③ a) $t(n) = \begin{cases} 1 & n=1 \\ 2\cdot t(n-1)+n+1 & n>1 \end{cases}$

n	1	2	3	4	5
$t(n)$	1	5	14	33	72

$$T(5) = 2\cdot t(4)+5+1 = 2(2\cdot(t(3))+4+1)+5+1 = 2\cdot(2\cdot(2\cdot \overbrace{t(2)}^{2+2+1}+4)+5)+6$$

$$= 2^4 + 2^3\cdot 3 + 2^2\cdot 4 + 2\cdot 5 + 6$$

$\left[\sum_{i=2}^{n}((i+1)\cdot 2^{n-i}) + 2^{n-1}\right]$ $t(1)$ ✓ $t(3)$ ✓ $T(5)$ ✓ $t(2)$ ✓ $t(4)$ ✓] verified

↳ guess

$$t(n) = 2^{n-1} + \sum_{i=2}^{n} i\cdot 2^{n-i} + 2^{n-i} = 2^{n-1} + 2^n \sum \frac{i+1}{2^i}$$

$$= 2^{n-1} + 2^n\left(2^{-n}\left(-n+2^{n+1}-3\right)\right) = 2^{n-1} + 2^{n+1} - n - 3 \left[= 2{,}5\cdot 2^n - n - 3\right]$$ $t(1)$ ✓ $t(2)$ ✓ $T(3)$ ✓

b) base case

$t(1) = 2{,}5\cdot 2^1 - 1 - 3 = 1$ ✓

induction step assume $t(n) = 2{,}5\cdot 2^n - n - 3$

$$t(n+1) = 2\cdot t(n) + (n+1) + 1 = 2{,}5\cdot 2^{n+1} - n - 3 - 1$$

$$t(n) = \frac{2{,}5\cdot 2^{n+1} - 2n - 3 - 3}{2}$$

$$t(n) = 2{,}5\cdot 2^n - n - 3 \quad \blacksquare$$

4) Prove $\log(n!) \in \Theta\, n\log(n) \Rightarrow \log(n!) \in \Omega\, n\log(n)$

$\log(n!) \in O\, n\log(n)$

$\log(n!) \le C \cdot n\log(n)$

$\log(n!) = \log(n \cdot (n-1) \cdot (n-2) \dots) = \log(1) + \log(2 \dots + \log(n) \le \underbrace{\log(n) + \log(n) \dots}_{n \text{ times}}$

$\log(n!) \le C \cdot n\log(n)$ for $N_0 \ge 1$ and $C = 1$

$\therefore \log(n!) \ge n\log(n)$

$n! = n \times (n-1) \dots \times 1 \le n^n$

$\left(\frac{n}{2}\right)^{n/2} \le n! \Rightarrow \frac{n}{2} \log \frac{n}{2} \le \log(n!)$

$\frac{n}{2} \ge \sqrt{n}$

$N_0 \ge 2$

$\frac{n}{2} \log(\sqrt{n}) \le \log(n!)$

$\frac{n}{4} \log(n) \le \log(n!)$

$n\log(n) \le c \log(n!)$ with $C = 4$ and $N_0 = 2$