University of Balamand

Department of Mathematics

Instructor: Al-Kosseifi, Aouf, Barakeh, Maatouk — **Examination**: First

Course: Linear Algebra — **Date**: Oct 18

Semester: Fall 2023 — **Duration**: 75 minutes

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1. (a) [**15%**] Use **inverse of a matrix** to solve the system

$$\begin{cases} x & +1y & +z & = 4 \\ 2x & +3y + 6z & & = 10 \\ 3x & +6y + 10z & & = 17 \end{cases}$$

   (b) [**10%**] Use Cramer's rule to solve for z in the above syst

   (c) [**5%**] Discuss all possible solution(s) for $AX = 0$ where A is the coefficient matrix of the above system.

2. (a) [**20%**] Determine all values of k for which the linear system

$$\begin{array}{ccccccc} x & + & y & - & z & = & 2 \\ x & + & 2y & + & z & = & 3 \\ x & + & y & + & (k^2-5)z & = & k \end{array}$$

   admits **(a)** no solution **(b)** a unique solution and **(c)** infinitely many solutions.

   (b) [**5%**] Deduce the solution of

$$\begin{array}{ccccccc} x & + & y & - & z & = & 2 \\ x & + & 2y & + & z & = & 3 \\ x & + & y & - & 4z & = & 1 \end{array}$$

3. [**30%**] Determine wether the following statements are true or false, justify your answer:

   (a) Suppose that $\det(A_{3\times 3}) = 2$, then $\det(adj(A)) = 8$.

   (b) Suppose that $\det(A_{3\times 3}) = 7$, then $\det(2A^TA^{-1}) = 2$.

   (c) If $A_{n\times n}$ is invertible matrix, then for all positive integer m, A^m is also invertible.

4. [**15%**] **Do not evaluate determinant, use properties only**.
   For all $x, y, z \neq 0$, prove that $\det(A) = \det(B)$ where A and B are given by:

$$A = \begin{pmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ x & y & z \\ yz & zx & xy \end{pmatrix}$$