Impact of Media Advertising on Sales Performance: A Comprehensive Analysis Across TV, Radio, and Newspaper Channels

Authors: David Teng, Hao Shen

Date: May 1, 2025

---

1. Introduction

Effective allocation of advertising budgets across different media platforms is a critical factor influencing product sales. In this study, we aim to analyze the impact of advertising expenditures on TV, Radio, and Newspaper channels on product sales using statistical modeling. The primary research questions guiding this analysis are: (1) Which advertising channels significantly contribute to increased sales? (2) What model best captures the relationship between advertising investments and sales performance? (3) How should advertisers allocate their budgets across media channels to maximize sales outcomes? Using multiple linear regression and advanced modeling techniques, we seek to answer these questions and provide actionable insights for marketing strategies.

2. Data Description

The dataset used in this analysis is the "Advertisement Sales Dataset," publicly available on Kaggle. It consists of 200 observations and five variables. The response variable is **Sales** (in thousands of units). The predictor variables are **TV** (advertising expenditure in thousands of dollars), **Radio** (advertising expenditure in thousands of dollars), and **Newspaper** (advertising expenditure in thousands of dollars). Each row represents an independent observation.

Summary Statistics:

The following is a summary of the descriptive statistics for the current study. The main findings include that TV advertising budgets range from \$0.7k to \$296.4k while radio advertising budgets range from \$0k to \$49.6k, and newspaper advertising budgets range from \$0.3k to \$114k. As for sales, they range from 1.6k to 27k units across the three different categories.

Exploratory Data Analysis:

Scatterplots reveal a strong positive linear relationship between Sales and TV advertising expenditures, indicating that higher investments in TV campaigns are generally associated with increased Sales. A moderate positive relationship is also observed between Sales and Radio advertising expenditures, albeit with more spread in the data. In contrast, the relationship between Newspaper advertising and Sales appears weaker and less consistent. See Figure 1.

Boxplots further illustrate these patterns, showing that Newspaper advertising budgets exhibit greater variability and include several high-end outliers, whereas TV and Radio budgets are more tightly and consistently distributed. These exploratory findings suggest that TV and Radio are likely to be more reliable predictors of Sales in comparison to Newspaper advertising, motivating their prioritization in subsequent modeling. See Figure 2 below.

Figure 1: Pairwise Scatterplot of Sales and Advertising Channels

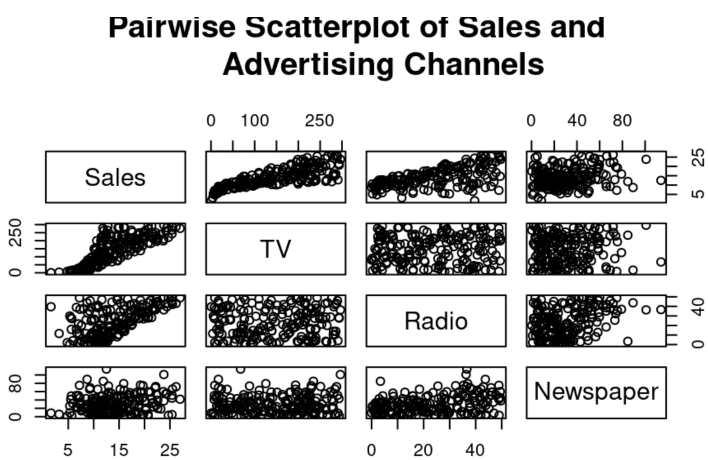
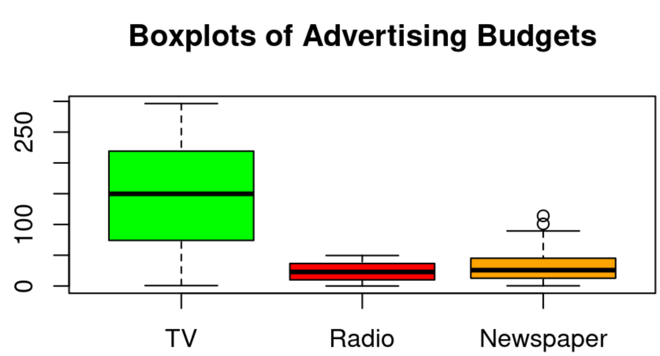


Figure 2: Boxplots of Advertising Budgets



3. Methods and Results

3.1. Base Multiple Linear Regression

A multiple linear regression model was fitted with Sales as the response and TV, Radio, and Newspaper as predictors. Both TV and Radio had significant positive coefficients ($p < 0.001$), while the coefficient for Newspaper did not show statistical significance ($p = 0.822$). The model achieved an Adjusted R^2 of 0.894, indicating that about 89.4% of the variance in Sales was explained.

3.2. Model Simplification

Given the insignificance of the results for Newspaper, a reduced model excluding Newspaper was fitted. The Partial F-test confirmed that the exclusion of Newspaper did not significantly worsen the model fit ($p = 0.822$). In fact, the reduced model had an Adjusted R^2 of 0.895, which was slightly higher than the full model, signifying that it is a more accurate model.

