Structural Design Optimization

Practice Midterm

McGill University

Problem 1 (20 points):

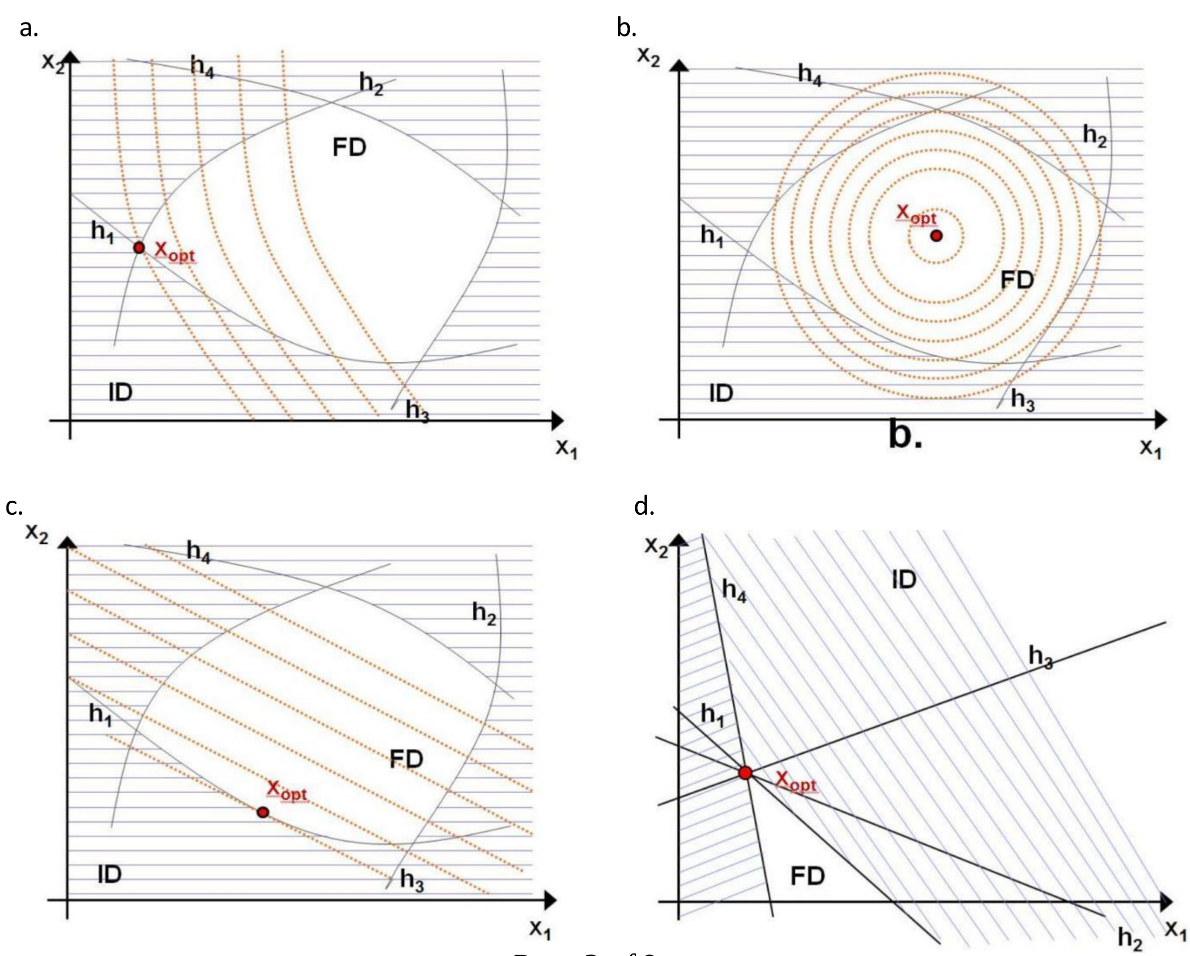
Question 1: Which of the methods below is NOT a gradient-based optimization method?

a. Steepest descent method
b. Genetic algorithm method
c. Newton's method
d. Quasi-Newton's method

Question 2: Which of the statements is incorrect for gradient-based optimization algorithms?

a. Linearization is performed using differentiation
b. Relatively intolerant of difficulties such as noisy objective function and inaccurate gradients
c. Find global minimum / maximum
d. Rapid convergence in comparison with genetic algorithms
e. All are correct

Question 3: Which of the following has all inactive constraints for the optimal solution?



