

# **CIVE 546 Structural Design Optimization**

**(3 units)**

**Instructor: Prof. Yi Shao**

**Class will start at 11:40am**

**TA: Xiaoning Sun**

**Winter 2025**



# Yi SHAO

Assistant Professor  
[yi.shao2@mcgill.ca](mailto:yi.shao2@mcgill.ca)



## Expertise:

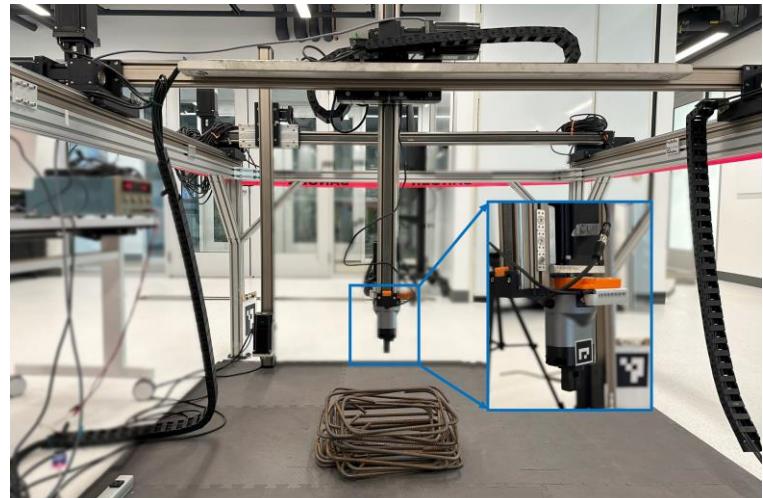
Structural Engineering | Construction Robotics

## Research:

- Application of Advanced Materials (CIVE 512)
- Structural Optimization (CIVE 546)
- Robotic Construction (Upcoming!)
- AI-assisted Structural Design (Upcoming!)



Structural  
Engineering

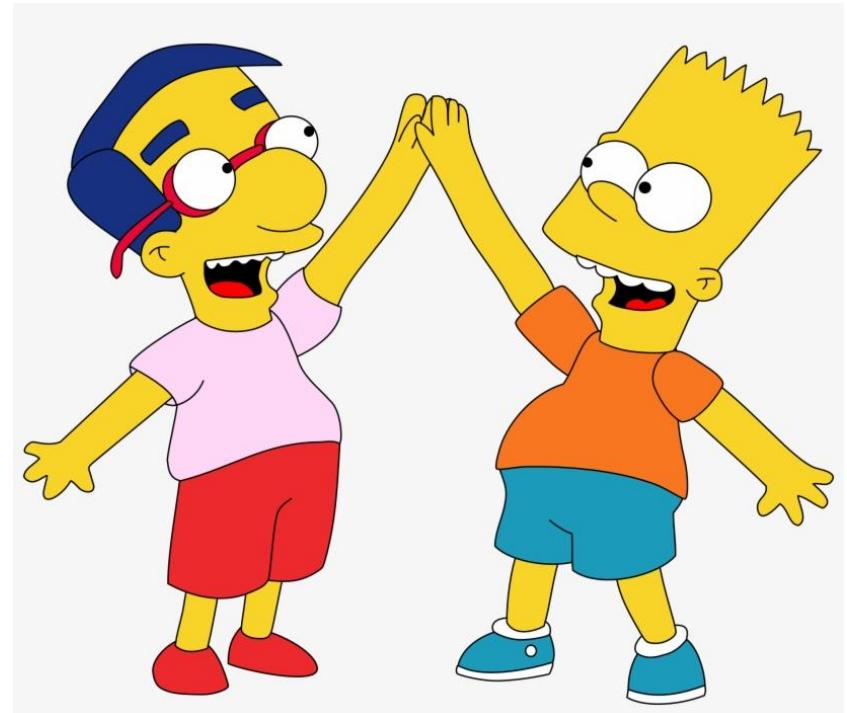


McGill

Department of  
Civil Engineering

# Introduce yourself

- Your name
- Year (undergrad or MS or PHD?)
- Your major
- Your research topic (if any)



# Course interactions

## In-class:

- Questions? Confusions? Speed? Other? Hands up!
- Group discussions/activities
- Respectful and Inclusive Environment
- Polling@McGill
- Class notes (PDF, myCourses) Don't redistribute, don't post online

## After-class:

- Office hours
- Emails

## Office Hour Survey:

**Please fill your general availability, not specific to the dates**

<https://www.when2meet.com/?28150990-xGdzZ>

**If you have any special needs or suggestions, please let me know!**



## Lecture structure

- Beginning (10 min) Slido to review key concepts
- Session 1 (50 min) lecture
- Break (15 min) Review + question answering
- Session 2 (50 min) lecture
- Rest: review + question answering + tutorial



Critical way for getting  
timely feedbacks and  
self-assessment

## Lecture tech

- The theoretical content will be delivered primarily through hand-writing, assisted by slides/animation for enhanced understanding. Be prepared to take notes.

# Why do we need “Structural Design Optimization”

***Sstructural Engineering: Develop a functional design***

**Functionality: costs, efficiency, environmental impacts, disaster resistance, etc.**

**Need careful analysis to verify the functionality**

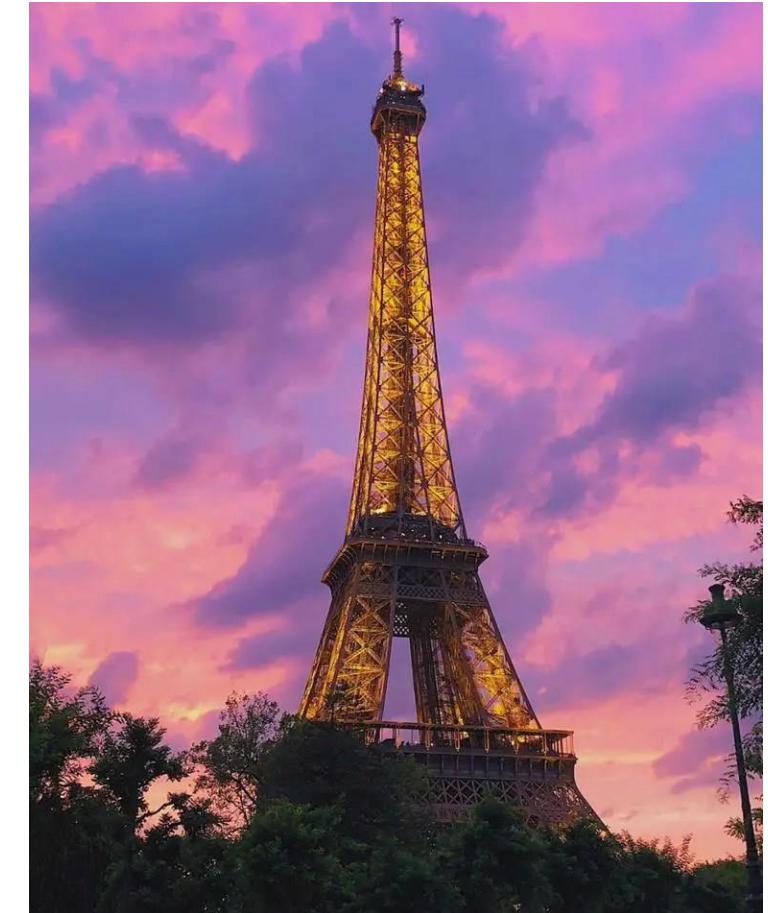
- CIVE 205 Statics
- CIVE 207 Solid Mechanics
- CIVE 317 Structural Engineering I
- CIVE 318 Structural Engineering 2
- CIVE 462 Design of Steel Structures
- CIVE 463 Design of Concrete Structures
- CIVE 507 Wind Engineering
- CIVE 525 Design of Wood Structures
- CIVE 545 LCA and the Circular Economy

Many courses that teach analysis methods

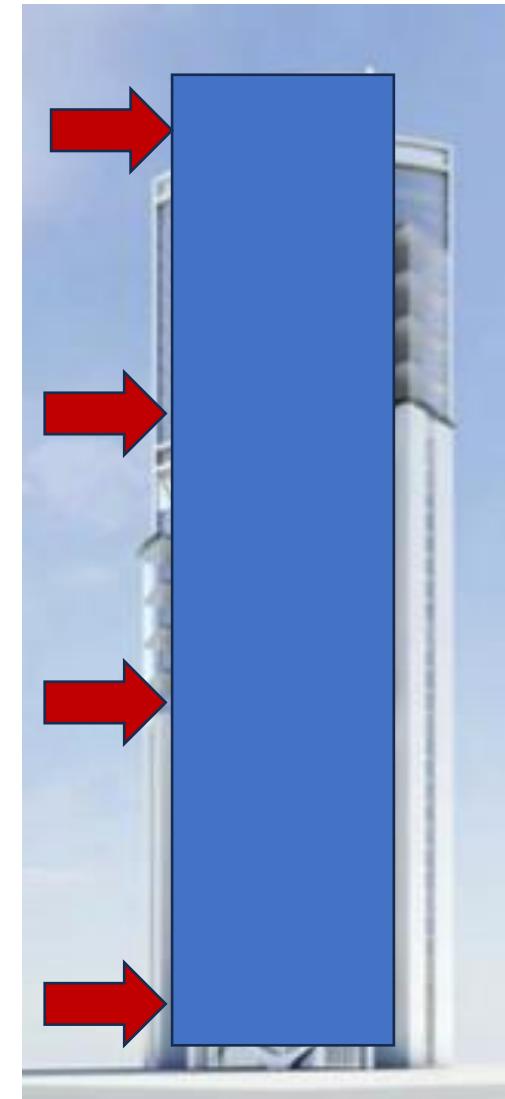
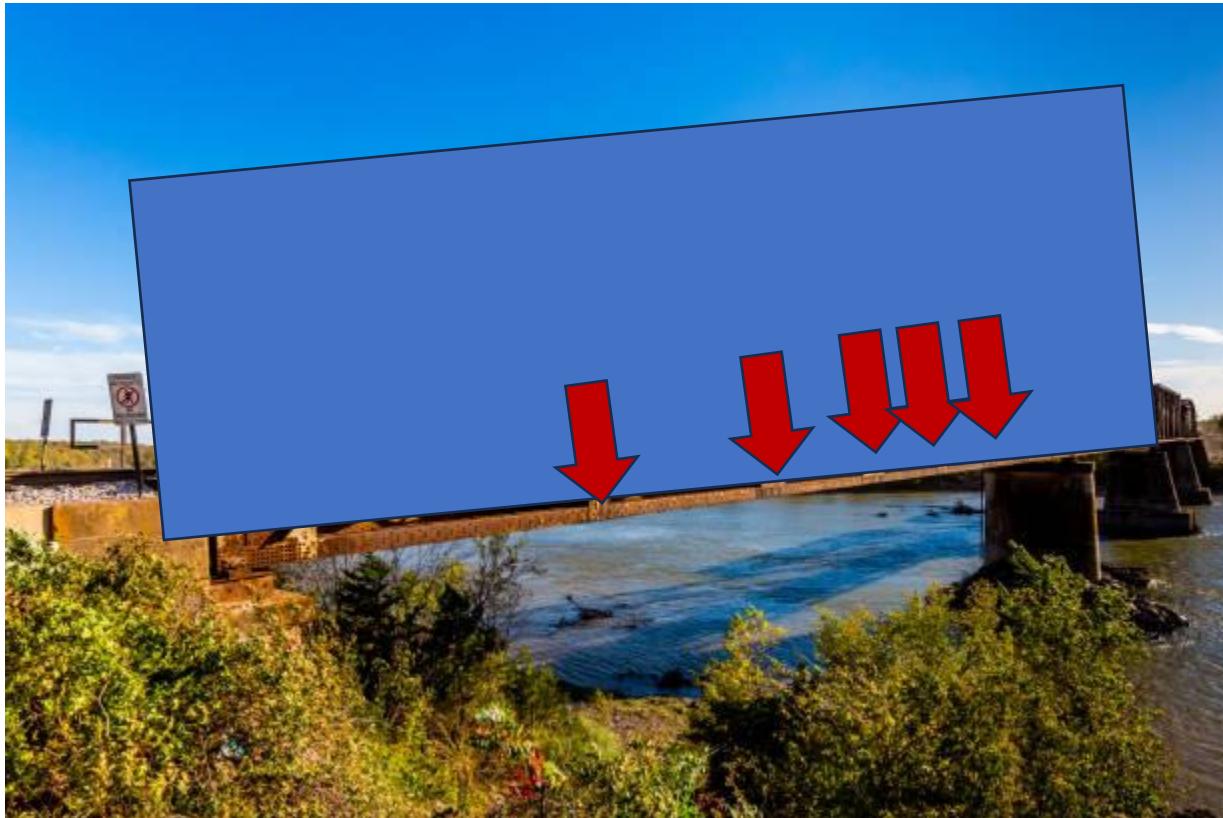
**First-step: How to achieve the “BEST” design for analysis?**  
**Critical for functionality**



The Bout-de-l'Île Railway Bridge, Montreal



Eiffel tower, Paris



# Why do we need “Structural Design Optimization”

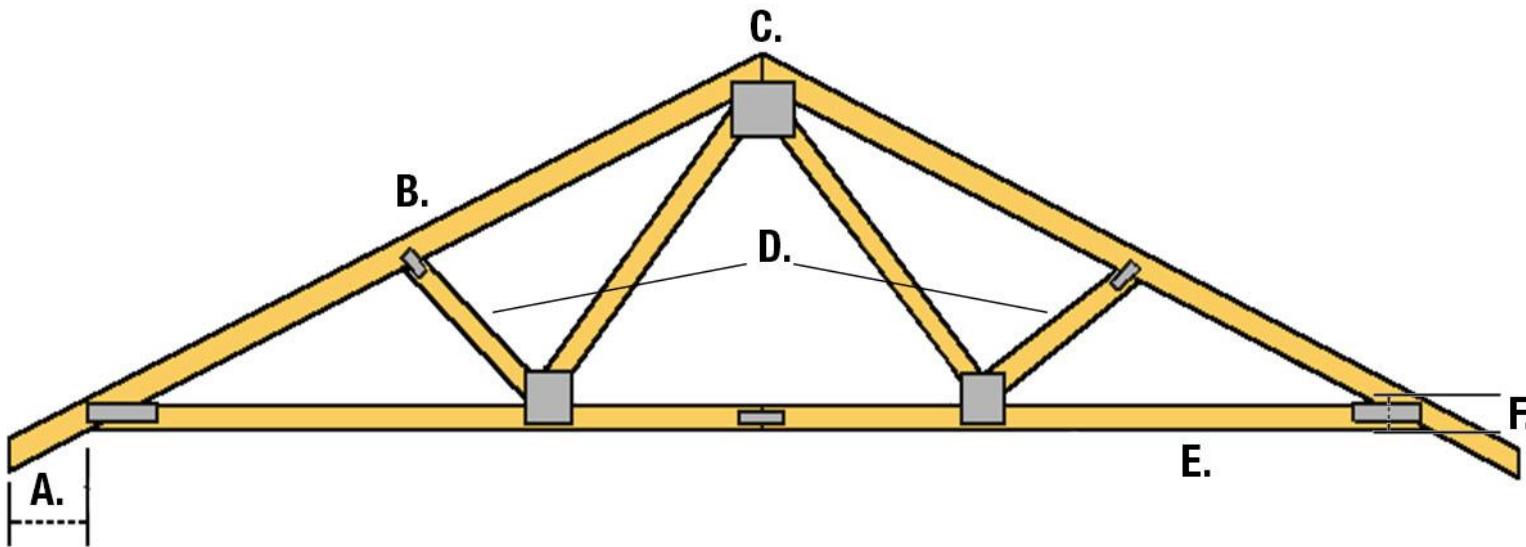
## *A simple example*

Consider a problem with 3 Design Variables (DVs), assume 10 values for each design variables and each design needs 0.1s to calculate.

