UNIVERSITY OF BALAMAND

Instructors: Dr. Fares
MATH 211,Final examination
Date:28-7-2021 **Duration**: 1h30min including submission

<u>Question 1.</u> (15 points) Use the **Gauss-Jordan reduction** method to solve the following linear system

$$\begin{cases} x+y-2z+t=0 \\ 2x+3y+2t=-1 \\ -x+y-z+t=-2 \end{cases}$$

<u>Question 2.</u> (20 points) Given the following matrix

$$\boldsymbol{A}=\begin{bmatrix} \mathbf{1} & \mathbf{-1} & \mathbf{2} \\ \mathbf{2} & \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{2} & \mathbf{0} \\ \mathbf{-1} & \mathbf{1} & \mathbf{-2} \end{bmatrix}$$

Answer by true or false and **justify** (any answer without justification will not be considered)

1. Null(A) is subspace of $\mathbb{R}^4$.
2. S=$\left\{u_1=\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, u_2=\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$ is a basis of null (A).
3. Rank(A)=3
4. The column space of A is a subspace of $\mathbb{R}^3$of dimension 2.

<u>Question 3.</u> (30 points) Given $L: P_2 \to P_1$ defined by

$$L(a_2t^2+a_1t+a_0)=3a_2t^2+(a_1+a_0)t+a_1$$

1. Prove that $S=\{t^2+1, t^2+t, t-2\}$ is a basis of P_2.
2. Find A the representative matrix of L with respect to S.
3. Prove that L is diagonalizable and find D.

<u>Question 4.</u> (35 points) Given $L: \mathbb{R}^3 \longrightarrow \mathbb{R}^2$ defined by $L\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right)=\begin{bmatrix} x_1+2x_2-x_3 \\ x_2-x_3 \end{bmatrix}$

1. Show that L is a linear transformation.
2. Find a basis of Ker(L) denote it by S.
3. Is L one-to-one.
4. Deduce if L is onto.
5. Complete S into a basis T of $\mathbb{R}^3$.
6. Transform T into an orthonormal basis T'.
7. Find the coordinates of $v=\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ into the basis T'.