STAT632 Project (Advertisement Sales Dataset)

David Teng

2025-05-15

About Dataset

Advertisement Sales Dataset

The Advertisement Sales dataset is a collection of data points used to analyze the impact of advertising on sales. This dataset consists of 200 entries, each representing a unique observation with data on various types of media advertising and corresponding sales figures.

Load Libraries and Data

```r
# Load necessary libraries
library(MASS)          # for boxcox
library(glmnet)        # for LASSO
```

```
Loading required package: Matrix
```

```
Loaded glmnet 4.1-8
```

```r
library(randomForest)  # for Random Forest
```

```
randomForest 4.7-1.2
```

```
Type rfNews() to see new features/changes/bug fixes.
```

```r
library(car)           # for VIF
```

```
Loading required package: carData
```

```r
library(ggplot2)       # for nice plots
```

```

Attaching package: 'ggplot2'

The following object is masked from 'package:randomForest':

    margin
```

```r
library(caret)         # for model validation
```

```
Loading required package: lattice
```

```r
library(dplyr)         # for data manipulation
```

```

Attaching package: 'dplyr'

The following object is masked from 'package:car':

    recode

The following object is masked from 'package:randomForest':

    combine

The following object is masked from 'package:MASS':

    select

The following objects are masked from 'package:stats':

    filter, lag
```

```
The following objects are masked from 'package:base':

    intersect, setdiff, setequal, union
```

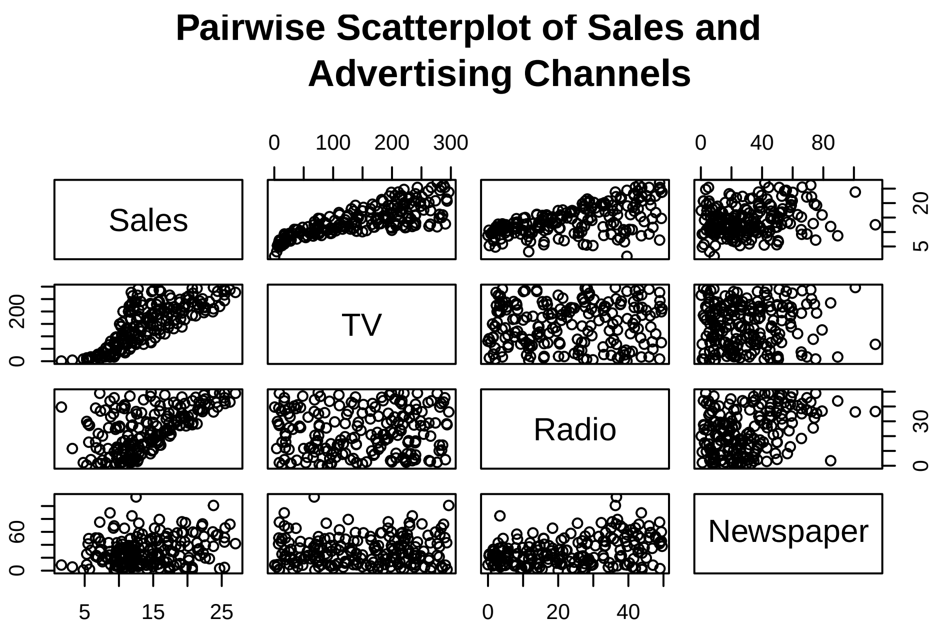
```
# Load data
adver <- read.csv("Advertising And Sales.csv")

# Quick overview
summary(adver)
```

```
       ID               TV             Radio          Newspaper     
 Min.   :  1.00   Min.   :  0.70   Min.   : 0.00   Min.   :  0.30  
 1st Qu.: 50.75   1st Qu.: 74.38   1st Qu.:10.07   1st Qu.: 12.75  
 Median :100.50   Median :149.75   Median :22.90   Median : 25.75  
 Mean   :100.50   Mean   :147.03   Mean   :23.29   Mean   : 30.55  
 3rd Qu.:150.25   3rd Qu.:218.82   3rd Qu.:36.52   3rd Qu.: 45.10  
 Max.   :200.00   Max.   :296.40   Max.   :49.60   Max.   :114.00  
     Sales      
 Min.   : 1.60  
 1st Qu.:10.40  
 Median :12.90  
 Mean   :14.04  
 3rd Qu.:17.40  
 Max.   :27.00  
```

```
str(adver)
```

```
'data.frame':   200 obs. of  5 variables:
 $ ID       : int  1 2 3 4 5 6 7 8 9 10 ...
 $ TV       : num  230.1 44.5 17.2 151.5 180.8 ...
 $ Radio    : num  37.8 39.3 45.9 41.3 12.8 48.9 32.8 19.6 2.1 2.6 ...
 $ Newspaper: num  69.2 45.1 69.3 58.5 58.4 75 23.5 11.6 1 21.2 ...
 $ Sales    : num  22.1 10.4 9.3 18.5 12.9 7.2 11.8 13.2 4.8 10.6 ...
```

```
# Pairwise scatterplot with title
pairs(Sales ~ TV + Radio + Newspaper, data = adver,
      main = "Pairwise Scatterplot of Sales and
      Advertising Channels")
```



Data Preparation

```r
# Check for missing values
colSums(is.na(adver))
```

```
       ID        TV     Radio Newspaper     Sales 
        0         0         0         0         0 
```

Base Multiple Linear Regression

```r
# Base Multiple Linear Regression
lm_full <- lm(Sales ~ TV + Radio + Newspaper, data = adver)
summary(lm_full)
```

```

Call:
lm(formula = Sales ~ TV + Radio + Newspaper, data = adver)

Residuals:
    Min      1Q  Median      3Q     Max 
```

```
-8.8335 -0.8662  0.2411  1.1927  3.4411

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.003556   0.313296   9.587   <2e-16 ***
TV           0.045686   0.001402  32.583   <2e-16 ***
Radio        0.187110   0.008649  21.634   <2e-16 ***
Newspaper   -0.001330   0.005905  -0.225    0.822    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.695 on 196 degrees of freedom
Multiple R-squared:  0.8958,	Adjusted R-squared:  0.8942 
F-statistic: 561.4 on 3 and 196 DF,  p-value: < 2.2e-16
```

Interpretation of Base Multiple Linear Regression:

We fitted a multiple linear regression model to predict **Sales** based on **TV**, **Radio**, and **Newspaper** advertising budgets.

- The **Intercept** is estimated at **3.00** ($p < 0.001$), meaning that when advertising budgets are zero, the expected sales would be about 3,000 units.
- The **TV advertising budget** has a **positive and significant** effect on Sales.

  Each additional thousand dollars spent on TV is associated with an **increase of about 45.7 units** in Sales, **holding other factors constant** ($p < 0.001$).
- The **Radio advertising budget** also has a **positive and significant** effect on Sales.

  Each additional thousand dollars spent on Radio is associated with an **increase of about 187.1 units** in Sales ($p < 0.001$).
- The **Newspaper advertising budget** is **not statistically significant** ($p = 0.822$), suggesting that spending on Newspaper ads does **not have a meaningful effect** on Sales in this model.

Goodness of Fit:

- The **Multiple R-squared** is **0.8958**, meaning the model explains about **89.6%** of the variance in Sales.
- The **Adjusted R-squared** is **0.8942**, which adjusts for the number of predictors and confirms the model still fits the data very well.

- The overall **F-statistic** is highly significant ($p < 2.2e\text{-}16$), indicating that the model provides a better fit than a model with no predictors.

Conclusion:

The model shows that TV and Radio advertising significantly increase sales, while Newspaper advertising does not. The model explains 89.6% of the variance in sales, and overall, it fits the data very well (F-test $p < 0.001$).

Reduced Model (remove Newspaper)

```r
# Reduced Model (remove Newspaper)
lm_reduced <- lm(Sales ~ TV + Radio, data = adver)
summary(lm_reduced)
```

```
Call:
lm(formula = Sales ~ TV + Radio, data = adver)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.7951 -0.8621  0.2422  1.1749  3.4344 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 2.980757   0.295772   10.08   <2e-16 ***
TV          0.045674   0.001398   32.68   <2e-16 ***
Radio       0.186423   0.008073   23.09   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.691 on 197 degrees of freedom
Multiple R-squared:  0.8957,    Adjusted R-squared:  0.8947 
F-statistic: 846.2 on 2 and 197 DF,  p-value: < 2.2e-16
```

Interpretation of Reduced Model (TV + Radio only):

We fitted a reduced multiple linear regression model to predict **Sales** using only **TV** and **Radio** advertising budgets (after removing Newspaper).

- The **Intercept** is estimated at **2.98** ($p < 0.001$), meaning that when TV and Radio advertising expenditures are zero, the expected sales would be about **2,980 units**.
- The **TV advertising budget** remains a **positive and highly significant predictor** of Sales.

  Each additional thousand dollars spent on TV advertising is associated with an **increase of approximately 45.7 units** in Sales, **holding Radio constant** ($p < 0.001$).
- The **Radio advertising budget** also remains **positive and highly significant**.

  Each additional thousand dollars spent on Radio advertising is associated with an **increase of approximately 186.4 units** in Sales ($p < 0.001$).

Goodness of Fit:

- The **Multiple R-squared** is **0.8957**, indicating that about **89.6%** of the variance in Sales is explained by TV and Radio budgets.
- The **Adjusted R-squared** is **0.8947**, very close to the full model, suggesting that removing Newspaper **did not harm model fit**.
- The model's **F-statistic** is **highly significant** ($p < 2.2e\text{-}16$), showing the model overall is statistically significant.

Model Comparison:

- Compared to the full model (TV + Radio + Newspaper), the reduced model achieves **almost identical R-squared** with fewer predictors.
- Based on the **partial F-test** and **adjusted R-squared**, we conclude that **Newspaper** advertising is **not necessary** for predicting Sales.

Conclusion:

The reduced model including only TV and Radio advertising performs just as well as the full model. Both TV and Radio advertising expenditures have significant positive effects on Sales, while Newspaper advertising was found to be unnecessary. The reduced model explains about 89.6% of the variance in Sales and provides a simpler, equally effective prediction model.