Then we have \_\_\_\_ combinations  $\rightarrow$  total time: \_\_\_\_ \* 0.1s = \_\_\_\_.

$$10 \times 10 \times 10 = 10^3$$

100s



# Why do we need “Structural Design Optimization”

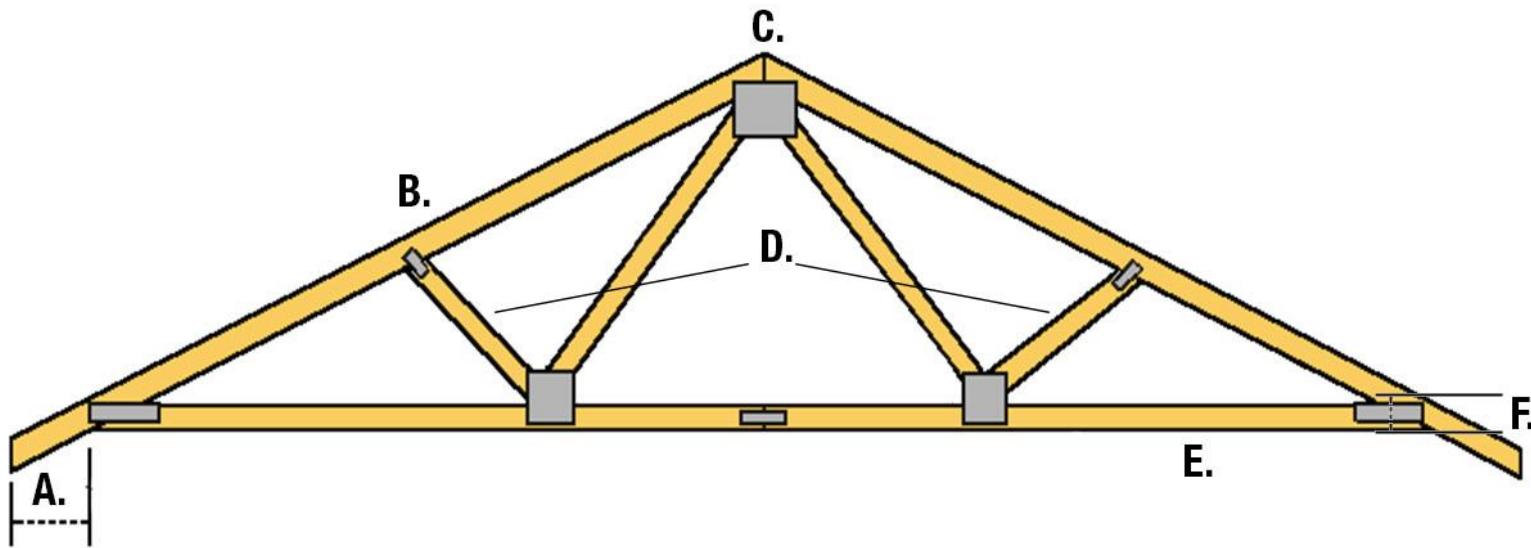
## *A simple example*

Consider a problem with 3 Design Variables (DVs), assume 10 values for each design variables and each design needs 1s to calculate.

Then we have  $10^3$  combinations  $\rightarrow$  total time:  $10^3 * 1s = \underline{\underline{100s}}$

If we have 10 DVs, still 10 values for each design and 1s to calculate each design

Then we have  $10^{10}$  combinations  $\rightarrow$  total time  $10^{10}s$  (317 years)



**We need a rational approach to search for efficient designs.**

# Structural Design Optimization

***A subject that involves a broad spectrum of science and tools***

## McGill University

CIVE 205 Statics

CIVE 207 Solid Mechanics

CIVE 208 System A

CIVE 317-318 Struc

CIVE 460 N

## Stanford U

MS&E111/211...

MS&E310: Linear Program

CEE 307 Structural Design Optimization  
(Topology Optimization)

CS334A Convex Optimization I

CS334B Convex Optimization II

## Technical University of Denmark (DTU)

Topology Optimization: Theory, Methods and  
Applications (with constraints!)

**The goal is to acquaint you with  
basic knowledge and tools of  
Structural Design Optimization to  
START your journey!**

Structural Design II

Computational Structural Design I

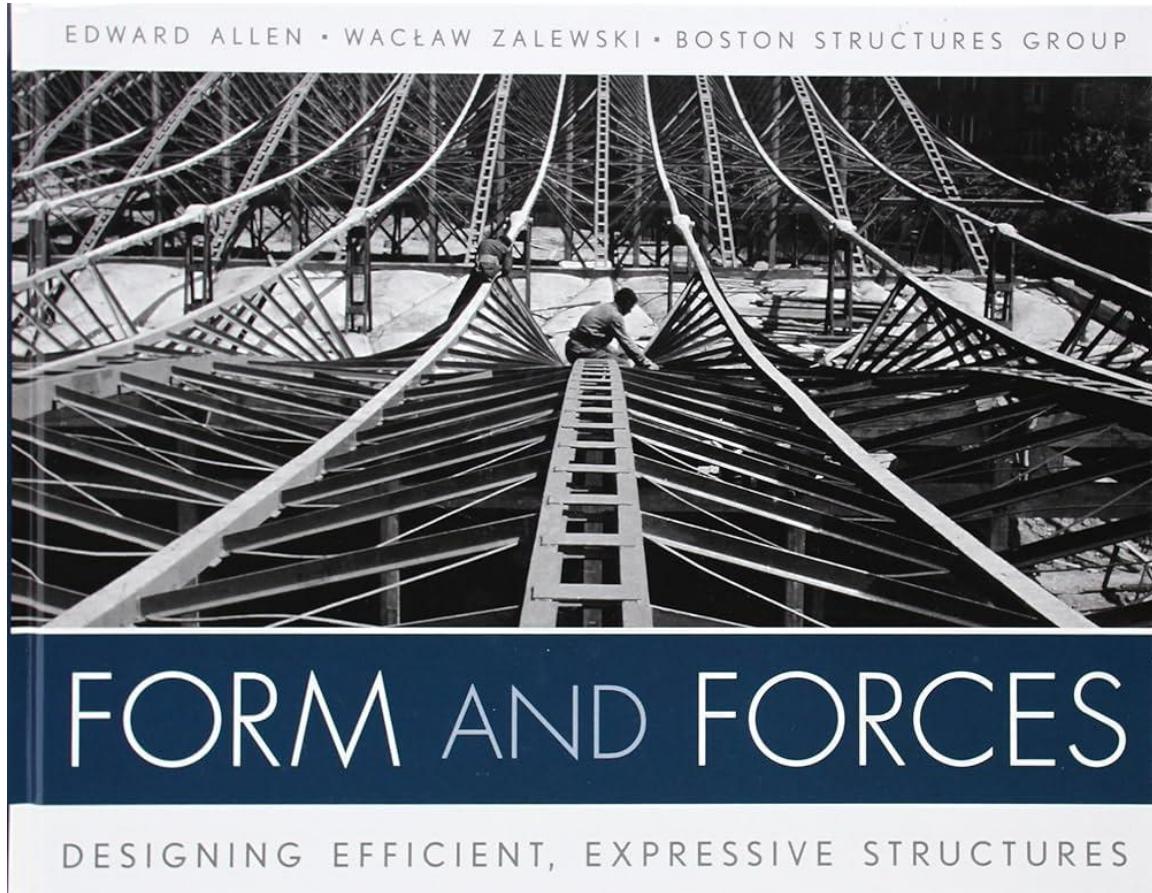
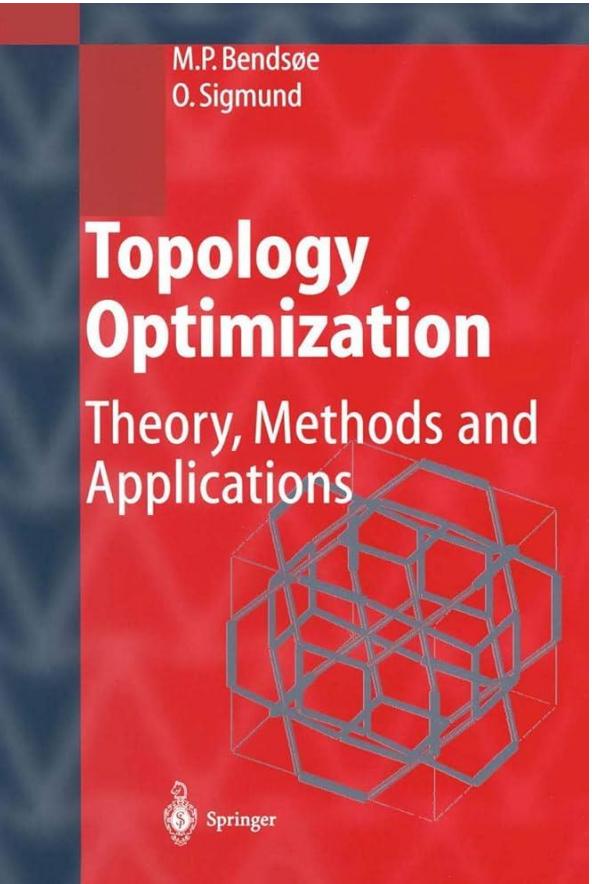
Computational Structural Design II

# Structural Design Optimization

***Two reference books***

**No textbook for this class.**

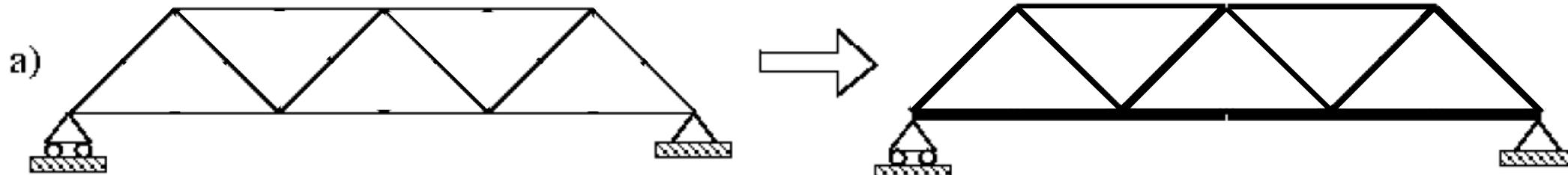
**These two books provide many useful information and extensions!**



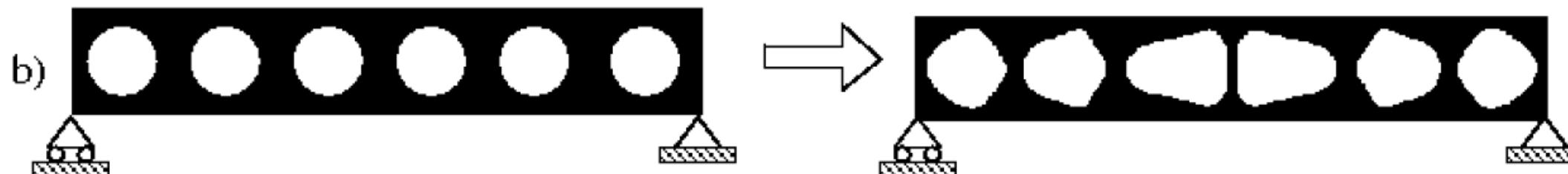
# Structural Design Optimization

**Numerical Optimization: Choice of design variable**

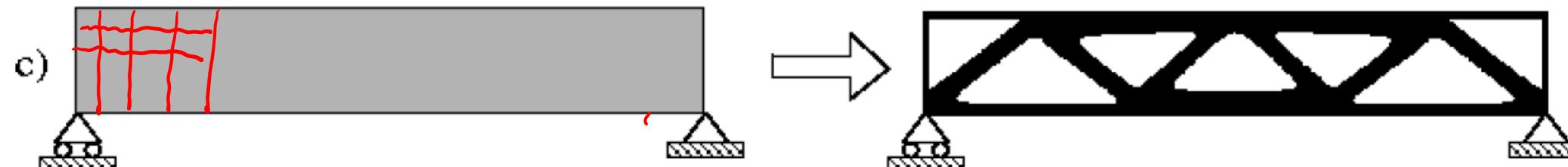
## Sizing Optimization



## Shape Optimization *→ level set*



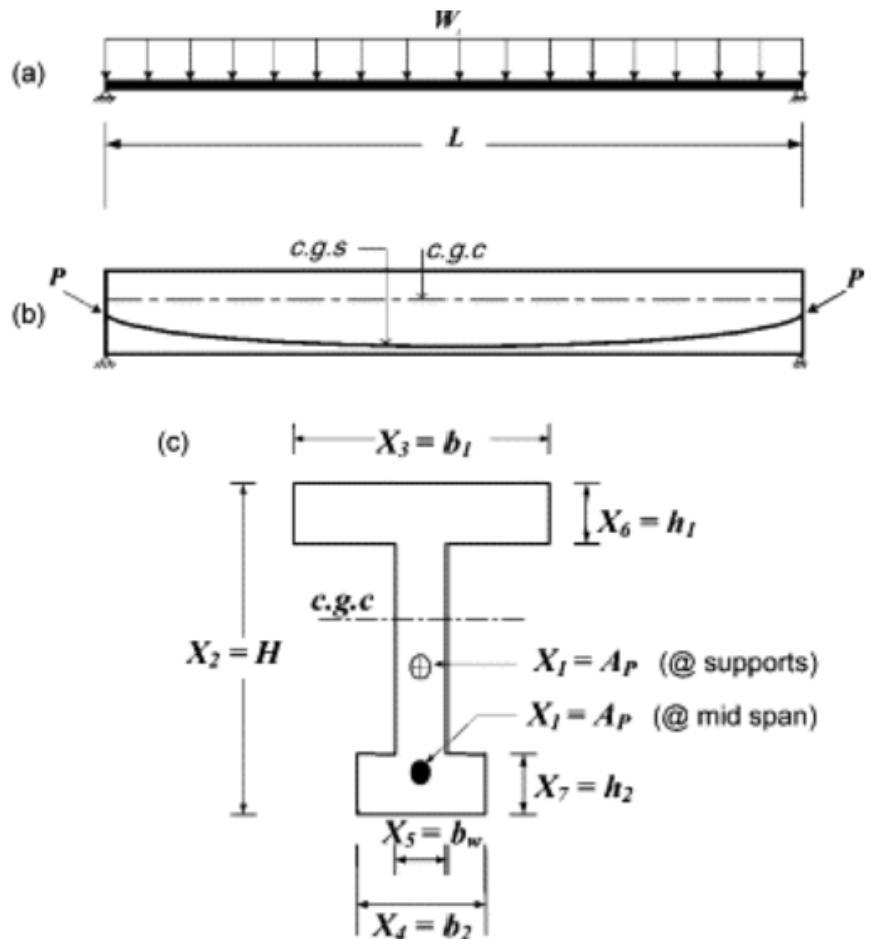
## Topology Optimization



# Structural Design Optimization

## Numerical Optimization: Choice of design variable

### Concrete Beam Cross-section and reinforcement layout



### Concrete Frame Reinforcement layout

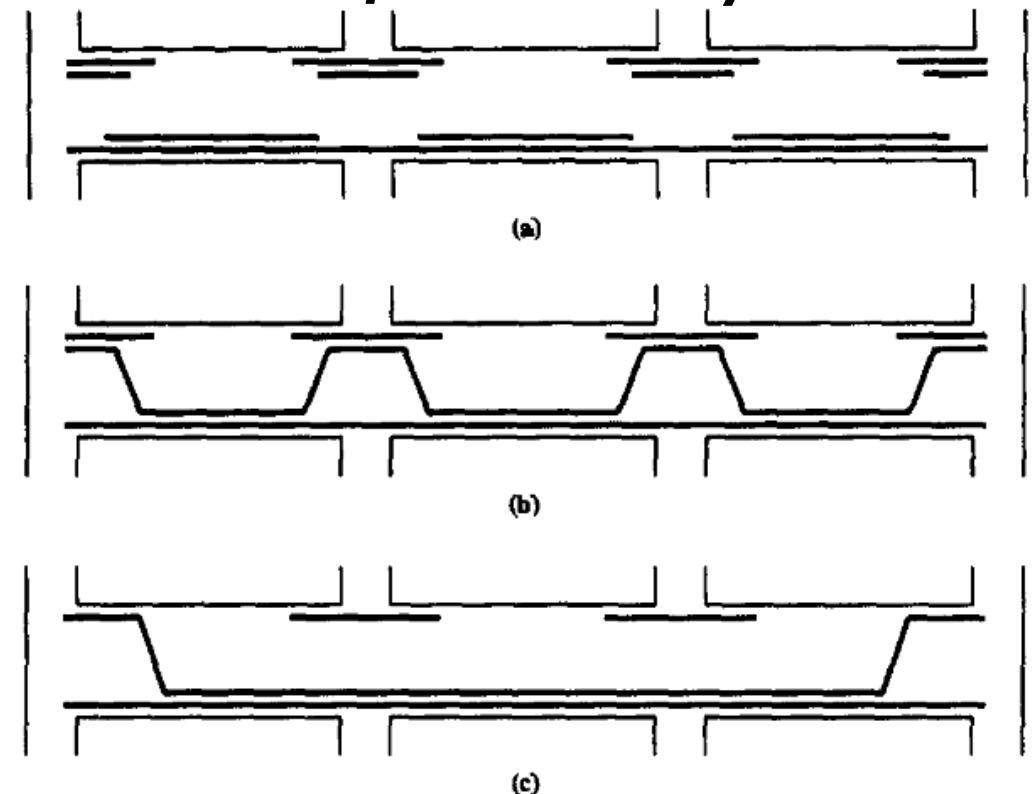
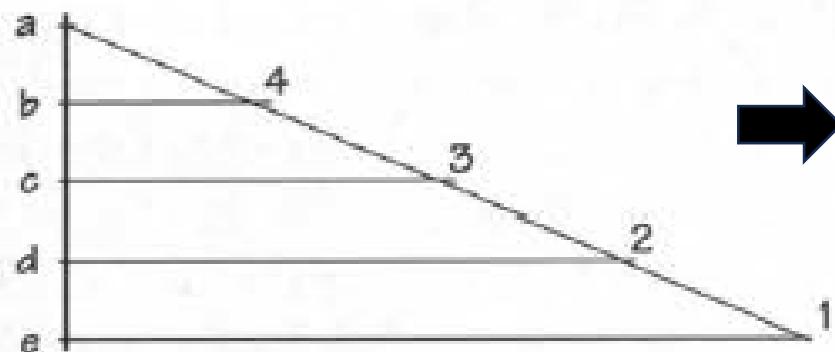


