EECS 498-007/598-005 Assignment 2-1: Linear Classifiers

Before we start, please put your name and UMID in following format

: Firstname LASTNAME, #00000000 // e.g.) Justin JOHNSON, #12345678

Your Answer:
Hello WORLD, #XXXXXXXX

Install starter code

We will continue using the utility functions that we've used for Assignment 1: `coutils` package. Run this cell to download and install it.

```
[19] !pip install git+https://github.com/deepvision-class/starter-code
```

```
Looking in indexes: https://pypi.org/simple, https://us-python.pkg.dev/colab-wheels/public/simple/
Collecting git+https://github.com/deepvision-class/starter-code
  Cloning https://github.com/deepvision-class/starter-code to /tmp/pip-req-build-5afdn8ib
  Running command git clone -q https://github.com/deepvision-class/starter-code /tmp/pip-req-build-5afdn8ib
Requirement already satisfied: pydrive in /usr/local/lib/python3.7/dist-packages (from Colab-Utils==0.1.dev0) (1.3.1)
Requirement already satisfied: google-api-python-client>=1.2 in /usr/local/lib/python3.7/dist-packages (from pydrive->Colab-Utils==0.1.dev0) (1.12.11)
Requirement already satisfied: oauth2client>=4.0.0 in /usr/local/lib/python3.7/dist-packages (from pydrive->Colab-Utils==0.1.dev0) (4.1.3)
Requirement already satisfied: PyYAML>=3.0 in /usr/local/lib/python3.7/dist-packages (from pydrive->Colab-Utils==0.1.dev0) (3.13)
Requirement already satisfied: google-auth-httplib2>=0.0.3 in /usr/local/lib/python3.7/dist-packages (from google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (0.0.4)
Requirement already satisfied: six<2dev,>=1.13.0 in /usr/local/lib/python3.7/dist-packages (from google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (1.15.0)
Requirement already satisfied: google-auth<3dev,>=1.16.0 in /usr/local/lib/python3.7/dist-packages (from google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (1.35.0)
Requirement already satisfied: httplib2<1dev,>=0.15.0 in /usr/local/lib/python3.7/dist-packages (from google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (0.17.4)
Requirement already satisfied: uritemplate<4dev,>=3.0.0 in /usr/local/lib/python3.7/dist-packages (from google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (3.0.1)
Requirement already satisfied: google-api-core<3dev,>=1.21.0 in /usr/local/lib/python3.7/dist-packages (from google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (1.31.6)
Requirement already satisfied: requests<3.0.0dev,>=2.18.0 in /usr/local/lib/python3.7/dist-packages (from google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (2.23.0)
Requirement already satisfied: pytz in /usr/local/lib/python3.7/dist-packages (from google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (2022.1)
Requirement already satisfied: protobuf<4.0.0dev,>=3.12.0 in /usr/local/lib/python3.7/dist-packages (from google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (3.17.3)
Requirement already satisfied: setuptools>=40.3.0 in /usr/local/lib/python3.7/dist-packages (from google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (57.4.0)
Requirement already satisfied: packaging>=14.3 in /usr/local/lib/python3.7/dist-packages (from google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (21.3)
Requirement already satisfied: googleapis-common-protos<2.0dev,>=1.6.0 in /usr/local/lib/python3.7/dist-packages (from google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.
Requirement already satisfied: pyasn1-modules>=0.2.1 in /usr/local/lib/python3.7/dist-packages (from google-auth<3dev,>=1.16.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (0.2.8)
Requirement already satisfied: rsa<5,>=3.1.4 in /usr/local/lib/python3.7/dist-packages (from google-auth<3dev,>=1.16.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (4.9)
Requirement already satisfied: cachetools<5.0,>=2.0.0 in /usr/local/lib/python3.7/dist-packages (from google-auth<3dev,>=1.16.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1.dev0) (4.2.4)
Requirement already satisfied: pyasn1>=0.1.7 in /usr/local/lib/python3.7/dist-packages (from oauth2client>=4.0.0->pydrive->Colab-Utils==0.1.dev0) (0.4.8)
Requirement already satisfied: pyparsing!=3.0.5,>=2.0.2 in /usr/local/lib/python3.7/dist-packages (from packaging>=14.3->google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1
Requirement already satisfied: certifi>=2017.4.17 in /usr/local/lib/python3.7/dist-packages (from requests<3.0.0dev,>=2.18.0->google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils
Requirement already satisfied: urllib3!=1.25.0,!=1.25.1,<1.26,>=1.21.1 in /usr/local/lib/python3.7/dist-packages (from requests<3.0.0dev,>=2.18.0->google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2
Requirement already satisfied: chardet<4,>=3.0.2 in /usr/local/lib/python3.7/dist-packages (from requests<3.0.0dev,>=2.18.0->google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils
Requirement already satisfied: idna<3,>=2.5 in /usr/local/lib/python3.7/dist-packages (from requests<3.0.0dev,>=2.18.0->google-api-core<3dev,>=1.21.0->google-api-python-client>=1.2->pydrive->Colab-Utils==0.1
```

Setup code

Run some setup code for this notebook: Import some useful packages and increase the default figure size.

```
[20] from __future__ import print_function
     from __future__ import division

     import torch
     import coutils
     import random
     import time
     import math
     import matplotlib.pyplot as plt
     from torchvision.utils import make_grid

     %matplotlib inline
     plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
     plt.rcParams['image.interpolation'] = 'nearest'
     plt.rcParams['image.cmap'] = 'gray'
```

Starting in this assignment, we will use the GPU to accelerate our computation. Run this cell to make sure you are using a GPU.

```
[21] if torch.cuda.is_available:
       print('Good to go!')
     else:
       print('Please set GPU via Edit -> Notebook Settings.')
```

```
Good to go!
```

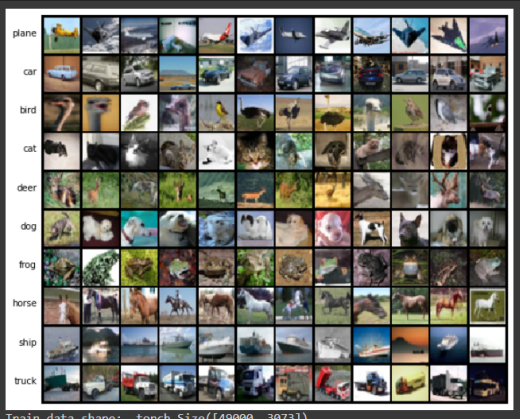
Now, we will load CIFAR10 dataset, with normalization.

In this notebook we will use the **bias trick**: By adding an extra constant feature of ones to each image, we avoid the need to keep track of a bias vector; the bias will be encoded as the part of the weight matrix that interacts with the constant ones in the input.

In the `two_layer_net.ipynb` notebook that follows this one, we will not use the bias trick.

```
[22] def get_CIFAR10_data(validation_ratio = 0.02):
       """
       Load the CIFAR-10 dataset from disk and perform preprocessing to prepare
       it for the linear classifier. These are the same steps as we used for the
       SVM, but condensed to a single function.
       """
       X_train, y_train, X_test, y_test = coutils.data.cifar10()

       # Move all the data to the GPU
       X_train = X_train.cuda()
       y_train = y_train.cuda()
       X_test = X_test.cuda()
       y_test = y_test.cuda()

       # 0. Visualize some examples from the dataset.
       class_names = [
           'plane', 'car', 'bird', 'cat', 'deer',
           'dog', 'frog', 'horse', 'ship', 'truck'
       ]
       img = coutils.utils.visualize_dataset(X_train, y_train, 12, class_names)
       plt.imshow(img)
```

```
    plt.axis('off')
    plt.show()

    # 1. Normalize the data: subtract the mean RGB (zero mean)
    mean_image = X_train.mean(dim=0, keepdim=True).mean(dim=2, keepdim=True).mean(dim=3, keepdim=True)
    X_train -= mean_image
    X_test -= mean_image

    # 2. Reshape the image data into rows
    X_train = X_train.reshape(X_train.shape[0], -1)
    X_test = X_test.reshape(X_test.shape[0], -1)

    # 3. Add bias dimension and transform into columns
    ones_train = torch.ones(X_train.shape[0], 1, device=X_train.device)
    X_train = torch.cat([X_train, ones_train], dim=1)
    ones_test = torch.ones(X_test.shape[0], 1, device=X_test.device)
    X_test = torch.cat([X_test, ones_test], dim=1)

    # 4. Carve out part of the training set to use for validation.
    # For random permutation, you can use torch.randperm or torch.randint
    # But, for this homework, we use slicing instead.
    num_training = int( X_train.shape[0] * (1.0 - validation_ratio) )
    num_validation = X_train.shape[0] - num_training

    # Return the dataset as a dictionary
    data_dict = {}
    data_dict['X_val'] = X_train[num_training:num_training + num_validation]
    data_dict['y_val'] = y_train[num_training:num_training + num_validation]
    data_dict['X_train'] = X_train[0:num_training]
    data_dict['y_train'] = y_train[0:num_training]

    data_dict['X_test'] = X_test
    data_dict['y_test'] = y_test
    return data_dict

# Invoke the above function to get our data.
data_dict = get_CIFAR10_data()
print('Train data shape: ', data_dict['X_train'].shape)
print('Train labels shape: ', data_dict['y_train'].shape)
print('Validation data shape: ', data_dict['X_val'].shape)
print('Validation labels shape: ', data_dict['y_val'].shape)
print('Test data shape: ', data_dict['X_test'].shape)
print('Test labels shape: ', data_dict['y_test'].shape)
```




```
Train data shape:  torch.Size([49000, 3073])
Train labels shape:  torch.Size([49000])
Validation data shape:  torch.Size([1000, 3073])
Validation labels shape:  torch.Size([1000])
Test data shape:  torch.Size([10000, 3073])
Test labels shape:  torch.Size([10000])
```

