TMME50 Assignment IV

David Wiman
20000120-8495

December 1, 2023

Instructions 2023 for reporting the computer assignments

The computer assignments are reported in writing, and submitted *printed on paper*. The assignments are performed individually. The use of ChatGPT or any similar system is not allowed. It is permissible to discuss the assignments and to show parts of solutions in that context, but *copying of Matlab code or sections of reports is not allowed.* Further, it is not allowed to possess copies of other students reports or Matlab code, either electronically or on paper, or to supply this to another student; this also means that you hand in and pick up your assignments yourself, not with the help of a friend. The reports shall contain:

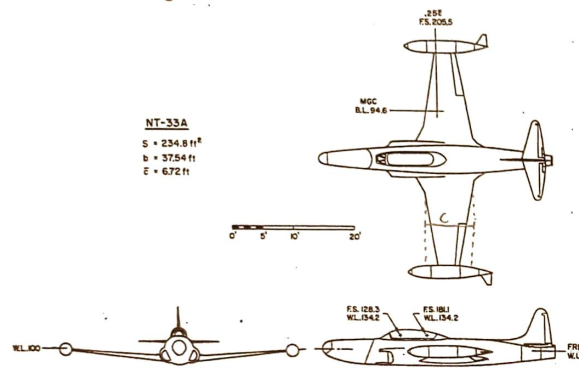
- *A copy of this page of instructions.*
- Name and complete (10 digits) civic registration number of the student (sometimes called p-number among exchange students).
- *Which aeroplane and which reference condition* that has been used. Specify the number of the column on the data sheet that has been used.
- Answers to all the questions appearing under the headings "Assignment I:a" etc. and all requested plots.
- A complete set of Matlab files for each computer assignment. Choose the most complete set, such as the one for part I:c in assignment I. In assignment II, also also include root loci and a graphic representation of your Simulink model for the final version of your model with all numerical values shown explicitly.
- The ODE system implemented in assignments I, III, IV and V must be given in the report in the order actually implemented and written in a *single* frame containing *all* the equations of the ODE and *nothing* else.

Further, note:

- With the exeption of flying qualities tables, illustrations from the lab-PM defining the problem and this page of instructions, no copying of text, figures, equations or code from another document is allowed (unless it is a document you have created yourself).
- It must be clear what data has been used in what way. Data is converted from American to SI units, and this should be done in a way that can be followed in detail either in the text of the report or in the Matlab files, so that misstakes can be found at a glance without making any calculations.
- Nothing written in Matlab syntax or pseudocode is allowed in the main text of the reports: your Matlab code is appended at the end of the report.
- The report must contain sufficient text and illustrations such that it is possible to understand without ever having seen the lab-PM.
- Use the simulation time given in the assignments. For a small number of datasets it is necessary to use a longer simulation time than 100 s in order to to see a full phugoid period, but the time should never be shorter than the time given and never longer than 400 s.

T-33 Shooting Star

David Wiman



	1	2	3	4	5	6	7	8
X_u s^{-1}	−0,0391	−0,00484	−0,0104	−0,0415	0,00477	−0,00735	−0,0511	−0,0355
X_α ft/s^2	18,58	35,37	25,12	−16,50	20,43	22,29	7,67	24,59
Z_u s^{-1}	−0,248	−0,153	−0,128	−0,162	−0,114	−0,107	−0,0703	−0,0766
Z_α ft/s^2	−213,41	−267,57	−773,31	−2807	−140,26	−712,5	−1400	−437,8
M_u $ft^{-1}s^{-1}$	0,000318	0,000603	0,000283	−0,00076	0,000114	−0,000183	−0,00151	−0,000183
M_α s^{-2}	−1,89	−1,81	−9,21	−33,7	−0,23	−9,95	−18,59	−5,42
$M_{\dot\alpha}$ s^{-1}	−0,35	−0,40	−0,63	0	−0,24	−0,31	−0,16	−0,06
M_q s^{-1}	−0,694	−0,806	−1,37	−2,80	−0,50	−0,981	−1,56	−0,535
X_{δ_e} ft/s^2	0,516	1,47	0,62	−2,65	1,88	0,50	−0,432	0,996
Z_{δ_e} ft/s^2	−13,4	−16,2	−44,4	−152	−11,3	−40,9	−82,4	−23,8
M_{δ_e} s^{-2}	−4,19	−5,83	−16,0	−52,7	−4,13	−14,2	−28,7	−8,28

