Sample Student
Intro to Examples
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Homework 1

Question 1. Simple Theorems and Proofs Consider the following basic examples:

Theorem 1. *For any integer n, $n^2 \geq 0$.*

Proof. Any integer squared is either zero or positive. *quod erat dem*■

a) Even or Odd

Theorem 2. *Every integer is either even or odd.*

Proof. Let n be any integer. Define:
- Case 1: $n = 2k$ for some $k \in \mathbb{Z}$, then n is even.
- Case 2: $n = 2k + 1$ for some $k \in \mathbb{Z}$, then n is odd.

These cases cover all integers. *quod erat dem*■

b) Positive Numbers

Theorem 3. *There exists a positive integer less than 10.*

Proof. Simply choose $n = 5$, which is positive and less than 10. *quod erat dem*■

Question 2. Identifying Mistakes

a) Consider a faulty induction attempt to prove that $n! > 0$ for $n \geq 0$. Spot the error. The error occurs if the base case is skipped. For induction, we must verify $0! = 1 > 0$ first.
b) Another example: Assuming $n^2 > n$ for all n without checking $n = 0$. The mistake is not considering $n = 0$, for which $0^2 = 0 \not> 0$.

Question 3. Simple State Machine Consider a counter that halves a number until it reaches zero.

a) Starting from $n = 10$, show the sequence of states. Sequence: $10 \to 5 \to 2 \to 1 \to 0$.
b) Define invariant $P(n) := n \geq 0$ and verify. Base case: $n = 10 \geq 0$. Step: Halving preserves non-negativity, so $P(n)$ holds for all transitions.
c) Explain termination. Each transition decreases n, and the process stops when $n = 0$. Hence, the state machine always terminates.