McGill University

CIVE 546 — Structural Design Optimization (SDO)

2025 Winter

HOMEWORK #3 — Assigned 02/03/2025;

Recommended completion date: 02/17/2025

Problem 1

Given the unconstrained function,

$$F(\mathbf{x}) = 3x_1 + \frac{1}{x_1 - 1} + {x_2}^2 + \frac{1}{x_2}$$

a) At $\mathrm{x}_1 = 3$ and $\mathrm{x}_2 = 0.5$, calculate the gradient of $\mathrm{F}(\mathbf{x})$.

b) At $x_1 = 3$ and $x_2 = 0.5$, calculate the direction of steepest descent.

c) Using the direction of steepest descent calculated in part (b), update the design by the standard formula

$$\mathbf{x}^1 = \mathbf{x}^0 + \alpha \mathbf{S}^1$$

Evaluate x_1, x_2, and $F(\mathbf{x})$ for α ranging between 0.0 and 1.0 and plot the curve of F versus α in MATLAB. Hint: You may need to adjust the axis. Usage is *axis([XMIN XMAX YMIN YMAX])*. Use help to see axis command.

d) Write the equation for F in terms of α alone. Discuss the character of this function.

e) From part (d), calculate $\mathrm{d}F/\mathrm{d}\alpha$ at $\alpha = 0$.

f) Calculate the scalar product $\nabla\mathbf{F} \cdot \mathbf{S}$ using the results of parts (a) and (b) and compare this with the result of part (e).

Problem 2

The two-spring system shown in Figure 2 is loaded by two forces P_1 and P_2. The system in equilibrium is deformed as in Figure 3, with the displacement defined by x_1 and x_2.

Springs are assumed to be linearly elastic and the loads P_1 and P_2 are constant. Nonetheless, the system will experience large deformations.

Given the following parameters: $B = 7\ cm$, $H_1 = 12\ cm$, $H_2 = 10\ cm$, $k_1 = 1\ N/cm$, $k_2 = 8\ N/cm$, $P_1 = 4\ N$ and $P_2 = 1\ N$.

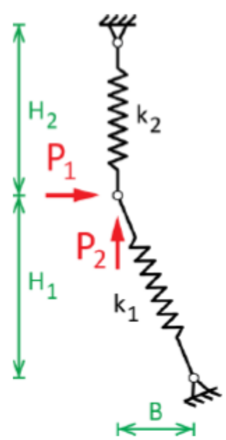

Figure 2: Undeformed 2-spring system

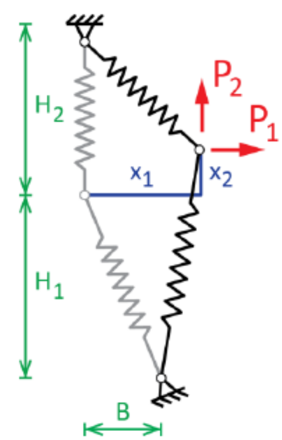

Figure 3: Deformed 2-spring system

a) Write the expression for the total potential energy $\mathrm{PE}(x_1, x_2)$ of the system for a given deformation x_1 and x_2, and obtain the gradient $\nabla\mathrm{PE}(x_1, x_2)$. [Hint: The potential energy for a spring is $\frac{1}{2}k\Delta^2$]

b) Write a MATLAB function that returns the potential energy and its gradient. Use *nargin* to specify an optional plotting parameter *plotopt*, that plots the point $\{x_1, x_2\}$ upon request with *plotopt* as the *linetype*. Use the following stub as a guide[3]

```
function [PE,dPE]=Spring2D(x,plotopt)

x1=x(1); x2=x(2);

b=7; h1=12; h2=10;
k1=1; k2=8;
P1=4; P2=1;
L1=sqrt(b^2+h1^2);

dPE=zeros(2,1);
PE= ;
dPE(1)= ;
dPE(2)= ;

if nargin==2, plot(x1,x2,plotopt,'LineWidth',2), end
```

c) Using the function from part (b), plot the contours for the total potential energy (objective function) for the range $x_1 = \{-2\,,12\}$ and $x_2 = \{-2\,,8\}$. Don't supply an *plotopt* parameter to the function at this stage. Put a hold on this plot with **hold on.**

d) Find the structural equilibrium minimizing the potential energy function. Solve the problem using the function **fminunc** from MATLAB with a starting point $x_1 = 0$ and $x_2 = 0$.
Make fminunc operate with the objective function only (no gradient) using the following options

```
options=optimset('Largescale','off','Display','off');
```

When passing the function to fminunc, specify the *plotopt* parameter as **'b^'** (with quotes), so that every time fminunc calls the function, a point gets plotted on the window from (c).

e) Repeat (d) using setting the *plotopt* parameter to **'ro'** (with quotes). This time, use the gradient in fminunc by specifying the following options

```
options=optimset('GradObj','on','Display','off');
```

f) The end result should be a single plot with parts (c), (d) and (e).
Attach the plot and comment on the results: Compare the points from parts (d) and (e).
What do you think would happen if the number of variables increases dramatically?