CIVE 546 Structural Design Optimization

(3 units)

Sensitivity Analysis (Continued)
Load Path

Instructor: Prof. Yi Shao

Winter 2025

McGill Poll
Or hands up!

slido

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Select finite difference approximation based on geometric intepretations

ⓘ Start presenting to display the poll results on this slide.

Topology Optimization

General Setup for Density Based Approach

$$\min_{\mathbf{u},\rho_e} \mathbf{f}^T\mathbf{u}$$

Compliance $:= \frac{1}{\text{Stiffness}}$

$$\text{s.t. : } \left(\sum_{e=1}^{N} \rho_e^p \mathbf{K}_e\right)\mathbf{u} = \mathbf{f}\ ,$$

$$\sum_{e=1}^{N} v_e \rho_e \le V, \quad 0 < \rho_{\min} \le \rho_e \le 1, \quad e = 1, \ldots, N\ .$$

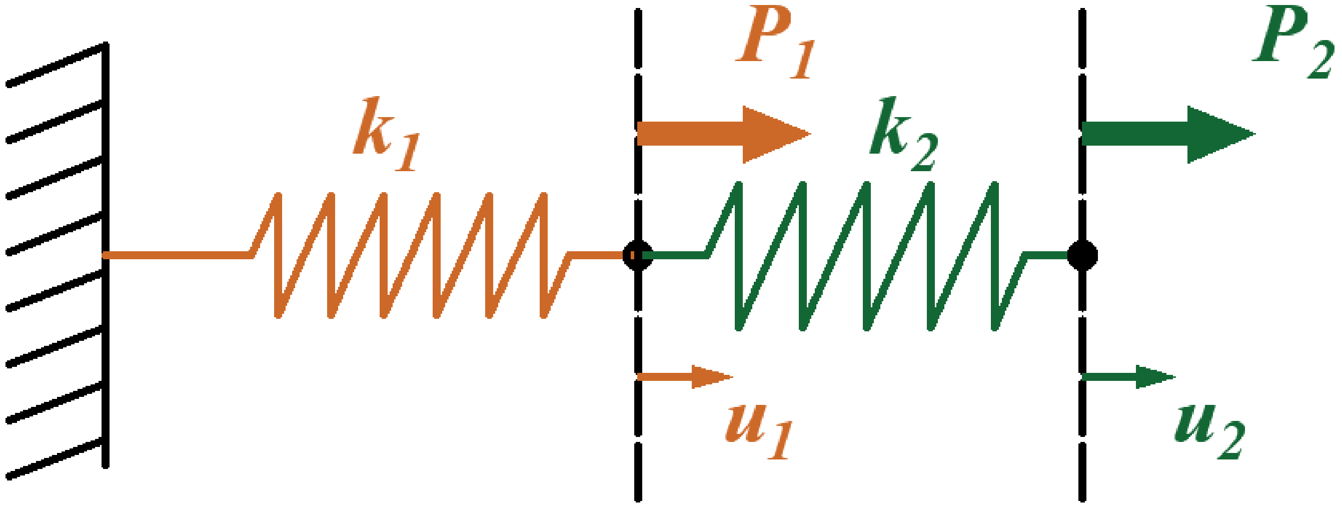
1:20

Given material volume, how can we distribute materials to minimize compliance (i.e., maximize stiffness) ?

Sensitivity Analysis
A powerful example

12:50

Consider a 2 DOF spring system shown below, let's try to determine the sensitivity of the compliance to spring stiffness using (1) analytical method (2) direct differentiation method (3) adjoint method.

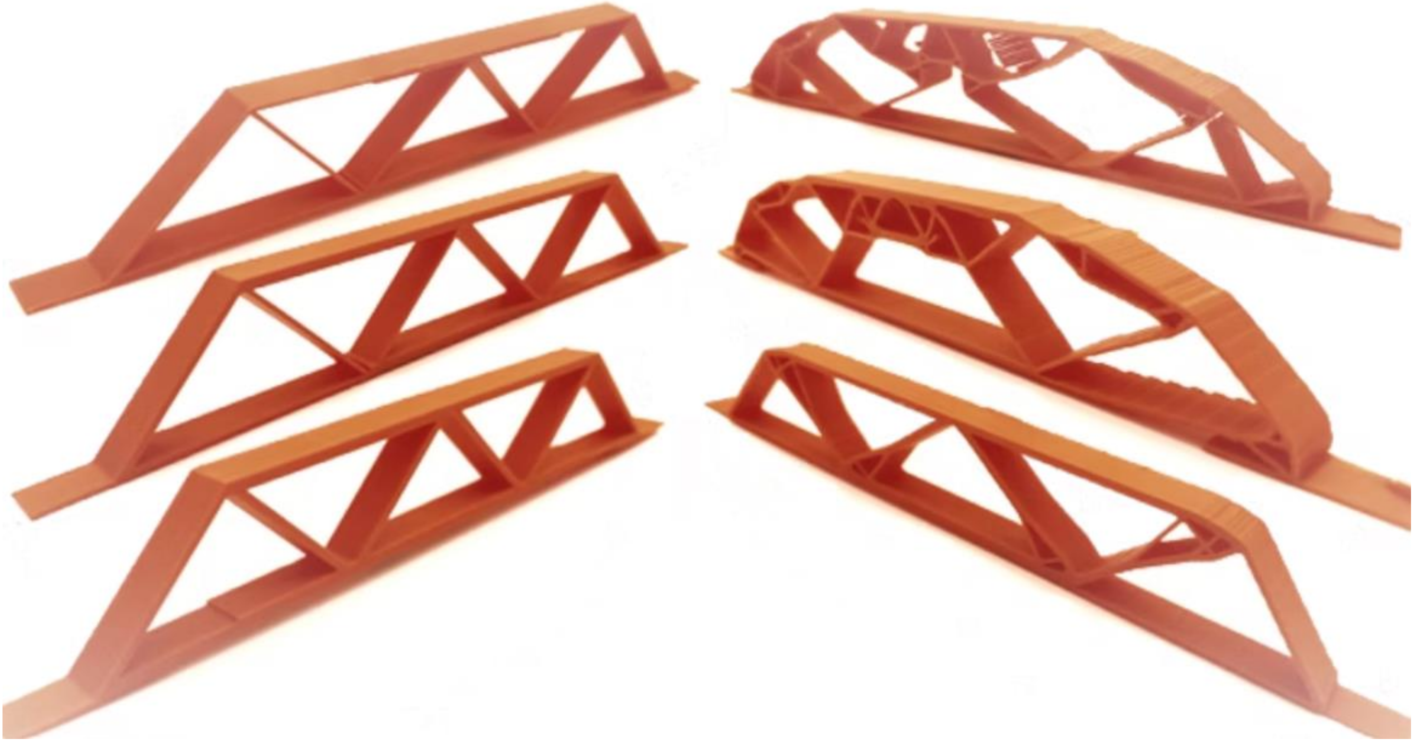


Current value: k_1=1 (N/m), k_2=2, (N/m), P_1=1 (N), P_2=2 (N)
External force P is independent of stiffness.

Administrative announcement

Monday (Mar 17) 1:30pm-3:30pm, MD 497

Guest lecture from **Prof. Josephine Carstensen (MIT)** on Structural Design Optimization



