CIVE 546 Structural Design Optimization

(3 units)

Instructor: Prof. Yi Shao

TA: Xiaoning Sun

Class will start at 11:40am

Winter 2025

Yi SHAO

Assistant Professor

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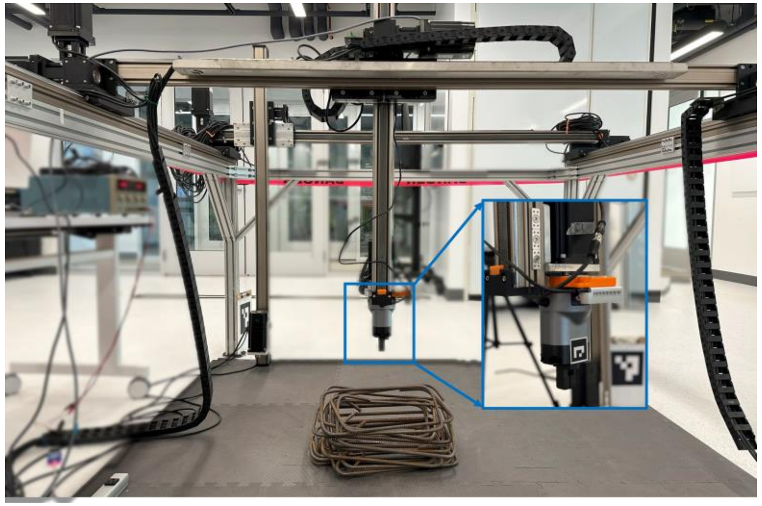
Structural
Engineering



Expertise:

Structural Engineering | Construction Robotics

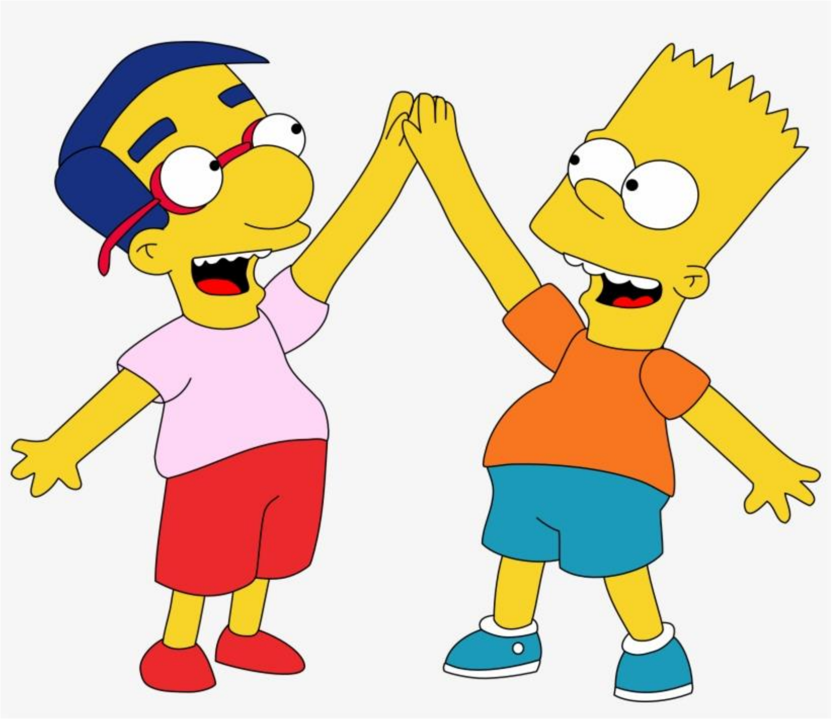
Research:

- Application of Advanced Materials (CIVE 512)
- Structural Optimization (CIVE 546)
- Robotic Construction (Upcoming!)
- AI-assisted Structural Design (Upcoming!)

McGill
Department of
Civil Engineering

Introduce yourself

- Your name
- Year (undergrad or MS or PHD?)
- Your major
- Your research topic (if any)



Course interactions

In-class:

- Questions? Confusions? Speed? Other? Hands up!
- Group discussions/activities
- Respectful and Inclusive Environment
- Polling@McGill
- Class notes (PDF, myCourses) *Don’t redistribute, don’t post online*

After-class:

- Office hours
- Emails

If you have any special needs or suggestions, please let me know!

Office Hour Survey:
Please fill your general availability, not specific to the dates
https://www.when2meet.com/?28150990-xGdzZ

Lecture structure

- Beginning (10 min) Slido to review key concepts
- Session 1 (50 min) lecture
- Break (15 min) Review + question answering
- Session 2 (50 min) lecture
- Rest: review + question answering + tutorial

Critical way for getting timely feedbacks and self-assessment

Lecture tech

- The theoretical content will be delivered primarily through hand-writing, assisted by slides/animation for enhanced understanding. Be prepared to take notes.

Why do we need "Structural Design Optimization"

Sstructural Engineering: Develop a functional design

Functionality: costs, efficiency, environmental impacts, disaster resistance, etc.

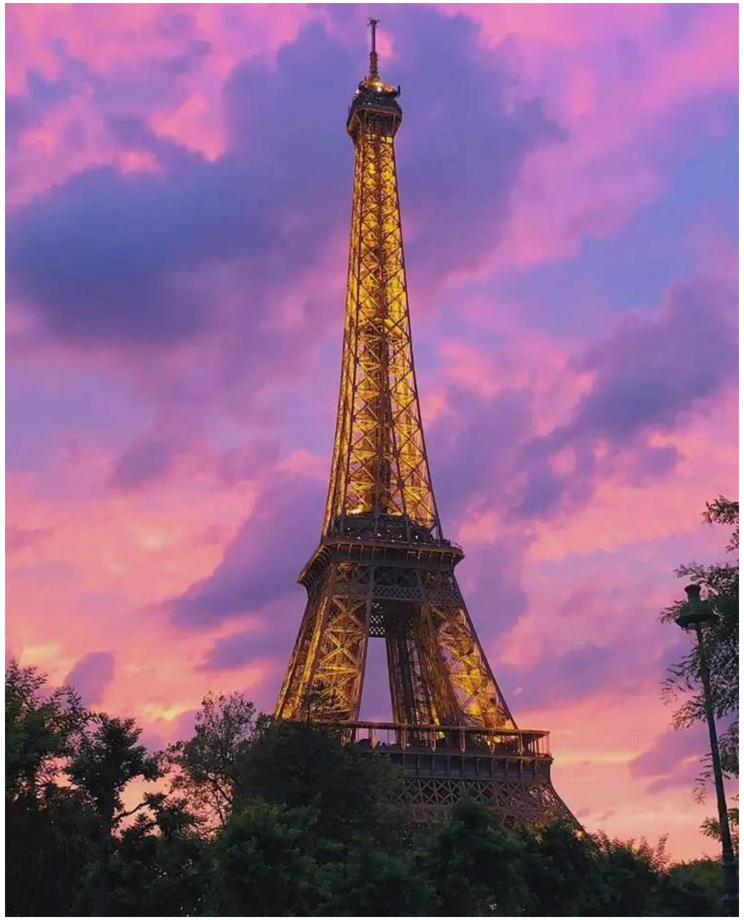
Need careful analysis to verify the functionality

- CIVE 205 Statics
- CIVE 207 Solid Mechanics
- CIVE 317 Structural Engineering 1
- CIVE 318 Structural Engineering 2
- CIVE 462 Design of Steel Structures
- CIVE 463 Design of Concrete Structures
- CIVE 507 Wind Engineering
- CIVE 525 Design of Wood Structures
- CIVE 545 LCA and the Circular Economy

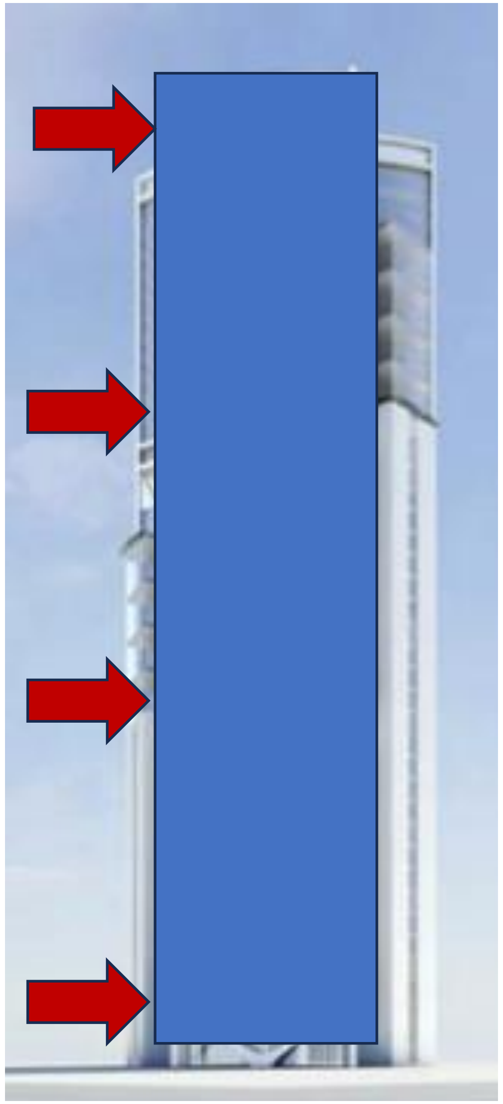
Many courses that teach analysis methods

First-step: How to achieve the "BEST" design for analysis?
Critical for functionality

The Bout-de-l'Île Railway Bridge, Montreal



Eiffel tower, Paris



Why do we need "Structural Design Optimization"

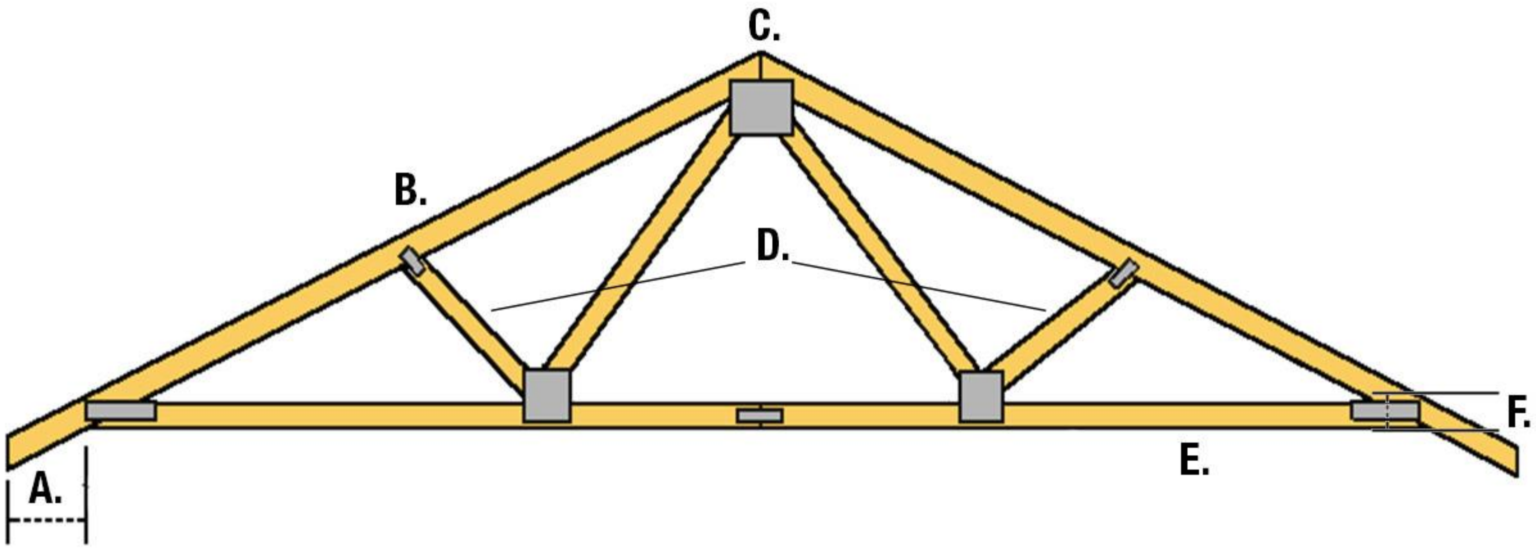
A simple example

Consider a problem with 3 Design Variables (DVs), assume 10 values for each design variables and each design needs 0.1s to calculate.

Then we have ____ combinations → total time: ______*0.1s= ______.

$100s$

$10 \times 10 \times 10 = 10^3$

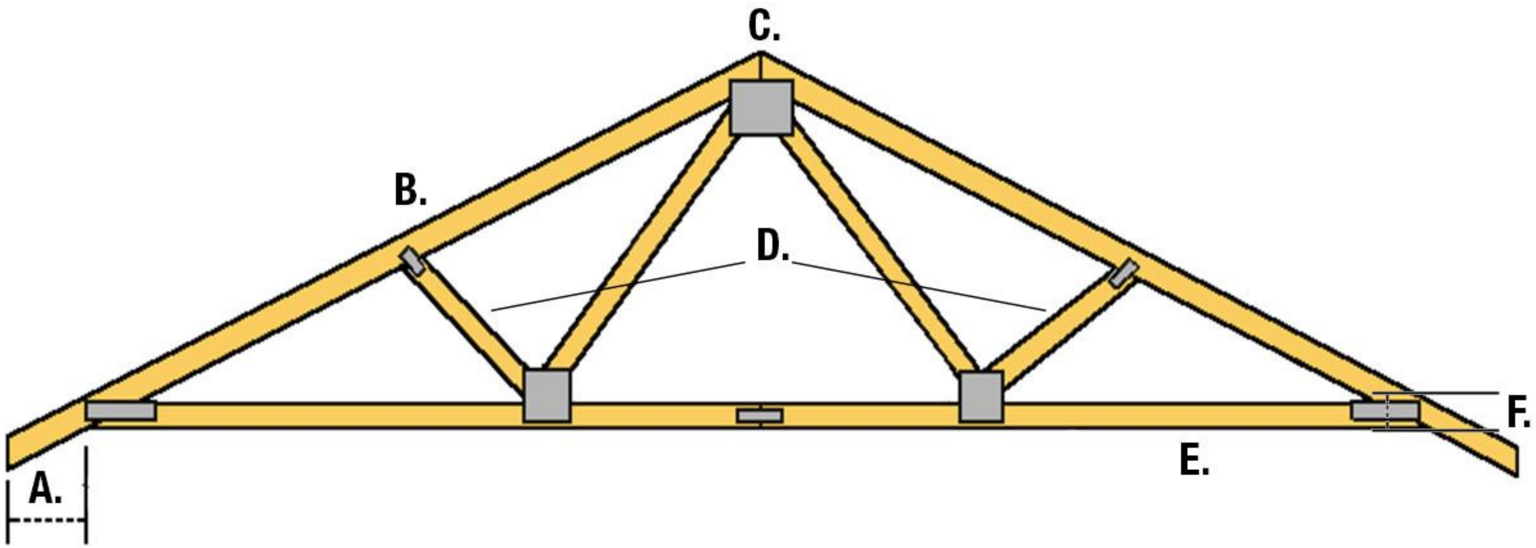


Why do we need "Structural Design Optimization"

A simple example

Consider a problem with 3 Design Variables (DVs), assume 10 values for each design variables and each design needs 0.1s to calculate.
Then we have 10^3 combinations → total time: 10^3*0.1s=**100s**

If we have 10 DVs, still 10 values for each design and 1s to calculate each design
Then we have 10^{10} combinations → total time **10^{10}s (317 years)**



We need a rational approach to search for efficient designs.

Structural Design Optimization

A subject that involves a broad spectrum of science and tools

McGill University

CIVE 205 Statics

CIVE 207 Solid Mechanics

CIVE 208 System A

CIVE 317-318 Str

CIVE 460 N

Stanford U

MS&E111/211:

MS&E310: Linear Program

CEE 307 Structural Design Optimization (Topology Optimization)

CS334A Convex Optimization I

CS334B Convex Optimization II

Technical University of Denmark (DTU)

Topology Optimization: Theory, Methods and

ctors!)