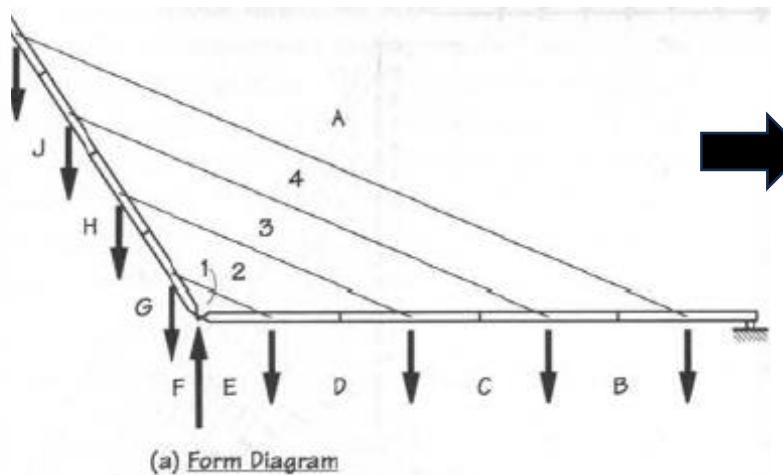
FIG. 2. Beam-Reinforcement Topologies: (a) Topology with 12 Bar Groups; (b) Topology with Six Bar Groups; (c) Topology with Four Bar Groups

# Structural Design Optimization

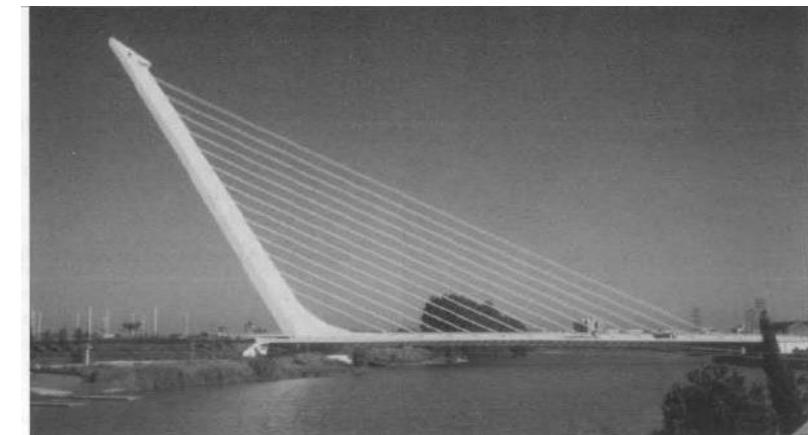
**Graphic Statics → form-finding**



(b) Partial Force Polygon



(a) Form Diagram



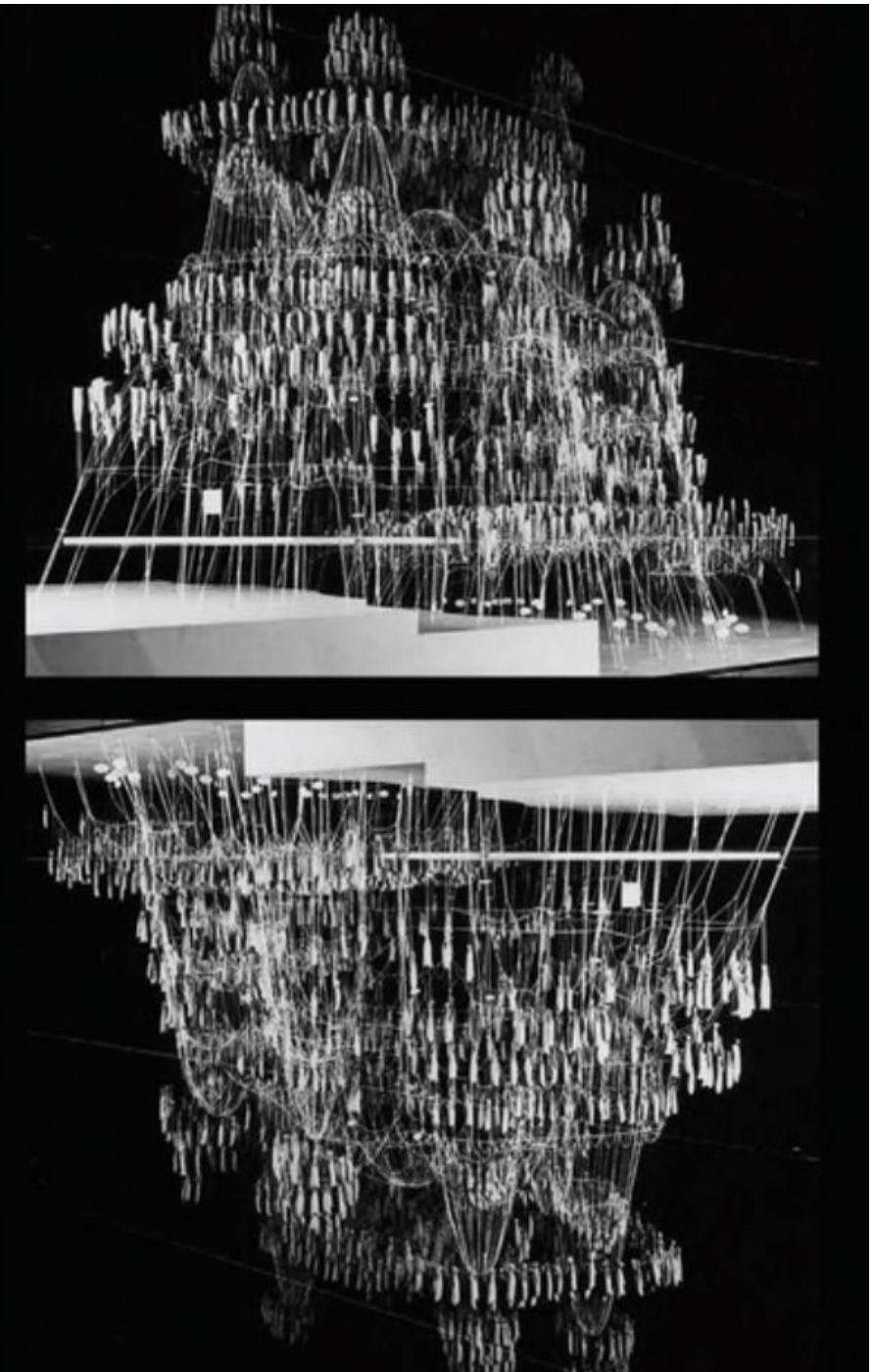
Santiago Calatrava: Alamillo Bridge, Sevilla, 1992

# Form Finding

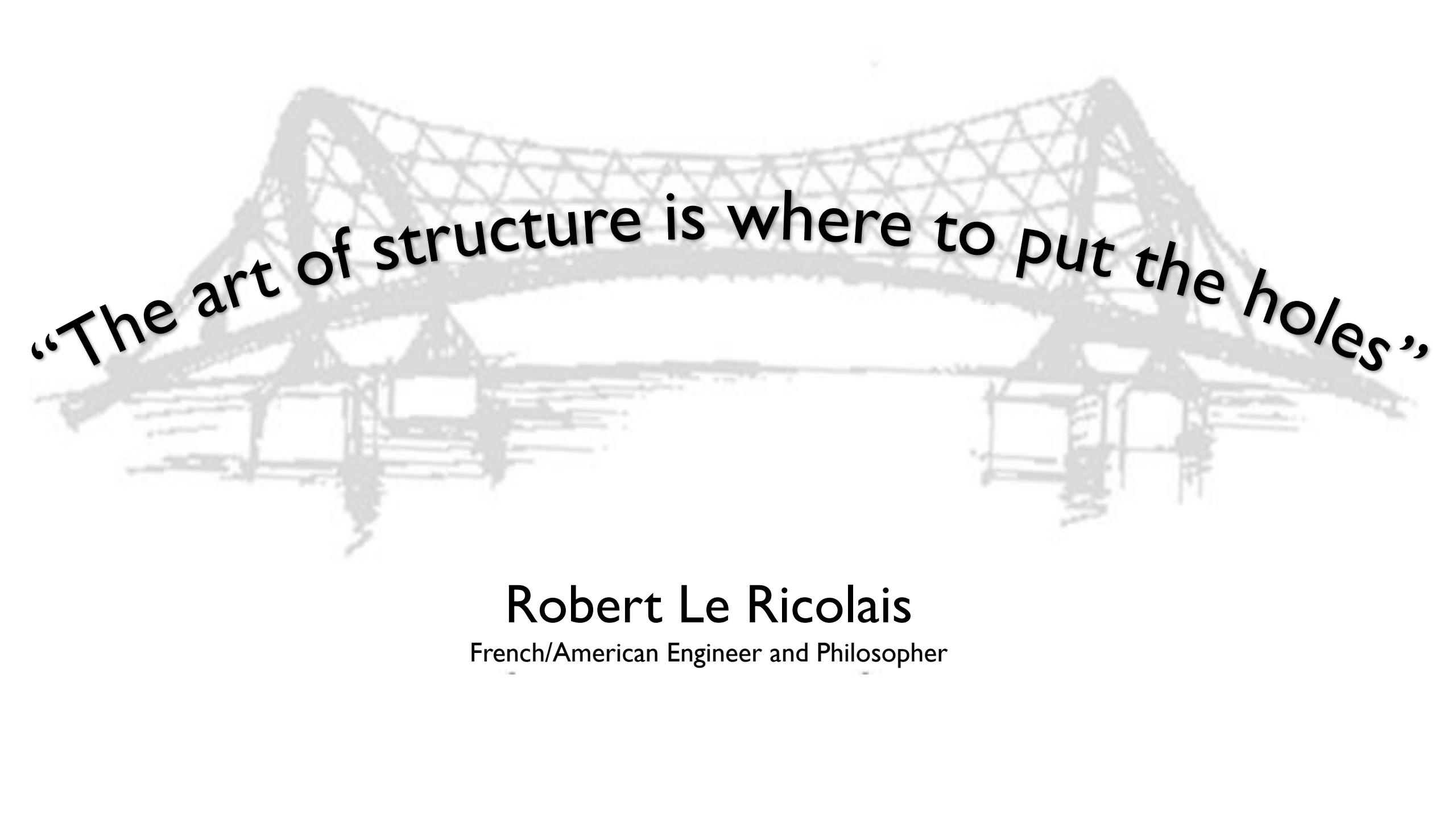
*Physical models*

1889 – Gaudí's Hanging Chain Models





[Crypt, Colònia Güell, Barcelona](#)



**“The art of structure is where to put the holes,”**

**Robert Le Ricolais**

French/American Engineer and Philosopher

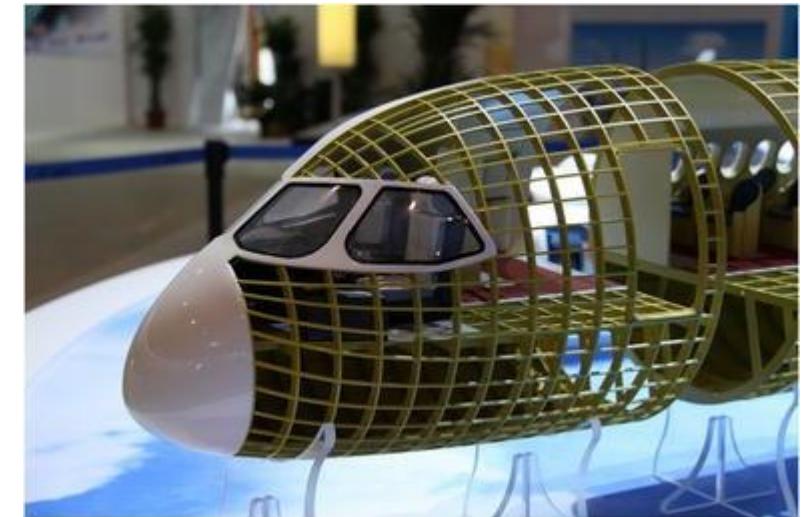
# “Holy Structures”



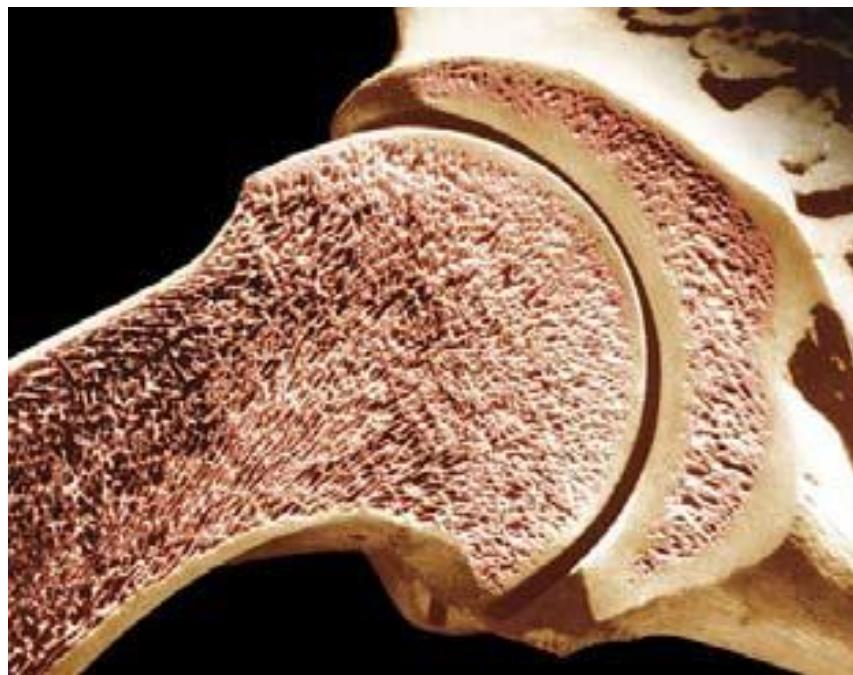
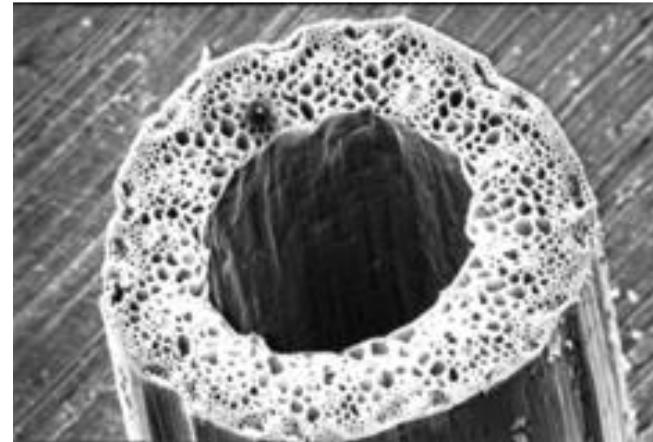
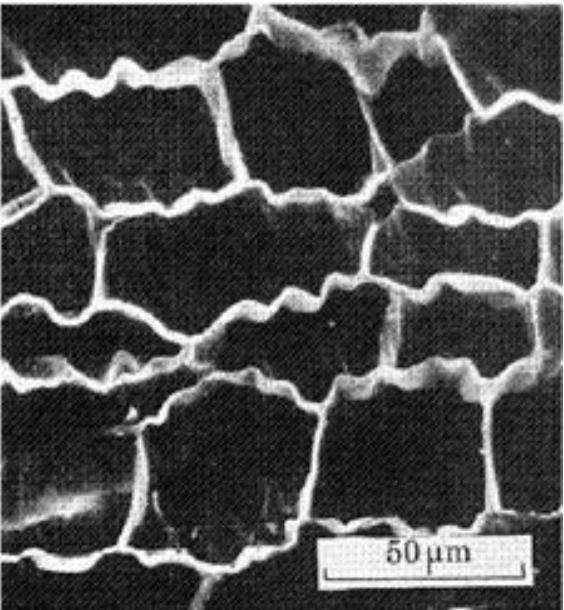
## Optimal use of material

