University of Balamand
Faculty of Arts and Sciences

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Date: Spring 2021 **Duration**: 1 hour 30 minutes (including submission)

Question 1- (50pts: 6-5-5-6-2-4-10-12)

Consider the matrix $A = \begin{pmatrix} 1 & 4 & 2 \\ 2 & 4 & 2 \\ 3 & 4 & 2 \end{pmatrix}$

a) Find the null space of A.
b) Find a basis for the null space of A. Call this basis S_1.
c) Deduce the nullity and the rank of A.
d) Complete S_1 into a basis of $\mathbb{R}^3$. Call it S
e) Verify that $T = \left\{ u1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}; u2 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}; u3 = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} \right\}$ is a basis of $\mathbb{R}^3$.
f) Find $[v]_T \; for \;\; v = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$
g) Find the transition matrix from T to S and deduce $[v]_S$
h) Transform T into an orthonormal basis W.

Question 2- (25pts: 8-8-9)

Let $L: \mathbb{R}^2 \rightarrow \mathbb{R}^3 \; such \, that \; L\left(\begin{pmatrix} u_1 \\ u_2 \end{pmatrix}\right) = \begin{pmatrix} u_1 \\ u_1 + 3u_2 \\ u_1 - u_2 \end{pmatrix}$

a) Prove the L is a linear transformation.
b) Find Ker L and check whether L is one-to-one.
c) Find Range L and check if L is onto.

Question 3- (25pts: 20-5)

Consider the ***symmetric*** matrix $A = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{pmatrix}$

a) Find a diagonal matrix *D* and an orthogonal matrix *P* such that $A = PDP^T$
b) Find A^{100}