McGill University

CIVE 546 — Structural Design Optimization (SDO)

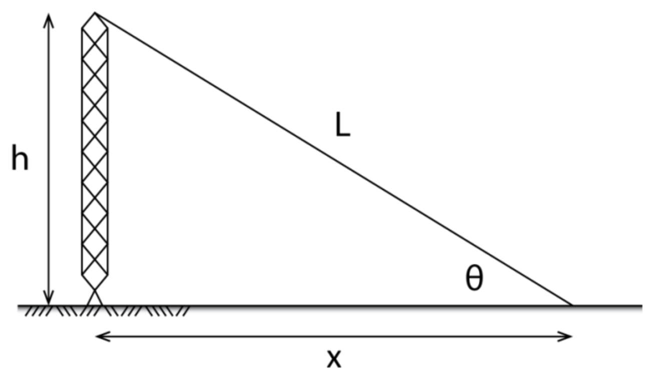
2025 Winter

HOMEWORK #1 — Assigned 01/13/2025;

Recommended completion date: 1/27/2025

Problem 1

A slender tower is anchored by a cable that provides the required stiffness in the horizontal direction.



The horizontal stiffness on the top is given by

$$k = \frac{EA\cos^2\theta}{l}$$

Where E is the Young's modulus of the cable, A is the cross-sectional area and L is its length.

What is the anchoring angle θ with respect to the horizontal line such that the ratio of horizontal stiffness by the cable volume, i.e. k/V, is maximized? Show that the result obtained is the max value of the ratio.

Both the cable area as well as the inferior anchoring point can be chosen freely. The anchoring point at the top as well as the Young's modulus of the material are fixed. Note that $V = AL$ and $L = h/\sin\theta$.

Problem 2

Start playing with MATLAB. For fun, verify the following properties numerically for any "random" square matrices $[A]$, $[B]$ and $[C]$ of your own choice. Turn in just one (and only one) MATLAB output for each problem below:

a) If $[A]$ is symmetric, the matrix $[D] = [A][B]$ is, in general, not symmetric, even if $[B]$ is also symmetric.
b) If $[A]$ is symmetric, the matrix $[D] = [B]^T[A][B]$ is always symmetric.
c) If $[A][B] = [C][B]$, it does not necessarily follow that $[A] = [C]$.
d) If $[D] = [A][B]$, then $[D]^T = [B]^T[A]^T$
e) $det[A] = \prod \lambda_j$ where λ_j is the j-th eigenvalue of $[A]$ (use help in MATLAB to see det, prod, eig)

Problem 3

Verify if each statement below is true or not. If not true, provide a simple counter-example using MATLAB. The counter-example that you provide should clearly prove the point that you want to make.

a) If $[A][B] = [0]$, a zero matrix, then either $[A]$ or $[B]$ is a zero matrix.
b) $[A][B]^{-1} = [C]$, then $[A] = [C][B]$.
c) If $[A]$ is an 5 x 5 matrix with $det[A] = 10$, then $det(10[A]) = 10\ det[A] = 100$.

Problem 4

Given the unconstrained function

$$f = f(x_1, x_2) = x_1^2 - \ln x_1 + x_2 + \frac{2}{x_2^2}$$

a) Obtain the gradient vector and the Hessian matrix of f.
b) For which values of x_1 and x_2 is the gradient null?
c) Discuss the behavior of f for each of the points computed in (b), e.g. max, min or neither.
d) Plot the function using MATLAB (use **help** in MATLAB to see linspace, meshgrid, surf and contour).