Maximal stiffness/mass

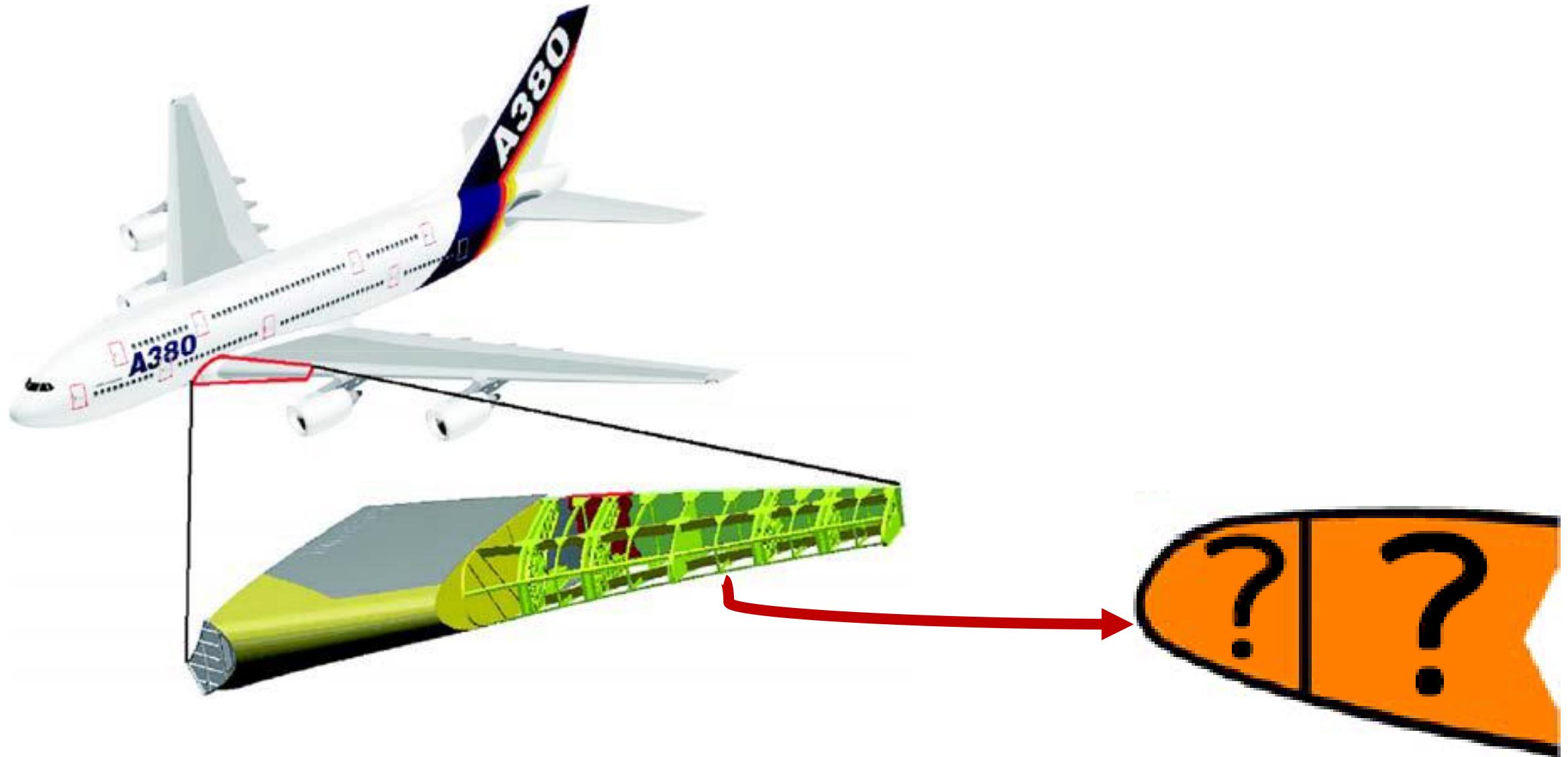
Maximal strength/mass



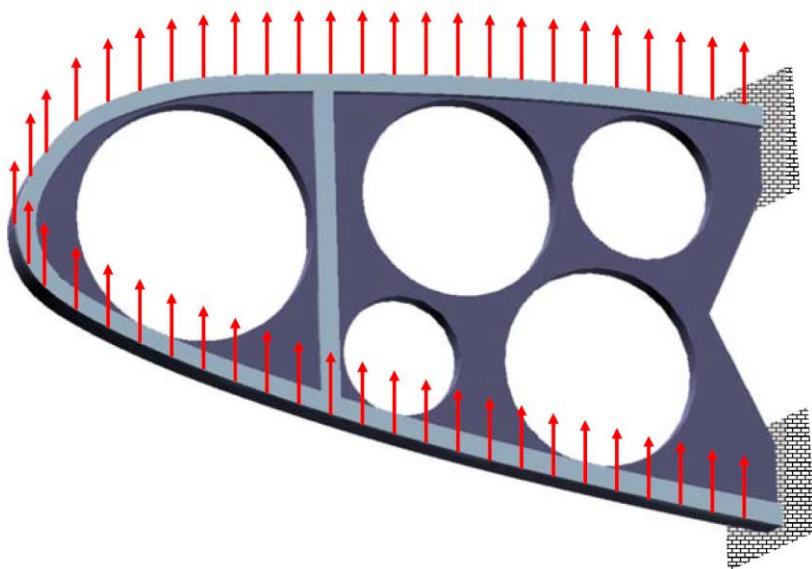
# Holes in nature



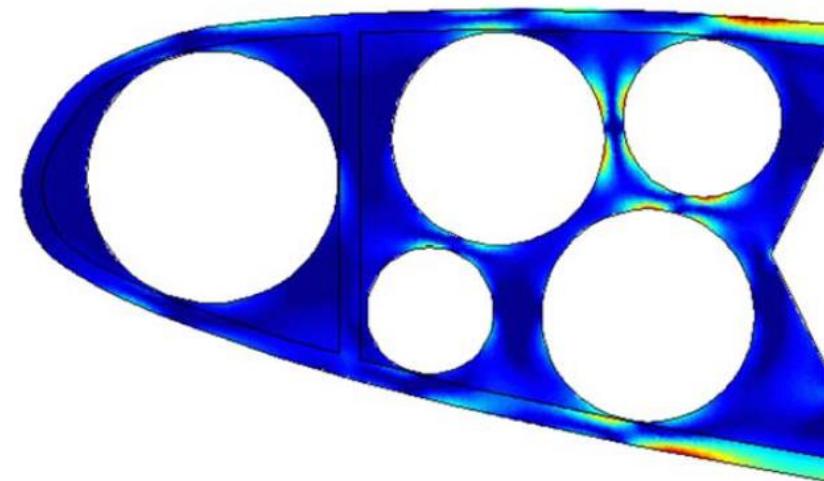
# The design of the web frame of aircraft wings



# Circular holes

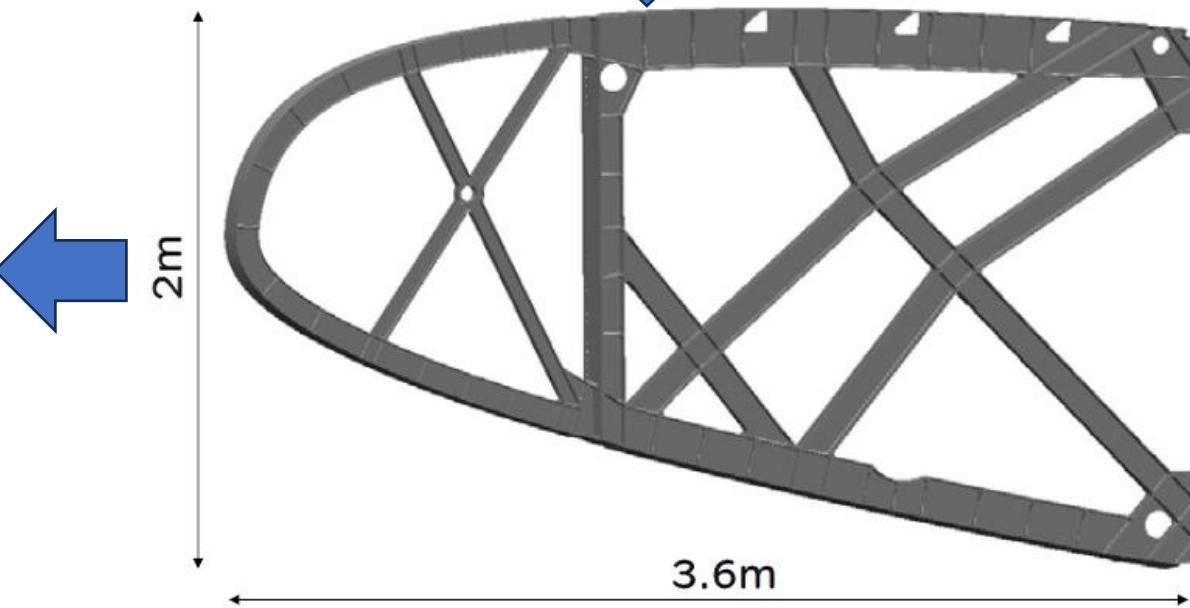
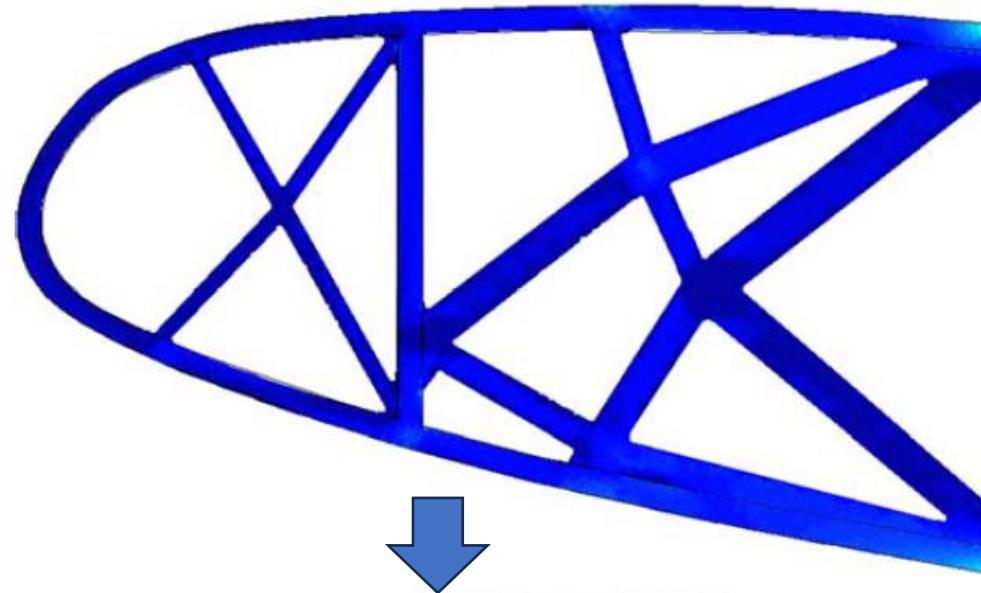
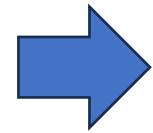
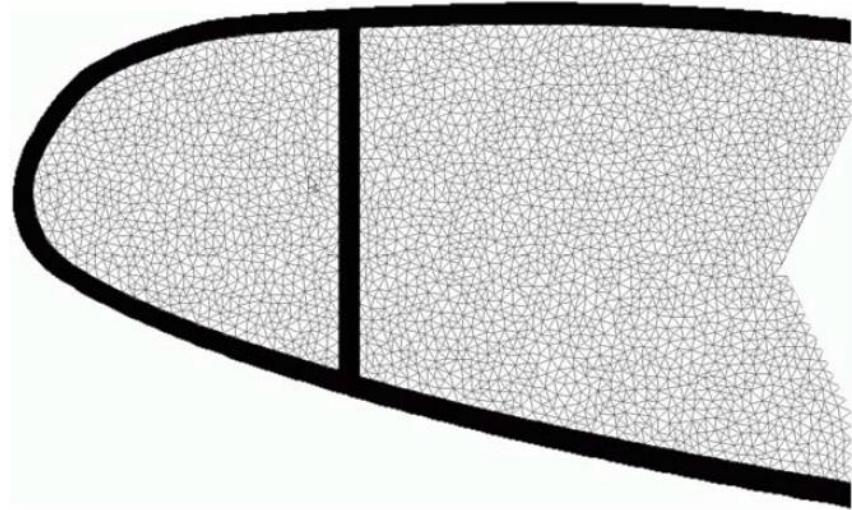


Strain Energy Density Function



# A more systematic approach: topology optimization

Strain Energy Density Function



Courtesy of "EADS Military Aircraft"

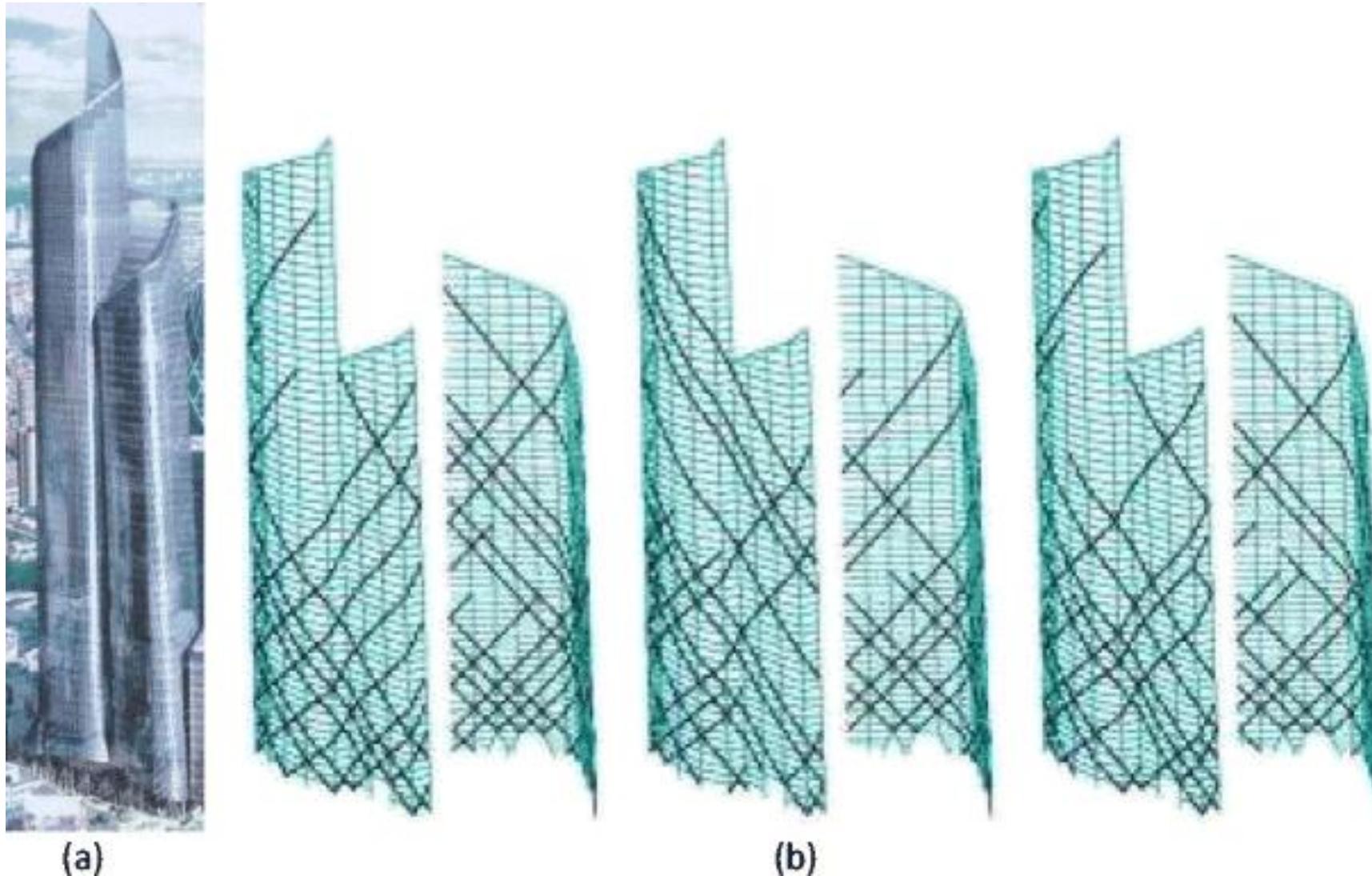
# The design of lateral-force-resisting system in buildings



Stromberg, L. L., Beghini, A., Baker, W. F., & Paulino, G. H. (2012). *Engineering Structures*, 37, 106-124.