Compare Models: Full vs Reduced:

Hypotheses for Model Comparison:

$H_0 : \beta_{Newspaper} = 0$ (The coefficient for **Newspaper** is equal to zero which means Newspaper does **not** improve the model.)

vs. $H_1 : \beta_{Newspaper} \neq 0$

(The coefficient for **Newspaper** is **not** equal to zero which means Newspaper **does** improve the model.)

```r
# Compare Models: Full vs Reduced
anova(lm_reduced, lm_full) # partial F-test
```

```
Analysis of Variance Table

Model 1: Sales ~ TV + Radio
Model 2: Sales ~ TV + Radio + Newspaper
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1    197 563.09
2    196 562.95  1   0.14567 0.0507 0.8221
```

Interpretation of Model Comparison (Full vs Reduced):

We conducted a **partial F-test** to formally compare the full model (**Sales** ~ **TV** + **Radio** + **Newspaper**) with the reduced model (**Sales** ~ **TV** + **Radio**).

From the ANOVA table:

- The test statistic is **F = 0.0507**, with a corresponding **p-value = 0.8221**.
- The p-value is **much greater than 0.05**, meaning we **fail to reject** the null hypothesis.

Interpretation:

- There is **no significant evidence** that adding **Newspaper** as a predictor improves the model.
- Therefore, the **simpler model with only TV and Radio** is preferred.

Additional Note:

- The Residual Sum of Squares (RSS) only **slightly decreased** from **563.09** to **562.95** after adding Newspaper, which is not meaningful.
- This further confirms that **Newspaper** is not a useful predictor for Sales.

Conclusion:

Since the partial F-test (p = 0.8221), we **fail to reject** H_0.

This means **Newspaper does not significantly improve** the model. Thus, the reduced model with only TV and Radio is sufficient.

Adjusted R-squared comparison

```r
# Adjusted R-squared comparison
summary(lm_full)$adj.r.squared # Full model (TV + Radio + Newspaper
```

```
[1] 0.8941635
```

```r
summary(lm_reduced)$adj.r.squared # Reduced model (TV + Radio only)
```

```
[1] 0.8946735
```

Adjusted R-squared Comparison Interpretation:

The **adjusted R-squared** for the reduced model (**0.8947**) is **slightly higher** than that of the full model (**0.8942**).

- **Adjusted R-squared** adjusts for the number of predictors in the model.
- A **higher adjusted R-squared** suggests that the reduced model **fits the data better**, even though it uses **fewer predictors**.
- Therefore, the model including only **TV** and **Radio** provides a **better and simpler fit** than the model that also includes **Newspaper**.

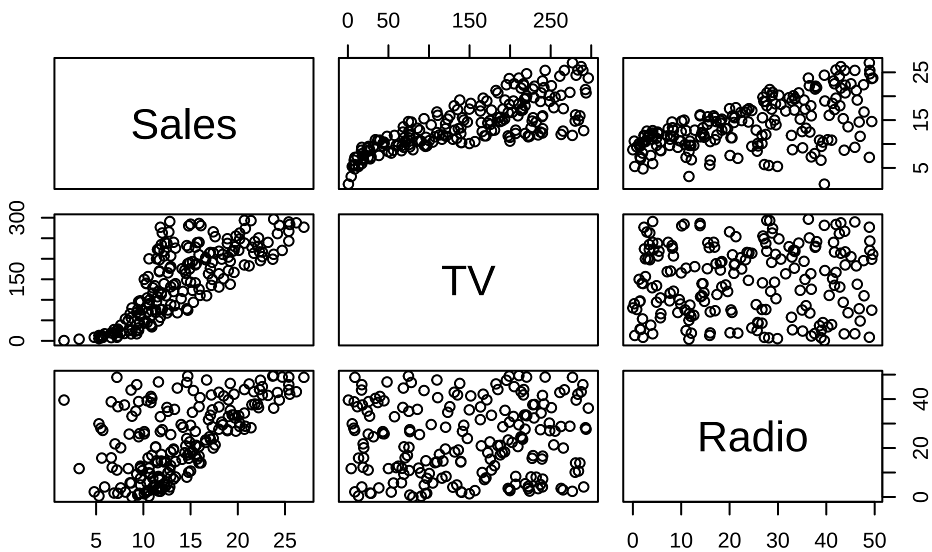
Conclusion:

The reduced model (TV + Radio) has a slightly higher adjusted R-squared than the full model, indicating a better fit with fewer predictors.

Result: Slightly better adjusted R^2 for reduced model → remove Newspaper

Pairwise scatterplot

```
pairs(Sales ~ TV + Radio, data = adver)
```



Interpretation of Pairwise Scatterplot (Sales, TV, Radio):

The pairwise scatterplot shows the relationships between **Sales**, **TV**, and **Radio** advertising:

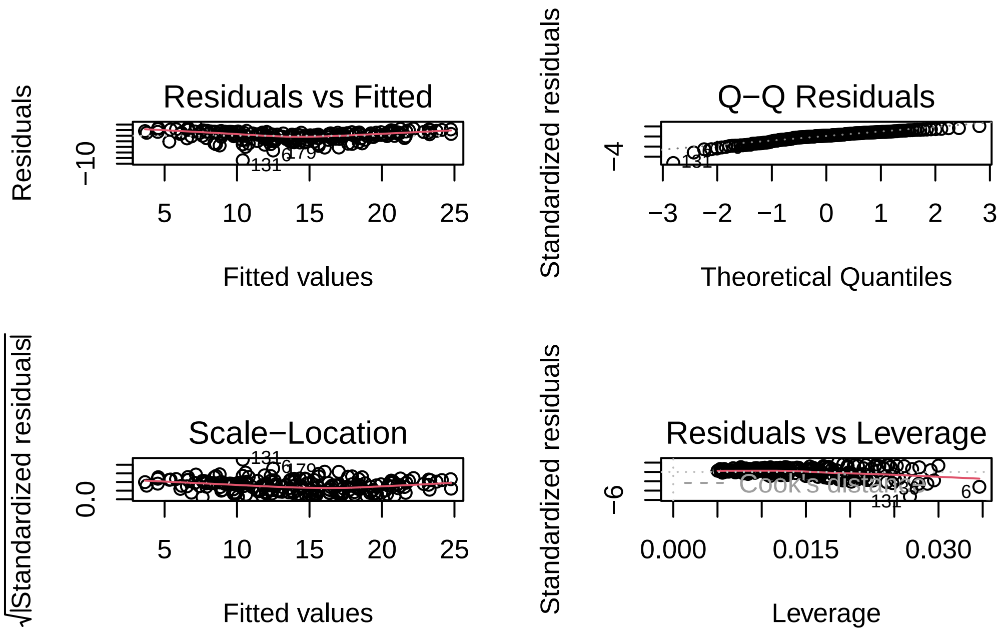
- **Sales vs TV**: There is a **strong positive linear relationship**. As spending on TV advertising increases, Sales also tend to increase. The pattern is clear and linear, supporting the use of TV as a predictor in a linear regression model.
- **Sales vs Radio**: A **moderate positive linear relationship** is also observed. Though more spread out than the TV relationship, the trend is still upward, suggesting Radio advertising has a meaningful impact on Sales.
- **TV vs Radio**: The scatterplot shows **no strong correlation** between TV and Radio advertising budgets. The points are scattered without a clear pattern, suggesting that TV and Radio are **not highly collinear**, which is good for regression modeling.

Conclusion:

Sales shows strong positive correlation with TV advertising and moderate positive correlation with Radio advertising. TV and Radio budgets appear to be largely independent.

Diagnostic Plots for lm_reduced

```
# Diagnostic Plots for lm_reduced
par(mfrow=c(2,2))
plot(lm_reduced)
```



Diagnostic Plots Interpretation (for Reduced Model):

These diagnostic plots help assess the assumptions of the multiple linear regression model:

1. Residuals vs Fitted

- This plot checks for **linearity** and **homoscedasticity**.

- The residuals appear to be randomly scattered around the horizontal line, indicating that:

  The relationship between predictors and response is likely **linear**.

  There is **no clear pattern**, suggesting **constant variance** (no heteroscedasticity).

2. Normal Q-Q Plot

- This plot checks for **normality of residuals**.
- The residual points mostly fall along the straight line, indicating that the residuals are **approximately normally distributed**.

3. Scale-Location Plot

- This plot also checks for **homoscedasticity**, using standardized residuals.
- The red line is mostly flat and the spread of residuals is consistent across fitted values, suggesting **homogeneity of variance**.

4. Residuals vs Leverage

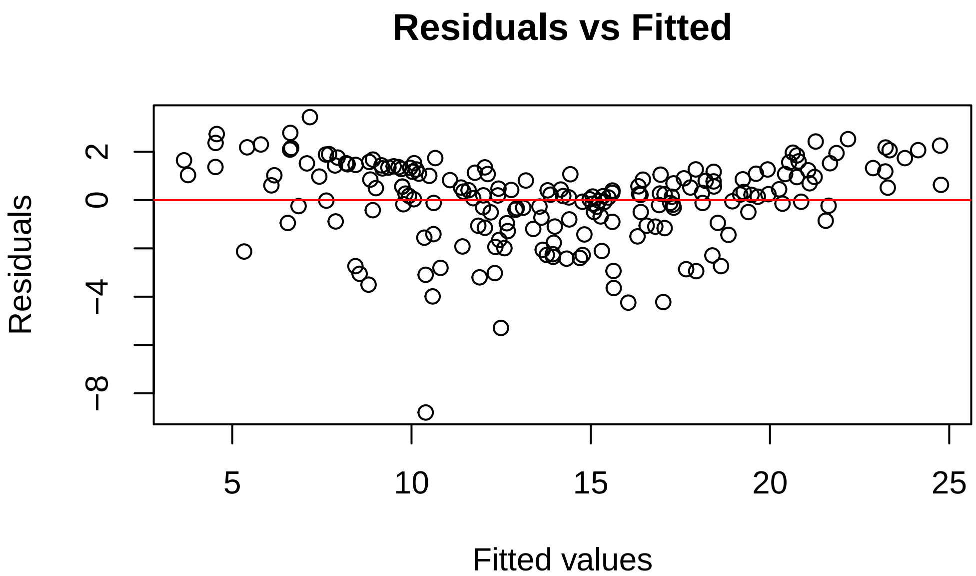
- This plot identifies **influential points** that may disproportionately affect the model.
- There are no points with **unusually high leverage** or **extreme residuals**, indicating that there are **no strong outliers or influential observations**.

