

Part One:

COMP307 Assignment 3 Part One : David Burrell - 300209541.

1.

Joint probability:	X\Y	Y=0	Y=1
	X=0	0.090	0.210
	X=1	0.560	0.140

$$\begin{aligned}
 P(X=0, Y=0) &= P(X=0) * P(Y=0 | X=0) \text{ [product rule]} \\
 &= 0.300 * 0.300 \\
 &= 0.090
 \end{aligned}$$

$$\begin{aligned}
 P(X=1, Y=0) &= P(X=1) * P(Y=0 | X=1) \text{ [product rule]} \\
 &= 0.700 * 0.800 \\
 &= 0.560
 \end{aligned}$$

$$\begin{aligned}
 P(X=0, Y=1) &= P(X=0) * P(Y=1 | X=0) \text{ [product rule]} \\
 &= 0.300 * 0.700 \\
 &= 0.210
 \end{aligned}$$

$$\begin{aligned}
 P(X=1, Y=1) &= P(X=1) * P(Y=1 | X=1) \text{ [product rule]} \\
 &= 0.700 * 0.200 \\
 &= 0.140
 \end{aligned}$$

2. Joint Probability (x,y,z):

X	Y	Z	P(x,y,z)
0	0	0	0.054
0	0	1	0.036
0	1	0	0.168
0	1	1	0.042
1	0	0	0.336
1	0	1	0.224
1	1	0	0.112
1	1	1	0.028

$$P(X=0) = P(X=0,Y=0,Z=0) + P(X=0,Y=0,Z=1) + P(X=0,Y=1,Z=0) + P(X=0,Y=1,Z=1) \text{ [Sum rule]}$$

$$0.300 = P(X=0,Y=0,Z=0) + 0.036 + 0.168 + 0.042$$

$$P(X=0,Y=0,Z=0) = 0.300 - 0.036 + 0.168 + 0.042$$

$$= 0.054$$

$$P(Z=1,Y=1,X=1) = P(Z=1|Y=1) * P(Y=1|X=1) * P(X=1) \text{ [Chain rule + independance]}$$

$$= 0.200 * 0.200 * 0.700.$$

$$= 0.028$$

$$P(Z=0,Y=1,X=1) = P(Z=0|Y=1) * P(Y=1|X=1) * P(X=1) \text{ [Chain rule + independance]}$$

$$= 0.800 * 0.200 * 0.700.$$

$$= 0.112$$

$$P(Z=1,Y=0,X=1) = P(Z=1|Y=0) * P(Y=0|X=1) * P(X=1) \text{ [Chain rule + independance]}$$

$$= 0.400 * 0.800 * 0.700.$$

$$= 0.224$$

3. (i)

$$P(Z=0) = P(X=0,Y=0,Z=0) + P(X=0,Y=1,Z=0) + P(X=1,Y=0,Z=0) + P(X=1,Y=1,Z=0) \text{ [sum rule]}$$

$$= 0.054 + 0.168 + 0.336 + 0.112$$

$$= 0.670$$

$$P(X=0,Z=0) = P(X=0,Y=0,Z=0) + P(X=0,Y=1,Z=0) \text{ [Sum rule]}$$

$$= 0.054 + 0.168$$

$$= 0.222$$

4. (i)  $P(X=1,Y=0|Z=1)$

$$(ii) P(Y=0) = P(X=0,Y=0,Z=0) + P(X=0,Y=0,Z=1) + P(X=1,Y=0,Z=0) + P(X=1,Y=0,Z=1) \text{ [sum rule]}$$

