If solution looks like $\xi^t e^{ikx}$,

leap frog scheme gives $\dfrac{e^{ikx}(\xi^{t+dt} - \xi^{t-dt})}{2\,dt}$

$$= \frac{e^{ikx}\xi^t}{2\,dt}\left(\xi^{dt} - \xi^{-dt}\right)$$

RHS : $-v \dfrac{\xi^t e^{ikx}(e^{ikdx} - e^{-ikdx})}{2\,dx}$.

Combine to get:

$$\xi^{dt} - \xi^{-dt} = -v\frac{dt}{dx}\sin(kdx)\cdot(2i)$$

$$(\xi^{dt})^2 - 1 + 2iv\frac{dt}{dx}\sin(kdx)\,\xi^{dt} = 0.$$

Solve for ξ^{dt}:

$$\xi^{dt} = \underbrace{-iv\frac{dt}{dx}\sin(kdx)}_{\text{imag.}} \pm \sqrt{-v^2\left(\frac{dt}{dx}\right)^2\sin^2(kdx) + 1}$$

If real, $|\xi^{dt}|^2 = 1$ due to term cancellations.

$\Rightarrow$ requires $v\dfrac{dt}{dx} < 1$ for solution to be stable.

If imaginary, $|\xi^{dt}|^2$ could be greater than 1 for some k. The solution becomes unstable.