For Softmax and SVM, we will analytically compute the gradient, as a sanity check.

```
def grad_check_sparse(f, x, analytic_grad, num_checks=10, h=1e-7):
  """
  Utility function to perform numeric gradient checking. We use the centered
  difference formula to compute a numeric derivative:

  f'(x) =~ (f(x + h) - f(x - h)) / (2h)

  Rather than computing a full numeric gradient, we sparsely sample a few
  dimensions along which to compute numeric derivatives.

  Inputs:
  - f: A function that inputs a torch tensor and returns a torch scalar
  - x: A torch tensor giving the point at which to evaluate the numeric gradient
  - analytic_grad: A torch tensor giving the analytic gradient of f at x
  - num_checks: The number of dimensions along which to check
  - h: Step size for computing numeric derivatives
  """
  # fix random seed for
  coutils.utils.fix_random_seed()

  for i in range(num_checks):

    ix = tuple([random.randrange(m) for m in x.shape])

    oldval = x[ix].item()
    x[ix] = oldval + h # increment by h
    fxph = f(x).item() # evaluate f(x + h)
    x[ix] = oldval - h # increment by h
    fxmh = f(x).item() # evaluate f(x - h)
    x[ix] = oldval     # reset

    grad_numerical = (fxph - fxmh) / (2 * h)
    grad_analytic = analytic_grad[ix]
    rel_error_top = abs(grad_numerical - grad_analytic)
    rel_error_bot = (abs(grad_numerical) + abs(grad_analytic) + 1e-12)
    rel_error = rel_error_top / rel_error_bot
    msg = 'numerical: %f analytic: %f, relative error: %e'
    print(msg % (grad_numerical, grad_analytic, rel_error))
```

SVM Classifier

SVM Classifier

In this section, you will:

- implement a fully-vectorized **loss function** for the SVM
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

```
[24] def svm_loss_naive(W, X, y, reg):
       """
       Structured SVM loss function, naive implementation (with loops).

       Inputs have dimension D, there are C classes, and we operate on minibatches
       of N examples. When you implment the regularization over W, please DO NOT
       multiply the regularization term by 1/2 (no coefficient).

       Inputs:
       - W: A PyTorch tensor of shape (D, C) containing weights.
       - X: A PyTorch tensor of shape (N, D) containing a minibatch of data.
       - y: A PyTorch tensor of shape (N,) containing training labels; y[i] = c means
         that X[i] has label c, where 0 <= c < C.
       - reg: (float) regularization strength

       Returns a tuple of:
       - loss as torch scalar
       - gradient of loss with respect to weights W; a tensor of same shape as W
       """
       dW = torch.zeros_like(W) # initialize the gradient as zero

       # compute the loss and the gradient
       num_classes = W.shape[1]
       num_train = X.shape[0]
       loss = 0.0
       for i in range(num_train):
         scores = W.t().mv(X[i])
         correct_class_score = scores[y[i]]
         for j in range(num_classes):
           if j == y[i]:
             continue
           margin = scores[j] - correct_class_score + 1 # note delta = 1
           if margin > 0:
             loss += margin
             #############################################################################
             # TODO:                                                                     #
             # Compute the gradient of the loss function and store it dW. (part 1)       #
             # Rather that first computing the loss and then computing the               #
             # derivative, it is simple to compute the derivative at the same time       #
             # that the loss is being computed.                                          #
             #############################################################################
             # Replace "pass" statement with your code



             dW[:,j] += X[i]
             dW[:,y[i]] -= X[i]




             #############################################################################
             #                       END OF YOUR CODE                                    #
             #############################################################################


       # Right now the loss is a sum over all training examples, but we want it
       # to be an average instead so we divide by num_train.
       loss /= num_train

       # Add regularization to the loss.
       loss += reg * torch.sum(W * W)

       #############################################################################
       # TODO:                                                                     #
       # Compute the gradient of the loss function and store it in dW. (part 2)    #
       #############################################################################
       # Replace "pass" statement with your code


       dW /= num_train
       dW += 2*reg*W




       #############################################################################
       #                       END OF YOUR CODE                                    #
       #############################################################################

       return loss, dW
```

Evaluate the naive implementation of the loss we provided for you. You will get around 9.000175.

```
[25] # generate a random SVM weight tensor of small numbers
     coutils.utils.fix_random_seed()
     W = torch.randn(3073, 10, device=data_dict['X_val'].device) * 0.0001

     loss, grad = svm_loss_naive(W, data_dict['X_val'], data_dict['y_val'], 0.000005)
     print('loss: %f' % (loss, ))
```

```
loss: 9.000433
```

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function `svm_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you (The relative errors should be less than `1e-6`).

```
[26] # Once you've implemented the gradient, recompute it with the code below
     # and gradient check it with the function we provided for you

     # Use a random W and a minibatch of data from the val set for gradient checking
     # For numeric gradient checking it is a good idea to use 64-bit floating point
     # numbers for increased numeric precision; however when actually training models
     # we usually use 32-bit floating point numbers for increased speed.
     coutils.utils.fix_random_seed()
     W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()
     batch_size = 64
     X_batch = data_dict['X_val'][:64].double()
     y_batch = data_dict['y_val'][:64]

     # Compute the loss and its gradient at W.
     loss, grad = svm_loss_naive(W.double(), X_batch, y_batch, reg=0.0)

     # Numerically compute the gradient along several randomly chosen dimensions, and
     # compare them with your analytically computed gradient. The numbers should
     # match almost exactly along all dimensions.
     f = lambda w: svm_loss_naive(w, X_batch, y_batch, reg=0.0)[0]
     grad_numerical = grad_check_sparse(f, W.double(), grad)
```

```
numerical: -0.034577 analytic: -0.034577, relative error: 2.372452e-07
numerical: 0.126951 analytic: 0.126951, relative error: 5.721190e-08
numerical: -0.068597 analytic: -0.068597, relative error: 2.249695e-07
numerical: 0.025717 analytic: 0.025717, relative error: 4.774223e-07
numerical: 0.048266 analytic: 0.048266, relative error: 2.668362e-07
numerical: 0.052260 analytic: 0.052260, relative error: 2.475153e-07
numerical: 0.096133 analytic: 0.096133, relative error: 4.690208e-09
numerical: 0.032702 analytic: 0.032702, relative error: 3.644517e-07
numerical: -0.117158 analytic: -0.117158, relative error: 4.006759e-08
numerical: -0.154093 analytic: -0.154093, relative error: 7.809949e-08
```

Let's do the gradient check once again with regularization turned on. (You didn't forget the regularization gradient, did you?)

You should see relative errors less than `1e-5`.

```
[27] # Use a minibatch of data from the val set for gradient checking
     coutils.utils.fix_random_seed()
     W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()
     batch_size = 64
     X_batch = data_dict['X_val'][:64].double()
     y_batch = data_dict['y_val'][:64]

     # Compute the loss and its gradient at W.
     loss, grad = svm_loss_naive(W.double(), X_batch, y_batch, reg=1e3)

     # Numerically compute the gradient along several randomly chosen dimensions, and
     # compare them with your analytically computed gradient. The numbers should
     # match almost exactly along all dimensions.
     f = lambda w: svm_loss_naive(w, X_batch, y_batch, reg=1e3)[0]
     grad_numerical = grad_check_sparse(f, W.double(), grad)
```

```
numerical: -0.121624 analytic: -0.121624, relative error: 7.160875e-08
numerical: 0.020923 analytic: 0.020923, relative error: 3.331613e-07
numerical: -0.076604 analytic: -0.076604, relative error: 1.800879e-07
numerical: 0.256069 analytic: 0.256069, relative error: 6.121908e-08
numerical: -0.330089 analytic: -0.330089, relative error: 4.165267e-08
numerical: 0.004713 analytic: 0.004713, relative error: 3.512375e-06
numerical: 0.101968 analytic: 0.101968, relative error: 1.802643e-08
numerical: 0.097235 analytic: 0.097235, relative error: 1.618033e-07
numerical: -0.117112 analytic: -0.117112, relative error: 2.948518e-08
numerical: -0.257260 analytic: -0.257260, relative error: 4.474401e-08
```

