Adapting to Climate Change: An analysis under uncertainty

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Abstract

Climate change is a phenomenon beset with major uncertainties and researchers should include them in Integrated Assessment Models. However, including further dimensions in IAM models comes at a cost. In particular, it makes most of these models suffer from the curse of dimensionality. In this study we benefit from a state-reduced framework to overcome those problems. In an attempt to advance in the modelling of adaptation within IAM models, we apply this methodology to shed some light on how the optimal balance between mitigation and adaptation changes under different stochastic scenarios. We find that stochastic technology growth hardly affects the optimal bundle of mitigation and adaptation whereas uncertainty about the value of climate sensitivity and the possibility of tipping points hitting the system change substantially the composition of the optimal mix as both persuade the risk-averse social planner to invest more in mitigation. Overall, we identify that including uncertainty into the model tends to favour (long-lasting) mitigation with respect to (instantaneous) adaptation. Further research should address the properties of the optimal mix when a stock of adaptation can be built.

Keywords: climate change, adaptation, mitigation, dynamic programming, uncertainty, Integrated Assessment, DICE

JEL Classification: C61, D58, D90, O44, Q01, Q54, Q56

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1 Introduction

Climate change is all about uncertainty: uncertainty governing the natural processes involved, uncertainty arisen due to its long-term nature, uncertainty on how agents react to those phenomena or uncertainty on how every event is modelled. Most IAMs (including DICE) are deterministic and way too complex to permit a proper incorporation of uncertainty. Monte-Carlo methods are the most common approach to addressing uncertainty in the integrated assessment literature. However, Monte-Carlo methods, as implemented in this strand of literature, do not model decision making under uncertainty. They present a sensitivity analysis that averages over deterministic simulations. The quantitative analysis of optimal climatic policies under uncertainty requires a recursive dynamic programming implementation of IAMs. Such implementations are subject to the curse of dimensionality. Every increase in the dimension of the state space is paid for by a combination of (exponentially) increasing processor time, lower quality of the value or policy function approximations, and reductions of the uncertainty domain. However, Traeger (2014) has recently delivered a state-reduced, recursive dynamic programming implementation of the DICE-2007 model which, in its basic specification, has only 4 state variables.[1] This leaves us with some extra margin to enrich the model with new features or include uncertainty. In contrast to Monte-Carlo approaches, this model solves for the optimal policy under uncertainty. The present methodology incorporates uncertainty in every period and the decision maker optimally reacts to the anticipated future resolution of uncertainty.

DICE deals explicitly with rational economic agents operating through time in stochastic environments. In this model, a decision maker must choose a sequence of actions through time subject to some environmental restrictions. If the environment is subject to unpredictable outside shocks, it is clear that the best future actions depend on the magnitude of these shocks. The way of deciding on the immediate action to take as a function of the current situation is called a recursive formulation because it exploits the observation that a decision problem of the same general structure recurs each period.[2] The use of recursive methods makes it possible to treat a wide variety of dynamic economic problems-both deterministic and stochastic.

In the past some authors have managed to formulate the DICE model recursively. For instance Kelly and Kolstad (1999, 2001) implement the DICE-1994 model as a recursive dynamic programming model, analysing learning time in detail, but they do not consider the separate contributions of uncertainty, learning and stochasticity on near term optimal policies. Some years later Leach (2007) implements the same DICE version showing that learning slows down further under additional uncer-

[1] Basically, the reduction is achieved by simplifying the carbon cycle and the temperature delay equations.

[2] Recursive methods in economics were extensively introduced by Stokey et al. (1989).

tainty. These papers are seminal contributions to uncertainty assessment in climate change and careful implementations of the original DICE version. A different set of papers introduce uncertainty into non-recursive implementations of integrated assessment models. Closest to our implementation, Keller et al. (2004) introduce uncertainty and learning into an earlier version of DICE. However, the employed non-recursive methodology only allows for a few discrete uncertain events, or exogenous learning over three alternative scenarios. For many applications, such individual uncertain events deliver interesting insights. However, these studies cannot replace comprehensive uncertainty evaluations using state of the art stochastic dynamic programming methods. Finally, Monte-Carlo methods are the most common approach to addressing uncertainty in the integrated assessment literature. Monte-Carlo methods, though, do not model decision making under uncertainty as they are implemented in this strand of literature. They present a sensitivity analysis that averages over deterministic simulations.

As noted in the earlier chapter, adaptation to climate change is key to confront climate change impacts and the IPCC has made a plea for advancing in its comprehension and its integration within IAM models. In the last chapter we analysed how different schemes of adaptation interfere with the optimal amount of mitigation under deterministic conditions. Here we will expand the previous analysis to include different sources of uncertainty that may affect the model. Including uncertainty into the model may potentially distort results as we already know them. For instance, Lecocq and Shalizi (2010) find on their partial equilibrium model, when uncertainty is introduced into the model, the cost effectiveness of mitigation is found to increase with regard to adaptation.

In this study we conduct a series of experiments covering a wide menu of uncertainties that may affect our model. We divide these uncertainties into four broad categories and provide an example of each group. First, we identify uncertainties about the value of the parameters of the model (epistemic). We will study how an unknown value of climate sensitivity would affect optimal policies and basic magnitudes. Second, we allow the exogenous processes that govern the dynamics of the model to behave stochastically. We will present stochastic labour-augmenting technology growth as an example. Third, we will include uncertainties in the way individuals (social planner) learn from the past. In particular we will equip our model with bayesian learning, by which policymakers have a certain prior about the value of certain parameters of the model and update their beliefs in response to the observation of realised variables. Fourth, we will study the possibility of the occurrence of catastrophes, allowing for the existence of tipping points in an undetermined point of time.

The remainder of the paper is organised as follows. In Section 2 we will include uncertainty generically into the AD-DICE model and state the benchmark Bellman equation. Section 3 will draw some messages about the optimal adaptation decision if we are unsure about the outstanding climate sensitivity. In Section 4 we will experiment how a stochastic technology growth affects the basic magnitudes of our model. Section 5 will address how optimal decisions vary if individuals learn over the years about the uncertain parameters governing the model. Section 6 will explore the possibility of having irreversible tipping points that compromise the stability of the system. Finally, Section 7 will conclude.

2 The AD-DICE model with uncertainty

Our IAM combines a growing Ramsey-Cass-Koopmans economy with a simple climate model. It is based on the widespread DICE model by Nordhaus (2008) and its stochastic dynamic programming implementation following Kelly and Kolstad (1999). Our formulation resembles closely that of Traeger (2014), who extends the small set of existing state of the art implementations of stochastic dynamic programming integrated assessment models. His main contribution is to reduce the number of states needed to represent the climate side of the DICE model without sacrificing its benchmark in capturing the interaction between emissions and temperature increase. Such reduction of the state space is crucial to permit additional state variables needed to capture uncertainty and avoid the curse of dimensionality.[3] To model the adaptation behaviour, we rely primarily on the work by de Bruin et al. (2009), which allows us to separately choose between mitigation and adaptation at every optimisation stage. Check Figure 1 to have a glimpse of the detailed workflow of the model. The equations and processes describing the model are fully described in Appendix A.

2.1 Bringing uncertainty into the picture

As noted, the power of dynamic programming, relative to an alternative such as nonlinear programming, is most evident with stochastic problems. Suppose that the stochastic equation of motion

$$x_{\tau+1} = g(a_\tau, x_\tau, \varepsilon_\tau)$$

replaces the deterministic equation of motion $x_\tau = g(a_\tau, x_\tau)$. Here, ε_τ is the time τ realisation of an independently identically distributed random variable with known distribution. The dynamic programming equation reads now as

$$V^s(x) = \max_a \mathbb{E}_\varepsilon \left\{ U(a, x) + \beta V^{s-1}(g(a, x, \varepsilon)) \right\}$$

[3]We cut the number of state variables almost to half with respect to DICE 2007 and DICE 2013.

The approximation proceeds as in the deterministic case, except now we have to take expectations at every stage. If ε is distributed continuously, Gaussian quadrature presents an efficient approximation to the expectation integral. Gauss- Legendre quadrature once more approximates a probability weighted integral by a weighted sum. The L quadrature nodes and the L weights in the sum are selected to match first $2L$ moments of the distribution

$$\int_Z z^k p(z)dz = \sum_{l=1}^{L} \omega_l x_l^k \quad \text{for } k = 0, \ldots, 2L-1$$

Hence we approximate the expectation

$$\mathbb{E}_\varepsilon \left\{ U(a,x) + \beta V^{s-1}(g(a,x,\varepsilon)) \right\} \approx U(a,x) + \beta \sum_{l=1}^{L} \omega_l V^{s-1}(g(a,x,\varepsilon_l))$$

Given an estimate of the value function at stage $s-1$, $\hat{V}^{s-1}(x) = \phi(x)c^{s-1}$, at stage s we obtain

$$V_j^s = \max_a U(a,x) + \beta \sum_{l=1}^{L} \omega_l \phi(g(a,x_j,\varepsilon_l))c^{s-1}$$

We calculate the stage s basis coefficients c^s as described in the (Value) Function approximation method in García-León (2015) to obtain an estimate of the stage s value function, $\hat{V}^s(x) = \phi(x)c^s$, and proceed to stage $s+1$. This strategy goes on recursively up to a desired break criterion for the vector of coefficients, c.[4]

2.2 The Bellman equation

An optimal decision under uncertainty has to anticipate all possible future realizations of the random variables together with the corresponding optimal future responses. The Bellman equation reduces the complexity of the decision tree by breaking it up into a trade-off between current consumption utility and future welfare, where future welfare is a function of the climatic and economic states in the next period. The best possible total value of present and future welfare is the so-called value function $V(k_t, M_t, T_t, t)$.[5] In the case of uncertainty, the value function generally relies on additional states summarized in the vector Φ_t, capturing uncertain (possibly formerly) exogenous states. We then would write the value function as $V(k_t, M_t, T_t, \Phi_t, t)$.

[4]We set this break criterion at 10^{-4}.

