

Spectral Graph Theory and High Performance Computing Literature Survey

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1 Related work study

1.1 Denoising

Background Even with higher quality cameras, denoising and improving the resulting picture remains important. The two main issues that still have to be addressed by denoising are blur and noise. The effect of blur is intrinsic to cameras and noise comes from the light acquisition system that fluctuates with the amount of incoming photons.

To model these problems, we can formulate the corrupted image as,

$$y = z + e,$$

where e is the noise vector of variance σ^2 , z the clean signal vector and y the noisy picture.

What we want is a high-performance denoiser, capable of scaling up in relation to increasing the image size and keeping reasonable performances. The output image should come as close as possible to the clean image. As an important matter, it is now accepted that image contain a lot redundancy. This means that, in a natural image, every small enough window has many similar windows in the same image.

Traditional methods - patch-based methods The image denoising algorithms review proposed by [1] suggests that the NL-means algorithm, compared to other reviewed methods, comes closest to the original image when applied to a noisy image. This algorithm takes advantage of the redundancy of natural images and for a given pixel i , we can predict its value by using the pixels in its neighbourhood.

In [2], the authors propose the BM3D algorithm, a denoising strategy based on grouping similar 2-D fragments of the image into 3-D data arrays. Then, collaborative filtering is performed on these groups and return 2-D estimates

of all grouped blocks. This algorithm exposed state-of-the-art performance in terms of denoising at that time. The results still are one of the best for a reasonable computational cost.

Global filter In the last couple of years, global image denoising filters came up, based on spectral decompositions [3]. This approach considers the image as a complete graph, where the filter of each pixel is approximated by all pixels in the image. We define the approximated clean image \hat{z} by,

$$\hat{z} = Wy,$$

where W is our global filter, a $n \times n$ matrix, n the number of pixels in the picture. W is computed from the graph affinity matrix¹ K , also of size $n \times n$, such as

$$K = \mathcal{K}_{ij},$$

where \mathcal{K} is a similarity function between two pixels i and j . As the size of K can grow very large, we sample the image and compute K on this subset, called the Nyström extension. K can be approximated as \hat{K} through its eigenvectors. Knowing that the eigenvalues of K decay very fast [4], the first eigenvectors of the subset of pixels, are enough to compute the approximated \hat{K} .

Generally, as proposed in [3] and [5], to improve the denoising performance of global filters, pre-filtering techniques are used. It is proposed to first apply a NL-means algorithm to the image to reduce the noise, but to compute the global filter on the noisy input but apply it to the pre-filtered image.

Comparison between patch-based and global methods As [5] suggests, global filter methods have the possibility to converge to a perfect reconstruction of the clean image, which seems to be impossible for techniques like BM3D. Global filtering also seems promising for creating more practical image processing algorithms.

The thesis [6] proposes a normalised iterative denoising algorithm which is patch-based. The work reports that this technique has slightly better results but essentially a better runtime performance.

¹Also called kernel matrix or similarity matrix

References

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