Homework 1. Markov chains

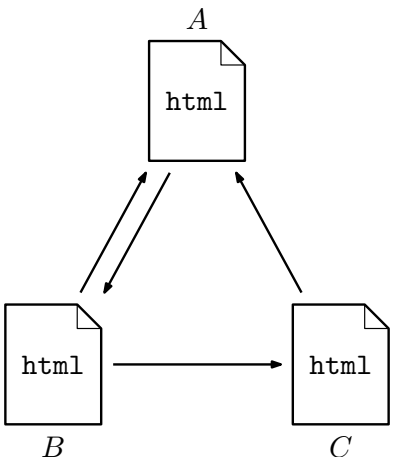

Figure 1: Web of linked documents. An arrow from a document x to a document y indicates that document x links to document y.

Consider the "web" of linked documents depicted in Fig. 1. In this representation, an arrow from a document x to a document y means that there is a link to document y in document x. In this homework, you will analyze the behavior of a bot navigating this web of documents. Upon arriving at a document, the bot will select, uniformly at random, one of the links in that document and follow that link.

Exercise 1.

(a) Write down a Markov chain model that describes the motion of the bot described above in that web.

(b) Let T_{xx} denote the average number of steps that the bot takes to return to document x when starting in x. Compute T_{AA}, T_{BB} and T_{CC}, indicating the relevant computations.

(c) Note that, the longer it takes the bot to return to a document, the less time it spends in that particular document. In fact, the percentage of time that a bot spends in document x—let us denote it as μ_x—is inversely proportional to T_{xx}. Compute μ_A, μ_B and μ_C using the results from question (b). Show that the resulting distribution is invariant for the chain.