h
V, u_0
m
I_x
I_y
I_z
I_{xz}
Q

surface, span, chord

$S = 234{,}8\ ft^2$, $b = 37{,}54\ ft$, $\bar{c} = 6{,}72\ ft$

	1	2	3	4	5	6	7	8
h ft	S/L	S/L	S/L	S/L	20000	20000	20000	40000
$V(u_0)$ ft/s	228	270	447	782	311	570	778	629
m slug	367	426	426	426	426	426	426	426
I_x $slug \cdot ft^2$	12700	23801	23801	23801	23801	23801	23801	23801
I_y $slug \cdot ft^2$	20700	21101	21101	21101	21101	21101	21101	21101
I_z $slug \cdot ft^2$	32001	43802	43802	43802	43802	43802	43802	43802
I_{xz} $slug \cdot ft^2$	480	480	480	480	480	480	480	480
Q lb/ft^2	61,7	86,7	247	726	61,3	206	383	117

	1	2	3	4	5	6	7	8
Y_β ft/s^2	−28,4	−30,1	−81,0	−264	−21,6	−72,7	−144	−42,4
L_β s^{-2}	−5,49	−4,72	−8,02	−18,0	−4,06	−7,42	−9,89	−5,08
L_p s^{-1}	−2,03	−1,32	−2,15	−4,51	−0,82	−1,56	−2,23	−0,877
L_r s^{-1}	0,641	0,305	0,320	0,495	0,214	0,256	0,328	0,179
N_β s^{-2}	0,667	0,94	2,71	10,6	0,54	2,60	6,24	1,68
N_p s^{-1}	−0,116	−0,112	−0,0512	0,0118	−0,103	−0,0383	−0,0141	−0,0428
N_r s^{-1}	−0,207	−0,173	−0,291	−0,561	−0,104	−0,204	−0,318	−0,110
Y_{δ_r} ft/s^2	0,0295	0,0301	0,0503	0,102	0,0185	0,0363	0,0571	0,0195
L_{δ_r} s^{-2}	−0,0125	0,443	1,57	5,89	0,287	1,39	3,20	0,408
N_{δ_r} s^{-2}	−1,24	−1,25	−3,50	−12,6	−0,883	−3,21	−6,99	−1,92
L_{δ_a} s^{-2}	6,01	4,53	12,6	47,0	3,14	11,7	24,0	7,13
N_{δ_a} s^{-1}	0,0286	0,134	0,165	0,260	0,164	0,121	0,195	0,118

Background

The longitudinal linearized equations of motion can be written, for constant thrust and $\theta_0 = 0$:

$$\begin{bmatrix} \Delta\dot{u} \\ \Delta\dot{w} \\ \Delta\dot{q} \\ \Delta\dot{\theta} \end{bmatrix} = \begin{bmatrix} X_u & X_w & 0 & -g \\ Z_u & Z_w & u_0 & 0 \\ M_u + M_{\dot{w}} Z_u & M_w + M_{\dot{w}} Z_w & M_q + M_{\dot{w}} u_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} + \begin{bmatrix} 0 \\ Z_{\delta_e} \\ M_{\delta_e} + M_{\dot{w}} Z_{\delta_e} \\ 0 \end{bmatrix} \Delta\delta_e \tag{1}$$

where, as in previous exercises, $u_0 = V$ taken from the data sheet. The data sheet for the tasks at hand were given in imperial units and must therefore be converted to SI units before use. One of the values given were:

$$V = 228 \text{ ft/s} \cdot 0.3048 = 69.4944 \text{ m/s}. \tag{2}$$

I use data from a T-33 Shooting Star at sea level flight (column one on the data sheet).

The lateral linearized equations of motion are, for constant thrust, $\theta_=0$ and with the simplification $I_{xz} = 0$:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{p} \\ \Delta\dot{r} \\ \Delta\dot{\varphi} \end{bmatrix} = \begin{bmatrix} Y_\beta/u_0 & Y_p/u_0 & -(1 - Y_r/u_0) & g/u_0 \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\varphi \end{bmatrix} + \begin{bmatrix} 0 & Y_{\delta_r}/u_0 \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta\delta_a \\ \Delta\delta_r \end{bmatrix} \tag{3}$$

IV:a

The homogeneous solution to the linearized equations can be written as

$$x(t) = C_1 e^{\lambda_1 t} v_1 + C_2 e^{\lambda_2 t} v_2 + \dots C_N e^{\lambda_N t} v_N \tag{4}$$

where C_i are constants depending on the initial conditions, λ_i and v_i are eigenvalues and eigenvectors of the dynamics matrix and N is the number of eigenvalues.

