CIVE 546 – Structural Design Optimization (SDO)

HW3-Solution

Problem 1

Given the unconstrained function,

$$F(\mathbf{x}) = 3x_1 + \frac{1}{x_1 - 1} + x_2{}^2 + \frac{1}{x_2}$$

a) The gradient for this function is

$$\nabla F(\mathbf{x}) = \left(3 - \frac{1}{(x_1 - 1)^2} \quad , \quad 2x_2 - \frac{1}{x_2^2}\right)$$

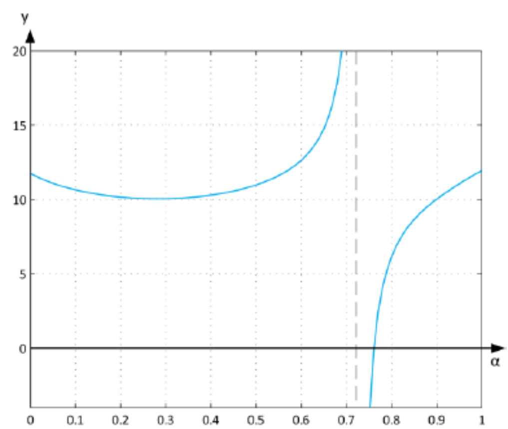
, evaluated at $x_1 = 3$ and $x_2 = 0.5$

$$\nabla F(\mathbf{x}^0) = \nabla F(3{,}0.5) = \left(\frac{11}{4} \quad , \quad -3\right)$$

b) The direction of the steepest descent is $\mathbf{S}^1 = -\nabla F(\mathbf{x}^0) = \left(-\frac{11}{4} \quad , \quad 3\right)$

c) The updated design is given by

$$\mathbf{x}^1 = \mathbf{x}^0 + \alpha\mathbf{S}^1 = (3{,}0.5) + \alpha\left(-\frac{11}{4}, 3\right) = \left(3 - \frac{11}{4}\alpha \quad , \quad \frac{1}{2} + 3\alpha\right)$$



d) The equation F in terms of α alone is

$$F(\mathbf{x}(\alpha)) = 3\left(3 - \frac{11}{4}\alpha\right) + \frac{1}{2 - \frac{11}{4}\alpha} + \left(\frac{1}{2} + 3\alpha\right)^2 + \frac{1}{\frac{1}{2} + 3\alpha}$$

Simple inspection of this equation gives two values of α where the function becomes indeterminate

$$\alpha_1^* = \frac{8}{11} = 0.7273 \qquad \alpha_2^* = -\frac{1}{6} = -0.1666$$

Starting from a step of length zero ($\alpha = 0$), the function $F(\alpha)$ takes an extreme value first at $\frac{8}{11}$, therefore indicating that we overshot and left the well region where the minimum is located. It is also clear from the range between 0 and $\frac{8}{11}$ that there is an optimum value for α: when α is too small, we make little progress towards the optimum, if α is too big overshoot and diverge.

e) The derivative with respect to α is

$$\frac{dF}{d\alpha} = 18\alpha + \frac{11}{4\left(2 - \frac{11}{4}\alpha\right)^2} - \frac{3}{\left(\frac{1}{2} + 3\alpha\right)^2} - \frac{21}{4}$$

evaluated at $\alpha = 0$

$$\frac{dF}{d\alpha}(0) = \frac{11}{16} - 12 - \frac{21}{4} = -\frac{265}{16}$$

f) The scalar product $\nabla \mathbf{F} \cdot \mathbf{S}$ is

$$\nabla \mathbf{F} \cdot \mathbf{S} = \left(\frac{11}{4} \quad , \quad -3\right) \cdot \left(-\frac{11}{4} \quad , \quad 3\right) = -\frac{121}{16} - \frac{144}{16} = -\frac{265}{16}$$

that is equal to the result from part (e).