[5]For numerical considerations, we will work with the normalised version of this Bellman equation.

$$V(k_t, M_t, T_t, \Phi_t, t) = \max_{C_t,\mu_t,p_t} L_t \frac{\left(\frac{C_t}{L_t}\right)^{1-\eta}}{1-\eta}\Delta t + \beta_{t,\Delta t}\,\mathbb{E}\left[V(k_{t+\Delta t}, M_{t+\Delta t}, T_{t+\Delta t}, \Phi_{t+\Delta t}, t+\Delta t)\right] \quad (2.1)$$

Given the value function, we can analyse the control rules and simulate different representations of the optimal policy over time. For the simulation, we either fit a continuous control rule, or we forward-solve the Bellman equation, knowing the value function, starting from the initial state. Under uncertainty, we can quickly simulate a large set of runs and depict statistical properties. Without normalising capital to effective labour units we would need a much larger state space for capital to cover at least a reasonably long time horizon, even without growth uncertainty.

We solve the model using the algorithm presented in García-León (2015). We run the resulting code in Matlab using the compecon optimiser as presented in Miranda and Fackler (2002). Since each of the optimisation at the different Chebychev nodes is independent conditional on the time step, we can compute each of them independently. Hence, we make use of the Parallel Programming Toolbox in Matlab to parallelise that process so that the whole process speeds up nearly 4 times.[6]

3 Uncertain climate sensitivity

A paradigmatic source of uncertainty is that arisen due to imperfect knowledge of the parameters governing the dynamics of the model. We call it parametric (or *epistemic*) uncertainty. The present system is represented by a very large set of parameters. One whose calibration arises more controversy is climate sensitivity. The reader should recall that climate sensitivity is the equilibrium temperature response to doubling of atmospheric CO_2 concentration with respect to preindustrial levels and is represented by s in our model. Despite significant advances in climate science, the "likely" range has been 1.5°C to 4.5°C for over three decades, with a "most likely" value of 3°C. In 2007, the IPCC narrowed the likely range to 2-4.5°C. It reversed its decision in 2013, reinstating the old range. The AR5 also removed the 3°C "most likely" value.

We start with a simple but powerful exercise. We ignore the actual value of climate sensitivity but here we assume a likely distribution to describe its value. In particular, we believe that the *true* climate sensitivity follows from the realisation of a normal variable

$$s \sim N(\mu_s, \sigma_s^2)$$

[6]In a Windows 10, Intel i7-2600 @3.40GHz PC. Matlab R2011a

centered around $\mu_z = 3.08$ and standard deviation $\sigma_z = 2.7\%$.[7] Typically, this parameterisation will yield values belonging to the interval $(1.5, 4.5)$.

We run a set of $N = 100$ simulations projecting the model forward for each of the respective values of climate sensitivity. A description of some of the basic results can be found in Figure 2. The model features essentially the basic properties of the standard AD-DICE model but now the degree of variability of the basic magnitudes increase in response to climate sensitivity uncertainty. As depicted in the various panels of FIgure 2 the main variables of the model behave similarly to the benchmark specification and most of them fluctuate symmetrically around the median values.

Lower (higher) climate sensitivity will decrease (increase) mitigation relative to adaptation. The rationale behind is that if emissions cause less climate change, there will be lower damages. This will lead to lower levels of mitigation and adaptation. These latter results are gently summarised in Figure 3. Specifically, a slow climatic response diverts resources to instantly adapt to climate change in the short-run (more than half of the resources are devoted to adaptation) although the long-term optimal behaviour still yields a stable equilibrium of the mix (60% mitigation - 40% adaptation). Conversely, if high climate sensitivity applies, mitigation is relatively more benefical to combat climate change. As a result, a ratio of 75-25% quickly becomes optimal. In this case, we can find optimal full abatement of emissions in the very long run.

4 Stochastic technology growth

Now we present an analysis of the optimal mix between mitigation and adaptation when major processes governing the dynamics of the model are stochastic. In this area, we can find randomness in shocks affecting the growth of technology, the accumulation of carbon, the evolution of temperatures, etc. In order to preserve computability we will study only one random source of variation.

In the present environment, the most important uncertain variable for climatic outcomes is by far the growth in total factor productivity (TFP). The reason is that TFP is the main driver of economic growth in the long run, and output tends to dominate emissions trends and therefore climate change. In this experiment, we will assume that the rate of technological progress is uncertain. The technology level enters the Cobb–Douglas production function and determines the overall productivity of the economy. A shock in the growth rate permanently affects the technology level in the economy. The technology level A_t in the economy follows the equation of motion

[7] The social planner solves the problem as if she is certain about its value. More precisely, she assumes that climate sensitivity takes a deterministic value of 3.08.

$$\tilde{A}_{t+1} = A_t \exp[\tilde{g}_{A,t}] \qquad \text{with} \quad \tilde{g}_{A,t} = g_{A,0} \exp[\delta_A t] + \tilde{z}_t \tag{4.1}$$

Our set of simulations analyses the consequences of an iid shock

$$\tilde{z}_t \sim N(\mu_z, \sigma_z^2)$$

We set the standard deviation to $\sigma_z = 2.6\%$ which corresponds to twice our initial technology growth rate.

4.1 Normalised Bellman equation under stochastic technology growth

As in the benchmark formulation, it is convenient to normalise the Bellman equation 2.1 to ease the numerical calculations. In this respect, we follow closely Jensen and Traeger (2014). To express variables in effective labour units, we normalise by the deterministic technology level, A^{det}.[8] Its dynamic behaviour responds to the following equation

$$A_{t+1} = A_t^{det} \exp[\bar{g}_{A,t}] \qquad \text{with} \quad \bar{g}_{A,t} = g_{A,0} \exp[\delta_A t]$$

If we define $a_t = \frac{A_t}{A_t^{det}}$ as the deviation of the current technology level away from its respective deterministic value, then

$$\tilde{a}_{t+1} = \frac{\tilde{A}_{t+1}}{A_{t+1}^{det}} = \frac{\exp\left(\tilde{g}_{A,t}\right) A_t}{\exp\left(g_{A,t}\right) A_t^{det}} = \exp\left(\tilde{z}_t\right) a_t$$

which leads to a new normalised Bellman equation

$$V^*(k_t, M_t, T_t, a_t, t) = \max_{c_t, \mu_t, p_t} \frac{c_t^{1-\eta}}{1-\eta} \Delta t + \beta_{t,\Delta t} \mathbb{E}\left[V^*(k_{t+\Delta t}, M_{t+\Delta t}, T_{t+\Delta t}, \tilde{a}_{t+\Delta t}, t+\Delta t)\right]$$

At the same time and, for the sake of comparison between specifications, when calibrating the model, we choose the mean of the technology shock so that the uncertain technology path would match in expected terms the deterministic path. On the one hand, we have that

$$\tilde{A}_{t+j} = \tilde{a}_{t+j} A_{t+j}^{det} = \exp\left(\tilde{z}_{t+j-1}\right) \tilde{a}_{t+j-1} A_{t+j}^{det} =$$

[8] $A^{det} \equiv$ level of technology in the certainty scenario ($z_t = 0$, $\forall t$)

$$= \exp\left(\tilde{z}_{t+j-1} + \tilde{z}_{t+j-2}\right)\tilde{a}_{t+j-2}A_{t+j}^{det} =$$

$$= \exp\left(\sum_{j'=0}^{j-1} \tilde{z}_{t+j'}\right) a_t A_{t+j}^{det}$$

so

$$\mathbb{E}\left[\tilde{A}_{t+j}\right] = \mathbb{E}\left[\exp\left(\sum_{j'=0}^{j-1} \tilde{z}_{t+j'}\right) a_t A_{t+j}^{det}\right] =$$

$$= \exp\left(\sum_{j'=0}^{j-1}\left(\mu_z + \frac{\sigma_z^2}{2}\right)\right) a_t A_{t+j}^{det} =$$

$$= \exp\left(j\left(\mu_z + \frac{\sigma_z^2}{2}\right)\right) a_t A_{t+j}^{det}$$

Hence, if we set $\mu_z = -\frac{\sigma_z^2}{2}$, we have that

$$\mathbb{E}\left[\tilde{A}_{t+j}\right] = a_t A_{t+j}^{det}$$

which is not more than the deterministic value of technology at time $t+j$.

After this renormalisation some equations will difer slightly from those presented in Chapter 2. In particular, the gross product per effective unit of labour now reads

$$y_t^{gross} = a_t^{1-\kappa} k_t^{\kappa}$$

where, it now incorporates the stochastic deviattions of technology away from its deterministic level. Accordingly, yearly CO_2 emissions derived from industrial emission will change so that total CO_2 emissions now follow

$$E_t = (1-\mu_t)\sigma_t A_t^{det} a_t^{(1-\kappa)} L_t k_t^{\kappa} + B_t$$

where B_t represent total emissions from land use change. The rest of equation hold as presented in Chapter 2.

4.2 Results

In Figure 4 we observe the overall results after running 100 simulations with $\sigma_z = 2.6\%$ and $\mu_z = 3.38 \cdot 10^{-4}$.[9] As it emerges from the upper left panel of FIgure 4,[10] technology deviations do not represent a major force to deviate from the optimal mix under a potential deterministic scenario. Only subtle changes in response to transitory deviations are observed in the optimal allocation between mitigation and adaptation. As a consequence, the observed atmospheric CO_2 stock gravitates also around the deterministic values, as noted in 5(b). The bottom panels of FIgure 4 depict the evolution of the stock of capital in our simulated states of the world, which respond directly to the uncertain path of technology

Meanwhile, Figure 5 compares the deterministic optimal path of the mix with the median values resulting from our simulated series. In general, we may state that including uncertainty in the technology level favours adaptation as it derives from the parallel shift of the line downwards. Particular levels of the mix will depend on the magnitude of the shocks.

