COMP 2012 — A1 JIANG Guanlin (21093962D)

1. (a)

let $a_1, a_2, a_3, \cdots a_n > m$

$\therefore a_1 + a_2 + a_3 + \cdots + a_n > nm$

But this is contradictive

$\therefore$ there have some number smaller than m.

(b) $\underbrace{\neg(p \vee q \vee r)}_{①}$ and $\underbrace{\neg p \wedge \neg q \wedge \neg r}_{②}$

p	q	r	$p \vee q$	$p \vee q \vee r$	$\neg p$	$\neg q$	$\neg r$	$\neg p \wedge \neg q$	①	②
T	T	T	T	T	F	F	F	F	F	F
T	T	F	T	T	F	F	T	F	F	F
T	F	T	T	T	F	T	F	F	F	F
T	F	F	T	T	F	T	T	F	F	F
F	T	T	T	T	T	F	F	F	F	F
F	T	F	T	T	T	F	T	F	F	F
F	F	T	F	T	T	T	F	T	F	F
F	F	F	F	F	T	T	T	T	T	T

$\therefore$ There're logically equivalent

(C)

$Dom: \{2, 4, 6, 8, 10, 12, 14, 16, 18\}$

$\therefore P = \frac{x}{4} \ (P \in \mathbb{Z})$

$Q = \frac{x}{3} \ (Q \in \mathbb{Z})$

① $\underline{\exists x \in Dom \ P(x)}$

$4P = x \Rightarrow x$ in Dom $\Rightarrow 4, 8, 12, 16$

$\therefore$ True

② $\underline{\exists y \in Dom \ Q(y)}$

$3y = x \Rightarrow x$ in Dom $\Rightarrow 6, 12, 18$

$\therefore$ True

③ $\underline{\neg(\exists z \in Dom \ P(z) \wedge Q(z))}$

$3 \times 4 = 12$

$\therefore$ only 12 in Dom

$\therefore \neg$

$\therefore$ Other in Dom

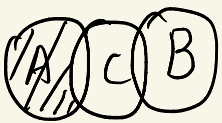
$\therefore$ True

2.(a)

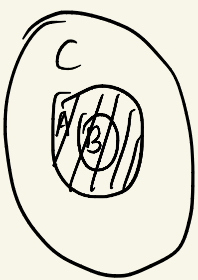
$$(A-C)-B$$

Way 1:

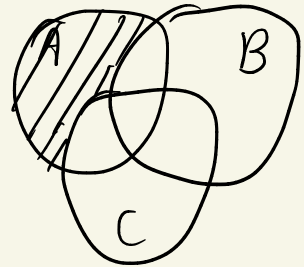
Way 2:



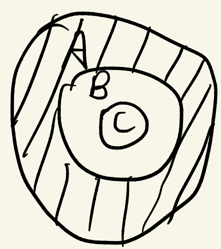
Way 3:



Way 4:



Way 5:



(b)

$A = \{a, b, c, d, e\}$

(I)

for a: Set $A \Longrightarrow n$ elements $(n=5)$

$A \times A \Longrightarrow n^2$ elements $(n^2 = 5^2 = 25)$

for subsets of $A \times A \Longrightarrow 2^{n^2}$ relations, $2^{25} = 33554432$

$\therefore$ There're 33554432 relations.

(II)

① $\{a, d, e\}, \{b, c\}$ ④ $\{a, d, e, c\}$ $\{b\}$

② $\{a, d, e\}, \{b\}, \{c\}$ ⑤ $\{a, d, e, b, c\}$

③ $\{a, d, e, b\}, \{c\}$

3(a)、

$$f(n) = \log_2 n^{\log_2 23} \Rightarrow \log_2 23 \cdot \log_2 n$$

$$g(n) = \log_2 23^{\log_2 n} \Rightarrow \log_2 n \cdot \log_2 23$$

$\therefore f(n) = g(n)$

$\therefore f(n) = O(g(n))$

$f(n) = \Omega(g(n))$

$f(n) = \Theta(g(n))$

(b)

$$f(n) \prec g(n) \equiv f(n) = O(g(n))$$

$$f(n) = O(g(n)) \text{ and } g(n) = O(h(n))$$

$$n \geq n_0 \Rightarrow f(n) \leq c_1 \cdot g(n)$$

$$n \geq n_0' \Rightarrow g(n) \leq c_2 \cdot h(n)$$

$$f(n) \leq c_1 \cdot c_2 \cdot h(n)$$

$$\therefore n \geq n_0'' \Rightarrow f(n) \leq c_3 \cdot h(n)$$

$$\therefore f(n) = O(h(n))$$

(C)

```
FindMedian(Array A[1...n])
    for integer i ← 1 to n-1
        k ← i
        for integer j ← i+1 to n
            if A[k] > A[j] then
                k ← j
            swap(A[i] and A[j])
    if n mod 2 == 0 then
        median ← A
    else
        median ← A[ ]
    return median
```

4(a).

$(S_1 \cup T) \cap (S_2 \cup T) \cap \cdots \cap (S_n \cup T)$

$= (S_1 \cap S_2 \cdots \cap S_n) \cup T$

let $n=1$:

$LHS = S_1 \cup T = S_1 \cup T = RHS$

let $n=k$:

$LHS = (S_1 \cup T) \cap (S_2 \cup T) \cap \cdots (S_k \cup T) = (S_1 \cap S_2 \cdots \cap S_k) \cup T = RHS$

Let $n=k+1$:

$LHS = (S_1 \cup T) \cap (S_2 \cup T) \cap \cdots \cap (S_{k+1} \cup T) = (S_1 \cap S_2 \cdots \cap S_k \cap S_{k+1}) \cup T$

$= RHS$

$\therefore$ $P(n)$ is ~~tre~~ true for all n.

(b)