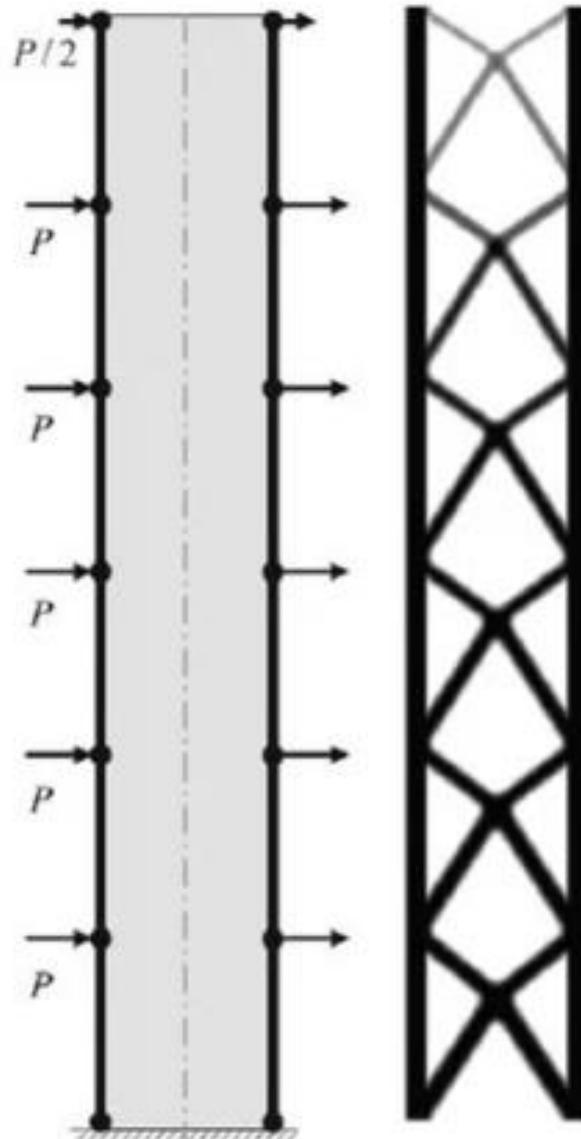
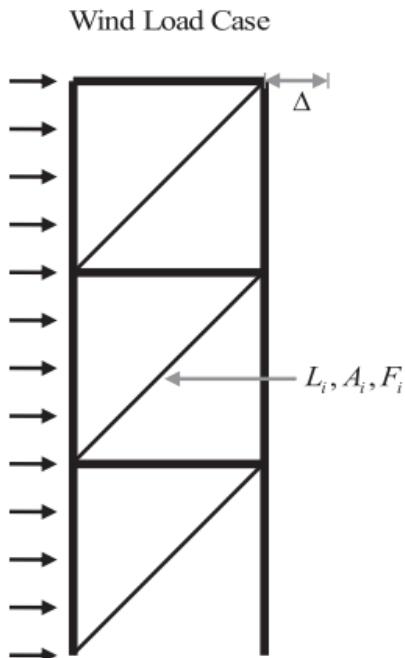
# The design of lateral-force-resisting system in buildings

The Pinnacle Tower (a) General view (b) Three possible profiles for the framing spirals of steel tubes



# The design of lateral-force-resisting system in buildings

What are other idealization possibilities or loading cases?

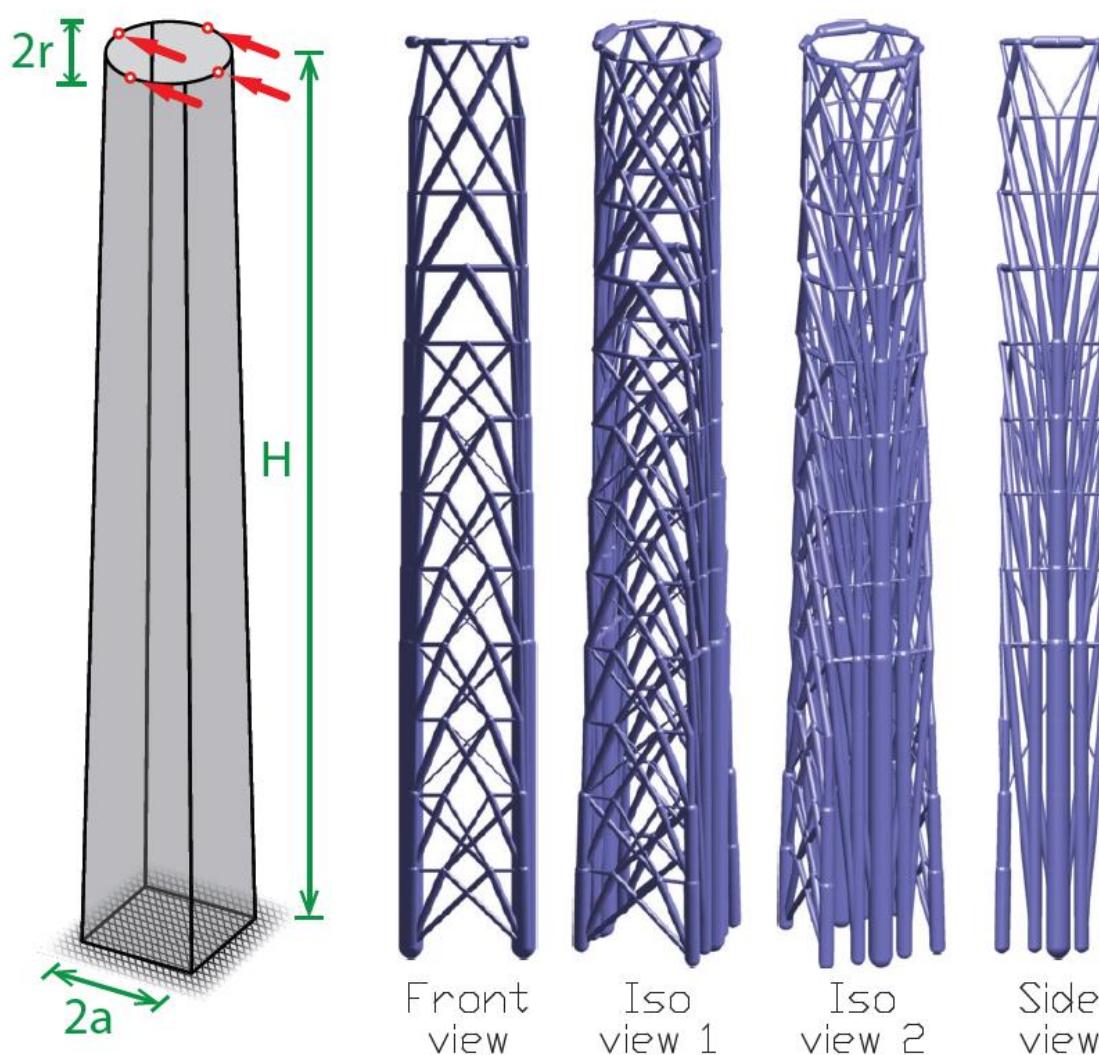


**Idealize** the lateral forces as concentrated forces at each floor

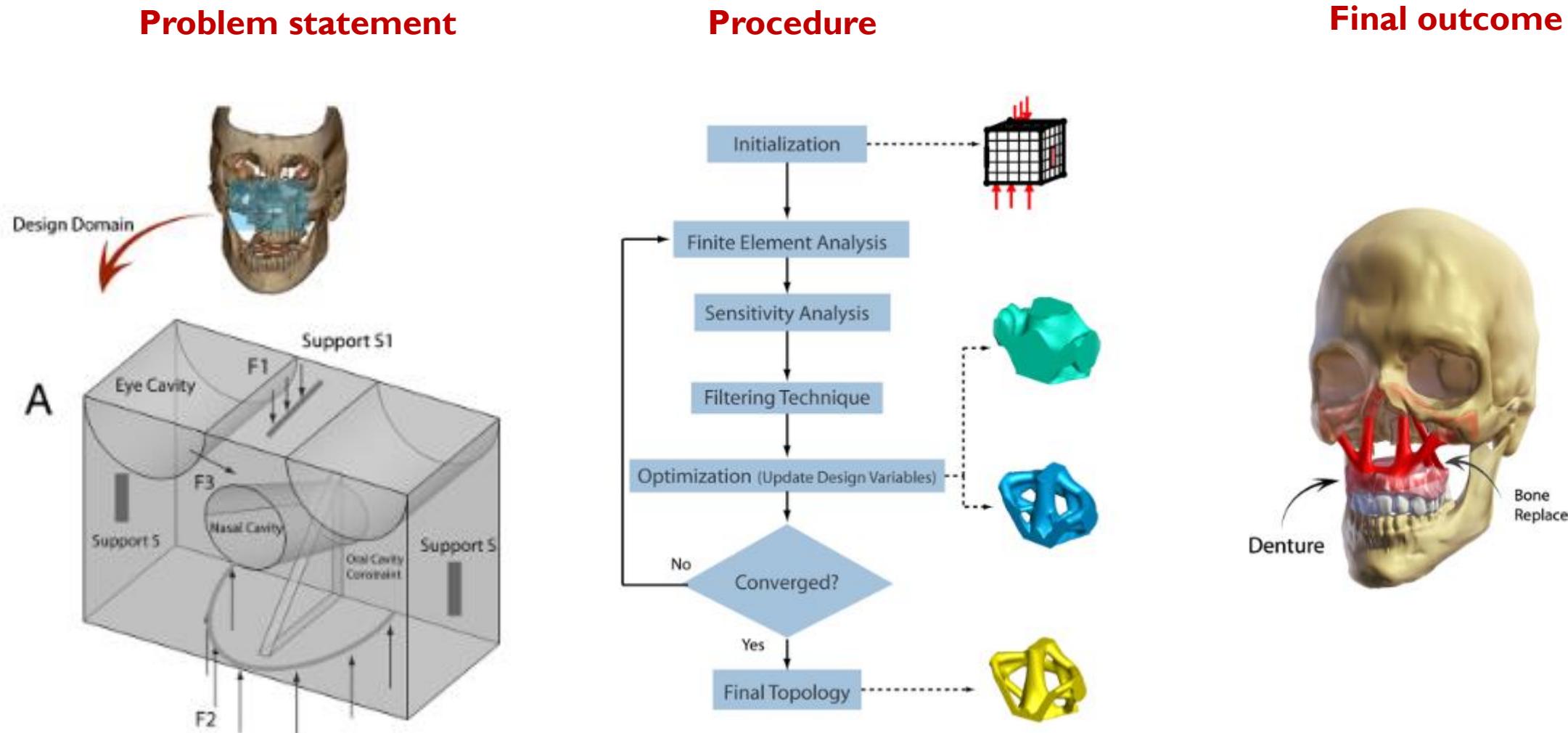


SOM design

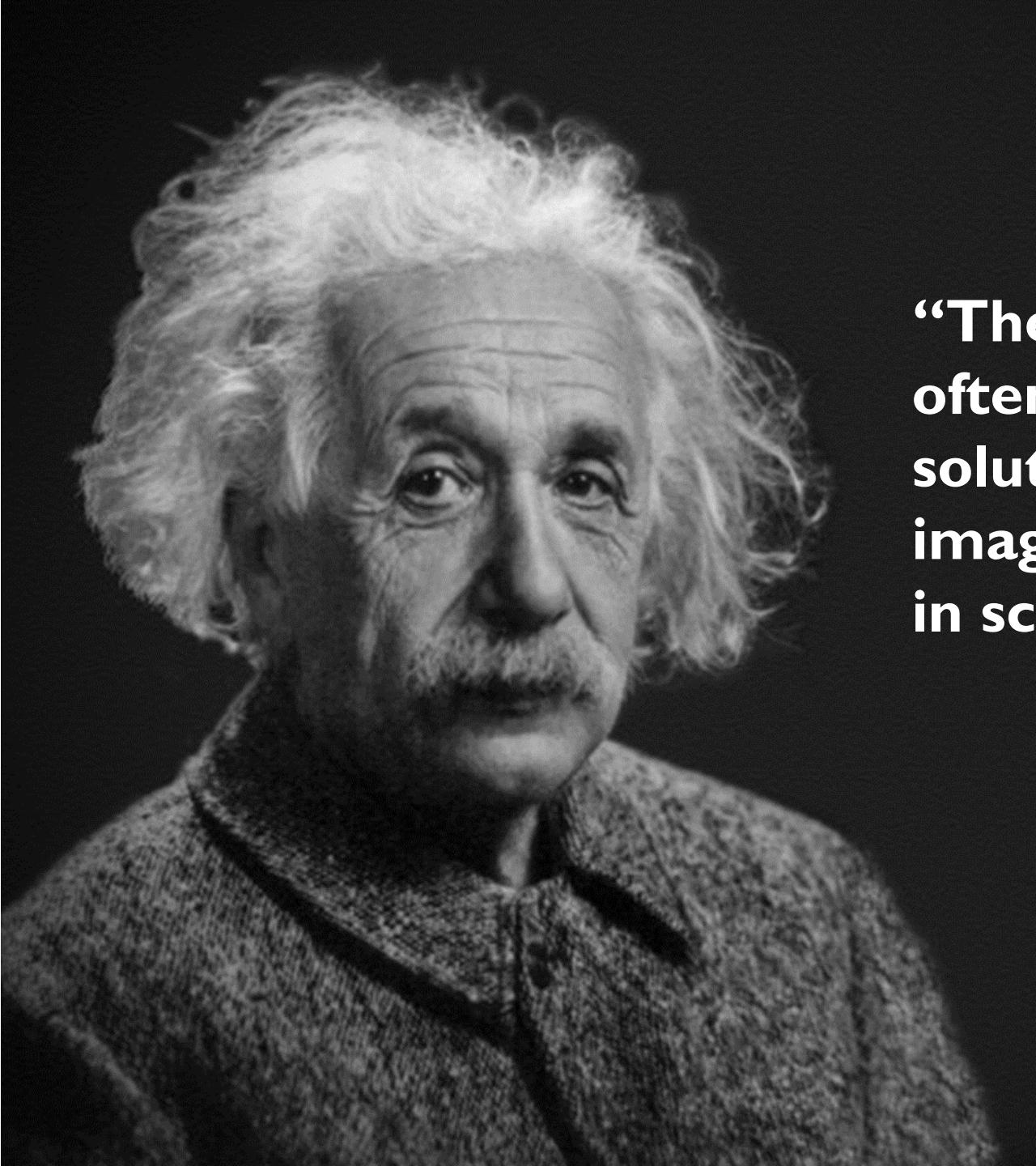
# The design of lateral-force-resisting system in buildings



# Topology optimization for segmental bone replacement



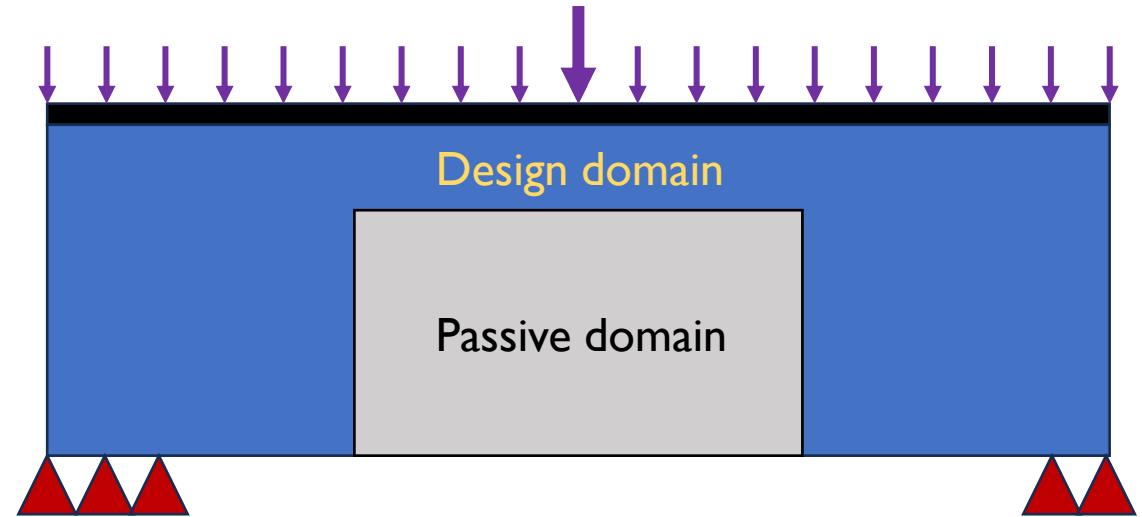
Sutradhar et al. (2010). Proceedings of the National Academy of Sciences, 107(30), 13222-13227.