The goal is to acquaint you with basic knowledge and tools of Structural Design Optimization to <u>START</u> your journey!

rm-finding)

ural Design II

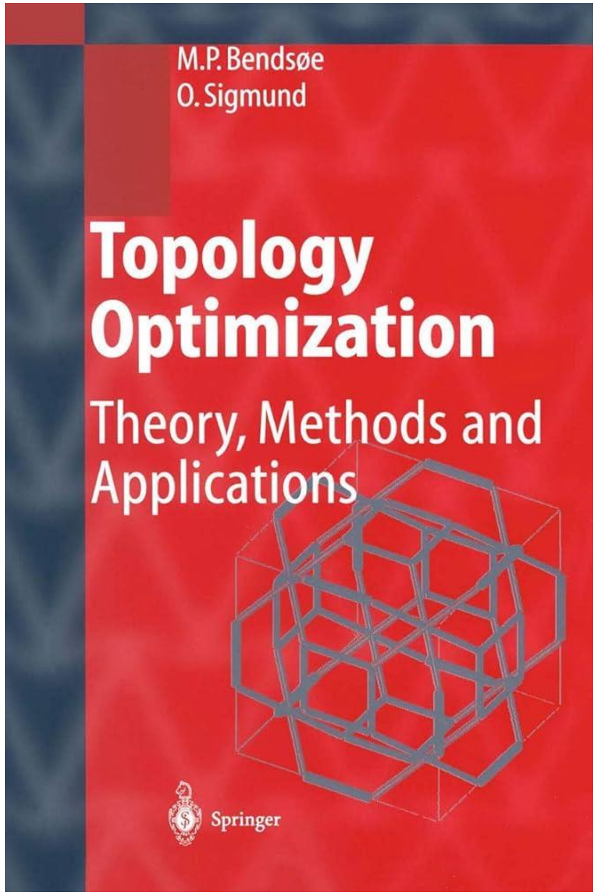
Computational Structural Design I

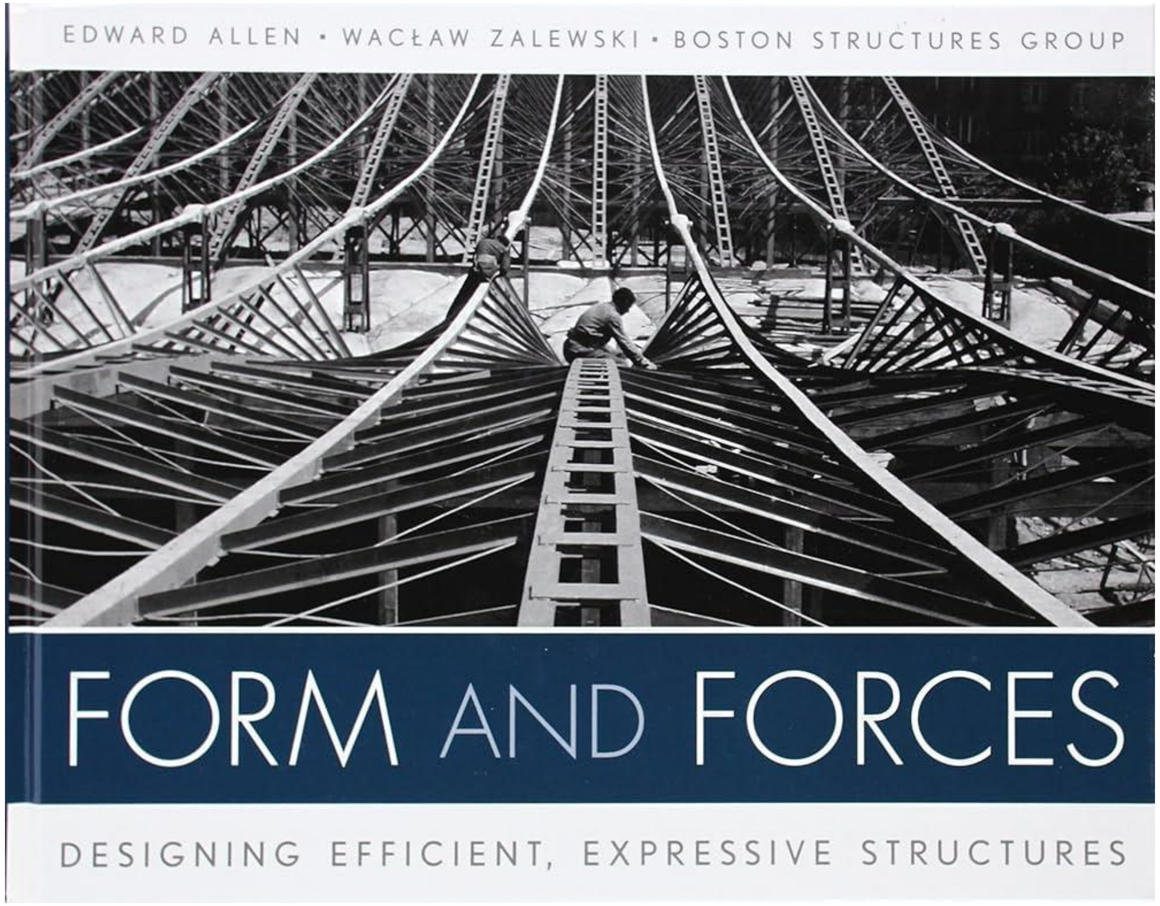
Computational Structural Design II

Structural Design Optimization

Two reference books

No textbook for this class.
These two books provide many useful information and extensions!



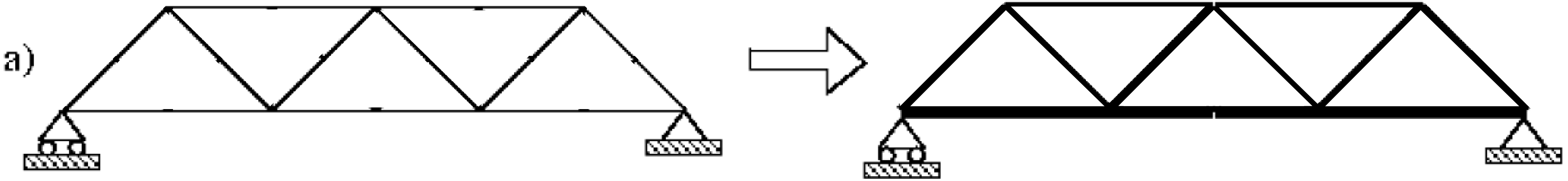


Structural Design Optimization

Numerical Optimization: Choice of design variable

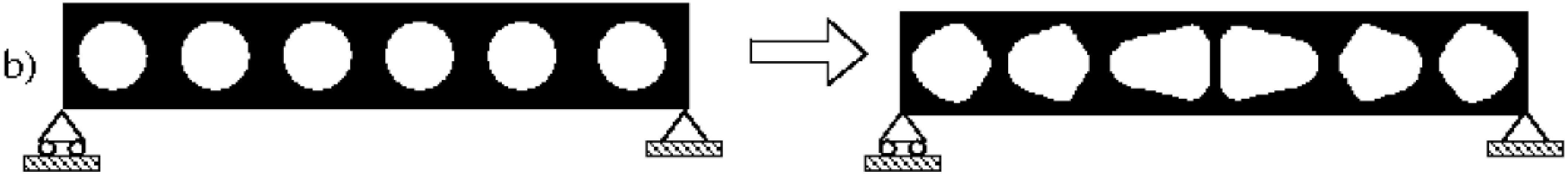
Sizing Optimization

a)



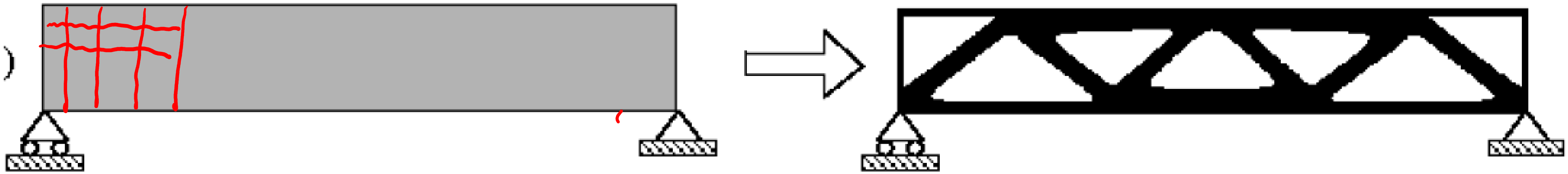
Shape Optimization → level set

b)



Topology Optimization

c)

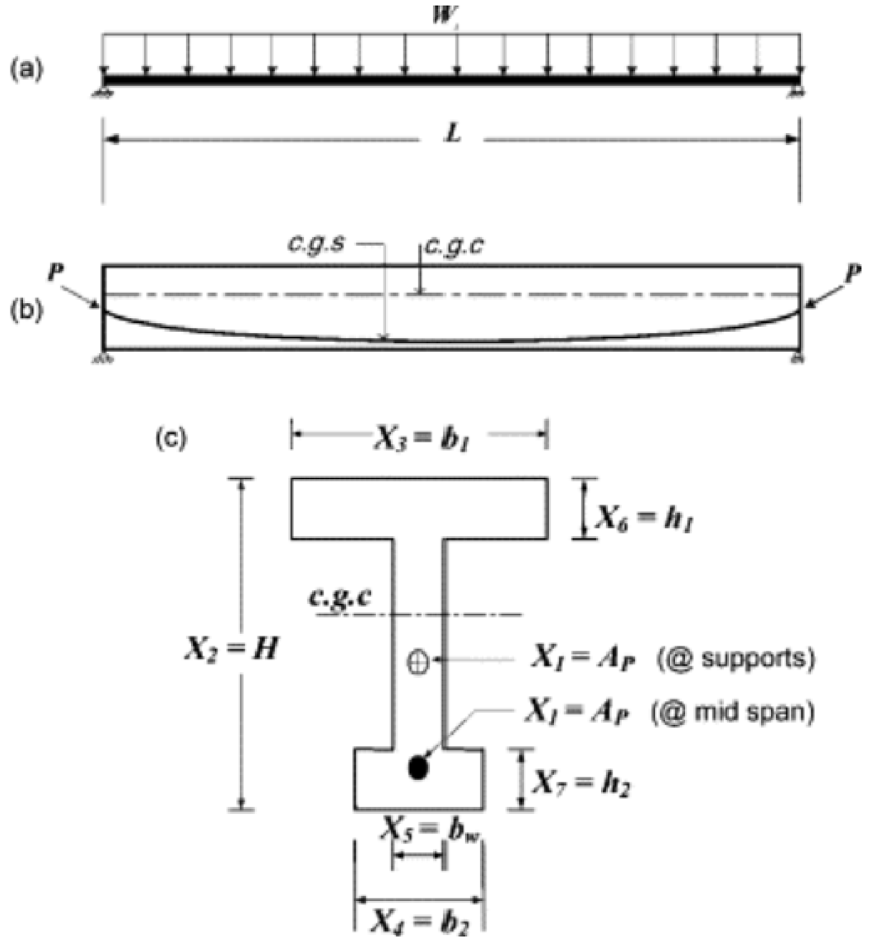


Structural Design Optimization

Numerical Optimization: Choice of design variable

Concrete Beam

Cross-section and reinforcement layout



Concrete Frame

Reinforcement layout

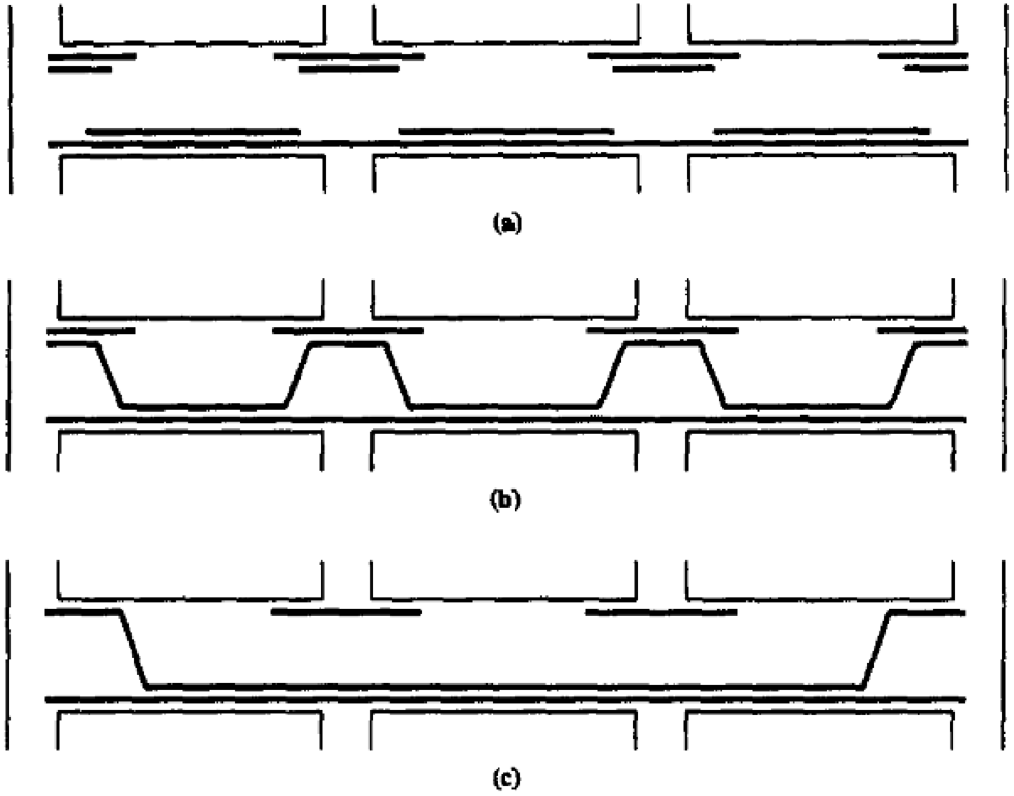
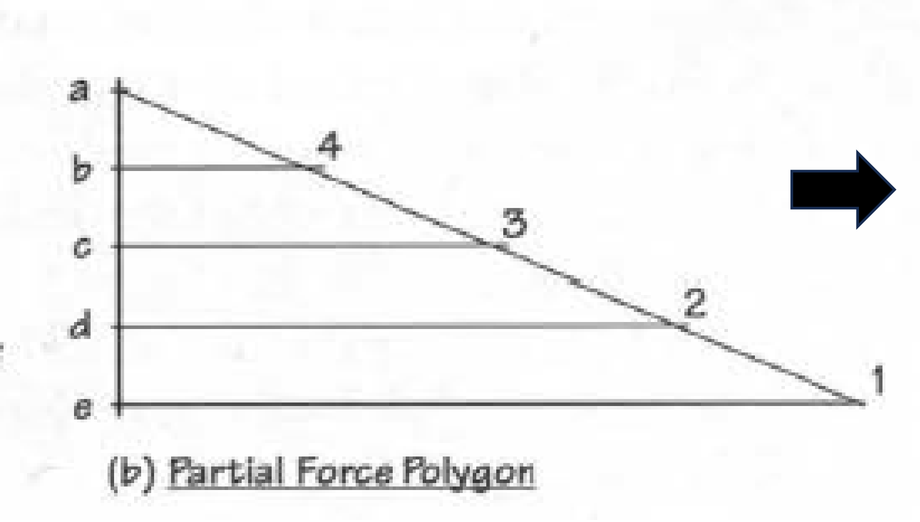


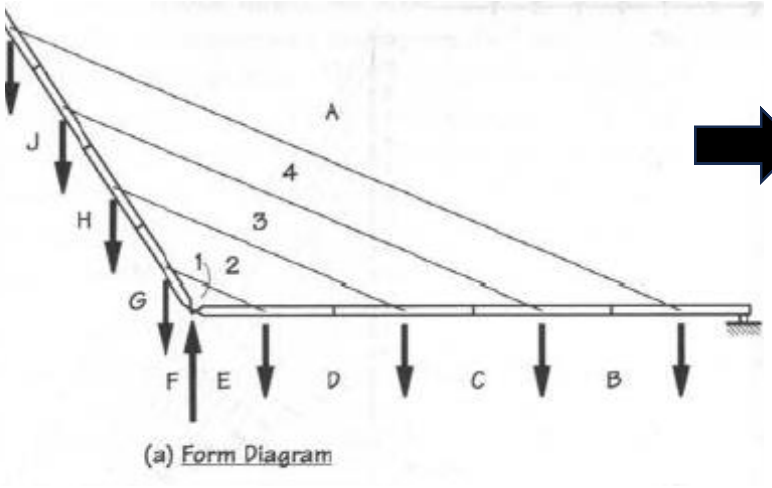
FIG. 2. Beam-Reinforcement Topologies: (a) Topology with 12 Bar Groups; (b) Topology with Six Bar Groups; (c) Topology with Four Bar Groups