5 Learning about uncertain climate sensitivity

In this section we will address how optimal decisions vary if individuals learn over the years about the uncertain parameters governing the model. Similarly to Section 3 we will assume that climate sensitivity is not known but in this case we will have a guess of its value.[11] This guess will be updated at each iteration through Bayesian learning once the stock of carbon and temperature are observed. In particular we will assume that the social planner is unsure about the true value of climate sensitivity, s, but holds the following prior

$$\tilde{s}_0 \sim \Pi(s) = \mathcal{N}(\mu_{s,0}, \sigma^2_{s,0}) \tag{5.1}$$

In addition to its uncertain nature due to unknown climate sensitivity, atmospheric temperature is also stochastic, insofar as it responds to random weather fluctuations. These "weather fluctuations" are normally distributed with mean zero. Thus, for a given value of climate sensitivity, s, temperature

[9] This value is chosen so as to mimic on average the behaviour of technology under deterministic conditions.

[10] Each point in these pictures is visually weighted according its probability density, that is, darkest shaded areas represent locations most probably visited whereas lighter areas denote less likely outcomes.

[11] Recall that climate sensitivity captures the equilibrium warming from doubling the CO_2 concentration with respect to preindustrial levels.

behaves according to the following law of motion

$$\tilde{T}_{t+1} = (1-\sigma_{forc})T_t + \sigma_{forc} s \left[\frac{\ln \frac{M_t}{M_{pre}}}{\ln 2} + \frac{EF_t}{\eta_{forc}} \right] - \sigma_{ocean} \triangle T_t + \tilde{\epsilon}_t \tag{5.2}$$

For a given value of s, since ϵ follows a normal distribution, so will do temperature

$$\tilde{T}_{t+1} \sim \mathcal{N}(\mu_{T,t+1}(s), \sigma_T^2)$$

with variance σ_T^2, known and exogenous.[12]

The temperature mean is obtained from taking expectations in equation (5.2) is

$$\mu_{T,t+1} = s\chi_t(M_t, t) + \xi_t(T_t, t)$$

where

$$\chi_t(M_t, t) = \sigma_{forc} \left(\frac{\ln \frac{M_t}{M_{pre}}}{\ln 2} + \frac{EF_t}{\eta_{forc}} \right),$$
$$\xi_t(T_t, t) = (1-\sigma_{forc})T_t - \sigma_{ocean} \triangle T_t$$

Assuming the above prior, and update rule for the prior as well as a predictive rule for temperatures can be obtained.[13] In particular, the mean of the prior at time $t+1$ is

$$\mu_{s,t+1} = \frac{\chi_t^2 \sigma_{s,t}^2 \frac{\hat{T}_{t+1} - \xi_t}{\chi_t} + \sigma_T^2 \mu_{s,t}}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2} \tag{5.3}$$

whereas the variance is updated through

$$\sigma_{s,t+1}^2 = \frac{\sigma_T^2 \sigma_{s,t}^2}{\chi_t^2 \sigma_{s,t}^2 + \sigma_T^2} \tag{5.4}$$

As a result, the decision maker learns faster the lower the temperature stochasticity and the larger the carbon stock. We must add to the state variables governing the model (k, M, T and t) those responsible for updating the climate sensitivity prior $\Pi(s)$, namely, $\mu_{s,t}$ and $\sigma_{s,t}^2$. Meanwhile, the predicitve equation of temperature governs the realisation of temperature in $t+1$ accounting for stochasticity and climate sensitivity uncertainty. More precisely, $\hat{T}_{t+1} \sim \mathcal{N}(\chi_t \mu_{s,t}, \chi_t^2 \sigma_{s,t}^2 + \sigma_T^2)$.

The Bellman equation reads as follows

[12] Empirical estimates suggest annual volatility in global mean temperature in $\sigma_T^2 = 0.042$

[13] See Jensen and Traeger (2013) for the full derivation of these results

$$V^*(k_t, M_t, T_t, t, \mu_{s,t}, \sigma^2_{s,t}) = \max_{c_t,\mu_t,p_t} \frac{c_t^{1-\eta}}{1-\eta}\Delta t + \beta_{t,\Delta t}\,\mathbb{E}\left[V^*(k_{t+\Delta t}, M_{t+\Delta t}, \tilde{T}_{t+\Delta t}, t+\Delta t, \tilde{\mu}_{s,t+\Delta t+}, \tilde{\sigma}^2_{s,t+\Delta t})\right] \tag{5.5}$$

The new Bellman equation has now six state variables: three physical state variables (k, M, T) and three informational variables $(t, \mu_{s,t}, \sigma^2_{s,t})$ that characterise the state of the system.

As in the previous section, we simulate the system forward 300 steps using the optimal control obtained in the estimation phase. In this exercise, we assume that the social planner holds a prior about the mean of climate sensitivity equal to its actual value ($\mu_{s,0} = 3$) but she is unsure about her belief ($\sigma^2_{s,0} = 3$). These beliefs are updated each period so that the social planner gradually learns about how correct are her beliefs through the observation of realised variables. Additionally, each year an exogenous, additive shock ϵ affect global temperature, $\epsilon_t \sim \mathcal{N}(0, \sigma^2_T)$ with $\sigma^2_T = 0.042$. Random fluctuations of temperature will add an extra degree of complexity to how the planner disentangles the actual value of climate sensitivity.

The compendium of simulated optimal mix strategies are depicted in Figure 6. The risk-averse social planner now decides to mitigate relatively more as she is uncertain about whether the desired mitigation level will be able to cope with the expected increase in temperatures. As the prior becomes more certain (decrease in the prior variance shown in Figure 7) the optimal mix returns to more balanced values. However, it does not recover the values shown in the deterministic case. The median behaviour of the optimal mix is shown in Figure 8, where a notable shift in the relative importance of mitigation is observed.

6 Tipping points

The evolution of climate variables entails different sources of uncertainty inherent to the climate system. Despite most climate change models predict an overall robust increase in global temperatures at the end of the present century, it is not excluded that the occurrence of certain climatic phenomena could provoke a series of abrupt, sudden changes in the system that may end being non-reversible. Tipping points are understood as irreversible shifts in system dynamics that occur upon crossing a threshold in the state space. In a climate change context, tipping points have been modelled differently by several authors. Some examples are the work by Lontzek et al. (2012) and Diaz (2015). We will follow the approach by Lemoine and Traeger (2014), in which the social planner does not know the exact location of the threshold. The probability of a tipping point occurring, known as the hazard rate, is endogenous and depends on the evolution of the state variables, which in turn depends on policy choices as well as on the stochastics governing the system. The social planner learns that regions that she has already visited are free of tipping points. Crossing the threshold shifts the world from the *pre-threshold* regime to a *post-threshold* regime with permanently altered system dynamics. Optimal pre- and post-threshold policies together determine the welfare loss triggered by the tipping point.

We evaluate a tipping point of prominent concern in the climate change literature: this tipping point increases the climate feedbacks that amplify global warming, that is, it increases the effect of emissions on temperature. In particular climate sensitivity will shift from 3ºC after doubling CO_2 concentrations in the pre-threshold regime to 4ºC, 5ºC or 6ºC in the post-threshold regime. The new dynamics include melted ice sheets, large methane releases, or disruptive forest ecosystems; lowering temperature would not undo any of these changes. Optimal policy in the pre-threshold regime must consider its effect on both the pre- and post-threshold value functions, but once the state variables cross the threshold, optimal policy depends only on post-threshold dynamics. Therefore, we solve the model recursively, starting with the post-threshold problem and then substituting the solution into the pre-threshold problem

The system passes from the pre-threshold level ($\psi_t = 0$) into the post-threshold regime ($\psi_{t+1} = 1$) when cumulative temperature change $T :_{t+1}$ crosses an unknown threshold $\hat{T}$. We assume a uniform prior distribution for thresholds. This distribution recognises that more warming entails more threshold risk. The uniform distribution for T means that every temperature between the maximum temperature previously reached and an upper bound $\bar{T}$ has an equal chance of being the threshold. The probability of crossing the threshold between periods t and $t+1$ conditional on not having crossed the threshold by time t is

$$h(T_t, T_{t+1}) = \max\left\{\frac{\min\{T_{t+1}, \bar{T}\} - T_t}{\bar{T} - T_t}\right\} \tag{6.1}$$

This expression is the hazard of crossing the tipping point. As the world reaches higher temperatures without reaching a threshold, the social planner learns that the threshold is above the current temperature and updates his beliefs by moving probability density from the newly safe region to the remaining unexplored temperatures.

In the post-threshold world the Bellman equation would be

$$V_1^*(S_t) = \max_{x_t} u(x_t, S_t) + \beta_t \int V_1^*(S_{t+1})dP \tag{6.2}$$

whereas the pre-threshold Bellman equation reads as follows

$$V_0^*(S_t) = \max_{x_t} u(x_t, S_t) + \beta_t \int \left[(1 - h(T_t, T_{t+1}))\, V_0^*(S_{t+1}) + h(T_t, T_{t+1}) V_1^*(S_{t+1}) \right] d\mathbb{P} \tag{6.3}$$

and the usual restrictions. Because of the stochasticity in the equations of motion, we take expectations over the next period's value functions and over the hazard rate (via the integral).[14] Once we have solved for V_1 in equation (6.2), we find V_0 as the fixed point in (6.3).

6.1 Deterministic

The first step of this exercise involves the resolution of the Bellman equation under deterministic conditions. With this experiment we try to estimate the effect that the sole inclusion of this undetermined trigger point may have in the inferred optimal policies, paying special attention to the composition of the optimal mix. We will check whether the social planner insures herself against this potential danger. Accordingly, the Bellman equation that the planner faces takes the form

$$V_0^*(S_t) = \max_{x_t} u(x_t, S_t) + \beta_t \left[(1 - h(T_t, T_{t+1}))\, V_0^*(S_{t+1}) + h(T_t, T_{t+1}) V_1^*(S_{t+1}) \right] \tag{6.4}$$

where V_1^* represents the optimal response in the post-threshold scenario and V_0^* corresponds to the optimal pre-threshold response if we take into account the possibility of an undetermined tipping point in time.[15] We approximate both value functions, feeding the values of V_1^* into the resolution of V_0^* and then simulate the system as if the tipping point never occurs. We also assume expected draws of the weather shock. The mere inclusion of the possibility of tipping points in the model result in an increase of the mitigation motive as shown in Figure 9. In this way, the social planner prevents the occurrence of the tipping point by mitigating relatively more as compared to the benchmark scenario.

