Thick Lenses

BACKGROUND

You are given a sphere and two sizes of hemispheres made of a clear plastic. You have to determine n: the index of refraction of the plastic. In this experiment, we assume that all the objects are made from the same material, and the radii of the larger hemisphere and the sphere are the same.

Two refracting surfaces having a common axis form a simple lens. A ray passing through a simple lens undergoes refraction at both surfaces, except when it is incident normally on one of the surfaces. For a spherical interface of radius R as shown below, the relationship between object distance s and image distance s' is given by:

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{R} \tag{1}$$

In Figure 1, as the light travels from left to right: R is positive for a convex surface, and negative for a concave surface.

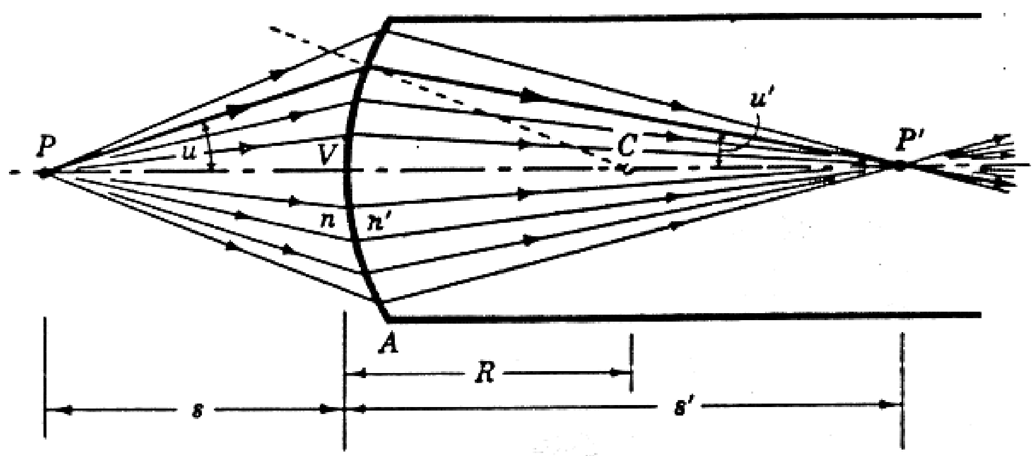

Figure 1: Refraction at a spherical surface. V is the vertex

Refraction by a thick lens

The general problem of refraction by a thick lens is solved by applying the equation for the refraction at one surface to each surface in turn. The image formed by the first surface is treated as an object for the second surface. By using this approach, you can easily obtain the formulae for the focal distances f for both hemisphere and sphere in terms of R and n. As an alternative, you can try to derive these equations using ray tracing. Assume that the angles are small so that $sin\theta \approx \theta$.

The first focal point of a lens is defined as the object point on the lens axis which is imaged by the lens at infinity. Rays diverging from the first focal point are parallel to the axis of the lens after refraction as seen in Figure 2(a):

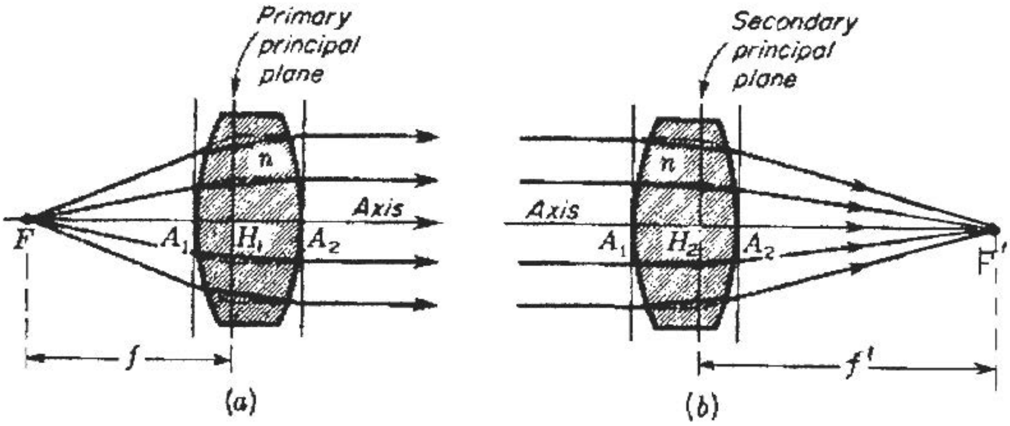

Figure 2: Primary and secondary principal planes of a thick lens. A1 and A2 are the vertices.

The second focal point of a lens is defined as the image point of an infinitely distant point object on the axis lens. Rays incident on the lens and parallel to the lens axis pass through the second focal point after the refraction (Figure 2 (b)).
Computation of the position of the image of a given object at a given distance from the thick lens is facilitated by determining the positions of two planes inside the lens, known as the principal planes. They are defined as following:
- **First principal plane:** For a cone of rays diverging from the first focal point, when the incident and emergent rays are projected ahead and back, their points of intersection lie in a common plane known as the first (or primary) principal plane (Figure 2(a)) . The intersection of the first principal plane and the axis of the lens is the first principal point H_1. The two deviations at the two surfaces are equivalent to a single deviation in the first principal plane.
- **Second principal plane**: For a bundle of rays incident on a lens parallel to its axis, when the incident and emergent rays are projected ahead and back, the points of intersection lie in a common plane known as the second (or secondary) principal plane (Figure 2(b)).
The intersection of the second principal plane and the axis of the lens is the second principal point H_2. The distance from the first focal point F to the first principal point H_1 is the first focal distance f, and the distance from the second principal point H_2 to the second focal point F' is the second focal distance f'. However, if the medium on both sides of the lens has the same index of refraction, which is the case if the lens is in air, the two focal lengths are equal.
The equation derived for a thin lens and relating two conjugated points is:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \tag{2}$$

