A6_512

November 10, 2022

[230]:
```
import numpy as np
import matplotlib.pylab as plt
import scipy.signal as sig
plt.style.use(['classic','science'])

%matplotlib inline

plt.rcParams["figure.figsize"] = [6.4, 4.8]
%config InlineBackend.figure_format='retina'
```

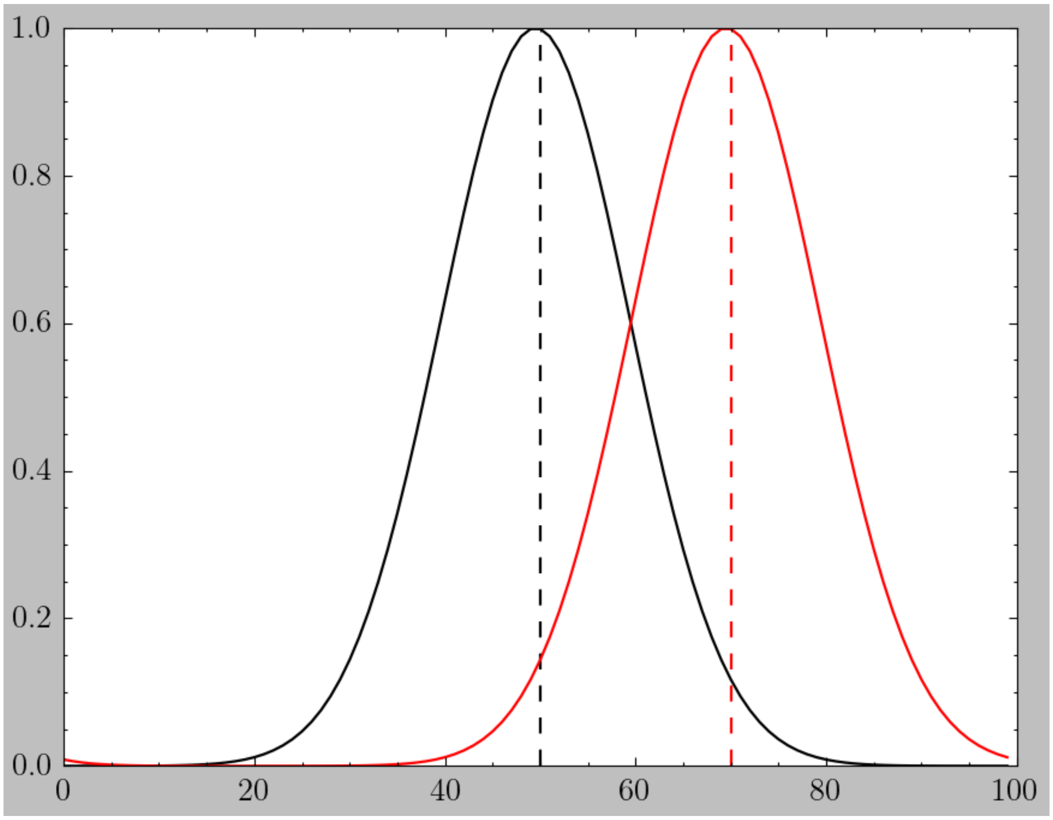
1 Question 1

Define a shifting function. We convolve the shifted delta distribution with the original function to achieve this goal. **Impulse shifting property**

[231]:
```
def shift(arr, steps):
    Farr = np.fft.fft(arr)
    delta = np.zeros(len(arr))
    delta[steps] = 1
    Fdelta = np.fft.fft(delta)
    return np.fft.ifft(Fdelta * Farr)
```

[232]:
```
xx = np.linspace(-5,5,100)
yy = np.exp(-1/2*xx**2)

plt.plot(np.abs(yy), c='k')
plt.axvline(len(yy)//2, c='k', ls = '--')
plt.plot(np.abs(shift(yy,20)), c = 'r')
plt.axvline(len(yy)//2+20, c='r', ls = '--', label = "shifted")
```

[232]:
```
<matplotlib.lines.Line2D at 0x2ba32e910>
```



2 Question 2

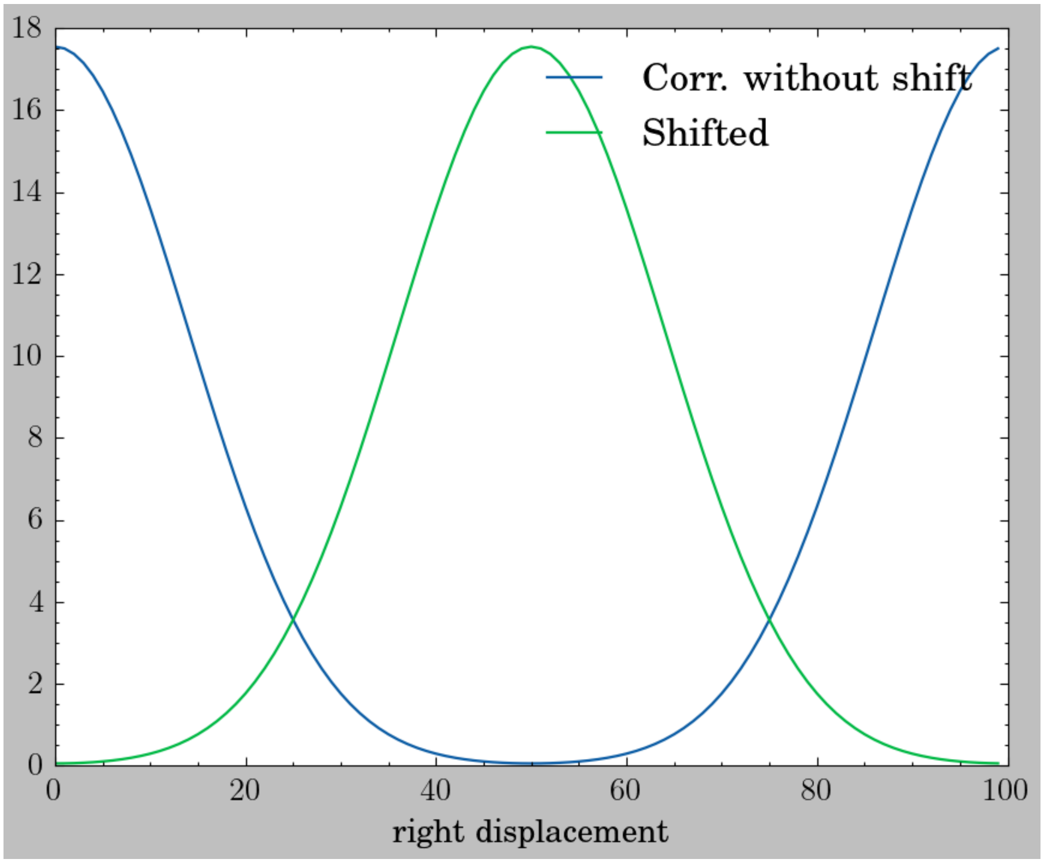
[233]:
```
def corr(a, b):
    return np.fft.ifft(np.fft.fft(a) * np.conj(np.fft.fft(b)))
```

2.1 Notice that our correlation function is subject to boundary condition. Therefore, we get another symmetric piece across the Gaussian.

We can use fftshift to bring the center to zero.

[234]:
```
plt.plot(np.abs(corr(yy,yy)), label = 'Corr. without shift')
plt.plot(np.abs(np.fft.fftshift(corr(yy,yy))), label = 'Shifted')
plt.xlabel('right displacement')
plt.legend()
```

[234]: <matplotlib.legend.Legend at 0x2ba3665b0>

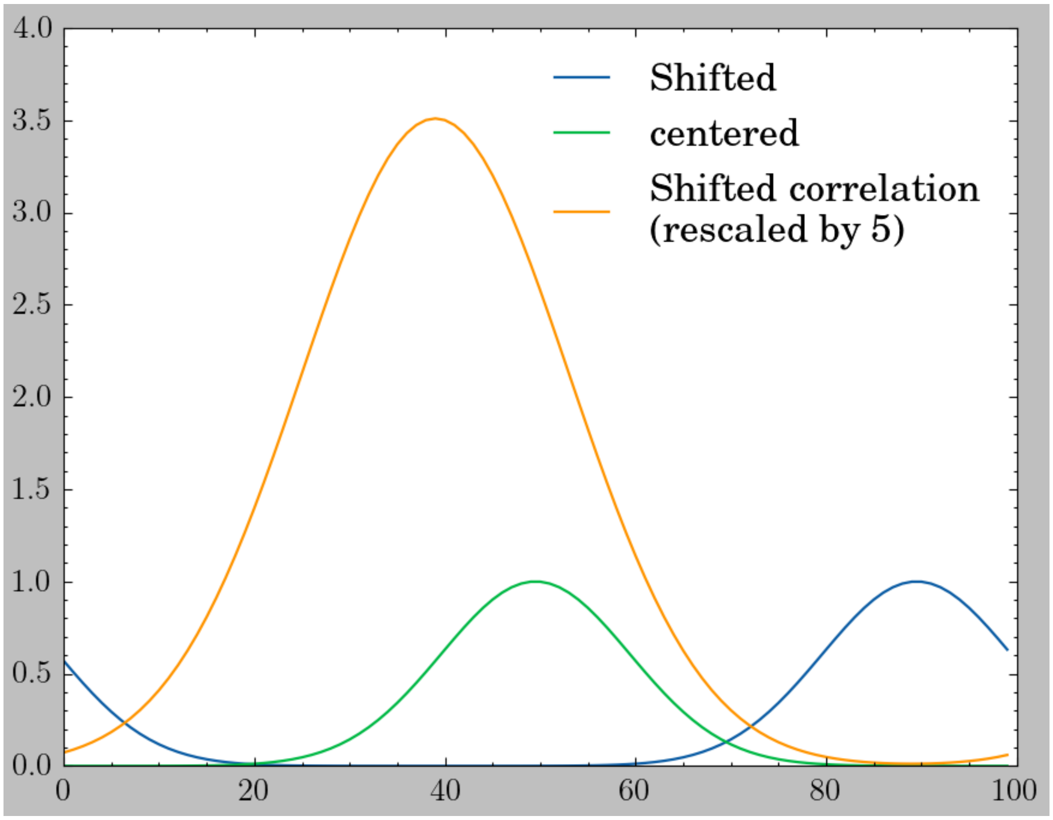


2.2 Correlation between gaussian and shifted.

[235]:
```
def s_corr(a, steps):
    return np.fft.ifft(np.fft.fft(a) *np.fft.fft(shift(a, steps)))
```

[236]:
```
step = 40
plt.plot(np.abs(shift(yy,step)), label='Shifted')
plt.plot(np.abs(shift(yy,0)), label = 'centered')
plt.plot(np.abs(s_corr(yy, step))/5, label = 'Shifted correlation \n(rescaled␣
↪by 5)')
plt.legend()
```

[236]: <matplotlib.legend.Legend at 0x2ba12d220>



In general, the maximum amplitude of the correlation remains unchainged.
But the peak of the correlation function shifts by the same amount. This is no surprise. We get max overlap if we bring the shifted Gaussian back.

