CIVE 546 – Structural Design Optimization (SDO)

HW2-Solution

Problem 1

a) The forces in all members of truss are obtained using the method of the joints (Figure 1).

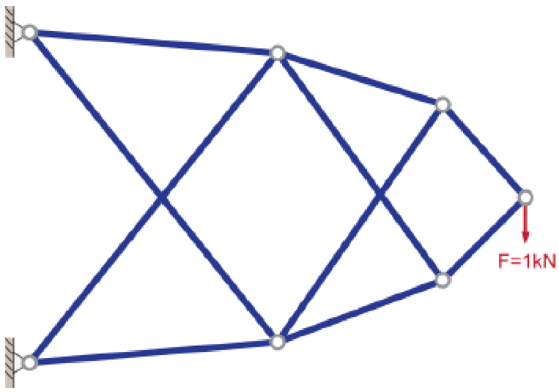

Figure 1. Virtual force

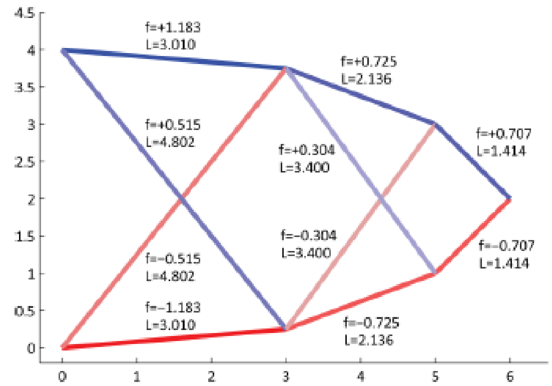

Figure 2. Virtual internal forces

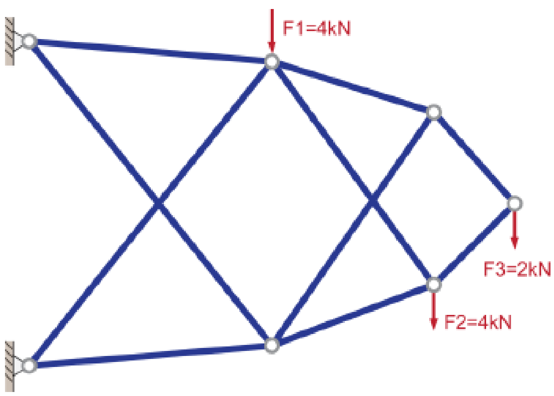

Figure 3. Actual forces

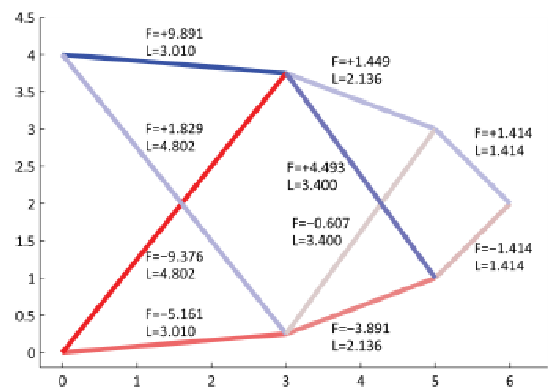

Figure 4. Actual internal forces

b) Virtual forces, actual forces, material and element properties are tabulated as:

Member	f (kN)	F (kN)	L (m)	f FL (kN²m)	A (m²)	E (kN/m²)	EA(kN)
1	-1.183	-5.161	3.010	18.380	0.0900	20000	1800
2	-0.725	-3.891	2.136	6.026	0.0400	20000	800
3	-0.707	-1.414	1.414	1.414	0.0225	20000	450
4	1.183	9.891	3.010	35.225	0.0900	20000	1800
5	0.725	1.449	2.136	2.244	0.0400	20000	800
6	0.707	1.414	1.414	1.414	0.0225	20000	450
7	-0.514	-9.376	4.802	23.144	0.0100	20000	200
8	0.514	1.829	4.802	4.515	0.0100	20000	200
9	-0.304	-0.607	3.400	0.627	0.0100	20000	200
10	0.304	4.493	3.400	4.644	0.0100	20000	200

From the table above

$$\Delta = \sum_{i=1}^{n} \frac{f_i F_i L_i}{A_i E_i} = 0.2111m$$

c) We can rewrite an auxiliary objective function by introducing the Lagrange multiplier as

$$\mathcal{L}(\boldsymbol{A}, \lambda) = \sum_{i=1}^{n} A_i L_i + \lambda \left(\sum_{i=1}^{n} \frac{f_i F_i}{A_i E_i} L_i - \Delta_{max} \right)$$

Based on the necessary condition for extremum, we can compute

$$\frac{\partial \mathcal{L}(\boldsymbol{A}, \lambda)}{\partial A_i} = L_i - \lambda \frac{f_i F_i}{{A_i}^2 E_i} L_i = 0$$

$$\frac{\partial \mathcal{L}(\boldsymbol{A}, \lambda)}{\partial \lambda} = \sum_{i=1}^{n} \frac{f_i F_i}{A_i E_i} L_i - \Delta_{max} = 0$$

