# CIVE 546 – Structural Design Optimization (SDO)

## HW1-Solution

### Problem 1

From geometry we know that

$$\sin\theta = \frac{H}{L} \quad \rightarrow \quad L = \frac{H}{\sin\theta}$$

The problem is then formulated as

$$\max_{\theta} f(\theta) = \max_{\theta} \frac{k}{V} = \max_{\theta} \frac{EA\cos^2\theta}{AL^2} = \max_{\theta} E\cos^2\theta\sin^2\theta/H^2$$

The derivative is then

$$\frac{df}{d\theta} = \frac{E}{H^2}[-2\cos\theta\sin^3\theta + 2\cos^3\theta\sin\theta] = \frac{2E\cos\theta\sin\theta}{H^2}[\cos^2\theta - \sin^2\theta] = \frac{E}{2H^2}\sin 4\theta$$

The physics of the problem indicate that the only viable solutions are in the range $0 \leq \theta \leq \frac{\pi}{2}$. Thus, making $\frac{df}{d\theta} = 0$ yields the following solutions

$$\theta_1 = 0 \qquad \theta_2 = \frac{\pi}{4} \qquad \theta_3 = \frac{\pi}{2}$$

To evaluate them, the second derivative is needed (despite the fact that in this simple case, the physics of the problem make it obvious to interpret these solutions)

$$\frac{d^2 f}{d\theta^2} = \frac{2E}{H^2}\cos 4\theta$$

Evaluating for all three solutions

$$\frac{d^2 f}{d\theta^2}(\theta_1) = +\frac{2E}{H^2} \quad \rightarrow \quad minimum$$
$$\frac{d^2 f}{d\theta^2}(\theta_2) = -\frac{2E}{H^2} \quad \rightarrow \quad maximum$$
$$\frac{d^2 f}{d\theta^2}(\theta_3) = +\frac{2E}{H^2} \quad \rightarrow \quad minimum$$

Thus, the stiffness over volume ratio is maximized for $\theta = \frac{\pi}{4}$, or $x = H$ .

## Problem 2

Note: When needed, the following 2 matrices will be used to solve all the parts of Problem 1

```
>> A = [1 2 3; 2 3 4; 3 4 5]
A =
     1     2     3
     2     3     4
     3     4     5

>> B = [2 3 4; 3 5 6; 4 6 7]
B =
     2     3     4
     3     5     6
     4     6     7
```

Part (a)

```
>> D = A*B
D =
    20    31    37
    29    45    54
    38    59    71
```

Though $[A]$ and $[B]$ are symmetric, $[D] = [A][B]$ is not symmetric.

Part (b)

```
>> D = B'*A*B
D =
   279   433   520
   433   672   807
   520   807   969
```

$[A]$ is symmetric, therefore $[D] = [B]^T[A][B]$ is also symmetric.

Part (c)

```
>> A = [1 2; 0 0]
A =
     1     2
     0     0

>> B = [3 4; 3 4]
B =
     3     4
     3     4
>> C = [2 1; 0 0]
C =
     2     1
     0     0

>> A*B
ans =
     9    12
     0     0
>> C*B
ans =
     9    12
     0     0
```

False. These matrices prove that $[A][B] = [C][B]$ do not necessary imply $[A] = [C]$.

Part (d)

```
>> A = [1 2;3 4]
A =
     1     2
     3     4
>> B = [5 6;7 8]
B =
     5     6
     7     8
>> D = A*B
D =
    19    22
    43    50
>> D'
ans =
    19    43
    22    50
>> B'*A'
ans =
    19    43
    22    50
```

The transpose of $[D] = [A][B]$ is the same as $[D]^T = [B]^T[A]^T$.