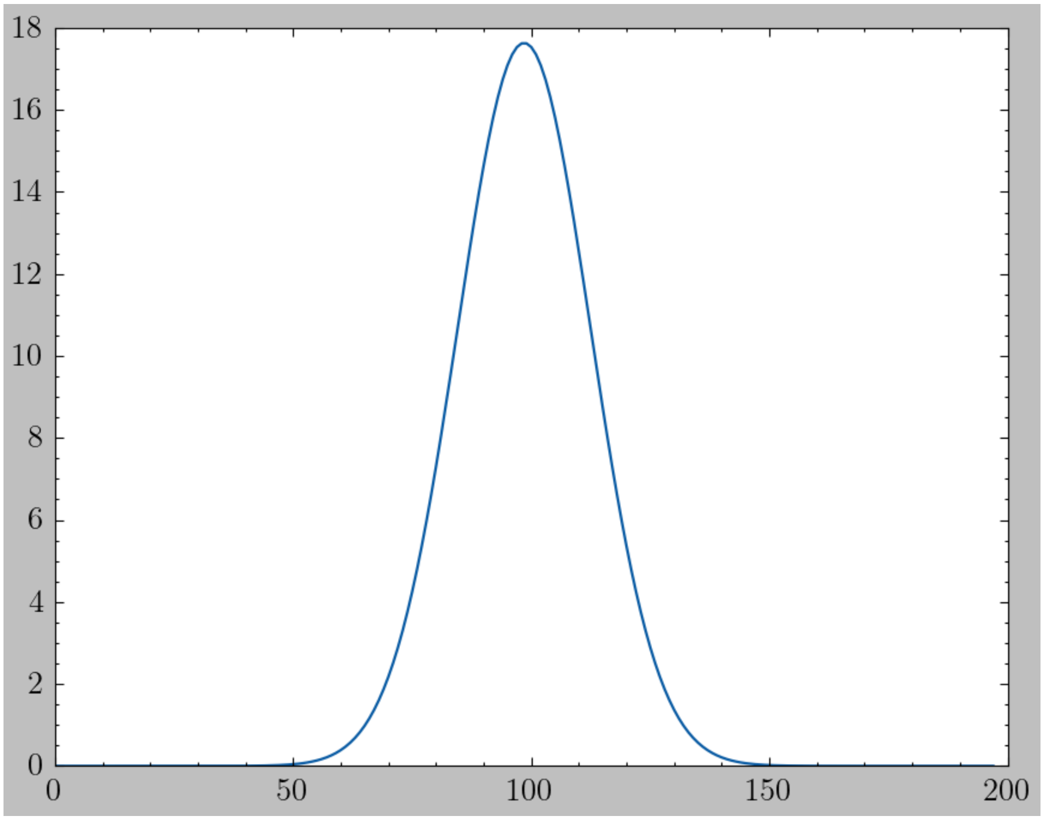
3 Question 3

The total length of two signals with non-zero overlaps is at most **N+M-1**.
If the two signals are of length N, M, we pad each of them with zeros at the end to length N+M-1. Then there is a buffer zone at the end for signal to NOT wrap around.

[237]:
```
def conv(a, b):
    lena = len(a)
    lenb = len(b)

    a = np.pad(a, (0, lenb - 1))
    b = np.pad(b, (0, lena - 1))

    return np.fft.irfft(np.fft.rfft(a) * np.fft.rfft(b))
```

[238]:
```
plt.plot(conv(yy,yy))
```

[238]:
```
[<matplotlib.lines.Line2D at 0x2ba837490>]
```



4 Question 5 LIGO data

We analyze GW151226 as an example, followed up by looping over the other three events in similar style.

[239]:
```
import h5py
import glob
import json

dat_fold = './LOSC_Event_tutorial/'
event_file = dat_fold + "BBH_events_v3.json"
```

[240]:
```
def read_template(filename):
    dataFile=h5py.File(filename,'r')
    template=dataFile['template']
    tp=template[0]
    tx=template[1]
    return tp,tx

def read_file(filename):
    dataFile=h5py.File(filename,'r')
```

```
    dqInfo = dataFile['quality']['simple']
    qmask=dqInfo['DQmask'][...]

    meta=dataFile['meta']
    #gpsStart=meta['GPSstart'].value
    gpsStart=meta['GPSstart'][()]
    #print meta.keys()
    #utc=meta['UTCstart'].value
    utc=meta['UTCstart'][()]
    #duration=meta['Duration'].value
    duration=meta['Duration'][()]
    #strain=dataFile['strain']['Strain'].value
    strain=dataFile['strain']['Strain'][()]
    dt=(1.0*duration)/len(strain)

    dataFile.close()
    return strain,dt,utc


def smooth_vector(vec,sig):
    n=len(vec)
    x=np.arange(n)
    x[n//2:]=x[n//2:]-n
    kernel=np.exp(-0.5*x**2/sig**2) #make a Gaussian kernel
    kernel=kernel/kernel.sum()
    vecft=np.fft.rfft(vec)
    kernelft=np.fft.rfft(kernel)
    vec_smooth=np.fft.irfft(vecft*kernelft) #convolve the data with the kernel
    return vec_smooth
```

[241]:
```
event_codes = json.load(open(event_file, "r")).keys()
event_codes
```

[241]: dict_keys(['GW150914', 'LVT151012', 'GW151226', 'GW170104'])

[242]:
```
event = json.load(open(event_file, "r"))["GW150914"]

sL, dtL, utcL = read_file(dat_fold + event['fn_L1'])
sH, dtH, utcH = read_file(dat_fold + event['fn_H1'])
tp, tx = read_template(dat_fold + event['fn_template'])
```

5 Frequency space x-axis

The time steps between Livingston and Hanford data are the same. Because of the inverse proportionality between dt and df, we can calculate our frequencies.

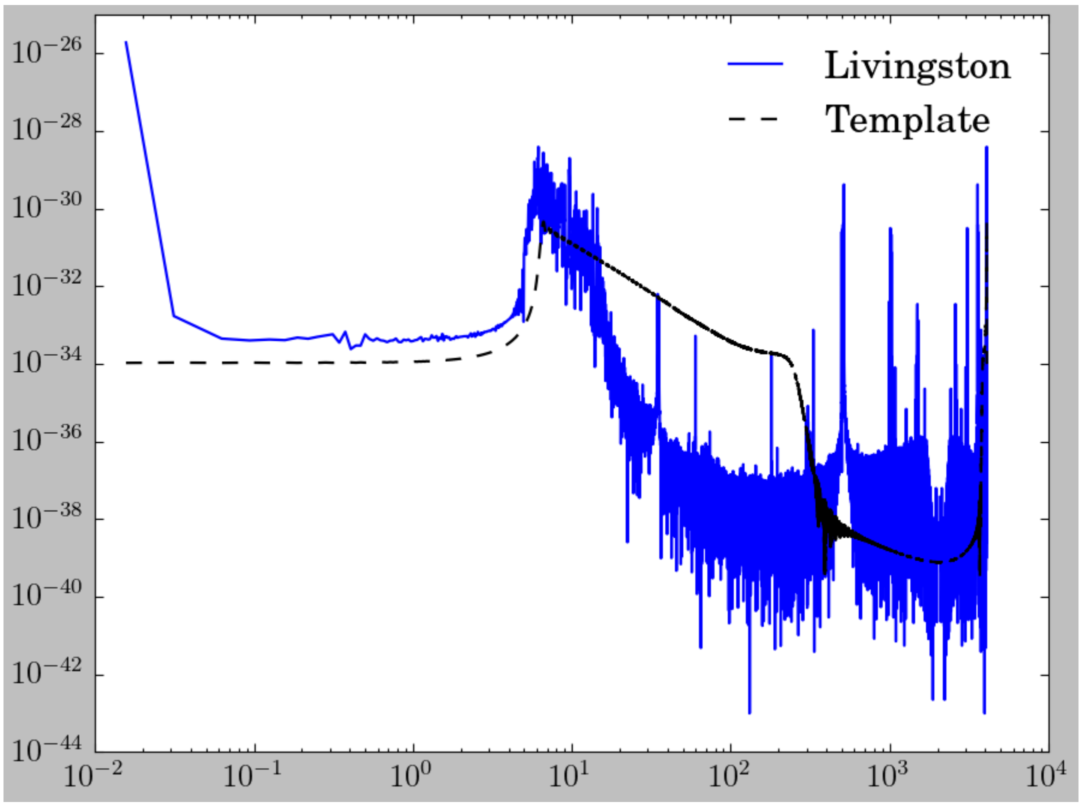
[243]:
```
time = dtL *len(sH)
df = 1/time
f = np.arange(len(sL))*df
f[0]=0.5*f[1]
t = dtL * np.arange(len(sH))
```

6 Power spectrum without windowing and smoothing

[244]:
```
PSsL = np.abs(np.fft.fft(sL))**2
PSsH = np.abs(np.fft.fft(sH))**2
PStp = np.abs(np.fft.fft(tp))**2
```

[245]:
```
plt.loglog(f, PSsL, c='b', label = 'Livingston')
plt.loglog(f, PStp, c='k', ls = '--', label = 'Template')
plt.legend()
```