Using Matlab to compute the eigenvalues and eigenvectors of both system of equations (ignoring initial conditions), we get the resulting functions of time:

$$\begin{aligned} \begin{bmatrix} \Delta u \\ \Delta w \\ \Delta q \\ \Delta\theta \end{bmatrix} =& C_1 \begin{bmatrix} -0.0356 + 0.0214i \\ -0.9989 \\ 0.0007 - 0.0179i \\ -0.0090 + 0.0066i \end{bmatrix} e^{(-0.9957+1.2494i)t} + C_2 \begin{bmatrix} -0.0356 - 0.0214i \\ -0.9989 \\ 0.0007 + 0.0179i \\ -0.0090 - 0.0066i \end{bmatrix} e^{(-0.9957-1.2494i)t} + \\ & C_3 \begin{bmatrix} -0.9990 \\ 0.0403 + 0.0117i \\ -0.0031 + 0.0003i \\ 0.0029 + 0.0176i \end{bmatrix} e^{(-0.0139+0.1717i)t} + C_4 \begin{bmatrix} -0.9990 \\ 0.0403 - 0.0117i \\ -0.0031 - 0.0003i \\ 0.0029 - 0.0176i \end{bmatrix} e^{(-0.0139-0.1717i)t} \end{aligned} \tag{5}$$

as well as

$$\begin{aligned} \begin{bmatrix} \Delta\beta \\ \Delta p \\ \Delta r \\ \Delta\varphi \end{bmatrix} =& C_1 \begin{bmatrix} -0.0439 \\ -0.9135 \\ -0.0372 \\ 0.4028 \end{bmatrix} e^{-2.2677t} + C_2 \begin{bmatrix} -0.2631 - 0.1138i \\ 0.6887 \\ -0.1031 + 0.2142i \\ -0.0163 - 0.6219i \end{bmatrix} e^{(-0.0289+1.1065i)t} + \\ & C_3 \begin{bmatrix} -0.2631 + 0.1138i \\ 0.6887 \\ -0.1031 - 0.2142i \\ -0.0163 + 0.6219i \end{bmatrix} e^{(-0.0289-1.1065i)t} + C_4 \begin{bmatrix} 0.0290 \\ -0.0356 \\ 0.1371 \\ 0.9895 \end{bmatrix} e^{-0.0360t} \end{aligned} \tag{6}$$

IV:b

By looking at Figure 2 and comparing to the previously computed eigenvalues, we can characterize the flying qualities of our aeroplane in different modes. However, we must first determine what class and category our aeroplane belongs to. The descriptions can be seen in Figure 1

TABLE 4.7
Classification of airplanes

Class I	Small, light airplanes, such as light utility, primary trainer, and light observation craft
Class II	Medium-weight, low-to-medium maneuverability airplanes, such as heavy utility/search and rescue, light or medium transport/cargo/tanker, reconnaissance, tactical bomber, heavy attack and trainer for Class *II*
Class III	Large, heavy, low-to-medium maneuverability airplanes, such as heavy transport/cargo/tanker, heavy bomber and trainer for Class III
Class IV	High-maneuverability airplanes, such as fighter/interceptor, attack, tactical reconnaissance, observation and trainer for Class IV

(a)

TABLE 4.8
Flight phase categories

Nonterminal flight phase	
Category A	Nonterminal flight phase that require rapid maneuvering, precision tracking, or precise flight-path control. Included in the category are air-to-air combat ground attack, weapon delivery/launch, aerial recovery, reconnaissance, in-flight refueling (receiver), terrain-following, antisubmarine search, and close-formation flying
Category B	Nonterminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in the category are climb, cruise, loiter, in-flight refueling (tanker), descent, emergency descent, emergency deceleration, and aerial delivery.
Terminal flight phases	
Category C	Terminal flight phases are normally accomplished using gradual maneuvers and usually require accurate flight-path control. Included in this category are takeoff, catapult takeoff, approach, wave-off/go-around and landing.

(b)

Figure 1: Table for determining class and category.