Question 4: Which description is <u>incorrect</u>?

A 3 x 3- diagonal matrix

$$\boldsymbol{A} = \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix}$$

a. positive definite if $d_i > 0$ for i = 1, 2, 3
b. positive semidefinite if $d_1 > 0,\ d_2 > 0$ and $d_3 = 0$
c. negative definite if $d_i < 0$ for i = 1, 2, 3
d. indefinite if at least one d_i is positive and at least one d_i is negative for i = 1, 2, 3
e. All are correct

Question 5: Classify the following matrices according to whether they are positive or negative definite or semidefinite or indefinite

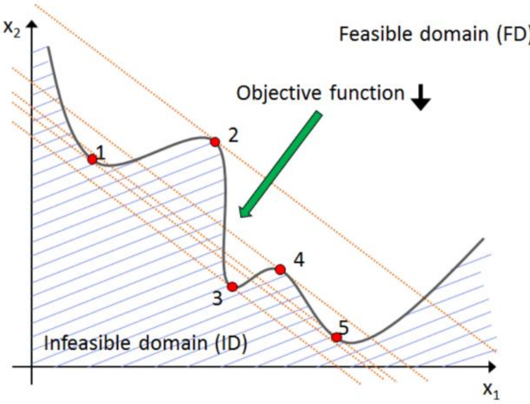
$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad C = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix} \quad E = \begin{bmatrix} -2.5 & 0 & 0 \\ 0 & -6.3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

a. *A*-ID, *B*-ND, *C*-ND, *D*=PSD, *E*=NSD
b. *A*-PD, *B*-ID, *C*-ND, *D*=PSD, *E*=NSD
c. *A*-PSD, *B*-ID, *C*-NSD, *D*=PD, *E*=ND
d. *A*-ID, *B*-PSD, *C*-NSD, *D*=PD, *E*=ND
e. *A*-ID, *B*-PSD, *C*-ND, *D*=PD, *E*=ND

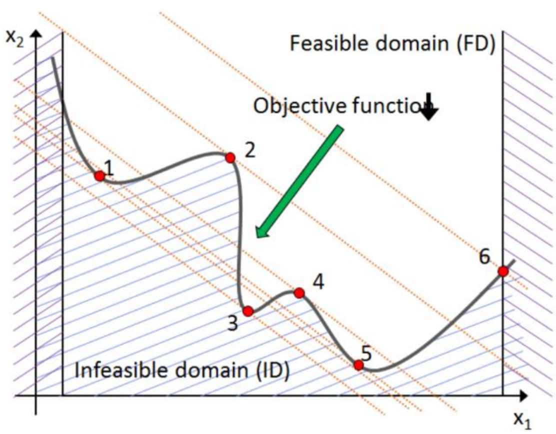
PD	: Positive definite
PSD	: Positive semidefinite
ND	: Negative definite
NSD	: Negative semidefinite
ID	: Indefinite

Question 6: Which of the points represent the global minimum and maximum points?



a. 1, 2   b. 3, 2   c. 5, 4   d. 3, 4   e. None

Question 7: Which of the points represent the global maximum points?



a. 2   b. 4   c. 6   d. None

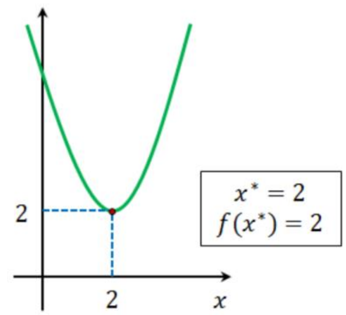
Question 8: Which of the statements is <u>incorrect</u>?