[245]:
```
<matplotlib.legend.Legend at 0x2ba938af0>
```

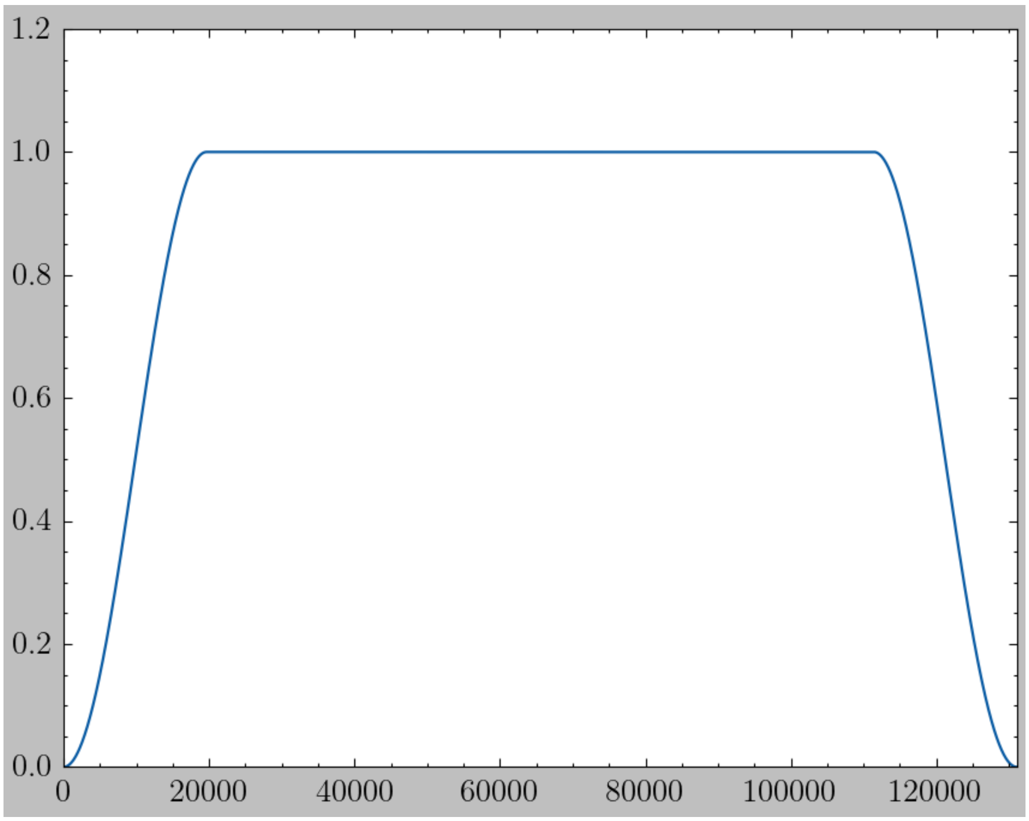


7 Choice of window function

Here we use Tukey window funtion since it has a flat top so it doesn't taper the data.

[246]:
```
win = sig.tukey(len(tp), alpha = .3)
plt.plot(win)
plt.axis([0, len(tp), 0,1.2])
```

[246]: (0.0, 131072.0, 0.0, 1.2)



8 Power spectrum Smoothing

We used a Gaussian with width of ten similar to what Jon did in class. Borrowing his code, which convolves a given spectrum with a Gaussian…

The whitening step will convolve the power spectrum with a Gaussian filter, make it less fluctuating.

This also gets rid of spectral features, and we can use the frequency independent part as our noise spectrum.

[247]:
```
width = 30
# strain
PSsL_win = np.abs(np.fft.fft(sL * win)**2)
```

```
PSsL_smooth = smooth_vector(PSsL_win, width)


PSsH_win = np.abs(np.fft.fft(sH * win)**2)
PSsH_smooth = smooth_vector(PSsH_win, width)

# template
PStp_win = np.abs(np.fft.fft(tp * win)**2)
PStp_smooth = smooth_vector(PStp_win, width)
```

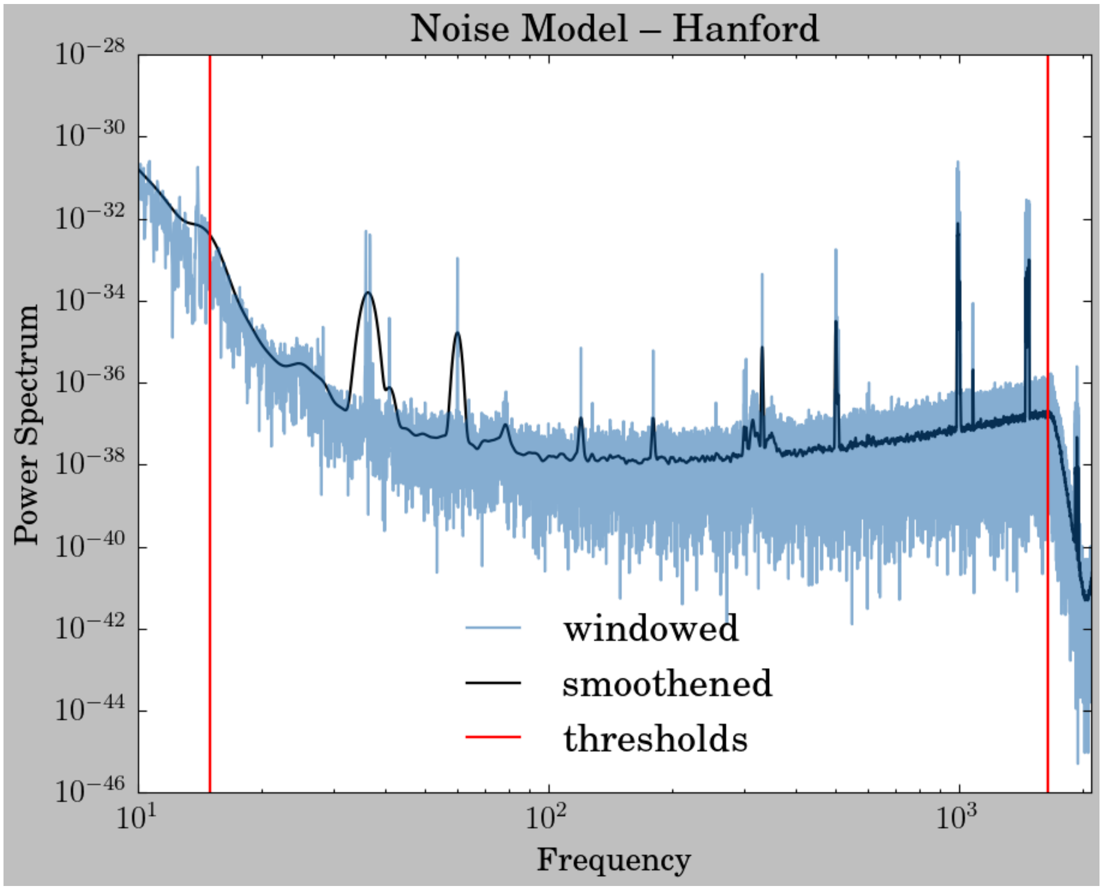
9 Let's have a look at the windowed P.S.. We will infer our noise from this.

The instrument has a digital filter at 1650 Hz. Hence we want to reference this threshold when setting the noise matrix.

10 Noise Models

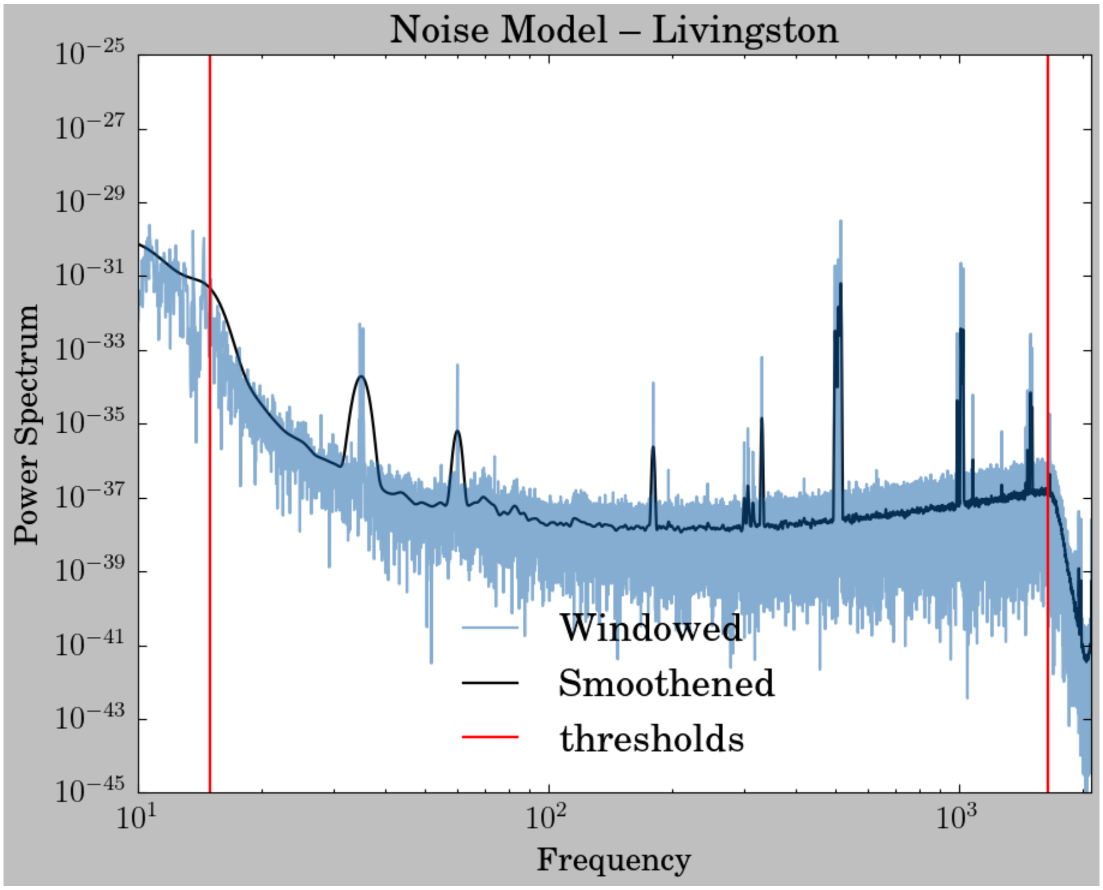
[248]:
```
plt.loglog(f, PSsH_win, alpha = .5, label = 'windowed')
plt.loglog(f, PSsH_smooth, c='k', zorder = 1,  label = 'smoothened')
plt.axvline(15,c='r', label = 'thresholds')
plt.axvline(1650, c ='r')
plt.xlim([10,2100])


plt.ylabel("Power Spectrum")
plt.xlabel('Frequency')
plt.title('Noise Model -- Hanford')
plt.legend(loc = 8)
```

[248]:
```
<matplotlib.legend.Legend at 0x2babaa520>
```



[249]:
```
plt.loglog(f, PSsL_win, alpha = .5, label = 'Windowed')
plt.loglog(f, PSsL_smooth, c='k', zorder = 1,  label = 'Smoothened')
plt.axvline(15,c='r', label = 'thresholds')
plt.axvline(1650, c ='r')
plt.xlim([10,2100])

plt.ylabel("Power Spectrum")
plt.xlabel('Frequency')
plt.title('Noise Model -- Livingston')
plt.legend(loc = 8)
```

[249]: <matplotlib.legend.Legend at 0x2bac2fa60>



11 Inverse matrix conditioning

Setting entries of the `Ninv` matrix to zero is equivalent to downweighting the frequencies corresponding to these entries.
The greater the uncertainties, the smaller Ninv is.

The left cutoff is due to blow up towards low frequency. The right cutoff is due tot he digital instrumental filter.