Solving two equations in terms of the Lagrange multiplier, we can obtain

$$\lambda = \left[\frac{1}{\Delta_{max}} \sum_{i=1}^{n} \left[\frac{f_i F_i}{E_i} \right]^{0.5} L_i \right]^2 = 11.033$$

d) Optimal cross-sectional areas are

$$A_i^* = \sqrt{\lambda \frac{f_i F_i}{E_i}} = 3.322 \cdot \sqrt{\frac{f_i F_i}{E_i}}$$

$$\boldsymbol{A}^* = [0.0580\ \ 0.0394\ \ 0.0235\ \ 0.0803\ \ 0.0241\ \ 0.0235\ \ 0.0516\ \ 0.0228\ \ 0.0101\ \ 0.0274]^{\mathrm{T}} (m^2).$$

Problem 2

The second moment of inertia and the first moment of area are:

$$I(b,t_w) = 2 \cdot b \cdot t_f \cdot \left(\frac{d-t_f}{2}\right)^2 + \frac{t_w}{12} \cdot \left(d - 2 \cdot t_f\right)^3$$
$$Q(b,t_w) = b \cdot t_f \cdot \left(\frac{d-t_f}{2}\right) + \frac{t_w}{2} \cdot \left(d - 2 \cdot t_f\right)^2$$

a) The objective function is volume:

$$F(b,t_w) = 2 \cdot b \cdot t_f + \left(d - 2 \cdot t_f\right) \cdot t_w$$

The flange must be larger than the web thickness:

$$g_1(b,t_w) = -b + t_w \leq 0$$

The minimum web thickness is $0.25\ in$:

$$g_2(b,t_w) = 0.25 - t_w \leq 0$$

The flange width cannot be negative:

$$g_3(b,t_w) = -b \leq 0$$

Stress due to bending moment has to be lower than σ_y:

$$g_4(b,t_w) = \frac{M}{I(b,t_w)}\left(\frac{d}{2}\right) - \sigma_y \leq 0$$

Stress due to shear has to be lower than τ_y:

$$g_5(b,t_w) = \frac{V \cdot Q(b,t_w)}{t_w \cdot I(b,t_w)} - \tau_y \leq 0$$

Classification:

1. Constraints g_2 and g_3 are box constraints (lower bound)
2. Constraint g_1 is a linear inequality
3. Constraints g_4 and g_5 are nonlinear inequalities

b) Taking the unknown vector to be

$$\mathbf{x} = \begin{Bmatrix} b \\ t_w \end{Bmatrix}$$

The input data for *fmincon* is

$$\underbrace{\begin{bmatrix} -1 & 1 \end{bmatrix}}_{\mathbf{A}} \mathbf{x} \leq \underbrace{0}_{\mathbf{b}}$$
$$\mathbf{A}_{eq} = \emptyset \quad \mathbf{b}_{eq} = \emptyset$$
$$\mathbf{lb} = \begin{bmatrix} 0 & 0.25 \end{bmatrix}$$
$$\mathbf{ub} = \emptyset$$

With g_4 and g_5 defined in a separate nonlinear constraint function. Note that there are no nonlinear equalities, only inequalities.

SectionNLCON.m

```
function [c,ceq]=SectionNLCON(d,b,tw,tf,M,V)

sigma_y = 36;
tau_y = sigma_y / sqrt(3);

[I,Q]=SectionIQ(d,b,tw,tf);
sigma=M/I*(d/2);
tau=V*Q/(I*tw);

c = [ sigma-sigma_y tau-tau_y ]';
ceq = [];

function [I,Q]=SectionIQ(d,b,tw,tf)
I = 2*b*tf*(d/2-tf/2)^2 + tw/12*(d-2*tf)^3;
Q = b*tf*(d/2-tf/2) + tw/2*(d/2-tf)^2;
return
```

The main function to solve the problem is

```
clc, clear, close all

M = 9200; V = 223;
tf = 0.750; d = 24.06;

SectionA=@(h,b,tw,tf)(2*b*tf+(h-2*tf)*tw); % Area function

x0 = [1 1]'; % Starting point
A = [-1 1]; b = 0; % b >= tw
Aeq = []; beq = [];
lb=[0 0.25]'; ub=[];

% options = optimoptions('fmincon','Display','off','Algorithm','active-set');
options = optimset('Display','off','Algorithm','active-set');
[x,fval,flag] = fmincon(@(x)SectionA(d,x(1),x(2),tf),x0,A,b,Aeq,beq,...
            lb,ub,@(x)SectionNLCON(d,x(1),x(2),tf,M,V),options);
if flag<0, error('Optimization encountered problems. Check exitflag.'), end
fprintf('The optimal values are\nb  = %f in\ntw = %f in\n',x);
```

The output for the problem is

```
The optimal values are
b  = 12.741877 in
tw = 0.499635 in
>>
```