David Hatcher

COT4400

Homework 2

1. Suppose we have some arbitrary and positive integer n.
   a. For the first loop:
      i. Line 2: x = 1;
      ii. Line 3: y = 0;
      iii. Line 4: True, go into while loop
      iv. Line 5: y = y + x; //y = 1
      v. Line 6: x = 3x; //x = 3
      vi. Current value of x, x = 3, y = 1, x = 2y + 1 = 2(1) + 1 = 3
         1. So after the first loop x = 2y + 1 is true
   b. Suppose that for the k^{th} iteration x = 2y + 1
      i. For the $(k+1)^{th}$ iteration
      ii. Line 5: y_{k+1} = y + x; // y_{k+1} = y + (2y + 1) = 3y + 1
      iii. Line 6: x_{k+1} = 3x // x_{k+1} = 3(2y + 1) = 6y + 3
      iv. With these values we can use algebra to prove that x = 2y + 1 is correct
         1. X_{k+1} = 6y + 3
         2. $2y_{k+1}$ + 1 = 2(3y + 1) = 6y + 2 + 1 = 6y + 3
         3. Thus $x_{k+1} = 2y_{k+1} + 1$ does hold true
   c. Therefore, after every iteration of this loop, x will equal 2y + 1
2. Base Case, n = 11, F(n) = 89, 1.5^n = 86.498
   a. Assume that F(k) >= c(1.5)^k for some k >= 11
      i. F(k+1) = F(k) + F(k-1)
      ii. = c(1.5)^k + c(1.5)^(k-1)
      iii. = c(1.5 +1)(1.5)^(k-1)
      iv. = c(2.5)(1.5)^(k-1)
      v. >= c(2.25)(1.5)^(k-1)
      vi. >= c(1.5)^(k+1)
      vii. = Omega(c(1.5)^(k+1)), for all n >= 11
      viii. Therefore, there exists a constants greater than 0 c and n_0, where c = 1 and n_0 = 11, such that F(n) >= c1.5^n, for all n >= n_0
3. h(n) = O(f(n)g(n)), f(n) = O(j(n)), g(n) = O(k(n))
   a. h(n) = O(f(n))O(g(n))   Reverse envelopment
   b. = O(j(n))O(k(n))   Transitive properties
   c. = O(j(n)k(n))   Envelopment
   d. Therefore, h(n) does equal O(j(n)k(n))