[250]:
```
NL_inv = 1/PSsL_smooth
NH_inv = 1/PSsH_smooth

NH_inv[f>1650] = 0
NL_inv[f>1650] = 0

NH_inv[f<15] = 0
NL_inv[f<15] = 0
```

12 Matched Filtering

Matched filter is done via

$$m = (A^T N^{-1} A)^{-1} (A^T N^{-1} d)$$

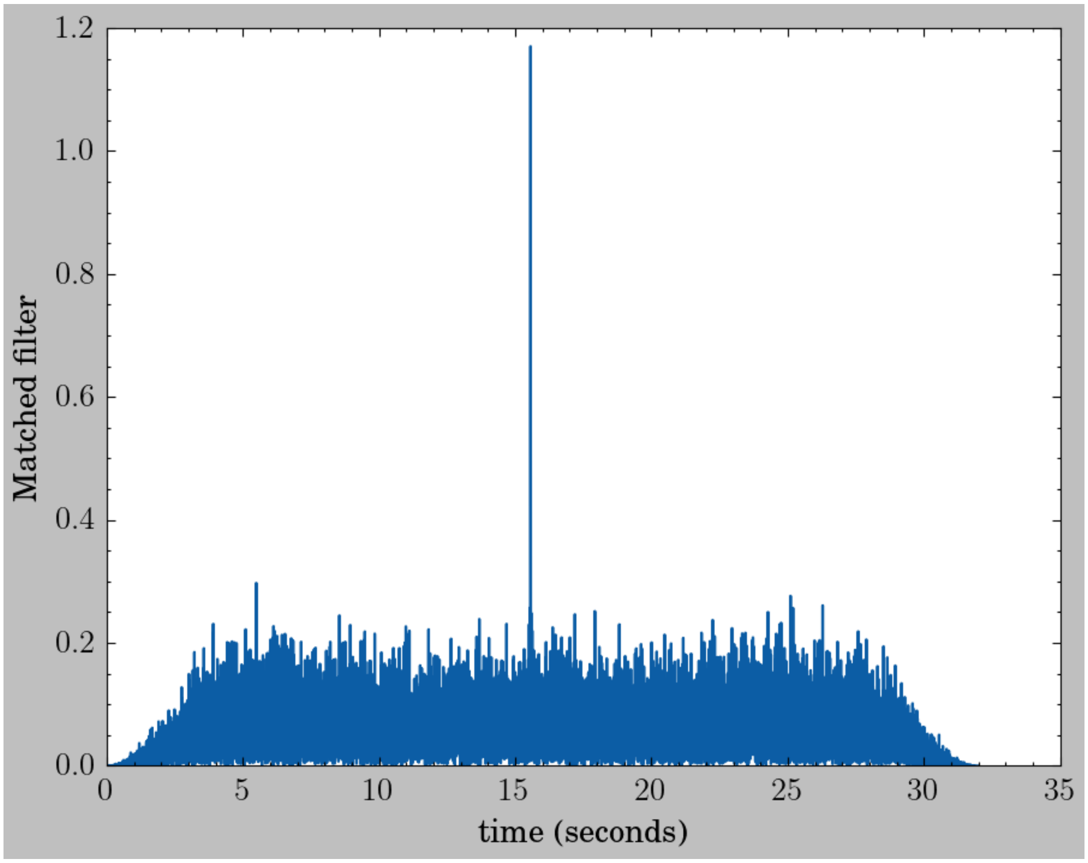
. The pre-whitening process requires us to split the noise matrix into $N^{-\frac{1}{2}}$ in the numerators. Following this process, the matched filter (right hand side) becomes

$$(N^{-\frac{1}{2}} A)^T (N^{-\frac{1}{2}} d)$$

(discarding the normalization term).
This can be done in Fourier spacewhere using windowed data and windowed template (according to Jon's notes on M.F.).

[251]:
```
FH_white = np.sqrt(NH_inv)* np.fft.fft(sH*win)
FL_white = np.sqrt(NL_inv)* np.fft.fft(sL*win)
FL_tp_white = np.sqrt(NL_inv)*np.fft.fft(tp*win)
FH_tp_white = np.sqrt(NH_inv)*np.fft.fft(tp*win)

sigL = np.fft.ifft(FL_tp_white * np.conj(FL_white))

sigH = np.fft.ifft(FH_tp_white * np.conj(FH_white))

plt.plot(t,np.abs(np.fft.fftshift(sigH)), label = 'Hanford')
plt.xlabel("time (seconds)")
plt.ylabel("Matched filter")
```

[251]:
```
Text(0, 0.5, 'Matched filter')
```



13 Signal to Noise ratio

The signal is our MF result (1D array). The noise is the covariance

$$(A^T N^{-1} A)^{-1} = N^{-1} A^2 = H^{-1}$$

, which is just the inverse of Hessian matrix (curvature) and **Noramlization of the M.F.**. This is equivalent to **auto-correlation of the pre-whitened version of the template**.

Note that variance is additive for independent measurements while standard deviation is not.

[252]:
```
varL = np.abs(np.fft.ifft(FL_tp_white * FL_tp_white.conjugate()))
varH = np.abs(np.fft.ifft(FH_tp_white * FH_tp_white.conjugate()))

noiseL = np.mean(np.sqrt(varL))
noiseH = np.mean(np.sqrt(varH))
noiseC = np.sqrt(noiseL**2+noiseH**2)

print("Noise for Livingston detector: ".ljust(30), noiseL)
print("Noise for Livingston detector: ".ljust(30), noiseH)
print("Combined noise: ".ljust(30), noiseC)
```

```
Noise for Livingston detector:  0.11946417036983117
Noise for Livingston detector:  0.1537860148136391
Combined noise:                 0.19473527249682548
```

14 SNR

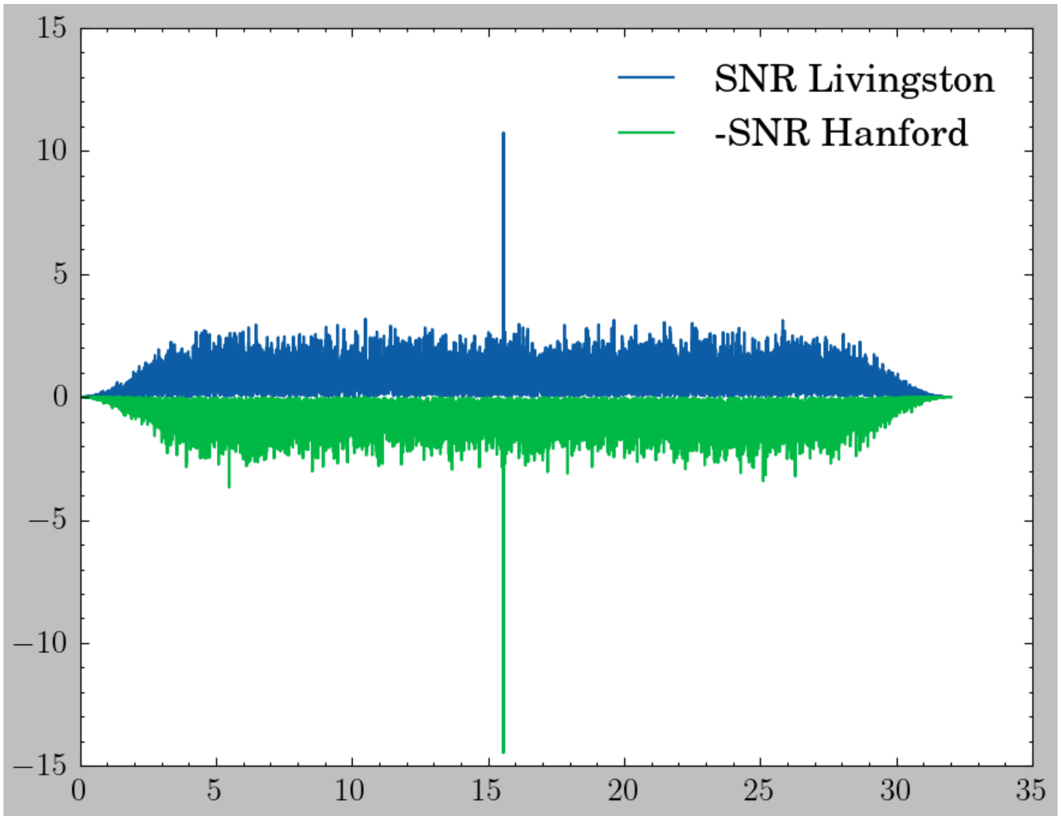
Similar to χ^2, the ratio of $E[X]^2$ and σ_X^2 is additive for independent events. We can sum up $\mathbf{SNR}^2$.

For the normalization, since we have enough data, the signaled domain is relatively short, and the window is wide, we can just take the standard deviaton across our M.F. to get a rough normalization estimate.

[253]:
```
normH = np.std(sigH[200:-200])
normL = np.std(sigL[200:-200])

snrH = np.fft.fftshift(np.abs(sigH/normH))
snrL = np.fft.fftshift(np.abs(sigL/normL))
# They can appear at different time
snrC = np.sqrt(snrH.max()**2 + snrL.max()**2)

print("Max SNR for Livingston detector: ".ljust(30), snrL.max())
print("Max SNR for Livingston detector: ".ljust(30), snrH.max())
print("Combined SNR: ".ljust(30), snrC)
plt.plot(t, snrL, label = "SNR Livingston")
plt.plot(t, -snrH, label = "-SNR Hanford")
plt.legend()
```

```
Max SNR for Livingston detector:  10.746929409886919
Max SNR for Livingston detector:  14.468889831600736
Combined SNR:                   18.023464281322045
```

