Q1 a) the worst case would be if the array A would be sorted in decreasing order since every A[j] will have to be swap and for every i g. the I for loop will have to go through the entire array n times, every time putting only the biggest value at its place and leaving all the rest of the array again sorted in decreasing order.

b)/c) :next page

to scan

b)

5 for $i \leftarrow 1$ to $n-1$ do | assi + arith · 2 + cond + comp.

6 for $j \leftarrow 0$ to $n-1-i$ do | comp + 3·arith + cond + assi

3 if $(A[j+1] < A[j])$ then | cond + comp + arith

1 $tmp \leftarrow [j]$ | index + assign

2 $A[j] \leftarrow A[j+1]$ | index + arith + index

2 $A[j+1] \leftarrow tmp$ | assignment + arith + index

repeat $(n-1-i)$ times

repeat $(n-2)$ times

$\cancel{(15\cdot(n-1)+5)(n-2) = T(n)}$

$$T(n) = \sum_{0}^{n-2}\left(5 + 14\sum_{j=0}^{n-1-i} 1\right) = \sum_{i=0}^{n-2}\left(5 + 14n - 14i\right) = 7n^2 + 12n - 19$$

$$= \sum^{n-2} 5 + \sum 14n - 14\sum i$$

$$= (n-1)5 + 14n(n-1) - 14\cdot\frac{(n-2)(n-1)}{2} = \cancel{5n + 10 + 14n^2 - 28n - 1 \mid n^2 - 3n +}$$

$$= 5n - 5 + 14n^2 - 14n - 7(n^2 - 3n + 3) = 7n^2 + 12n - 19$$

c) big-O $g(n) = n^2$

$T(n)$ is $O(g(n))$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = \lim \frac{7n^2 + 12n - 19}{n^2} = \lim \frac{7 + 12\frac{1}{n} - 19\cdot\frac{1}{n^2}}{1} = 7$$

therefore $f(n) \in O(g(n))$ $\qquad 7n^2 + 12n - 19 \leq 7n^2 + 12n^2 = 19n^2$

$f(n) \leq 19 \cdot g(n)$ for all $n \geq 1$ $\qquad c = 19$, $N_0 = 1$

Q.2

Algorithm	Running time in big-Oh notation
Algorithm Example(n) $x \leftarrow 0$ **for** $i \leftarrow 1$ **to** n **do** . $x \leftarrow x + 1$	**O(n)**
Algorithm algo1(n) $i \leftarrow 1$ **while** $i < n$ **do** . $i \leftarrow i + 100$	**O(n)**
Algorithm algo2(n) $x \leftarrow 0$ **for** $i \leftarrow 1$ **to** n **do** . **for** $j \leftarrow 1$ **to** i **do** . $x \leftarrow x + 1$	O(n^2)
Algorithm algo3(n) $i \leftarrow n$ **while** $(i > 1)$ **do** . $i \leftarrow i/2$	$O(\log_2 n)$
Algorithm algo4(n) $k \leftarrow 1$ **for** $i \leftarrow 1$ **to** 1000 . **for** $j \leftarrow 1$ to i . $k \leftarrow (k + i - j) * (2 + i + j)$	O(1)

③ $4^n < n!$ for $n \geq 9$

To scan

base case $n=9$ $\quad 4^9 \approx 2{,}6 \times 10^5 < 9! \approx 3{,}6 \times 10^5$

$4^9 < 9!$ ✓

induction: assume $4^n < n!$

prove $4^{n+1} < (n+1)!$

$4 \cdot 4^n \;\big|\; (n+1) \cdot n! \quad \Rightarrow \quad 4^n < n!$ by induction

$4 < n+1$ for all $(n > 5) \subset n > 9$

$4 \cdot 4^n < (n+1) \cdot n!$ ∎

④ $T(n) = \begin{cases} 1 & \text{if } n=1 \\ 3 \cdot T(n-1)+2 & \text{if } n>1 \end{cases}$

n	t(n)		
1	1		1
2	$3 \cdot 1+2$	$3 \cdot t(1)+2$	5
3	$3 \cdot 5+2$	$3 \cdot t(2)+2$	17
4	$3 \cdot 17+2$	$3 \cdot t(3)+2$	53

4, 12, 36

$4 \times 3^{(n-1)}$

$1 + 4 + 12 + 36$

$1 + \sum_{i=0}^{n-1} 4 \cdot 3^i = 1 + 4 \cdot \frac{3^n - 1}{3-1} = 1 + 2 \cdot 3^{n-1} - 2$

$4 + 4 \cdot 3 + 4 \cdot 3 \cdot 3$

$T(n) = 1 + 2 \cdot 3^{n-1} - 2$ ✓

Q5. prove $25n+5$ is $O(n)$

$25n+5 \leq 25n+5n = 30n = c \cdot n$ if $n \geq 1$

since we have a value for c and N_0

$25n+5 \leq 30 \cdot n$ if $n \geq 1$ $c=30$, $n=1$ $25n+5$ is $O(n)$

Q6 prove $(n+10)^{2,5}+n^2+1$ is $O(n^{2,5})$

$(n+10n)^{2,5}+n^2+1 \leq (n+10n)^{2,5}+n^{2,5}+n^{2,5} = 13n^{2,5}$ if $n \geq 1$

there exist a value for c and N_0

such that $(n+10)^{2,5}+n^2+1 \leq c \cdot n^{2,5}$ for $N_0=1$ and $c=13$

Q7 prove $(n+1)^2$ is not $O(n)$: assuming it is

then there exist a c and an N_0 such that

$n^2+2n+1 \leq c \cdot n$ if ~~me~~ $n \geq N_0$

$n \leq c$ c is suppose to be constant

~~so for~~

for any value of c there will be a value of $n = c+1$ which contradict the ~~def~~ definition

Q.8:

10 people are told at least 1 have a blue face. In a simpler case we have 2 people, if we take both to have blue face, each of them know that the other has a blue face. On the first day neither are sure what color their face is so they don’t die , because of that they both know they have blue face since if one didn’t have a blue face then surely the other did and he would have known and therefore being the only one left would have killed himself but since neither did, they both know they have blue face. The same principle can be extended to any number of people with the day they realize it being equal to the number of people there is.