Structural Design Optimization

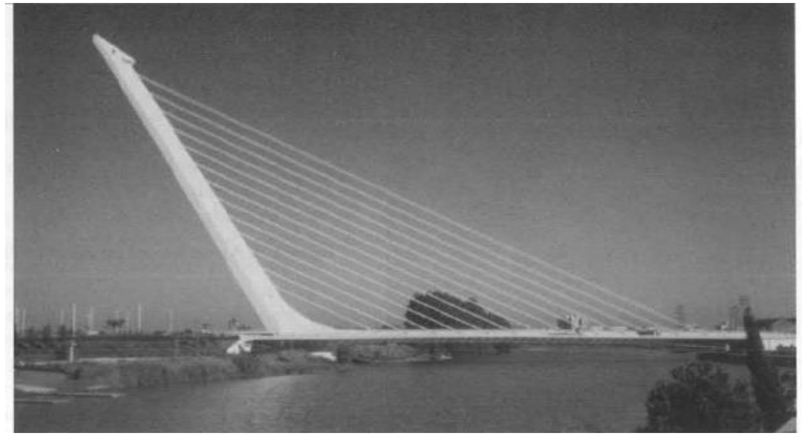
Graphic Statics → form-finding



(b) Partial Force Polygon



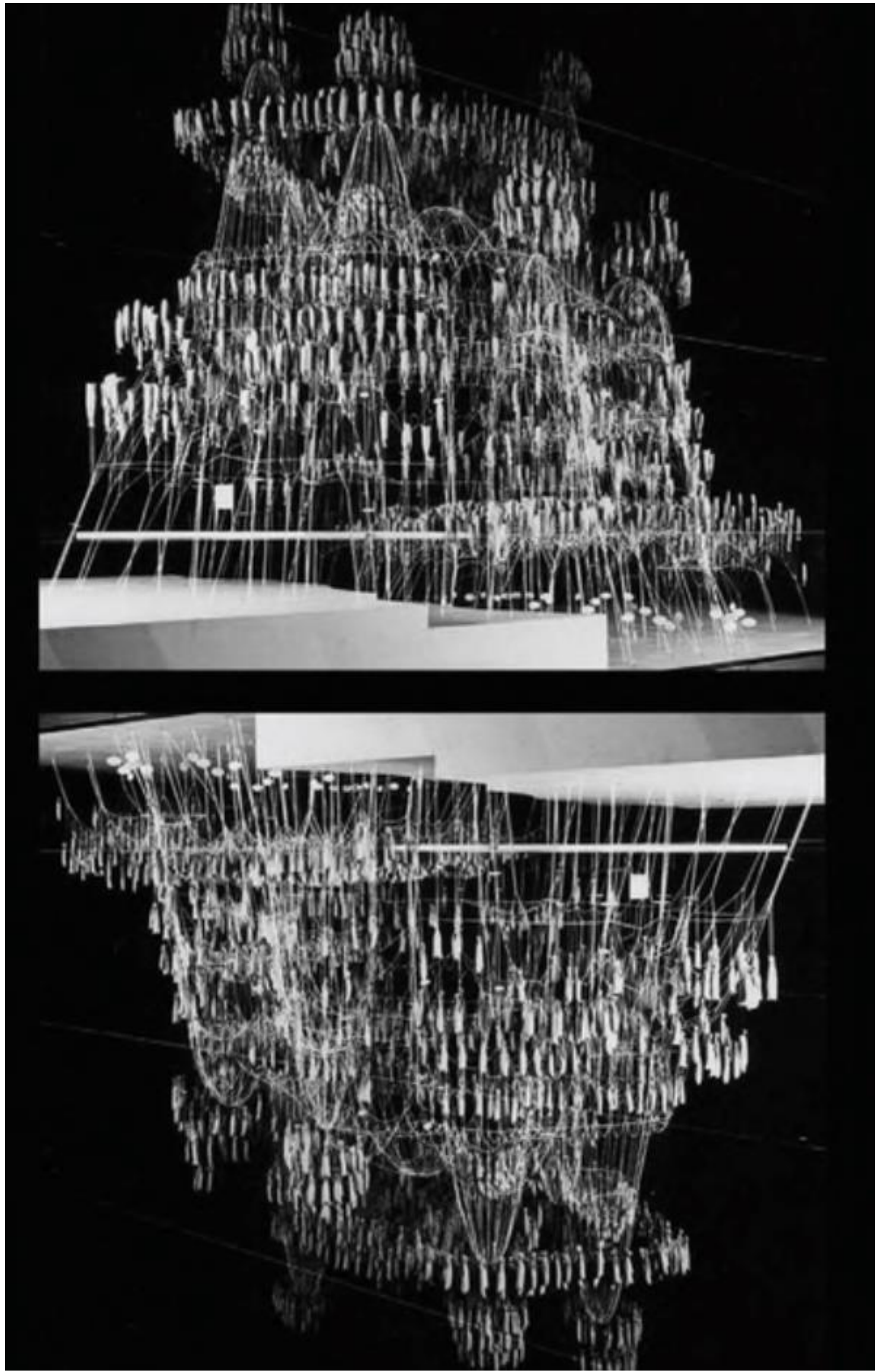
(a) Form Diagram



Santiago Calatrava: Alamillo Bridge, Sevilla, 1992

Form Finding
Physical models

1889 – Gaudí's Hanging Chain Models



Crypt, Colònia Güell, Barcelona

"The art of structure is where to put the holes"

Robert Le Ricolais

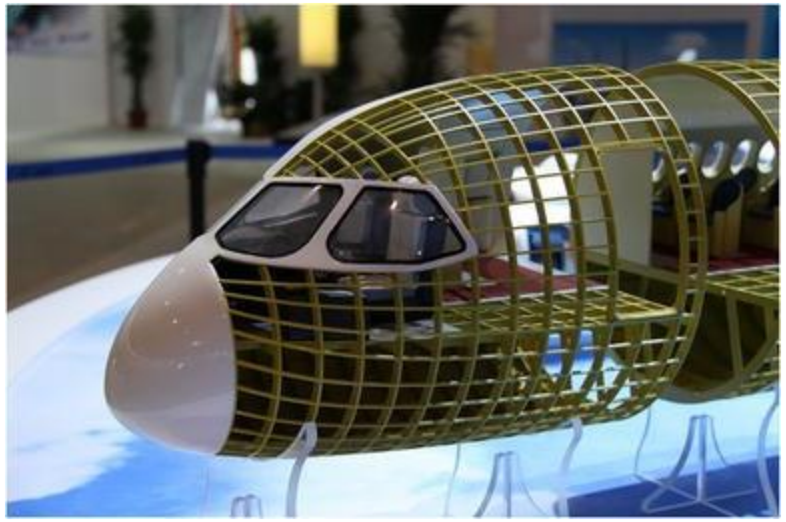
French/American Engineer and Philosopher

“Holy Structures”

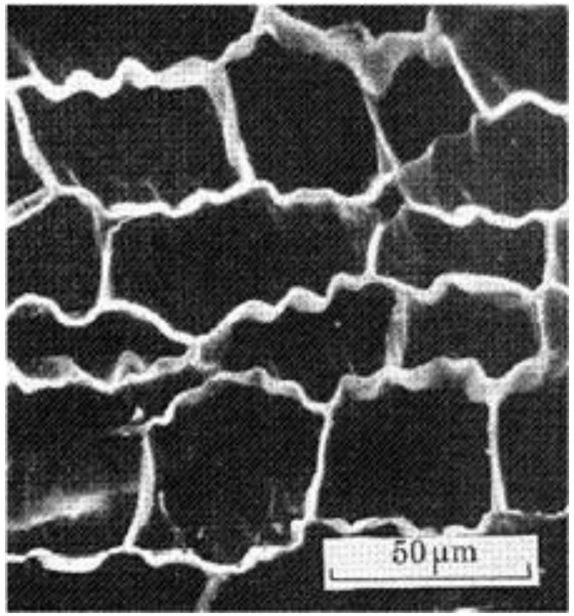
Optimal use of material

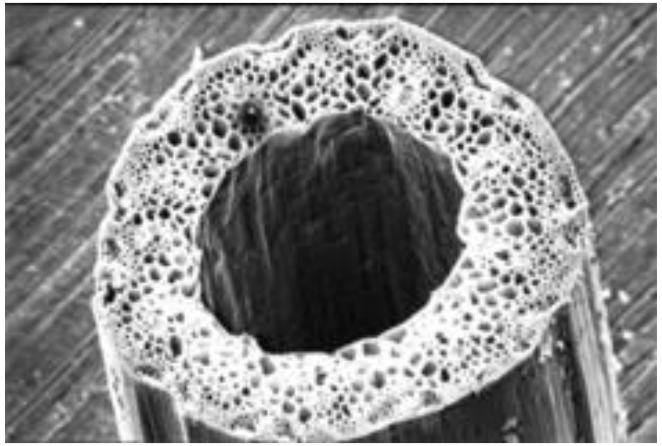
Maximal stiffness/mass

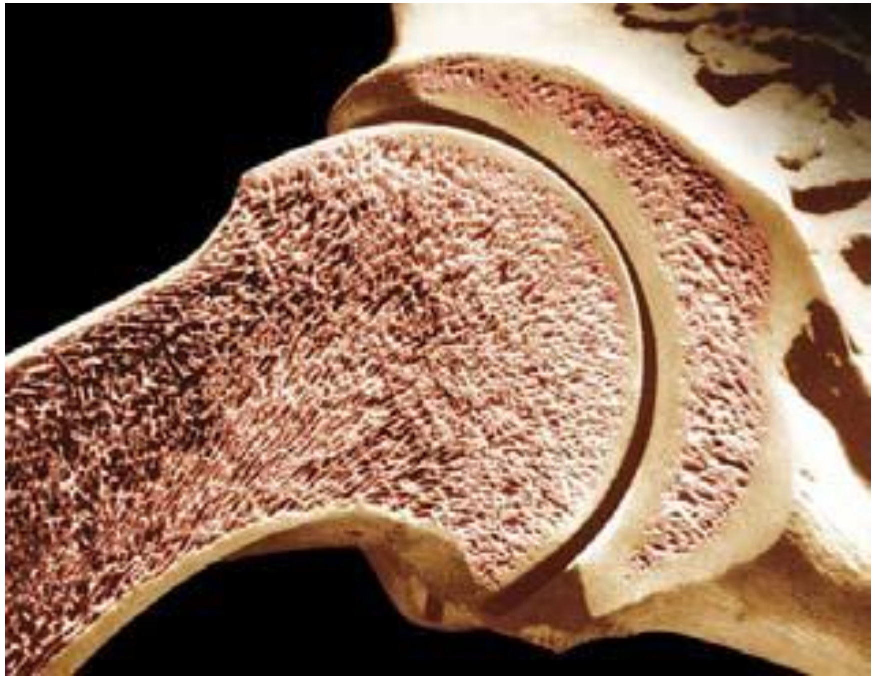
Maximal strength/mass



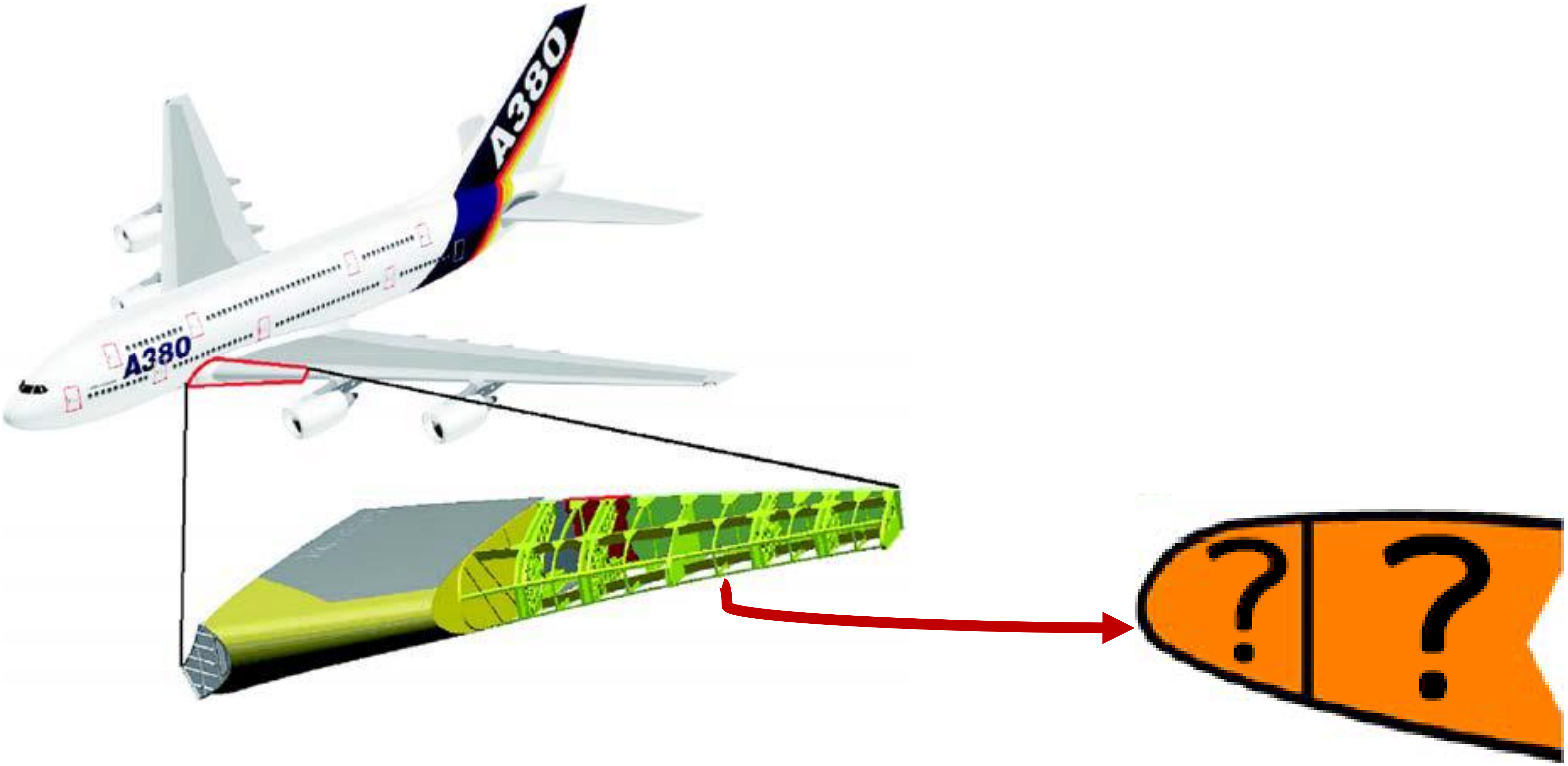
Holes in nature



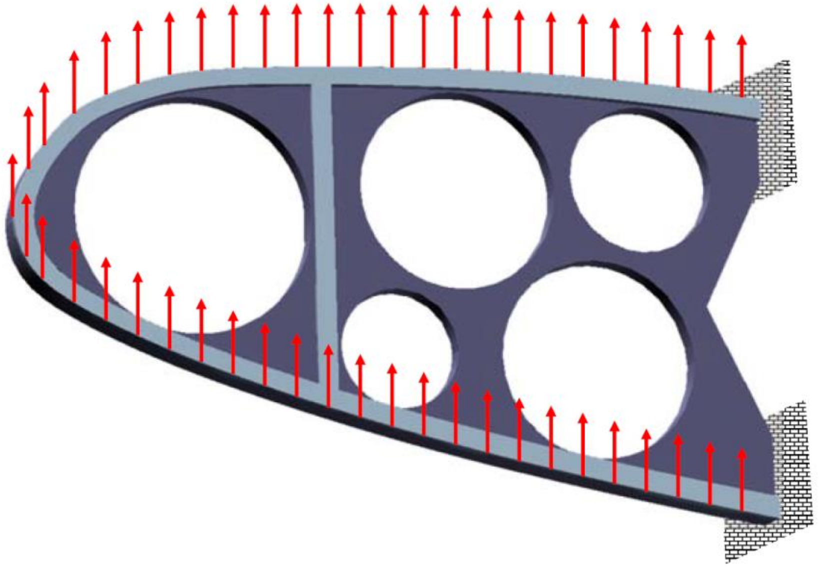




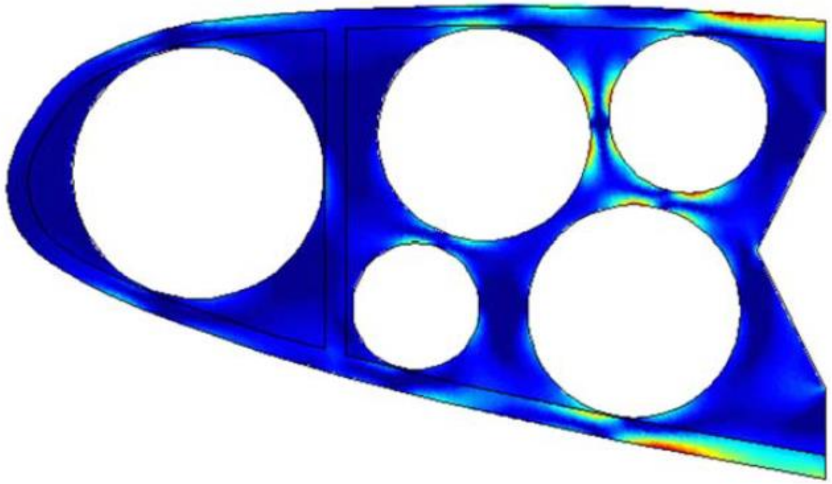
The design of the web frame of aircraft wings