[253]:
```
<matplotlib.legend.Legend at 0x2bba1fc70>
```



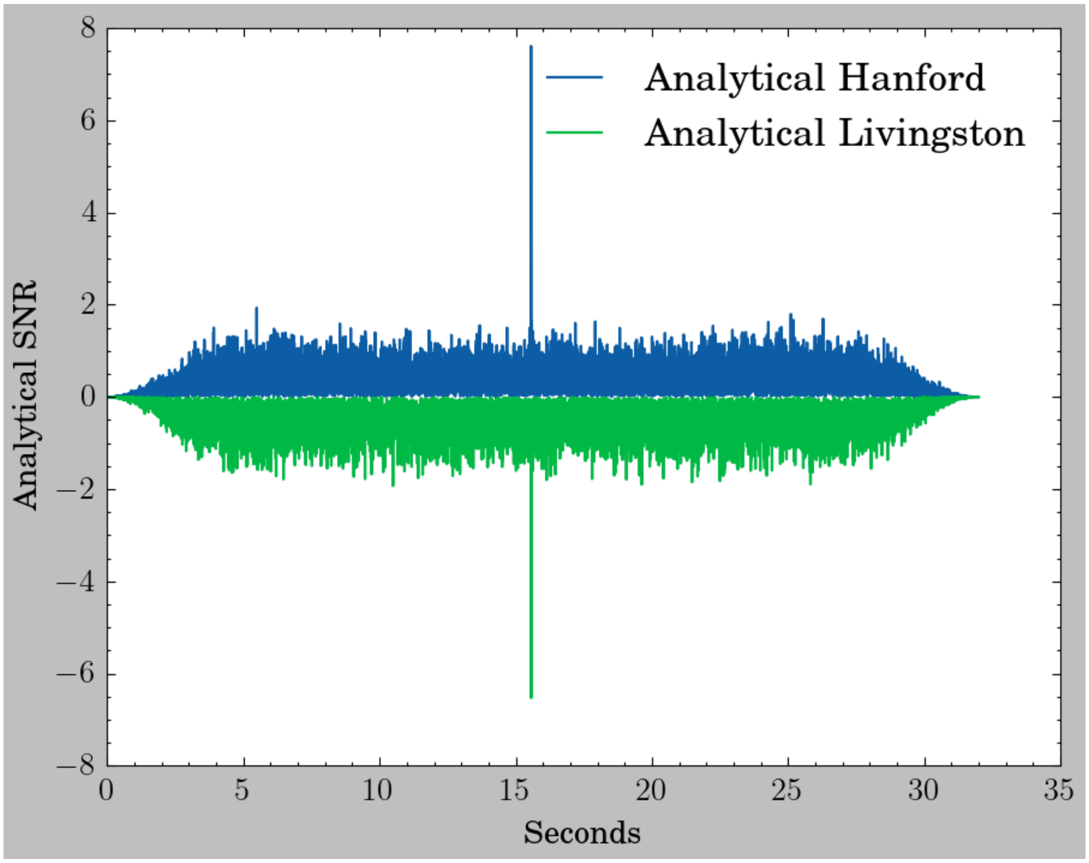
15 Analytic SNR vs. SNR

[254]:
```
AsnrH = np.fft.fftshift(np.abs(sigH/noiseH))
AsnrL = np.fft.fftshift(np.abs(sigL/noiseL))
AsnrC = np.sqrt(AsnrH.max()**2 + AsnrL.max()**2)

print("--"*20, "analytical", "--"*20)
print("Max Analytical SNR for Livingston detector: ".ljust(50), AsnrL.max())
print("Max Analytical SNR for Hanford detector: ".ljust(50), AsnrH.max())
print("Combined Analytical SNR: ".ljust(50), AsnrC)

print("--"*20, "computed", "--"*20)
print("Max SNR for Livingston detector: ".ljust(50), snrL.max())
print("Max SNR for Livingston detector: ".ljust(50), snrH.max())
print("Combined SNR: ".ljust(50), snrC)

print("--"*20, "ratio", "--"*20)
print("Analytic vs. Numerical -- Livingston: ".ljust(50), AsnrL.max()/snrL.
 ↪max())
print("Analytic vs. Numerical -- Hanford: ".ljust(50), AsnrH.max()/snrH.max())
```

```
print("Analytic vs. Numerical -- Combined SNR: ".ljust(50), AsnrC/ snrC)
plt.plot(t, AsnrH, label = 'Analytical Hanford')
plt.plot(t, -AsnrL, label = 'Analytical Livingston')
plt.xlabel("Seconds")
plt.ylabel("Analytical SNR")
plt.legend()
```

```
---------------------------------------- analytical
----------------------------------------
Max Analytical SNR for Livingston detector:       6.525279718941973
Max Analytical SNR for Hanford detector:          7.610173745774942
Combined Analytical SNR:                          10.024670560737528
---------------------------------------- computed
----------------------------------------
Max SNR for Livingston detector:                  10.746929409886919
Max SNR for Livingston detector:                  14.468889831600736
Combined SNR:                                     18.023464281322045
---------------------------------------- ratio
----------------------------------------
Analytic vs. Numerical -- Livingston:             0.6071761961085248
Analytic vs. Numerical -- Hanford:                0.5259680482986306
Analytic vs. Numerical -- Combined SNR:           0.5562010945435294
```

```
[254]: <matplotlib.legend.Legend at 0x2bbb8f160>
```



16 Comparison

The analytical SNR is, in general, smaller compared to our numerical SNR obtained by directly dividing by the variance. This can be because our noise model is not exact in terms of amplitude, and $\sqrt{(A^T N^{-1} A)}$ becomes larger.

Maybe Gaussian filter is not the best way to smoothen out the power spectrum.

17 Half cumulative amplitude Frequency

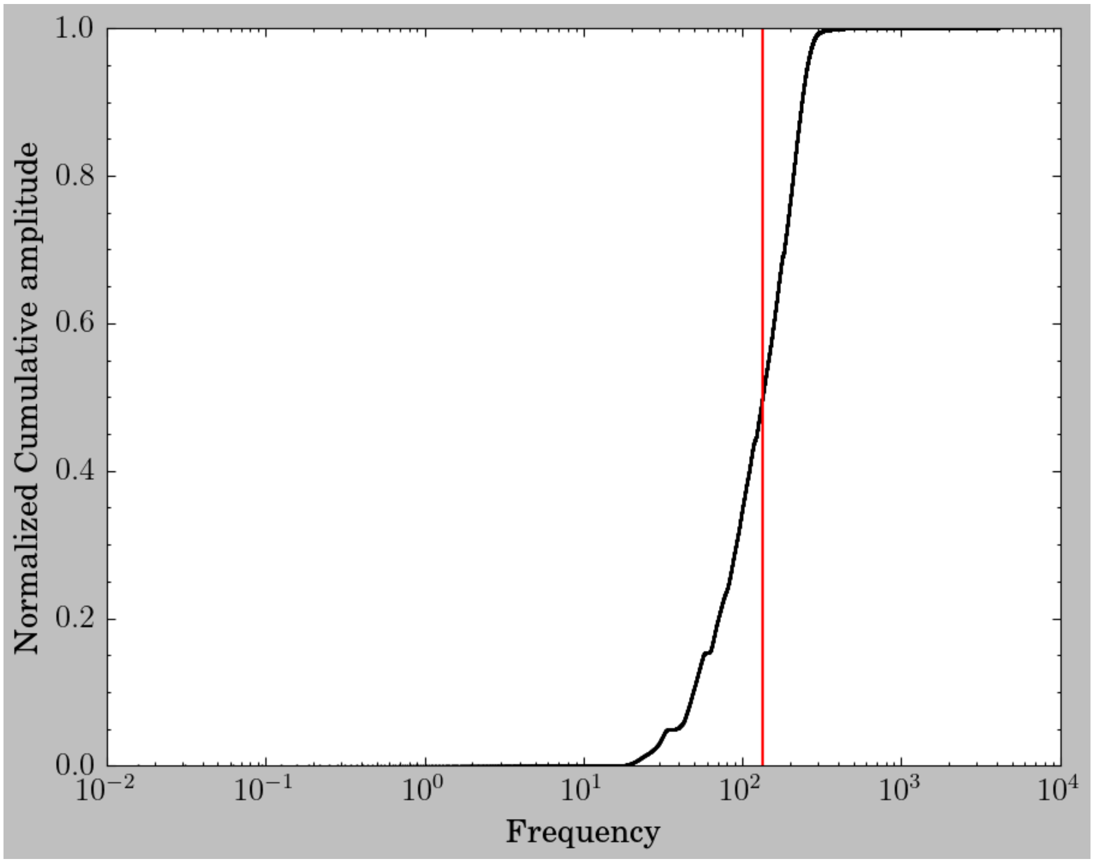
[255]:
```
idx_half = np.argwhere(np.cumsum(np.abs(FH_tp_white))/np.sum(np.
    ↪abs(FH_tp_white))>.5).flatten().min()
print("Half-intensity frequency is:".ljust(30), f"{f[idx_half]:.0f}", "Hz")
```

```
Half-intensity frequency is:   134 Hz
```

[256]:
```
plt.plot(f, np.cumsum(np.abs(FH_tp_white))/np.sum(np.abs(FH_tp_white)), 'k.',␣
    ↪ms = 1)
plt.axvline(f[idx_half], c = 'r', label = 'half-amp. freq.')
```

```
plt.xscale('log')
plt.xlabel('Frequency')
plt.ylabel('Normalized Cumulative amplitude')
```

[256]: Text(0, 0.5, 'Normalized Cumulative amplitude')



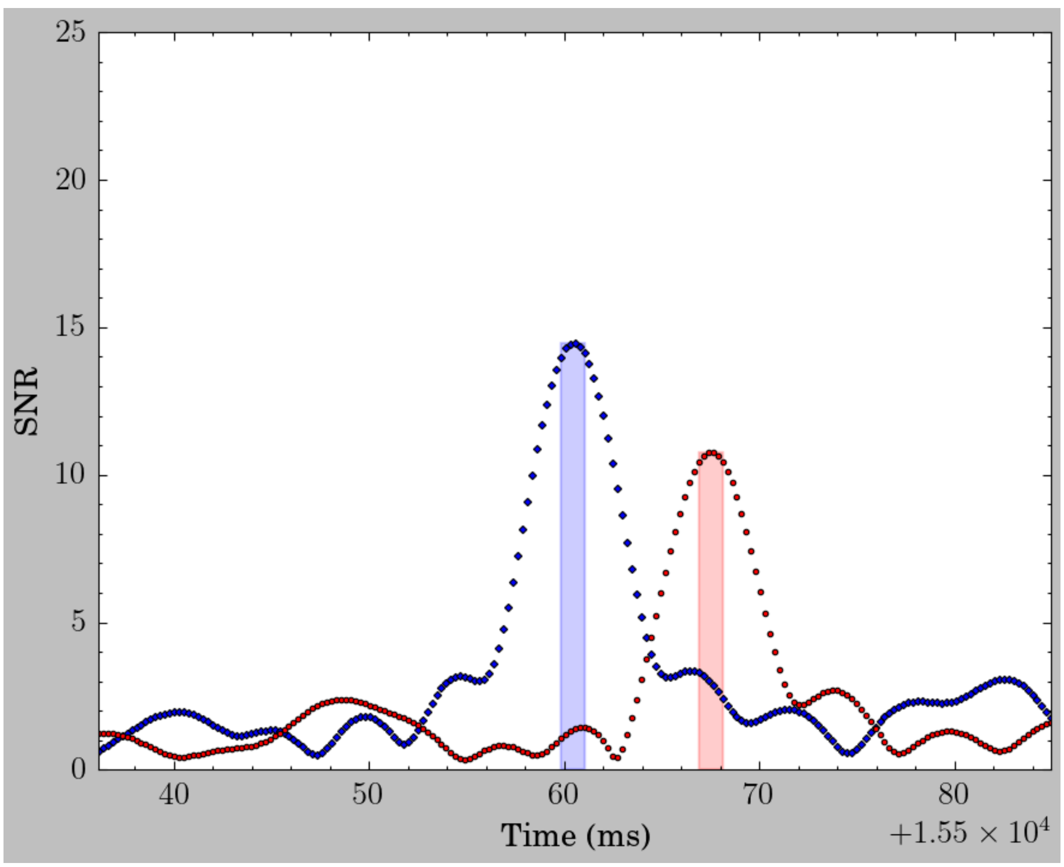
18 Time Comparison

For the uncertainty, we use interval within which SNR decreases by 1.