Conclusion:

The diagnostic plots suggest that the reduced model meets the assumptions of linearity, normality, constant variance, and no influential outliers. Thus, the model appears appropriate for inference and prediction.

Residuals vs Fitted for lm2

```r
# Residuals vs Fitted for lm_reduced
plot(fitted(lm_reduced), resid(lm_reduced),
     xlab="Fitted values", ylab="Residuals",
     main="Residuals vs Fitted")
abline(h=0, col="red")
```

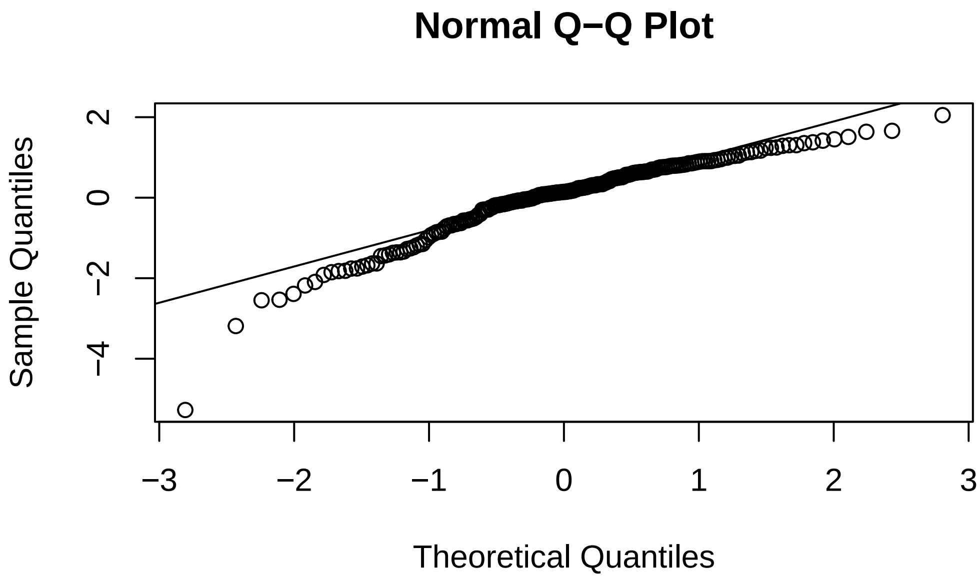


Interpretation:

The Residuals vs Fitted plot shows that residuals are randomly scattered around zero with no strong pattern, supporting the assumptions of linearity and constant variance.

QQ plot for residuals (Normality)

```r
# QQ plot for residuals (Normality)
qqnorm(rstandard(lm_reduced))
qqline(rstandard(lm_reduced))
```

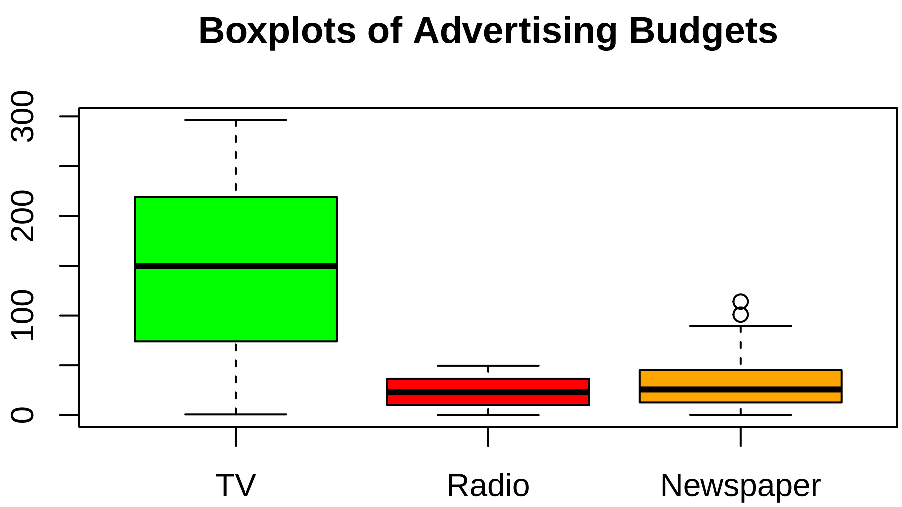


Interpretation:

The Q-Q plot suggests that the residuals are **approximately normally distributed**, with **minor deviations at the tails**. This does **not seriously violate** the normality assumption required for multiple linear regression.

Boxplots to check for outliers in predictors

```
# Boxplots to check for outliers in predictors
boxplot(adver[, c("TV", "Radio", "Newspaper")],
        main="Boxplots of Advertising Budgets",
        col=c("green", "red", "orange"))
```



Interpretation:

The boxplots show that **TV has the largest budget range**, while **Newspaper advertising contains a few outliers**. No extreme values are observed for TV or Radio. This insight helps explain why Newspaper may not be a strong predictor in the regression model — its distribution is more scattered and includes outlying values.

Why Use a Box-Cox Transformation for `lm_full`?

For Model (`lm_full: Sales ~ TV + Radio + Newspaper`)

- We already checked the **residuals vs fitted plot** and **QQ plot**, which were **mostly okay**, but:

  There was **some non-linearity** and **slight skewness** in the residuals.

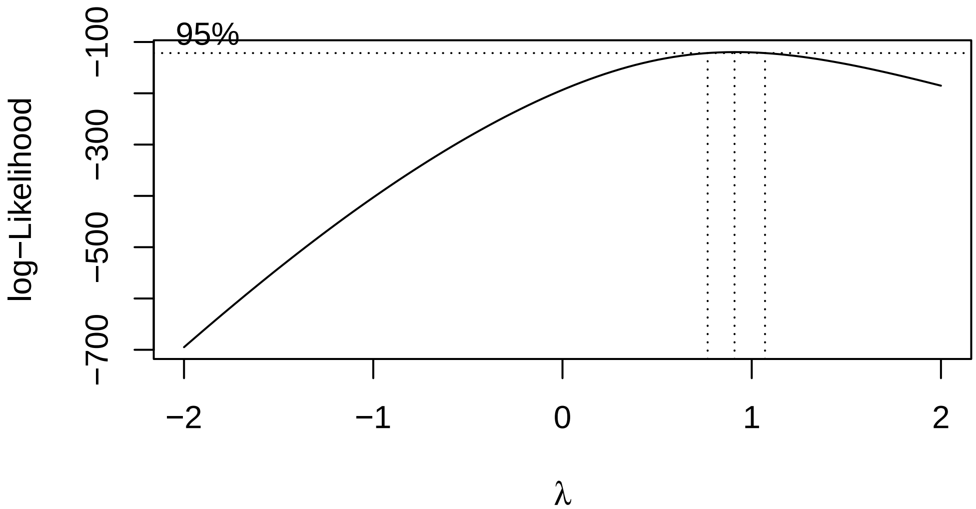
- So using `boxcox()` helps :

  Confirm whether transformation is needed, and

  Find the best power transformation (e.g., log, sqrt, etc.) to improve the model.

```
# Box-Cox transformation for lm_full
boxcox(lm_full, lambda = seq(-2, 2, 0.1),
       main = "Box-Cox Transformation for Sales")
```

```
Warning: In lm.fit(x, y, offset = offset, singular.ok = singular.ok, ...) :
 extra argument 'main' will be disregarded
```



Interpretation:

The Box-Cox transformation plot indicates that the optimal λ is close to 1, and **no transformation of the response variable is necessary**. This supports using the original Sales variable in the multiple linear regression model.

Check Influential Observations

```
# Check Influential Observations
influence.measures(lm_reduced)
```

```
Influence measures of
     lm(formula = Sales ~ TV + Radio, data = adver) :

       dfb.1_    dfb.TV  dfb.Radi    dffit cov.r   cook.d     hat inf
1   -5.43e-02  6.07e-02  6.13e-02  0.11080 1.016 4.09e-03 0.01398
2   -4.54e-02  1.04e-01 -9.51e-02 -0.16031 1.014 8.55e-03 0.01877
3   -6.47e-02  2.10e-01 -2.11e-01 -0.31829 0.995 3.34e-02 0.02949
4   -1.35e-02 -6.43e-04  4.62e-02  0.05987 1.024 1.20e-03 0.01237
5   -1.73e-02 -1.33e-02  2.23e-02 -0.03966 1.021 5.26e-04 0.00845
6   -1.03e-01  4.03e-01 -4.28e-01 -0.61818 0.897 1.21e-01 0.03462   *
7    2.35e-03 -3.59e-03  2.32e-03  0.00532 1.029 9.49e-06 0.01292
8    3.43e-02 -1.35e-02 -1.04e-02  0.04847 1.015 7.86e-04 0.00576
9    1.03e-01 -6.83e-02 -5.95e-02  0.10326 1.037 3.57e-03 0.02702
10  -7.05e-02 -5.91e-02  1.21e-01 -0.15710 1.011 8.21e-03 0.01715
```

