Spectral Graph Theory and High Performance Computing
Literature Survey

David Wobrock `david.wobrock@gmail.com`

October 13, 2017

1 Related work study on Image Processing using Graph Laplacian Operator

1.1 Denoising

Background   Even with high quality cameras, denoising and improving the taken picture remains important. The two main issues that have to be addressed by denoising are blur and noise. The effect of blur is internal to cameras since the number of samples of the signal and it has to hold the Shannon-Nyquist theorem [1], and noise comes from the light aquisition system that fluctuates in relation to the amount of incoming photons.

To model these problems, we can formulate the corrupted image as,

$$y = z + e,$$

where e is the noise vector of variance σ^2, z the clean signal vector and y the noisy picture.

What we want is a high-performance denoiser, capable of scaling up in relation to increasing the image size and keeping reasonable performances. The output image should come as close as possible to the clean image. As an important matter, it is now accepted that images contain a lot of redundancy. This means that, in a natural image, every small enough window has many similar windows in the same image.

Traditional, patch-based methods   The image denoising algorithms review proposed by [1] suggests that the NL-means algorithm, compared to other reviewed methods, comes closest to the original image when applied to a noisy image. This algorithm takes advantage of the redundancy of natural images and for a given pixel i, we can predict its value by using the pixels in its neighbourhood.

In [2], the authors propose the BM3D algorithm, a denoising strategy based on grouping similar 2-D fragments of the image into 3-D data arrays. Then, collaborative filtering is performed on these groups and return 2-D estimates of all grouped blocks. This algorithm exposed state-of-the-art performance in terms of denoising at that time. The results still are one of the best for a reasonable computational cost.

Global filter In the last couple of years, global image denoising filters came up, based on spectral decompositions [3]. This approach considers the image as a complete graph, where the filter value of each pixel is approximated by all pixels in the image. We define the approximated clean image $\hat{z}$ by,

$$\hat{z} = Wy,$$

where W is our data-dependent global filter, a $n \times n$ matrix, n the number of pixels in the picture. W is computed from the graph affinity matrix[1] K, also of size $n \times n$, such as

$$K = \mathcal{K}_{ij},$$

where $\mathcal{K}$ is a similarity function between two pixels i and j. As the size of K can grow very large, we sample the image and compute an approximated $\hat{K}$ on this subset, using the Nyström extension. K can in fact be approximated through its eigenvectors. Knowing that the eigenvalues of K decay very fast [4], the first eigenvectors of the subset of pixels are enough to compute the approximated $\hat{K}$.

Generally, as proposed in [3] and [5], to improve the denoising performance of global filters, pre-filtering techniques are used. It is proposed to first apply a NL-means algorithm to the image to reduce the noise, but to still compute the global filter on the noisy input and apply it to the pre-filtered image.

Comparison between patch-based and global methods As [5] suggests, global filter methods have the possibility to converge to a perfect reconstruction of the clean image, which seems to be impossible for techniques like BM3D. Global filtering also seems promising for creating more practical image processing algorithms.

The thesis [6] proposes a normalised iterative denoising algorithm which is patch-based. The work reports that this technique has slightly better results than the global filter but essentially has a better runtime performance.

1.2 Sharpening

Background Sharpening consists basically in a high pass filter which will magnify high frequency details [6]. The two main issues again with this approach are that noise often displays high frequency attributes, which will be amplified by the filter. Secondly, the concept of overshoot and undershoot,

---

[1]Also called kernel matrix or similarity matrix

which appears when sharpening already sharp parts (edges for example), will exhibit unpleasant artifacts.

Modeling the problem is the same as for denoising, $\hat{z} = Fy$, F being our filter.

Classical Difference of Gaussian (DoG) operator This technique consists of computing the difference between two gaussian kernels, which will produce a range of different kernels with various frequencies.

Structure-aware sharpening We know from its definition [7] that the smoothing filter W is a symmetric and doubly stochastic matrix[2]. So the largest eigenvalue $\lambda_1 = 1$ shows that it has low pass filter characteristics. With the definition of the Laplacian, in relation to the matrix W, as $\mathcal{L} = I - W$, we can consider that it is a data-adaptive high pass filter [6]. So we can define a data-adaptive unsharp mask filter as

$$F_1 = I + \beta(I - W),$$

with $\beta > 0$. This is basically a weighted high pass filtered version of the input image [4].

But with this approach, the noise amplifying and overshoot problems remain. The proposed solution in [7] applies first a smoothing filter to the image, then the unsharp mask filter and finally the same smoothing filter again. The smoothing and sharping filters use possibly two different kernels matrices as basis. The first smoothing filter aims to reduce the noise, while we still avoid over-smoothing. The second and final smoothing controls the effect of the amplified noise and the overshoot artifacts. This gives us the filter

$$F = W_1(I + \beta(I - W_2))W_1,$$

where W_1 and W_2 are constructed from the affinity matrices K_1 and K_2, which can differ in relation to the parameters of their respective kernel function.

---

[2]Depending on the used Laplacian definition. For example the one obtained by "Sinkhorn" iterations [8]

References

[1] Antoni Buades, Bartomeu Coll, and Jean-Michel Morel. "A Review of Image Denoising Algorithms, with a New One". In: *SIAM Journal on Multiscale Modeling and Simulation* 4 (Jan. 2005). DOI: 10.1137/040616024.

[2] K. Dabov et al. "Image Denoising by Sparse 3-D Transform-Domain Collaborative Filtering". In: *IEEE Transactions on Image Processing* 16.8 (Aug. 2007), pp. 2080–2095. ISSN: 1057-7149. DOI: 10.1109/TIP.2007.901238.

[3] Hossein Talebi and Peyman Milanfar. "Global Image Denoising". In: *IEEE Transactions on Image Processing* 23.2 (Feb. 2014), pp. 755–768. ISSN: 1057-7149, 1941-0042. DOI: 10.1109/TIP.2013.2293425. URL: http://ieeexplore.ieee.org/document/6678291/ (visited on 10/03/2017).

[4] Peyman Milanfar. *Non-conformist Image Processing with Graph Laplacian Operator*. 2016. URL: https://www.pathlms.com/siam/courses/2426/sections/3234.

[5] Hossein Talebi and Peyman Milanfar. "Asymptotic Performance of Global Denoising". en. In: *SIAM Journal on Imaging Sciences* 9.2 (Jan. 2016), pp. 665–683. ISSN: 1936-4954. DOI: 10.1137/15M1020708. URL: http://epubs.siam.org/doi/10.1137/15M1020708 (visited on 10/12/2017).

[6] Amin Kheradmand. "Graph-based image restoration". PhD thesis. University of California, Santa Cruz, 2016. URL: http://search.proquest.com/openview/29f820ea2c8cd1f23d36a6a2bc4d3e7b/1?pq-origsite=gscholar&cbl=18750&diss=y (visited on 10/12/2017).

[7] Amin Kheradmand and Peyman Milanfar. "Non-Linear Structure-Aware Image Sharpening with Difference of Smoothing Operators". In: *Frontiers in ICT* 2 (Oct. 2015). DOI: 10.3389/fict.2015.00022.

[8] Peyman Milanfar. "Symmetrizing Smoothing Filters". en. In: *SIAM Journal on Imaging Sciences* 6.1 (Jan. 2013), pp. 263–284. ISSN: 1936-4954. DOI: 10.1137/120875843. URL: http://epubs.siam.org/doi/10.1137/120875843 (visited on 10/04/2017).