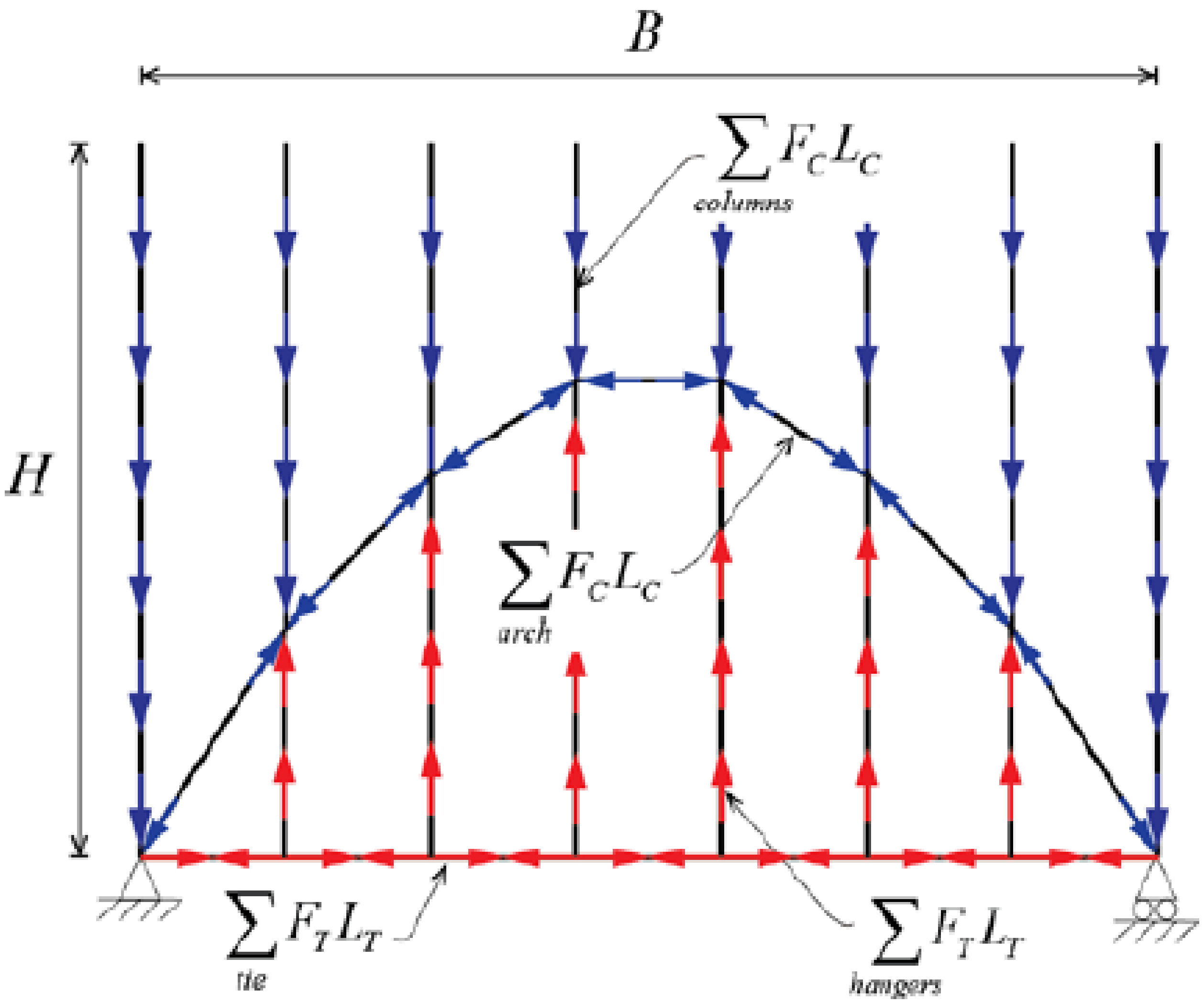
Administrative announcement

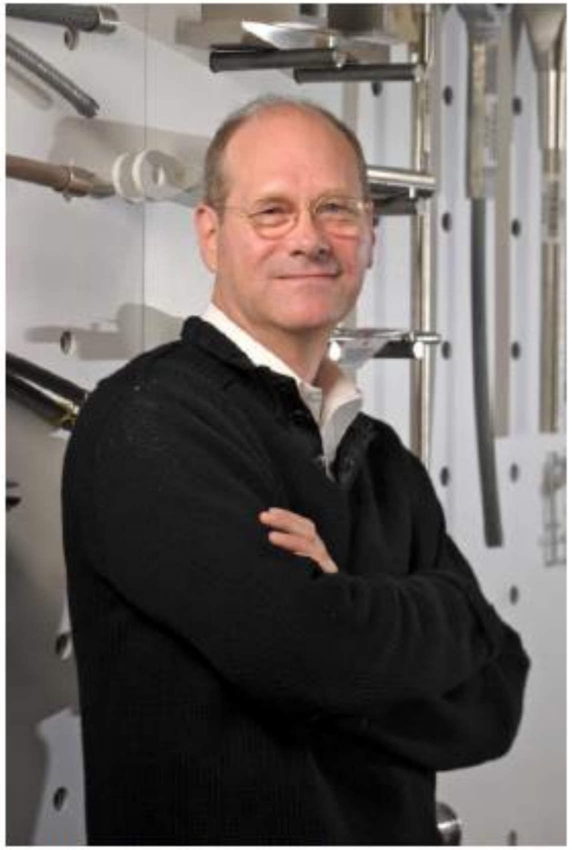
Practice midterm has been uploaded

Load Path

Load Path

Exchange house in London





William F. Baker, PE, SE, FASCE, FIStructE, NAE

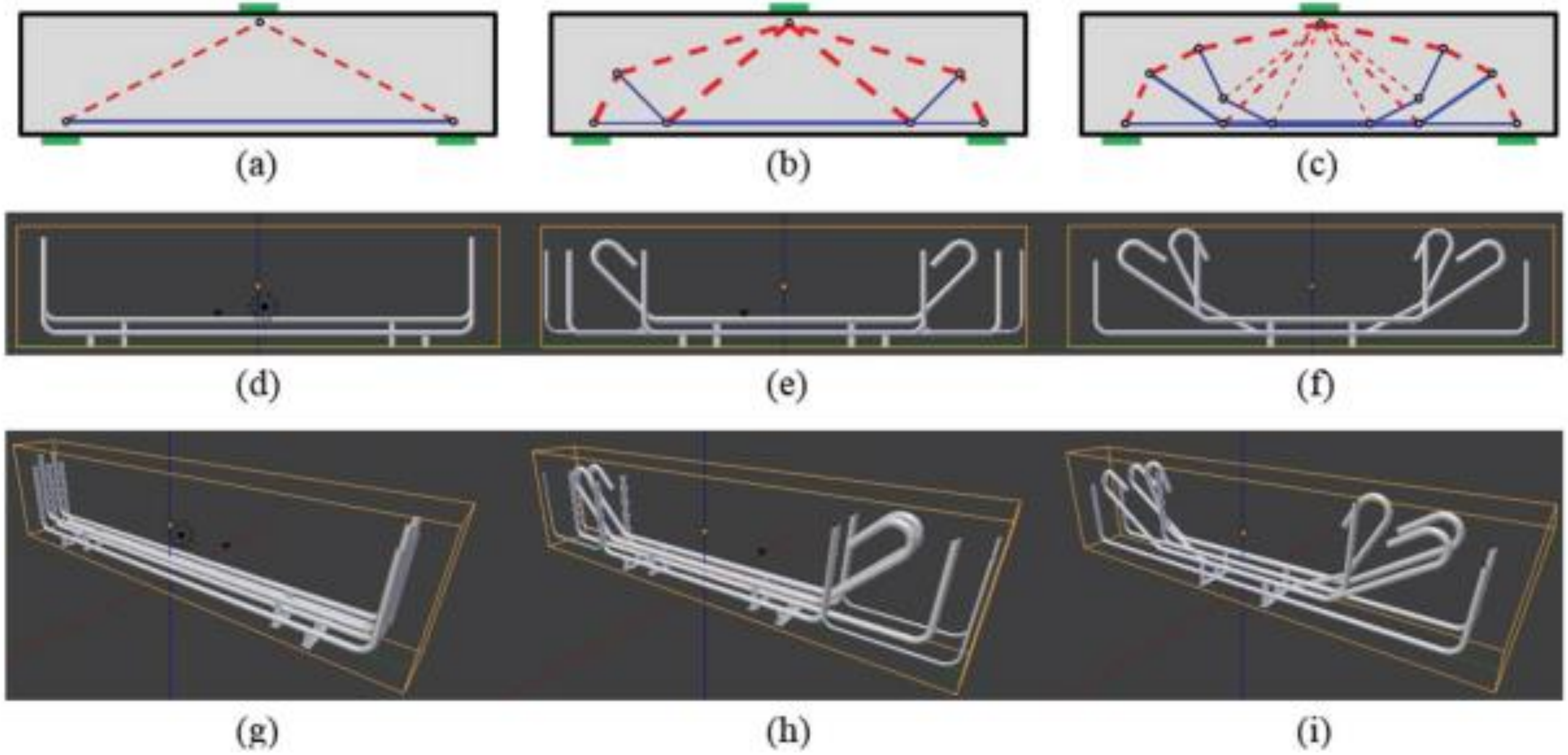
STRUCTURAL INNOVATION:
COMBINING CLASSIC THEORIES
WITH NEW TECHNOLOGIES

Load Path

Reinforced concrete

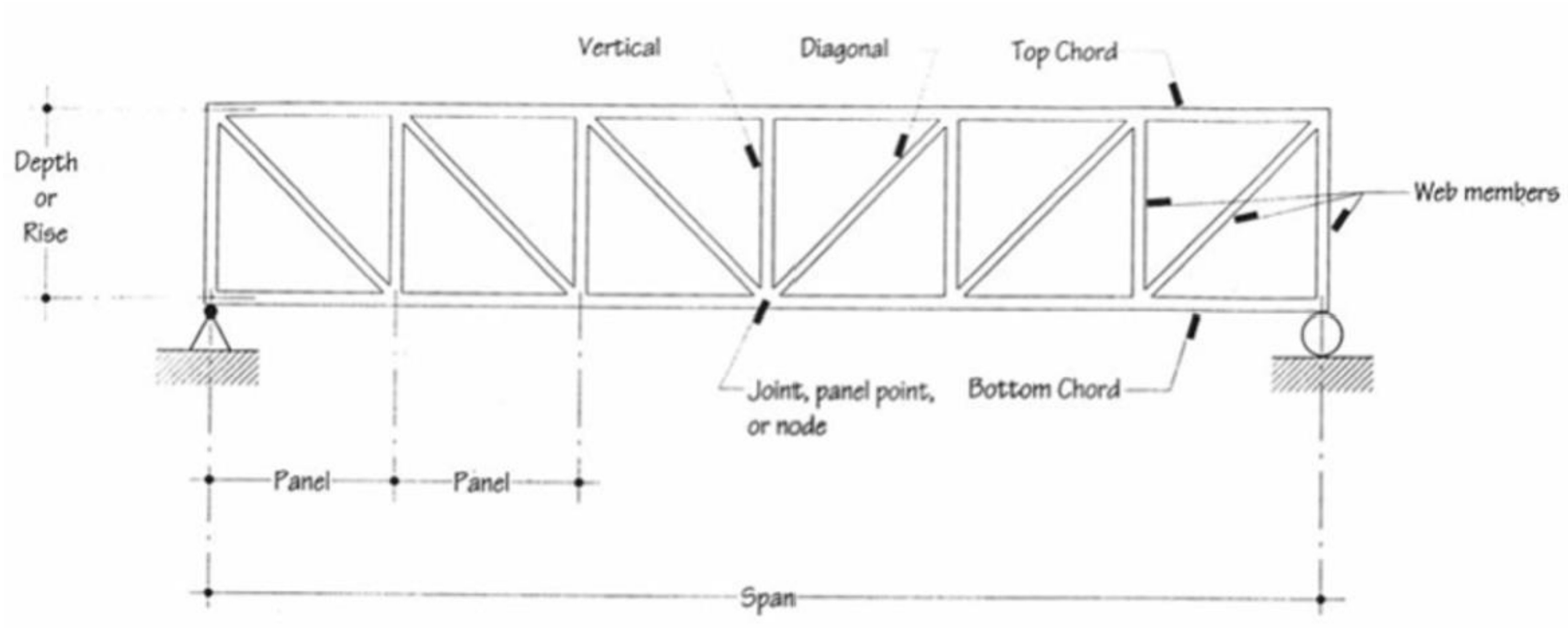
$$Z = \sum_e |F_e| L_e = \sum_{e \in G^T} |F_e| L_e + \sum_{e \in G^C} |F_e| L_e$$

$$V = \frac{\sum_{e \in G^T} |F_e| L_e}{\sigma^T} + \frac{\sum_{e \in G^C} |F_e| L_e}{\sigma^C} = \frac{(\sigma^C + \sigma^T) Z + (\sigma^C - \sigma^T) C}{2\sigma^C \sigma^T} \quad (2)$$

