University of Balamand
Department of Mathematics

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Course: Linear Algebra **Date**: 27 February
Semester: Spring 2024 **Duration**: 75 min

Question 1. **[15%]** Solve the following linear system by using the Gauss-Jordan method:

$$\begin{cases} x+3y+z+t & =1 \\ y+2z+t & =1 \\ -3y-6z-3t & =-3 \\ -x-9y+2z-t & =-1 \end{cases}$$

Question 2. **[20%]** Determine all values of m for which the following linear system has:

$$\begin{cases} (m+1)x+y+2z & =0 \\ (m+1)x+my+2z & =0 \\ (2m+2)x+2my+4mz & =2m \end{cases}$$

No solution, a unique solution, infinitely many solutions.

Question 3. Let $A=\begin{pmatrix} 1 & 0 & 1 \\ -1 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix}$

(a) **[05%]** Show that A is invertible.

(b) **[12%]** Use **the cofactors method** to find the inverse of the A.

(c) **[08%]** Deduce the solution of the system $AX=B$ where $B=\begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix}$.

(d) **[05%]** Use Cramer's rule to find the value of x.

Question 4. Given

$$\det(A)=\begin{vmatrix} \alpha_1 & \alpha_2 & \alpha_3 \\ \beta_1 & \beta_2 & \beta_3 \\ \gamma_1 & \gamma_2 & \gamma_3 \end{vmatrix}=-2$$

(a) **[05%]** Deduce $\det(B)=\begin{vmatrix} \alpha_1-3\gamma_1 & \alpha_3-3\gamma_3 & \alpha_2-3\gamma_2 \\ 2\beta_1 & 2\beta_3 & 2\beta_2 \\ \gamma_1-2\beta_1 & \gamma_3-2\beta_3 & \gamma_2-2\beta_2 \end{vmatrix}=4.$

(b) **[05%]** Find $\det(A^2)$ and $\det(adj(A^2))$.

(c) **[05%]** Find $det(A(2AB)^{-1})+det(BA(A^T)^{-1})$

(d) **[05%]** Show that $A^{-1}B$ is invertible.

Question 5. Tell whether the following statements are True or False (Justify):

(a) **[05%]** If B has the columns of A in reverse order, then matrix $A-B$ is invertible.

(b) **[05%]** If A is an invertible matrix, then A^TA is also invertible.

(c) **[05%]** Let A be a 6×4 matrix and B a 4×6 matrix, then the matrix AB is invertible.