McGill University

CIVE 546 — Structural Design Optimization (SDO)

2025 Winter

HOMEWORK #2 — Assigned 01/20/2025;

Recommended completion date: 02/03/2025

Problem 1

Consider a 10-bar truss structure subjected to concentrated forces acting at the nodes (see Figure 1). Size the members in order to minimize the total weight of the truss while satisfying the gap (displacement) constraint at the tip. Truss geometry, loading and boundary conditions are shown in Figure 1.

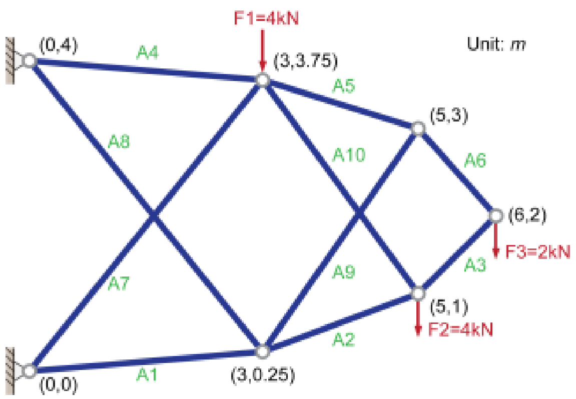


Figure 1. Geometry, loading and boundary conditions.

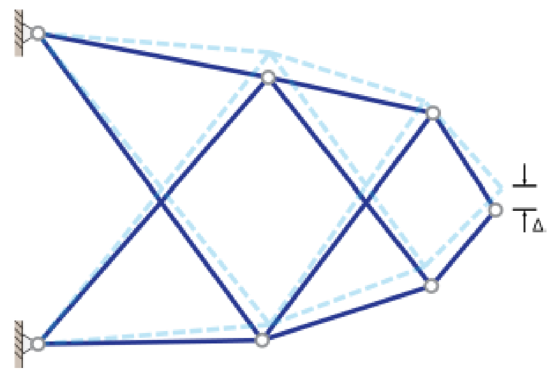


Figure 2. Deformed shape

The tip displacement of the truss can be computed as follows (see class notes):

$$\Delta = \sum_{i=1}^{n} \frac{f_i F_i L_i}{A_i E_i}$$

where f_i denotes an internal virtual force in the *i*-th member – it is obtained by applying a unit virtual force at the tip along the degree of freedom of interest; F_i is an actual internal force in the *i*-th member under the given forces; A_i and E_i are the cross-sectional area and elastic modulus, respectively. In this problem, the elastic modulus $\mathrm{E} = 20000\ \mathrm{kN/m^2}$ (constant) is assumed for all members and the maximum displacement $\Delta_{\max}$ at the tip node is 0.1m. The following initial guess is chosen for the areas:

$$\boldsymbol{A_0} = [0.09\ \ 0.04\ \ 0.0225\ \ 0.09\ \ 0.04\ \ 0.0225\ \ 0.01\ \ 0.01\ \ 0.01\ \ 0.01]^{\mathrm{T}} (m^2).$$

The constrained optimization problem can be formulated as:

$$\min_{\mathbf{A}} \qquad V(\mathbf{A}) = \sum_{i=1}^{n} A_i L_i$$

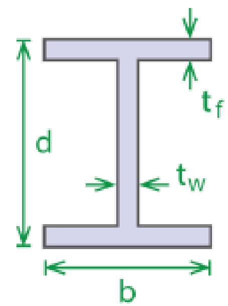
$$\text{s. t.} \quad \sum_{i=1}^{n} \frac{f_i F_i L_i}{A_i E_i} - \Delta_{max} = 0$$

a) Obtain virtual and actual internal forces in the truss.
b) Compute actual displacement at the tip.
c) Obtain the Lagrange multiplier of the optimization problem.
d) Compute the optimum values of the bar areas and volume.
e) Solve the optimization problem using MATLAB and report your result.
f) Play with the initial truss area vector and solve the problem again using MATLAB. How do the above answers change as you change $\boldsymbol{A}_0$? Please comment on the influence of the initial guess.

Problem 2

Given a required moment $M = 9200\ in \cdot kips$ and shear $V = 223\ kips$ that act independently. Optimize the weight (area) of an I-beam with depth $d = 24.06\ in$, flange thickness $t_f = 0.750\ in$, and unknown flange width b and web thickness t_w.

The limit stresses in bending and shear are $\sigma_y = 36\ ksi$ and $\tau_y = \sigma_y/\sqrt{3}$.



$$M = 9200\ in \cdot kips \qquad V = 223\ kip$$
$$b = ? \qquad t_w = ?$$
$$d = 24.06\ in \qquad t_f = 0.750\ in$$
$$\sigma_y = 36\ ksi \qquad \tau_y = \sigma_y/\sqrt{3}$$

In addition, the flange width must be larger than the web thickness, and the minimum web thickness is 0.25 in.

a) Clearly define the objective function and constraints: Separate the box constraints, linear constraints (matrix form) and nonlinear constraints.
b) Using *fmincon* to optimize the I-beam section.

Hint: The shear stress is $v = \frac{VQ}{t\,I}$, where t is the thickness and Q is the first moment of area, that at the neutral axis (maximum shear) is:

$$Q = (b)(t_f)\left(\frac{d - t_f}{2}\right) + \frac{1}{2}(t_w)\left(\frac{d}{2} - t_f\right)^2$$

Problem 2

Start playing with MATLAB. For fun, verify the following properties numerically for any "random" square matrices $[A]$, $[B]$ and $[C]$ <u>of your own choice</u>. Turn in just one (and only one) MATLAB output for each problem below:

a) If $[A]$ is symmetric, the matrix $[D] = [A][B]$ is, in general, not symmetric, even if $[B]$ is also symmetric.
b) If $[A]$ is symmetric, the matrix $[D] = [B]^T[A][B]$ is always symmetric.
c) If $[A][B] = [C][B]$, it does not necessarily follow that $[A] = [C]$.
d) If $[D] = [A][B]$, then $[D]^T = [B]^T[A]^T$
e) $det[A] = \prod \lambda_j$ where λ_j is the j-th eigenvalue of $[A]$ (use help in MATLAB to see det, prod, eig)

Problem 3

Verify if each statement below is true or not. If not true, provide a <u>simple</u> counter-example using MATLAB. The counter-example that you provide should clearly prove the point that you want to make.

a) If $[A][B] = [0]$, a zero matrix, then either $[A]$ or $[B]$ is a zero matrix.
b) $[A][B]^{-1} = [C]$, then $[A] = [C][B]$.
c) If $[A]$ is an 5 x 5 matrix with $det[A] = 10$, then $det(10[A]) = 10\ det[A] = 100$.

Problem 4

Given the unconstrained function

$$f = f(x_1, x_2) = x_1^2 - \ln x_1 + x_2 + \frac{2}{x_2^2}$$

a) Obtain the gradient vector and the Hessian matrix of f.
b) For which values of x_1 and x_2 is the gradient null?
c) Discuss the behavior of f for each of the points computed in (b), e.g. max, min or neither.
d) Plot the function using MATLAB (use **help** in MATLAB to see linspace, meshgrid, surf and contour).