1 Groups

Definition 1.1. Let $G = (G, e)$ be a monoid,
G is a group $\iff \forall a \in G : \exists a^{-1} \in G : a^{-1}a = aa^{-1} = e$

Proposition 1.2. *Let G be a group,* $\forall c \in G : (cc = c \implies c = e)$

Proof.

1 $\quad|\quad \forall c \in G : cc = c$
2 $\quad|\quad \implies c^{-1}cc = c^{-1}c$
3 $\quad|\quad \implies (c^{-1}c)c = c^{-1}c$
4 $\quad|\quad \implies ec = e$
5 $\quad|\quad \implies c = e$
6 $\quad \forall c \in G : (cc = c \implies c = e)$ $\square$

Proof. $\forall c \in G :$

1 $\quad|\quad cc = c$
2 $\quad|\quad c^{-1}(cc) = c^{-1}c$
3 $\quad|\quad (c^{-1}c)c = c^{-1}c$
4 $\quad|\quad ec = e$
5 $\quad|\quad c = e$
6 $\quad (cc = c \implies c = e)$ $\square$

Proposition 1.3. *Let G be a group,*