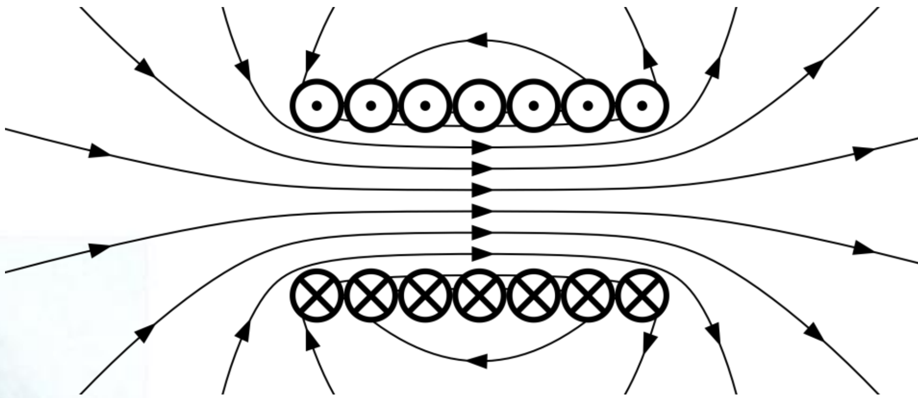


Zhao, T., Alshannaq, A. A., Scott, D. W., & Paulino, G. H. (2023). Strut-and-Tie Models Using Multi-Material and Multi-Volume Topology Optimization: Load Path Approach. *ACI Structural Journal*, *120*(6).

Is it possible to design stiffer structure with less material?

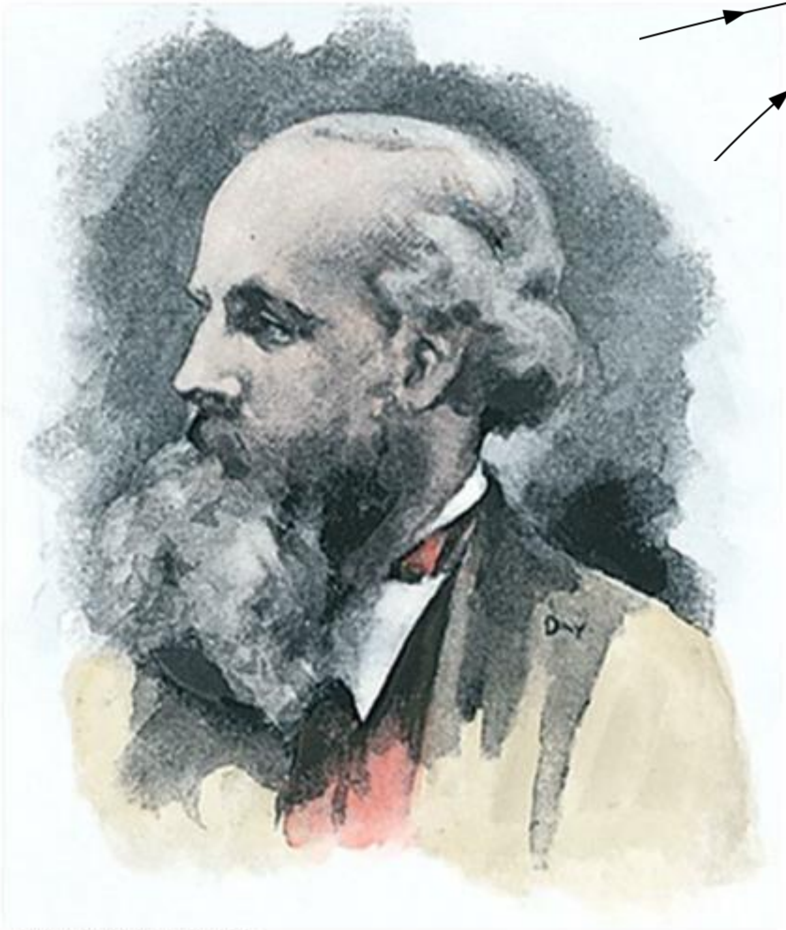


James Clerk Maxwell



- In 1865, James Clerk Maxwell provided a mathematical theory that showed a close relationship between all electric and magnetic phenomena

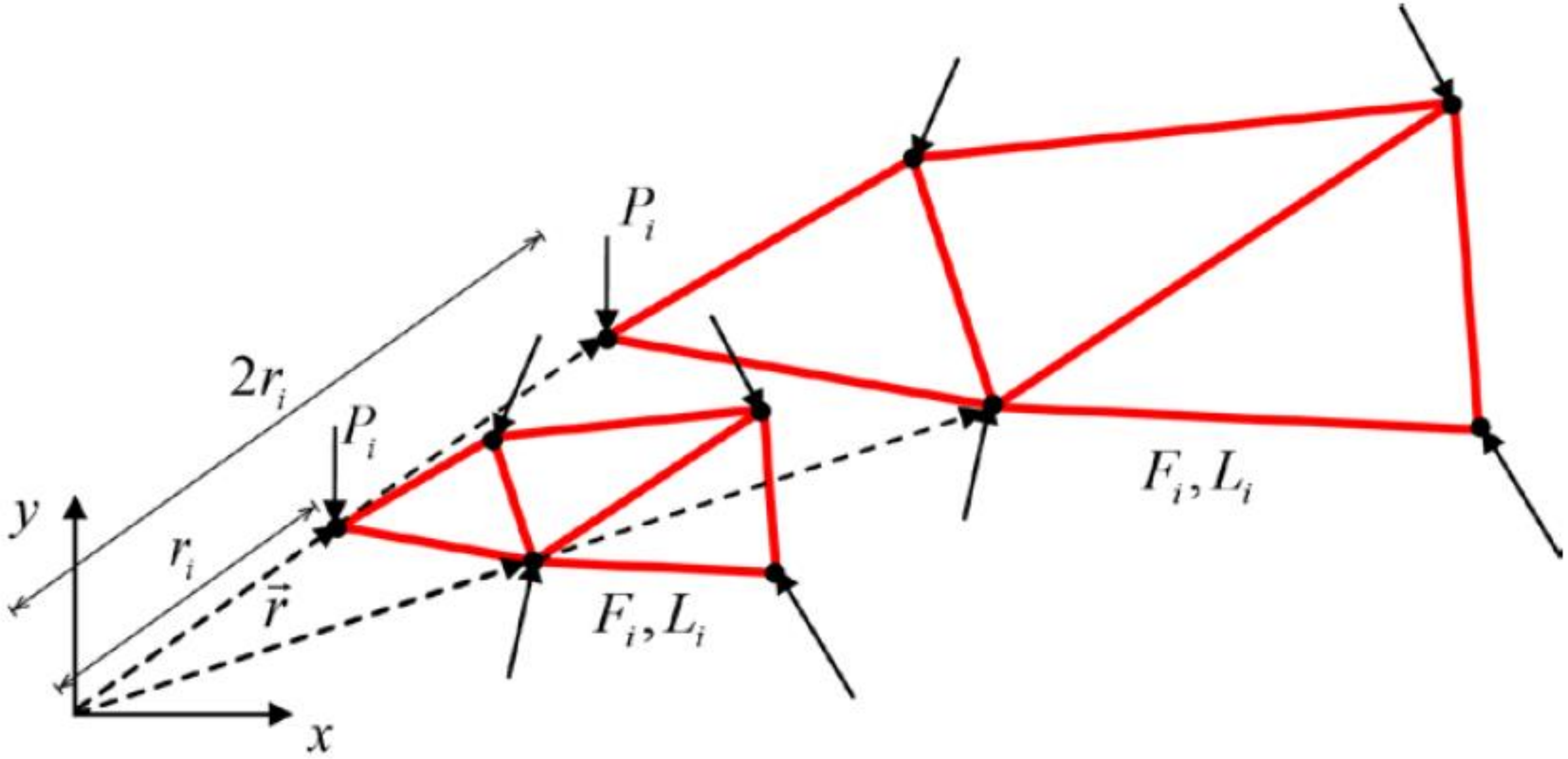
- Maxwell's equations also predict the existence of electromagnetic waves that propagate through space



Maxwell's Theorem
Geometric Proof

Internal force
Internal length
External { Loads / Reaction

$$\text{Maxwell number: } C = \sum F_i L_i = \sum \underline{P_l} \cdot \underline{r_l}$$