6.2 Stochastic temperature

In the next experiment, we include a new source of uncertainty represented by an exogenous random additive shock which impacts global temperatures each period. This shock will be distributed as $\epsilon_T \sim \mathcal{N}(0, \sigma_\epsilon^2)$ with $\sigma_\epsilon^2 = 0.042$[16] Each period, exogenous weather fluctuations affect global atmospheric temperatures and thus, the law of motion of temperatures behaves analogously to equation (5.2). The general Bellman equation described in equation (6.3) applies and temperature is stochastic.

[14]We approximate expectations using a Gauss-Legendre quadrature rule with 8 nodes.
[15]The post-threshold scenario involves an increase of climate sensitivity from 3 to 4.
[16]Further details in Lemoine and Traeger (2014).

In general, and qualitatively similar to the results presented in Section 5, the social planner decides to react to the uncertainty created by both tipping points and stochastic temperatures by favouring emissions abatement with respect to adaptation to climate change impacts. This behaviour is clearly visible in FIgure 10, where an upward parallel shift of the optimal mix curve is observed. Optimal mitigation grows steadily relative to adaptation over the first years until it stabilises in later years around a range of 75% of climate investments.

6.3 Stochastic Damage function

Lastly, we explore the possibility of facing an uncertain damage function. This would be the equivalent of having an imperfect estimate of the functional form of the damage function. In this sense, we enable some deviations in its realisation each period. With this experiment, we try to measure the degree of sensitivity of the social planner against uncertainties in the effect on output of temperature changes. Conceptually, this is very similar to the case where we are unsure about the true value of climate sensitivity but this time the effect is manifested through the damage function. Hence, we modify the shape of the climate damage function and let a multiplicative random shock in temperature intervene each period. The new gross damage function reads

$$GD_t = 1 + b_1(\epsilon_t T_t)^{b_2}, \tag{6.5}$$

where the independent, normally distributed multiplicative shock $\epsilon_t \sim \mathcal{N}(1, \sigma_\epsilon^2)$ with $\sigma_\epsilon^2 = 0.0068$[17] has probability measure $\mathbb{P}$.

The results are qualitatively analogous to those derived with stochastic temperatures as it can be observed in Figure 11. In this case, though, given the calibration of the damage shock, the planner can acommodate easily the variations in the damage function so that results are numerically close to those presented in the deterministic case.

7 Conclusion

Incorporating new features into IAM models is highly desirable but comes at a cost. In particular, it makes most of these models suffer from the curse of dimensionality. To overcome this problem we adapt a recent methodology proposed by Traeger (2014) which casts the well established Nordhaus' DICE model in a recursive way, making it particularly suitable for uncertainty analysis. At the same

[17] See Lemoine and Traeger (2014) for further details on the calibration of this parameter.

time, it reduces the state space to only 4 state variables, thus, making the model accessible to be solved in a regular computer.

Adopting this methodology, we echo the IPCC's call for greater integration of adaptation within Integrated Assessment modelling and extend the original DICE model by incorporating adaptation *à la de Bruin*, that is, specifying adaptation as a separate decision variable. Then we perform a thorough analysis of the optimal balance between mitigation and adaptation under various stochastic scenarios. First, we solve the model for an assorted amount of different climate sensitivities. Recall that climate sensitivity is the reaction of the system, in terms of mean air temperature, to a doubling of the CO_2 concentration. That climate sensitivity is an unknown parameter, reportedly said to be a positive number around 3. We confirm that climate sensitivity is crucial in the way that the system behaves. Very high values cannot be easily accommodated by efficiently mitigating nor adapting damages. On a second exercise, we assume a random path for technology. Technology enters directly into the production function and is reported to be the major source of distortion in the basic properties of the DICE model. Our results suggest that, indeed, technology growth amounts to be a great source of distortion. If we look at the mitigation-adaptation mix, we can infer that adaptation in more efficient dealing with an uncertain technology scenario.

Next, we include the possibility of dealing with an unknown climate sensitivity. The planner, though, holds a prior of its value and learns gradually about its certain value through time thanks to the observation of realised climatic variables. Hence, we equip our model with bayesian learning about climate sensitivity. We even add further complications to the planner by enabling temperatures to oscillate randomly each period. Consequently, the observation of realised temperature will be itself imperfect and so will be the updates of our priors. The results show that, the higher the degree of ignorance of the social planner about the true value of climate sensitivity (higher variance), the more will she try to protect herself with the help of more mitigation relative to adaptation.

Lastly, we feed our model with a very interesting feature in the context of climate change: the possibility of crossing a determined (unknown) temperature threshold or *tipping point* after which the dynamics of the system behaves notably different. In our example, this change in the dynamics is manifested by an increase in climate sensitivity from 3 to 4. We analyse the effect in the optimal mix under three different scenarios: deterministic, stochastic temperatures and stochastic damages. The results are all qualitatively similar and all aim at favouring mitigation with respect to adaptation as a method of insurance against potentially future adverse scenarios.

The present study represents a new approach to the dynamic analysis of adaptation to climate change within a simplified recursive IAM model fed with various potential sources of uncertainty. Many other additional features can be further incorporated into this model: uncertainty in the parameters governing the damage function, alternative damage function specifications, persistent effects of technology shocks,... Additionally, different types of adaptation could be jointly modelled. For example, Bosello et al. (2010) construct a more involved framework in which different types of adaptation can be found. In particular, they distinguish between anticipatory adaptation (modelled as a stock variable), reactive adaptation (modelled as a flow variable) and accumulation of reactive adaptation knowledge. These extensions are left for future research papers.

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A The benchmark AD-DICE model

Our IAM combines a growing Ramsey-Cass-Koopmans economy with a simple climate model. It is based on the widespread DICE model by Nordhaus (2008) and its stochastic dynamic programming implementation following Kelly and Kolstad (1999). Our formulation resembles closely that of Traeger (2014), who extends the small set of existing state of the art implementations of stochastic dynamic programming integrated assessment models. His main contribution is to reduce the number of states needed to represent the climate side of the DICE model without sacrificing its benchmark in capturing the interaction between emissions and temperature increase. Such reduction of the state space is crucial to permit additional state variables needed to capture uncertainty and avoid the curse of dimensionality.[18] To model the adaptation behaviour, we rely primarily on the work by de Bruin et al. (2009), which allows us to separately choose between mitigation and adaptation at every optimisation stage. Check Figure 1 to have a glimpse of the detailed workflow of the model.

Exogenous processes

Six exogenous processes derive straight from DICE-2007. The exogenous processes in the **economy** determine population growth, technological progress, the carbon intensity of production, and an abatement cost coefficient. Population L_t simultaneously represents labour. We denote the annual growth rate of labour and technology in period t by $g_{L,t}$ and $g_{A,t}$, respectively. The difference equations defining annual population growth in DICE have the continuous time approximation

$$g_{L,t} = \frac{g_L^*}{\frac{L_\infty}{L_\infty - L_0}\exp(g_L^* t) - 1} \tag{A.1}$$

corresponding to the analytic continuous time solution characterising period t population

$$L_t = L_0 + (L_\infty - L_0)(1 - \exp(-g_L^* t)) \tag{A.2}$$

Here, L_0 denotes the initial and L_∞ the asymptotic population. The parameter g_L^* characterises the speed of convergence from initial to asymptotic population.

The technology level A_t in the economy grows at an exponentially declining rate

$$g_{A,t} = g_{A,0}\exp(-\delta_A t) \tag{A.3}$$

[18]We cut the number of state variables almost to half with respect to DICE 2007 and DICE 2013.

leading to the analytic continuous time solution

$$A_t = A_0 \left(\exp g_{A,0} \frac{1 - \exp(-\delta_A t)}{\delta_A} \right) \tag{A.4}$$

The DICE model assumes an exogenous decrease of the carbon intensity of production. The decarbonization factor of production grows at the (decreasing) rate $g_{\sigma,t} = g_{\sigma,0} \exp(-\delta_\sigma t)$, leading to the continuous time representation

$$\sigma_t = \sigma_0 \left(\exp g_{\sigma.0} \frac{1 - \exp(-\delta_\sigma t)}{\delta_\sigma} \right) \tag{A.5}$$

In addition, the economy can pay for abating emissions. The abatement cost coefficient Ψ_t falls exogenously over time and is given by

$$\Psi_t = \frac{\sigma_t}{a_2} a_0 \left(1 - \frac{1 - \exp(g_\Psi^* t)}{a_1} \right) \tag{A.6}$$

The parameter a_0 denotes the initial cost of the backstop (in 2005), a_1 denotes the ratio of initial over final backstop, and a_2 denotes the cost exponent. The rate g_Ψ^* captures the speed of convergence from the initial to the final cost of the backstop.