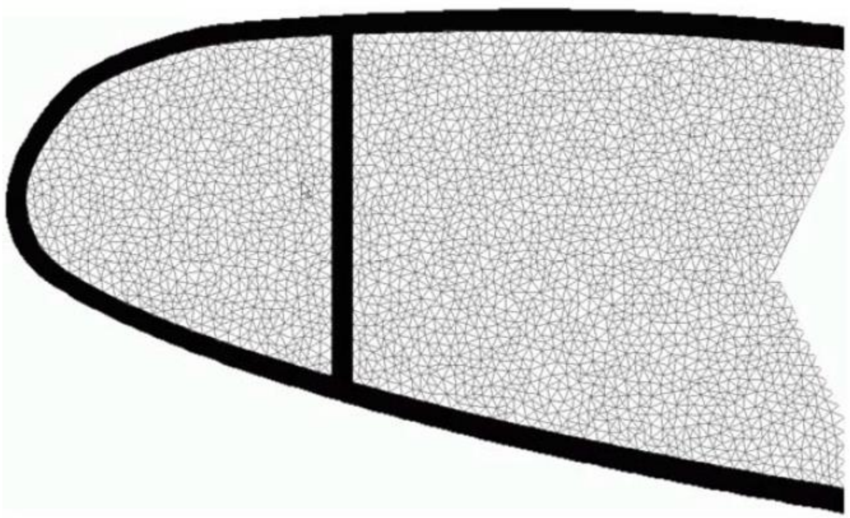
Circular holes



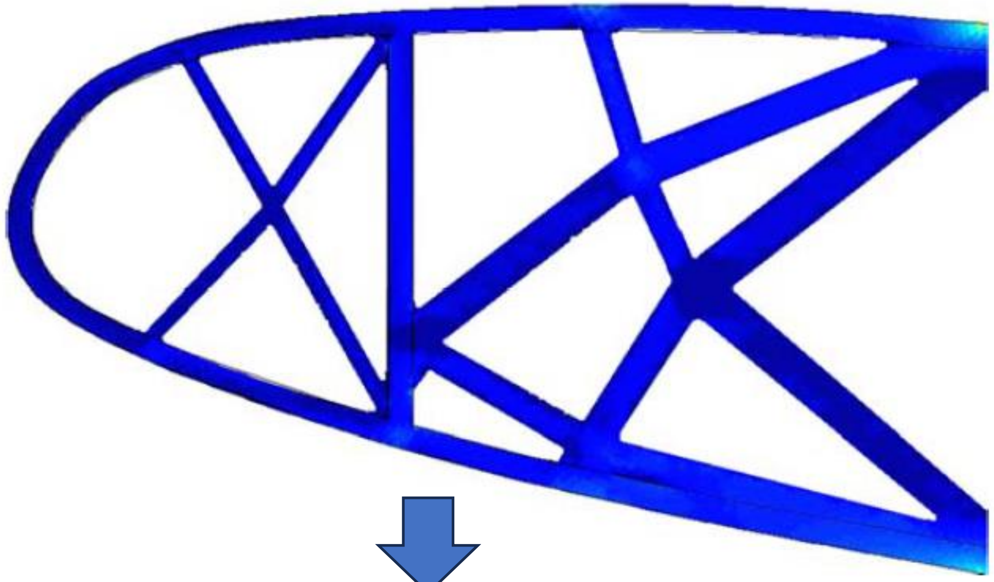
Strain Energy Density Function

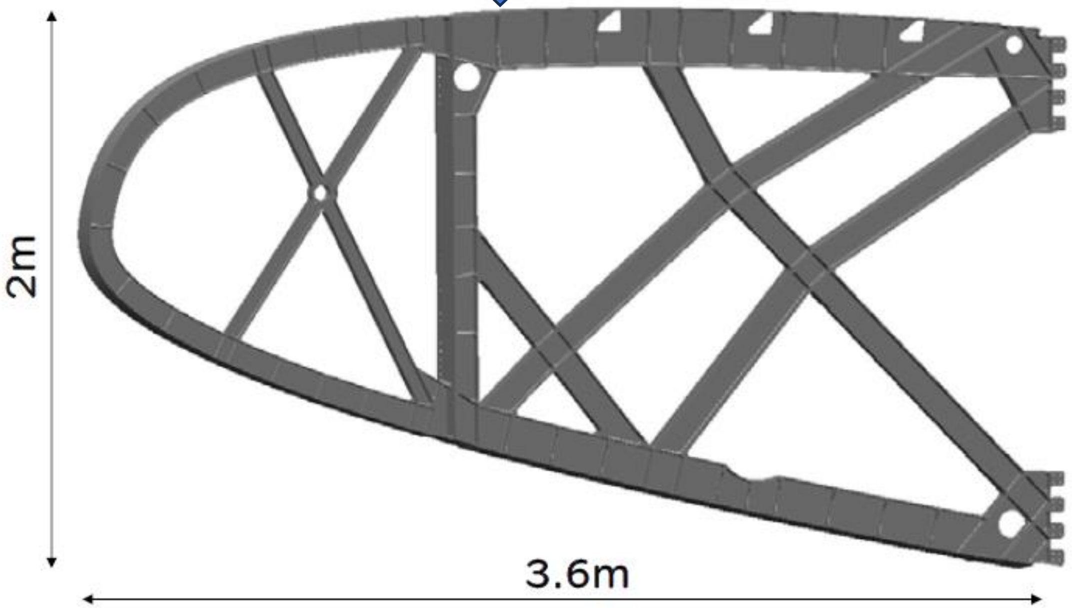


A more systematic approach: topology optimization



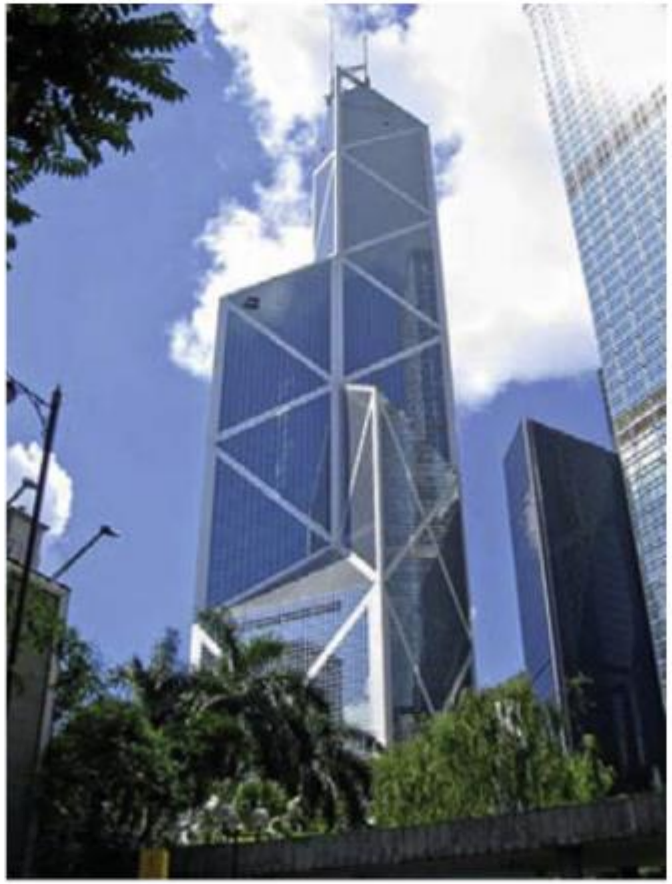
Strain Energy Density Function





Courtesy of ”EADS Military Aircraft”

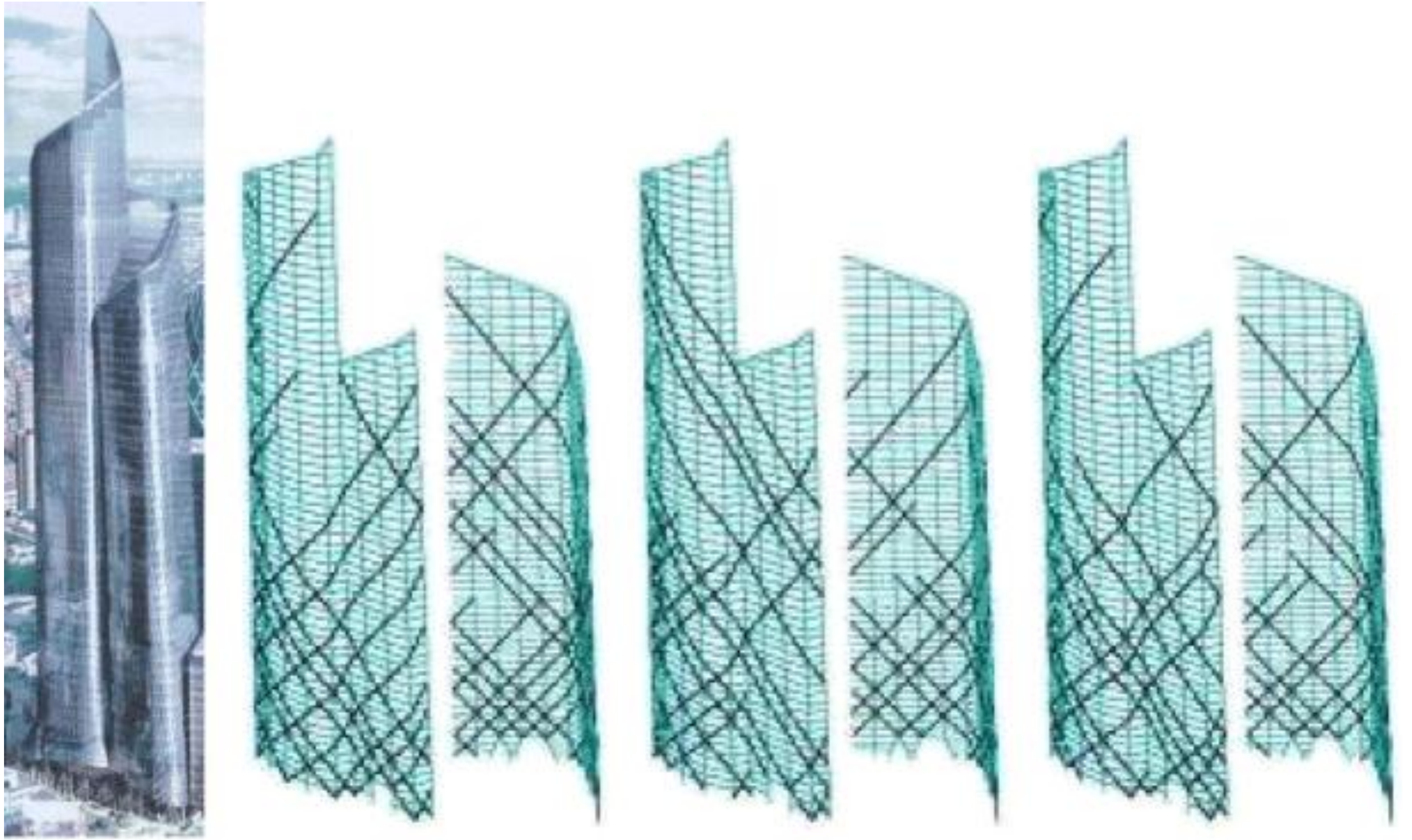
The design of lateral-force-resisting system in buildings



Stromberg, L. L., Beghini, A., Baker, W. F., & Paulino, G. H. (2012). *Engineering Structures*, *37*, 106-124.

The design of lateral-force-resisting system in buildings

The Pinnacle Tower (**a**) General view (**b**) Three possible profiles for the framing spirals of steel tubes

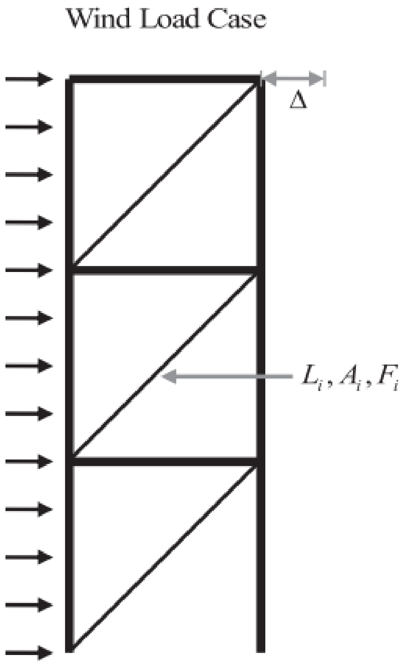


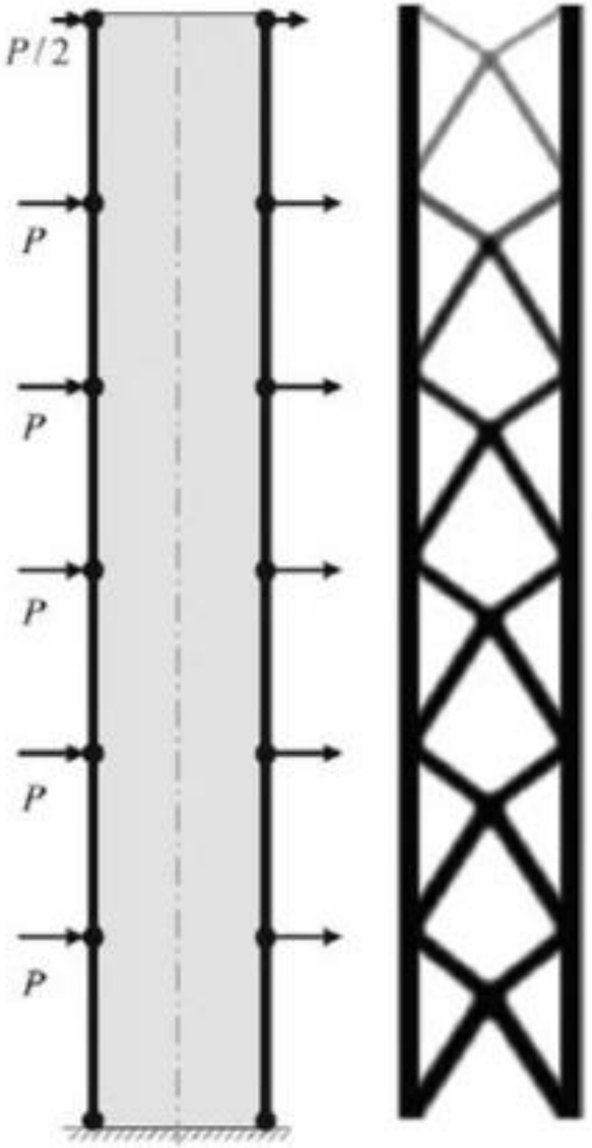
(a) (b)

Aldwaik, M., Adeli, H. *Struct Multidisc Optim* **50**, 899–919 (2014).

The design of lateral-force-resisting system in buildings

What are other idealization possibilities or loading cases?