Now, let's implement vectorized version of SVM: `svm_loss_vectorized`. It should compute the same inputs and outputs as the naive version before, but it should involve **no explicit loops**.

```
def svm_loss_vectorized(W, X, y, reg):
  """
  Structured SVM loss function, vectorized implementation. When you implment
  the regularization over W, please DO NOT multiply the regularization term by
  1/2 (no coefficient).

  Inputs and outputs are the same as svm_loss_naive.
  """
  loss = 0.0
  dW = torch.zeros_like(W) # initialize the gradient as zero

  #############################################################################
  # TODO:                                                                     #
  # Implement a vectorized version of the structured SVM loss, storing the    #
  # result in loss.                                                           #
  #############################################################################
  # Replace "pass" statement with your code
#   print(X.shape)
#   print(W.shape)
  scores = X.mm(W)
  num_train=X[0]

  correct_label_score_idxes = (range(scores.shape[0]), y)

  correct_label_scores = scores[correct_label_score_idxes]
  scores_diff = scores - torch.reshape(correct_label_scores, (-1, 1))

  scores_diff += 1

  scores_diff[correct_label_score_idxes] = 0
  indexes_of_neg_nums = torch.nonzero(scores_diff < 0)
  scores_diff[indexes_of_neg_nums] = 0

  loss = scores_diff.sum()
  num_train = X.shape[0]
  loss /= num_train
  # add in the regularization part.
  loss += reg * (W * W).sum()
  #############################################################################
  #                             END OF YOUR CODE                              #
  #############################################################################


  #############################################################################
  # TODO:                                                                     #
  # Implement a vectorized version of the gradient for the structured SVM     #
  # loss, storing the result in dW.                                           #
  #                                                                           #
  # Hint: Instead of computing the gradient from scratch, it may be easier    #
  # to reuse some of the intermediate values that you used to compute the     #
```

```
    # loss.                                                                #
    ###########################################################################
    # Replace "pass" statement with your code
    scores_diff[scores_diff > 0] = 1
    correct_label_vals = torch.sum(scores_diff , 1) * -1
    scores_diff[correct_label_score_idxes] = correct_label_vals

    dW = X.t().mm(scores_diff)
    dW /= num_train
    # add the regularization contribution to the gradient
    dW += reg * 2* W
    ###########################################################################
    #                           END OF YOUR CODE                              #
    ###########################################################################

    return loss, dW
```

Let's first check the speed and performance bewteen the non-vectorized and the vectorized version. You should see a speedup of more than 100x.

(Note: It may have some difference, but should be less than 1e-6)

```
[29] # Next implement the function svm_loss_vectorized; for now only compute the loss;
     # we will implement the gradient in a moment.

     # Use random weights and a minibatch of val data for gradient checking
     coutils.utils.fix_random_seed()
     W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()
     X_batch = data_dict['X_val'][:128].double()
     y_batch = data_dict['y_val'][:128]
     reg = 0.000005

     # Run and time the naive version
     torch.cuda.synchronize()
     tic = time.time()
     loss_naive, grad_naive = svm_loss_naive(W, X_batch, y_batch, reg)
     torch.cuda.synchronize()
     toc = time.time()
     ms_naive = 1000.0 * (toc - tic)
     print('Naive loss: %e computed in %.2fms' % (loss_naive, ms_naive))

     # Run and time the vectorized version
     torch.cuda.synchronize()
     tic = time.time()
     loss_vec, _ = svm_loss_vectorized(W, X_batch, y_batch, reg)
     torch.cuda.synchronize()
     toc = time.time()
     ms_vec = 1000.0 * (toc - tic)
     print('Vectorized loss: %e computed in %.2fms' % (loss_vec, ms_vec))

     # The losses should match but your vectorized implementation should be much faster.
     print('Difference: %.2e' % (loss_naive - loss_vec))
     print('Speedup: %.2fX' % (ms_naive / ms_vec))
```

```
Naive loss: 9.000144e+00 computed in 520.02ms
Vectorized loss: 9.000144e+00 computed in 6.23ms
Difference: -1.07e-14
Speedup: 83.48X
```

Then, let's compute the gradient of the loss function. We can check the difference of gradient as well. (The error should be less than 1e-6)

Now implement a vectorized version of the gradient computation in `svm_loss_vectorize` above. Run the cell below to compare the gradient of your naive and vectorized implementations. The difference between the gradients should be less than `1e-6`, and the vectorized version should run at least 100x faster.

```
[30] # The naive implementation and the vectorized implementation should match, but
     # the vectorized version should still be much faster.

     # Use random weights and a minibatch of val data for gradient checking
     coutils.utils.fix_random_seed()
     W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()
     X_batch = data_dict['X_val'][:128].double()
     y_batch = data_dict['y_val'][:128]
     reg = 0.000005

     # Run and time the naive version
     torch.cuda.synchronize()
     tic = time.time()
     _, grad_naive = svm_loss_naive(W, X_batch, y_batch, 0.000005)
     torch.cuda.synchronize()
     toc = time.time()
     ms_naive = 1000.0 * (toc - tic)
     print('Naive loss and gradient: computed in %.2fms' % ms_naive)

     # Run and time the vectorized version
     torch.cuda.synchronize()
     tic = time.time()
     _, grad_vec = svm_loss_vectorized(W, X_batch, y_batch, 0.000005)
     torch.cuda.synchronize()
     toc = time.time()
     ms_vec = 1000.0 * (toc - tic)
     print('Vectorized loss and gradient: computed in %.2fms' % ms_vec)

     # The loss is a single number, so it is easy to compare the values computed
     # by the two implementations. The gradient on the other hand is a tensor, so
     # we use the Frobenius norm to compare them.
     grad_difference = torch.norm(grad_naive - grad_vec, p='fro')
     print('Gradient difference: %.2e' % grad_difference)
     print('Speedup: %.2fX' % (ms_naive / ms_vec))
```

```
Naive loss and gradient: computed in 416.44ms
Vectorized loss and gradient: computed in 8.71ms
Gradient difference: 0.00e+00
Speedup: 47.79X
```

Now that we have an efficient vectorized implementation of the SVM loss and its gradient, we can implement a training pipeline for linear classifiers.

Complete the implementation of the following function:

```
[97] def train_linear_classifier(loss_func, W, X, y, learning_rate=1e-3,
                                 reg=1e-5, num_iters=100, batch_size=200, verbose=False):
       """
```

```
    Train this linear classifier using stochastic gradient descent.

    Inputs:
    - loss_func: loss function to use when training. It should take W, X, y
      and reg as input, and output a tuple of (loss, dW)
    - W: A PyTorch tensor of shape (D, C) giving the initial weights of the
      classifier. If W is None then it will be initialized here.
    - X: A PyTorch tensor of shape (N, D) containing training data; there are N
      training samples each of dimension D.
    - y: A PyTorch tensor of shape (N,) containing training labels; y[i] = c
      means that X[i] has label 0 <= c < C for C classes.
    - learning_rate: (float) learning rate for optimization.
    - reg: (float) regularization strength.
    - num_iters: (integer) number of steps to take when optimizing
    - batch_size: (integer) number of training examples to use at each step.
    - verbose: (boolean) If true, print progress during optimization.

    Returns: A tuple of:
    - W: The final value of the weight matrix and the end of optimization
    - loss_history: A list of Python scalars giving the values of the loss at each
      training iteration.
    """
    # assume y takes values 0...K-1 where K is number of classes
    num_classes = torch.max(y) + 1
    num_train, dim = X.shape
    if W is None:
      # lazily initialize W
      W = 0.000001 * torch.randn(dim, num_classes, device=X.device, dtype=X.dtype)

    # Run stochastic gradient descent to optimize W
    loss_history = []
    for it in range(num_iters):
      X_batch = None
      y_batch = None
      ###########################################################################
      # TODO:                                                                   #
      # Sample batch_size elements from the training data and their             #
      # corresponding labels to use in this round of gradient descent.          #
      # Store the data in X_batch and their corresponding labels in             #
      # y_batch; after sampling, X_batch should have shape (batch_size, dim)    #
      # and y_batch should have shape (batch_size,)                             #
      #                                                                         #
      # Hint: Use torch.randint to generate indices.                            #
      ###########################################################################
      # Replace "pass" statement with your code

      inds = torch.randint(0, num_train, (batch_size,))
      X_batch = X[inds]
      y_batch = y[inds]




      ###########################################################################
      #                             END OF YOUR CODE                            #
      ###########################################################################

      # evaluate loss and gradient
      loss, grad = loss_func(W, X_batch, y_batch, reg)
      loss_history.append(loss.item())

      # perform parameter update
      ###########################################################################
      # TODO:                                                                   #
      # Update the weights using the gradient and the learning rate.            #
      ###########################################################################
      # Replace "pass" statement with your code

      W = W - learning_rate*grad



      ###########################################################################
      #                             END OF YOUR CODE                            #
      ###########################################################################

      if verbose and it % 100 == 0:
        print('iteration %d / %d: loss %f' % (it, num_iters, loss))

    return W, loss_history
```

