## PNAS

## Supporting Information for

- Nonreciprocal interactions give rise to fast cilium synchronisation in finite systems
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- 6 This PDF file includes:
- Figs. S1 to S2
- 8 Legends for Movies S1 to S2
- 9 Other supporting materials for this manuscript include the following:
- Movies S1 to S2

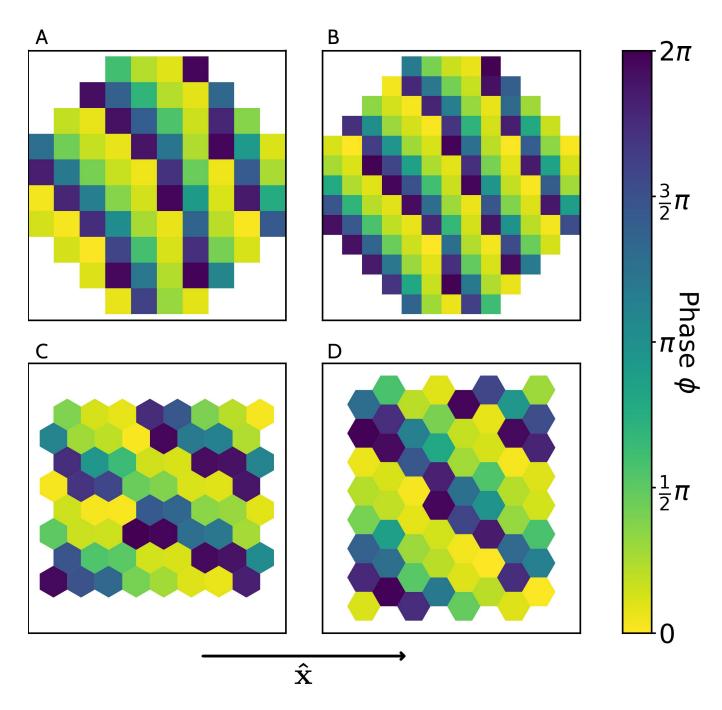


Fig. S1. Emergent metachronal waves on different lattices, with pumping direction in the x-direction (indicated). (A) 64 cilia arranged on a square lattice forming an octagon. (B) 144 cilia in the same arrangement. (C) 64 cilia on a hexagonal lattice, oriented such that one base vector is aligned with the direction of the power stroke. (D) as in (C), but with a lattice that is rotated by 90°.

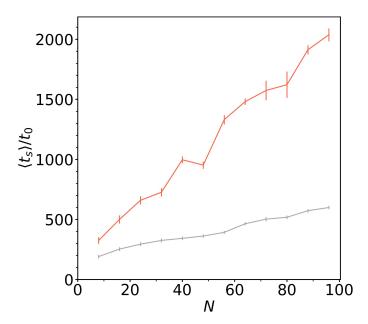


Fig. S2. Synchronisation times on a 1D lattice. The orange line shows cilia arranged along the y-axis (i.e. in the direction of weaker nonreciprocity). The grey line shows cilia arranged along the x-axis, as in Fig. 2C (main text). Synchronisation times still scale linearly when cilia are arranged in the y-direction, but are higher by some numerical factor, as expected from the weaker (but still present) nonreciprocity.

- 11 Movie S1. Animation of a metachronal wave in 2D.
- Movie S2. Emergence of a metachronal wave on a lattice of  $8\times8$  cilia.