Part (e)

```
>> A = [1 2 3;2 3 4;3 4 8]
A =
     1     2     3
     2     3     4
     3     4     8
>> lambda = eig(A)
lambda =
   -0.3069
    0.8536
   11.4533
>> prod(lambda)
ans =
   -3.0000
>> det(A)
ans =
   -3.0000
```

The determinant of $[A]$ equals $-3$. The product of the eigenvalues of $[A]$ also equals $-3$.

## Problem 3

Part (a)

```
>> A = [1 0;0 0]
A =
     1     0
     0     0
>> B = [0 0;0 1]
B =
     0     0
     0     1
>> A*B
ans =
     0     0
     0     0
```

False. Even though $[A]$ and $[B]$ are both not zero matrices, $[A][B]$ is a zero matrix.

Part (b)

True. This is a fundamental property of matrix inverses (provided that the inverse exists)

Part (c)

```
>> A = diag([1 1 1 1 10])
A =
     1     0     0     0     0
     0     1     0     0     0
     0     0     1     0     0
     0     0     0     1     0
     0     0     0     0    10
>> det(A)
ans =
    10
>> det(10*A)
ans =
     1000000
>> 10*det(A)
ans =
   100
```

False. $10\det([A]) \neq \det(10[A])$.

## Problem 4

Inspection of the function clearly indicate when the function becomes indeterminate

$$f = f(x_1, x_2) = x_1^2 - \ln x_1 + x_2 + \frac{2}{x_2^2}$$

The $\ln x_1$ makes the function indeterminate for $x_1 = 0$, and complex for $x_1 < 0$. The $\frac{1}{x_2^2}$ makes the function indeterminate for $x_2 = 0$.

The gradient and the Hessian are

$$\nabla f = \left\{2x_1 - \frac{1}{x_1} \quad , \quad 1 - \frac{4}{x_2^3}\right\}$$

$$\mathcal{H}f = \begin{bmatrix} 2 + \frac{1}{x_1^2} & 0 \\ 0 & \frac{12}{x_2^4} \end{bmatrix}$$

Making the gradient equal to zero, the following critical points are obtained

$$x_1 = \begin{cases} +\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{cases} \qquad\qquad x_2 = \begin{cases} \sqrt[3]{4} \\ \sqrt[3]{4}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}i\right) \\ \sqrt[3]{4}\left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right) \end{cases}$$

The complex solutions are not considered for this problem. In addition, the negative solution in $x_1$ would result in a complex value of the function $f$, and will not be considered either. Thus, there is a single critical point at

$$\{x_1 \quad , \quad x_2\} = \left\{\frac{\sqrt{2}}{2} \quad , \quad \sqrt[3]{4}\right\}$$

The Hessian for this point is

$$\mathcal{H}f\left(\frac{\sqrt{2}}{2} \quad , \quad \sqrt[3]{2}\right) = \begin{bmatrix} 4 & 0 \\ 0 & \frac{3}{2}\sqrt[3]{4} \end{bmatrix} > 0 \quad \rightarrow \quad minimum$$

The following MATLAB code plots the function and the critical point

```
clc, clear, close all

lim=[0 4 -1.5 2.5];
n1=100; n2=100;
top=10;

[x1,x2]=meshgrid(linspace(lim(1),lim(2),n1),linspace(lim(3),lim(4),n2));
y=x1.^2-log(x1)+x2+2./(x2.^2);

figure, hold on
[C,h] = contour(x1,x2,y,[1 2 4 6 8 12 16 50]);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2)
plot(1/sqrt(2),4^(1/3),'ro','LineWidth',1,'MarkerFaceColor',[1 0 0])
text(1/sqrt(2),4^(1/3),'$\; \leftarrow P_1=1/\sqrt{2},4^{1/3}$',...
     'FontSize',12,'Interpreter','latex')
xlabel('x_1'), ylabel('x_2'), set(gca,'CLim',[0 22]), box

figure
surf(x1,x2,y), shading interp
xlabel('x_1'), ylabel('x_2')
axis([lim 0 top]), set(gca,'CLim',[0 top]), box, view(55,75)
```