For the thick lens, s_o is the distance between the object and the first principal plane, and s_i is the distance between the second principal plane and the image.
While applying this equation to the thick lens, remember that the measurable values are the distances from the object to the vertex and from the image to the vertex (see Figure 3, below).

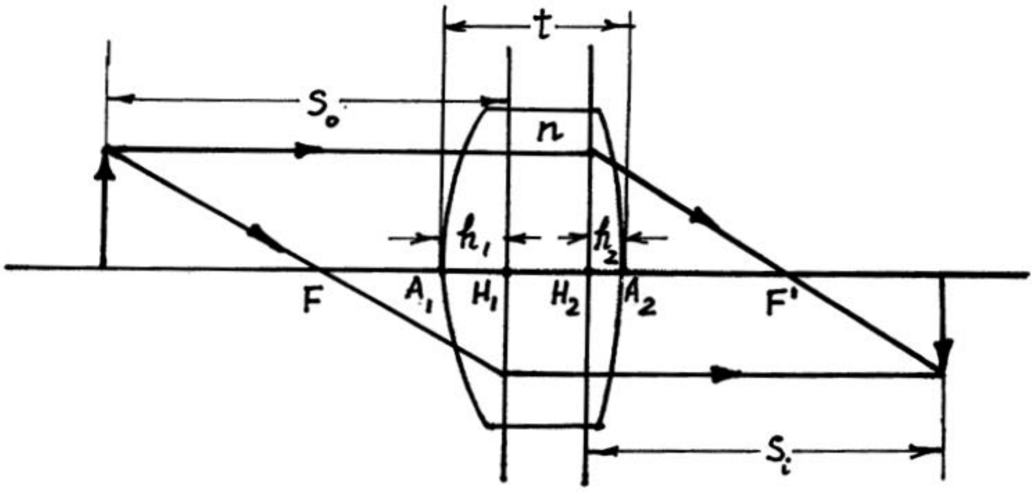

Figure 3: Image formation by a thick lens. Lens thickness is: $t = A_1A_2$.

EXERCISES

Note on distance measurements: In order to measure distances along the optical track, youll use the calibrated rod provided. It has 20.02 ± 0.01 cm. Figure out how to use the calibrated rod, together with positions of the holders on the track. You will not receive any other instrument (ruler, caliper or else).

1. **Principal planes**.
   To Do Sketch the locations of the second and first principal planes for the hemisphere when it is placed with its flat surface facing the light.
   Q1 Where are these planes located if the hemisphere is facing the light with its curved side?
   To Do Sketch the locations of the principal planes for the sphere.

2. **Focal distances**.
   Measure focal distances of both hemispheres (for both orientations) and that of the sphere. If you were allowed to measure the radius of the hemisphere or sphere, you could easily find the index of refraction from a single measurement of f. However, using the objects that are given to you, you can find n even without measuring R!
   Figure 3 shows the image formation by a thick lens. One of the ways to measure the focal distance is to find the arrangement when you observe the inverted real image of the same size as the object (magnification is $m = -1$).
   It follows from (2) that this situation takes place when $s_o = s_i = 2f$.
   Note that the measurable values are the distances from the object to the vertex and from the image to the vertex. There are other ways to measure the focal distance.
   Optional You can explore them as an optional exercise after you have completed the mandatory parts.

3. **Index of refraction**.
   To Do Using the Lens Makers Equation (3) and the appropriate sign for radii R_1 and R_2, determine the formulae for the focal distance of the hemisphere and the sphere in terms of R and n. Once you have these equations, you should be able to find n from the measurements of the focal distances for the sphere and the hemisphere of the same radius.

4. **Oliver's Law**: A student has demonstrated that the index of refraction can be found from two measurements (for the hemisphere facing light with its flat side and for the reversed hemisphere) without knowledge of R! This is "Oliver's Law", (although Oliver wanted to reject it as "obviously wrong").
To Do Try to determine n from the measurements using only one hemispherical lens.

Useful formulae:

- **The Lens Maker's Equation:**

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)t}{nR_1R_2}\right) \tag{3}$$

- **The focal distance of a ball (spherical) lens**:

$$f_{ball} = \frac{nR}{2(n-1)} \tag{4}$$

- **Distances from Figure 3**: A_1H_1 distance is h_1, H_2A_2 distance is h_2.

$$h_1 = -\frac{f(n-1)t}{nR_2} \tag{5}$$

$$h_2 = -\frac{f(n-1)t}{nR_1} \tag{6}$$

In equations (3-6), R_1 and R_2 are the radii of the first and the second surfaces, h_1 and h_2 are the distances from the principal planes to the corresponding verticess (see Figure 3), and $t = A_1A_2$ is the axial thickness of the lens (the distance between the vertices).

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