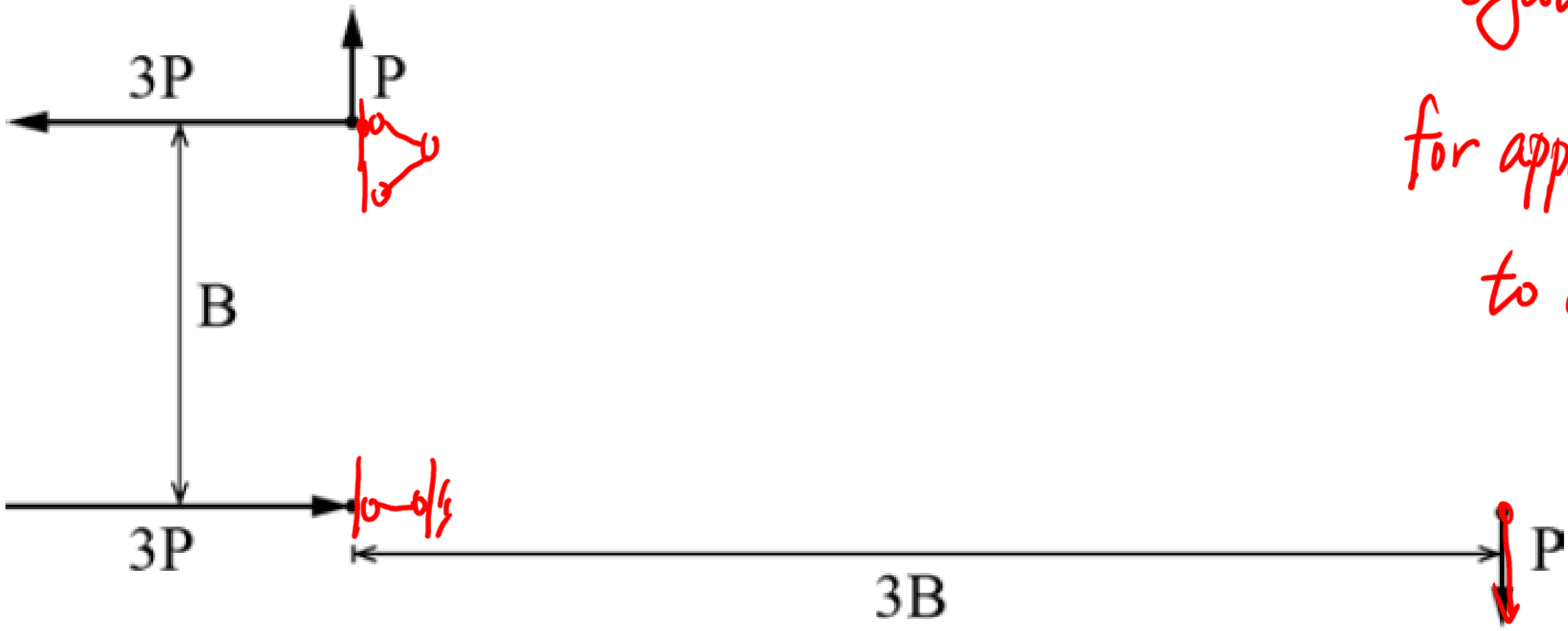
Maxwell's Theorem

A powerful example

Maxwell number: $C = \sum F_i L_i = \sum \underline{P_l} \cdot \underline{r_l} = P \cdot B$

Michell number: $Z = \sum |F_i| L_i$

Negative of the work for applied loads & reactions to cancel each other



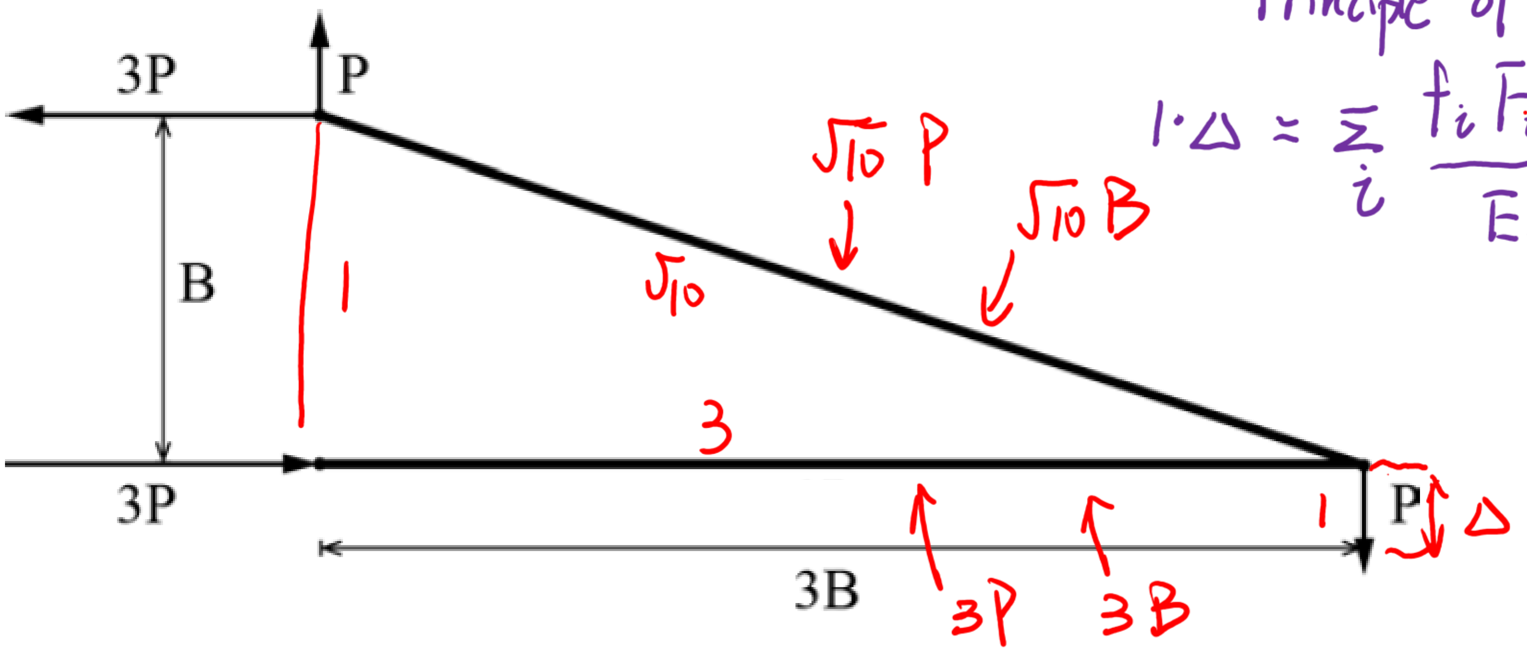
External force

Maxwell's Theorem

A powerful example: Option 1 Moment-diagram

Maxwell number: $C = \sum F_i L_i = \sum F_T \cdot L_T - \sum F_C \cdot L_C = 10PB - 9PB = PB$

Michell number: $Z = \sum |F_i| L_i = 10PB + 9PB = 19PB$



Principle of virtual work.

$$1 \cdot \Delta = \sum_i \frac{f_i F_i \cdot L_i}{E_i A_i} = \sum_i \frac{|f_i| \cancel{F_i} L_i}{E \cdot \frac{\cancel{F_i}}{\sigma_t}}$$