Once you have implemented the training function, run the following cell to train a linear classifier using some default hyperparameters:

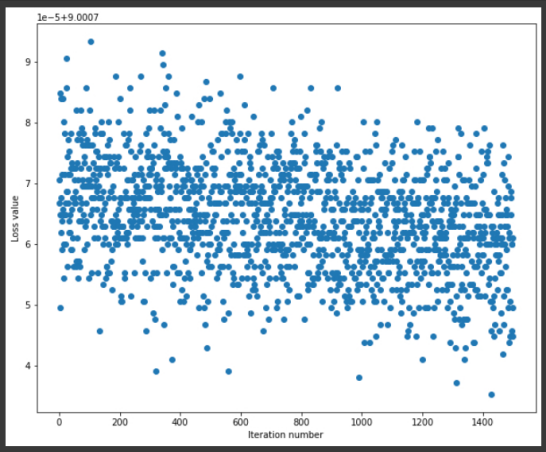
(You should see a final loss close to 9.0, and your training loop should run in about two seconds)

```
[36] # fix random seed before we perform this operation
     coutils.utils.fix_random_seed()

     torch.cuda.synchronize()
     tic = time.time()

     W, loss_hist = train_linear_classifier(svm_loss_vectorized, None,
                                            data_dict['X_train'],
                                            data_dict['y_train'],
                                            learning_rate=3e-11, reg=2.5e4,
                                            num_iters=1500, verbose=True)

     torch.cuda.synchronize()
     toc = time.time()
     print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 9.000767
iteration 100 / 1500: loss 9.000782
iteration 200 / 1500: loss 9.000784
iteration 300 / 1500: loss 9.000782
iteration 400 / 1500: loss 9.000751
iteration 500 / 1500: loss 9.000767
iteration 600 / 1500: loss 9.000767
iteration 700 / 1500: loss 9.000778
iteration 800 / 1500: loss 9.000764
iteration 900 / 1500: loss 9.000774
iteration 1000 / 1500: loss 9.000758
iteration 1100 / 1500: loss 9.000775
iteration 1200 / 1500: loss 9.000771
iteration 1300 / 1500: loss 9.000765
iteration 1400 / 1500: loss 9.000761
That took 2.739040s
```

A useful debugging strategy is to plot the loss as a function of iteration number. In this case it seems our hyperparameters are not good, since the training loss is not decreasing very fast.

```
[33] plt.plot(loss_hist, 'o')
     plt.xlabel('Iteration number')
     plt.ylabel('Loss value')
     plt.show()
```



Let's move on to the prediction stage:

```
[43] def predict_linear_classifier(W, X):
       """
       Use the trained weights of this linear classifier to predict labels for
       data points.

       Inputs:
       - W: A PyTorch tensor of shape (D, C), containing weights of a model
       - X: A PyTorch tensor of shape (N, D) containing training data; there are N
         training samples each of dimension D.

       Returns:
       - y_pred: PyTorch int64 tensor of shape (N,) giving predicted labels for each
         elemment of X. Each element of y_pred should be between 0 and C - 1.
       """
       y_pred = torch.zeros(X.shape[0])
       ###########################################################################
       # TODO:                                                                   #
       # Implement this method. Store the predicted labels in y_pred.            #
       ###########################################################################
       # Replace "pass" statement with your code


       W_times_X = torch.matmul(W.t(), X.t())
       y_pred = torch.argmax(W_times_X, dim = 0)




       ###########################################################################
       #                           END OF YOUR CODE                              #
       ###########################################################################
       return y_pred
```

Then, let's evaluate the performance our trained model on both the training and validation set. You should see validation accuracy less than 10%.

```
[44] # evaluate the performance on both the training and validation set
     y_train_pred = predict_linear_classifier(W, data_dict['X_train'])
     train_acc = 100.0 * (data_dict['y_train'] == y_train_pred).float().mean().item()
     print('Training accuracy: %.2f%%' % train_acc)
     y_val_pred = predict_linear_classifier(W, data_dict['X_val'])
     val_acc = 100.0 * (data_dict['y_val'] == y_val_pred).float().mean().item()
     print('Validation accuracy: %.2f%%' % val_acc)
```

```
Training accuracy: 10.23%
Validation accuracy: 10.20%
```

Unfortunately, the performance of our initial model is quite bad. To find a better hyperparamters, let's first modulize the functions that we've implemented.

```
[45] # Note: We will re-use `LinearClassifier' in Softmax section
     class LinearClassifier(object):

       def __init__(self):
         self.W = None

       def train(self, X_train, y_train, learning_rate=1e-3, reg=1e-5, num_iters=100,
                 batch_size=200, verbose=False):
         train_args = (self.loss, self.W, X_train, y_train, learning_rate, reg,
                       num_iters, batch_size, verbose)
         self.W, loss_history = train_linear_classifier(*train_args)
         return loss_history

       def predict(self, X):
         return predict_linear_classifier(self.W, X)

       def loss(self, W, X_batch, y_batch, reg):
         """
         Compute the loss function and its derivative.
         Subclasses will override this.

         Inputs:
         - W: A PyTorch tensor of shape (D, C) containing (trained) weight of a model.
```

```
      - X_batch: A PyTorch tensor of shape (N, D) containing a minibatch of N
        data points; each point has dimension D.
      - y_batch: A PyTorch tensor of shape (N,) containing labels for the minibatch.
      - reg: (float) regularization strength.

      Returns: A tuple containing:
      - loss as a single float
      - gradient with respect to self.W; an tensor of the same shape as W
      """
      pass
    def _loss(self, X_batch, y_batch, reg):
      self.loss(self.W, X_batch, y_batch, reg)


class LinearSVM(LinearClassifier):
  """ A subclass that uses the Multiclass SVM loss function """

  def loss(self, W, X_batch, y_batch, reg):
    return svm_loss_vectorized(W, X_batch, y_batch, reg)
```

Now, please use the validation set to tune hyperparameters (regularization strength and learning rate). You should experiment with different ranges for the learning rates and regularization strengths.

To get full credit for the assignment your best model found through cross-validation should achieve an accuracy of at least 37% on the validation set.

(Our best model got over 40% -- did you beat us?)