The exogenous processes on the **climate** side of DICE govern non-industrial CO_2 emissions and radiative forcing from non-CO_2 greenhouse gases. In addition, our state space reduction introduces an exogenous process governing the removal of excess carbon from the atmosphere and the cooling due to the ocean's heat capacity. DICE assumes an exponential decline of CO_2 emissions from land use change an forestry

$$B_t = B_0 \exp(-\delta_B t) \tag{A.7}$$

Non-CO_2 greenhouse gases are exogenous to the model and cause the radiative forcing

$$EF_t = EF_0 + 0.01(EF_{100} - EF_0) \times \min\{t, 100\} \tag{A.8}$$

Endogenous equations of motion

In its benchmark formulation, the model is endowed with four state variables, namely, produced capital K_t, the stock of atmospheric carbon M_t, atmospheric temperature T_t, and time t. Including time as a state variable is particularly convenient for a number of reasons: it makes it possible to contract the Bellman equation to an arbitrary precision and, more importantly, enables us to solve the model for an infinite time horizon with an arbirary time step.[19]

The production equation of the economy is a fairly standard Cobb-Douglas production function depending on capital K_t, labour L_t, and labour augmenting technology A_t. Hence, the gross (potential) output would amount to

$$Y_t^{gross} = (A_t L_t)^{1-\kappa} K_t^{\kappa} \tag{A.9}$$

where κ represents the share of capital in production. Given the current formulation, capital would grow by an order if magnitude over the centuries, which would result in a poor approximation of K_t along a constant, discrete grid. We therefore proceed by normalising capital and consumption in per effective labour units by defining

$$k_t = \frac{K_t}{A_t L_t} \quad \text{and} \quad c_t = \frac{C_t}{A_t L_t}$$

yielding our labour effective gross production $y_t^{gross} = k_t^{\kappa}$. Net production follows from gross production by subtracting abatement expenditure and climate damages

$$y_t = \frac{1 - \Lambda(\mu_t)}{1 + D(T_t)} k_t^{\kappa} = \frac{1 - \Psi_t \mu_t^{a_2}}{1 + D(T_t)} k_t^{\kappa} \tag{A.10}$$

One of the foundational components of the economic model of IAMs is the climate "damage function", which specifies how temperatures or other aspects of climate affect economic activity. For example, in the DICE model, the damage function is of the form

$$D(T_t) = \frac{1}{1 + b_1 T^{b_2}} \tag{A.11}$$

DICE calibrates the b parameters to match cross-sectional estimates of climate damages reviewed in Tol (1999) and then adjusts damages up by 25% to incorporate non-monetised damages, such as impacts on bio-diversity, and to account for potentially catastrophic scenarios, such as sea level rise, changes in ocean circulation, and accelerated climate change.[20] The DICE/RICE models use this common proportional damage function for the entire world.

Net production not consumed is invested in capital, implying the equation of motion

[19] Bear in mind that Nordhaus' DICE model is solved within a 10-year time step. However, given our time flexibility, we will calibrate and solve the model in a more illustrative 1-year time step.

[20] See Nordhaus and Sztorc (2013) for further details

$$k_{t+\Delta t} = [(1-\delta_k)\Delta t k_t + y_t \Delta t - c_t \Delta t] \exp[-(g_{A,t} + g_{L,t})\Delta t] \tag{A.12}$$

where δ_k is the annual rate of capital depreciation.

Anthropogenic emissions are the sum of industrial emissions and emissions from land use change and forestry B_t.

$$E_t = (1-\mu_t)\sigma_t A_t L_t k_t^\kappa + B_t \tag{A.13}$$

Industrial emissions are proportional to gross production $A_t L_t k_t^\kappa$, and the emission intensity of production σ_t, and they are reduced by the emission control rate μ_t. The flow of CO_2 emissions accumulates in the atmosphere. Atmospheric carbon in the next period is the sum of preindustrial carbon M_{pre}, current excess carbon in the atmosphere $M_t - M_{pre}$ net of its (natural) removal, and anthropogenic CO_2 emissions

$$M_{t+\Delta t} = M_{pre} + (M_t - M_{pre})(1-\delta_{M,t})\Delta t + E_t \Delta t \tag{A.14}$$

The pre-industrial emission stock M_{pre} is the steady state level in the absence of anthropogenic emissions. Equation (A.14) is our approximation to the carbon cycle in DICE-2007.

The atmpospheric temperature change is a delayed response to radiative forcing

$$F_{t+\Delta t} = \eta_{forc} \frac{\ln \frac{M_{t+\Delta t}}{M_{pre\ ind}}}{\ln 2} + EF_t \tag{A.15}$$

which is the sum of the forcing caused by atmospheric CO_2 and the non-CO_2 forcing that follows the exogenous process EF_t. Note that the forcing parameter η_{forc} contains the climate sensitivity parameter, which characterises the equilibrium warming response to a doubling of preindustrial CO_2 concentrations. The temperature state's equation of motion is

$$T_{t+\Delta t} = (1-\sigma_{forc})T_t + \sigma_{forc}\frac{F_{t+\Delta t}}{\lambda} - \sigma_{ocean}\Delta T_t \tag{A.16}$$

The parameter σ_{forc} captures the warming delay and σ_{ocean} quantifies the ocean cooling in a given time step that derives from the atmospheric ocean temperature difference ΔT_t. This last term in equation (A.16) replaces the oceanic temperature state in DICE-2007.

Instantaneous adaptation

Adaptation would directly decrease the total damages of climate change. But this reduction comes at a cost. These costs are referred to as protection costs. But adaptation choices are potentially quite different to mitigation decisions and differ in cost. While the original DICE model assumes that adaptation is included in the damage function and is implicitely assumed to be optimal, we rather include adaptation explicitely in the model. Following de Bruin et al. (2009), we model adaptation as a decision by the planner that has some benefits and costs. Accordingly, total damages of climate change are split into the sum of residual damages and protection costs

$$D_t = RD_t(GD_t, p_t) + PC_t(p_t) \tag{A.17}$$

where residual damages RD_t are the "unprotected" part of total damages[21]

$$RD_t = GD_t(1 - p_t)$$

whereas gross damages amount to

$$GD_t = 1 + b_1 T_t^{b_2},$$

and protection costs take the form

$$PC_t = \gamma_1 p_t^{\gamma_2}$$

with p_t being the optimal level of protection chosen each period. In this setup, optimal mitigation and adaptation are jointly modelled and both decisions are separable. In this setup, adaptation and mitigation will behave as economic substitutes. In the original DICE model, mitigation is set by the marginal damage cost. In this framework, the adaptation level is chosen so as to minimize net damages plus adaptation costs, while the mitigation level is chosen to minimize the aggregate of net damages and adaptation costs plus mitigation costs.

The Bellman equation

An optimal decision under uncertainty has to anticipate all possible future realizations of the random variables together with the corresponding optimal future responses. The Bellman equation reduces the complexity of the decision tree by breaking it up into a trade-off between current consumption utility and future welfare, where future welfare is a function of the climatic and economic states in the

[21] We can play along with another alternative specifications of the damage function. For example, $RD_t = \frac{GD_t}{p_t}$.

next period. The best possible total value of present and future welfare is the so-called value function $V(K_t, M_t, T_t, t)$.[22] In the case of uncertainty, the value function generally relies on additional states summarized in the vector Φ_t, capturing uncertain (possibly formerly) exogenous states. We then would write the value function as $V(K_t, M_t, T_t, \Phi_t, t)$.

$$
\begin{aligned}
&V(K_t, M_t, T_t, \Phi_t, t) = \max_{C_t, \mu_t, p_t} L_t \frac{\left(\frac{C_t}{L_t}\right)^{1-\eta}}{1-\eta} \Delta t + \exp\left(-\delta_u \Delta t\right) \mathbb{E}\left[V(K_{t+\Delta t}, M_{t+\Delta t}, T_{t+\Delta t}, \Phi_{t+\Delta t}, t + \Delta t)\right] \\
&\text{s.t.} \\
&K_{t+\Delta t} = [(1 - \delta_k A_t)K_t + y_t A_t - c_t A_t] \exp[-(g_{A,t} + g_{L,t})A_t] \\
&M_{t+\Delta t} = M_{pre} + (M_t - M_{pre})(1 - \delta_{M,t} A_t) + E_t A_t \\
&T_{t+\Delta t} = (1 - \sigma_{forc})T_t A_t + \sigma_{forc} \frac{F_{t+\Delta t}}{\lambda} A_t - \sigma_{ocean} \Delta T_t A_t + \tilde{\epsilon}_t \\
&0 \le \mu_t \le 1 \\
&0 \le p_t \le 1 \\
&0 \le C_t \le Y_t
\end{aligned}
\tag{A.18}
$$

Given the value function, we can analyse the control rules and simulate different representations of the optimal policy over time. For the simulation, we either fit a continuous control rule, or we forward-solve the Bellman equation, knowing the value function, starting from the initial state. Under uncertainty, we can quickly simulate a large set of runs and depict statistical properties. Without normalising capital to effective labour units we would need a much larger state space for capital to cover at least a reasonably long time horizon, even without growth uncertainty.

[22] For numerical considerations, we will work with the normalised version of this Bellman equation. See Appendix A for details.

B Parameters

Table 1: Parameters of the model (Economic)

	Economic parameters	
η	-2	Intertemporal consumption smoothing preference
b_1	0.284%	Damage coefficient
b_2	2	Damage exponent
γ_1	0.115	Protection coefficient
γ_2	3.6	Protection exponent
β_p	10%	depreciation of Stock of Adaptation
α	20%	percentage of unavoidable damage
r	1.43	Stock of Adaptation's discount factor
δ_u	1.5%	Pure rate of time preference per year
L_0	6514	In millions, population in 2005
L_∞	8600	In millions, asymptotic population
g_L^*	3.5%	Rate of convergence to asymptotic population
K_0	137	In trillion 2005-USD, initial global capital stock
δ_K	10%	Depreciation rate of capital per year
κ	0.3	Capital elasticity in production
A_0	0.0058	Initial labor productivity; corresponds to total factor productivity of 0.02722 used in DICE
$g_{A,0}$	1.31%	Initial growth rate of labor productivity, corresponds to total factor productivity of 0.9% used in DICE, per year
δ_A	0.1%	Rate of decline of productivity growth rate per year
σ_0	0.1342	CO_2 emissions per unit of output in 2005
$g_{\sigma,0}$	-0.73%	Initial rate of decarbonization per year
δ_σ	0.3%	Rate of decline of the rate of decarbonization per year
a_0	1.17	Cost of backstop in 2005
a_1	2	Ratio of initial over final backstop cost
a_2	2.8	Cost exponent
g_Ψ^*	-0.5%	Rate of convergence from initial to final backstop cost

Table 2: Parameters of the model (Climatic)

