

**University of Balamand**  
**Faculty of Arts and Sciences**

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**MATH 211, 1st examination**  
**Duration:** 1h +10 minutes(submission)

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**Question 1 (25 pts)**

Solve the following system:

$$\begin{cases} x_1 - x_2 - x_3 + x_4 = 0 \\ x_2 + 2x_3 = 8 \\ 3x_1 + 3x_3 = 12 \\ 3x_1 - 3x_2 - 2x_3 + 4x_4 = 7 \end{cases}$$

**Question 2 (25 pts)**

$$\text{Let } A = \begin{pmatrix} -1 & 2 & -1 \\ 1 & 3 & 4 \\ 2 & 5 & 6 \end{pmatrix}$$

- a. Show that A is invertible.
- b. Find  $A^{-1}$  using adjoint (A).
- c. Deduce the solution of the system  $A.X = B$  for  $B = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$
- d. Use Cramer's rule to compute  $x_2$ .

**Question 3 (25 pts)**

Determine all values of "m" for which the following linear system has:

- a. A unique solution
- b. No solution
- c. Infinitely many solutions.

**Note:** do not find the solutions for parts a and c.

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 2 \\ 2x_1 - 2x_2 + 5x_3 = 9 \\ 3x_1 - 8x_2 + (m^3 - m + 10)x_3 = m + 2 \end{cases}$$

**Question 4 (25 pts)**

a. Evaluate the determinant of the matrix  $A = \begin{bmatrix} -1 & -1 & -3 & -3 \\ 3 & 1 & 1 & 1 \\ 4 & 2 & 1 & 2 \\ 3 & 1 & 3 & 3 \end{bmatrix}$ .

b. Deduce the determinant of the matrix  $B = \begin{bmatrix} -1 & -3 & -3 & -3 \\ 3 & 3 & 1 & 1 \\ 4 & 6 & 2 & 1 \\ 3 & 3 & 3 & 3 \end{bmatrix}$

- c. Let A and B be  $3 \times 3$  matrices. Find all values of  $\det(B)$  if  $\det[A^{-1}(2B)I_3B^T A] = 200$ .