Q1.1 1) $FV(\text{Pair}(e_1, e_2)) = FV(e_1) \cup FV(e_2)$

2) $FV(\text{Let Pair}(x,y) = e_1 \text{ in } e_2) = FV(e_1) \cup FV(e_2) \setminus \{x, y\}$

Q1.3 1) $[e/x](\text{Pair } e_1, e_2) = \text{Pair}([e/x]e_1, [e/x]e_2)$

2) $[e/x](\text{let Pair}(x,y) = e_1 \text{ in } e_2)$

rename $x \to a$ to avoid confusion

$\Rightarrow [e/x](\text{let Pair}(a,y) = e_1 \text{ in } e_2) =$

$\text{let Pair}(a,y) = [e/x]e_1 \text{ in } [e/x]e_2$, if $\begin{cases} y \neq x \\ a \neq x \\ y \notin FV(e) \\ a \notin FV(e) \end{cases}$

Q1.5 1) $\text{Pair}(e_1, e_2)$:

$$\frac{\Gamma \vdash e_1 : T_1 \qquad \Gamma \vdash e_2 : T_2}{\Gamma \vdash \text{Pair}(e_1, e_2) : (T_1 \times T_2)}$$

2) $\text{let Pair}(x,y) = e_1 \text{ in } e_2$:

$$\frac{\Gamma(x) = T_1 \qquad \Gamma(y) = T_2}{\dfrac{\Gamma, x:T_1, y:T_2 \vdash e_1 : (T_1 \times T_2)}{\dfrac{\Gamma, x:T_1, y:T_2, e_1 : T_1 \times T_2 \vdash e_2 : T}{\Gamma \vdash \text{let Pair}(x,y) = e_1 \text{ in } e_2 : T}}}$$

Q 1.7 1) Pair (e_1, e_2):

$$\frac{e_1 \Downarrow v_1 \qquad e_2 \Downarrow v_2}{\text{Pair}(e_1, e_2) \Downarrow \text{Pair}(v_1, v_2)}$$

2) let Pair $(x, y) = e_1$ in e_2 :

$$\frac{e_1 \Downarrow (v_1, v_2) \qquad [v_1, x]\, e_2 \Downarrow e_2' \qquad [v_2, y]\, e_2' \Downarrow v}{\text{let Pair}(x, y) = e_1 \text{ in } e_2 \Downarrow v}$$

Q 2.1 1) $FV(\text{Fst}(e)) = FV(e)$

2) $FV(\text{Snd}(e)) = FV(e)$

Q 2.2 1) $[e', x](\text{Fst}(e)) = \text{Fst}([e', x]e)$

2) $[e', x](\text{Snd}(e)) = \text{Snd}([e', x]e)$