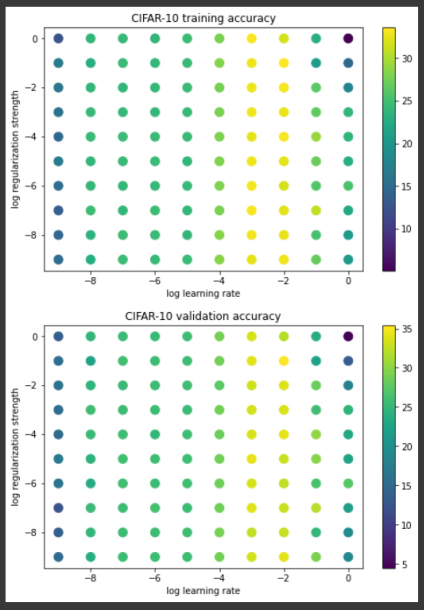
```
# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.
results = {}
val_results = {}
best_val = -1   # The highest validation accuracy that we have seen so far.
best_svm = None # The LinearSVM object that achieved the highest validation rate.
learning_rates = [] # learning rate candidates, e.g. [1e-3, 1e-2, ...]
regularization_strengths = [] # regularization strengths candidates e.g. [1e0, 1e1, ...]

################################################################################
# TODO:                                                                        #
# Write code that chooses the best hyperparameters by tuning on the validation #
# set. For each combination of hyperparameters, train a linear SVM on the      #
# training set, compute its accuracy on the training and validation sets, and  #
# store these numbers in the results dictionary. In addition, store the best   #
# validation accuracy in best_val and the LinearSVM object that achieves this  #
# accuracy in best_svm.                                                        #
#                                                                              #
# Hint: You should use a small value for num_iters as you develop your         #
# validation code so that the SVMs don't take much time to train; once you are #
# confident that your validation code works, you should rerun the validation   #
# code with a larger value for num_iters.                                      #
################################################################################
# Replace "pass" statement with your code
# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.


import numpy as np

learning_rates = np.logspace(-4,-1,10)

regularization_strengths = np.logspace(-4,1,10)

arr = [[LinearSVM() for i in range(len(regularization_strengths))] for j in range(len(learning_rates))]


it = 0

for i,rate in enumerate(learning_rates):
  for j,reg in enumerate(regularization_strengths):
    arr[i][j].train( data_dict['X_train'] , data_dict['y_train'], rate, reg , num_iters = 100)

    y_pred_train = arr[i][j].predict(data_dict['X_train'])
    y_pred_val = arr[i][j].predict(data_dict['X_val'])

    train_acc = 100.0 * (data_dict['y_train'] == y_pred_train).float().mean().item()
    val_acc = 100.0 * (data_dict['y_val'] == y_pred_val).float().mean().item()

    results[rate,reg] = train_acc,val_acc

    if it>0:
      if val_acc > max(val_results.values()):
        best_val = val_acc
        best_svm = arr[i][j]

    val_results[rate,reg] = val_acc

    if val_acc == max(val_results.values()):
        best_val = val_acc
        best_svm = arr[i][j]
    it +=1




################################################################################
#                              END OF YOUR CODE                                #
################################################################################

# Print out results.
for lr, reg in sorted(results):
  train_accuracy, val_accuracy = results[(lr, reg)]
  print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
        lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' % best_val)
```

```
lr 1.000000e-09 reg 1.000000e-09 train accuracy: 15.159184 val accuracy: 15.100001
lr 1.000000e-09 reg 1.000000e-08 train accuracy: 14.597958 val accuracy: 13.900001
lr 1.000000e-09 reg 1.000000e-07 train accuracy: 13.704081 val accuracy: 12.700000
lr 1.000000e-09 reg 1.000000e-06 train accuracy: 15.263265 val accuracy: 16.400000
lr 1.000000e-09 reg 1.000000e-05 train accuracy: 16.889796 val accuracy: 17.100000
lr 1.000000e-09 reg 1.000000e-04 train accuracy: 14.685714 val accuracy: 15.000001
```

```
lr 1.000000e-09 reg 1.000000e-03 train accuracy: 15.869386 val accuracy: 15.500000
lr 1.000000e-09 reg 1.000000e-02 train accuracy: 16.387755 val accuracy: 16.300000
lr 1.000000e-09 reg 1.000000e-01 train accuracy: 16.924489 val accuracy: 15.600000
lr 1.000000e-09 reg 1.000000e+00 train accuracy: 12.630612 val accuracy: 13.700001
lr 1.000000e-08 reg 1.000000e-09 train accuracy: 22.759183 val accuracy: 23.700002
lr 1.000000e-08 reg 1.000000e-08 train accuracy: 23.444897 val accuracy: 25.000000
lr 1.000000e-08 reg 1.000000e-07 train accuracy: 24.759182 val accuracy: 25.500003
lr 1.000000e-08 reg 1.000000e-06 train accuracy: 23.973468 val accuracy: 23.500001
lr 1.000000e-08 reg 1.000000e-05 train accuracy: 23.989795 val accuracy: 24.500000
lr 1.000000e-08 reg 1.000000e-04 train accuracy: 24.395917 val accuracy: 25.500003
lr 1.000000e-08 reg 1.000000e-03 train accuracy: 24.312244 val accuracy: 25.900000
lr 1.000000e-08 reg 1.000000e-02 train accuracy: 23.940815 val accuracy: 24.600001
lr 1.000000e-08 reg 1.000000e-01 train accuracy: 23.018366 val accuracy: 22.400001
lr 1.000000e-08 reg 1.000000e+00 train accuracy: 23.948979 val accuracy: 25.000000
lr 1.000000e-07 reg 1.000000e-09 train accuracy: 24.493876 val accuracy: 25.700000
lr 1.000000e-07 reg 1.000000e-08 train accuracy: 24.479590 val accuracy: 25.700000
lr 1.000000e-07 reg 1.000000e-07 train accuracy: 24.051020 val accuracy: 25.800002
lr 1.000000e-07 reg 1.000000e-06 train accuracy: 24.130611 val accuracy: 25.900000
lr 1.000000e-07 reg 1.000000e-05 train accuracy: 24.465305 val accuracy: 26.000002
lr 1.000000e-07 reg 1.000000e-04 train accuracy: 24.520408 val accuracy: 25.800002
lr 1.000000e-07 reg 1.000000e-03 train accuracy: 24.200000 val accuracy: 25.800002
lr 1.000000e-07 reg 1.000000e-02 train accuracy: 23.975509 val accuracy: 25.500003
lr 1.000000e-07 reg 1.000000e-01 train accuracy: 24.475509 val accuracy: 26.000002
lr 1.000000e-07 reg 1.000000e+00 train accuracy: 24.312244 val accuracy: 25.600001
lr 1.000000e-06 reg 1.000000e-09 train accuracy: 24.344897 val accuracy: 25.700000
lr 1.000000e-06 reg 1.000000e-08 train accuracy: 24.367346 val accuracy: 25.700000
lr 1.000000e-06 reg 1.000000e-07 train accuracy: 24.740815 val accuracy: 26.300001
lr 1.000000e-06 reg 1.000000e-06 train accuracy: 24.236734 val accuracy: 25.600001
lr 1.000000e-06 reg 1.000000e-05 train accuracy: 24.561223 val accuracy: 25.900000
lr 1.000000e-06 reg 1.000000e-04 train accuracy: 24.365306 val accuracy: 25.000000
lr 1.000000e-06 reg 1.000000e-03 train accuracy: 24.065305 val accuracy: 25.700000
lr 1.000000e-06 reg 1.000000e-02 train accuracy: 24.206121 val accuracy: 25.600001
lr 1.000000e-06 reg 1.000000e-01 train accuracy: 24.138774 val accuracy: 25.700000
lr 1.000000e-06 reg 1.000000e+00 train accuracy: 24.010204 val accuracy: 25.400001
lr 1.000000e-05 reg 1.000000e-09 train accuracy: 24.497959 val accuracy: 25.700000
lr 1.000000e-05 reg 1.000000e-08 train accuracy: 23.973468 val accuracy: 25.900000
lr 1.000000e-05 reg 1.000000e-07 train accuracy: 24.124488 val accuracy: 25.700000
lr 1.000000e-05 reg 1.000000e-06 train accuracy: 24.330612 val accuracy: 26.000002
lr 1.000000e-05 reg 1.000000e-05 train accuracy: 23.897958 val accuracy: 25.700000
lr 1.000000e-05 reg 1.000000e-04 train accuracy: 24.379592 val accuracy: 25.800002
lr 1.000000e-05 reg 1.000000e-03 train accuracy: 24.271427 val accuracy: 25.600001
lr 1.000000e-05 reg 1.000000e-02 train accuracy: 24.426530 val accuracy: 25.900000
lr 1.000000e-05 reg 1.000000e-01 train accuracy: 24.383673 val accuracy: 25.800002
lr 1.000000e-05 reg 1.000000e+00 train accuracy: 24.506122 val accuracy: 25.900000
lr 1.000000e-04 reg 1.000000e-09 train accuracy: 28.469387 val accuracy: 28.800002
lr 1.000000e-04 reg 1.000000e-08 train accuracy: 28.295916 val accuracy: 29.100001
lr 1.000000e-04 reg 1.000000e-07 train accuracy: 27.910203 val accuracy: 28.700000
lr 1.000000e-04 reg 1.000000e-06 train accuracy: 28.120407 val accuracy: 28.600001
lr 1.000000e-04 reg 1.000000e-05 train accuracy: 28.146937 val accuracy: 28.700000
lr 1.000000e-04 reg 1.000000e-04 train accuracy: 27.967346 val accuracy: 28.500003
lr 1.000000e-04 reg 1.000000e-03 train accuracy: 27.871427 val accuracy: 28.800002
lr 1.000000e-04 reg 1.000000e-02 train accuracy: 27.606121 val accuracy: 27.600002
```

Visualize the cross-validation results. You can use this as a debugging tool -- after examining the cross-validation results here, you may want to go back and rerun your cross-validation from above.

```
[103] x_scatter = [math.log10(x[0]) for x in results]
      y_scatter = [math.log10(x[1]) for x in results]

      # plot training accuracy
      marker_size = 100
      colors = [results[x][0] for x in results]
      plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap='viridis')
      plt.colorbar()
      plt.xlabel('log learning rate')
      plt.ylabel('log regularization strength')
      plt.title('CIFAR-10 training accuracy')
      plt.gcf().set_size_inches(8, 5)
      plt.show()

      # plot validation accuracy
      colors = [results[x][1] for x in results] # default size of markers is 20
      plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap='viridis')
      plt.colorbar()
      plt.xlabel('log learning rate')
      plt.ylabel('log regularization strength')
      plt.title('CIFAR-10 validation accuracy')
      plt.gcf().set_size_inches(8, 5)
      plt.show()
```



Evaluate the best svm on test set. To get full credit for the assignment you should achieve a test-set accuracy above 35%.