11	1.12e-01	-5.67e-02	-7.27e-02	0.11475	1.019	4.39e-03	0.01582	
12	-8.50e-04	4.61e-03	1.76e-05	0.00745	1.024	1.86e-05	0.00812	
13	-5.28e-02	8.94e-02	-5.29e-02	-0.11788	1.024	4.64e-03	0.01921	
14	5.09e-02	-1.86e-02	-3.68e-02	0.05545	1.024	1.03e-03	0.01194	
15	-1.00e-02	1.50e-02	1.45e-02	0.03201	1.023	3.43e-04	0.00909	
16	-6.50e-02	3.22e-02	1.10e-01	0.13550	1.021	6.12e-03	0.01965	
17	-8.11e-03	1.66e-02	-1.61e-02	-0.02815	1.029	2.65e-04	0.01380	
18	-6.51e-02	7.63e-02	5.12e-02	0.10716	1.031	3.84e-03	0.02242	
19	6.29e-02	-5.07e-02	-7.70e-03	0.07657	1.015	1.96e-03	0.00922	
20	6.90e-03	1.43e-05	7.50e-04	0.01830	1.019	1.12e-04	0.00501	
21	1.63e-03	-4.12e-03	-1.26e-03	-0.00669	1.024	1.50e-05	0.00878	
22	-4.28e-02	-1.09e-01	1.25e-01	-0.18873	1.006	1.18e-02	0.01886	
23	-6.98e-02	6.19e-02	1.65e-02	-0.07655	1.029	1.96e-03	0.01804	
24	-1.59e-03	-4.36e-02	2.17e-02	-0.06530	1.020	1.43e-03	0.01068	
25	9.60e-02	-6.13e-02	-4.29e-02	0.10041	1.015	3.36e-03	0.01211	
26	-4.86e-02	-2.25e-01	2.23e-01	-0.34616	0.967	3.92e-02	0.02413	
27	2.46e-04	-9.06e-05	5.19e-04	0.00137	1.021	6.31e-07	0.00585	
28	2.32e-03	-5.46e-02	2.49e-02	-0.07659	1.020	1.96e-03	0.01219	
29	1.12e-02	-2.46e-02	-3.98e-03	-0.03281	1.027	3.60e-04	0.01224	
30	7.12e-02	-4.79e-02	-2.44e-02	0.07812	1.016	2.04e-03	0.00995	
31	9.19e-03	-1.68e-02	-2.41e-03	-0.01985	1.036	1.32e-04	0.01979	
32	1.97e-02	-8.20e-03	-8.17e-03	0.02484	1.021	2.07e-04	0.00650	
33	1.35e-01	-4.04e-02	-1.16e-01	0.14934	1.013	7.42e-03	0.01703	
34	2.30e-02	-8.55e-02	1.84e-02	-0.10596	1.019	3.75e-03	0.01503	
35	1.35e-01	-4.15e-02	-1.16e-01	0.14916	1.013	7.41e-03	0.01722	
36	-1.30e-02	-3.23e-01	2.57e-01	-0.44209	0.946	6.33e-02	0.02874	*
37	-1.19e-01	1.17e-01	1.16e-01	0.19105	1.016	1.21e-02	0.02331	
38	3.47e-03	3.65e-02	-6.99e-02	-0.08629	1.037	2.49e-03	0.02497	
39	7.81e-03	-8.99e-03	2.19e-03	0.01172	1.028	4.60e-05	0.01281	
40	-3.65e-02	4.07e-02	4.19e-02	0.07549	1.023	1.91e-03	0.01369	
41	1.51e-04	5.84e-03	-9.28e-04	0.01071	1.022	3.84e-05	0.00715	
42	1.88e-03	-2.51e-03	-5.34e-03	-0.01007	1.023	3.39e-05	0.00781	
43	3.28e-02	-6.18e-02	-7.30e-03	-0.07249	1.032	1.76e-03	0.01984	
44	-2.52e-02	-3.51e-02	4.85e-02	-0.07454	1.022	1.86e-03	0.01290	
45	-2.21e-02	2.54e-02	-4.32e-03	-0.03113	1.030	3.25e-04	0.01543	
46	-2.52e-03	-3.79e-03	8.24e-04	-0.01207	1.021	4.88e-05	0.00556	
47	9.82e-02	-4.39e-02	-6.14e-02	0.10512	1.011	3.68e-03	0.01099	
48	-6.78e-02	6.60e-02	7.58e-02	0.12214	1.021	4.98e-03	0.01770	
49	-5.61e-03	-6.14e-02	3.55e-02	-0.09399	1.014	2.95e-03	0.01094	
50	9.33e-02	-5.60e-02	-4.57e-02	0.09731	1.016	3.16e-03	0.01203	
51	-4.42e-02	-3.79e-02	7.63e-02	-0.09965	1.023	3.32e-03	0.01667	
52	7.46e-02	-2.80e-02	-5.07e-02	0.08213	1.016	2.25e-03	0.01047	
53	-7.27e-02	6.18e-02	9.98e-02	0.14659	1.010	7.15e-03	0.01544	

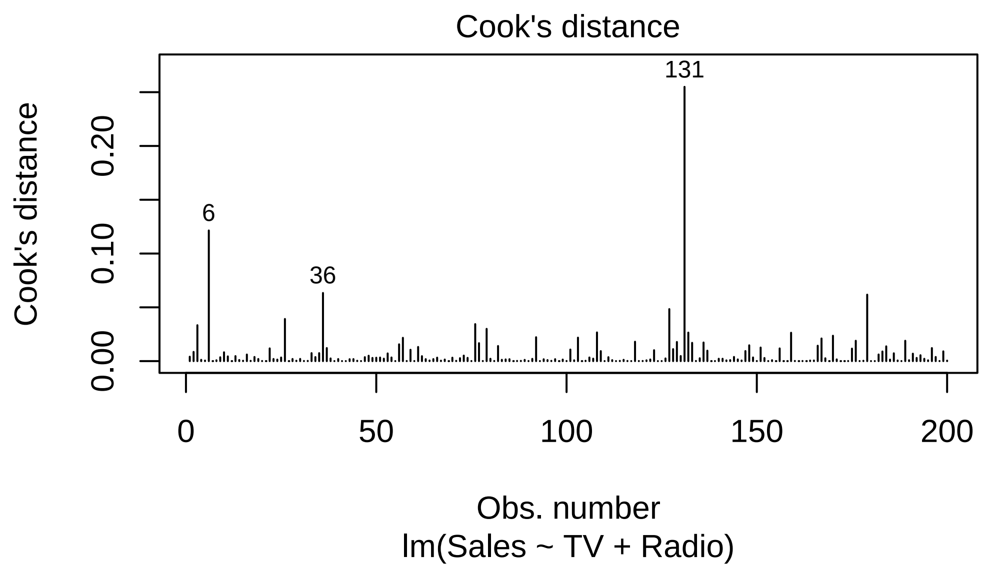
54	-4.27e-02	1.77e-02	8.20e-02	0.10065	1.024	3.38e-03	0.01747
55	4.45e-03	-8.36e-03	-1.85e-03	-0.01071	1.030	3.84e-05	0.01455
56	-1.09e-01	5.26e-02	1.80e-01	0.21702	1.005	1.56e-02	0.02177
57	-1.68e-01	2.16e-01	-5.47e-02	-0.25643	0.984	2.17e-02	0.01917
58	1.15e-02	-1.95e-03	-4.73e-03	0.01834	1.020	1.13e-04	0.00544
59	-9.55e-02	5.37e-02	1.44e-01	0.17793	1.018	1.05e-02	0.02281
60	-4.19e-03	8.98e-03	4.70e-03	0.01623	1.024	8.82e-05	0.00847
61	1.95e-01	-9.97e-02	-1.36e-01	0.19915	1.007	1.32e-02	0.02042
62	-7.11e-02	7.13e-02	6.98e-02	0.11733	1.028	4.60e-03	0.02152
63	-3.36e-04	-5.16e-02	2.73e-02	-0.07364	1.022	1.81e-03	0.01252
64	1.68e-02	-1.85e-02	1.55e-02	0.04140	1.019	5.73e-04	0.00738
65	-1.15e-02	-1.16e-02	5.92e-02	0.07457	1.024	1.86e-03	0.01399
66	9.52e-02	-5.21e-02	-5.42e-02	0.09843	1.018	3.23e-03	0.01313
67	2.60e-02	-2.84e-02	3.46e-03	0.03531	1.029	4.18e-04	0.01423
68	4.66e-02	-3.23e-03	-3.35e-02	0.06628	1.012	1.47e-03	0.00677
69	9.80e-04	-2.21e-03	-4.76e-04	-0.00312	1.026	3.27e-06	0.01081
70	-5.05e-02	3.86e-02	7.05e-02	0.09771	1.025	3.19e-03	0.01737
71	-6.38e-03	1.27e-02	1.01e-02	0.02755	1.022	2.54e-04	0.00790
72	7.71e-02	-2.94e-02	-4.27e-02	0.09058	1.007	2.73e-03	0.00764
73	-6.22e-02	9.56e-02	-4.87e-02	-0.12413	1.020	5.14e-03	0.01755
74	7.60e-02	-7.63e-03	-7.46e-02	0.09505	1.019	3.02e-03	0.01354
75	2.11e-03	-1.02e-02	-5.86e-04	-0.01669	1.023	9.33e-05	0.00801
76	-8.10e-02	2.22e-01	-2.03e-01	-0.32344	0.986	3.44e-02	0.02726
77	2.23e-01	-1.34e-01	-1.41e-01	0.22420	1.009	1.67e-02	0.02433
78	6.74e-03	-5.58e-03	6.24e-03	0.01876	1.021	1.18e-04	0.00616
79	-1.85e-01	2.53e-01	-8.15e-02	-0.30247	0.969	3.00e-02	0.02013
80	6.90e-02	-1.65e-02	-5.61e-02	0.08049	1.018	2.16e-03	0.01098
81	1.20e-02	-1.24e-02	4.11e-03	0.01965	1.024	1.29e-04	0.00878
82	-4.74e-02	-1.19e-01	1.39e-01	-0.20562	1.004	1.40e-02	0.02008
83	4.97e-02	-3.82e-02	-7.12e-03	0.06062	1.018	1.23e-03	0.00862
84	-5.32e-03	3.41e-02	-5.06e-02	-0.06862	1.033	1.58e-03	0.02023
85	-3.64e-02	2.83e-02	5.20e-02	0.07291	1.027	1.78e-03	0.01629
86	-3.62e-04	-8.25e-04	5.33e-04	-0.00176	1.023	1.04e-06	0.00710
87	1.34e-02	-1.45e-02	5.69e-03	0.02297	1.024	1.77e-04	0.00896
88	-2.15e-04	-8.19e-03	1.99e-02	0.02690	1.028	2.42e-04	0.01302
89	3.66e-02	-3.31e-02	8.97e-03	0.05847	1.016	1.14e-03	0.00753
90	2.60e-03	4.66e-03	-1.48e-02	-0.01763	1.036	1.04e-04	0.02005
91	6.16e-02	-3.88e-03	-6.11e-02	0.07906	1.021	2.09e-03	0.01272
92	2.58e-01	-1.53e-01	-1.64e-01	0.25915	0.999	2.22e-02	0.02425
93	-5.36e-03	7.66e-03	6.25e-03	0.01408	1.026	6.64e-05	0.01047
94	-3.76e-02	4.72e-02	3.34e-02	0.07189	1.027	1.73e-03	0.01574
95	4.58e-02	-1.81e-02	-2.54e-02	0.05303	1.018	9.40e-04	0.00788
96	-1.33e-03	3.79e-03	1.32e-02	0.02770	1.020	2.57e-04	0.00669