Climatic parameters		
T_0	0.76	In $^\circ\mathrm{C}$, temperature increase of preindustrial in 2005
M_{pre}	596	In GtC, preindustiral stock of CO_2 in the atmosphere
M_0	808.9	In GtC, stock of atmospheric CO_2 in 2005
$\delta_{M,0}$	1.4%	initial rate of CO_2 removal from the atmosphere per year
$\delta_{M,\infty}$	0.4%	Asymptotic rate of CO_2 removal from the atmosphere per year
δ_M^*	1%	Rate of convergence to asymptotic rate of atmospheric CO_2 removal
B_0	1.1	In GtC, initial CO_2 emissions from LUCF
δ_B	1.05%	Growth rate of CO_2 emission from LUCF per year
s	3.08	Climate sensitivity (equilibrium temperature response to doubling of atmospheric CO_2 concentration w.r.t. preindustrial)
η_{forc}	3.8	Forcing of CO_2-doubling
λ	1.23	Ratio of forcing to temperature increase under CO_2-doubling
EF_0	-0.06	External forcing in year 2000
EF_{100}	0.3	External forcing in year 2100 and beyond
σ_{forc}	3.2%	Warming delay, heat capacity atmosphere, annual
σ_{ocean}	0.7%	Parameter governing oceanic temperature feedback, annual

C Figures

Figure 1: AD-DICE model workflow

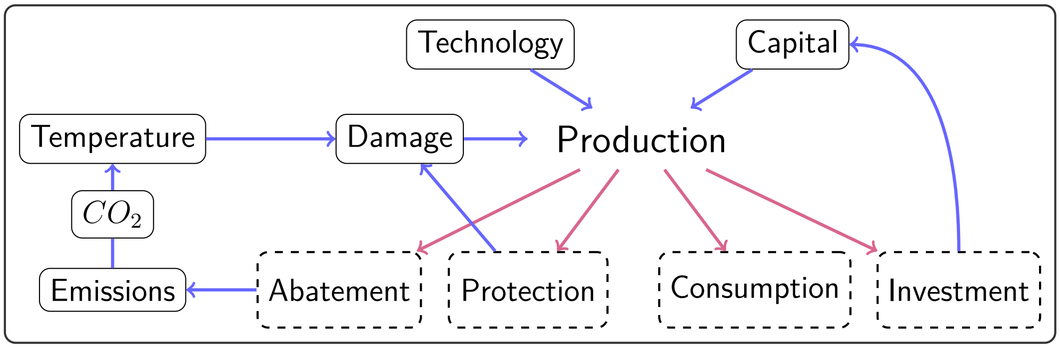
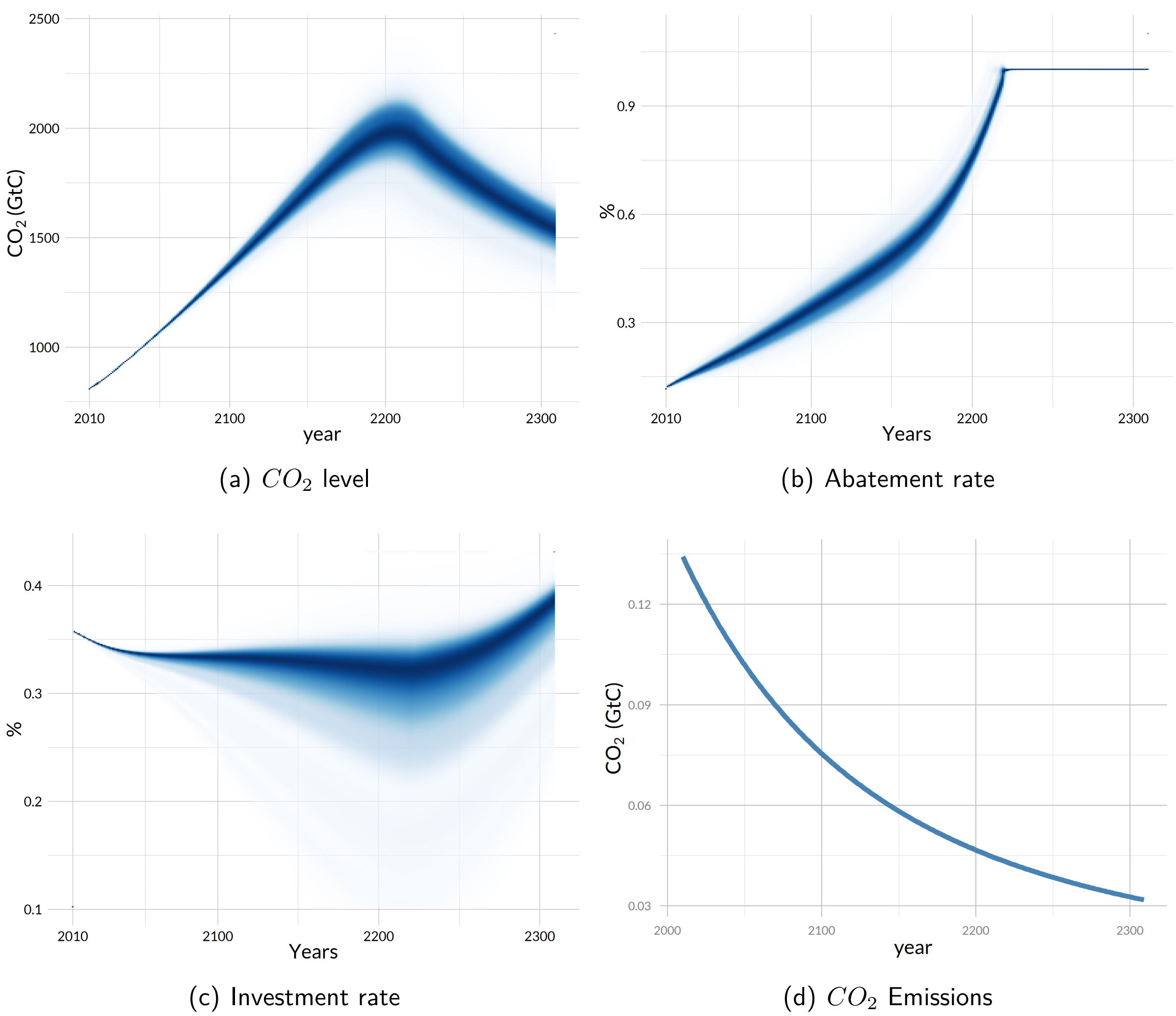


Figure 2: AD-DICE (uncertain climate sensitivity)



(a) CO_2 level

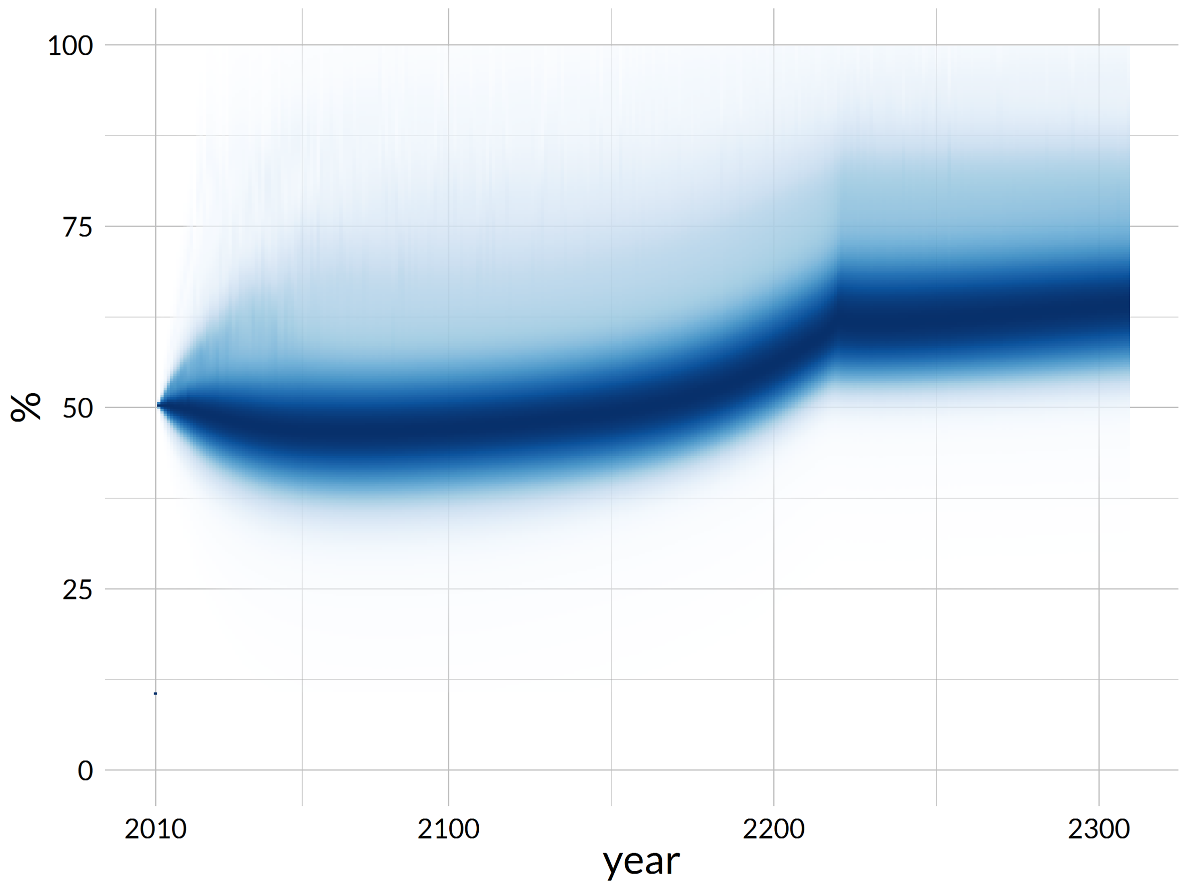
(b) Abatement rate

(c) Investment rate

(d) CO_2 Emissions

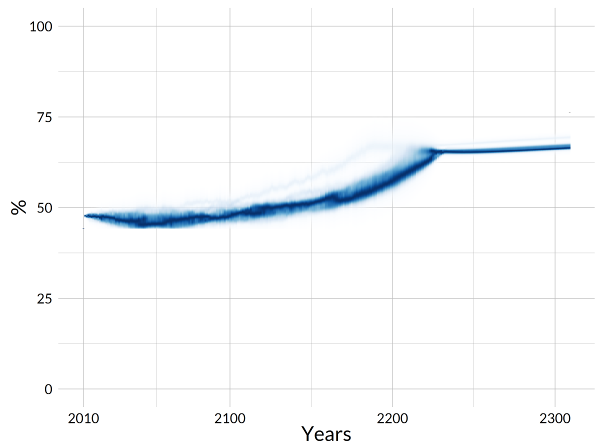
All pictures feature shaded areas according to probability density.