$$= \frac{\sigma_t}{E} \underbrace{\sum |f_i| L_i}_{Z}$$

$$= 19 \cdot \frac{\sigma B}{E}$$

Maxwell's Theorem

A powerful example: Option 2 Pratt Truss

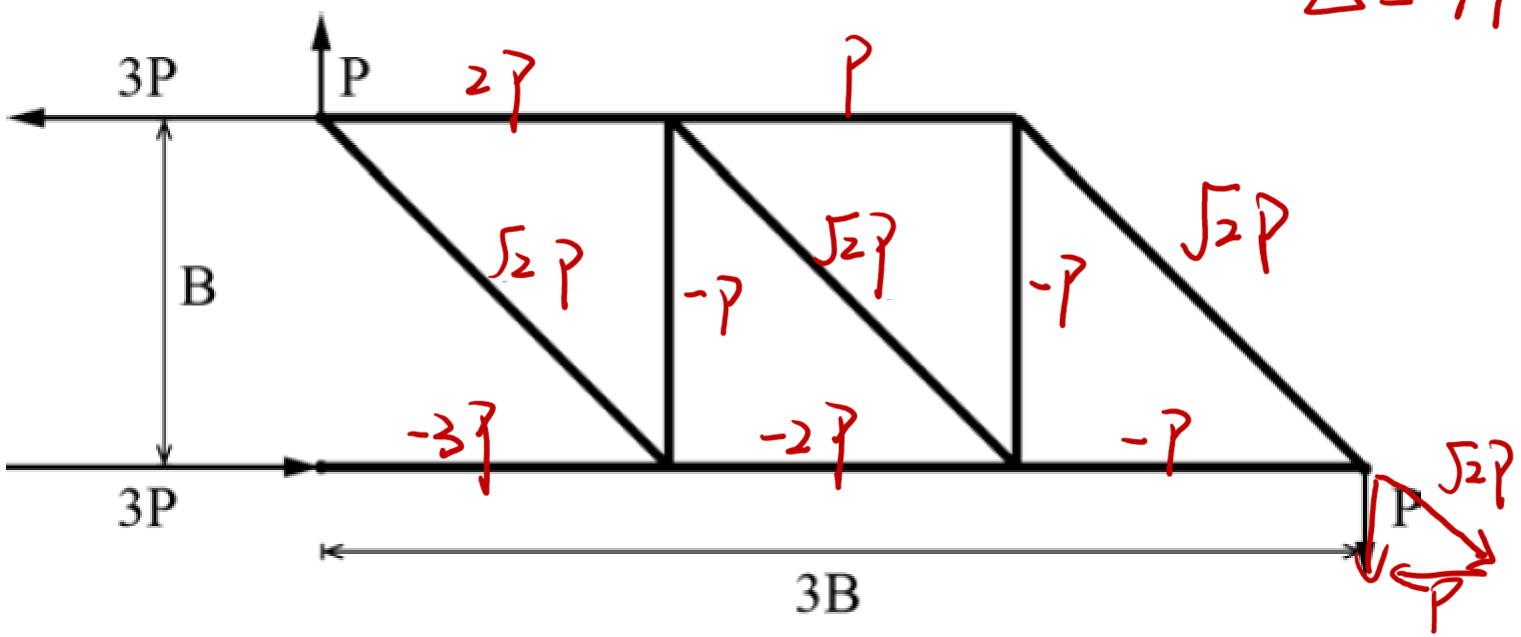
$\Sigma F_T \cdot L_T = 9PB$

$\Sigma F_C L_C = 8PB$

Maxwell number: $C = \sum F_i L_i = 9PB - 8PB = PB$

Michell number: $Z = \sum |F_i| L_i = 9PB + 8PB = 17PB$

$\Delta = 17 \cdot \frac{\sigma B}{E}$



Maxwell's Theorem

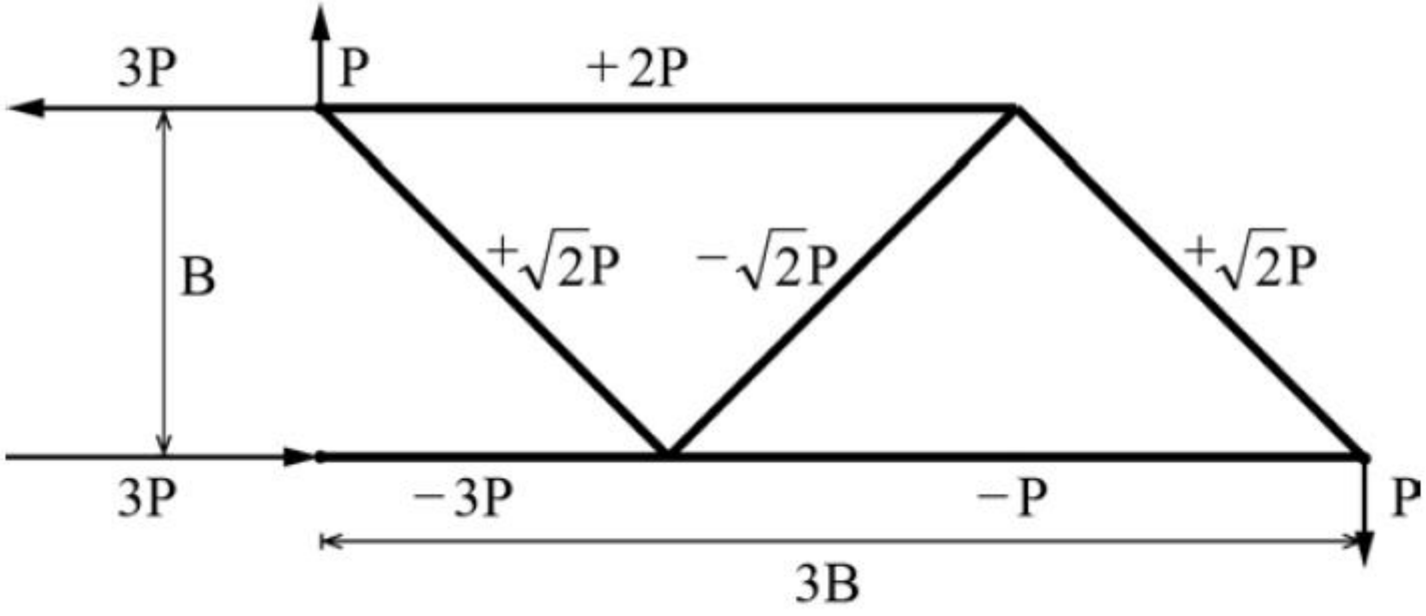
A powerful example: Option 3 Warren Truss

$\sum F_T \cdot L_T = 8PB$

$\sum F_c \cdot L_c = 7PB$

Maxwell number: $C = \sum F_i L_i = 8PB - 7PB = PB$

Michell number: $Z = \sum |F_i| L_i = 8PB + 7PB = 15PB$



Maxwell's Theorem

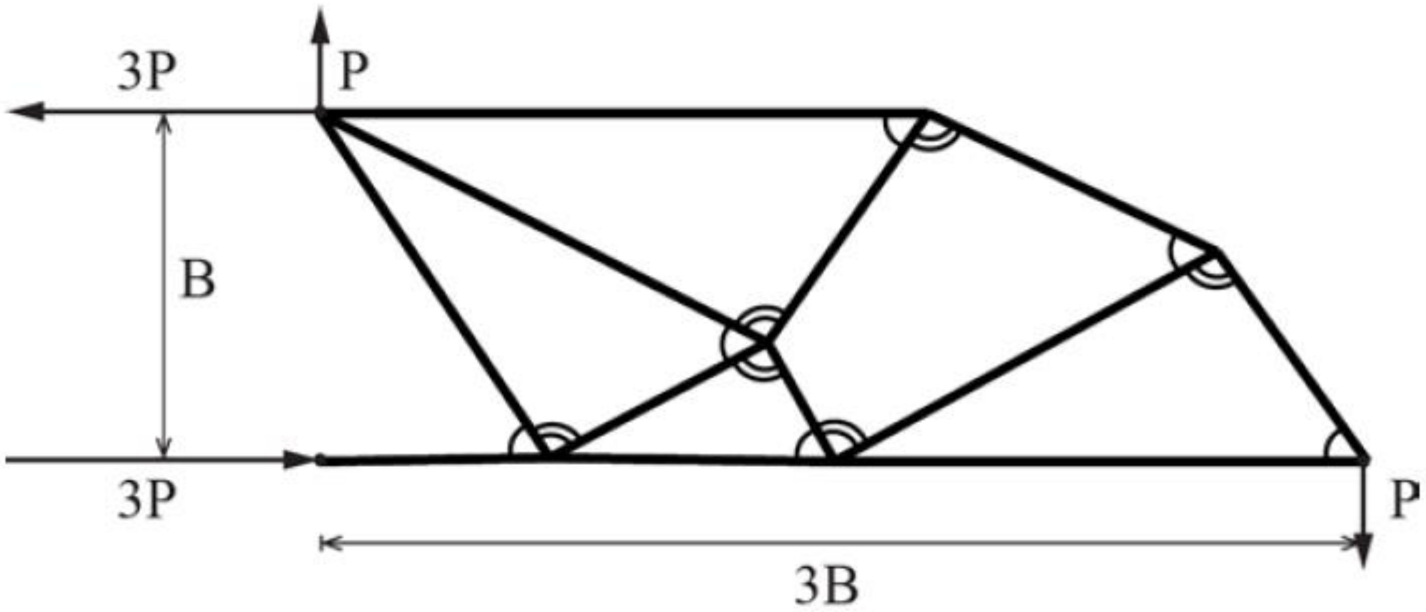
A powerful example: Option 3 Optimized Truss

$\Sigma F_T \cdot L_T = 7.7PB$

$\Sigma F_C \cdot L_C = 6.7PB$

Maxwell number: $C = \sum F_i L_i =$ $7.7PB - 6.7PB = 1PB$

Michell number: $Z = \sum |F_i| L_i$ $= 7.7PB + 6.7PB = 14.47PB$



Maxwell's Theorem

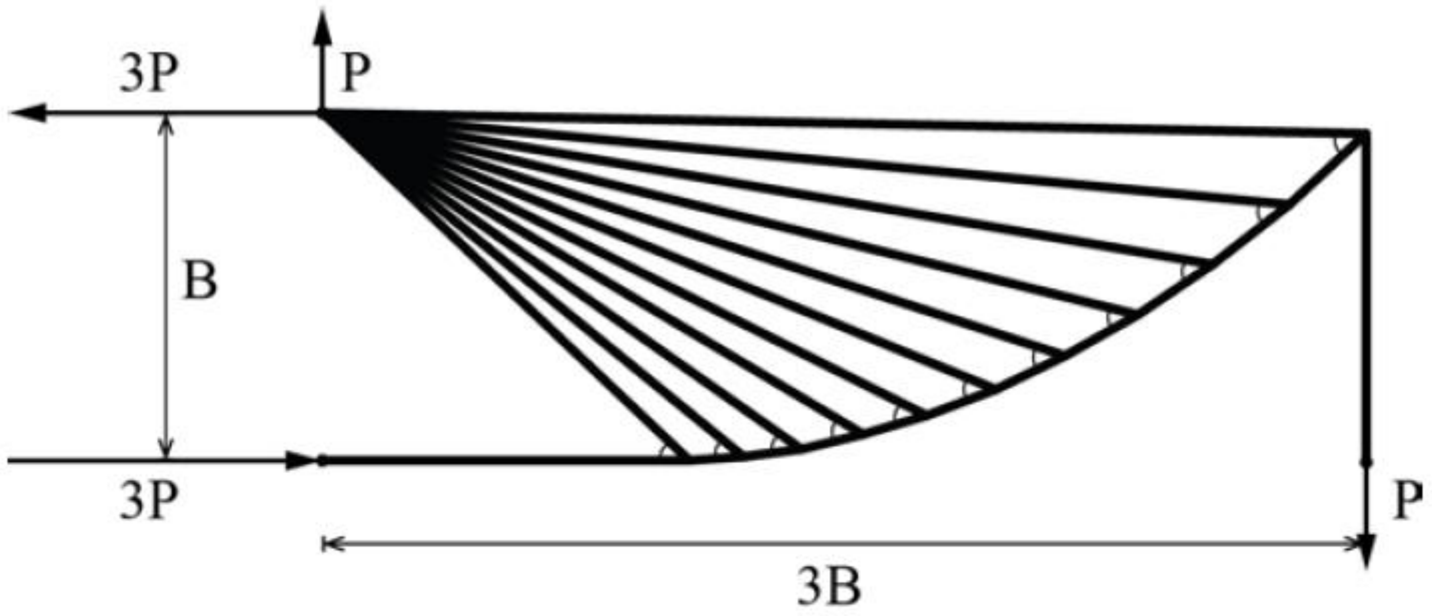
A powerful example: Option 4 Only Compression Chord