[257]:
```
plt.plot(t*1e3, snrH,'bD', ms = 2 , label = "Livingston")
plt.plot(t*1e3, snrL,'ro', ms =2 , label = 'Hanford')

tmaxH = t[np.argmax(snrH)]
tmaxL = t[np.argmax(snrL)]

binH = np.argwhere(np.abs(snrH - snrH.max())<1)
binL = np.argwhere(np.abs(snrL - snrL.max())<1)

tleftH = t[np.min(binH)]
```

```
trightH = t[np.max(binH)]
tleftL = t[np.min(binL)]
trightL = t[np.max(binL)]

etH = (trightH-tleftH)/2
etL = (trightL-tleftL)/2

edt = np.sqrt(etH**2 + etL**2)
dt = np.abs(tmaxH - tmaxL)

plt.axis([(tmaxH-100*dtH)*1e3, (tmaxH+100*dtH)*1e3, 0,25])
plt.fill_between(t*1e3 , 0, snrH.max(),where=(t<trightH) & (t>tleftH)␣
 ↪,color='b', alpha = .2)
plt.fill_between(t*1e3 , 0, snrL.max(),where=(t<trightL) & (t>tleftL)␣
 ↪,color='r', alpha = .2)
plt.xlabel("Time (ms)")
plt.ylabel("SNR")

print("Time difference is :".ljust(30), f"{dt*1e3:.1f} +/- {edt*1e3:.1f} ms")
```

```
Time difference is :           7.1 +/- 1.2 ms
```



Distance between Hanford and Livingston (LHO and LLO) is 3002 km (source: https://www.ligo.caltech.edu/page/ligo-detectors).
If GW propagates parallelly to our plane of detection, and strikes the two detectors at maximal time difference, the time difference we expect is Δt = distance/speed of light.

[258]:
```
Adt = 3002*1e3/3/1e8
print("We expect time difference of: ", f"{Adt*1000:.3f} ms \
if GW propagates parallelly to our plane of detection.")
```

```
We expect time difference of:  10.007 ms if GW propagates parallelly to our
plane of detection.
```

[ ]:

[ ]:

19 For the other three events, we simply loop over them below.

The waves seem to be travelling almost perpendicular for all these three, as indicated by the relatively short Δt.

[259]:
```
event_codes = json.load(open(event_file, "r")).keys()
```

[261]:
```
for e in list(event_codes)[1:]:
    event = json.load(open(event_file, "r"))[e]

    print("-"*20, f"EVENT {e}","-"*20)
    sL, dtL, utcL = read_file(dat_fold + event['fn_L1'])
    sH, dtH, utcH = read_file(dat_fold + event['fn_H1'])
    tp, tx = read_template(dat_fold + event['fn_template'])

    time = dtL *len(sH)
    df = 1/time
    f = np.arange(len(sL))*df
    f[0]=0.5*f[1]
    t = dtL * np.arange(len(sH))

    PSsL = np.abs(np.fft.fft(sL))**2
    PSsH = np.abs(np.fft.fft(sH))**2
    PStp = np.abs(np.fft.fft(tp))**2

    win = sig.tukey(len(tp), alpha = .3)

    width = 30
    # strain
    PSsL_win = np.abs(np.fft.fft(sL * win)**2)
```

```
PSsL_smooth = smooth_vector(PSsL_win, width)


PSsH_win = np.abs(np.fft.fft(sH * win)**2)
PSsH_smooth = smooth_vector(PSsH_win, width)

# template
PStp_win = np.abs(np.fft.fft(tp * win)**2)
PStp_smooth = smooth_vector(PStp_win, width)


NL_inv = 1/PSsL_smooth
NH_inv = 1/PSsH_smooth

NH_inv[f>1650] = 0
NL_inv[f>1650] = 0

NH_inv[f<15] = 0
NL_inv[f<15] = 0

FH_white = np.sqrt(NH_inv)* np.fft.fft(sH*win)
FL_white = np.sqrt(NL_inv)* np.fft.fft(sL*win)
FL_tp_white = np.sqrt(NL_inv)*np.fft.fft(tp*win)
FH_tp_white = np.sqrt(NH_inv)*np.fft.fft(tp*win)

sigL = np.fft.ifft(FL_tp_white * np.conj(FL_white))
sigH = np.fft.ifft(FH_tp_white * np.conj(FH_white))

varL = np.abs(np.fft.ifft(FL_tp_white * FL_tp_white.conjugate()))
varH = np.abs(np.fft.ifft(FH_tp_white * FH_tp_white.conjugate()))

noiseL = np.mean(np.sqrt(varL))
noiseH = np.mean(np.sqrt(varH))
noiseC = np.sqrt(noiseL**2+noiseH**2)

print("Noise for Livingston detector: ".ljust(30), noiseL)
print("Noise for Livingston detector: ".ljust(30), noiseH)
print("Combined noise: ".ljust(30), noiseC)

normH = np.std(sigH[200:-200])
normL = np.std(sigL[200:-200])

snrH = np.fft.fftshift(np.abs(sigH/normH))
snrL = np.fft.fftshift(np.abs(sigL/normL))

snrC = np.sqrt(snrH.max()**2 + snrL.max()**2)
```

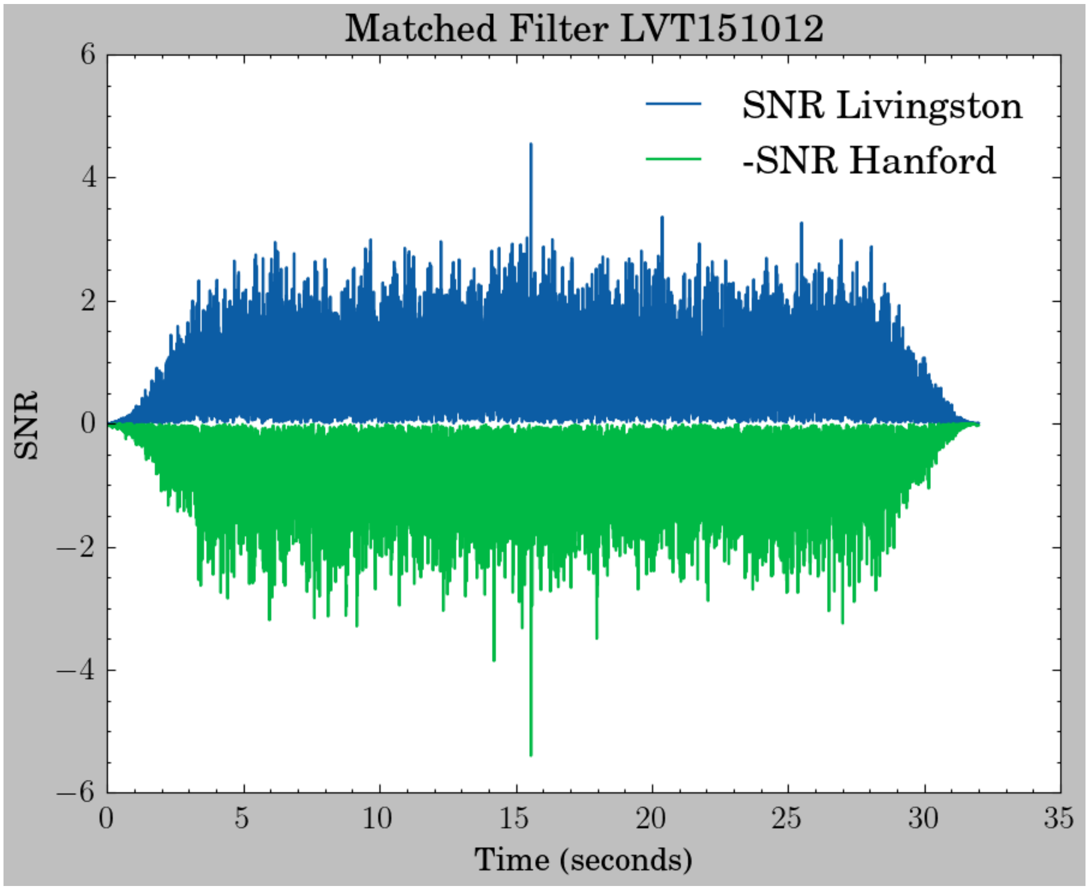
```
    plt.figure()
    print("Max SNR for Livingston detector: ".ljust(30), snrL.max())
    print("Max SNR for Livingston detector: ".ljust(30), snrH.max())
    print("Combined SNR: ".ljust(30), snrC)
    plt.title(f"Matched Filter {e}")
    plt.plot(t, snrL, label = "SNR Livingston")
    plt.plot(t, -snrH, label = "-SNR Hanford")
    plt.xlabel('Time (seconds)')
    plt.ylabel('SNR')
    plt.legend()


    AsnrH = np.fft.fftshift(np.abs(sigH/noiseH))
    AsnrL = np.fft.fftshift(np.abs(sigL/noiseL))
    AsnrC = np.sqrt(AsnrH.max()**2 + AsnrL.max()**2)

    print("--"*20, "analytical", "--"*20)
    print("Max Analytical SNR for Livingston detector: ".ljust(50), AsnrL.max())
    print("Max Analytical SNR for Hanford detector: ".ljust(50), AsnrH.max())
    print("Combined Analytical SNR: ".ljust(50), AsnrC)

    print("--"*20, "computed", "--"*20)
    print("Max SNR for Livingston detector: ".ljust(50), snrL.max())
    print("Max SNR for Livingston detector: ".ljust(50), snrH.max())
    print("Combined SNR: ".ljust(50), snrC)

    print("--"*20, "ratio", "--"*20)
    print("Analytic vs. Numerical -- Livingston: ".ljust(50), AsnrL.max()/snrL.
↪max())
    print("Analytic vs. Numerical -- Hanford: ".ljust(50), AsnrH.max()/snrH.
↪max())
    print("Analytic vs. Numerical -- Combined SNR: ".ljust(50), AsnrC/ snrC)
    idx_half = np.argwhere(np.cumsum(np.abs(FH_tp_white))/np.sum(np.
↪abs(FH_tp_white))>.5).flatten().min()
    print("\nHalf-intensity frequency is:".ljust(30), f"{f[idx_half]:.0f}",␣
↪"Hz")


    plt.figure()
    plt.plot(t*1e3, snrH,'bD', ms = 2 , label = "Livingston")
    plt.plot(t*1e3, snrL,'ro', ms =2 , label = 'Hanford')

    tmaxH = t[np.argmax(snrH)]
    tmaxL = t[np.argmax(snrL)]

    binH = np.argwhere(np.abs(snrH - snrH.max())<1)
    binL = np.argwhere(np.abs(snrL - snrL.max())<1)
```

