University of Balamand
Faculty of Arts and Sciences

Instructors: K. Hitti, M. Dib — **MATH** 211, Final examination
Date: Fall 2020 — **Duration**: 1.5 h +10 minutes (submission)

Question 1 [25pts]

Let L: $P_2 \to R^3$ defined as L (u) = $\begin{bmatrix} a+b-c \\ -3b+6c \\ a+c \end{bmatrix}$, for u= $at^2 + bt + c$ in P_2.

- **a.** Show that L is a linear transformation.
- **b.** Determine Ker L.
- **c.** Find a Basis for Ker L and deduce rank L.
- **d.** Is L one-to-one?
- **e.** Find a Basis for range L.
- **f.** Determine whether L is onto?

Question 2 [25pts]

Let L: $R^3 \to R^2$ be the linear transformation whose representation is A = $\begin{bmatrix} 2 & -1 & 3 \\ 3 & 1 & 0 \end{bmatrix}$

With respect to the ordered bases:

S = $\left\{ v1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}, v3 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \right\}$ and T = $\left\{ w1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, w2 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$

- ➢ Find the representation matrix of L with respect to the natural bases S′ and T′ for R^3 and R^2 respectively.

Question 3[25pts]

Let $A = \begin{bmatrix} 0 & 2 & -3 \\ -1 & 3 & -3 \\ -1 & 2 & -2 \end{bmatrix}$, Find A^{100}

Question 4 [25 pts]

Let $A = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$

- **a.** Show that A is symmetric.
- **b.** Compute the eigenvalues and associated eigenvectors of A.
- **c.** Find a diagonal matrix *D* and an orthogonal matrix *P* such that $A = PDP^T$.

Good Luck!