97	-3.30e-02	-2.71e-02	5.57e-02	-0.07304	1.027	1.78e-03	0.01613		
98	1.37e-03	3.02e-03	-1.20e-03	0.00741	1.022	1.84e-05	0.00614		
99	-4.35e-02	4.29e-02	3.88e-02	0.06543	1.044	1.43e-03	0.02912		
100	-2.81e-03	-2.39e-03	1.43e-02	0.01837	1.028	1.13e-04	0.01292		
101	-5.62e-02	-9.12e-02	1.27e-01	-0.18007	1.006	1.08e-02	0.01775		
102	-2.79e-02	3.73e-02	1.71e-02	0.04737	1.038	7.51e-04	0.02324		
103	1.11e-02	-1.98e-01	1.20e-01	-0.25719	0.992	2.18e-02	0.02187		
104	-9.75e-04	-1.46e-03	1.28e-03	-0.00347	1.023	4.04e-06	0.00710		
105	-1.40e-02	1.93e-02	1.29e-02	0.03036	1.028	3.09e-04	0.01299		
106	-2.46e-02	-1.06e-02	8.46e-02	0.10058	1.024	3.38e-03	0.01733		
107	8.03e-02	-6.02e-02	-3.27e-02	0.08272	1.028	2.29e-03	0.01795		
108	2.61e-01	-8.47e-02	-2.24e-01	0.28509	0.970	2.66e-02	0.01866		
109	1.67e-01	-1.05e-01	-1.03e-01	0.16709	1.029	9.31e-03	0.02783		
110	-3.39e-03	7.38e-03	1.01e-03	0.00955	1.029	3.06e-05	0.01315		
111	-2.44e-02	-5.91e-02	6.47e-02	-0.10442	1.020	3.64e-03	0.01497		
112	-2.70e-02	3.10e-02	2.74e-02	0.05182	1.029	8.99e-04	0.01543		
113	4.70e-03	3.43e-03	-5.18e-03	0.01117	1.022	4.18e-05	0.00708		
114	-6.37e-04	-1.54e-02	4.64e-03	-0.02616	1.022	2.29e-04	0.00792		
115	3.53e-04	2.58e-02	-4.72e-02	-0.05998	1.035	1.20e-03	0.02157		
116	-6.90e-03	1.26e-02	-1.19e-02	-0.02200	1.027	1.62e-04	0.01204		
117	6.79e-03	-4.69e-04	-4.94e-03	0.00962	1.022	3.10e-05	0.00685		
118	2.21e-01	-8.81e-02	-1.75e-01	0.23359	0.992	1.80e-02	0.01923		
119	2.59e-04	-3.80e-03	1.18e-02	0.01752	1.025	1.03e-04	0.00966		
120	-1.78e-02	1.55e-02	4.32e-03	-0.01949	1.033	1.27e-04	0.01693		
121	1.38e-02	-3.61e-03	1.08e-02	0.04628	1.015	7.16e-04	0.00531		
122	-5.47e-02	5.61e-02	8.31e-04	-0.06759	1.028	1.53e-03	0.01620		
123	-5.74e-02	-8.62e-02	1.29e-01	-0.17470	1.012	1.01e-02	0.01972		
124	8.21e-04	-1.99e-03	4.81e-03	0.00801	1.024	2.15e-05	0.00843		
125	-5.40e-03	8.46e-03	5.04e-03	0.01359	1.027	6.19e-05	0.01117		
126	8.05e-02	-3.98e-02	-4.47e-02	0.08642	1.014	2.49e-03	0.01015		
127	-1.47e-01	2.92e-01	-1.99e-01	-0.38550	0.953	4.83e-02	0.02478	*	
128	1.71e-01	-6.38e-02	-1.41e-01	0.18301	1.010	1.11e-02	0.01972		
129	-1.29e-01	8.21e-02	1.83e-01	0.23181	1.004	1.78e-02	0.02290		
130	1.14e-01	-7.30e-02	-5.25e-02	0.11880	1.011	4.70e-03	0.01269		
131	-3.56e-01	7.21e-01	-4.88e-01	-0.94133	0.661	2.55e-01	0.02677	*	
132	-3.99e-02	-1.85e-01	1.84e-01	-0.28357	0.993	2.65e-02	0.02510		
133	-1.54e-01	1.92e-01	-4.17e-02	-0.22658	0.993	1.70e-02	0.01873		
134	-8.01e-03	1.15e-02	9.06e-03	0.02063	1.026	1.43e-04	0.01066		
135	-2.88e-02	6.09e-02	-5.01e-02	-0.08907	1.029	2.65e-03	0.01939		
136	-2.12e-02	1.26e-01	-1.68e-01	-0.22836	1.011	1.73e-02	0.02553		
137	-5.81e-02	1.22e-01	-9.38e-02	-0.17061	1.017	9.69e-03	0.02156		
138	2.34e-03	-4.29e-03	-8.67e-04	-0.00532	1.032	9.48e-06	0.01636		
139	-8.05e-03	8.97e-03	-1.79e-03	-0.01164	1.028	4.54e-05	0.01267		