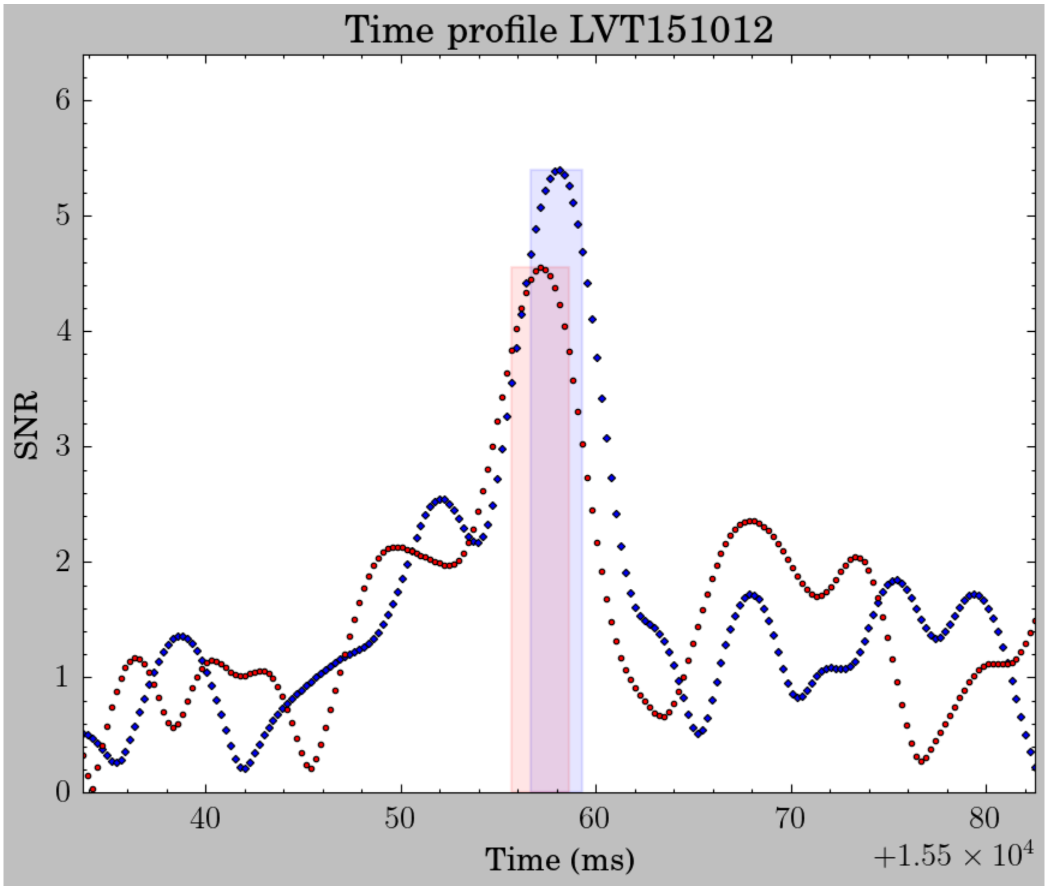
```
    tleftH = t[np.min(binH)]
    trightH = t[np.max(binH)]
    tleftL = t[np.min(binL)]
    trightL = t[np.max(binL)]

    etH = (trightH-tleftH)/2
    etL = (trightL-tleftL)/2

    edt = np.sqrt(etH**2 + etL**2)
    dt = np.abs(tmaxH - tmaxL)

    plt.title(f"Time profile {e}")
    plt.axis([(tmaxH-100*dtH)*1e3, (tmaxH+100*dtH)*1e3, 0, np.max([snrH.max(),␣
↪snrL.max()])+1])
    plt.fill_between(t*1e3 , 0, snrH.max(),where=(t<trightH) & (t>tleftH)␣
↪,color='b', alpha = .1)
    plt.fill_between(t*1e3 , 0, snrL.max(),where=(t<trightL) & (t>tleftL)␣
↪,color='r', alpha = .1)
    plt.xlabel("Time (ms)")
    plt.ylabel("SNR")

    print("\nTime difference is :".ljust(30), f"{dt*1e3:.1f} +/- {edt*1e3:.1f}␣
↪ms")
    print("\n"*5)
```

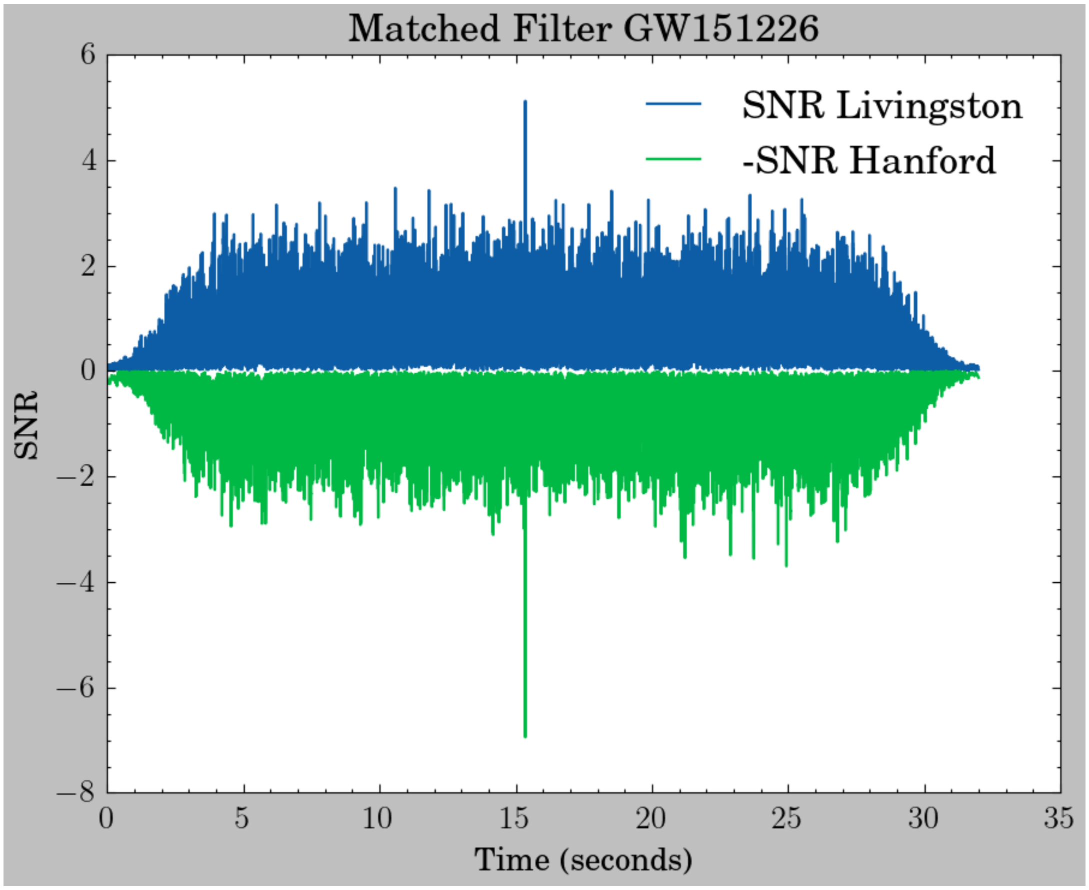
```
-------------------- EVENT LVT151012 --------------------
Noise for Livingston detector:  0.08346468304013799
Noise for Livingston detector:  0.12277159874020832
Combined noise:                0.14845611732837896
Max SNR for Livingston detector:  4.551292486553042
Max SNR for Livingston detector:  5.396140980683283
Combined SNR:                  7.059220975544236
---------------------------------------- analytical
----------------------------------------
Max Analytical SNR for Livingston detector:       2.7454784722302477
Max Analytical SNR for Hanford detector:          2.248612634855601
Combined Analytical SNR:                          3.5487900505823085
---------------------------------------- computed
----------------------------------------
Max SNR for Livingston detector:                  4.551292486553042
Max SNR for Livingston detector:                  5.396140980683283
Combined SNR:                                     7.059220975544236
---------------------------------------- ratio
----------------------------------------
Analytic vs. Numerical -- Livingston:             0.6032305065740914
Analytic vs. Numerical -- Hanford:                0.4167075402412618
```

```
Analytic vs. Numerical -- Combined SNR:          0.5027169517538317

Half-intensity frequency is:  126 Hz

Time difference is :          1.0 +/- 2.3 ms




-------------------- EVENT GW151226 --------------------
Noise for Livingston detector:  0.042121273005258475
Noise for Livingston detector:  0.07854958860994357
Combined noise:                0.08913046342511012
Max SNR for Livingston detector:  5.11918583587116
Max SNR for Livingston detector:  6.944408905165313
Combined SNR:                  8.627333230165808
---------------------------------------- analytical
----------------------------------------
Max Analytical SNR for Livingston detector:       3.2809999343555596
Max Analytical SNR for Hanford detector:          3.0796963129550528
Combined Analytical SNR:                          4.499943327339816
---------------------------------------- computed
----------------------------------------
Max SNR for Livingston detector:                  5.11918583587116
Max SNR for Livingston detector:                  6.944408905165313
Combined SNR:                                     8.627333230165808
---------------------------------------- ratio
----------------------------------------
Analytic vs. Numerical -- Livingston:             0.6409222168425566
Analytic vs. Numerical -- Hanford:                0.44347853863621817
Analytic vs. Numerical -- Combined SNR:           0.521591459062412

Half-intensity frequency is:  146 Hz

Time difference is :          1.0 +/- 1.4 ms




-------------------- EVENT GW170104 --------------------
Noise for Livingston detector:  0.15493065044716983
Noise for Livingston detector:  0.1379976559947835
Combined noise:                0.20747737107462522
Max SNR for Livingston detector:  7.71909445304796
```