The T-33 Shooting Star belongs to class I due to its small size (37.54 ft = 11.44 m wingspan) and low weight (367 slug = 5356 kg). The "T" in T-33 also suggests that it is a trainer aeroplane, which is in the definition of class I.

As for category, our aeroplane in the current flying conditions (sea level flight at 100 m) belongs to category A. An examples of category A activities are terrain-following, which is necessary at 100 m altitude. Other examples of category A are anti-submarine search and A2G weapon deployment which can both require low altitude, low speed (228 ft/s = 69.49 m/s = 135 kt) flying.

The eigenvalues can be written as

$$\lambda = a + bi = -\zeta\omega \pm i\omega\sqrt{1-\zeta^2}$$

From this we get

$$\begin{aligned} a &= -\zeta\omega \\ b &= \omega\sqrt{1-\zeta^2} \end{aligned} \tag{7}$$

$$\begin{aligned} \zeta &= \frac{-a}{\omega} \\ b &= \omega\sqrt{1-\frac{a^2}{\omega^2}} \\ b^2 &= \omega^2(1-\frac{a^2}{\omega^2}) \\ b^2 + a^2 &= \omega^2 \\ \omega &= \sqrt{a^2+b^2} \\ \zeta &= \frac{-a}{\sqrt{a^2+b^2}} \end{aligned} \tag{8}$$

For longitudinal motion, we study two modes.

- In the phugoid mode, our aeroplane has the eigenvalues -0.0139 ± 0.1717i, which give it $\zeta = 0.0807$ and $\omega = 0.1723$. This constitutes level 1 flying qualities.

- In the short-period mode, our aeroplane has the eigenvalues -0.9957 ± 1.2494i, which give it $\zeta = 0.6232$ and $\omega = 1.5976$. This constitutes level 1 flying qualities.

For lateral motion, we study three modes.

- In the roll mode, our aeroplane has the eigenvalue -2.2677 which gives us

$$T_r = \frac{-1}{-2.2677} = 0.44.$$

  This in turn give us level 1 flying qualities.

- In the Dutch roll mode, our aeroplane has the eigenvalues -0.0289 ± 1.1065i which gives it $\zeta = 0.0261$ and $\omega = 1.1069$ and therefore $\zeta\omega = 0.0289$. This constitutes level 3 flying qualities.

- In the spiral mode, our aeroplane has the eigenvalue -0.0360 which gives us

$$T_2 = \frac{\ln(2)}{-0.0360} = -19.25.$$

  Since T_2 is negative, our aeroplane is stable and has level 1 flying qualities. If we had the same eigenvalue but with flipped sign, T_2 could be interpreted as "time to double amplitude" with a value of 19.25 s which would cause an unstable spiral mode. The flying qualities would still however belong to level 1 due to the high value.

TABLE 4.10
Longitudinal flying qualities

Phugoid mode	
Level 1	$\zeta > 0.04$
Level 2	$\zeta > 0$
Level 3	$T_2 > 55$ s

Short-period mode				
	Categories A and C		Category B	
	ζ_{sp}	ζ_{sp}	ζ_{sp}	ζ_{sp}
Level	min	max	min	max
1	0.35	1.30	0.3	2.0
2	0.25	2.00	0.2	2.0
3	0.15	—	0.15	—

(a)

TABLE 5.4
Spiral mode (minimum time to double amplitude) flying qualities

Class	Category	Level 1	Level 2	Level 3
I and IV	A	12 s	12 s	4 s
	B and C	20 s	12 s	4 s
II and III	All	20 s	12 s	4 s

TABLE 5.5
Roll mode (maximum roll time constant) flying qualities (in seconds)

Class	Category	Level 1	Level 2	Level 3
I, IV	A	1.0	1.4	10
II, III		1.4	3.0	
All	B	1.4	3.0	10
I, IV	C	1.0	1.4	10
II, III		1.4	3.0	

(b)

TABLE 5.6
Dutch roll flying qualities

Level	Category	Class	Min ζ*	Min $\zeta\omega_n$,* rad/s	Min ω_n, rad/s
1	A	I, IV	0.19	0.35	1.0
		II, III	0.19	0.35	0.4
	B	All	0.08	0.15	0.4
	C	I, II-*C* IV	0.08	0.15	1.0
		II-*L*, III	0.08	0.15	0.4
2	All	All	0.02	0.05	0.4
3	All	All	0.02	—	0.4

(c)

Figure 2: Tables from Nelson.