A black and white portrait of Albert Einstein, showing him from the chest up. He has his characteristic wild, white hair and a full, bushy white beard. He is looking slightly to the right of the camera with a thoughtful expression. The background is dark and out of focus.

**“The mere formulation of a problem is often far more essential than its solution, which [...] requires creative imagination and marks real advances in science”**

**-Albert Einstein, 1921**

# Topology optimization: Problem setup

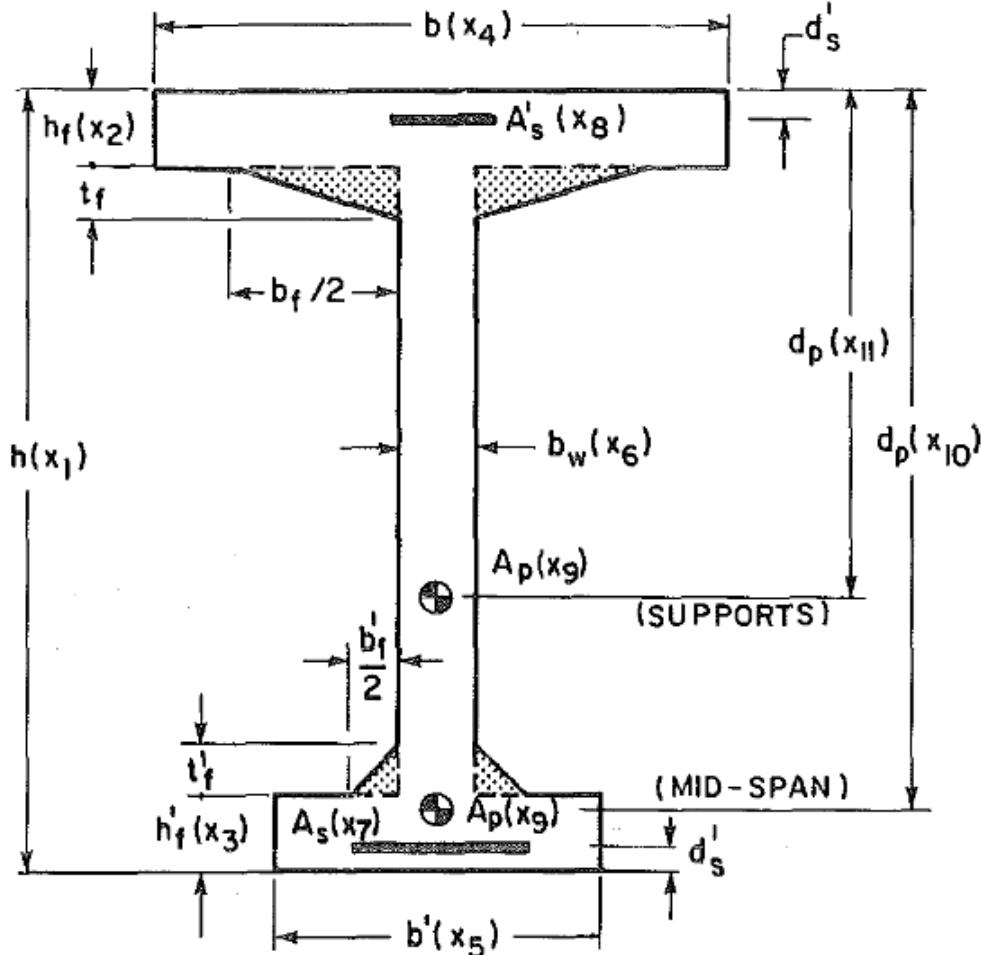


- **Objective (maximize stiffness or others)**
- **Domain condition**
  - Enforced domain: region where materials must be present
  - Design domain: region where materials can be freely arranged
  - Passive domain: region where there must be no material
- **Loading condition**
- **Support condition**

# Structural Design Optimization in a Broader Sense

# Concrete Beam

## ***Cross-section and reinforcement layout***



## What's my objective(s)?

1. Minimum member cost:  $c_c, c_s, c_p, c_f \neq 0$ .
  2. Minimum weight:  $c_c \neq 0; c_s = c_p = c_s = 0$ .
  3. Minimum reinforcement:  $c_c = c_f = 0; c_s, c_p \neq 0$ .
  4. Minimum prestressing:  $c_c = c_f = c_s = 0; c_p \neq 0$ .

## What contributes to the objective?

Material, formwork, labor, transportation, foundation, etc.?

## What are the design variables?

## Dimensions, locations, amount, concrete mix, etc. ?

## What are the constraints?

Loading, strength, stiffness, ductility, fatigue, cracking, construction, etc.?

## General form of an optimization problem

*Objective function  
(merit/cost function)*

$$\min f(\underline{x})$$

*Subject to:*

*Inequality constraints*

$$g_j(\underline{x}) \leq 0 \quad j = 1, \dots, p$$

*Equality constraints*

$$h_k(\underline{x}) = 0 \quad k = 1, \dots, m$$

*Box constraints*

$$x_i^L \leq x_i \leq x_i^U$$

$$\begin{aligned} \underline{x} &\rightarrow \text{Vectors} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \\ \underline{A} &\rightarrow \text{Matrix} \end{aligned}$$

$n \times 1$

DVs

(Design  
Variables)