$\sum F_i L_i = 8.52\,PB$

$\sum F_c \cdot L_c = 7.52\,PB$

Maxwell number: $C = \sum F_i L_i =$ $1PB$

Michell number: $Z = \sum |F_i| L_i$ $= 16.04\ PB$



Maxwell's Theorem
Comparison

	Tensile Load Path, $\sum F_T L_T$	Compressive Load Path, $\sum F_C L_C$	Difference in Load Paths, $\sum F_T L_T - \sum F_C L_C$	Sum of Load Paths, $\sum F_T L_T + \sum F_C L_C$	Deflection, Δ
Moment Diagram Truss	$10PB$	$9PB$	PB	$19PB$	$19\frac{\sigma B}{E}$
Pratt Truss	$9PB$	$8PB$	PB	$17PB$	$17\frac{\sigma B}{E}$
Warren Truss	$8PB$	$7PB$	PB	$15PB$	$15\frac{\sigma B}{E}$
Bounded Optimal Truss	$7.7PB$	$6.7PB$	PB	$14.47PB$	$14.47\frac{\sigma B}{E}$
Comp. Chord Cantilever	$8.52PB$	$7.52PB$	PB	$16.04PB$	$16.04\frac{\sigma B}{E}$

Maxwell's Theorem

Remark

$$\Delta = \underbrace{Z_1 \cdot \frac{\sigma_1 \cdot B}{E}}_{\text{1st system}} = Z_2 \frac{\sigma_2 B}{E} \Rightarrow \sigma_2 = \frac{Z_1}{Z_2} \sigma_1$$

Let's compare two structures that are uniformly stressed and designed to achieve the same target deflection.

$$V_1 = Z_1 \cdot \frac{PB}{\sigma_1}$$

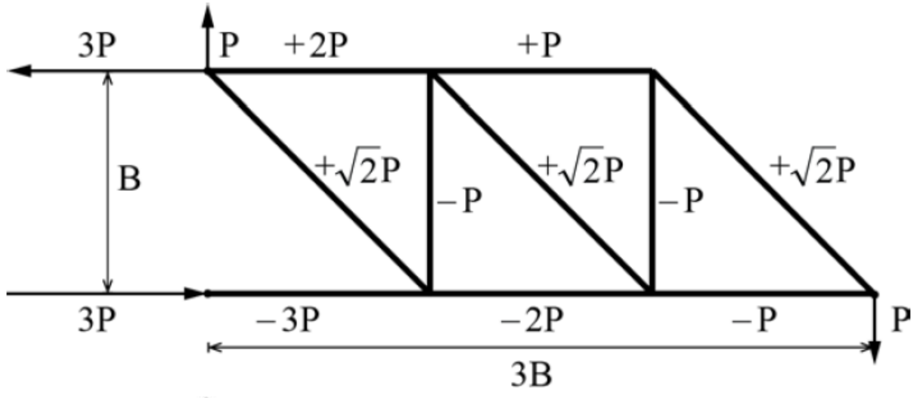
$$V_2 = Z_2 \frac{PB}{\sigma_2}$$

$$\frac{V_1}{V_2} = \frac{Z_1}{Z_2} \cdot \left(\frac{\sigma_2}{\sigma_1}\right) = \left(\frac{Z_1}{Z_2}\right)^2$$

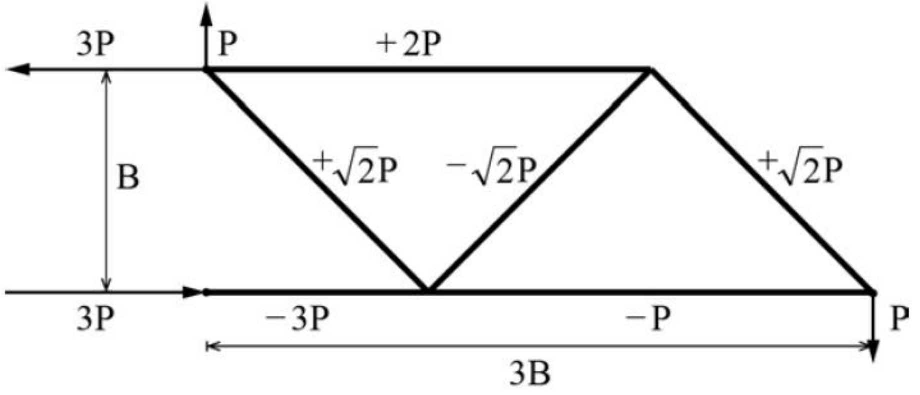
$$\frac{\sigma_2}{\sigma_1} = \frac{Z_1}{Z_2}$$