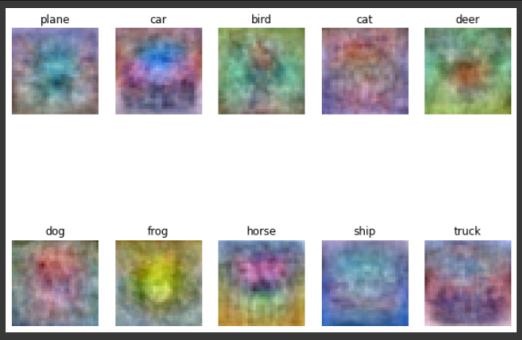
(Our best was over 38% -- did you beat us?)

```
[104] y_test_pred = best_svm.predict(data_dict['X_test'])
      test_accuracy = torch.mean((data_dict['y_test'] == y_test_pred).float())
      print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

```
linear SVM on raw pixels final test set accuracy: 0.329400
```

Visualize the learned weights for each class. Depending on your choice of learning rate and regularization strength, these may or may not be nice to look at.

```
w = best_svm.W[:-1,:] # strip out the bias
w = w.reshape(3, 32, 32, 10)
w = w.transpose(0, 2).transpose(1, 0)

w_min, w_max = torch.min(w), torch.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
  plt.subplot(2, 5, i + 1)

  # Rescale the weights to be between 0 and 255
  wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
  plt.imshow(wimg.type(torch.uint8).cpu())
  plt.axis('off')
  plt.title(classes[i])
```



Softmax Classifier

Similar to the SVM, you will:

- implement a fully-vectorized **loss function** for the Softmax classifier
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** with numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

First, let's start from implementing the naive softmax loss function with nested loops.

```
def softmax_loss_naive(W, X, y, reg):
  """
  Softmax loss function, naive implementation (with loops).  When you implment
  the regularization over W, please DO NOT multiply the regularization term by
  1/2 (no coefficient).

  Inputs have dimension D, there are C classes, and we operate on minibatches
  of N examples.

  Inputs:
  - W: A PyTorch tensor of shape (D, C) containing weights.
  - X: A PyTorch tensor of shape (N, D) containing a minibatch of data.
  - y: A PyTorch tensor of shape (N,) containing training labels; y[i] = c means
    that X[i] has label c, where 0 <= c < C.
  - reg: (float) regularization strength

  Returns a tuple of:
  - loss as single float
  - gradient with respect to weights W; an tensor of same shape as W
  """
  # Initialize the loss and gradient to zero.
  loss = 0.0
  dW = torch.zeros_like(W)

  ############################################################################
  # TODO: Compute the softmax loss and its gradient using explicit loops.     #
  # Store the loss in loss and the gradient in dW. If you are not careful     #
  # here, it is easy to run into numeric instability (Check Numeric Stability #
  # in http://cs231n.github.io/linear-classify/). Plus, don't forget the      #
  # regularization!                                                           #
  ############################################################################
  # Replace "pass" statement with your code


  scores = torch.matmul(X, W)

  scores = scores - (torch.max(scores, dim = 1)[0]).reshape(-1, 1)

  scores = torch.exp(scores)

  scores = scores / (torch.sum(scores, dim = 1).reshape(-1, 1))

  D, C = W.shape
  N, D = X.shape

  for i in range(N):
      loss+= -torch.log(scores[i][y[i]]).item()
      temp = X[i].reshape(-1, 1).repeat(1, C)*scores[i].reshape(1, -1)
      temp[:,y[i]] += -X[i]
      dW+= temp

  loss /= N
  loss += reg * torch.sum(W * W)
```

```
    dW /=N
    dW += 2*reg*W




    ##############################################################################
    #                          END OF YOUR CODE                                  #
    ##############################################################################

    return loss, dW
```

As a sanity check to see whether we have implemented the loss correctly, run the softmax classifier with a small random weight matrix and no regularization. You should see loss near log(10) = 2.3

```
[197] # Generate a random softmax weight tensor and use it to compute the loss.
    coutils.utils.fix_random_seed()
    W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()

    X_batch = data_dict['X_val'][:6].double()
    y_batch = data_dict['y_val'][:6]

    # Complete the implementation of softmax_loss_naive and implement a (naive)
    # version of the gradient that uses nested loops.
    loss, grad = softmax_loss_naive(W, X_batch, y_batch, reg=0.0)

    # As a rough sanity check, our loss should be something close to log(10.0).
    print('loss: %f' % loss)
    print('sanity check: %f' % (math.log(10.0)))
```

```
loss: 2.302621
sanity check: 2.302585
```

Next, we use gradient checking to debug the analytic gradient of our naive softmax loss function. If you've implemented the gradient correctly, you should see relative errors less than `1e-6`.

```
    coutils.utils.fix_random_seed()
    W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()
    X_batch = data_dict['X_val'][:128].double()
    y_batch = data_dict['y_val'][:128]

    loss, grad = softmax_loss_naive(W, X_batch, y_batch, reg=0.0)

    f = lambda w: softmax_loss_naive(w, X_batch, y_batch, reg=0.0)[0]
    grad_check_sparse(f, W, grad, 10)
```

```
numerical: 0.008387 analytic: 0.008387, relative error: 1.366549e-07
numerical: 0.009227 analytic: 0.009227, relative error: 1.339656e-07
numerical: -0.002471 analytic: -0.002471, relative error: 1.718331e-07
numerical: -0.003144 analytic: -0.003144, relative error: 2.403080e-06
numerical: 0.006011 analytic: 0.006011, relative error: 6.813253e-08
numerical: 0.005936 analytic: 0.005936, relative error: 2.473992e-07
numerical: 0.015703 analytic: 0.015703, relative error: 2.149831e-08
numerical: 0.006452 analytic: 0.006452, relative error: 2.068055e-09
numerical: -0.015533 analytic: -0.015533, relative error: 2.569959e-07
numerical: -0.010170 analytic: -0.010170, relative error: 4.657956e-07
```

Let's perform another gradient check with regularization enabled. Again you should see relative errors less than `1e-6`.

```
[199] coutils.utils.fix_random_seed()
    W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device).double()
    reg = 10.0

    X_batch = data_dict['X_val'][:128].double()
    y_batch = data_dict['y_val'][:128]

    loss, grad = softmax_loss_naive(W, X_batch, y_batch, reg)

    f = lambda w: softmax_loss_naive(w, X_batch, y_batch, reg)[0]
    grad_check_sparse(f, W, grad, 10)
```

```
numerical: 0.007517 analytic: 0.007517, relative error: 2.286123e-07
numerical: 0.008167 analytic: 0.008167, relative error: 1.238116e-07
numerical: -0.002551 analytic: -0.002551, relative error: 3.147037e-08
numerical: -0.000841 analytic: -0.000841, relative error: 8.976250e-06
numerical: 0.002228 analytic: 0.002228, relative error: 4.070641e-07
numerical: 0.005460 analytic: 0.005460, relative error: 1.891176e-07
numerical: 0.015762 analytic: 0.015762, relative error: 7.076879e-08
numerical: 0.007097 analytic: 0.007097, relative error: 1.474077e-07
numerical: -0.015532 analytic: -0.015532, relative error: 2.149161e-07
numerical: -0.011201 analytic: -0.011201, relative error: 3.391712e-07
```

Then, let's move on to the vectorized form

```
[207] def softmax_loss_vectorized(W, X, y, reg):
        """
        Softmax loss function, vectorized version.  When you implment the
        regularization over W, please DO NOT multiply the regularization term by 1/2
        (no coefficient).

        Inputs and outputs are the same as softmax_loss_naive.
        """
        # Initialize the loss and gradient to zero.
        loss = 0.0
        dW = torch.zeros_like(W)

        ##############################################################################
        # TODO: Compute the softmax loss and its gradient using no explicit loops.  #
        # Store the loss in loss and the gradient in dW. If you are not careful     #
        # here, it is easy to run into numeric instability (Check Numeric Stability #
        # in http://cs231n.github.io/linear-classify/). Don't forget the            #
        # regularization!                                                           #
        ##############################################################################
        # Replace "pass" statement with your code
        D, C = W.shape
        N, D = X.shape
        scores = torch.matmul(X,W)
        scores = scores - (torch.max(scores, dim = 1)[0]).reshape(-1, 1)
        scores = torch.exp(scores)
        scores = scores / (torch.sum(scores, dim = 1).reshape(-1, 1))
        log_scores = -torch.log(scores[torch.arange(N), y])
        loss = torch.sum(log_scores)
```

```
    loss /= N
    loss += reg * torch.sum(W * W)

    dW1 = torch.matmul(X.t(),scores)
    dW2 = torch.zeros_like(scores)
    dW2[torch.arange(N), y] = 1
    dW2 = -torch.matmul(X.t(),dW2)

    dW = dW1 + dW2
    dW /=N
    dW += 2 * reg * W
    ##############################################################################
    #                          END OF YOUR CODE                                  #
    ##############################################################################

    return loss, dW
```

Now that we have a naive implementation of the softmax loss function and its gradient, implement a vectorized version in softmax_loss_vectorized. The two versions should compute the same results, but the vectorized version should be much faster.

The differences between the naive and vectorized losses and gradients should both be less than `1e-6`, and your vectorized implementation should be at least 100x faster than the naive implementation.