# General form of an optimization problem

*Objective function*

$$\min f(\underline{x})$$

*(merit/cost function)*

*Subject to:*

*Inequality constraints*

$$g_j(\underline{x}) \leq 0 \quad j = 1, \dots, p$$

*Equality constraints*

$$h_k(\underline{x}) = 0 \quad k = 1, \dots, m$$

*Box constraints*

$$x_i^L \leq x_i \leq x_i^U$$

Type I: Linear Programming: Both objective AND constraints are linear

Type II: Nonlinear Programming: Objective AND/OR constraints are nonlinear

# Course Project

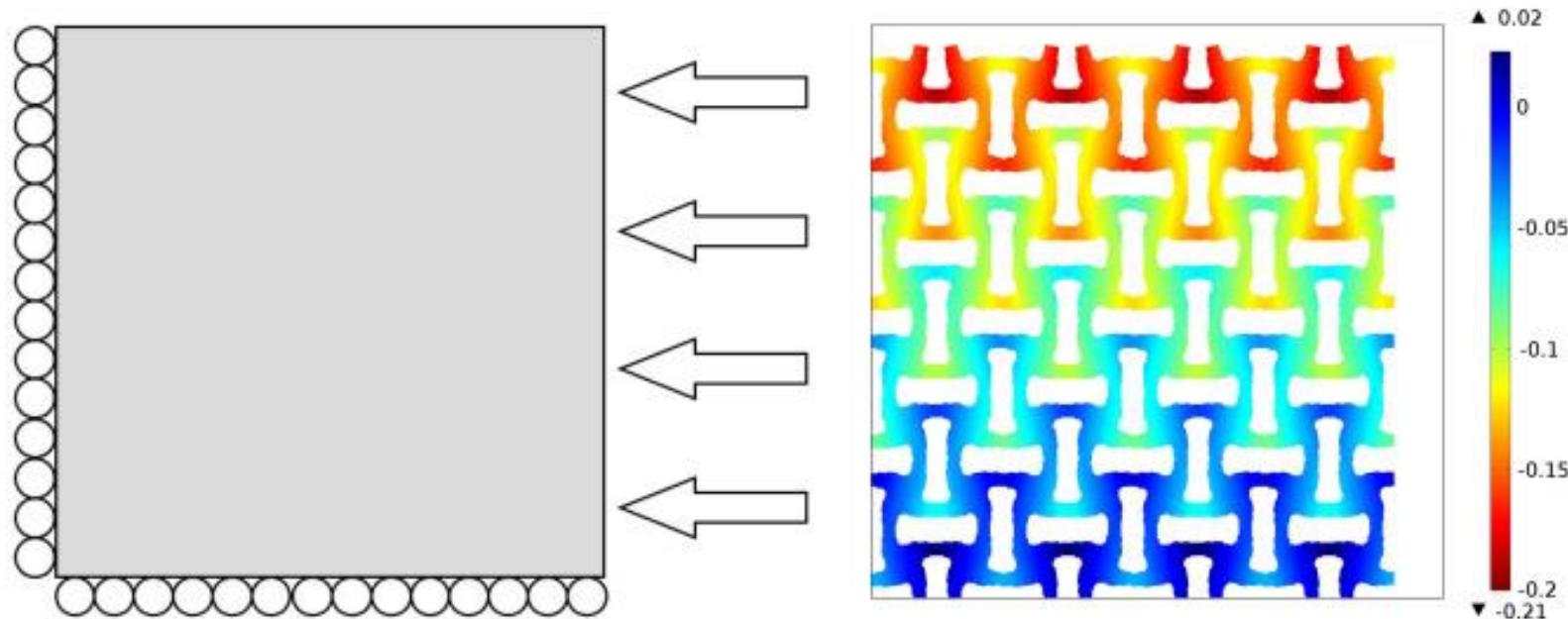
## *Topic*

**Optimize any structures of your choice!**

# Course Project

## Various scale

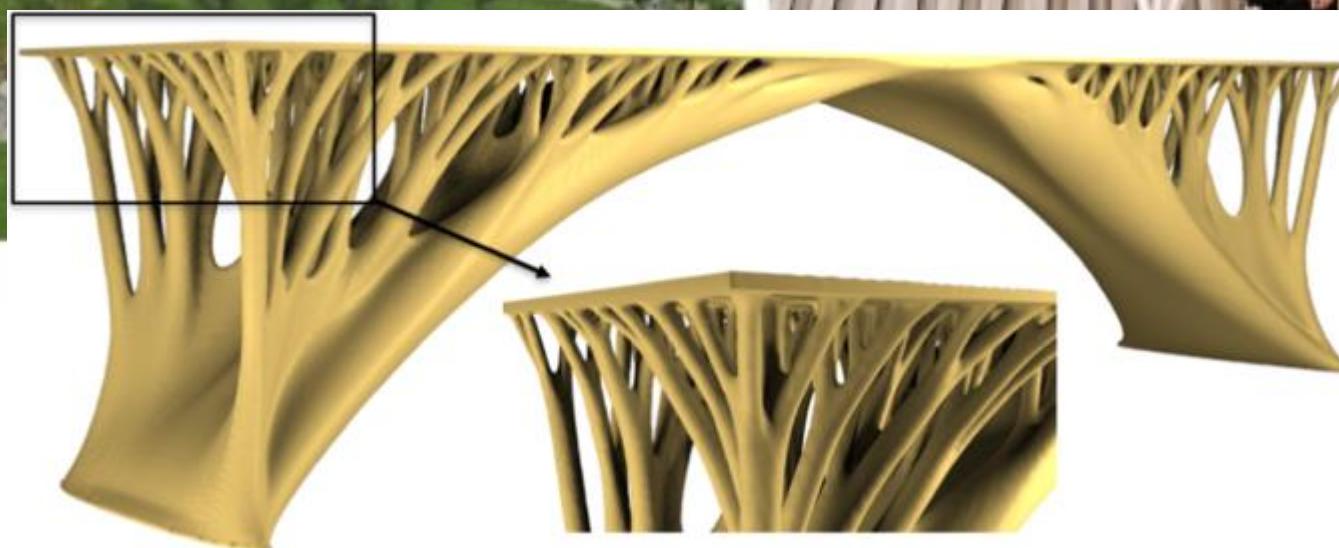
### Material scale



# Course Project

*Various scale*

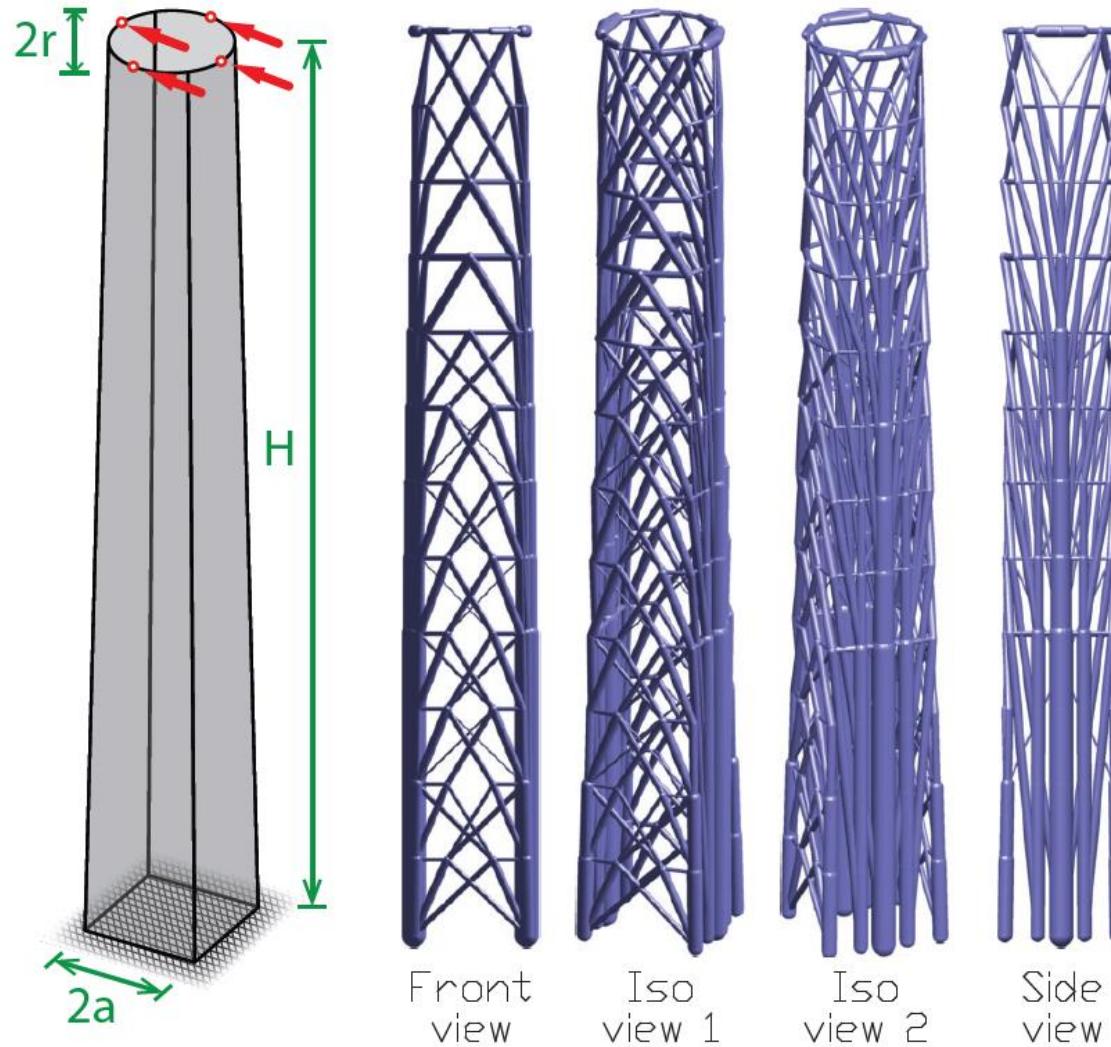
## Component scale



# Course Project

**Various scale**

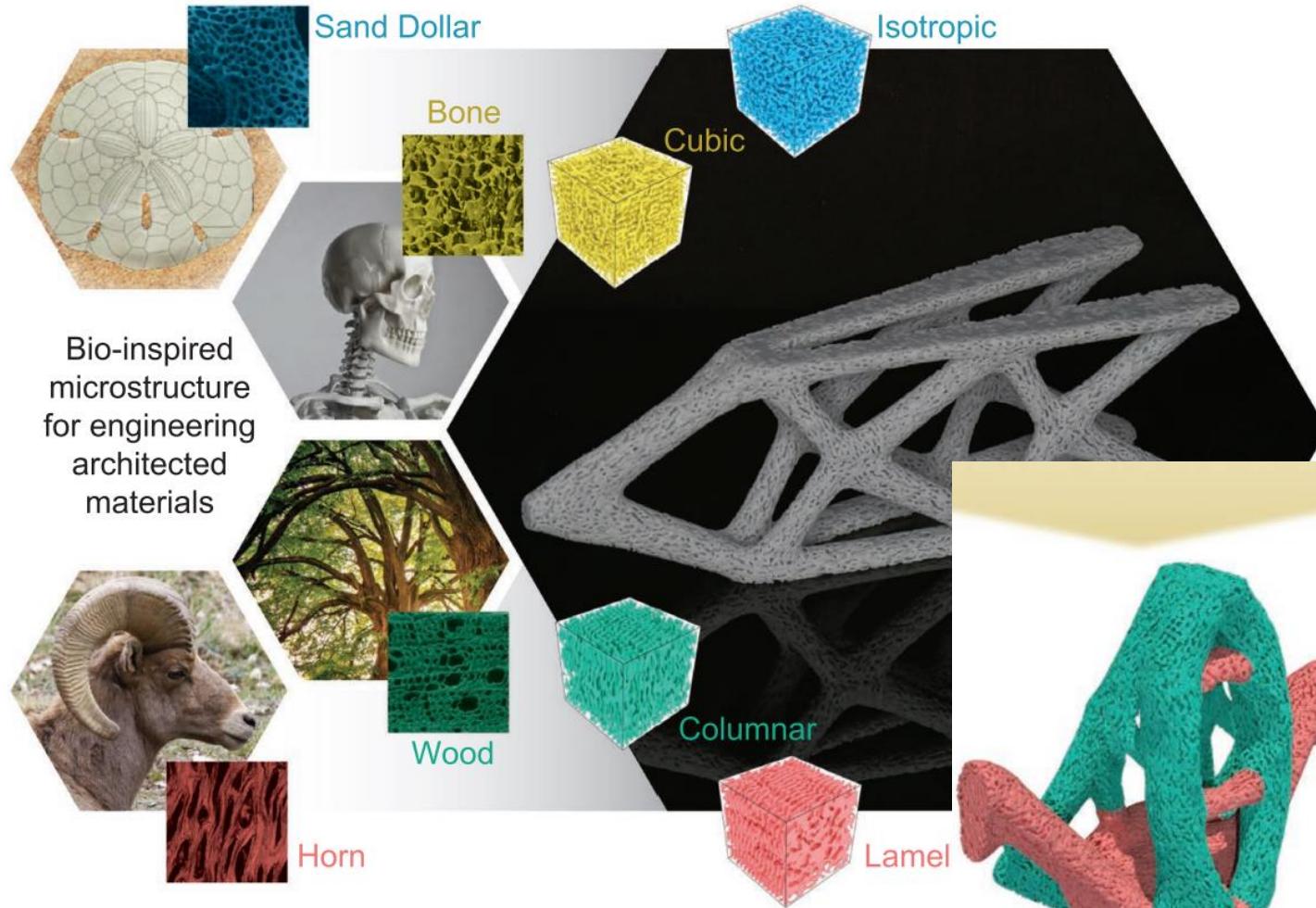
## System scale



# Course Project

## Various scale

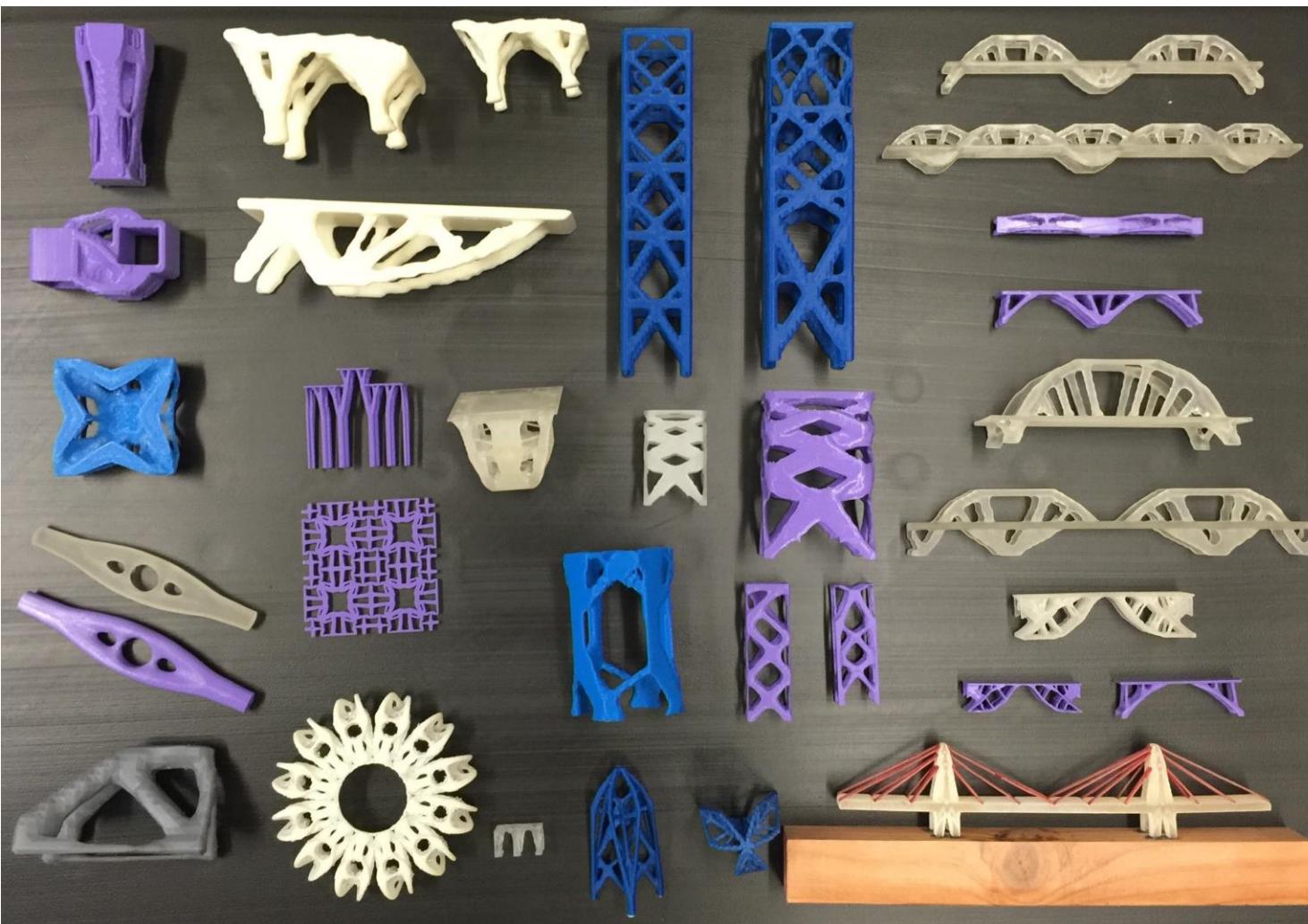
### Across scale



# Course Project

## *Past Examples*

12:45



# Course Project

**Think about something cool that you could  
put in your CV or Website ☺**

# Course Project

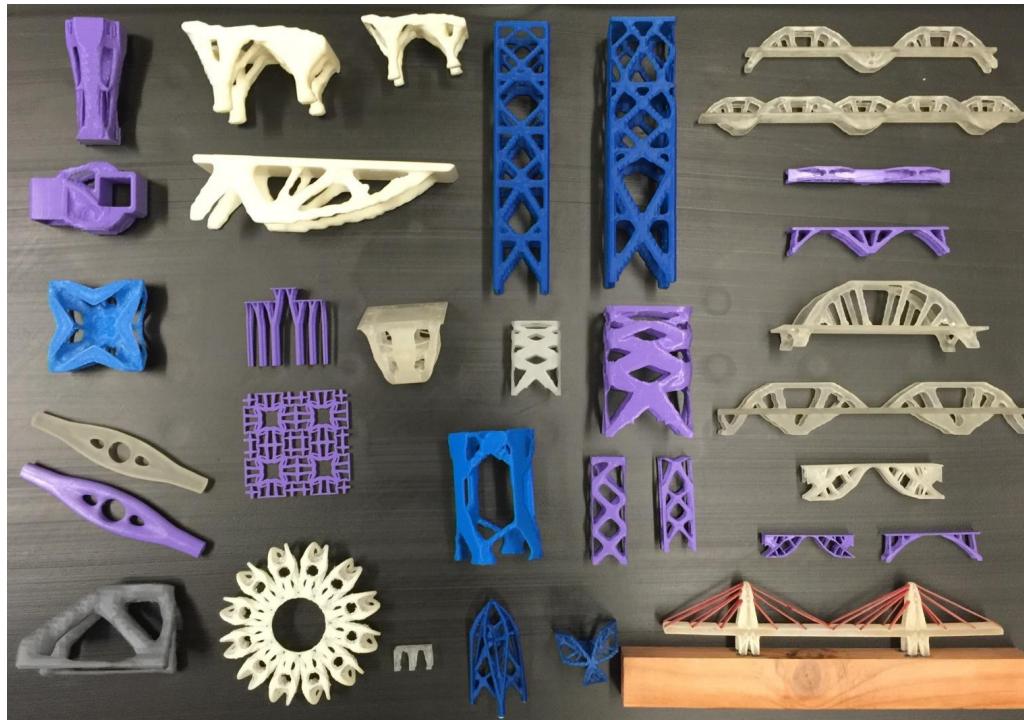
## *Scope*

1. Background (design background, state-of-the-art practice, typical loading and support patterns)
2. Simplify into optimization setting
3. Optimization (any existing packages or write your own)
4. Results and discussions
5. Fabrication (we will only provide suggestions, you need to determine your fabrication method and arrange them)

# Course Project

## ***Path 1***

(Individual) Optimize any structures of your choice. You can use any existing topology optimization programs (e.g., in MATLAB, ANSYS, ABAQUS). You will be asked to fabricate a demonstration model of your optimized structures (e.g., using 3D printing or by hand). You are encouraged to be creative in using other materials and fabrication methods.



# Course Project

## Path 2

(A team of 2 students) Optimize a simply-supported truss bridge. You can use any existing topology optimization programs (e.g., in MATLAB or ABAQUS) and choose your testing protocol. You will be asked to design, fabricate, and test a conventional design for comparison. You will need to fabricate a reduced-scale model and then test on using weights under a simply-supported option. Fabrication and testing may follow a [Spaghetti](#) fabrication and tests.



# Course Project

## Path 3

(Individual or a team of 2 students) Optimize any structures of your choice. You write your own optimization program (no need to be restricted to topology optimization). You need to justify the necessity of using an optimization algorithm for the selected topic. No need to fabricate.

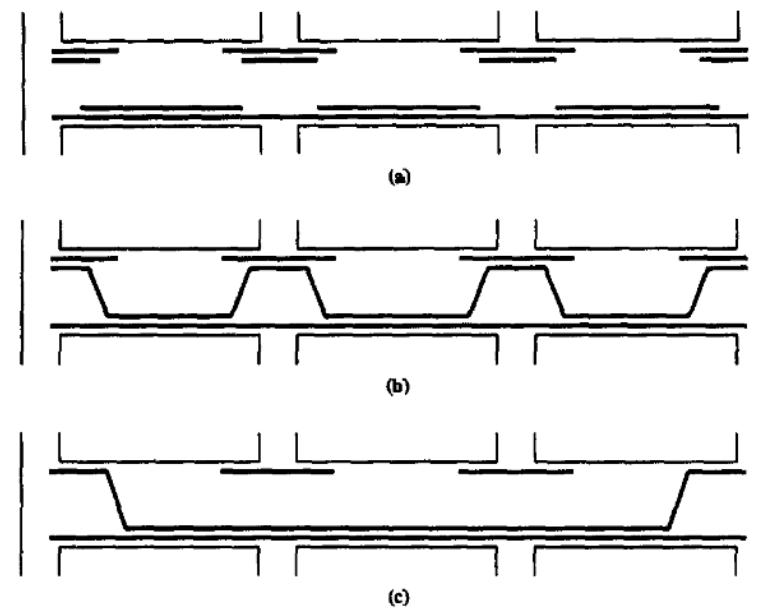
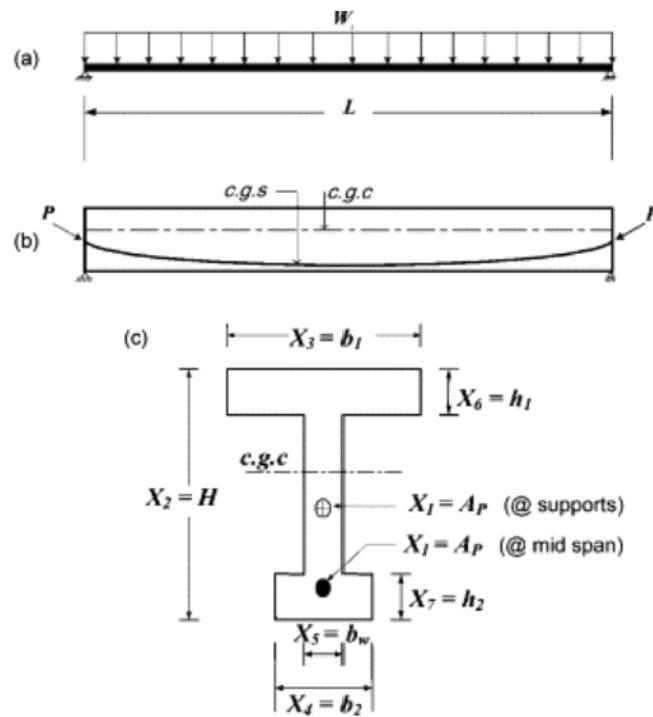
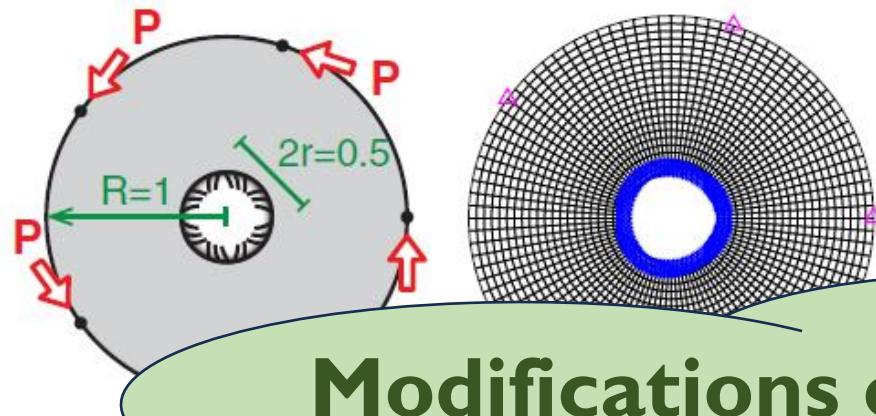


FIG. 2. Beam-Reinforcement Topologies: (a) Topology with 12 Bar Groups; (b) Topology with Six Bar Groups; (c) Topology with Four Bar Groups

