Разглеждаме данните $x_1, x_2, \ldots, x_n$ като наблюдения над случайните величини $X_1, X_2, \ldots, X_n$, които са независими и еднакво разпределени.

$$X_1, X_2, \ldots, X_n \sim F(x), \qquad F(x) = \mathbb{P}(X_i \leq x)$$

Нека τ е число (константа, неслучайна величина), което може да се определи ако се знае $F(x)$.

Примери:

1. $X_1, X_2, \ldots, X_n \sim \mathrm{Exp}(\lambda)$
   $\tau = \mathrm{E}(X_i) = 1/\lambda$
   $\tau = \mathrm{Var}(X_i) = 1/\lambda^2$
   $\tau = \mathrm{Med}(X_i) = (1/\lambda)\log(2)$
   $\tau = \mathbb{P}(X_i > 3) = e^{-3\lambda}$

2. $X_1, X_2, \ldots, X_n \sim \mathcal{N}(\mu, \sigma^2)$
   $\tau = \mathrm{E}(X_i) = \mu$
   $\tau = \mathrm{Var}(X_i) = \sigma^2$
   $\tau = \mathbb{P}(X_i > 0) = \int_0^\infty \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/2\sigma^2} dx$

Нека $\widehat{\tau}$ е функция на $X_1, X_2, \ldots, X_n$,
$\widehat{\tau} = \widehat{\tau}(X_1, X_2, \ldots, X_n)$.

Пример: $\tau = \mathrm{E}(X_i) = \mu$, $\quad \widehat{\tau} = \overline{X} = \frac{1}{n}(X_1 + \ldots + X_n)$.
($\widehat{\tau}$ е оценка на τ.)

Казваме, че $\widehat{\tau}$ е **неизместена оценка** на τ ако $\mathrm{E}(\widehat{\tau}) = \tau$.

Казваме, че $\widehat{\tau}$ е **асимптотично неизместена оценка** на τ ако $\mathrm{E}(\widehat{\tau}) \longrightarrow \tau$ при $n \longrightarrow \infty$.

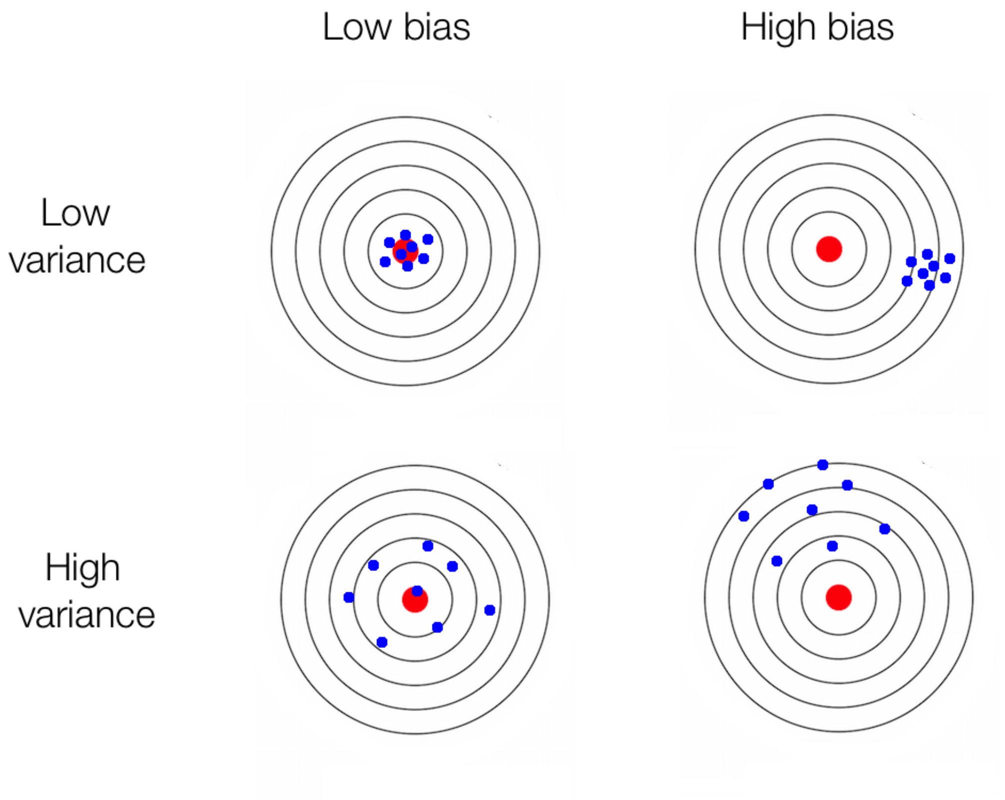
Изместване на оценката $\widehat{\tau}$:

$\mathrm{Bias}(\widehat{\tau}) = \mathrm{E}(\widehat{\tau}) - \tau$

Средноквадратична грешка на оценката $\widehat{\tau}$:

$\mathrm{MSE}(\widehat{\tau}) = \mathrm{E}(\widehat{\tau} - \tau)^2$

$\mathrm{MSE}(\widehat{\tau}) = \mathrm{Var}(\widehat{\tau}) + [\mathrm{Bias}(\tau)]^2$



$$
\begin{array}{cccc}
x_1^{(1)} & x_1^{(2)} & \dots & x_1^{(10000)} \\
x_2^{(1)} & x_2^{(2)} & \dots & x_2^{(10000)} \\
\dots & \dots & & \dots \\
x_n^{(1)} & x_n^{(2)} & \dots & x_n^{(10000)} \\
\widehat{\tau}^{(1)} & \widehat{\tau}^{(2)} & \dots & \widehat{\tau}^{(10000)}
\end{array}
$$

$$
\mathrm{E}(\widehat{\tau}) = \tau \quad \Longrightarrow \quad \frac{\widehat{\tau}^{(1)} + \widehat{\tau}^{(2)} + \ldots + \widehat{\tau}^{(10000)}}{10000} \approx \tau
$$

$$
\mathrm{E}(\widehat{\tau}) \approx \texttt{mean(c(}\widehat{\tau}^{(1)}, \widehat{\tau}^{(2)}, \ldots, \widehat{\tau}^{(10000)}\texttt{))}
$$

$$
\sqrt{\mathrm{Var}(\widehat{\tau})} \approx \texttt{sd(c(}\widehat{\tau}^{(1)}, \widehat{\tau}^{(2)}, \ldots, \widehat{\tau}^{(10000)}\texttt{))}
$$