

Cálculo II

3º Exercício Avaliativo - 2ª Nota

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a) $\int \frac{x-4}{x^2-10x+25} dx$

$$\int \frac{x-4}{(x-5)^2} dx$$

$$\int \frac{t+1}{t^2} dt \rightarrow \int \frac{t}{t^2} + \frac{1}{t^2} dt$$

$$\int \frac{1}{t} dt + \int \frac{1}{t^2} dt \rightarrow \ln(|t|) - \frac{1}{t}$$

$$\boxed{\ln(|x-5|) - \frac{1}{x-5} + C}$$

b) $\int \frac{x-2}{x^2+4x+4} dx$

$$\int \frac{x-2}{(x+2)^2} dx$$

$$\int \frac{t-4}{t^2} dt \rightarrow \int \frac{t}{t^2} - \frac{4}{t^2} dt$$

$$\int \frac{1}{t} dt - \int \frac{4}{t^2} dt \rightarrow \ln(|t|) + \frac{4}{t}$$

$$\boxed{\ln(|x+2|) + \frac{4}{x+2} + C}$$

$$c) \int \frac{x-2}{x^2-4x+3} dx$$

$$\int \frac{1}{2t} dt \rightarrow \frac{1}{2} \int \frac{1}{t} dt$$

$$\frac{1}{2} \cdot \text{Im}(|t|) \rightarrow \boxed{\frac{1}{2} \cdot \text{Im}(|x^2-4x+3|) + C}$$

$$d) \int \frac{x+3}{x^2-6x+8} dx$$

$$\int -\frac{5}{2(x-2)} + \frac{7}{2(x-4)} dx$$

$$\boxed{-\frac{5}{2} \text{Im}(|x-2|) + \frac{7}{2} \text{Im}(|x-4|) + C}$$

$$e) \int \frac{x+1}{x^2-5x+6} dx$$

$$\int -\frac{3}{x-2} + \frac{4}{x-3} dx \rightarrow -\int \frac{3}{x-2} dx + \int \frac{4}{x-3} dx$$

$$\boxed{-3 \text{Im}(|x-2|) + 4 \text{Im}(|x-3|) + C}$$

$$f) \int \frac{x+4}{x^2-3x-4} dx$$

$$\int -\frac{3}{5(x+1)} + \frac{8}{5(x-4)} dx$$

$$= -\int \frac{3}{5(x+1)} dx + \int \frac{8}{5(x-4)} dx$$

$$\boxed{-\frac{3}{5} \text{Im}(|x+1|) + \frac{8}{5} \text{Im}(|x-4|) + C}$$

$$g) \int \frac{3x-2}{x^2-8x+7} dx \rightarrow \frac{3x-2}{(x-1)(x-7)} \rightarrow \begin{cases} -2 = -7A - B \\ 3 = A + B \end{cases}$$

$$\int -\frac{1}{6(x-1)} + \frac{19}{6(x-7)} dx$$

$$= \int -\frac{1}{6(x-1)} dx + \int \frac{19}{6(x-7)} dx$$

$$\boxed{-\frac{1}{6} \ln(|x-1|) + \frac{19}{6} \ln(|x-7|) + C}$$

$$h) \int \frac{2x+2}{x^2-x-20} dx \rightarrow \frac{2x+2}{x^2-4x-5x-20} \rightarrow \frac{2x+2}{(x+4)(x-5)}$$

$$\frac{2x+2}{(x+4)(x-5)} = \frac{A}{x+4} + \frac{B}{x-5} \rightarrow 2x+2 = (A+B)x + (-5A+4B)$$

$$\begin{cases} 2 = -5A + 4B \\ 2 = A + B \end{cases} \rightarrow \frac{2}{3} + \frac{4}{3}$$

$$\int \frac{2}{3(x+4)} + \frac{4}{3(x-5)} dx \rightarrow \int \frac{2}{3(x+4)} dx + \int \frac{4}{3(x-5)} dx$$