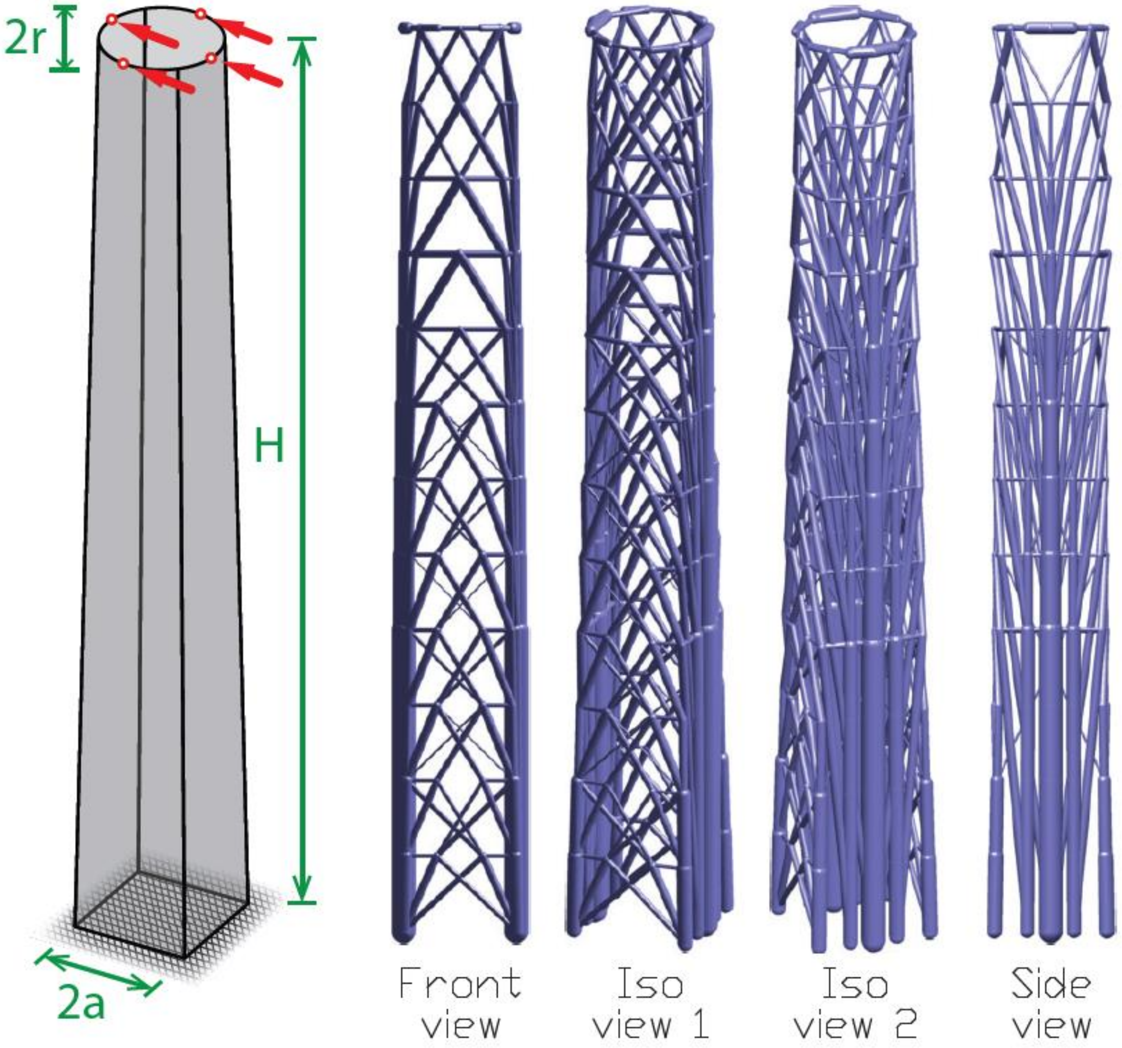


Idealize the lateral forces as concentrated forces at each floor

SOM design

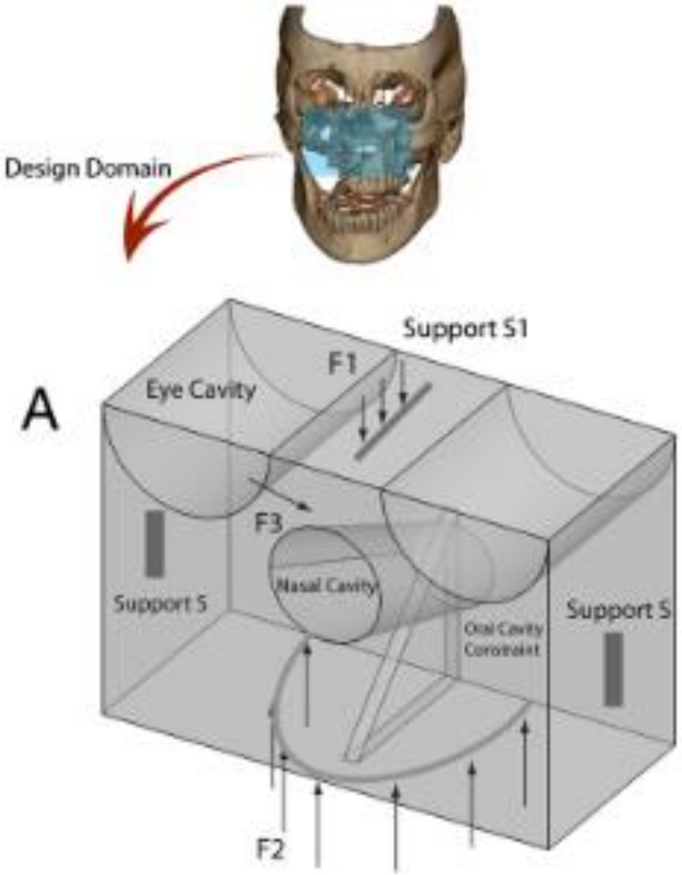
Stromberg, L. L., Beghini, A., Baker, W. F., & Paulino, G. H. (2012). *Engineering Structures*, *37*, 106-124.

The design of lateral-force-resisting system in buildings

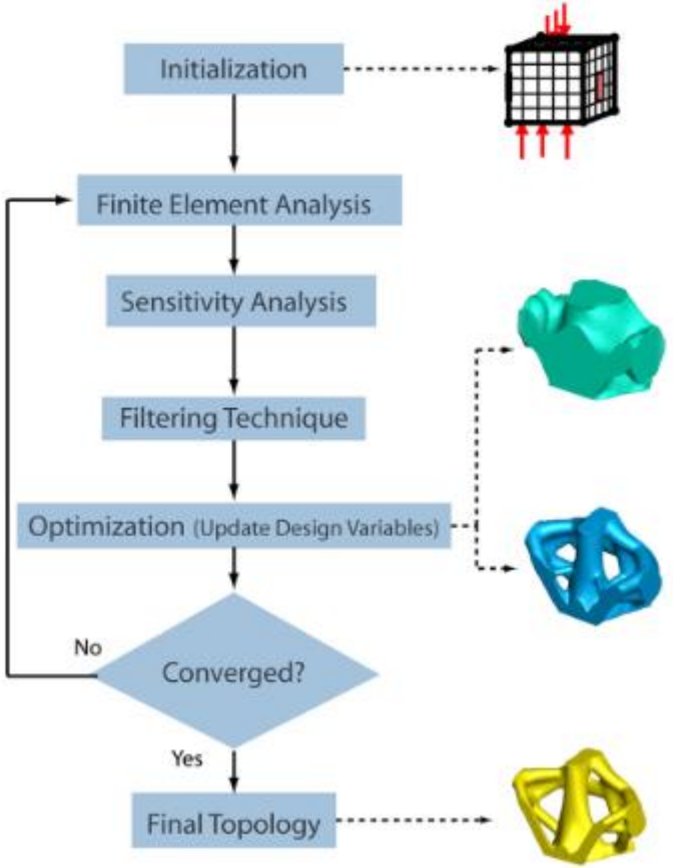


Topology optimization for segmental bone replacement

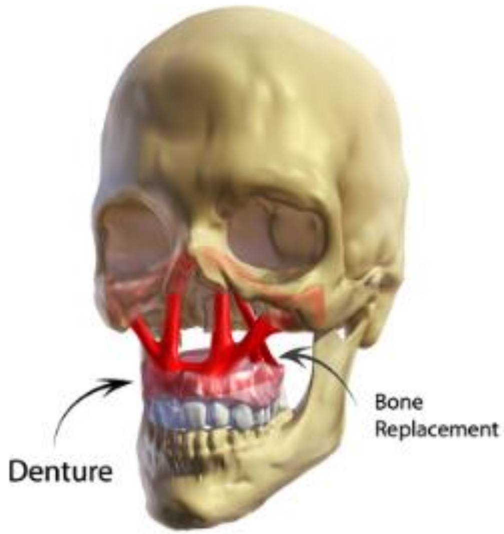
Problem statement



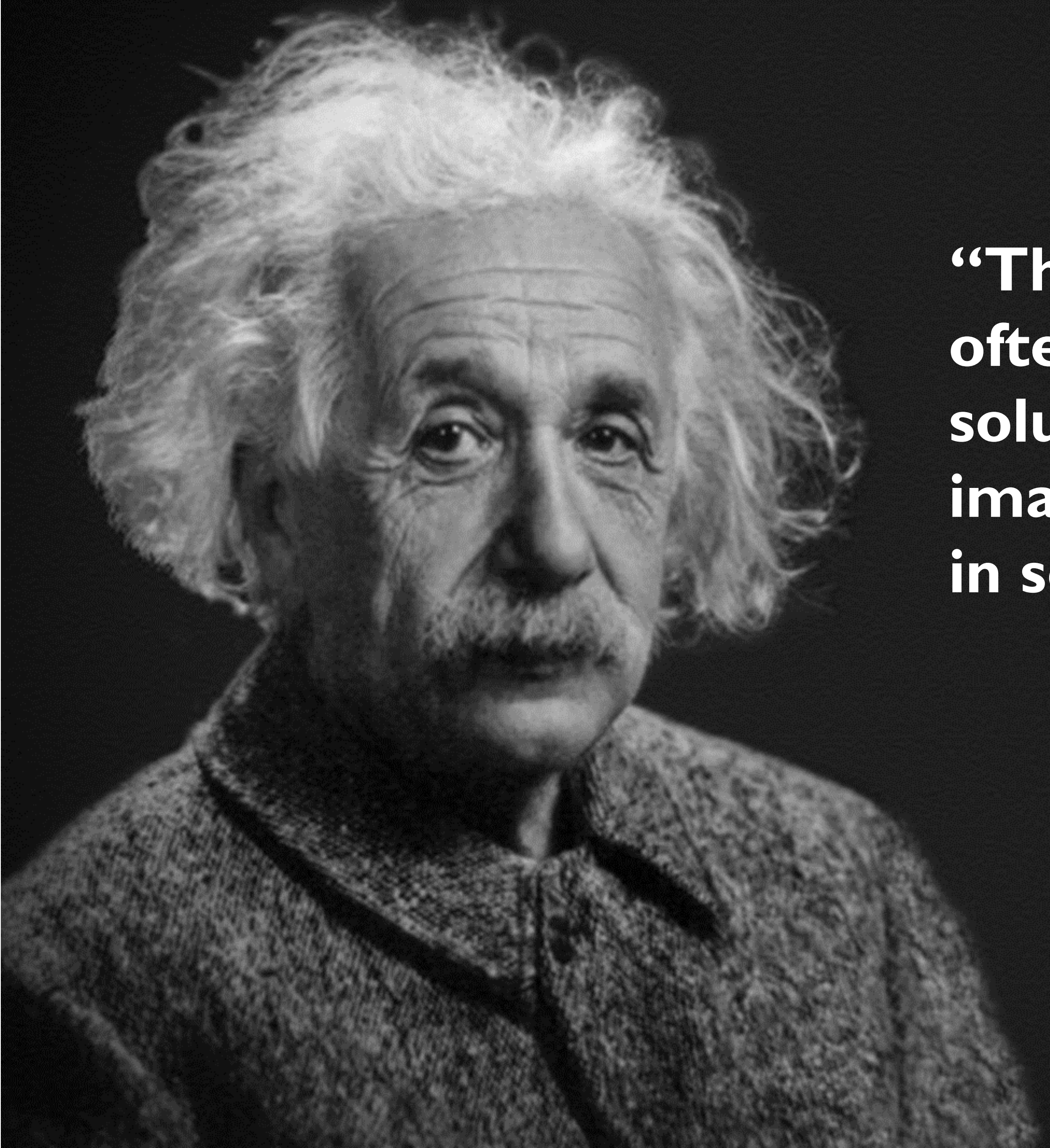
Procedure



Final outcome



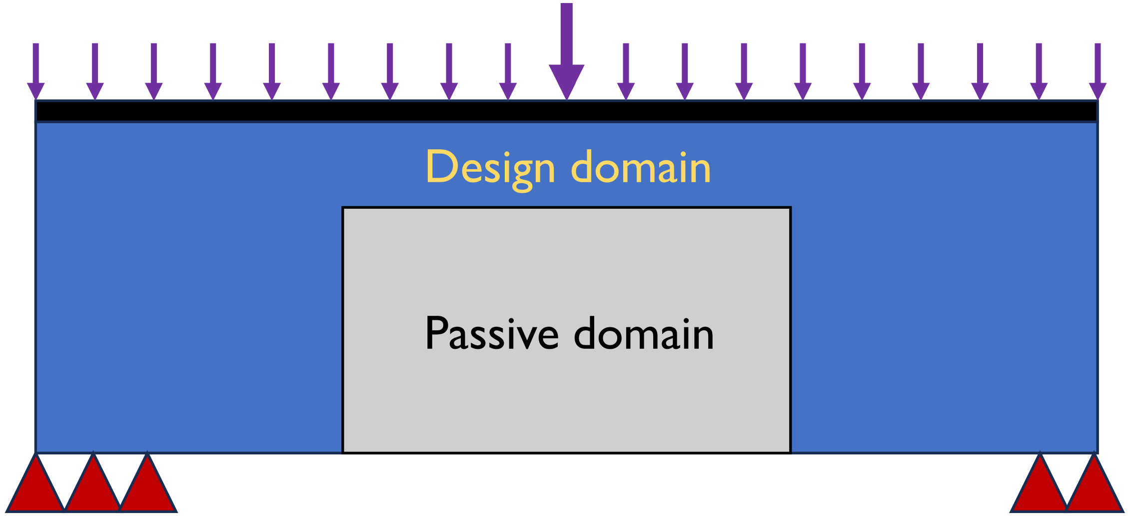
Sutradhar et al. (2010). Proceedings of the National Academy of Sciences, 107(30), 13222-13227.



"The mere formulation of a problem is often far more essential than its solution, which [...] requires creative imagination and marks real advances in science"

-Albert Einstein, 1921

Topology optimization: Problem setup

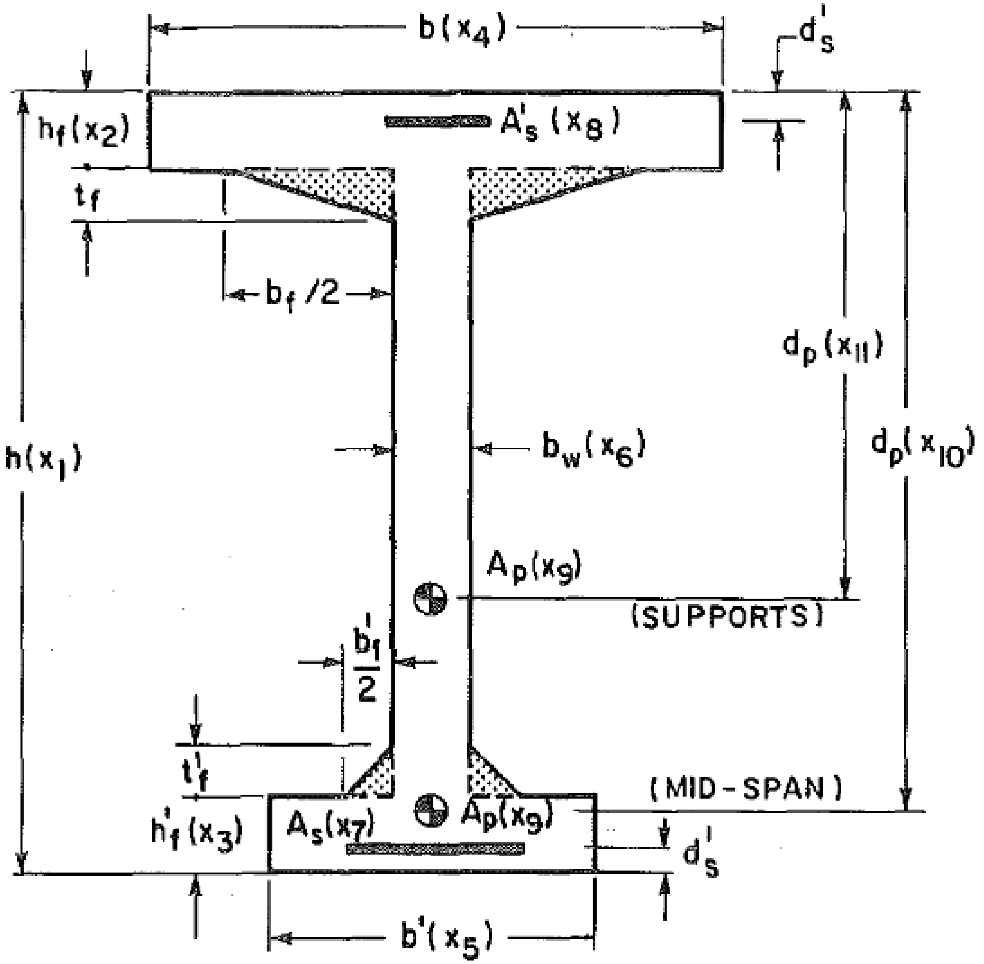


- **Objective (maximize stiffness or others)**
- **Domain condition**
  - Enforced domain: region where materials must be present
  - Design domain: region where materials can be freely arranged
  - Passive domain: region where there must be no material
- **Loading condition**
- **Support condition**

Structural Design Optimization in a Broader Sense

Concrete Beam

Cross-section and reinforcement layout



What's my objective(s)?

