

ChatGPT To design a multilayer perceptron (MLP) with a single hidden layer containing two hidden units that computes the given Boolean function $f(x)$, we need to construct a network that can distinguish between sums of inputs that fall within the interval $[A, B]$ and those that do not. The hard threshold activation function $g(z) = \text{sign}(z)$ will be used for the neurons in the hidden layer and the output layer.

We need two hidden units: one to detect if the sum of inputs is greater than or equal to A , and the other to detect if it is less than or equal to B . Then, the output layer will combine these to determine if the sum is within the interval $[A, B]$.

Hidden Layer

- Hidden Unit 1 (H1): This unit will fire (+1) if $\sum_{i=1}^D x_i \geq A$. To achieve this, we set the weights to 1 for each input and set the bias to $-(A - 0.5)$ (the 0.5 offset ensures robustness to infinitesimal perturbation).
- Weights: $w_{1i} = 1$ for all i in $[1, D]$
- Bias: $b_1 = -(A - 0.5)$
- Hidden Unit 2 (H2): This unit will fire (+1) if $\sum_{i=1}^D x_i \leq B$. We can implement this by setting the weights to -1 for each input and setting the bias to $B + 0.5$.
- Weights: $w_{2i} = -1$ for all i in $[1, D]$
- Bias: $b_2 = B + 0.5$

Output Layer

- The output neuron combines the outputs of H1 and H2. It should fire (+1) if both H1 and H2 fire (+1). This can be achieved by setting the weights from H1 and H2 to the output neuron to 1 and setting the bias to -1.5 (requiring both units to fire).
- Weights: $w_{o1} = 1, w_{o2} = 1$
- Bias: $b_o = -1.5$

Summary

- Hidden Unit 1 (H1): Weights = $[1, 1, \dots, 1]$, Bias = $-(A - 0.5)$
- Hidden Unit 2 (H2): Weights = $[-1, -1, \dots, -1]$, Bias = $B + 0.5$
- Output Unit: Weights = $[1, 1]$, Bias = -1.5

This network will compute the function $f(x)$ as desired, being robust to infinitesimal perturbations of the inputs. The added 0.5 in the biases ensures that slight changes in the input will not change the output, thus maintaining $\lim_{t \rightarrow 0} h(\mathbf{x} + t\mathbf{v}) = h(\mathbf{x}) = f(\mathbf{x})$ for any $\mathbf{x} \in \{-1, +1\}^D$ and $\mathbf{v} \in \mathbb{R}^D$.