Natural Selection

BIOL 1435

April 6, 2023

Overview

1. Drift (WF Model Review)
2. Single Locus Diploid Model
3. Selection's Impact on Allele Frequency Trajectories
4. Selection + Drift

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Modeling evolution

Q: Why do we need to model evolution?

Modeling evolution

A: To assess departures from neutrality.

Assumptions of the Wright Fisher model

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- Panmictic population
- Constant population size of N
- Total of $2N$ allele copies
- Discrete time process with non-overlapping generations
- All mutations are neutral

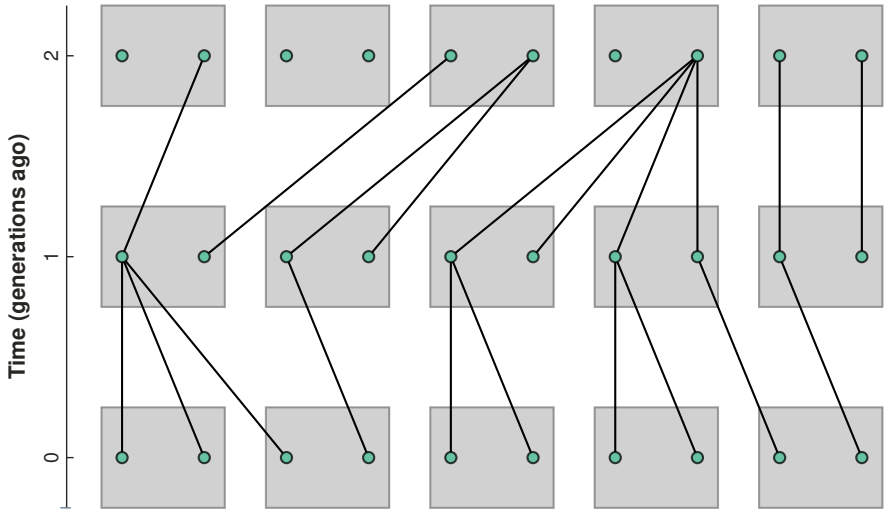
WF population ($N = 5$)

Simulating under the WF model

Simulating Reproduction

Sample with replacement an allelic copy (i) in the current generation (t) with a probability of $\frac{1}{2N}$ to produce an offspring in the next generation ($t+1$), until there are $2N$ allelic copies in the next generation.

WF population ($N = 5$)



WF Model & Genetic Drift

Definition

$$Pr(j) = \binom{2N}{j} \left(\frac{j}{2N}\right)^{j} \left(\frac{2N-j}{2N}\right)^{2N-j} \tag{1}$$

Where j represents the number of allelic copies of a particular allele.

WF Model & Genetic Drift

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Where j represents the number of allelic copies of a particular allele.

$$Pr(k) = \binom{2N}{k} p^{k} (1-p)^{2N-k} \tag{2}$$

Where k represents the number of offspring with a specific allelic copy and is binomially distributed with $p = \frac{j}{2N}$.

