

Homework 3

1.

a.

- i. Line 5 – $O(1)$
- ii. Line 6 – $O(1)$
- iii. Line 7 – $O(1)$
- iv. Line 4 – $O(n - \text{delta} - 1)$ or $O(n - \text{delta})$
- v. **Total worst-time** is $O(n - \text{delta})$

b. $O(\lg n - 1) \rightarrow O(\lg n)$

c.

- i. $O(n) - O(\text{delta}) = O(n) - O(n/2) + O(n) - O(n/4) + \dots + O(n) - O(1)$
- ii. $\sum_{i=0}^{\lg n} n - \frac{n}{2^i} = n * \sum_{i=0}^{\lg n} 1 - \frac{1}{2^i} = n * (O(\lg n) - O(1))$
- iii. $= n * (O(\lg n - 1)) = n * O(\lg n) = O(n * \lg n)$

2.

$\Theta(1)$ mid = $\lfloor n/2 \rfloor$
 $T(\frac{n}{2}) \text{ left} = \text{RecursionMystery}(\text{data}[\text{low}, \text{mid}])$
 $T(\frac{n}{2}) \text{ right} = \text{RecursionMystery}(\text{data}[\text{mid}+1, n])$
 $\Theta(1)$ return $\max(\text{left}, \text{right}, \text{data}[\text{mid}+1] - \text{data}[\text{mid}])$

a) $T(n) = 2T(\frac{n}{2}) + \Theta(1)$

b)

$h = \lg n$

$\sum_{i=0}^{\lg n} 2^i \Theta(1) = \Theta(1) \Theta(2^{\lg n})$
 $= \Theta(1) \Theta(n) = \Theta(n)$
 checks out!

c) $T(n) = 2T(\frac{n}{2}) + \Theta(1)$

$c = \log_b(a) = \log_2(2) = 1$
 $f(n) = \Theta(1)$
 $n^c = n$
 $F(n) < \Theta(n^c) = \Theta(n')$
 $f(n) = \Theta(n^{1-c}) = \Theta(n^{1-1}) = \Theta(n^0) = \Theta(1)$
 $T(n) = \Theta(n)$