Suppose that $f(x)$, $f'(x)$, $f''(x)$ are all continuous on an interval I and that point x^* in I is a critical point :

a. If $f''(x^*) \geq 0$,then x^* is a strict local minimizer of $f(x)$
b. If $f''(x) > 0$ for all $x \in I,\ x \neq x^*$, then x^* is a strict global minimizer of $f(x)$ on I
c. If $f''(x) \geq 0$ for all $x \in I$, then x^* is a global minimizer of $f(x)$ on I
d. $f'(x^*) = 0$
e. If $f''(x) < 0$ for all $x \in I,\ x \neq x^*$, then x^* is a strict global maximizer of $f(x)$ on I

The function below has a minimum at $x^* = 2$

$$\boxed{f(x) = x^2 - 4x + 6}$$



Question 9: Find a minimum point x^* and a minimum value of the following function

$$\boxed{g(x) = (x-1)^2 - 4(x-1) + 6}$$

a. $x^* = 2, \quad g(x^*) = 1$ b. $x^* = 3, \quad g(x^*) = 3$ c. $x^* = 3, \quad g(x^*) = 2$
d. $x^* = 2, \quad g(x^*) = 2$ e. $x^* = 1, \quad g(x^*) = 3$

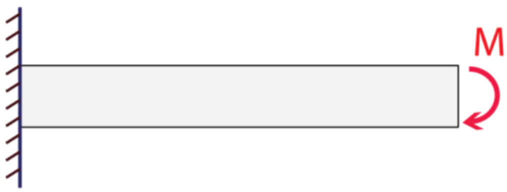
Question 10: Find a maximum point x^* and a maximum value of the following function

$$\boxed{g(x) = -(x^2 - 4x + 6)}$$

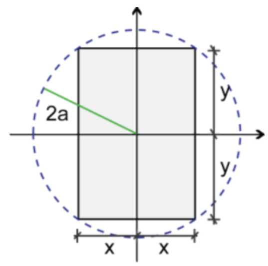
a. $x^* = 2, \quad g(x^*) = -2$ b. $x^* = 2, \quad g(x^*) = 3$ c. $x^* = 2, \quad g(x^*) = 2$
d. $x^* = 2, \quad g(x^*) = -3$ e. $x^* = 1, \quad g(x^*) = -3$

Problem 2 (40 points):

A cantilever beam of uniform rectangular cross section is subjected to a bending moment M at the tip. The beam is to be cut from a log having a circular cross section of diameter 4a. Find the dimensions of the beam that maximize the resulting bending stress.

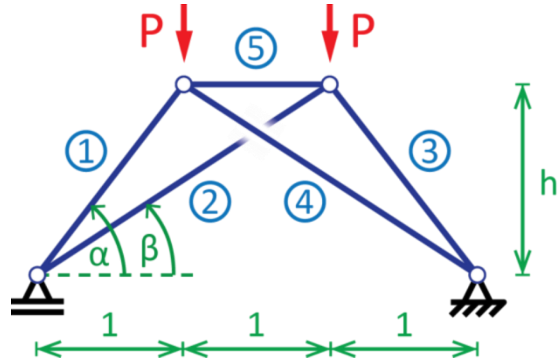


Cantilever beam



Cross section of the log

Problem 3 (40 points):



The 5-bar truss in the Figure is loaded by two symmetrical loads. Find the optimal height h that minimizes the volume (weight) of the structure, assuming the structure is fully stressed. The limit in tension and compression is $\bar{\sigma}$. In addition, you should verify that this optimum is indeed a minimum.

<u>Hint:</u> When possible, do not expand the member lengths in terms of h until the very end.