1. Minimum member cost: c_c , c_s , c_p , $c_f \neq 0$.
2. Minimum weight: $c_c \neq 0$; $c_s = c_p = c_s = 0$.
3. Minimum reinforcement: $c_c = c_f = 0$; c_s , $c_p \neq 0$.
4. Minimum prestressing: $c_c = c_f = c_s = 0$; $c_p \neq 0$.

What contributes to the objective?

Material, formwork, labor, transportation, foundation, etc.?

What are the design variables?

Dimensions, locations, amount, concrete mix, etc. ?

What are the constraints?

Loading, strength, stiffness, ductility, fatigue, cracking, construction, etc.?

General form of an optimization problem

Objective function	$\min f(\underset{\sim}{x})$
(merit/cost function)	*Subject to:*
Inequality constraints	$g_j(\underset{\sim}{x}) \leq 0 \qquad j = 1,,,,p$
Equality constraints	$h_k(\underset{\sim}{x}) = 0 \qquad k = 1,,,,m$
Box constraints	$x_i^L \leq x_i \leq x_i^U$

$\underset{\sim}{x} \rightarrow$ Vectors $= \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$\underset{\sim}{A} \rightarrow$ Matrix

$n \times 1$

DVs

(Design Variables)

General form of an optimization problem

Objective function	$\min f(\underline{x})$
(merit/cost function)	*Subject to:*
Inequality constraints	$g_j(\underline{x}) \leq 0 \qquad j = 1,,,,p$
Equality constraints	$h_k(\underline{x}) = 0 \qquad k = 1,,,,m$
Box constraints	$x_i^L \leq x_i \leq x_i^U$

Type I: Linear Programming: Both objective AND constraints are linear

Type II: Nonlinear Programming: Objective AND/OR constraints are nonlinear

Course Project

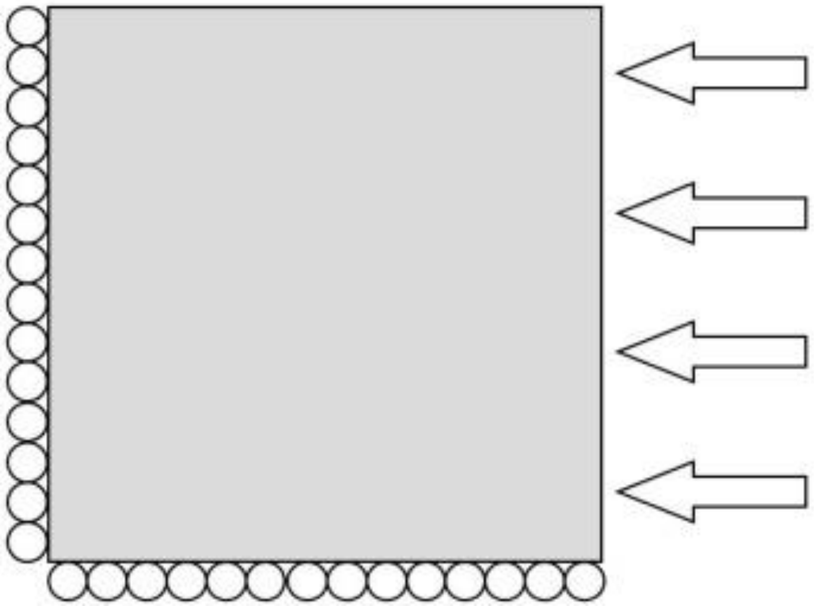
Topic

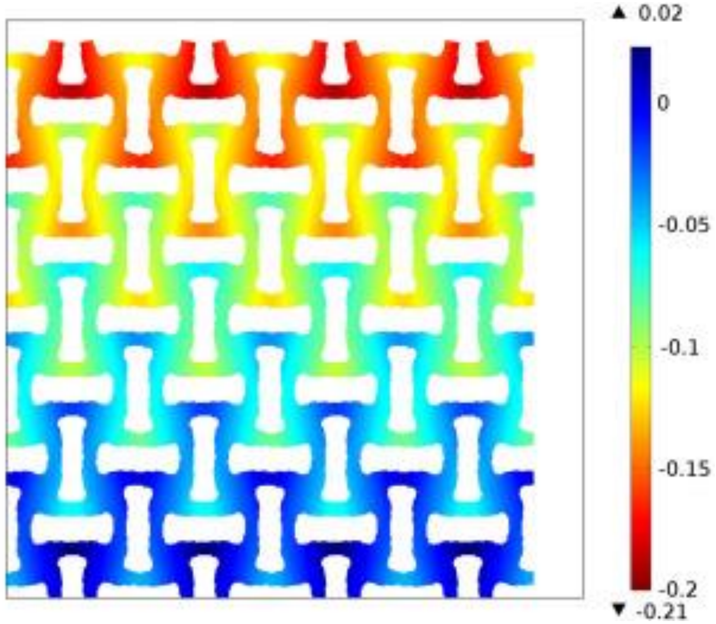
Optimize any structures of your choice!