$$= \left(\frac{17}{15}\right)^2$$

$$= 1.28$$

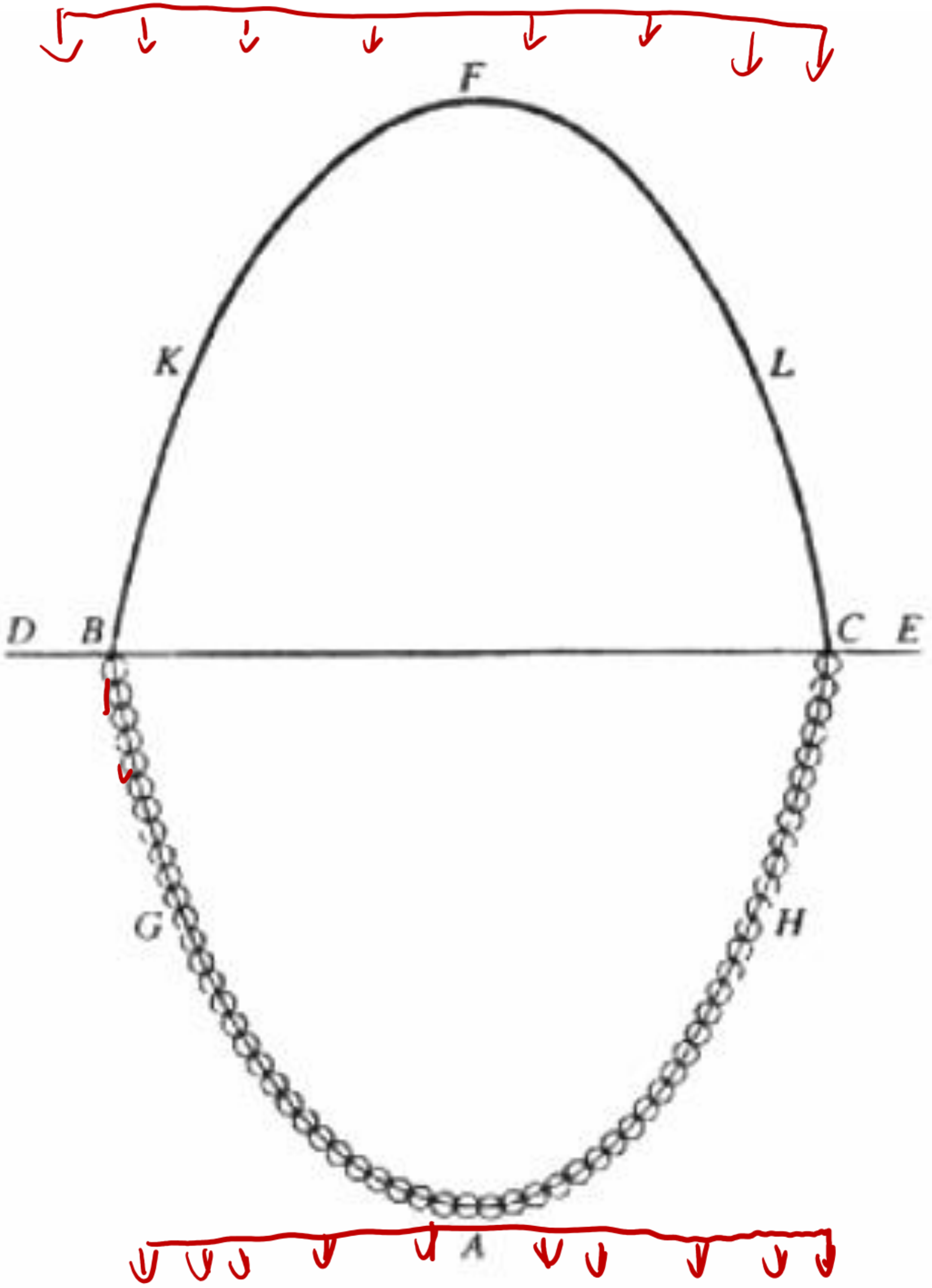


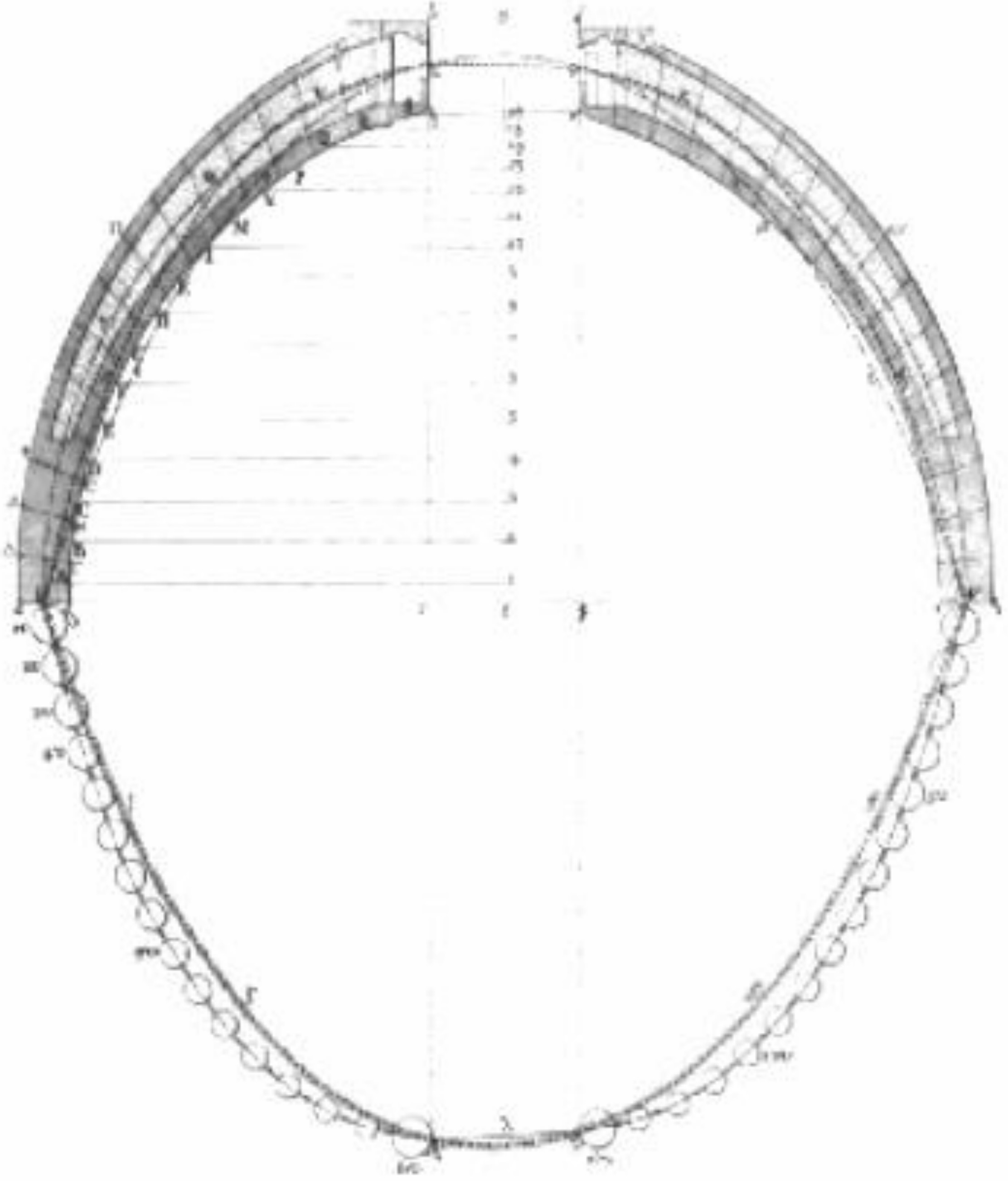
Pratt Truss



Warren Truss

Reichenau
Bridge



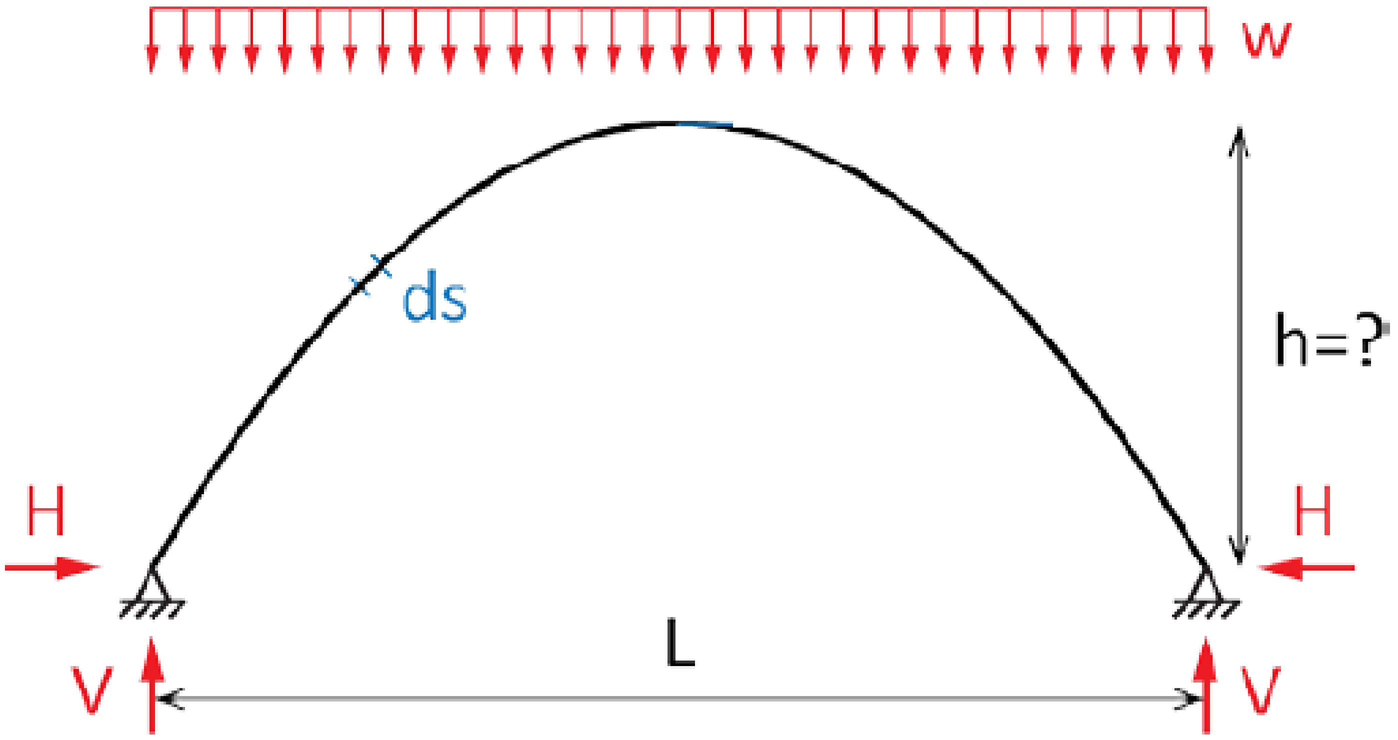


https://www.youtube.com/watch?v=wc0uUJ5zm9k

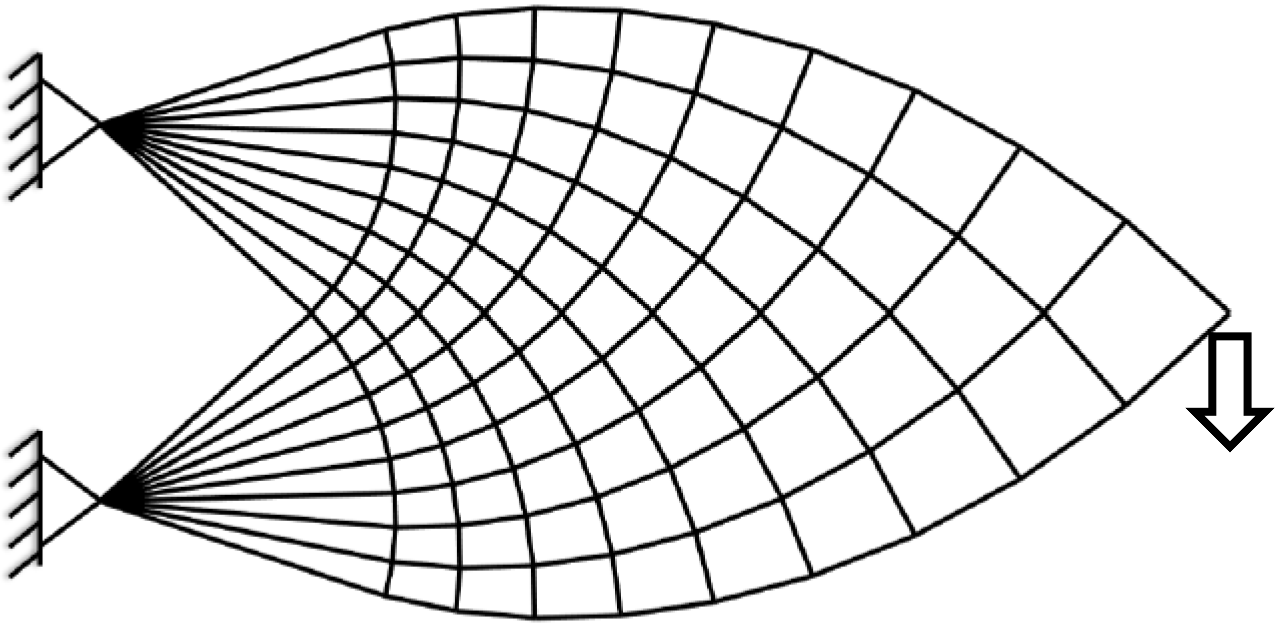
Maxwell's Theorem

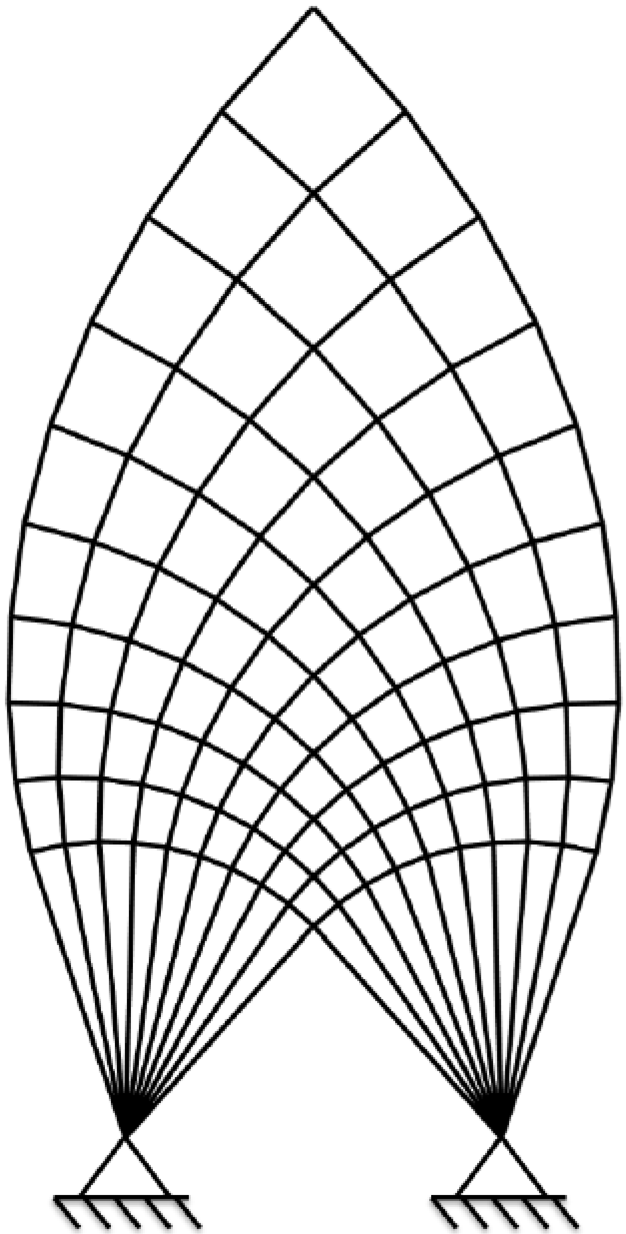
Practice

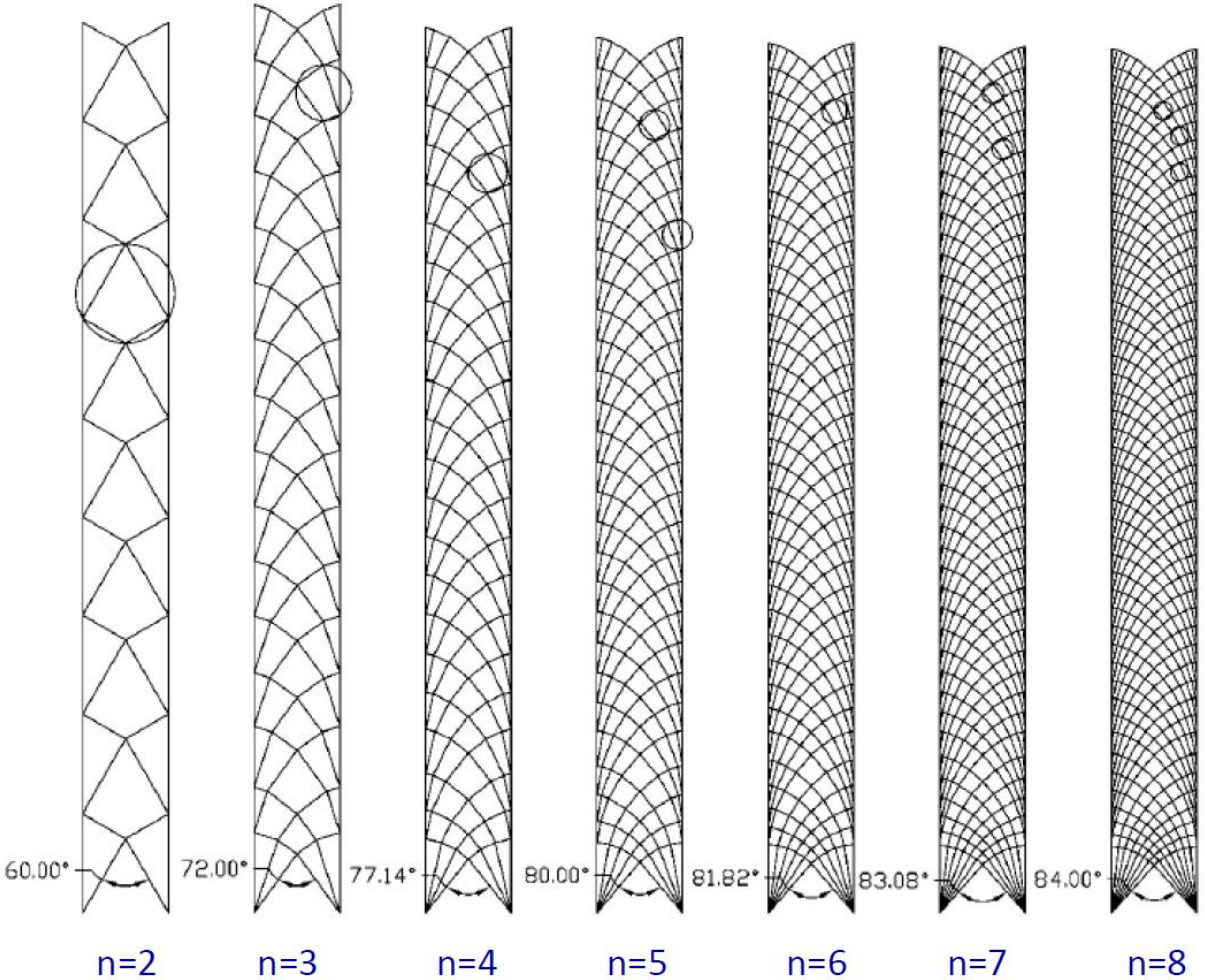
If a parabolic cable (zero bending stiffness) under uniform load is inverted, the shape is a funicular arch (under pure compression). What