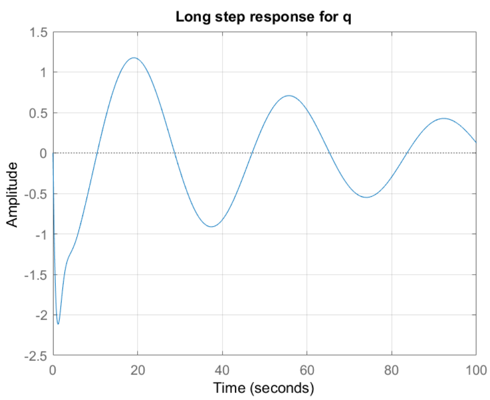
IV:c

From (1), we can determine the transfer function from $\Delta\tilde{\delta}_e$ to $\tilde{q}$.

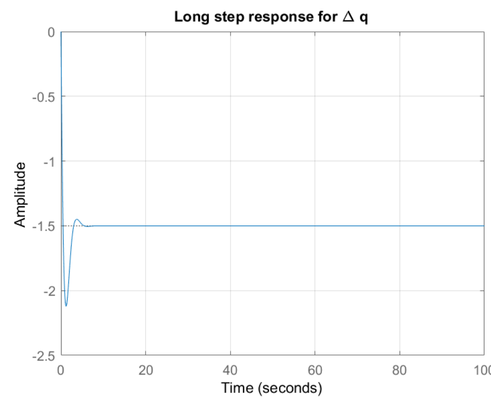
$$\tilde{q} = \frac{-4.169s^3 - 3.974s^2 - 0.234s}{s^4 + 2.019s^3 + 2.637s^2 + 0.1299s + 0.07575}\Delta\tilde{\delta}_e \tag{9}$$

We can then compare the step response of this transfer function to the step response computed in assignment II:b.

There is a big difference in the two step responses seen in Figure 3. The one from assignment II:b has a sharp oscillation in the beginning but then levels out. This is because it uses a model based on the short-period mode. The step response from this assignment behaves similarly in the beginning but then continues to oscillate. This is because it is based on a model better suited for long simulation times including the phugoid mode.



(a) Step response for q.



(b) Step response for q from assignment II:b

Figure 3

IV:d

We can also determine the flying qualities of our aeroplane by looking at the CAP criterion, see Figure 4.

First, we look back at ω and ζ from the short-period mode, $\omega_{n,sp} = 1.5976$ and $\zeta_{sp} = 0.6232$.

Secondly, we look at (9) and cast the transfer function onto the form

$$G_q = \frac{K_\theta s(s + \frac{1}{T_{\theta_1}})(s + \frac{1}{T_{\theta_2}})}{(s^2 + 2s\zeta_{sp}\omega_{n,sp} + \omega_{n,sp}^2)(s^2 + 2s\zeta_p\omega_{n,p} + \omega_{n,p}^2)}$$

to identify the constant T_{θ_2}. If we factorize the numerator of (9), we get the zeros: $s = 0, s = -0.89$ and $s = -0.0631$. This means, since $T_{\theta 2}$ is smaller than $T_{\theta 1}$, that $T_{\theta 2} = 1.1236$.

With $\omega_{n,sp}$ and T_{θ_2}, we can determine the CAP by

$$\text{CAP} = \frac{\omega_{n,sp}^2 g T_{\theta_2}}{V} = 0.4048$$

Finally, with $\zeta_{sp} = 0.6232$ and CAP $= 0.4048$, we can see that our aeroplane has level 1 flying qualities.

