Model for Estimating Abundance of Parous Females: Take 2

$$
\begin{aligned}
y_{it} &\sim \begin{cases} 0 & , z_{it} = 0 \\ \text{Bern}(p) & , z_{it} = 1 \end{cases} \\
z_{it} &\sim \begin{cases} 0 & , r_{it} = 0 \\ \text{Bern}(\phi_{it}) & , r_{it} = 1 \end{cases} \\
r_{it} &\sim \text{Bern}(\psi_{it})
\end{aligned}
$$

$$
\begin{aligned}
p &\sim \text{Beta}(\alpha_p, \beta_p) \\
\text{logit}(\phi_{it}) &= \mathbf{x}_{it}^T \boldsymbol{\beta} \\
\text{logit}(\psi_{it}) &= \mathbf{w}_{it}^T \boldsymbol{\alpha} \\
\boldsymbol{\beta} &\sim \text{N}(\boldsymbol{\mu}_\beta, \Sigma_\beta) \\
\boldsymbol{\alpha} &\sim \text{N}(\boldsymbol{\mu}_\alpha, \Sigma_\alpha)
\end{aligned}
$$

Notation:

parous females are individuals $i = 1, \ldots, n, \ldots, N, \ldots, M$
n is the number of observed parous females
N is the total abundance of parous females
M is the augmented data set
t is years

y_{it} indicates if individual i was sighted with a pup in year t
z_{it} indicates if individual i **survived** and had a pup in year t
p is probability of detecting an individual with her pup
ϕ_{it} is the probability of individual i surviving from year $t-1$ to year t and having a pup in year t
ψ_{it} is the probability that individual i was recruited into the reproducing population in year t

Derived quantity:

$N_t = \sum_{i=1}^{M} z_{it}$ is the abundance of parous females in year t