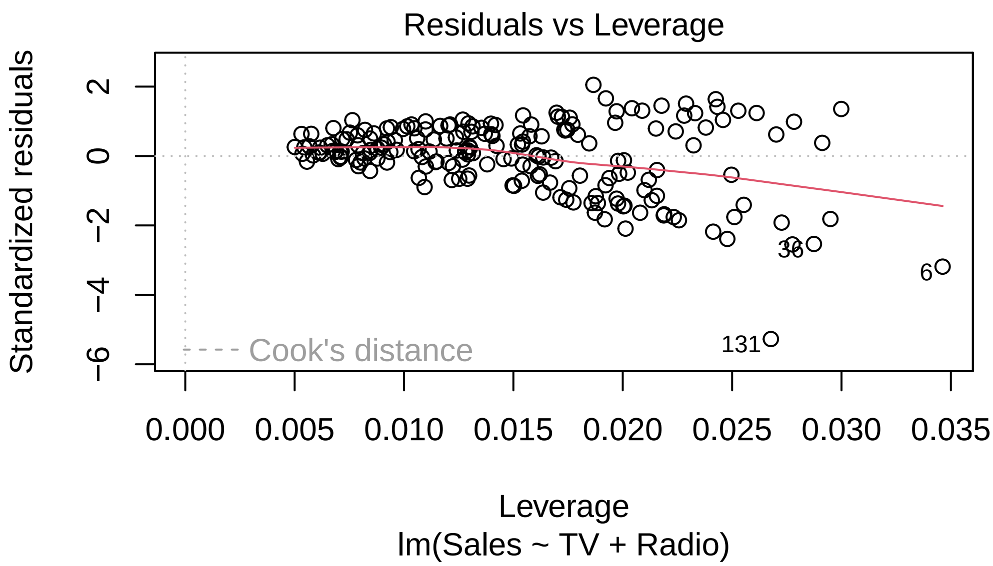
140	-3.32e-02	1.68e-02	6.32e-02	0.08092	1.025	2.19e-03	0.01532	
141	7.22e-02	-4.93e-02	-2.22e-02	0.08084	1.014	2.18e-03	0.00940	
142	-1.44e-02	1.63e-02	2.56e-02	0.04508	1.022	6.80e-04	0.00958	
143	-2.03e-02	2.97e-02	2.25e-02	0.05277	1.022	9.32e-04	0.01060	
144	9.63e-02	-2.87e-02	-7.76e-02	0.10765	1.015	3.87e-03	0.01295	
145	6.07e-02	-3.00e-02	-2.88e-02	0.06838	1.015	1.56e-03	0.00822	
146	3.11e-02	7.97e-05	-3.40e-02	0.04143	1.030	5.75e-04	0.01540	
147	-2.98e-02	-1.03e-01	1.03e-01	-0.16771	1.009	9.35e-03	0.01742	
148	-1.28e-01	9.60e-02	1.56e-01	0.21040	1.015	1.47e-02	0.02528	
149	-2.89e-02	6.78e-02	-5.99e-02	-0.10072	1.030	3.39e-03	0.02119	
150	1.24e-02	-1.37e-02	2.69e-03	0.01784	1.028	1.07e-04	0.01242	
151	2.42e-02	-1.57e-01	7.09e-02	-0.19503	1.007	1.26e-02	0.01978	
152	7.77e-02	-1.59e-02	-6.37e-02	0.09276	1.013	2.87e-03	0.01034	
153	1.51e-04	6.22e-03	-3.46e-04	0.01222	1.022	5.00e-05	0.00675	
154	-1.49e-02	7.40e-03	3.66e-02	0.05056	1.024	8.55e-04	0.01136	
155	8.10e-04	2.20e-03	-7.94e-04	0.00509	1.022	8.68e-06	0.00629	
156	-1.80e-01	1.48e-01	6.33e-02	-0.18835	1.012	1.18e-02	0.02133	
157	8.92e-07	2.34e-03	-4.69e-03	-0.00612	1.033	1.26e-05	0.01672	
158	1.88e-03	1.73e-04	-2.20e-03	0.00265	1.032	2.36e-06	0.01606	
159	-1.20e-01	2.18e-01	-1.34e-01	-0.28304	0.986	2.64e-02	0.02257	
160	1.43e-02	-3.19e-03	-6.36e-03	0.02118	1.020	1.50e-04	0.00567	
161	2.91e-03	2.22e-03	-2.56e-03	0.00772	1.021	2.00e-05	0.00612	
162	-4.23e-03	8.68e-03	-1.01e-02	-0.01720	1.027	9.91e-05	0.01148	
163	-1.77e-03	-3.35e-03	2.41e-03	-0.00752	1.022	1.90e-05	0.00700	
164	-7.76e-03	4.10e-03	2.62e-02	0.03960	1.022	5.25e-04	0.00925	
165	3.40e-02	-1.10e-02	-1.95e-02	0.04160	1.019	5.79e-04	0.00718	
166	-5.51e-02	-1.14e-01	1.45e-01	-0.20774	1.003	1.43e-02	0.02003	
167	-9.92e-02	1.89e-01	-1.27e-01	-0.25242	0.994	2.10e-02	0.02190	
168	-3.41e-02	-3.90e-02	6.40e-02	-0.08898	1.023	2.65e-03	0.01539	
169	6.72e-04	-3.98e-03	1.22e-04	-0.00638	1.024	1.36e-05	0.00819	
170	1.80e-02	-2.09e-01	1.20e-01	-0.26741	0.990	2.36e-02	0.02233	
171	6.68e-02	-4.49e-02	-2.99e-02	0.06905	1.025	1.59e-03	0.01404	
172	1.68e-03	9.81e-04	-7.95e-04	0.00476	1.021	7.59e-06	0.00536	
173	-1.48e-03	1.44e-03	1.28e-04	-0.00176	1.032	1.03e-06	0.01615	
174	-1.11e-02	-3.90e-03	1.38e-02	-0.01889	1.027	1.20e-04	0.01144	
175	-6.06e-02	-9.29e-02	1.35e-01	-0.18706	1.006	1.16e-02	0.01857	
176	-1.59e-01	1.38e-01	1.60e-01	0.23874	1.018	1.89e-02	0.02999	
177	-6.75e-03	1.19e-02	4.12e-03	0.01649	1.028	9.11e-05	0.01280	
178	-1.82e-02	-7.09e-03	2.28e-02	-0.03182	1.025	3.39e-04	0.01100	
179	-4.64e-02	-2.96e-01	2.79e-01	-0.43668	0.944	6.18e-02	0.02776	*
180	6.42e-03	2.16e-03	-7.33e-03	0.01104	1.025	4.08e-05	0.00937	
181	-5.81e-03	-9.56e-04	7.01e-03	-0.00863	1.031	2.49e-05	0.01491	
182	-4.31e-02	-6.80e-02	9.44e-02	-0.13596	1.015	6.16e-03	0.01636	

```
183  1.61e-01 -8.85e-02 -1.00e-01  0.16406 1.009 8.95e-03 0.01698
184 -1.31e-01  1.39e-01  1.10e-01  0.20319 1.018 1.37e-02 0.02612
185  1.35e-02 -5.01e-02  8.19e-03 -0.06430 1.024 1.38e-03 0.01298
186 -7.25e-02  4.68e-02  1.13e-01  0.14762 1.015 7.26e-03 0.01758
187  3.10e-02 -1.59e-04 -3.36e-02  0.04110 1.029 5.66e-04 0.01520
188 -1.55e-03  5.00e-03  3.40e-03  0.01186 1.022 4.72e-05 0.00689
189  3.38e-02 -1.95e-01  8.52e-02 -0.23924 0.995 1.89e-02 0.02079
190  4.78e-02 -3.78e-02 -1.74e-02  0.04980 1.032 8.30e-04 0.01847
191 -3.63e-02  9.33e-02 -8.96e-02 -0.14416 1.022 6.93e-03 0.02099
192  8.99e-02 -4.86e-02 -4.91e-02  0.09410 1.016 2.96e-03 0.01165
193  1.28e-01 -8.47e-02 -7.09e-02  0.12789 1.030 5.46e-03 0.02380
194 -2.48e-02  7.90e-03  6.20e-02  0.08017 1.021 2.15e-03 0.01308
195 -3.98e-03 -5.71e-04  2.95e-02  0.04625 1.020 7.16e-04 0.00845
196  1.90e-01 -1.12e-01 -1.17e-01  0.19109 1.010 1.21e-02 0.02089
197  9.89e-02 -3.49e-02 -7.69e-02  0.10736 1.017 3.85e-03 0.01418
198 -8.88e-03 -5.28e-03  1.13e-02 -0.01805 1.024 1.09e-04 0.00922
199 -1.04e-01  1.14e-01  8.73e-02  0.16512 1.024 9.08e-03 0.02458
200 -7.52e-03 -2.19e-02  2.26e-02 -0.03704 1.030 4.59e-04 0.01579
```

```r
plot(lm_reduced, which=4) # Cook's distance
```




```r
plot(lm_reduced, which=5) # Residuals vs Leverage
```



Influential Observations and Outlier Diagnostics:

We assessed potential influential data points using **Cook's Distance** and the **Residuals vs Leverage** plot.

Cook's Distance Plot:

- **Cook's Distance** measures how much a single observation influences the fitted regression coefficients.
- Points **6**, **36**, and especially **131** stand out with the **highest Cook's distances**.
- However, none of the Cook's distances exceed the common rule-of-thumb threshold of **1**, indicating **no extremely influential outliers**.

Residuals vs Leverage Plot:

- This plot highlights observations with both **high leverage** and **large residuals**, which can be particularly influential.
- Observations **6**, **36**, and **131** are again labeled and lie **furthest from the center**.
- Observation **131** shows **moderately high leverage and a notable residual**, suggesting it has **some influence**, but **not enough to distort the model**.

Conclusion:

While observations 6, 36, and 131 show some degree of influence, none exceed critical thresholds for Cook's distance or leverage. Therefore, we conclude that there are **no influential outliers** that threaten the validity of the model.

Check multicollinearity

```r
# Check multicollinearity
vif(lm_reduced)  # Variance Inflation Factors
```

```
     TV   Radio
1.00324 1.00324
```

Common rule of thumb:

- **VIF > 5** may indicate moderate multicollinearity.
- **VIF > 10** indicates serious multicollinearity problems.

Conclusion:

VIF values **close to 1** indicate **no multicollinearity**.

— Model Extensions —

1. Three-Way Interaction Model

```r
# Three-way interaction model
lm_interaction <- lm(Sales ~ TV * Radio * Newspaper, data = adver)
summary(lm_interaction)
```

```
Call:
lm(formula = Sales ~ TV * Radio * Newspaper, data = adver)

Residuals:
```

```
    Min      1Q  Median      3Q     Max 
-5.9139 -0.3535  0.1713  0.5706  1.9917 

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)    
(Intercept)         6.586e+00  4.798e-01  13.726  < 2e-16 ***
TV                  1.997e-02  2.801e-03   7.128 2.01e-11 ***
Radio               1.928e-02  1.688e-02   1.142    0.255    
Newspaper           1.322e-02  1.774e-02   0.745    0.457    
TV:Radio            1.150e-03  1.004e-04  11.447  < 2e-16 ***
TV:Newspaper       -6.036e-05  9.601e-05  -0.629    0.530    
Radio:Newspaper     1.013e-05  4.977e-04   0.020    0.984    
TV:Radio:Newspaper -7.067e-07  2.778e-06  -0.254    0.799    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.9695 on 192 degrees of freedom
Multiple R-squared:  0.9666,    Adjusted R-squared:  0.9654 
F-statistic: 793.4 on 7 and 192 DF,  p-value: < 2.2e-16
```

2. Three-Way Quadratic Model

```r
# Three-way Quadratic Model using lm()
lm_quadratic <- lm(
  Sales ~ (TV + Radio + Newspaper)^3 +
          I(TV^2) + I(Radio^2) + I(Newspaper^2),
  data = adver
)

# View model summary
summary(lm_quadratic)
```

```
Call:
lm(formula = Sales ~ (TV + Radio + Newspaper)^3 + I(TV^2) + I(Radio^2) + 
    I(Newspaper^2), data = adver)

Residuals:
    Min      1Q  Median      3Q     Max 
-4.7528 -0.3082  0.0029  0.3787  1.5388 
```

```
Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)         4.857e+00  3.405e-01  14.265   <2e-16 ***
TV                  5.455e-02  2.968e-03  18.377   <2e-16 ***
Radio               2.472e-02  1.465e-02   1.687   0.0932 .
Newspaper           1.678e-02  1.234e-02   1.360   0.1754
I(TV^2)            -1.127e-04  7.320e-06 -15.398   <2e-16 ***
I(Radio^2)          2.585e-04  2.596e-04   0.996   0.3207
I(Newspaper^2)      4.569e-05  7.939e-05   0.575   0.5657
TV:Radio            1.028e-03  6.940e-05  14.817   <2e-16 ***
TV:Newspaper       -1.213e-04  6.716e-05  -1.806   0.0725 .
Radio:Newspaper    -3.029e-04  3.714e-04  -0.816   0.4158
TV:Radio:Newspaper  2.291e-06  1.955e-06   1.172   0.2427
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6487 on 189 degrees of freedom
Multiple R-squared:  0.9853,    Adjusted R-squared:  0.9845
F-statistic:  1264 on 10 and 189 DF,  p-value: < 2.2e-16
```

Model Comparison Table

AIC Comparison

```r
# List of models
models <- list(
  "Full Model"        = lm_full,
  "Reduced Model"     = lm_reduced,
  "Interaction Model" = lm_interaction,
  "Quadratic Model"   = lm_quadratic
)

# Extract statistics
model_names <- names(models)
adj_r2 <- sapply(models, function(m) summary(m)$adj.r.squared)
rse <- sapply(models, function(m) summary(m)$sigma)
aic <- sapply(models, AIC)

# Combine into a data frame
aic_table <- data.frame(
```

```
  Model = model_names,
  Adjusted_R2 = round(adj_r2, 4),
  Residual_Std_Error = round(rse, 3),
  AIC = round(aic, 2)
)

# Print the table
print(aic_table, row.names = FALSE)
```

```
            Model Adjusted_R2 Residual_Std_Error    AIC
       Full Model      0.8942              1.695 784.55
    Reduced Model      0.8947              1.691 782.60
Interaction Model      0.9654              0.969 565.02
  Quadratic Model      0.9845              0.649 407.15
```