Figure 3: Mitigation-Adaptation mix (uncertain climate sensitivity)

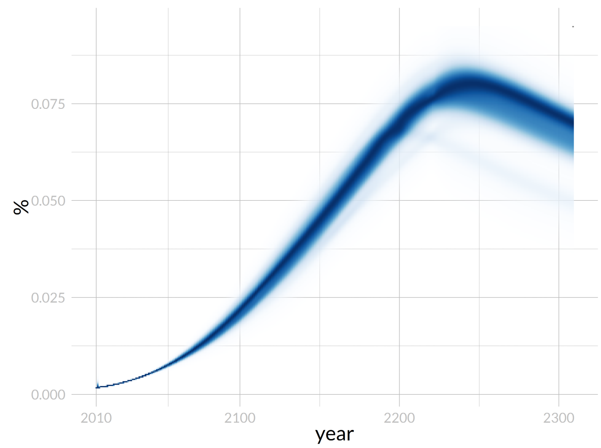


*This figure depicts the median ($N = 100$) optimal response of the social planner. The mix is defined as [mitigation/(mitigation+adaptation)]*100.*

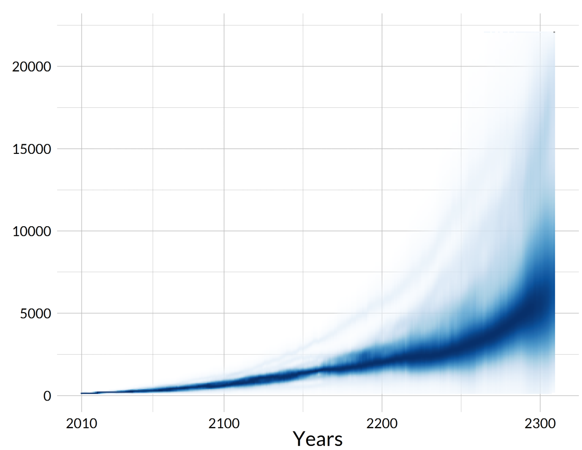
Figure 4: AD-DICE (uncertain technology)



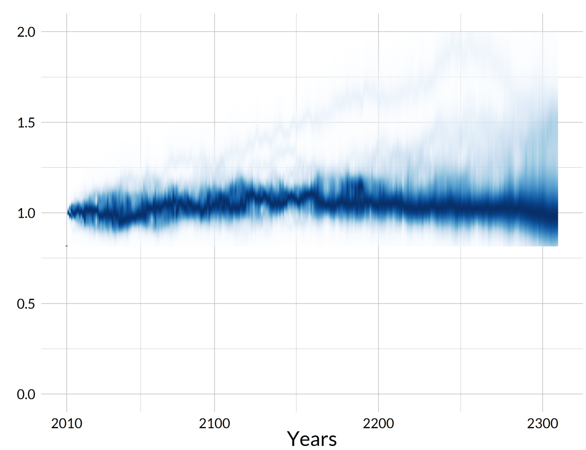
(a) Mitigation-Adaptation mix



(b) CO_2 level



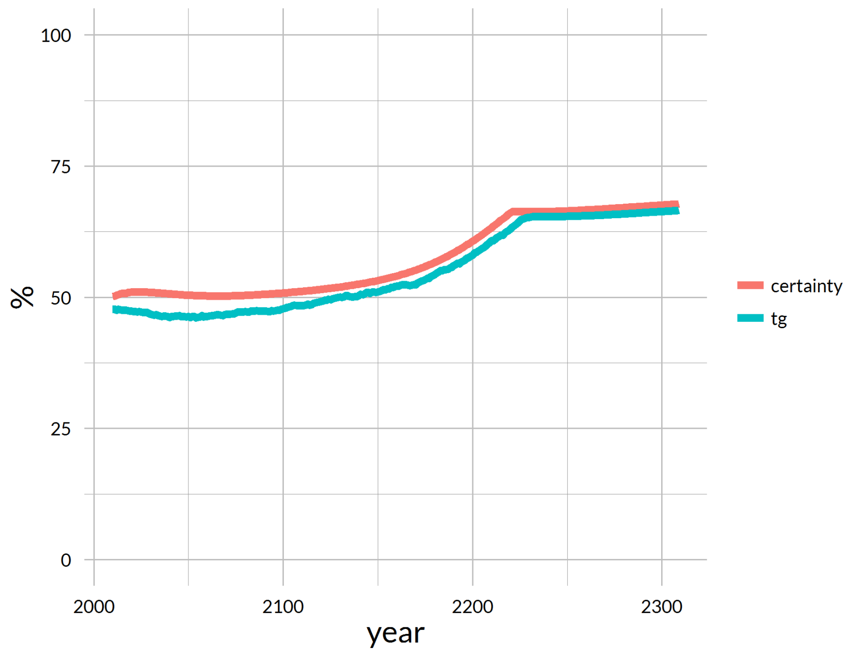
(c) Investment rate



(d) CO_2 Emissions

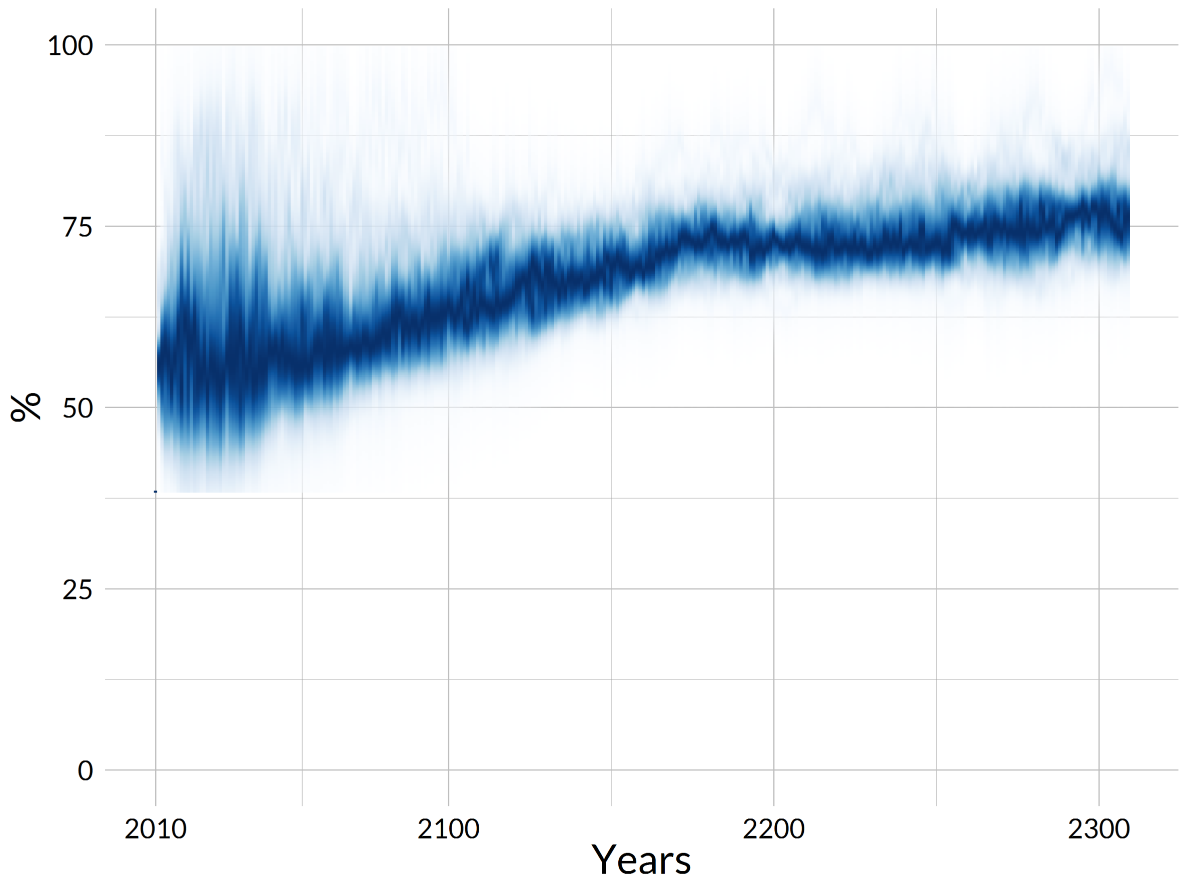
*All pictures feature shaded areas according to probability density. The mix is defined as [mitigation/(mitigation+adaptation)]*100.*

Figure 5: Mitigation-Adaptation mix (deterministic versus stochastic technology growth)



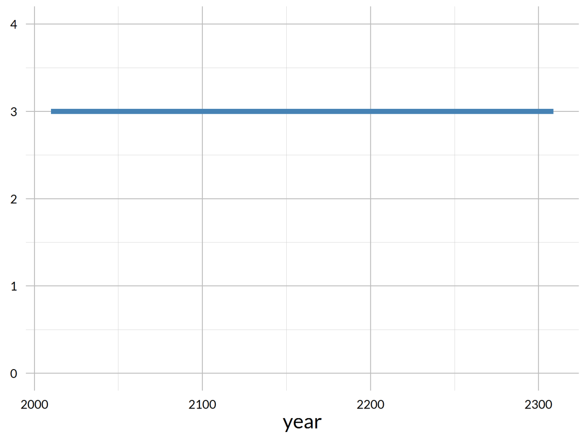
This figure depicts the median $(N = 100)$ *response of the social planner against the optimal response under the benchmark model. Each period technology deviates from its deterministic path according to an additive shock of standard deviation* $\sigma_z = 2.6\%$ *and mean* $\mu_z = 3.38 \cdot 10^{-4}$.

