

UNIVERSITY OF BALAMAND
DEPARTMENT OF MATHEMATICS

Instructors: , Abou Daher, Dib, Farah.

Course: Calculus I

Semester: Fall 2020-2021

Examination: Final

Date: 16 December 2020

Duration: 70 mins

Name: _____

ID: _____

Instructor: _____

Section: _____

1. Solve all questions and write your solution in clean, legible way.
2. Scan the papers on which the solution is written into a single PDF file.
3. The PDF file should be named as follows: LastName_FirstName_ID.
4. Upload the PDF file to Moodle before the deadline.
5. No late submissions will be allowed.

Question 1. [20%] Consider

$$\sum_{n=0}^{\infty} \frac{(-1)^n (2x - 5)^n}{3^n \sqrt{n^2 + 2}}$$

Find the radius and interval of convergence, write the interval where the series converges absolutely, and identify (if they exist) the points where the series converge conditionally.

Question 2. [15%] Reverse the order, then evaluate the integral

$$\int_1^2 \int_{y^2}^{y+2} (y+1) \, dx dy$$

Question 3.

(a)[10%] Find the power series representation of $\sin x$ at $x = \frac{3\pi}{2}$.

(b)[15%] Find the power series representation of $\tan^{-1}(3x^2)$ using $\frac{1}{1-x}$.

Question 4. [10%] Find the limit of the following functions using power series

$$\lim_{x \rightarrow 0} \frac{\sin x - x - \frac{x^3}{3!}}{x^3}$$

Question 5. [30%]

1. Evaluate the following integrals for some m integer number:

$$\text{a) } \int_{-\pi}^{\pi} (3x - 1) \cos(mx) dx \quad \text{b) } \int_{-\pi}^{\pi} (3x - 1) \sin(mx) dx$$

2. Deduce the Fourier series for $3x - 1$ for $-\pi < x < \pi$.