WF Model & Genetic Drift

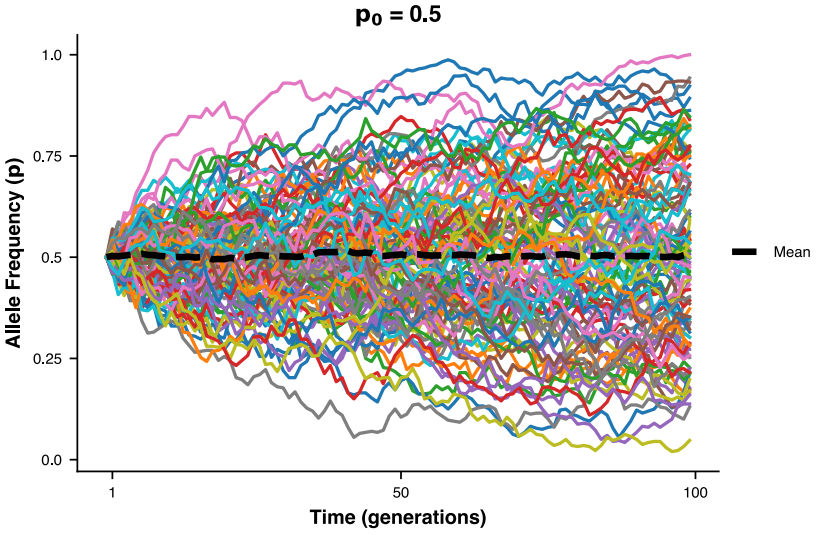
Definition

$$p' \sim \frac{B(2N, p)}{2N} \tag{3a}$$

$$\mathbb{E}(p') = \frac{2Np}{2N} = p \tag{3b}$$

$$Var(p') = \frac{2Np(1-p)}{2N} = p(1-p) \tag{3c}$$

WF Model & Genetic Drift



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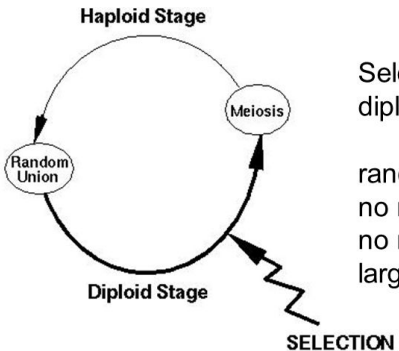
Single Locus Diploid Model

One-locus diploid model



Selection acts in the diploid phase

random mating,
no migration, and
no mutation
large population

Single Locus Diploid Model

Definition

Genotypes	A_1A_1	A_1A_2	A_2A_2

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Frequency Before Selection	p^2	$2pq$	q^2
Relative Fitness (Viability)	w_{11}	w_{12}	w_{22}
Frequency After Selection	$\frac{p^2 w_{11}}{\bar{w}}$	$\frac{2pqw_{12}}{\bar{w}}$	$\frac{q^2 w_{22}}{\bar{w}}$

Allele Frequency Trajectories

Definition

$$\text{Population Mean Fitness: } \bar{w} = p^2 w_{11} + 2pqw_{12} + q^2 w_{22} \tag{4}$$

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Population Mean Fitness: $\bar{w} = p^2 w_{11} + 2pqw_{12} + q^2 w_{22}$ (4)

Relative Frequencies of Genotypes After Selection: $1 = \frac{p^2 w_{11}}{\bar{w}} + \frac{2pqw_{12}}{\bar{w}} + \frac{q^2 w_{22}}{\bar{w}}$ (5)

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$$\text{Relative Frequencies of Genotypes After Selection: } 1 = \frac{p^2 w_{11}}{\bar{w}} + \frac{2pqw_{12}}{\bar{w}} + \frac{q^2 w_{22}}{\bar{w}} \tag{5}$$

$$\text{Frequency of the } A_1 \text{ Allele After Selection: } p' = \frac{p^2 w_{11} + pqw_{12}}{\bar{w}} \tag{6}$$

Allele Frequency Trajectories

Definition

Change in Allele Frequencies due to Selection: $\Delta p = p' - p$ (7a)

$$\Delta p = pq\left(p\left(w_{11} - w_{12}\right) + q\left(w_{12} - w_{22}\right)\right) \times \frac{1}{\bar{w}} \tag{7b}$$

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$$w_1 = pw_{11} + qw_{12} \tag{8a}$$

$$w_2 = pw_{12} + qw_{22} \tag{8b}$$

Change in Allele Frequencies due to Selection: $\Delta p = \dfrac{pq\left(w_1 - w_2\right)}{\bar{w}}$ (9)

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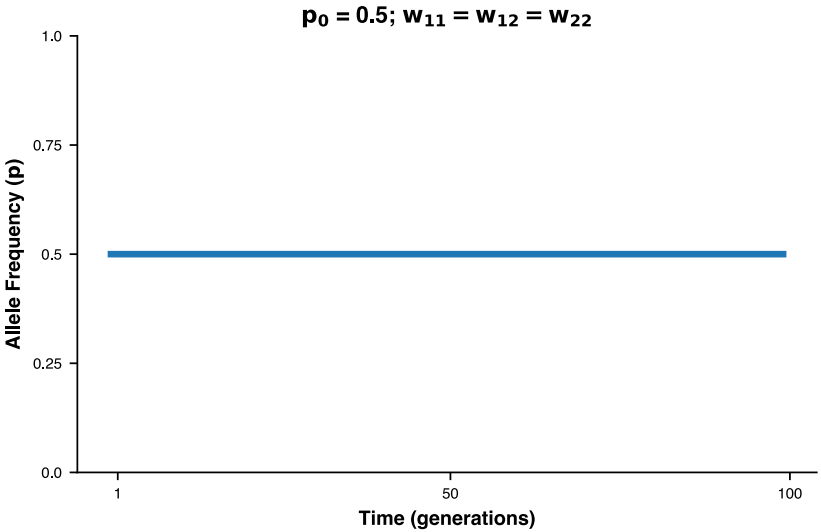