Course Project

Various scale

Material scale

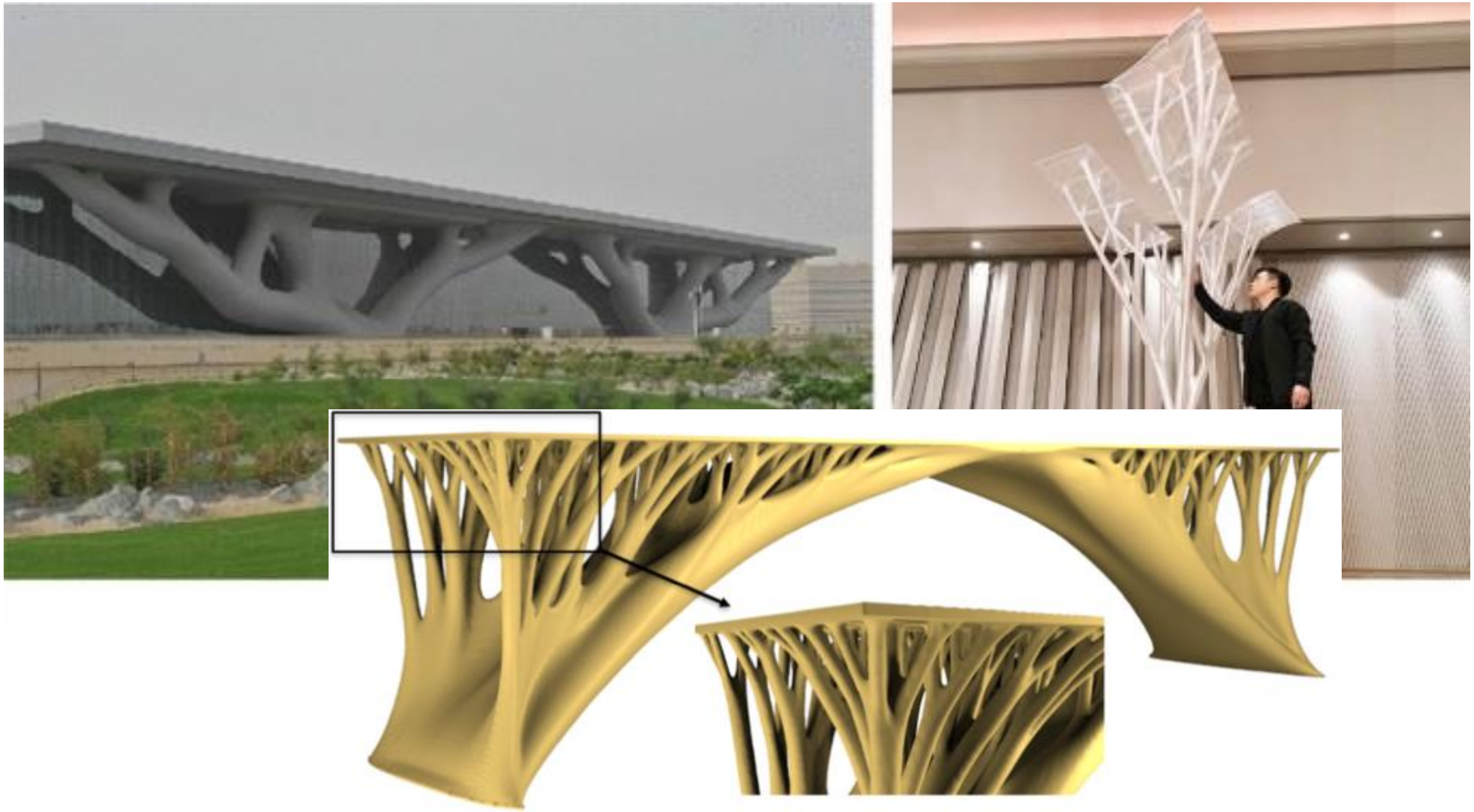




Course Project

Various scale

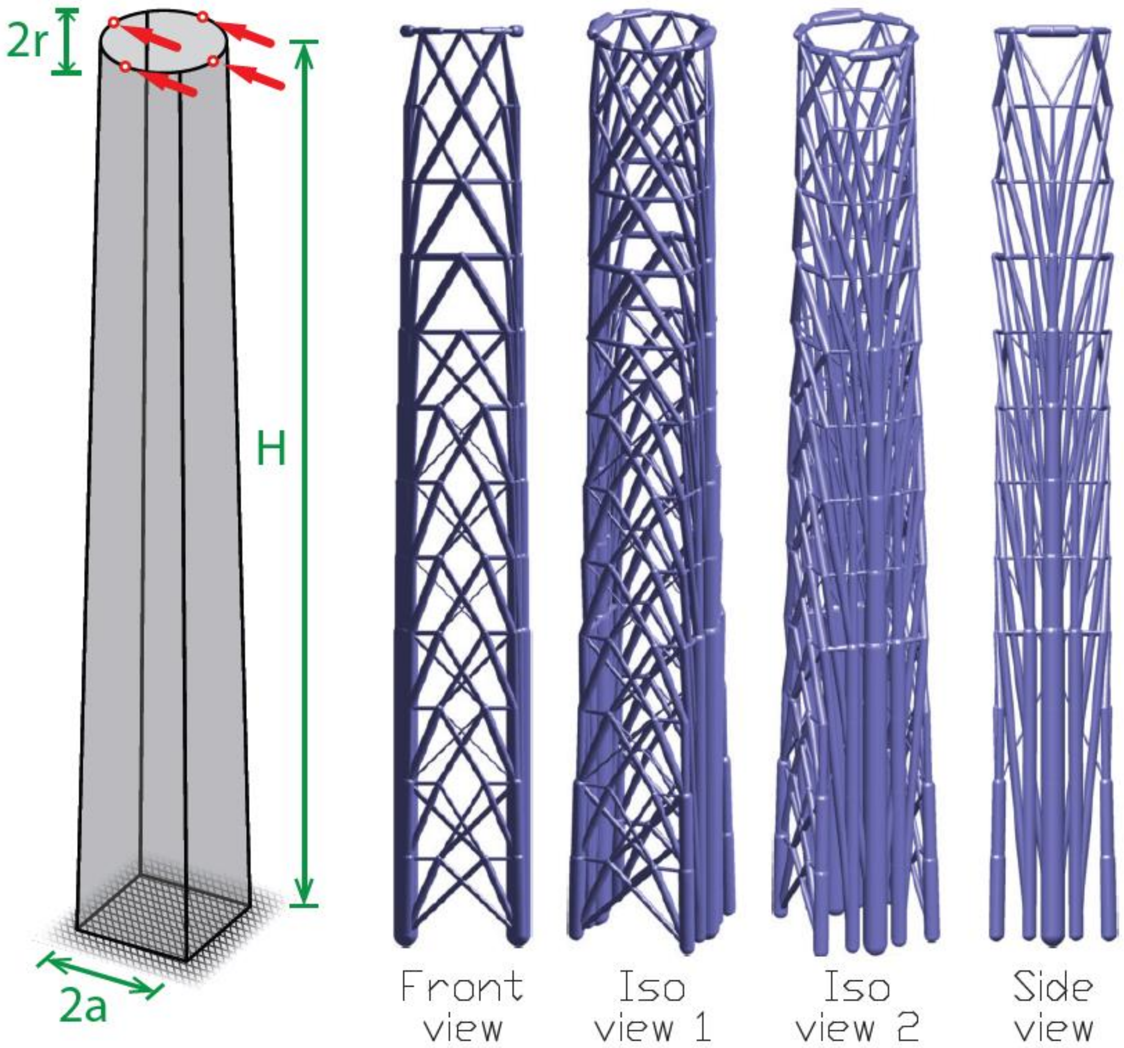
Component scale



Course Project

Various scale

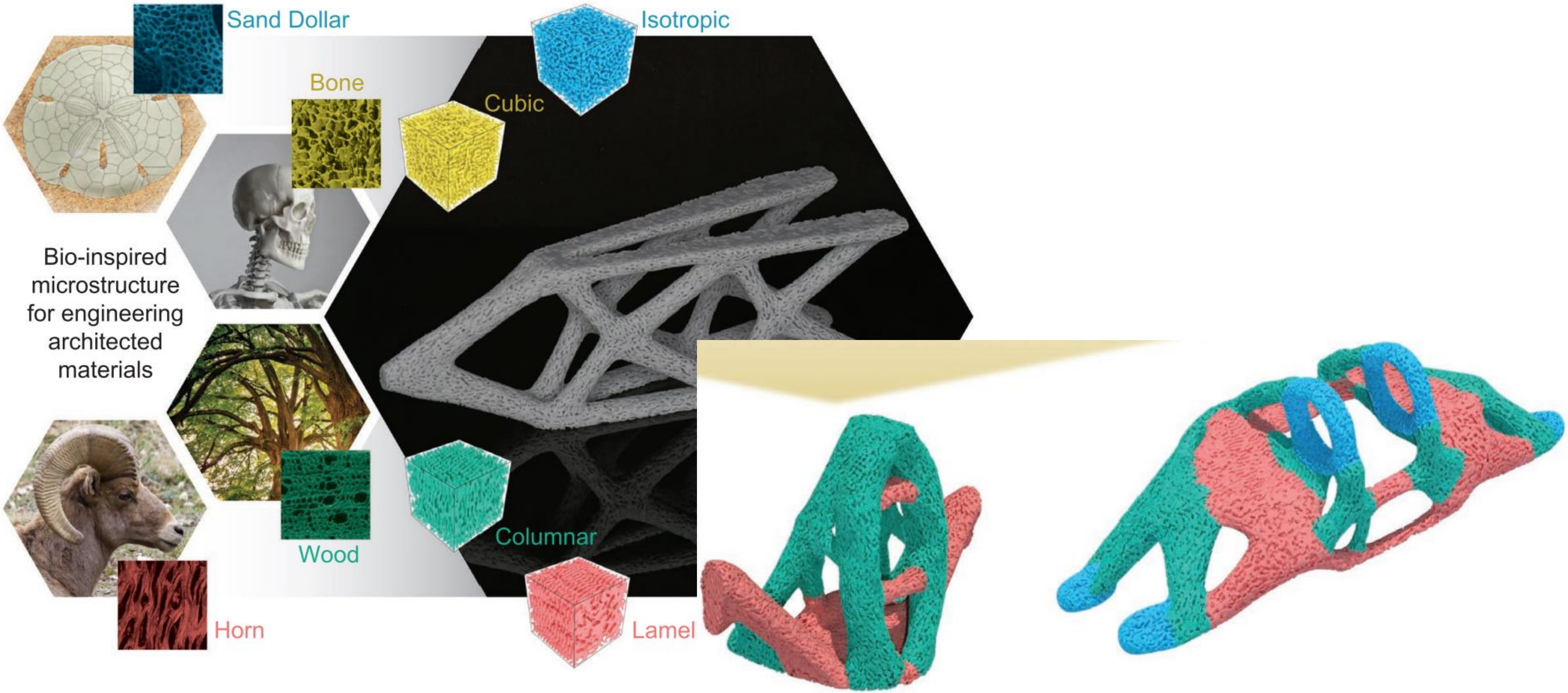
System scale



Course Project

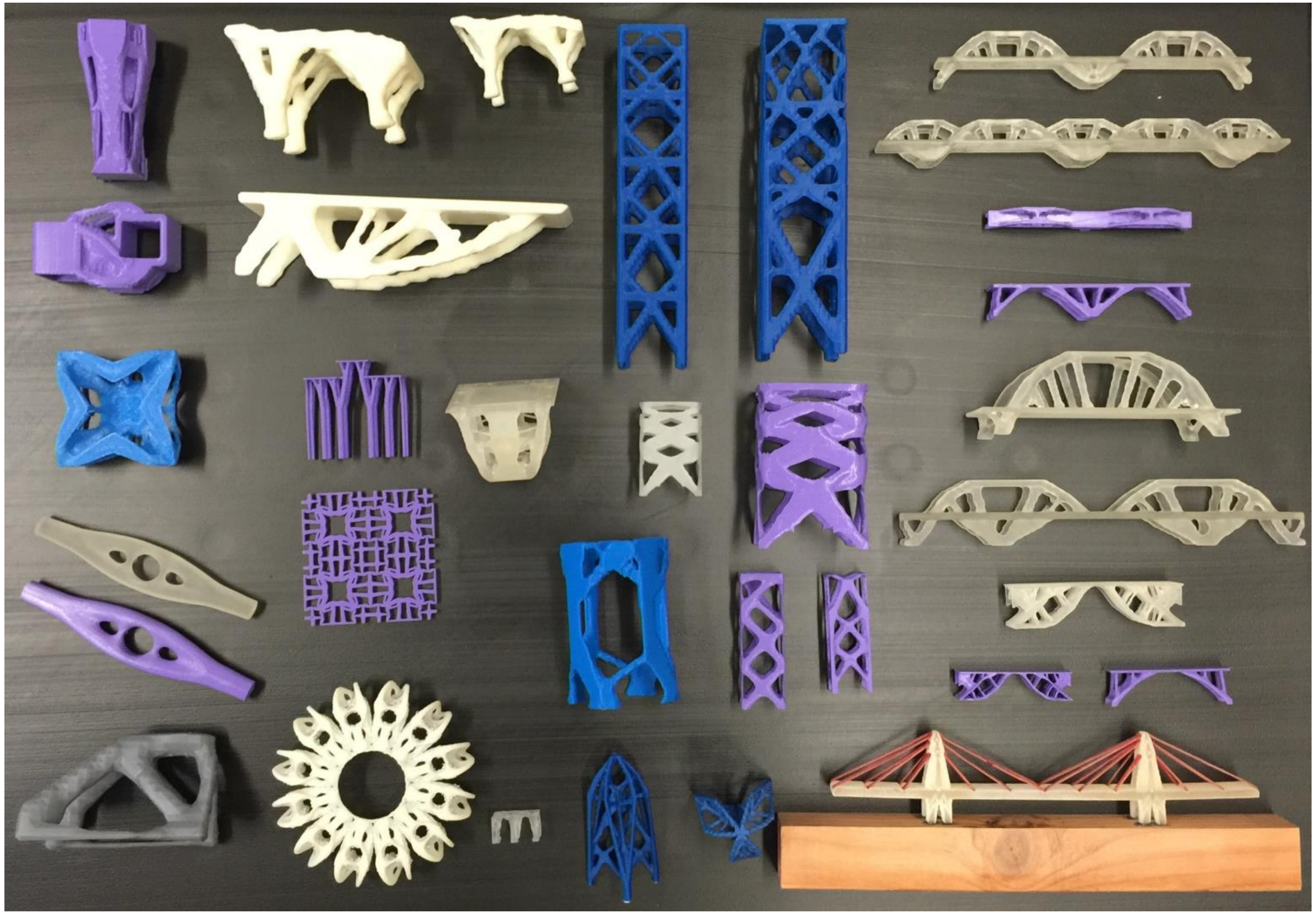
Various scale

Across scale



Course Project
Past Examples

12:45



Course Project

Think about something cool that you could put in your CV or Website ☺

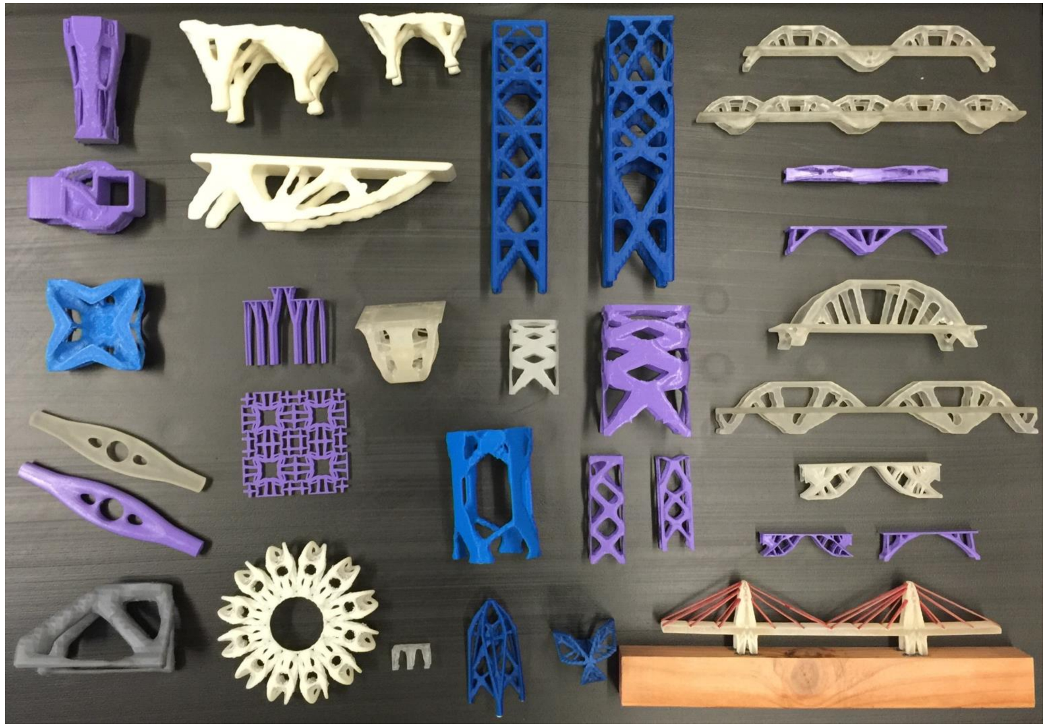
Course Project
Scope

1. Background (design background, state-of-the-art practice, typical loading and support patterns)
2. Simplify into optimization setting
3. Optimization (any existing packages or write your own)
4. Results and discussions
5. Fabrication (we will only provide suggestions, you need to determine your fabrication method and arrange them)

Course Project

Path I

(Individual) Optimize any structures of your choice. You can use any existing topology optimization programs (e.g., in MATLAB, ANSYS, ABAQUS). You will be asked to fabricate a demonstration model of your optimized structures (e.g., using 3D printing or by hand). You are encouraged to be creative in using other materials and fabrication methods.



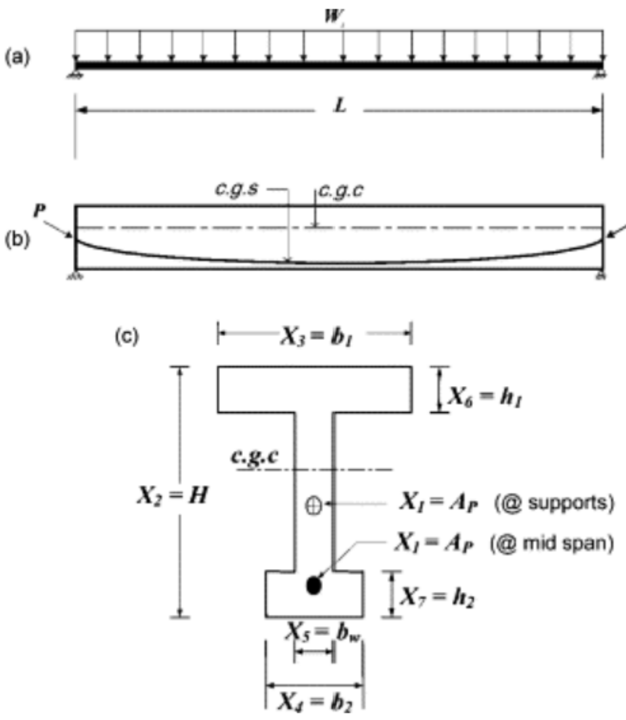
Course Project
Path 2

(A team of 2 students) Optimize a simply-supported truss bridge. You can use any existing topology optimization programs (e.g., in MATLAB or ABAQUS) and choose your testing protocol. You will be asked to design, fabricate, and test a conventional design for comparison. You will need to fabricate a reduced-scale model and then test on using weights under a simply-supported option. Fabrication and testing may follow a Spaghetti fabrication and tests.

Course Project

Path 3

(Individual or a team of 2 students) Optimize any structures of your choice. You write your own optimization program (no need to be restricted to topology optimization). You need to justify the necessity of using an optimization algorithm for the selected topic. No need to fabricate.



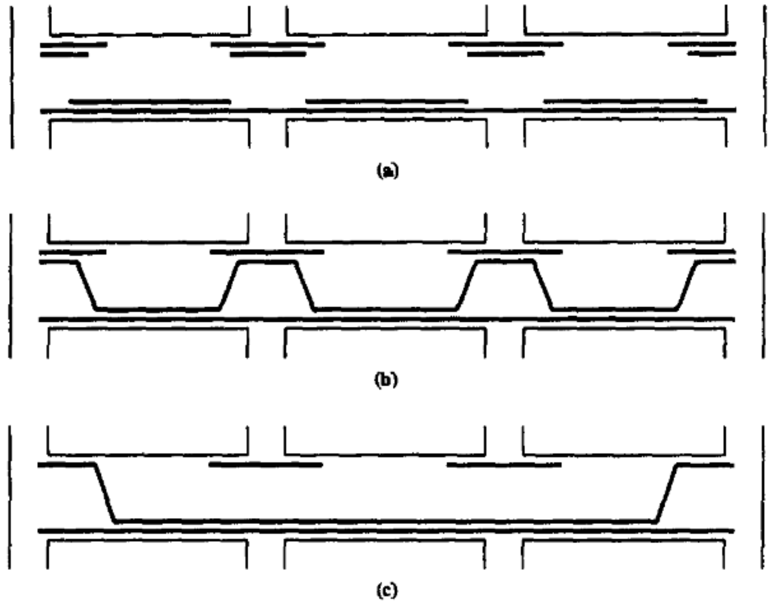
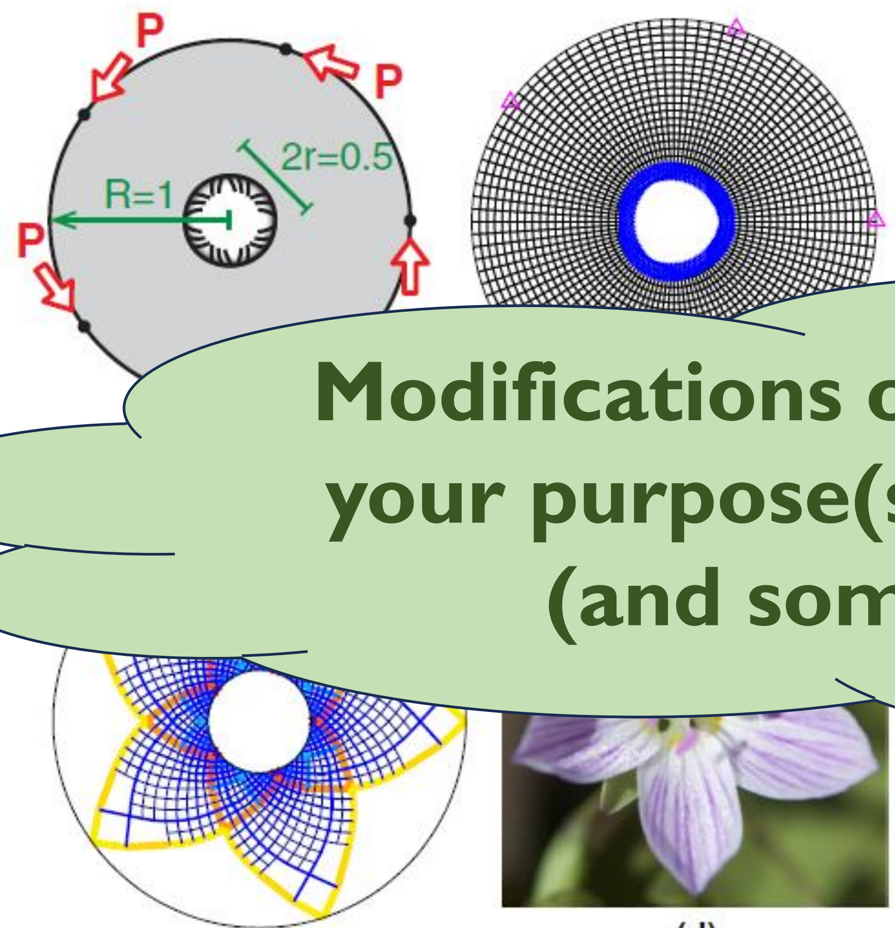


FIG. 2. Beam-Reinforcement Topologies: (a) Topology with 12 Bar Groups; (b) Topology with Six Bar Groups; (c) Topology with Four Bar Groups

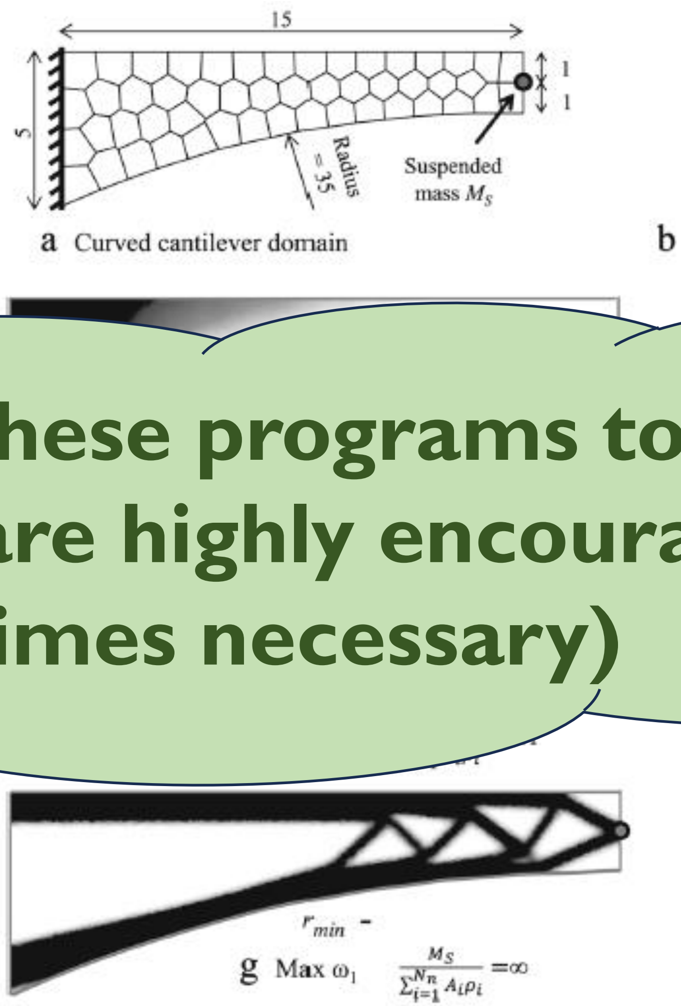
Course Project

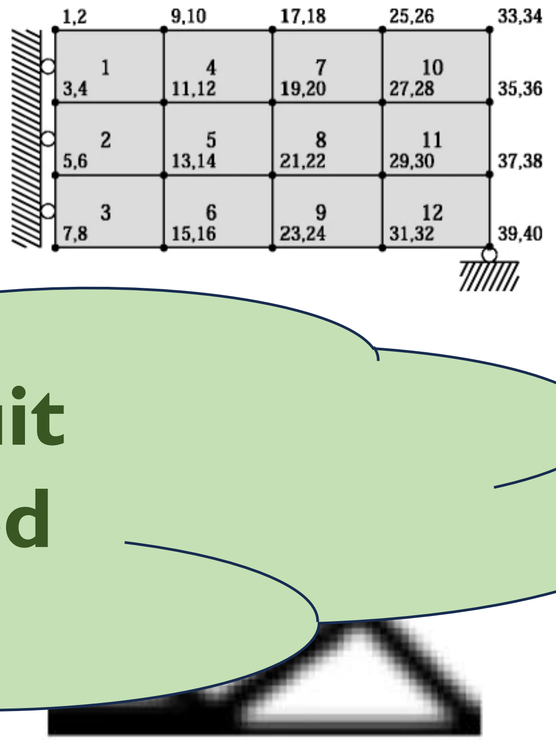
Possible tools in MATLAB that we will cover