```
[208] coutils.utils.fix_random_seed()
    W = 0.0001 * torch.randn(3073, 10, device=data_dict['X_val'].device)
    reg = 0.05

    X_batch = data_dict['X_val'][:128]
    y_batch = data_dict['y_val'][:128]

    # Run and time the naive version
    torch.cuda.synchronize()
    tic = time.time()
    loss_naive, grad_naive = softmax_loss_naive(W, X_batch, y_batch, reg)
    torch.cuda.synchronize()
    toc = time.time()
    ms_naive = 1000.0 * (toc - tic)
    print('naive loss: %e computed in %fs' % (loss_naive, ms_naive))

    # Run and time the vectorized version
    torch.cuda.synchronize()
    tic = time.time()
    loss_vec, grad_vec = softmax_loss_vectorized(W, X_batch, y_batch, reg)
    torch.cuda.synchronize()
    toc = time.time()
    ms_vec = 1000.0 * (toc - tic)
    print('vectorized loss: %e computed in %fs' % (loss_vec, ms_vec))

    # we use the Frobenius norm to compare the two versions of the gradient.
    loss_diff = (loss_naive - loss_vec).abs().item()
    grad_diff = torch.norm(grad_naive - grad_vec, p='fro')
    print('Loss difference: %.2e' % loss_diff)
    print('Gradient difference: %.2e' % grad_diff)
    print('Speedup: %.2fX' % (ms_naive / ms_vec))
```

```
naive loss: 2.302615e+00 computed in 21.522522s
vectorized loss: 2.302616e+00 computed in 1.731634s
Loss difference: 2.38e-07
Gradient difference: 3.40e-07
Speedup: 12.43X
```

Let's check that your implementation of the softmax loss is numerically stable.

If either of the following print `nan` then you should double-check the numeric stability of your implementations.

```
[210] device = data_dict['X_train'].device
    dtype = torch.float32
    D = data_dict['X_train'].shape[1]
    C = 10

    W_ones = torch.ones(D, C, device=device, dtype=dtype)
    W, loss_hist = train_linear_classifier(softmax_loss_naive, W_ones,
                                           data_dict['X_train'],
                                           data_dict['y_train'],
                                           learning_rate=1e-8, reg=2.5e4,
                                           num_iters=1, verbose=True)


    W_ones = torch.ones(D, C, device=device, dtype=dtype)
    W, loss_hist = train_linear_classifier(softmax_loss_vectorized, W_ones,
                                           data_dict['X_train'],
                                           data_dict['y_train'],
                                           learning_rate=1e-8, reg=2.5e4,
                                           num_iters=1, verbose=True)
```

```
iteration 0 / 1: loss 768249984.000000
iteration 0 / 1: loss 768249984.000000
```

Now lets train a softmax classifier with some default hyperparameters:

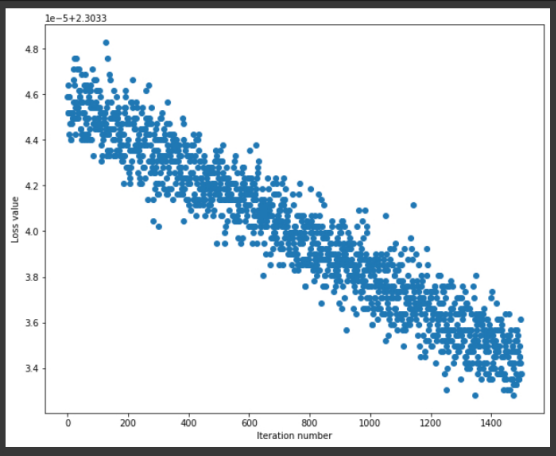
```
[211] # fix random seed before we perform this operation
    coutils.utils.fix_random_seed(10)

    torch.cuda.synchronize()
    tic = time.time()

    W, loss_hist = train_linear_classifier(softmax_loss_vectorized, None,
                                           data_dict['X_train'],
                                           data_dict['y_train'],
                                           learning_rate=1e-10, reg=2.5e4,
                                           num_iters=1500, verbose=True)

    torch.cuda.synchronize()
    toc = time.time()
    print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 2.303346
iteration 100 / 1500: loss 2.303344
iteration 200 / 1500: loss 2.303344
iteration 300 / 1500: loss 2.303342
iteration 400 / 1500: loss 2.303342
iteration 500 / 1500: loss 2.303342
iteration 600 / 1500: loss 2.303342
iteration 700 / 1500: loss 2.303341
```

```
iteration 800 / 1500: loss 2.303339
iteration 900 / 1500: loss 2.303339
iteration 1000 / 1500: loss 2.303339
iteration 1100 / 1500: loss 2.303336
iteration 1200 / 1500: loss 2.303335
iteration 1300 / 1500: loss 2.303337
iteration 1400 / 1500: loss 2.303334
That took 1.679042s
```

Plot the loss curve:

```
[212] plt.plot(loss_hist, 'o')
      plt.xlabel('Iteration number')
      plt.ylabel('Loss value')
      plt.show()
```



Let's compute the accuracy of current model. It should be less than 10%.

```
[213] # evaluate the performance on both the training and validation set
      y_train_pred = predict_linear_classifier(W, data_dict['X_train'])
      train_acc = 100.0 * (data_dict['y_train'] == y_train_pred).float().mean().item()
      print('training accuracy: %.2f%%' % train_acc)
      y_val_pred = predict_linear_classifier(W, data_dict['X_val'])
      val_acc = 100.0 * (data_dict['y_val'] == y_val_pred).float().mean().item()
      print('validation accuracy: %.2f%%' % val_acc)
```

```
training accuracy: 8.43%
validation accuracy: 8.00%
```

Now use the validation set to tune hyperparameters (regularization strength and learning rate). You should experiment with different ranges for the learning rates and regularization strengths.

To get full credit for the assignment, your best model found through cross-validation should achieve an accuracy above 0.37 on the validation set.

(Our best model was above 0.40 -- did you beat us?)

```
[214] class Softmax(LinearClassifier):
        """ A subclass that uses the Softmax + Cross-entropy loss function """
        def loss(self, W, X_batch, y_batch, reg):
          return softmax_loss_vectorized(W, X_batch, y_batch, reg)
```

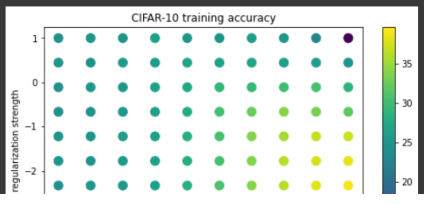
```
[221] results = {}
      best_val = -1
      best_softmax = None

      learning_rates = [] # learning rate candidates
      regularization_strengths = [] # regularization strengths candidates

      # As before, store your cross-validation results in this dictionary.
      # The keys should be tuples of (learning_rate, regularization_strength) and
      # the values should be tuples (train_accuracy, val_accuracy)
      results = {}

      ################################################################################
      # TODO:                                                                        #
      # Use the validation set to set the learning rate and regularization strength. #
      # This should be similar to the cross-validation that you used for the SVM,    #
      # but you may need to select different hyperparameters to achieve good         #
      # performance with the softmax classifier. Save your best trained softmax      #
      # classifer in best_softmax.                                                   #
      ################################################################################
      # Replace "pass" statement with your code



      learning_rates = np.logspace(-4,-1,10)
      regularization_strengths = np.logspace(-4,1,10)

      arr = [[Softmax() for i in range(len(regularization_strengths))] for j in range(len(learning_rates))]


      it = 0

      for i,rate in enumerate(learning_rates):
        for j,reg in enumerate(regularization_strengths):
          arr[i][j].train( data_dict['X_train'] , data_dict['y_train'], rate, reg , num_iters = 200)

          y_pred_train = arr[i][j].predict(data_dict['X_train'])
          y_pred_val = arr[i][j].predict(data_dict['X_val'])

          train_acc = 100.0 * (data_dict['y_train'] == y_pred_train).float().mean().item()
          val_acc = 100.0 * (data_dict['y_val'] == y_pred_val).float().mean().item()

          results[rate,reg] = train_acc,val_acc
```

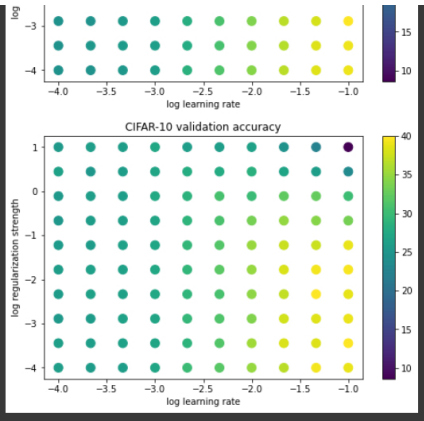
```
        if it>0:
          if val_acc > max(val_results.values()):
            best_val = val_acc
            best_softmax = arr[i][j]

        val_results[rate,reg] = val_acc

        if val_acc == max(val_results.values()):
            best_val = val_acc
            best_softmax = arr[i][j]
        it +=1