is the optimal height, h, that minimizes the weight of the cable, when the design is done under conditions of a maximum (constant) allowable stress σ?



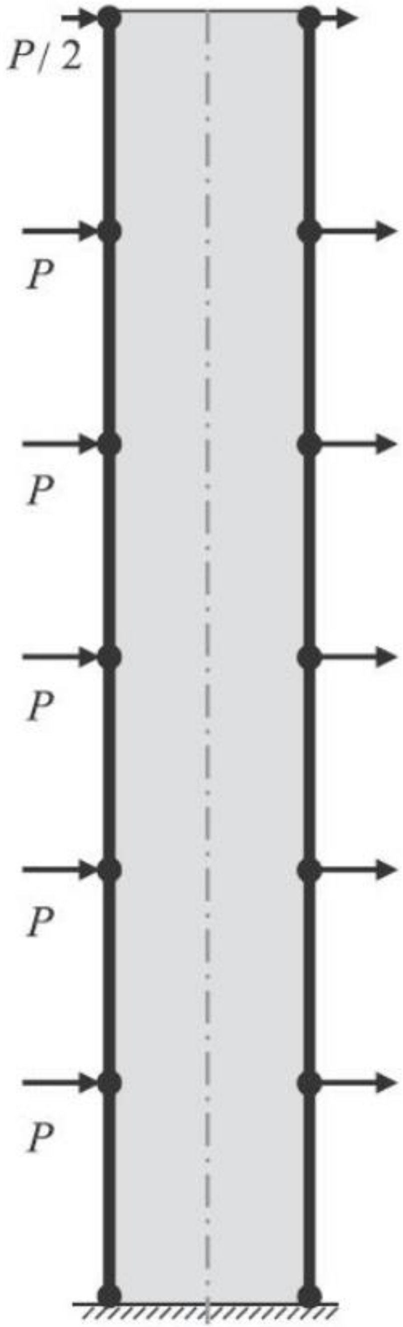
Mitchell Structure

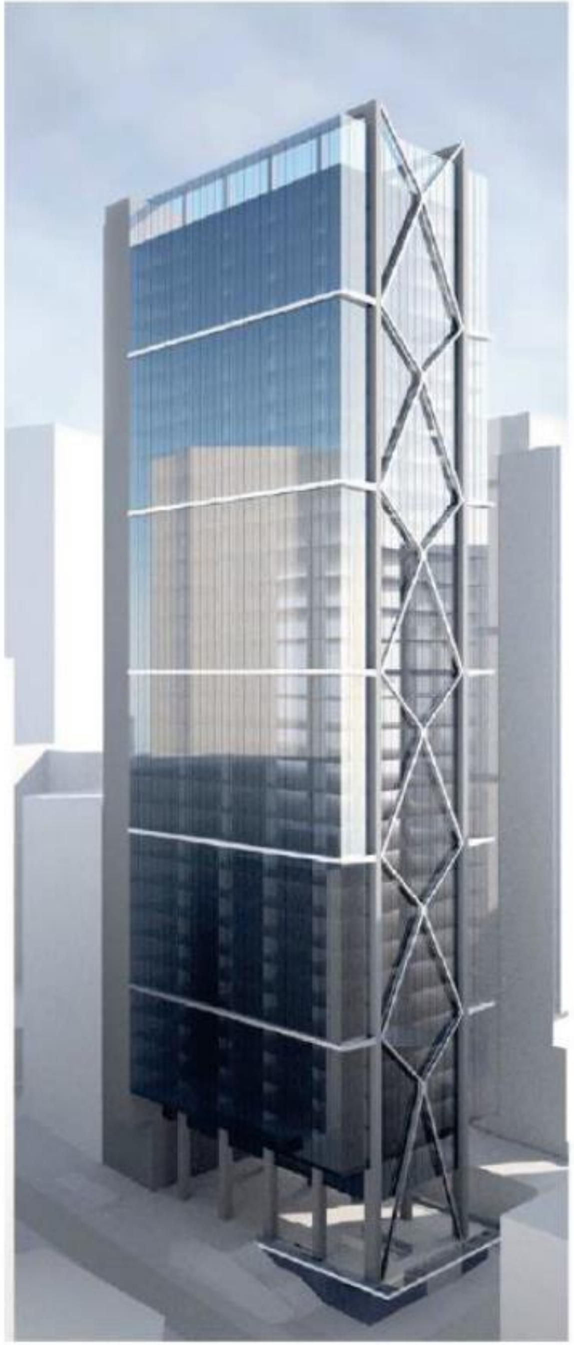






Beghini, Alessandro, et al. "Structural Optimization for Stiffness and Ductility of High-Rise Buildings."*Structures Congress 2015*.



Beghini, Lauren L., et al. "Connecting architecture and engineering through structural topology optimization."*Engineering Structures*59 (2014): 716-726.