$$\boxed{\frac{2}{3} \ln(|x+4|) + \frac{4}{3} \ln(|x-5|) + C}$$

$$i) \int e^{7x} - 5x^2 e^{3x^3} dx$$

$$\int e^{7x} dx - \int 5x^2 e^{3x^3} dx$$

$$\boxed{\frac{e^{7x}}{7} - \frac{5e^{3x^3}}{9} + C}$$

$$j) \int_1^2 3x^3 - 4x + 6 \, dx \rightarrow \int 3x^3 - 4x + 6 \, dx$$

$$\int 3x^3 \, dx - \int 4x \, dx + \int 6 \, dx$$

$$\left(\frac{3x^4}{4} - 2x^2 + 6x \right) \Big|_1^2$$

$$\frac{3 \cdot 2^4}{4} - 2 \cdot 2^2 + 6 \cdot 2 - \left(\frac{3 \cdot 1^4}{4} - 2 \cdot 1^2 + 6 \cdot 1 \right)$$

$$\frac{3 \cdot 2^4}{4} - 2^3 + 12 - \left(\frac{3}{4} - 2 + 6 \right) \rightarrow 16 - \frac{19}{4}$$

$$\left(\frac{45}{4} \right)$$

$$k) \int_1^2 7x/x^3 + 4x \, dx \rightarrow \int \frac{7x}{x^3} + 4x \, dx$$

$$\int 7 \cdot \frac{1}{x^2} + 4x \, dx \rightarrow \int \frac{7}{x^2} + 4x \, dx$$

$$\int \frac{7}{x^2} \, dx + \int 4x \, dx \rightarrow \left(-\frac{7}{x} + 2x^2 \right) \Big|_1^2$$

$$-\frac{7}{2} + 2 \cdot 2^2 - \left(-\frac{7}{1} + 2 \cdot 1^2 \right)$$

$$-\frac{7}{2} + 2^3 - (-7 + 2) \rightarrow -\frac{7}{2} + 8 - (-5)$$

$$-\frac{7}{2} + 13 \rightarrow \left(\frac{19}{2} \right)$$

$$l) \int x \sin 2x \, dx$$

$$u = x \quad dv = \sin 2x \, dx$$

$$du = dx \quad v = -\frac{\cos 2x}{2}$$

$$x \left(-\frac{\cos 2x}{2} \right) - \int -\frac{\cos 2x}{2} \, dx$$

$$x \left(-\frac{\cos 2x}{2} \right) - 1 \left(-\frac{1}{2} \right) \int \cos 2x \, dx$$

$$x \left(-\frac{\cos 2x}{2} \right) + \frac{1}{2} \int \frac{\cos(t)}{2} \, dt$$

$$x \left(-\frac{\cos 2x}{2} \right) + \frac{1}{2} \cdot \frac{1}{2} \cdot \int \cos(t) \, dt$$

$$x \left(-\frac{\cos 2x}{2} \right) + \frac{1}{4} \cdot \sin(2x)$$

$$\boxed{-\frac{x \cdot \cos 2x}{2} + \frac{\sin 2x}{4} + C}$$

$$m) \int (x+1) \sin 3x \, dx$$

$$\int x \sin 3x + \sin 3x \, dx$$

$$\int x \sin 3x \, dx + \int \sin 3x \, dx$$

$$-\frac{x \cdot \cos 3x}{3} + \frac{\sin 3x}{9} - \frac{\cos 3x}{3}$$

$$\boxed{-\frac{x \cdot \cos(3x)}{3} + \frac{\cos(3x)}{3} + \frac{\sin(3x)}{9} + C}$$

$$m) \int \frac{5x^3 + 5x - 1}{x^2 + 1} dx$$

$$\int \frac{5x^3}{x^2 + 1} + \frac{5x}{x^2 + 1} - \frac{1}{x^2 + 1} dx$$

$$\int \frac{5x^3}{x^2 + 1} dx + \int \frac{5x}{x^2 + 1} dx - \int \frac{1}{x^2 + 1} dx$$

$$\frac{5}{2}x^2 + \frac{5}{2} - \frac{5}{2} \cdot \text{Im}(x^2 + 1) + \frac{5}{2} \cdot \text{Im}(x^2 + 1) - \text{ARCTAN}(x)$$

$$\frac{5}{2}x^2 + \frac{5}{2} - \text{ARCTAN}(x) \rightarrow \boxed{\frac{5}{2}x^2 - \text{ARCTAN}(x) + C}$$

$$o) \int \frac{-x^2 + 2x - 3}{x} dx$$

$$\int -\frac{x^2}{x} + \frac{2x}{x} - \frac{3}{x} dx$$

$$\int -x + 2 - \frac{3}{x} dx$$

$$- \int x dx + \int 2 dx - \int \frac{3}{x} dx$$

$$\boxed{-\frac{x^2}{2} + 2x - 3 \text{Im}(|x|) + C}$$