AIC Model Comparison Table

Model	Adjusted R^2	Residual Std. Error	AIC	Notes
Full Model	0.8942	1.695	784.55	Includes Newspaper; not significant
Reduced Model	0.8947	1.691	782.60	Simpler; performs slightly better than full model
Interaction Model	0.9654	0.969	565.02	Captures strong synergy; includes three-way interaction
Quadratic Model	**0.9845**	**0.649**	**407.15**	Best statistical fit; includes all squared and interaction terms

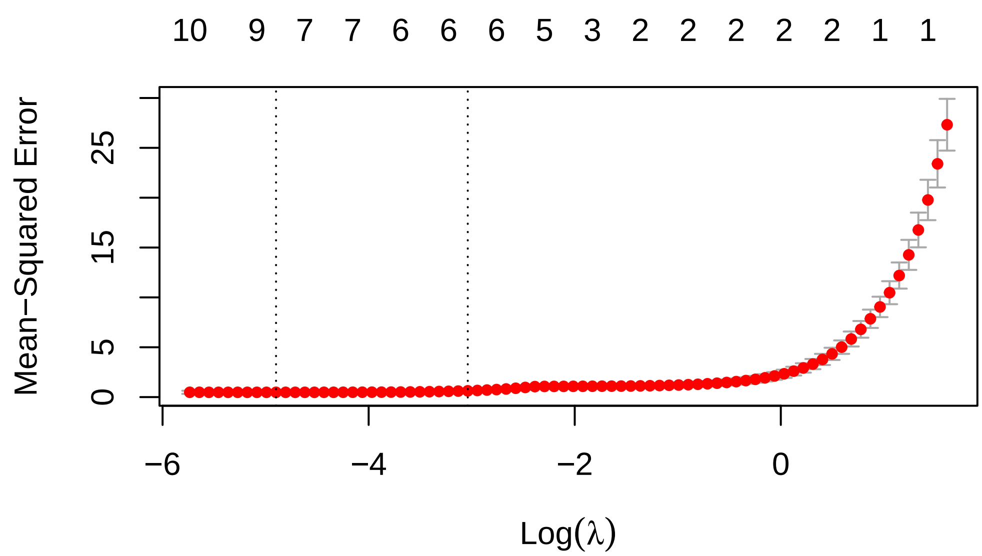
Recommended Final Model

Choose the **Quadratic Model** for our final project paper!

3. Three-Way Quadratic LASSO Model

```
library(glmnet)
library(caret)

# Build full quadratic + interaction design matrix
```

```r
design_formula <- ~ (TV + Radio + Newspaper)^3 + I(TV^2) + I(Radio^2) + I(Newspaper^2)

# Create model matrix (exclude intercept column)
x <- model.matrix(design_formula, data = adver)[, -1]
y <- adver$Sales

# Split into training and testing
set.seed(123)
train_idx <- createDataPartition(y, p = 0.8, list = FALSE)
x_train <- x[train_idx, ]
x_test <- x[-train_idx, ]
y_train <- y[train_idx]
y_test <- y[-train_idx]

# Fit LASSO with cross-validation
lasso_cv <- cv.glmnet(x_train, y_train, alpha = 1)
plot(lasso_cv)
```




```r
# Best lambda
best_lambda <- lasso_cv$lambda.min
cat("Best lambda:", best_lambda, "\n")
```

```
Best lambda: 0.007456631
```

```r
# Final LASSO model
lasso_model <- glmnet(x_train, y_train, alpha = 1, lambda = best_lambda)
print(coef(lasso_model))
```

```
11 x 1 sparse Matrix of class "dgCMatrix"
                              s0
(Intercept)         5.419356e+00
TV                  4.958871e-02
Radio               1.957177e-02
Newspaper           .
I(TV^2)            -1.070685e-04
I(Radio^2)          .
I(Newspaper^2)      .
TV:Radio            1.112765e-03
TV:Newspaper       -1.023794e-05
Radio:Newspaper     1.863151e-04
TV:Radio:Newspaper -2.255534e-07
```

```r
# Prediction and RMSE
lasso_pred <- predict(lasso_model, s = best_lambda, newx = x_test)
lasso_rmse <- sqrt(mean((y_test - lasso_pred)^2))
cat("Test RMSE:", round(lasso_rmse, 4), "\n")
```

```
Test RMSE: 0.6339
```

4. Three-Way Random Forest

```r
library(randomForest)
library(caret)

# Create interaction and quadratic terms manually
adver_rf <- adver |>
  mutate(
    TV2 = TV^2,
    Radio2 = Radio^2,
    Newspaper2 = Newspaper^2,
    TV_Radio = TV * Radio,
    TV_Newspaper = TV * Newspaper,
    Radio_Newspaper = Radio * Newspaper,
```

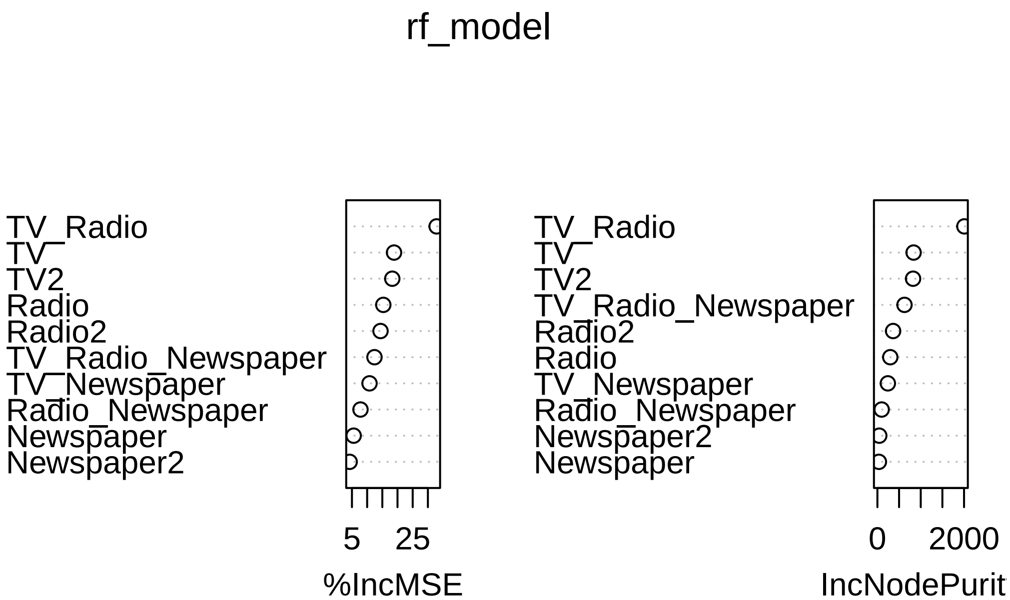
```
    TV_Radio_Newspaper = TV * Radio * Newspaper
  )

# Fit Random Forest with all terms
set.seed(123)
rf_model <- randomForest(
  Sales ~ TV + Radio + Newspaper +
          TV2 + Radio2 + Newspaper2 +
          TV_Radio + TV_Newspaper + Radio_Newspaper +
          TV_Radio_Newspaper,
  data = adver_rf,
  importance = TRUE
)

# Print model summary
print(rf_model)
```

```
Call:
 randomForest(formula = Sales ~ TV + Radio + Newspaper + TV2 +      Radio2 + Newspaper2 + TV_
               Type of random forest: regression
                     Number of trees: 500
No. of variables tried at each split: 3

          Mean of squared residuals: 0.4565567
                    % Var explained: 98.31
```

```
# Variable importance
importance(rf_model)
```

```
                     %IncMSE IncNodePurity
TV                 18.856691     835.51142
Radio              15.318380     296.77560
Newspaper           5.557338      34.43790
TV2                18.271546     823.12423
Radio2             14.409344     362.42121
Newspaper2          4.245022      42.72045
TV_Radio           32.874808    2007.16767
TV_Newspaper       10.776622     241.29112
Radio_Newspaper     7.802870      98.00334
TV_Radio_Newspaper 12.411719     624.68726
```

```r
varImpPlot(rf_model)
```




```r
# RMSE on full dataset
rf_pred <- predict(rf_model, adver_rf)
rf_rmse <- sqrt(mean((adver_rf$Sales - rf_pred)^2))
cat("Random Forest RMSE:", round(rf_rmse, 4), "\n")
```

```
Random Forest RMSE: 0.3093
```

Comparison Table

Model	Test RMSE / RSE	% Variance Explained / R²	Key Findings
Three-Way LASSO Model	0.6339	Not explicitly available	Newspaper & higher-order terms shrunk to zero
Three-Way Random Forest	0.3093	98.31%	Strongest performance; all variables contribute nonlinearly

Interpretation:

- **Random Forest** outperforms the LASSO model in both **RMSE** and **variance explained**, suggesting it is better at capturing complex non-linear patterns.
- **LASSO** is valuable for **feature selection**, showing that many higher-order terms (e.g., Newspaper^2, Radio^2) may not meaningfully contribute to prediction.