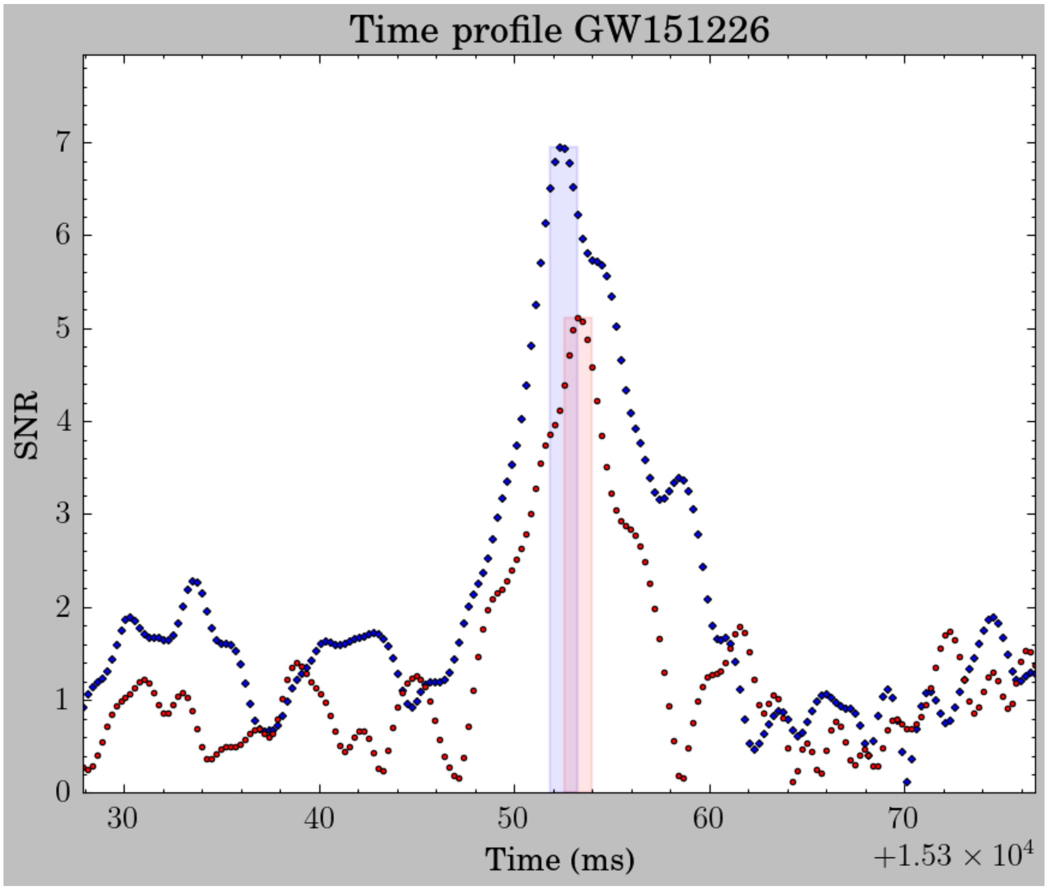
```
Max SNR for Livingston detector:  6.851174467552292
Combined SNR:                    10.320998535021513
---------------------------------------- analytical
----------------------------------------
Max Analytical SNR for Livingston detector:       4.430212012772098
Max Analytical SNR for Hanford detector:          3.2721630998121167
Combined Analytical SNR:                          5.507615621108852
---------------------------------------- computed
----------------------------------------
Max SNR for Livingston detector:                  7.71909445304796
Max SNR for Livingston detector:                  6.851174467552292
Combined SNR:                                     10.320998535021513
---------------------------------------- ratio
----------------------------------------
Analytic vs. Numerical -- Livingston:             0.5739289808823087
Analytic vs. Numerical -- Hanford:                0.47760615574881965
Analytic vs. Numerical -- Combined SNR:           0.5336320514357453

Half-intensity frequency is:  136 Hz

Time difference is :          2.7 +/- 1.6 ms
```

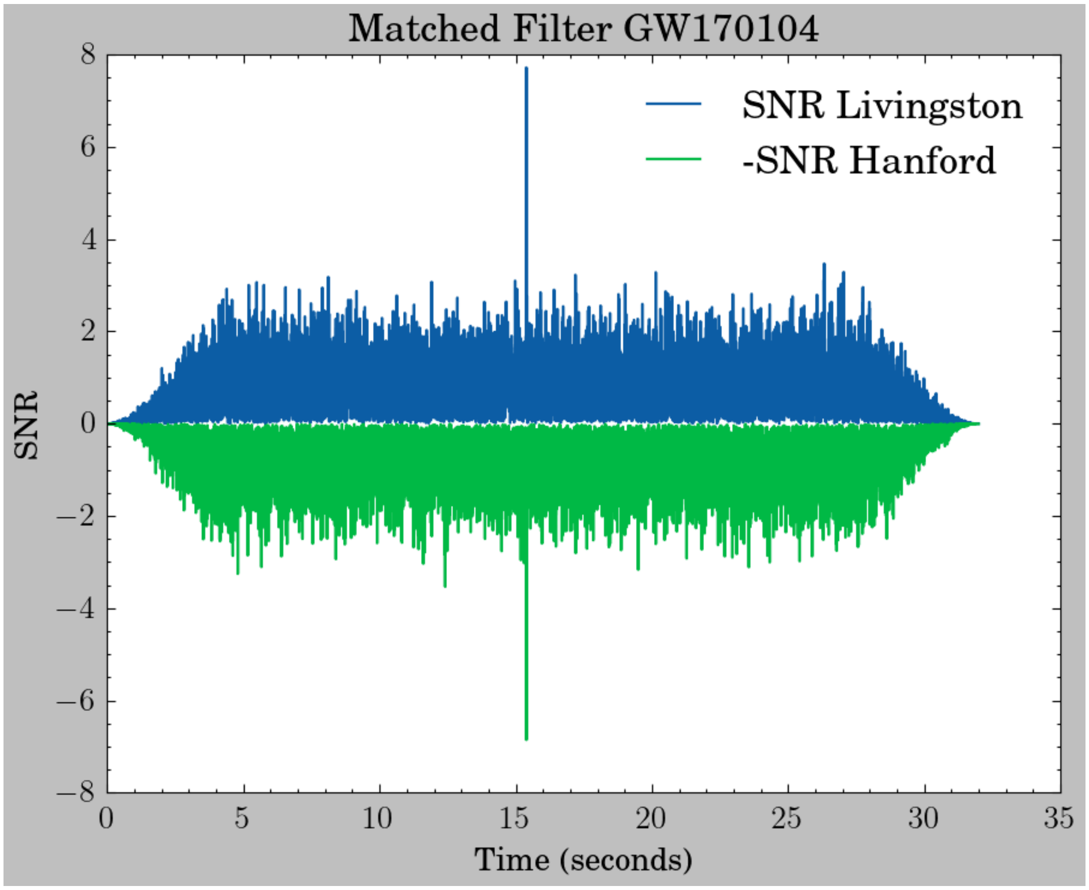

Matched Filter LVT151012
SNR Livingston
-SNR Hanford
SNR
Time (seconds)
6
4
2
0
−2
−4
−6
0
5
10
15
20
25
30
35

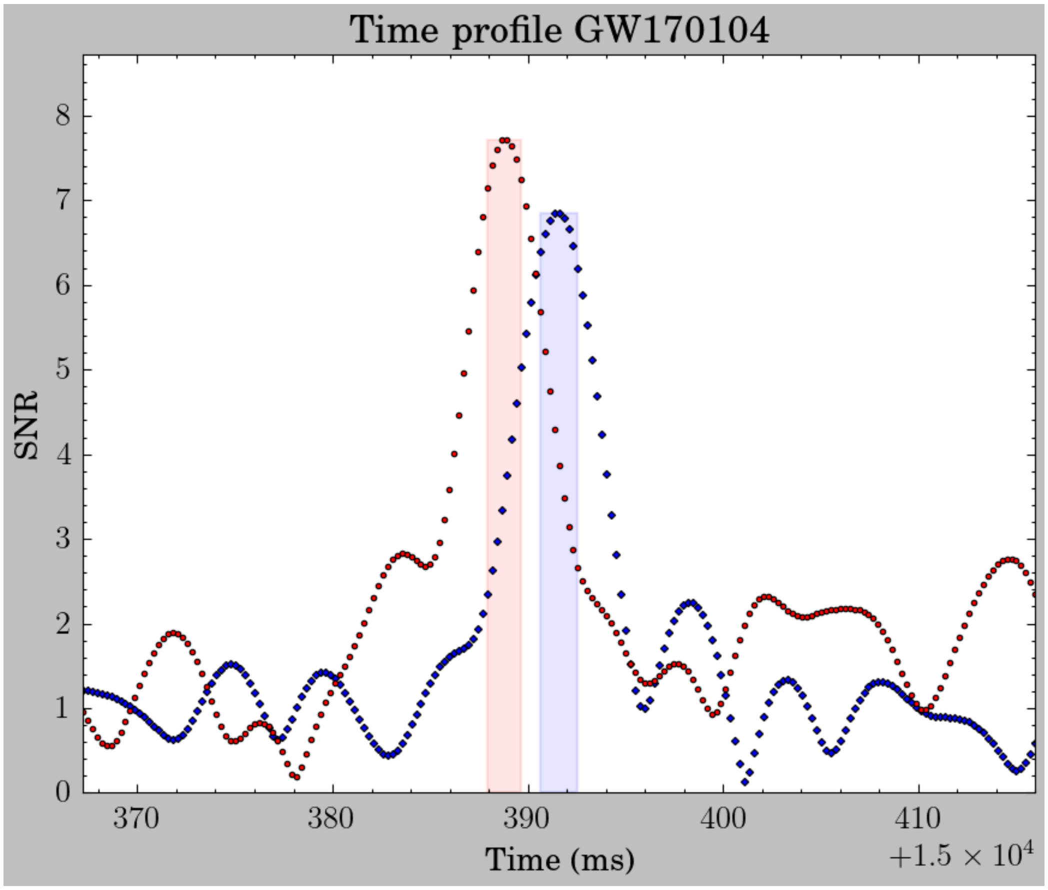

Time profile LVT151012
SNR
Time (ms)
$+1.55 \times 10^4$
0
1
2
3
4
5
6
40
50
60
70
80



Matched Filter GW151226
SNR Livingston
-SNR Hanford
SNR
Time (seconds)
6
4
2
0
−2
−4
−6
−8
0
5
10
15
20
25
30
35


Time profile GW151226
SNR
Time (ms)
$+1.53 \times 10^4$
0
1
2
3
4
5
6
7
30
40
50
60
70


Matched Filter GW170104
SNR
Time (seconds)
SNR Livingston
-SNR Hanford
8
6
4
2
0
−2
−4
−6
−8
0
5
10
15
20
25
30
35



Time profile GW170104
SNR
Time (ms)
$+1.5 \times 10^4$
0
1
2
3
4
5
6
7
8
370
380
390
400
410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[ ]:
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[ ]:
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[ ]:
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