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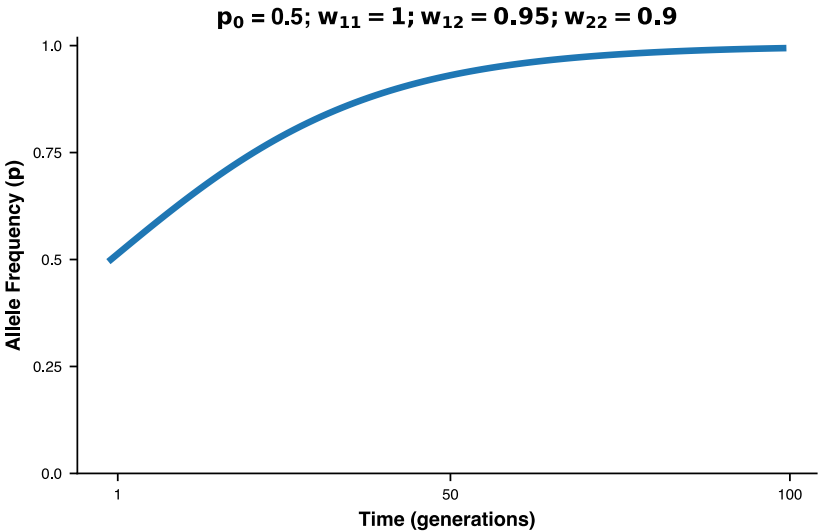

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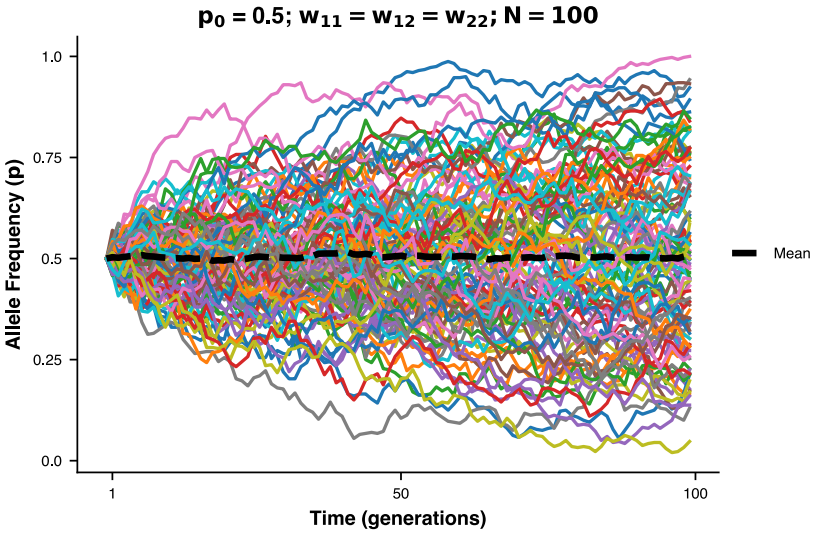

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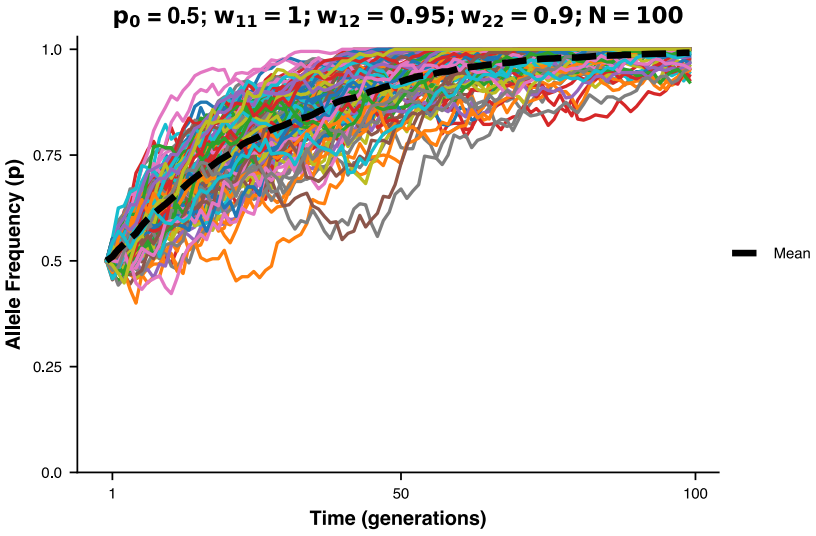

N's Impact on Drift & Selection

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