5. Cross validation

```
# 5. Cross-Validation of Updated Models

# Load libraries
library(caret)
library(glmnet)
library(randomForest)
library(dplyr)

# Set seed and split the data
set.seed(232)
train_idx <- createDataPartition(adver$Sales, p = 0.7, list = FALSE)
train_data <- adver[train_idx, ]
test_data <- adver[-train_idx, ]

# 1. Full Model (Three-way)
lm_full <- lm(Sales ~ TV + Radio + Newspaper, data = train_data)
full_pred <- predict(lm_full, newdata = test_data)

# 2. Reduced Model (Two predictors only)
lm_reduced <- lm(Sales ~ TV + Radio, data = train_data)
reduced_pred <- predict(lm_reduced, newdata = test_data)

# 3. Three-Way Interaction Model
lm_interaction <- lm(Sales ~ TV * Radio * Newspaper, data = train_data)
interaction_pred <- predict(lm_interaction, newdata = test_data)

# 4. Three-Way Quadratic Model
lm_poly <- lm(Sales ~ (TV + Radio + Newspaper)^3 +
                I(TV^2) + I(Radio^2) + I(Newspaper^2), data = train_data)
poly_pred <- predict(lm_poly, newdata = test_data)
```

```r
# 5. LASSO Model (Three-way)
x_train <- model.matrix(Sales ~ (TV + Radio + Newspaper)^3 +
                        I(TV^2) + I(Radio^2) + I(Newspaper^2), data = train_data)[, -1]
x_test <- model.matrix(Sales ~ (TV + Radio + Newspaper)^3 +
                       I(TV^2) + I(Radio^2) + I(Newspaper^2), data = test_data)[, -1]
y_train <- train_data$Sales
y_test <- test_data$Sales

lasso_cv <- cv.glmnet(x_train, y_train, alpha = 1)
lasso_model <- glmnet(x_train, y_train, alpha = 1, lambda = lasso_cv$lambda.min)
lasso_pred <- predict(lasso_model, newx = x_test)

# 6. Random Forest Model (Three-way)
train_data_rf <- train_data %>%
  mutate(
    TV2 = TV^2, Radio2 = Radio^2, Newspaper2 = Newspaper^2,
    TV_Radio = TV * Radio,
    TV_Newspaper = TV * Newspaper,
    Radio_Newspaper = Radio * Newspaper,
    TV_Radio_Newspaper = TV * Radio * Newspaper
  )

test_data_rf <- test_data %>%
  mutate(
    TV2 = TV^2, Radio2 = Radio^2, Newspaper2 = Newspaper^2,
    TV_Radio = TV * Radio,
    TV_Newspaper = TV * Newspaper,
    Radio_Newspaper = Radio * Newspaper,
    TV_Radio_Newspaper = TV * Radio * Newspaper
  )

rf_model <- randomForest(Sales ~ TV + Radio + Newspaper + TV2 + Radio2 + Newspaper2 +
                           TV_Radio + TV_Newspaper + Radio_Newspaper + TV_Radio_Newspaper,
                         data = train_data_rf)
rf_pred <- predict(rf_model, newdata = test_data_rf)

# Evaluation function
metrics <- function(pred, actual) {
  data.frame(
    RMSE = RMSE(pred, actual),
    R2 = R2(pred, actual),
    MAE = MAE(pred, actual)
```

```r
  )
}

# Combine results
cv_results <- bind_rows(
  metrics(full_pred, test_data$Sales)        %>% mutate(Model = "Full"),
  metrics(reduced_pred, test_data$Sales)     %>% mutate(Model = "Reduced"),
  metrics(interaction_pred, test_data$Sales) %>% mutate(Model = "Three-Way Interaction"),
  metrics(poly_pred, test_data$Sales)        %>% mutate(Model = "Three-Way Quadratic"),
  metrics(lasso_pred, y_test)                %>% mutate(Model = "Three-Way LASSO"),
  metrics(rf_pred, test_data$Sales)          %>% mutate(Model = "Three-Way Random Forest")
)

# Sort by RMSE
cv_results <- cv_results %>% select(Model, everything()) %>% arrange(RMSE)
print(cv_results)
```

```
                    Model      RMSE        R2       MAE        s0
1         Three-Way LASSO 0.5537281        NA 0.4513335 0.9899741
2     Three-Way Quadratic 0.5865020 0.9885835 0.4699852        NA
3 Three-Way Random Forest 0.6830802 0.9840980 0.5009243        NA
4   Three-Way Interaction 0.8299621 0.9777968 0.7156569        NA
5                 Reduced 1.6154295 0.9074125 1.3035471        NA
6                    Full 1.6185074 0.9069533 1.3078619        NA
```

Cross-Validation Summary Interpretation

- **Three-Way LASSO** achieved the **lowest RMSE (0.554)** and **lowest MAE**, indicating it had the **best predictive accuracy** on the test set, even though R^2 is not directly available.
- **Three-Way Quadratic** model also performed exceptionally well (**RMSE = 0.587**, **R^2 = 0.989**), capturing nearly **99% of the variance**, with strong predictive capability.
- **Three-Way Random Forest** showed slightly higher RMSE (**0.683**) but still **explained 98.4% of the variance**, making it a solid non-parametric alternative.
- **Three-Way Interaction** model performed well but was **less accurate** (RMSE = 0.83) than other three-way models.
- **Reduced** and **Full** linear models showed the **poorest performance**, with **RMSEs above 1.6**, confirming that more complex structures (interactions and nonlinearity) are necessary for optimal prediction.

Conclusion:

Three-way LASSO and **quadratic models** provide the best balance of accuracy and generalization. Simpler linear models underfit the data.

Final Model Comparison Table

Model	Adjusted R^2	Original RMSE / RSE	Cross-Validated RMSE	AIC	Included Predictors	Strength
Full Model	0.8942	1.695	1.619	784.55	TV, Radio, Newspaper	Newspaper not significant
Reduced Model	0.8947	1.691	1.615	782.60	TV, Radio	Simpler, slightly better
Three-Way Interaction	0.9654	0.9695	0.830	565.02	TV, Radio, Newspaper, TV×Radio, TV×Newspaper, Radio×Newspaper, TV×Radio×Newspaper	Strong synergy effects; good general fit
Three-Way Quadratic	**0.9845**	**0.6487**	0.587	**407.15**	TV, Radio, Newspaper, All 2-way & 3-way interactions, TV^2, $Radio^2$, $Newspaper^2$	**Best overall performance**; highest R^2
Three-Way LASSO	N/A	1.624	**0.6339**	N/A	Selected from all three-way + quadratic terms	**Best predictive accuracy**; automatic variable selection
Three-Way Random Forest	N/A	1.490	0.676	N/A	All three-way + quadratic terms	Strong nonlinear modeling; good generalization

Summary:

- **Three-Way Quadratic** is the best **statistical model** — explains most variance, interpretable.
- **Three-Way LASSO** is the best **predictive model** — best CV RMSE, automatic feature selection.

Optimal Allocation from Interaction model

Why the Interaction Model Is Best for Allocation?

The **Interaction Model** is the most appropriate choice for advertising budget allocation because it provides a **simple and interpretable formula** that still captures synergy between TV and Radio. Unlike the **Three-Way Quadratic Model**, which includes many higher-order and interaction terms (making symbolic optimization nearly impossible), the interaction model allows us to derive a **closed-form solution** using Lagrange multipliers.

Additionally, **Three-Way LASSO** is optimized for predictive accuracy — not interpretation — and shrinks some terms to zero, making it unreliable for marginal effect analysis. In contrast, the Interaction Model provides a **mathematically manageable** structure that supports meaningful economic insights and precise allocation strategies.

We aim to maximize the sales function:

$$\text{Sales}(TV, Radio) = 6.586 + 0.01997 \cdot TV + 0.01928 \cdot Radio + 0.00115 \cdot TV \cdot Radio$$

Subject to the constraint:

$$TV + Radio = B$$

We construct the Lagrangian function:

$$\mathcal{L}(TV, Radio, \lambda) = 6.586 + 0.01997 \cdot TV + 0.01928 \cdot Radio + 0.00115 \cdot TV \cdot Radio - \lambda(TV + Radio - B)$$

Take the partial derivatives:

$$\frac{\partial \mathcal{L}}{\partial TV} = 0.01997 + 0.00115 \cdot Radio - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial Radio} = 0.01928 + 0.00115 \cdot TV - \lambda = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = TV + Radio - B = 0$$

Subtracting the first two equations to eliminate λ:

$$(0.01997 + 0.00115 \cdot Radio) - (0.01928 + 0.00115 \cdot TV) = 0$$

Simplifying:

$$0.00115 \cdot (Radio - TV) = -0.00069$$

So the optimal allocation condition is:

$$\boxed{Radio = TV - 0.60}$$

Substitute into the budget constraint $TV + Radio = B$:

$$TV + (TV - 0.60) = B \Rightarrow 2TV = B + 0.60 \Rightarrow TV = \frac{B + 0.60}{2}$$

$$Radio = \frac{B - 0.60}{2}$$

Full Conclusion

In this study, we analyzed the Advertisement Sales dataset to investigate how advertising expenditures across TV, Radio, and Newspaper channels influence product sales. Multiple linear regression analysis initially confirmed that TV and Radio advertising budgets have statistically significant positive effects on sales, while Newspaper advertising was not a significant predictor. We then systematically compared a series of increasingly complex models—including full and reduced linear models, an interaction model, a three-way quadratic model, LASSO regression, and Random Forest—while checking model assumptions such as linearity, normality, constant variance, and absence of multicollinearity. No serious violations or outliers were found, supporting the reliability of our inferences.

Among all models, the **Three-Way Quadratic Model** demonstrated the best statistical performance, achieving the highest adjusted R^2 of **0.9845** and the lowest residual standard error of **0.6487**, indicating it explained the greatest proportion of sales variation. In terms of predictive accuracy, **Three-Way LASSO** achieved the lowest cross-validated RMSE of **0.554**, making it the most effective for generalization. However, when it comes to **strategic resource allocation**, the **Interaction Model** was most appropriate due to its mathematical simplicity and interpretability. It captures the synergy between TV and Radio while enabling symbolic optimization via Lagrange multipliers.

From this model, we derived an updated optimal allocation rule:

$$\boxed{Radio = TV - 0.60}$$

This means that to maximize marginal sales gains, the Radio budget should be approximately **$600 less** than the TV budget. Given a total advertising budget B, the optimal split is:

$$TV = \frac{B + 0.60}{2}, Radio = \frac{B - 0.60}{2}$$

These results offer meaningful guidance for marketing strategists. While complex models like three-way LASSO and quadratic regression provide high predictive power, the interaction model enables clear and actionable decisions for budget planning.

Future research could incorporate additional variables such as seasonality, competitive effects, or demographic segmentation to enhance predictive granularity. Causal inference techniques, including A/B testing or time-series modeling, would further validate advertising impact. Ultimately, this analysis highlights that coordinated investment in TV and Radio—guided by evidence-based allocation rules—can significantly improve advertising effectiveness and sales outcomes.