    ################################################################################
    #                              END OF YOUR CODE                                #
    ################################################################################

    # Print out results.
    for lr, reg in sorted(results):
        train_accuracy, val_accuracy = results[(lr, reg)]
        print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
                    lr, reg, train_accuracy, val_accuracy))

    print('best validation accuracy achieved during cross-validation: %f' % best_val)
```

```
lr 1.000000e-04 reg 1.000000e-04 train accuracy: 24.434693 val accuracy: 26.100001
lr 1.000000e-04 reg 3.593814e-04 train accuracy: 24.471428 val accuracy: 26.200002
lr 1.000000e-04 reg 1.291550e-03 train accuracy: 24.420407 val accuracy: 25.500003
lr 1.000000e-04 reg 4.641589e-03 train accuracy: 24.334693 val accuracy: 25.800002
lr 1.000000e-04 reg 1.668101e-02 train accuracy: 24.559183 val accuracy: 26.000002
lr 1.000000e-04 reg 5.994843e-02 train accuracy: 24.673468 val accuracy: 25.700000
lr 1.000000e-04 reg 2.154435e-01 train accuracy: 24.467346 val accuracy: 25.300002
lr 1.000000e-04 reg 7.742637e-01 train accuracy: 24.206121 val accuracy: 25.600001
lr 1.000000e-04 reg 2.782559e+00 train accuracy: 24.802040 val accuracy: 26.400000
lr 1.000000e-04 reg 1.000000e+01 train accuracy: 24.453060 val accuracy: 25.600001
lr 2.154435e-04 reg 1.000000e-04 train accuracy: 24.622448 val accuracy: 25.800002
lr 2.154435e-04 reg 3.593814e-04 train accuracy: 25.036734 val accuracy: 26.100001
lr 2.154435e-04 reg 1.291550e-03 train accuracy: 24.593876 val accuracy: 25.800002
lr 2.154435e-04 reg 4.641589e-03 train accuracy: 24.979591 val accuracy: 26.400000
lr 2.154435e-04 reg 1.668101e-02 train accuracy: 24.773468 val accuracy: 26.300001
lr 2.154435e-04 reg 5.994843e-02 train accuracy: 24.744897 val accuracy: 25.800002
lr 2.154435e-04 reg 2.154435e-01 train accuracy: 24.669386 val accuracy: 26.000002
lr 2.154435e-04 reg 7.742637e-01 train accuracy: 25.148979 val accuracy: 26.100001
lr 2.154435e-04 reg 2.782559e+00 train accuracy: 24.346937 val accuracy: 25.400001
lr 2.154435e-04 reg 1.000000e+01 train accuracy: 24.877550 val accuracy: 26.100001
lr 4.641589e-04 reg 1.000000e-04 train accuracy: 25.042856 val accuracy: 26.000002
lr 4.641589e-04 reg 3.593814e-04 train accuracy: 25.532651 val accuracy: 26.400000
lr 4.641589e-04 reg 1.291550e-03 train accuracy: 25.385714 val accuracy: 26.700002
lr 4.641589e-04 reg 4.641589e-03 train accuracy: 25.236735 val accuracy: 26.600000
lr 4.641589e-04 reg 1.668101e-02 train accuracy: 25.422448 val accuracy: 26.500002
lr 4.641589e-04 reg 5.994843e-02 train accuracy: 25.804082 val accuracy: 26.500002
lr 4.641589e-04 reg 2.154435e-01 train accuracy: 25.118366 val accuracy: 26.200002
lr 4.641589e-04 reg 7.742637e-01 train accuracy: 25.181630 val accuracy: 26.100001
lr 4.641589e-04 reg 2.782559e+00 train accuracy: 25.481632 val accuracy: 25.900000
lr 4.641589e-04 reg 1.000000e+01 train accuracy: 24.697958 val accuracy: 25.900000
lr 1.000000e-03 reg 1.000000e-04 train accuracy: 25.981632 val accuracy: 27.200001
lr 1.000000e-03 reg 3.593814e-04 train accuracy: 25.914285 val accuracy: 27.000001
lr 1.000000e-03 reg 1.291550e-03 train accuracy: 26.059183 val accuracy: 27.000001
lr 1.000000e-03 reg 4.641589e-03 train accuracy: 26.128569 val accuracy: 27.400002
lr 1.000000e-03 reg 1.668101e-02 train accuracy: 26.295918 val accuracy: 27.400002
lr 1.000000e-03 reg 5.994843e-02 train accuracy: 26.214284 val accuracy: 27.200001
lr 1.000000e-03 reg 2.154435e-01 train accuracy: 26.346937 val accuracy: 27.100003
lr 1.000000e-03 reg 7.742637e-01 train accuracy: 26.324490 val accuracy: 27.500001
lr 1.000000e-03 reg 2.782559e+00 train accuracy: 25.759181 val accuracy: 27.000001
lr 1.000000e-03 reg 1.000000e+01 train accuracy: 26.177549 val accuracy: 27.300000
lr 2.154435e-03 reg 1.000000e-04 train accuracy: 28.518367 val accuracy: 30.000001
lr 2.154435e-03 reg 3.593814e-04 train accuracy: 27.777550 val accuracy: 28.900000
lr 2.154435e-03 reg 1.291550e-03 train accuracy: 28.087753 val accuracy: 29.500002
lr 2.154435e-03 reg 4.641589e-03 train accuracy: 28.334692 val accuracy: 29.200003
lr 2.154435e-03 reg 1.668101e-02 train accuracy: 27.865306 val accuracy: 29.300001
lr 2.154435e-03 reg 5.994843e-02 train accuracy: 28.244898 val accuracy: 29.400000
lr 2.154435e-03 reg 2.154435e-01 train accuracy: 27.612245 val accuracy: 29.000002
lr 2.154435e-03 reg 7.742637e-01 train accuracy: 27.438775 val accuracy: 28.800002
lr 2.154435e-03 reg 2.782559e+00 train accuracy: 26.857141 val accuracy: 28.100002
lr 2.154435e-03 reg 1.000000e+01 train accuracy: 25.057143 val accuracy: 26.100001
lr 4.641589e-03 reg 1.000000e-04 train accuracy: 31.204081 val accuracy: 31.900001
lr 4.641589e-03 reg 3.593814e-04 train accuracy: 30.636734 val accuracy: 31.500003
lr 4.641589e-03 reg 1.291550e-03 train accuracy: 30.795917 val accuracy: 31.300002
lr 4.641589e-03 reg 4.641589e-03 train accuracy: 30.800000 val accuracy: 30.900002
lr 4.641589e-03 reg 1.668101e-02 train accuracy: 30.555102 val accuracy: 32.000002
lr 4.641589e-03 reg 5.994843e-02 train accuracy: 30.504081 val accuracy: 31.600001
lr 4.641589e-03 reg 2.154435e-01 train accuracy: 30.428571 val accuracy: 31.000000
lr 4.641589e-03 reg 7.742637e-01 train accuracy: 29.169387 val accuracy: 30.900002
```

Run the following to visualize your cross-validation results:

```
[223] x_scatter = [math.log10(x[0]) for x in results]
      y_scatter = [math.log10(x[1]) for x in results]

      # plot training accuracy
      marker_size = 100
      colors = [results[x][0] for x in results]
      plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap='viridis')
      plt.colorbar()
      plt.xlabel('log learning rate')
      plt.ylabel('log regularization strength')
      plt.title('CIFAR-10 training accuracy')
      plt.gcf().set_size_inches(8, 5)
      plt.show()

      # plot validation accuracy
      colors = [results[x][1] for x in results] # default size of markers is 20
      plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap='viridis')
      plt.colorbar()
      plt.xlabel('log learning rate')
      plt.ylabel('log regularization strength')
      plt.title('CIFAR-10 validation accuracy')
      plt.gcf().set_size_inches(8, 5)
      plt.show()
```





Them, evaluate the performance of your best model on test set. To get full credit for this assignment you should achieve a test-set accuracy above 0.36.

(Our best was just over 0.40 -- did you beat us?)

```
[224] y_test_pred = best_softmax.predict(data_dict['X_test'])
      test_accuracy = torch.mean((data_dict['y_test'] == y_test_pred).float())
      print('softmax on raw pixels final test set accuracy: %f' % (test_accuracy, ))
```

```
softmax on raw pixels final test set accuracy: 0.385500
```

Finally, let's visualize the learned weights for each class

```
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(3, 32, 32, 10)
w = w.transpose(0, 2).transpose(1, 0)

w_min, w_max = torch.min(w), torch.max(w)

classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse', 'ship', 'truck']
for i in range(10):
  plt.subplot(2, 5, i + 1)

  # Rescale the weights to be between 0 and 255
  wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
  plt.imshow(wimg.type(torch.uint8).cpu())
  plt.axis('off')
  plt.title(classes[i])
```