$$= 0.054 + 0.036 + 0.336 + 0.224$$

$$= 0.650$$

$$P(X=0|Y=0,Z=0) = P(X=0|Y=0) \text{ [independance]}$$

$$= P(Y=0|X=0) * P(X=0) / P(Y=0) \text{ [Bayes]}$$

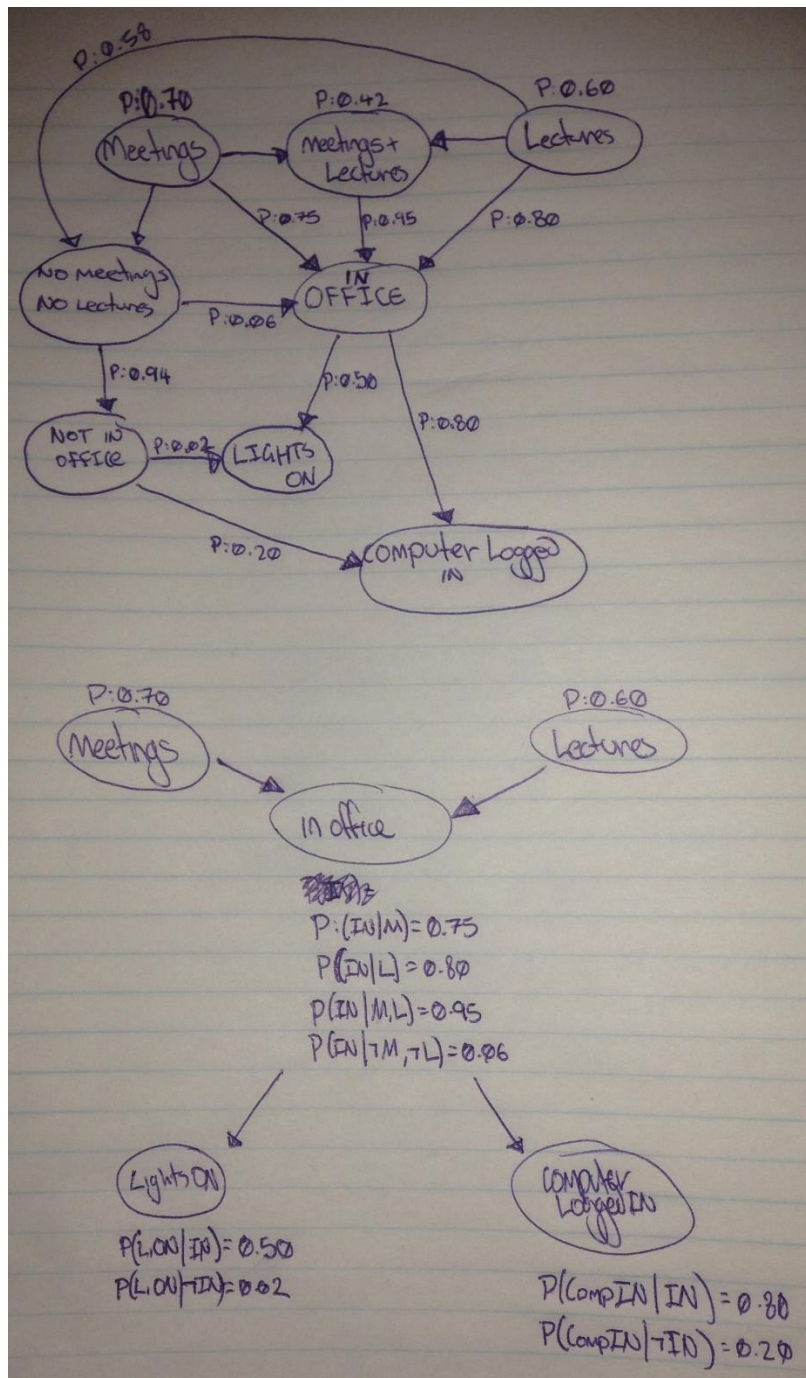
$$= 0.300 * 0.300 / 0.650$$

$$= 0.138$$

Part Two:

1.  $P(F1|T): 0.667$  (3.d.p)  $P(F1|F): 0.356$  (3.d.p)  
 $P(F2|T): 0.588$  (3.d.p)  $P(F2|F): 0.577$  (3.d.p)  
 $P(F3|T): 0.451$  (3.d.p)  $P(F3|F): 0.342$  (3.d.p)  
 $P(F4|T): 0.608$  (3.d.p)  $P(F4|F): 0.396$  (3.d.p)  
 $P(F5|T): 0.490$  (3.d.p)  $P(F5|F): 0.336$  (3.d.p)  
 $P(F6|T): 0.353$  (3.d.p)  $P(F6|F): 0.470$  (3.d.p)  
 $P(F7|T): 0.784$  (3.d.p)  $P(F7|F): 0.503$  (3.d.p)  
 $P(F8|T): 0.765$  (3.d.p)  $P(F8|F): 0.349$  (3.d.p)  
 $P(F9|T): 0.333$  (3.d.p)  $P(F9|F): 0.242$  (3.d.p)  
 $P(F10|T): 0.667$  (3.d.p)  $P(F10|F): 0.289$  (3.d.p)  
 $P(F11|T): 0.667$  (3.d.p)  $P(F11|F): 0.584$  (3.d.p)  
 $P(F12|T): 0.784$  (3.d.p)  $P(F12|F): 0.336$  (3.d.p)
2. Not quite sure what this is requesting of me, It isn't stating what needs to go in the report.
3. The spam data is not likely to contain information that is entirely conditionally independent. some conditions are likely to be dependent on each other and are mostly always included on the same types of emails. As we are not told the exact attributes we are dealing with I can only make an example such as the Vigara keyword would most likely have the word penis or sex attached to it in some way. As these are related subjects, unless the spam data only has one main keyword to sum up an entire subject, but even then there will likely be overlap. And on top of that the email with the keywords would also feature some other attribute such as invalid reply to or some such other attribute.

Part Three: Bayesian Networks



1.

2. Free Parameters: I can identify 2 potential free parameters:

Cleaners are coming, influences whether lights are on if she is not in the office.

If she has work to do (not meetings or lectures) influences if she is logged into the computer when not in the office, as well as the chances of her going into the office if no lectures or meetings.

$$\begin{aligned}
 3. \quad P(L, \neg M, IN, CompIN, LiOFF) &= P(IN|L, \neg M) \times P(CompIN|IN) \times P(LiOFF|IN) \\
 &= P(IN|L) \times P(CompIN|IN) \times P(LiOFF|IN) \\
 &= 0.80 \times 0.80 \times 0.50 \\
 &= 0.32
 \end{aligned}$$

4. 
$$\begin{aligned} P(IN) &= P(IN|M,L)P(M,L) + P(IN|M, \neg L)P(M, \neg L) + \\ &\quad P(IN|\neg M,L)P(\neg M,L) + P(IN|\neg M, \neg L)P(\neg M, \neg L) \\ &= 0.95*0.42 + 0.75*0.28 + 0.80*0.18 + 0.06*0.12 \\ &= 0.7602 \end{aligned}$$
5. 
$$\begin{aligned} P(\neg LiON, CompON|IN) &= P(\neg LiON|IN)P(IN) + P(CompON|IN)P(IN) \\ &= 0.50 * 0.7602 + 0.8 * 0.7602 \\ &= 0.98826 \end{aligned}$$
6. 
$$\begin{aligned} P(LiON|CompIN) &= P(CompIN|IN) * (LiON|IN) + P(CompIN|\neg IN) * P(LiON|\neg IN) \\ &= 0.8 * 0.5 + 0.2 * 0.02 \\ &= 0.4 + 0.004 \\ &= 0.404 \end{aligned}$$