Problem 2

a) The potential energy of the system is

$$\mathrm{PE}(x_1, x_2) = \frac{k_1}{2}\left[\sqrt{(x_1 - b)^2 + (x_2 + h_1)^2} - L_1\right]^2 + \frac{k_2}{2}\left[\sqrt{x_1^2 + (h_2 - x_2)^2} - L_2\right]^2 - P_1 x_1 - P_2 x_2$$

, and the gradient is

$$\nabla \mathrm{PE} = \begin{Bmatrix} k_1(x_1 - b)\left[1 - \frac{L_1}{\sqrt{(x_1 - b)^2 + (x_2 + h_1)^2}}\right] + k_2 x_1\left[1 - \frac{h_2}{\sqrt{x_1^2 + (h_2 - x_2)^2}}\right] - P_1 \\ k_1(x_2 + h_1)\left[1 - \frac{L_1}{\sqrt{(x_1 - b)^2 + (x_2 + h_1)^2}}\right] - k_2(h_2 - x_2)\left[1 - \frac{h_2}{\sqrt{x_1^2 + (h_2 - x_2)^2}}\right] - P_2 \end{Bmatrix}$$

b) The following MATLAB function returns the potential energy of the system and the gradient

```
function [PE,dPE]=Spring2D(x,plotopt)

x1=x(1); x2=x(2);

b=7; h1=12; h2=10;
k1=1; k2=8;
P1=4; P2=1;
L1=sqrt(b^2+h1^2);

dPE=zeros(2,1);
PE=1/2*k1*(sqrt((x1-b)^2+(x2+h1)^2)-L1)^2+1/2*k2*(sqrt(x1^2+(h2-x2)^2)-h2)^2-P1*x1-
P2*x2;
dPE(1)=k1*(x1-b)*(1-L1/sqrt((x1-b)^2+(x2+h1)^2))+k2*x1*(1-h2/sqrt(x1^2+(h2-x2)^2))-P1;
dPE(2)=k1*(x2+h1)*(1-L1/sqrt((x1-b)^2+(x2+h1)^2))-k2*(h2-x2)*(1-h2/sqrt(x1^2+(h2-
x2)^2))-P2;

if nargin==2, plot(x1,x2,plotopt,'LineWidth',2), end
```

c) The following MATLAB script plots the contours for the region requested

```
clc, clear, close all;

X0=[0 0]';

[mx1,mx2]=meshgrid(linspace(-2,12,100),linspace(-2,8,100));
mpe=zeros(size(mx1));
for i=1:100
    for j=1:100
        mpe(i,j)=Spring2D([mx1(i,j) mx2(i,j)]);
    end
end
figure, hold on, axis equal
[~,h]=contour(mx1,mx2,mpe,[-35 -30 -25 -15 0 25 50:50:200]);
set(h,'ShowText','on','TextStep',get(h,'LevelStep')*2)
plot([-4 12],[0 0],'k'), plot([0 0],[-2 12],'k')
axis([-2 12 -2 8]), xlabel('x_1'), ylabel('x_2')
```

d) The following script is executed after part (c) to generate the required plot

```
options=optimset('Largescale','off','Display','off');
[x,fval,exitflag,output,grad,hessian]=fminunc(@(x)Spring2D(x,'b^'),X0,options);
if exitflag<=0, disp('Problem did not converge - Check the ''exitflag'' for
troubleshooting')
else, fprintf('F(%.4f,%.4f)=%.4f\n',x,fval)
end
```

e) The following script is executed after parts (c) and (d) to generate the required plot

```
options=optimset('GradObj','on','Display','off');
[x,fval,exitflag,output,grad,hessian]=fminunc(@(x)Spring2D(x,'ro'),X0,options);
if exitflag<=0, disp('Problem did not converge - Check the ''exitflag'' for
troubleshooting')
```

```
else, fprintf('F(%.4f,%.4f)=%.4f\n',x,fval)
end
```

f) The solutions obtained are numerically equivalent

```
F(9.1555,4.8217)=-35.6659
F(9.1555,4.8217)=-35.6659
>>
```

, and the generated plot from parts (c), (d) and (e) is