3.3. Model Assumption Verification

Diagnostic plots (Residuals vs. Fitted, Q-Q plot, Scale-Location plot, Residuals vs. Leverage) confirmed the assumptions of linearity, normality of residuals, and homoscedasticity. Variance Inflation Factors (VIFs) for TV and Radio were both approximately 1.003, confirming no multicollinearity. Box-Cox transformation analysis indicated that no transformation of the response variable was necessary.

3.4. Quadratic Model

A quadratic regression model was fitted to explore potential non-linear relationships. The squared term for TV (TV^2) was highly significant ($p < 0.001$), indicating diminishing returns to TV advertising. However, the squared term for Radio was not significant ($p = 0.131$). The polynomial model improved Adjusted R^2 to 0.915.

3.5. Interaction Model (TV × Radio)

An interaction model including a TV × Radio term was fitted. The interaction term was highly significant ($p < 0.001$), and the model achieved the highest Adjusted R^2 of 0.965, substantially improving the model fit. This suggests a synergistic effect where the impact of TV advertising is amplified when Radio advertising expenditure is higher, and vice versa.

3.6. LASSO Regression

LASSO regression confirmed the insignificance of Newspaper by shrinking its coefficient to zero. However, the RMSE of the LASSO model (1.624) was higher than the interaction model. Random Forest regression achieved an approximate RMSE of 1.49 and explained 91.8% of the variance, but it did not outperform the interaction model.

3.7. Random Forest

A Random Forest regression model was also fitted using TV, Radio, and Newspaper as predictors. The Random Forest model achieved approximately 91.8% variance explained and an RMSE of about 1.49. The importance plots showed that TV and Radio were the most important predictors, while Newspaper had minimal influence. Although the Random Forest model captured non-linear patterns, its performance was slightly inferior to the interaction model based on RMSE and variance explained.

3.8. Cross-Validation

To evaluate the generalization performance of all six models, we conducted 70/30 cross-validation using RMSE, R^2, and MAE as performance metrics. The interaction model achieved the lowest cross-validated RMSE (0.799) and the highest test R^2 (0.979), reinforcing its superior predictive capability. Random Forest performed reasonably well (RMSE = 1.374), followed by the polynomial model (RMSE = 1.477), LASSO (RMSE = 1.606), and the reduced and full models (RMSE = 1.615 and 1.619, respectively). These results confirm that the interaction model not only fits the training data best but also generalizes most effectively to unseen data. See Table 1 for a summary of the results.

Table 1: Final Model Comparison

Model	**Adjusted R^2**	**Original RMSE / RSE**	**Cross-Validated RMSE**	**Note**
Full (TV + Radio + Newspaper)	0.894	1.695	1.619	Newspaper not significant
Reduced (TV + Radio)	0.895	1.691	1.615	Simpler, slightly better

Interaction (TV * Radio)	0.965	0.972	**0.799**	**Best fit overall**
Quadratic (TV + TV^2 + Radio + $Radio^2$)	0.915	1.518	1.477	TV^2 significant, $Radio^2$ not
LASSO Regression	N/A	1.624	1.606	Good for variable selection
Random Forest	N/A	1.490	1.374	Strong, non-linear, but not best

3.9. Optimal Budget Allocation

To determine the most effective allocation of a fixed advertising budget between TV and Radio, we applied marginal analysis through a constrained optimization approach using the method of Lagrange multipliers. This allowed us to derive a condition under which marginal returns from both channels are balanced, thereby maximizing predicted sales.

The interaction model indicated that the optimal condition is met when the Radio budget exceeds the TV budget by approximately $8.72k. This relationship is expressed as:

$$\text{Radio} = \text{TV} + 8.72$$

Given a total budget B, this translates to the following closed-form solution:

$$\text{TV} = (B - 8.72) / 2 \text{ and } \text{Radio} = (B + 8.72) / 2$$

This result offers a practical guideline for marketers: to maximize predicted sales under the interaction model, Radio advertising should be allocated exactly $8.72k more than TV, regardless of the overall budget size.

4. Conclusion

This study comprehensively analyzed the impact of advertising expenditures on Sales across TV, Radio, and Newspaper channels. TV and Radio advertising were found to be significant contributors to Sales, while Newspaper advertising was not. Model diagnostics confirmed that the reduced model excluding Newspaper was appropriate. Furthermore, a polynomial model indicated diminishing returns for TV advertising. The interaction model, including a TV × Radio term, provided the best model fit with an Adjusted R^2 of 0.965.

To assess predictive performance, we conducted a 70/30 train-test cross-validation for all models. The interaction model again achieved the best results, with the lowest cross-validated RMSE (0.799) and highest test R^2 (0.979), reinforcing its generalizability and robustness compared to the other models.

While these findings provide valuable guidance for strategic advertising budget allocation, limitations include the lack of time-series data, absence of control for external factors (e.g., market competition), and reliance on observational rather than experimental data. Future work should incorporate longitudinal data, additional covariates, and causal inference techniques to further validate and refine these findings.

Ultimately, this analysis underscores that optimizing the balance between TV and Radio advertising can significantly enhance sales performance, with a suggested optimal allocation in which the Radio budget should be approximately $8.72k higher than the TV budget, based on marginal effect analysis.

5. Code Appendix

All R scripts used in the data analysis and model building are publicly available at the following GitHub repository:
https://github.com/David-W-Teng/STAT-632-Linear-and-Logistic-Regression

---

End of Paper