GRAND in 2D and 3D | **PolyTop in 2D and 3D** | **88 line TopOpt**



(c) (d)





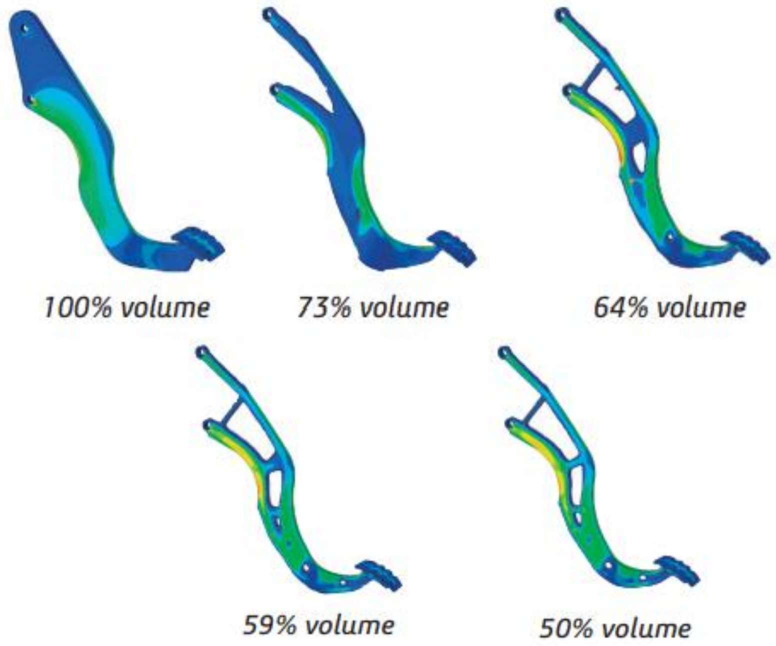
$c = 233.71$

Modifications of these programs to suit your purpose(s) are highly encouraged (and sometimes necessary)

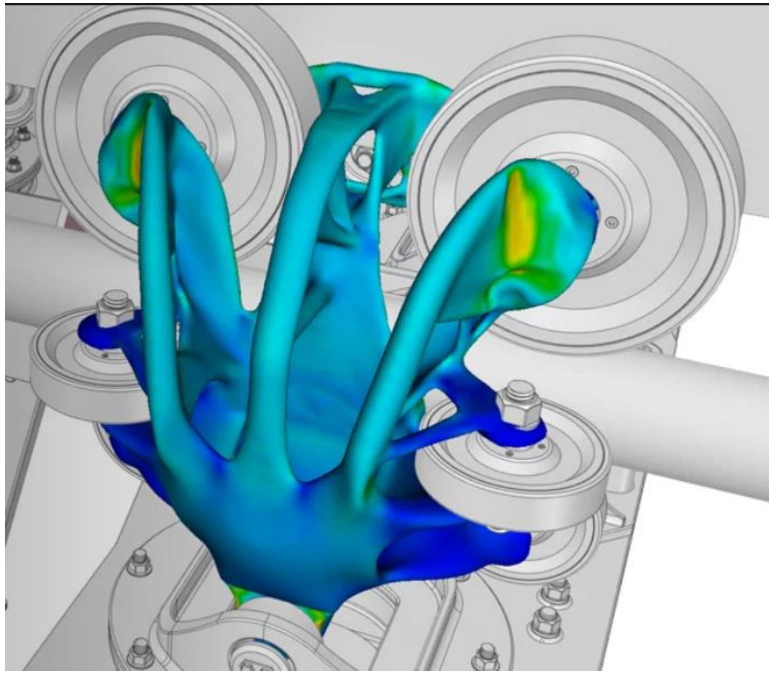
Course Project

Possible tools in other commercial software (that we won't cover)

ABAQUS



ANSYS



Course Project
Deliverables

[Preliminary Approval] For Paths 2&3, preliminary approval (team and topic) must be obtained from the instructor by **02/11/25.**

[Milestone] Upload the topic you have selected by the end of Monday, **03/11/25** on myCourses. The milestone should include the description of background and selection of program/software. If a team is formed, please include task divisions in the milestone.

The project presentations will be held near the end of semester. The exact date and time will be announced. Project report will be due one week after the presentations.

PechaKucha
12 X 20

If you need, a pitch session will be held on Jan 27 to help you find teammates.

3D Printing Guide

Course
Grading

<u>Grading Scheme:</u>	4-6 Assignments *(Not graded)*	0%
	Project (report + presentation)	20%
	Mid-term exam	30%
	Final exam	50%

Note: grades will not be curved so the entire class can all get A if you all perform well (85-100%). I encourage classmates to help each other improve their learning outcomes.

Tentative and subject to changes

Week	Class Date	Topic	Due
1	Jan 6	Introduction	
2	Jan 13	A Truss Optimization Example and Optimality Condition for Unconstrained Nonlinear Optimization	
3	Jan 20	Lagrange Multiplier, KKT Conditions, Structural Optimization of Concrete Structures	
4	Jan 27	Convexity, KKT Condition w/o Slacks, GRAND	
5	Feb 3	Sensitivity Analysis, Duality, Load Path	
6	Feb 10	Density-Based Method	Path 2&3 Approval
7	Feb 17	Robust Design Optimization, Multiple Load Cases	
8	Feb 24	Mid-term (covering wk1-6)	
9	Mar 3	Reading week, no lecture.	
10	Mar 10	Graphic Statics and Form Finding	Milestone
11	Mar 17	Multi-panel Truss Design Using Form Finding	
12	Mar 24	Reusing Structural Members Based On Graphic Statics	
13	Mar 31	Fabrication Break	
14	Apr 7	Project presentations.	

slido

Please download and install the Slido app on all computers you use

Do you have previous experience with MATLAB?

ⓘ Start presenting to display the poll results on this slide.

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What are your expectations of this course (anonymous)?

ⓘ Start presenting to display the poll results on this slide.

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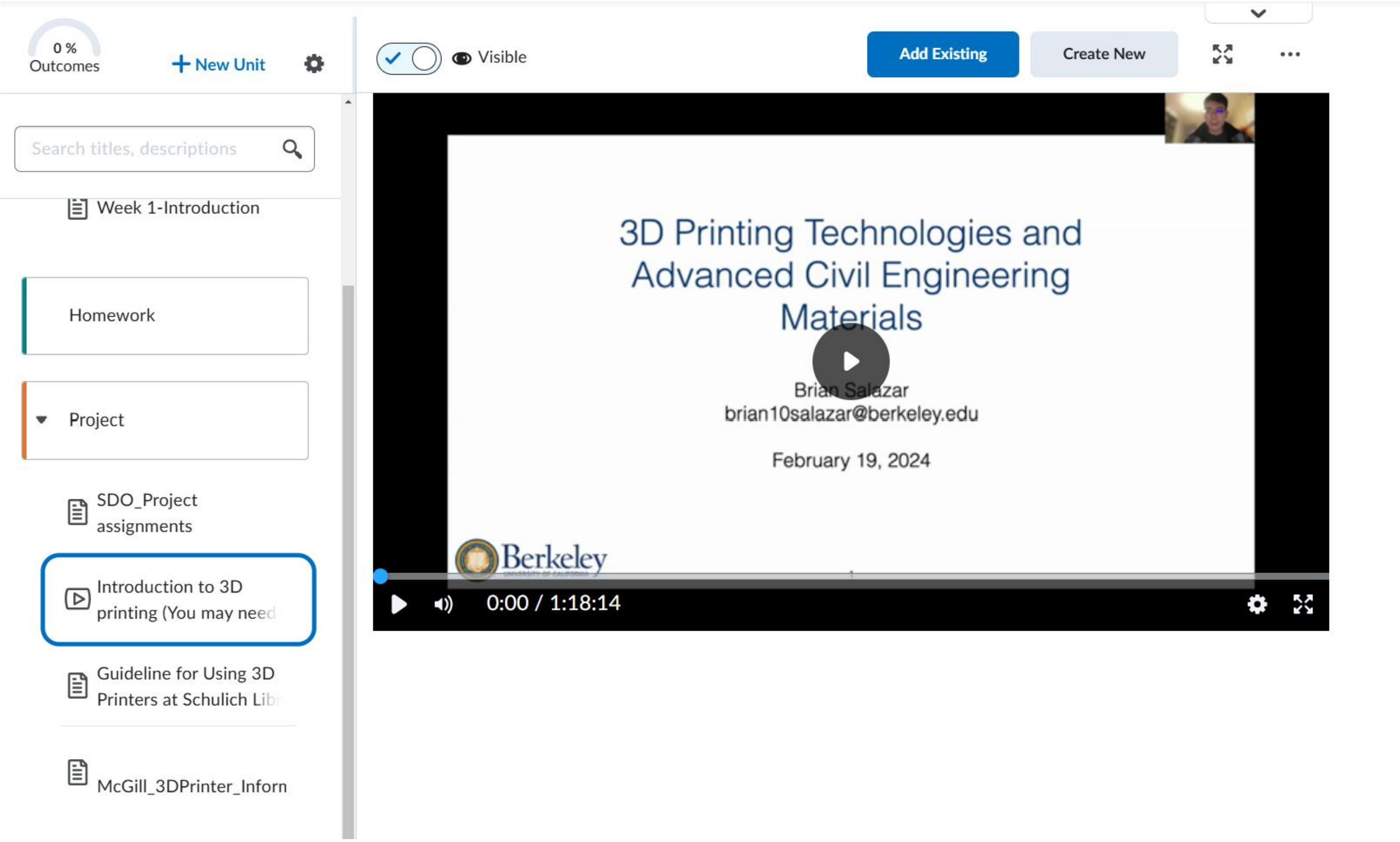
What are your fears or questions of this course (anonymous)?

ⓘ Start presenting to display the poll results on this slide.

HW 0:

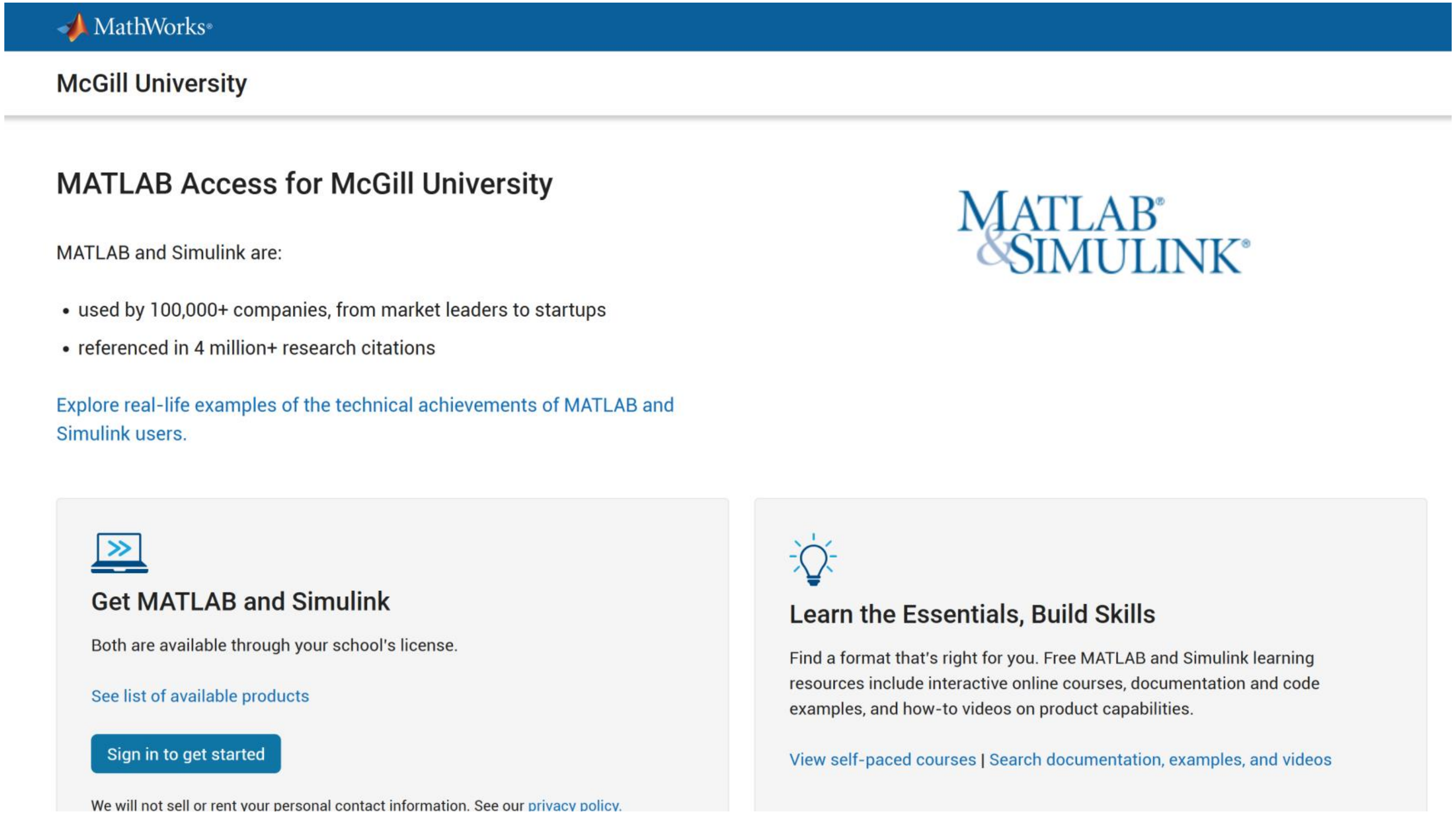
Review project assignment

Watch the 3D printing introduction video



HW 0: Please download MATLAB before next class

Familiarize with <u>fmincon</u>



- **Matlab (Basic function)**
- **Optimization Toolbox**

fmincon

Find minimum of constrained nonlinear multivariable function

Syntax

```
x = fmincon(fun,x0,A,b)
x = fmincon(fun,x0,A,b,Aeq,beq)
x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub)
x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon)
x = fmincon(fun,x0,A,b,Aeq,beq,lb,ub,nonlcon,options)
x = fmincon(problem)
[x,fval] = fmincon( __ )
[x,fval,exitflag,output] = fmincon( __ )
[x,fval,exitflag,output,lambda,grad,hessian] = fmincon( __ )
```

Description

Nonlinear programming solver.

Finds the minimum of a problem specified by

$$\min_x f(x) \text{ such that } \begin{cases} c(x) \le 0 \\ ceq(x) = 0 \\ A \cdot x \le b \\ Aeq \cdot x = beq \\ lb \le x \le ub, \end{cases}$$

b and beq are vectors, A and Aeq are matrices, $c(x)$ and $ceq(x)$ are functions that return vectors, and $f(x)$ is a function that returns a scalar. $f(x)$, $c(x)$, and $ceq(x)$ can be

x, lb, and ub can be passed as vectors or matrices; see Matrix Arguments.

Nonlinear Constraints

Find the minimum of a function subject to nonlinear constraints

Find the point where Rosenbrock's function is minimized within a circle, also subject to bound constraints.

Open in MATLAB Online

Copy Command

```
fun = @(x)100*(x(2)-x(1)^2)^2 + (1-x(1))^2;
```

Look within the region $0 \le x(1) \le 0.5,\ 0.2 \le x(2) \le 0.8$.

```
lb = [0,0.2];
ub = [0.5,0.8];
```

Also look within the circle centered at [1/3,1/3] with radius 1/3. Use this code for the nonlinear constraint function.

```
function [c,ceq] = circlecon(x)
c = (x(1)-1/3)^2 + (x(2)-1/3)^2 - (1/3)^2;
ceq = [];
end
```

There are no linear constraints, so set those arguments to `[]`.

```
A = [];
b = [];
Aeq = [];
beq = [];
```

Choose an initial point satisfying all the constraints.

```
x0 = [1/4,1/4];
```

Solve the problem.

Some quick play tools

- https://www.topopt.mek.dtu.dk/apps-and-software