Figure 6: Mitigation-Adaptation mix (Bayesian learning about climate sensitivity)

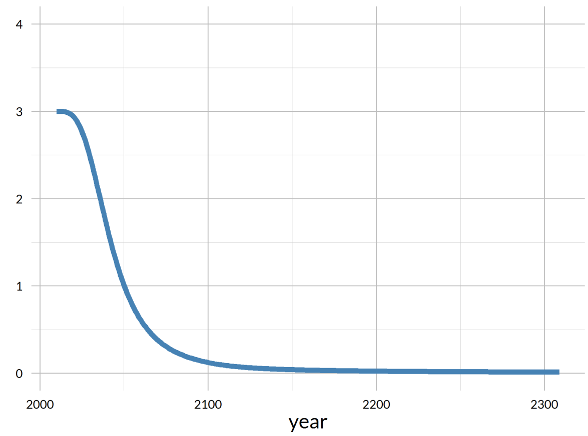


*This figure depicts the median ($N = 100$) optimal response of the social planner. The mix is defined as [mitigation/(mitigation+adaptation)]*100.*

Figure 7: Bayesian learning. Evolution of priors

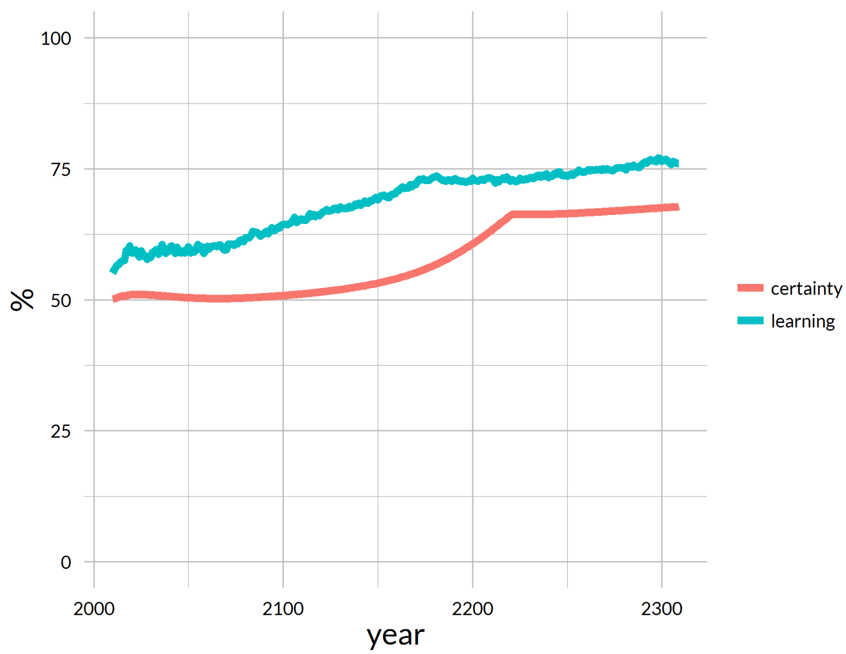


(a) Prior mean



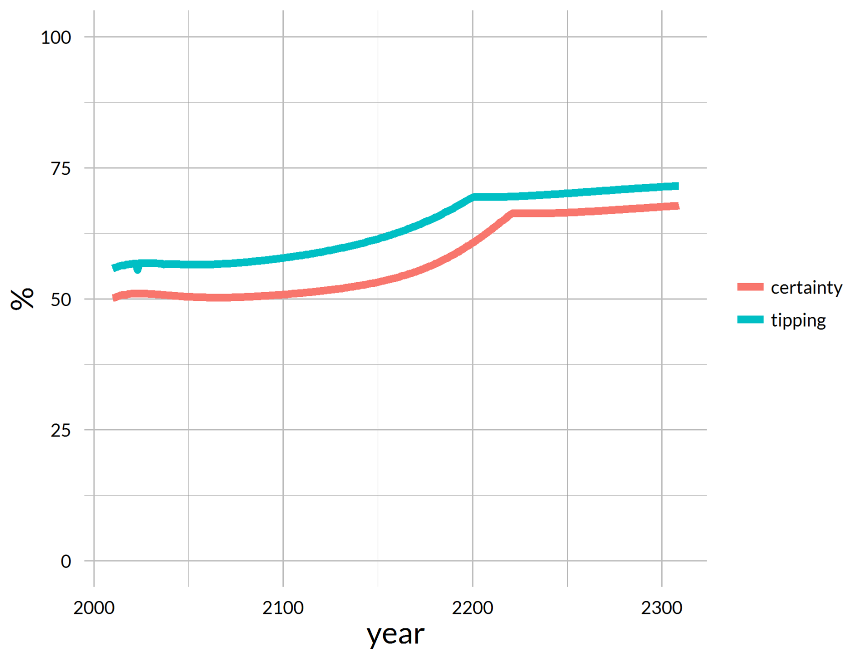
(b) Prior variance

Figure 8: Mitigation-Adaptation mix (deterministic versus bayesian learning)



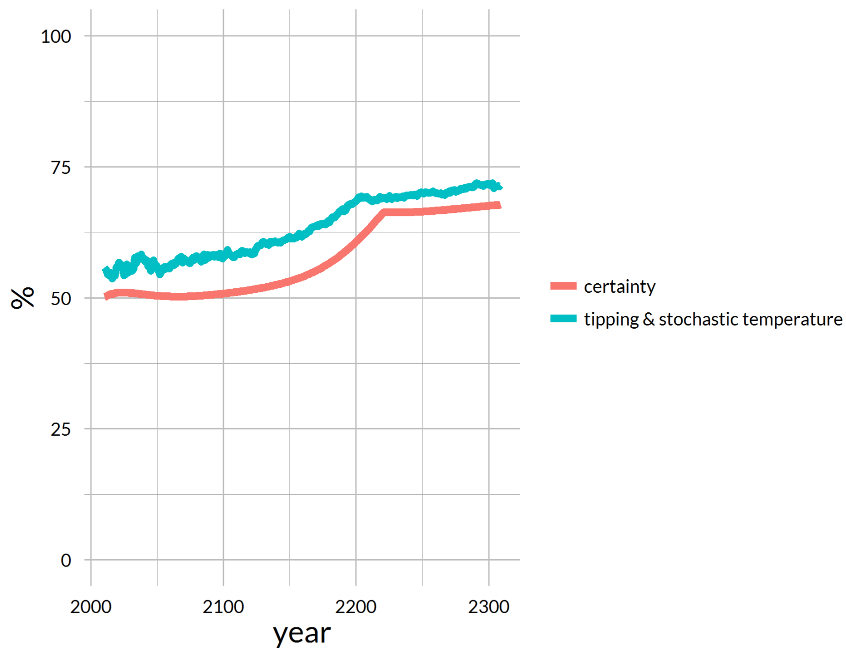
This figure depicts the median ($N = 100$) response of the social planner against the optimal response under the benchmark model. The planner holds an initial prior centered at the true value $\mu_{s,0} = 3$ and variance $\sigma^2_{s,0} = 3$. In addition temperatures oscillates each period in response to a shock of mean 0 and variance $\sigma^2_T = 0.42$.

Figure 9: Mitigation-Adaptation mix (deterministic versus presence of tipping points)



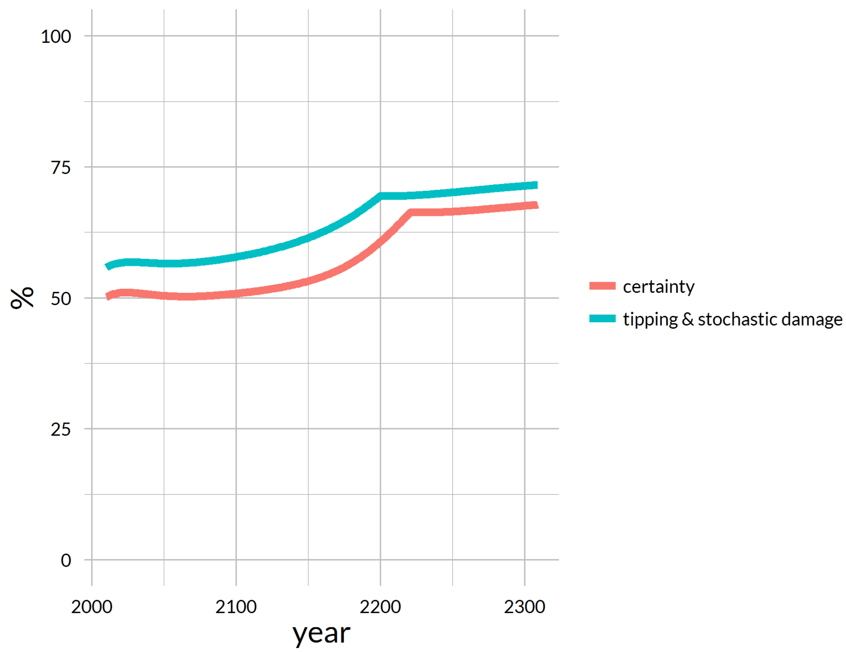
We simulate a path that happens to never cross a threshold in order to see how the social planner adjusts to the possibility over time. Results are for $\overline{T} = 3°\mathrm{C}$.

Figure 10: Mitigation-Adaptation mix (deterministic versus tipping & stochastic temprature)



This figure depicts the median ($N = 100$) response of the social planner against the optimal response under the benchmark model. We simulate a path that happens to never cross a threshold in order to see how the social planner adjusts to the possibility over time. Results are for $\overline{T} = 3°\mathrm{C}$.

Figure 11: Mitigation-Adaptation mix (deterministic versus tipping point & stochastic damage)



This figure depicts the median ($N = 100$) *response of the social planner against the optimal response under the benchmark model. We simulate a path that happens to never cross a threshold in order to see how the social planner adjusts to the possibility over time. Results are for* $\overline{T} = 3°\mathrm{C}$.