Monday, 4 Feb 2018

1 Phase DM

1.1 The Basic Search Method

With the complex structure of the sub-burst from FRB 121102, finding a effective method of determining a DM is becoming troublesome. The goal is to find an effective but universal way of determining the proper DM. The previous effort of taking the time derivative and then smoothed, was not robust and required increasing efforts to automate an effective smoothing kernel.

Yet we will start in a similar fashion. For most observations we will begin with a dynamic spectrum which is a function of emission-frequency and time $D(f,t)$. To correct for a given DM, we will shift the times series by the delay expected for that frequency. This is most simply shown in Fourier domain

$$FT[D(t',f)] \Rightarrow D(\omega,f)e^{\frac{i\omega CDM}{f^2}} \tag{1}$$

where FT is the Fourier transform, $D(t',f)$ is the shifted dynamic spectrum, $D(\omega,f)$ is the Fourier Transform of the initial dynamic spectrum, and C is the dispersion constant. Here we can see that all the dispersion "information" is contained solely in the phase angles. Therefore, we will only keep the phase angle information by dividing by the amplitudes. Then integrating over the emission-frequency will produce a coherent-spectrum where Fourier-frequencies that have similar phase angle (aka that are coherent) will sum to have max Amplitude. Therefore, we will search the coherent-power of the sum of the phase angles, which is written as:

$$Pow_{Co}(\omega,DM) = \Big|\int \frac{FT[D(t',f)]}{|FT[D(t',f)]|}df\Big|^2 \tag{2}$$

Yet as we have seen with the FRB profiles, we are interested in the time derivative. Knowing that

$$FT[\frac{dD(t',f)}{dt}] \Rightarrow i\omega D(\omega,f)e^{\frac{i\omega CDM}{f^2}} \tag{3}$$

The real power spectrum we are interested in is

$$Pow_{dCo}(\omega, DM) = \omega^2 \Big| \int \frac{FT[D(t', f)]}{|FT[D(t', f)]|} df \Big|^2 \tag{4}$$

Yet, we will treat the ω^2 term as a weighting when doing the final sum.

1.2 Error Math

Here we go through the math of determining in the S/N when looking at the sum of Pow_{dCo}. First we assume uniform distribution in phase angles between -π to π. Therefore, the probability distribution function is

$$PDF(\theta) = \frac{d\theta}{2\pi} \tag{5}$$

Since component X is

$$X = cos(\theta) \tag{6}$$

The PDF be comes

$$PDF(X) = \frac{dX}{2\pi sin(acos(X))} \tag{7}$$

Which has an expectation value of zero and a standard deviation of

$$\sigma_x = \frac{1}{2} \tag{8}$$

Since we will be summing components across the number of frequency channels (n_{chan}) means that central-limit-theorem is applied such that

$$\sigma_{\sum x} = \frac{\sqrt{n_{chan}}}{2} \tag{9}$$

After this we look at the power, which means the distribution becomes $\Gamma(2, n_{chan}/2)$. Making the expectation of the power distribution

$$< Pow_{Co} >= n_{chan} \tag{10}$$

with a deviation of

$$\sigma_{Pow_{Co}} = \frac{n_{chan}}{\sqrt{2}} \tag{11}$$

Finally we will sum across ω with the ω^2 "weighting" and we can apply the central-limit-theorem. Yet, to simplify things we replace ω^2 with $index^2$ since python has zero-indexing. This means that

$$< \sum Pow_{dCo} > = n_{chan} \sum_{0}^{index_{final}} index^2$$
$$< \sum Pow_{dCo} > = \frac{n_{chan}(index_{final})(index_{final}+1)(2*index_{final}+1)}{6} \tag{12}$$

and that

$$\sigma^2_{\sum Pow_{dCo}} = \frac{n^2_{chan}}{2} \sum_{0}^{index_{final}} index^4$$
$$\sigma^2_{\sum Pow_{dCo}} = \frac{n^2_{chan}(index_{final})(index_{final}+1)(2*index_{final}+1)(3*index^2_{final}+3*index_{final}-1)}{60} \tag{13}$$

Therefore

$$S/N = \frac{Max(\sum Pow_{dCo}) - < \sum Pow_{dCo} >}{\sigma_{\sum Pow_{dCo}}} \tag{14}$$