{-3t} = \left[\left(1+\frac{1}{t}\right)^t\right]^{-3} = \frac{1}{e^3}$

c) $1+\frac{7}{x} = 1+\frac{1}{t}$

$x = 7t$

$\lim_{x \to +\infty} \left(1+\frac{7}{7t}\right)^{14t} =$

$\left[\left(1+\frac{1}{7}\right)^t\right]^{14} = e^{-14}$

d) $1 + \frac{7}{5x} = 1 + \frac{1}{t}$

$5x = 7t$

$x = \frac{7t}{5}$

$\lim_{x \to +\infty} \left(1+\frac{7}{5x}\right)^x =$

$\left(1 - \frac{\cancel{7}}{\cancel{7}t}\right)^{\frac{7t}{5}} = \left[\left(1+\frac{1}{t}\right)^t\right]^{\frac{7}{5}} = e^{\frac{7}{5}}$

e) $1-\frac{2}{x} = 1+\frac{1}{t}$

$x = -2t$

$$\lim_{x \to -\infty}\left(1-\frac{\cancel{2}}{-\cancel{2}t}\right)^{-10t} =$$

$$\left[\left(1+\frac{1}{t}\right)^{t}\right]^{-10} = e^{-10} = \frac{1}{e^{10}}$$

f) $1+\frac{2}{x} = 1+\frac{1}{t}$

$x = 2t$

$$\lim_{x \to +\infty}\left(1+\frac{\cancel{2}}{\cancel{2}t}\right)^{3+2\cdot 2t} = \left(1+\frac{1}{t}\right)^{3+4t}$$

$$\left(1+\frac{1}{t}\right)^{4t} + \left(1+\frac{1}{t}\right)^{3} =$$

$$\left[\lim_{x \to +\infty}\left(1+\frac{1}{t}\right)^{2}\right]^{4} = \left(1+\frac{1}{\infty}\right)^{3} =$$

$$\left[\lim_{x \to +\infty}\left(1+\frac{1}{t}\right)^{2}\right]^{4} = (1+0)^{3} =$$

e^{4}

g) $\frac{x-3}{x+2} = 1+\frac{1}{t}$

$x-3 = x+2+\frac{x+2}{t}$

$t(-1) = x+2$

$x = -5t-2$

$$\lim_{x \to +\infty}\left(\frac{-5-5t}{-5t}\right)^{-5t-2} = \left(1+\frac{1}{t}\right)^{-5t-2} =$$

$$\left(1+\frac{1}{t}\right)^{-5t} \cdot \left(1+\frac{1}{t}\right)^{-2} =$$

$$e^{-5} \cdot 1^{-2} =$$

$$\frac{1}{e^{-5}} \cdot \frac{1}{1^{2}} = \frac{1}{e^{5}}$$

h) $1+2x = 1+t$

$x = \frac{t}{2}$

$$\left[\lim_{x \to 0}(1+t)^{\frac{1}{t}}\right]^{2} =$$

e^{2}

i) $1+4x = 1+t$

$x = \frac{t}{4}$

$$\left(1+\cancel{4}\frac{t}{\cancel{4}}\right)^{\frac{3}{2x}} =$$

$$(1+t)^{\frac{3}{2x}} = e^{6}$$