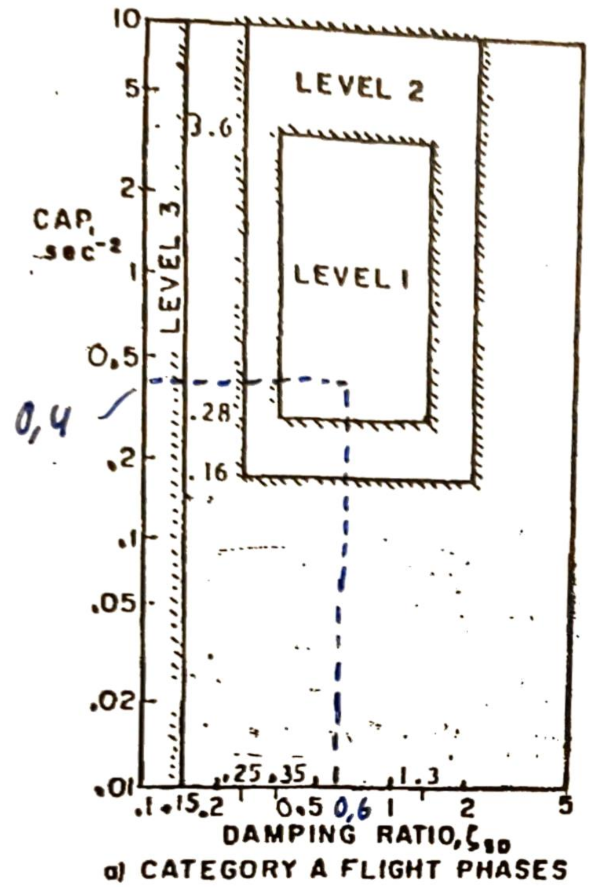


Figure 4: The CAP criterion for category A.

1 Matlab code

1.1 Assignment IV:a

```
% Set variables
run('T33_Shooting_Star_parameters.m');

% Longitudinal equations
long_eqs = [Xu , Xw , 0 , -g ;
            Zu , Zw , v , 0 ;
            Mu + Mw_dot*Zu , Mw + Mw_dot*Zw , Mq + Mw_dot*v , 0 ;
            0 , 0 , 1 , 0];

% Lateral equations
lat_eqs = [Ybeta/v , Yp/v , -(1-Yr/v) , g/v ;
           Lbeta , Lp , Lr , 0 ;
           Nbeta , Np , Nr , 0 ;
           0 , 1 , 0 , 0];

[long_eig_vectors, long_eig_values] = eig(long_eqs);
[lat_eig_vectors, lat_eig_values] = eig(lat_eqs);
```

1.2 Assignment IV:c

```
% Set variables
run('T33_Shooting_Star_parameters.m');

% Longitudinal equations
long_eqs = [Xu , Xw , 0 , -g ;
            Zu , Zw , v , 0 ;
            Mu + Mw_dot*Zu , Mw + Mw_dot*Zw , Mq + Mw_dot*v , 0 ;
            0 , 0 , 1 , 0];

linear_terms = [0;
                Zdelta_e;
                Mdelta_e + Mw_dot*Zdelta_e;
                0];

Gsys = ss(long_eqs, linear_terms, eye(4), 0);
G = minreal(tf(Gsys));
G_3 = G(3);

step(G_3, 100)
title('Long step response for q')
```

1.3 Unit conversion=

```
% Constants
g = 9.81;

% Unit conversions
feet_to_meter = 0.3048;
```

```
feet2_to_m2 = 0.09290;
lb_to_kg = 0.4536;
slug_to_kg = 14.59;
slug_feet2_to_kg_m2 = 1.356;
lb_to_N = 4.448;

% Parameters in SI units
h = 100;
v = 228*feet_to_meter;
m = 367*slug_to_kg;
I_x = 12700*slug_feet2_to_kg_m2;
I_y = 20700*slug_feet2_to_kg_m2;
I_z = 32001*slug_feet2_to_kg_m2;
I_xz = 480*slug_feet2_to_kg_m2;
Q = 61.7*lb_to_N/feet2_to_m2;

Xu = -0.0391;
Xalpha = 18.58*feet_to_meter;
Xw = Xalpha/v;
Zu = -0.248;
Zalpha = -213.41*feet_to_meter;
Zw = Zalpha/v;
Mu = 0.000318/feet_to_meter;
Malpha = -1.89;
Mw = Malpha/v;
Malpha_dot = -0.35;
Mw_dot = Malpha_dot/v;
Mq = -0.694;
Xdelta_e = 0.516*feet_to_meter;
Zdelta_e = -13.4*feet_to_meter;
Mdelta_e = -4.19;

Ybeta = -28.4*feet_to_meter;
Lbeta = -5.49;
Lp = -2.03;
Lr = 0.641;
Nbeta = 0.667;
Np = -0.116;
Nr = -0.207;
Ydelta_r = 0.0295*feet_to_meter;
Ldelta_r = -0.0125;
Ndelta_r = -1.24;
Ldelta_a = 6.01;
Ndelta_a = 0.0286;

S = 234.8*feet2_to_m2;
b = 37.54*feet_to_meter;
c = 6.72*feet_to_meter;

Yp = 0;
Yr = 0;
```