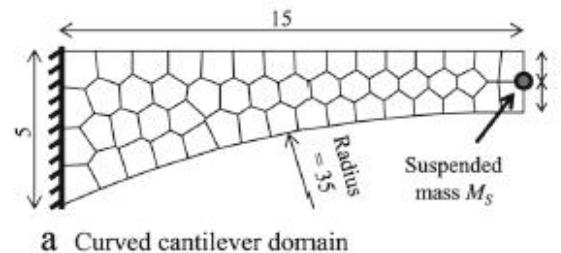
# Course Project

Possible tools in MATLAB that we will cover

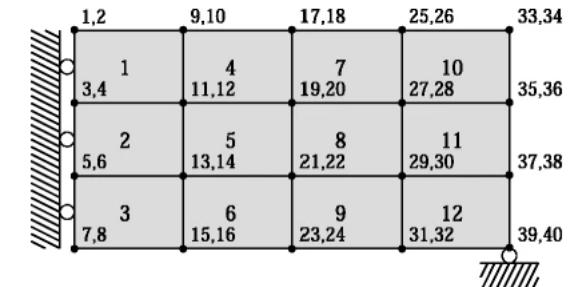
## GRAND in 2D and 3D



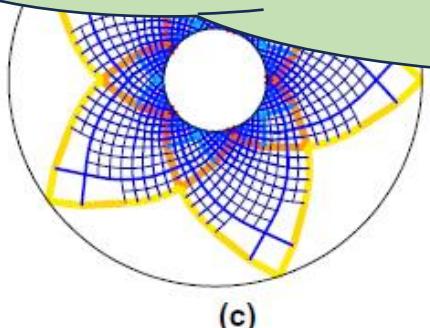
## PolyTop in 2D and 3D



## 88 line TopOpt

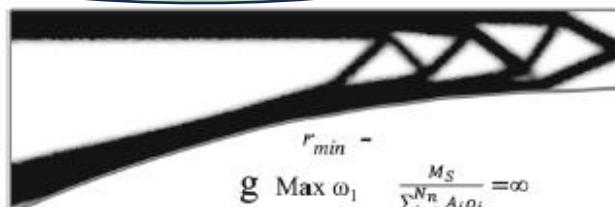


Modifications of these programs to suit your purpose(s) are highly encouraged (and sometimes necessary)



(c)

(d)



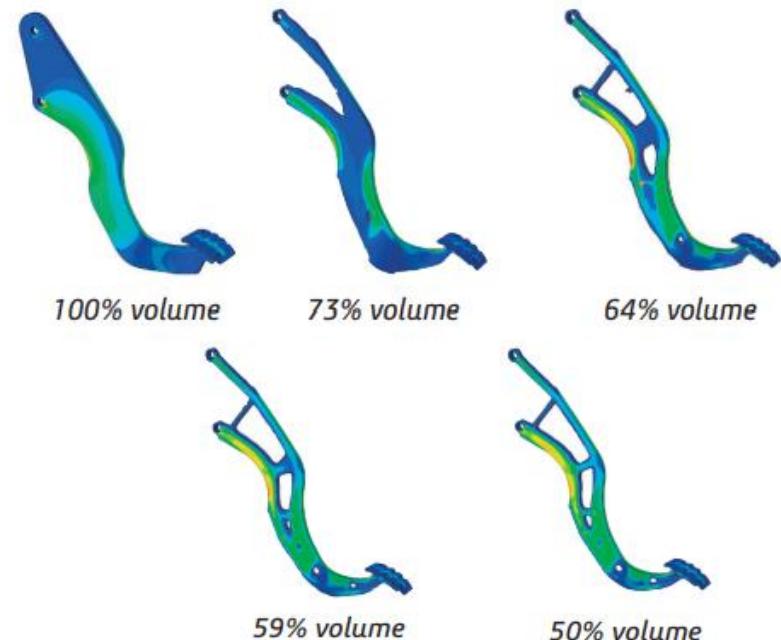
$$g \quad \text{Max } \omega_1 \quad \frac{M_S}{\sum_{i=1}^{N_n} A_i \rho_i} = \infty$$

$c = 233.71$

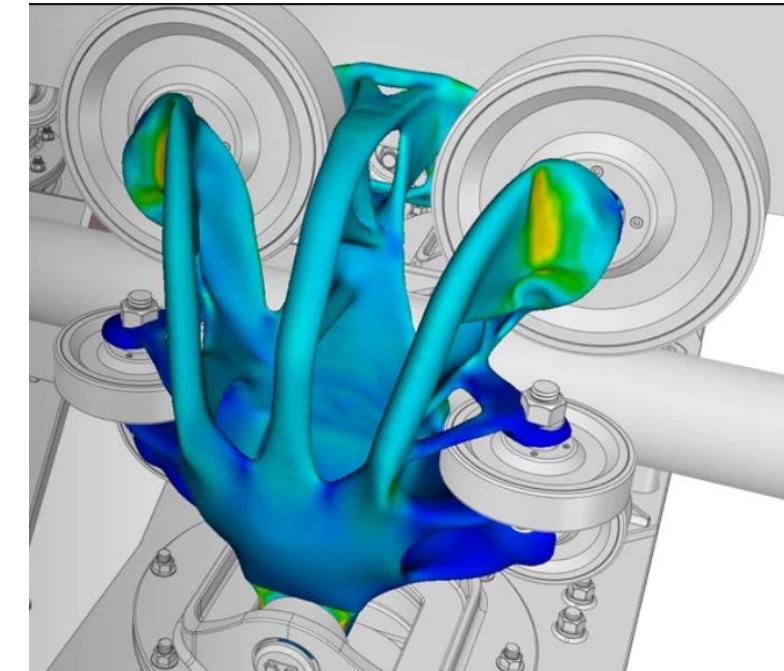
# Course Project

**Possible tools in other commercial software (that we won't cover)**

## ABAQUS



## ANSYS



# Course Project

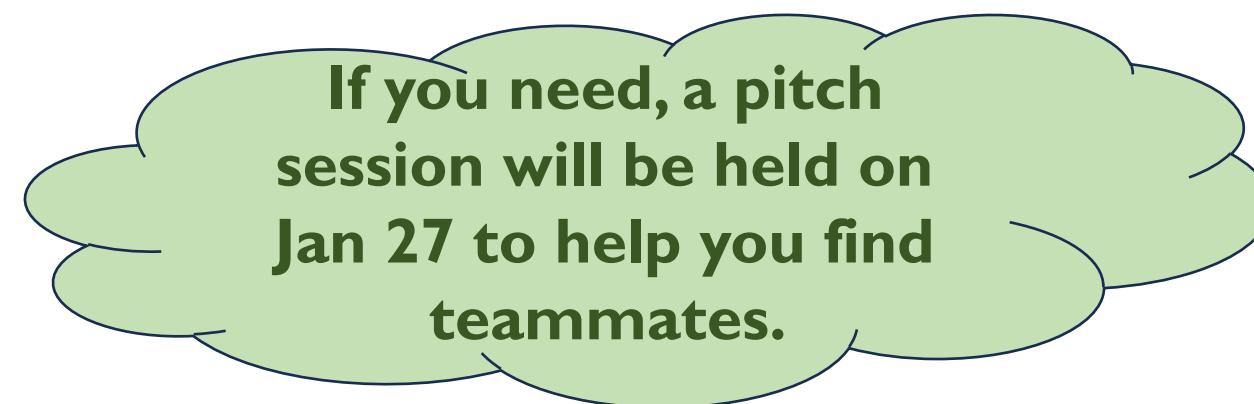
## Deliverables

[Preliminary Approval] For Paths 2&3, preliminary approval (team and topic) must be obtained from the instructor by **02/11/25**.

[Milestone] Upload the topic you have selected by the end of Monday, **03/11/25** on myCourses. The milestone should include the description of background and selection of program/software. If a team is formed, please include task divisions in the milestone.

The project presentations will be held near the end of semester. The exact date and time will be announced. Project report will be due one week after the presentations.

**PechaKucha**  
**12 X 20**



If you need, a pitch session will be held on Jan 27 to help you find teammates.

# 3D Printing Guide

# Course **Grading**

<u>Grading Scheme:</u>	4-6 Assignments <i>(Not graded)</i>	0%
	Project (report + presentation)	20%
	Mid-term exam	30%
	Final exam	50%

**Note:** grades will not be curved so the entire class can all get A if you all perform well (85-100%). I encourage classmates to help each other improve their learning outcomes.

## Tentative and subject to changes

Week	Class Date	Topic	Due
1	Jan 6	Introduction	
2	Jan 13	A Truss Optimization Example and Optimality Condition for Unconstrained Nonlinear Optimization	
3	Jan 20	Lagrange Multiplier, KKT Conditions, Structural Optimization of Concrete Structures	
4	Jan 27	Convexity, KKT Condition w/o Slacks, GRAND	
5	Feb 3	Sensitivity Analysis, Duality, Load Path	
6	Feb 10	Density-Based Method	Path 2&3 Approval
7	Feb 17	Robust Design Optimization, Multiple Load Cases	
8	Feb 24	Mid-term (covering wkl-6)	
9	Mar 3	Reading week, no lecture.	
10	Mar 10	Graphic Statics and Form Finding	Milestone
11	Mar 17	Multi-panel Truss Design Using Form Finding	
12	Mar 24	Reusing Structural Members Based On Graphic Statics	
13	Mar 31	Fabrication Break	
14	Apr 7	Project presentations.	

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**Do you have previous experience with MATLAB?**

① Start presenting to display the poll results on this slide.

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**What are your expectations of this course (anonymous)?**

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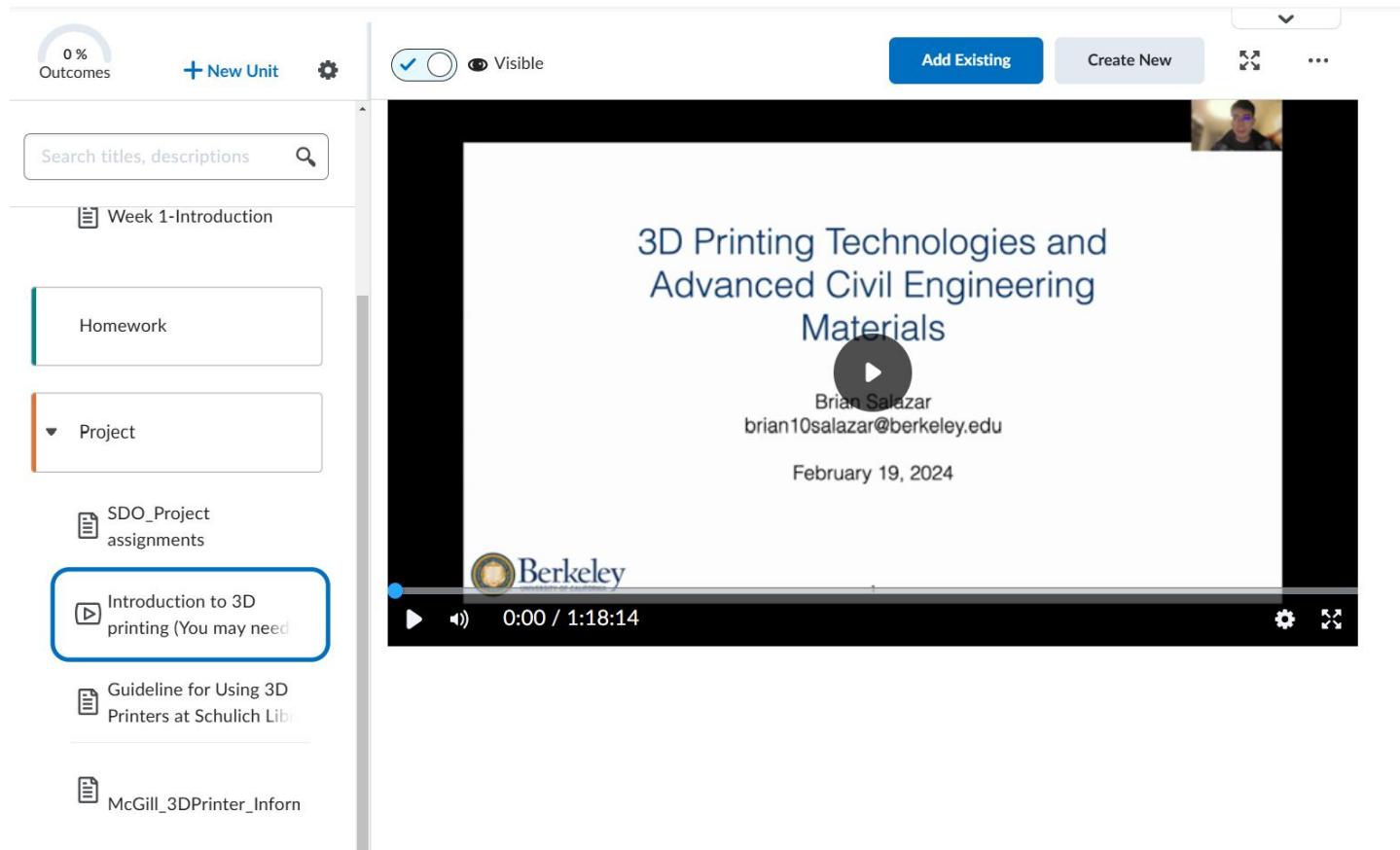
**What are your fears or  
questions of this course  
(anonymous)?**

- ① Start presenting to display the poll results on this slide.

**HW 0:**

**Review project assignment**

**Watch the 3D printing introduction video**



The image shows a screenshot of a learning management system (LMS) interface. On the left, there is a sidebar with the following items:

- 0 % Outcomes
- + New Unit
- Visible (checkbox checked)
- Search titles, descriptions
- Week 1-Introduction
- Homework
- Project (with a dropdown menu showing "SDO\_Project assignments")
- Introduction to 3D printing (You may need) (highlighted with a blue border)
- Guideline for Using 3D Printers at Schulich Lib.
- McGill\_3DPrinter\_Inform

The main content area displays a video player for a video titled "3D Printing Technologies and Advanced Civil Engineering Materials" by Brian Salazar. The video was recorded on February 19, 2024. The video player interface includes a play button, a progress bar showing 0:00 / 1:18:14, and a Berkeley logo. At the top of the video player, there are buttons for "Add Existing", "Create New", and a three-dot menu.

# HW 0: Please download MATLAB before next class

## ***Familiarize with fmincon***



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## fmincon

Find minimum of constrained nonlinear multivariable function

### Syntax

```
x = fmincon(fun,x0,A,b)
x = fmincon(fun,x0,A,b,Aeq,beq)
x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub)
x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon)
x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)
x = fmincon(problem)
[x,fval] = fmincon(__)
[x,fval,exitflag,output] = fmincon(__)
[x,fval,exitflag,output,lambda,grad,hessian] = fmincon(__)
```

### Description

Nonlinear programming solver.

Finds the minimum of a problem specified by

$$\min_x f(x) \text{ such that} \begin{cases} c(x) \leq 0 \\ ceq(x) = 0 \\ A \cdot x \leq b \\ Aeq \cdot x = beq \\ lb \leq x \leq ub, \end{cases}$$

*b* and *beq* are vectors, *A* and *Aeq* are matrices, *c(x)* and *ceq(x)* are functions that return vectors, and *f(x)* is a function that returns a scalar. *f(x)*, *c(x)*, and *ceq(x)* can be

*x*, *lb*, and *ub* can be passed as vectors or matrices; see [Matrix Arguments](#).

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**fmincon**

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Syntax

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Linear Inequality Constraint

Linear Inequality and Equality Constraint

Minimize with Bound Constraints

**Nonlinear Constraints**

Nondefault Options

Include Gradient

Use a Problem Structure

Obtain the Objective Function Value

Examine Solution Using Extra Outputs

Obtain All Outputs

Input Arguments

Output Arguments

Limitations

More About

Algorithms

[Documentation](#) [Examples](#) [Functions](#) [Apps](#) [Videos](#) [Answers](#)[Trial software](#) [Product updates](#)**Nonlinear Constraints**

Find the minimum of a function subject to nonlinear constraints

Find the point where Rosenbrock's function is minimized within a circle, also subject to bound constraints.

[Open in MATLAB Online](#)[Copy Command](#)

```
fun = @(x)100*(x(2)-x(1)^2)^2 + (1-x(1))^2;
```

[Get ▾](#)

Look within the region  $0 \leq x(1) \leq 0.5$ ,  $0.2 \leq x(2) \leq 0.8$ .

```
lb = [0,0.2];
ub = [0.5,0.8];
```

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Also look within the circle centered at  $[1/3,1/3]$  with radius  $1/3$ . Use this code for the nonlinear constraint function.

```
function [c,ceq] = circlecon(x)
c = (x(1)-1/3)^2 + (x(2)-1/3)^2 - (1/3)^2;
ceq = [];
end
```

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There are no linear constraints, so set those arguments to `[]`.

```
A = [];
b = [];
Aeq = [];
beq = [];
```

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Choose an initial point satisfying all the constraints.

```
x0 = [1/4,1/4];
```

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Solve the problem.



# Some quick play tools

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