Name: JIANG GUANLIN Intro to AI A1

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1.1、

$P(meltdown) = 0.01$

$P(late \mid meltdown) = 0.98$

$P(late \mid not\ meltdown) = 0.03$

$P(meltdown \mid late) = ?$

$$P(meltdown \mid late) = \frac{P(late \mid meltdown) \times P(meltdown)}{P(late)} = \frac{0.98 \times 0.01}{P(late)} = \frac{0.0098}{P(late)}$$

$$P(not\ meltdown \mid late) = \frac{P(late \mid not\ meltdown) \times (1 - P(meltdown))}{P(late)} = \frac{0.03 \times 0.99}{P(late)} = \frac{0.0297}{P(late)}$$

$$\frac{0.0098}{P(late)} + \frac{0.0297}{P(late)} = 1$$

$$P(late) = 0.0395$$

$$P(meltdown \mid late) = \frac{0.0098}{0.0395} = 0.2481 = 24.81\%$$

$\therefore$ The Probability: 24.81%

1.2、

(a) $$P(X \mid Y, E) = \frac{P(Y \mid X, E) P(X \mid E)}{P(Y \mid E)}$$

$$\cancel{= \frac{P(X,Y,E)}{P(Y,E)}}$$

$$\xlongequal{product} \frac{P(X, Y \mid E)}{P(Y \mid E)}$$

$$\xlongequal{product} \frac{P(X \mid Y, E)\ \cancel{P(Y \mid E)}}{\cancel{P(Y \mid E)}}$$

$$P(X, Y \mid E) = P(Y \mid X, E) P(X \mid E) = P(X \mid Y, E) P(Y \mid E)$$

$$\therefore P(X \mid Y, E) = \frac{P(Y \mid X, E) P(X \mid E)}{P(Y \mid E)}$$

(b) $$P(X \mid E) = \sum_y P(X, Y = y \mid E)$$

$$\sum_y P(X, Y = y \mid E) \xlongequal{Product} \sum_y \frac{P(X, Y = y, E)}{P(E)} \xlongequal{Marginal} \frac{P(X, E)}{P(E)} = P(X \mid E)$$

$$P(X \mid E) = \sum_y P(X, Y = y \mid E)$$

1.3、

(i) $P(X, Y \mid E) = P(X \mid E) P(Y \mid E)$

(ii) $P(X \mid Y, E) = P(X \mid E)$

(iii) $P(Y \mid X, E) = P(Y \mid E)$

For (i) to (ii):

$$P(X, Y \mid E) = P(X \mid E) P(Y \mid E)$$

$$\frac{P(X, Y, E)}{P(E)} \xlongequal{Bayes} \frac{P(X, E)}{P(E)} \cdot \frac{P(Y, E)}{P(E)}$$

$$P(X \mid Y, E) = \frac{P(X, Y, E)}{P(Y, E)} = \frac{P(X, E)\ \cancel{P(Y, E)}}{P(E)\ \cancel{P(Y, E)}} = \frac{P(X, E)}{P(E)} = P(X \mid E)$$

$\therefore (i) \Rightarrow (ii)$

For (ii) to (iii):

$$P(X|Y,E)=P(X|E)$$
$$\frac{P(X,Y,E)}{P(Y,E)}=\frac{P(X,E)}{P(E)}$$
$$P(X,Y,E)=\frac{P(X,E)P(Y,E)}{P(E)}$$

$$P(Y|X,E)=\frac{P(X,Y,E)}{P(X,E)}$$
$$P(Y|X,E)=\frac{\cancel{P(X,E)}P(Y,E)}{\cancel{P(X,E)}P(E)}=\frac{P(Y,E)}{P(E)}=P(Y|E)$$
$$\therefore (ii) \Rightarrow (iii)$$

For (iii) to (i):

$$P(Y|X,E)=P(Y|E)$$
$$\frac{P(X,Y,E)}{P(X,E)}=\frac{P(Y,E)}{P(E)}$$
$$P(X,Y,E)=\frac{P(X,E)P(Y,E)}{P(E)}$$

$$P(X,Y|E)=\frac{P(X,Y,E)}{P(E)}$$
$$P(X,Y|E)=\frac{P(X,E)P(Y,E)}{P(E)P(E)}=P(X|E)P(Y|E)$$
$$\therefore (iii) \Rightarrow (i)$$

1.4.

(a) $P(X=1|Z=1)>P(X=1)$

$P(X=1|Y=1,Z=1)<P(X=1|Z=1)$

if $X=1$ is Typhon, $Z=1$ is in summer, $Y=1$ is Chengdu

P(Typhon | in summer) > P(Typhon)

P(Typhon | in summer, Chengdu) < P(Typhon | in summer)

(b) ~~$P(X=1|Z=1)>P(X=1)$~~

~~$P(X=1|Z=1,Y=1)<P(X=1|Z=1)$~~

$P(X=1)<P(X=1|Y=1)<P(X=1|Z=1,Y=1)$

if $X=1$ is Typhon, $Y=1$ is in HK, $Z=1$ is in summer

P(Typhon) < P(Typhon | in HK) < P(Typhon | in HK, in summer)

(c) $P(X,Y|Z)=P(X|Z)P(Y|Z)$

$P(X=1,Y=1)<P(X=1)P(Y=1)$

$$\frac{P(X,Y,Z)}{P(Z)}=\frac{P(X,Z)}{P(Z)}\times\frac{P(Y,Z)}{P(Z)}\Rightarrow P(X,Y,Z)=P(X,Z)P(Y,Z)$$

$$P(X=1,Y=1)<P(X=1)P(Y=1)\Rightarrow P(X=1|Y=1)P(Y=1)<P(X=1)P(Y=1)\Rightarrow P(X=1|Y=1)<P(X=1)$$

X = rain $\begin{cases} X=1: \text{is rain} \\ X=0: \text{no rain}\end{cases}$

Y = snow $\begin{cases} Y=1 \ \text{is snow} \\ Y=0 \ \text{no snow}\end{cases}$

Z = cold $\begin{cases} Z=1 \ \text{is cold} \\ Z=0 \ \text{not cold}\end{cases}$

P(rain, snow | cold) = P(rain | cold) P(snow | cold)

P(is rain, is snow) < P(is rain) P(is snow)

1.5、

(a) $P(B=1|A=1)$

$$= \frac{P(B=1,A=1)}{P(A=1)}$$

$$= \frac{P(B=1)P(E=1)P(A=1|B=1,E=1)+P(B=1)P(E=0)P(A=1|B=1,E=0)}{P(A=1,B=1)+P(A=1,B=0)}$$

$$= \frac{0.001\times0.005\times0.98+0.001\times(1-0.005)\times0.96}{P(A=1,B=1)+P(B=0)P(E=1)P(A=1|B=0,E=1)+P(B=0)P(E=0)P(A=1|B=0,E=0)}$$

$$= \frac{0.0009601}{0.0009601+(1-0.001)\times0.005\times0.35+(1-0.001)\times0.995\times0.002} = \frac{0.0009601}{0.00469636}$$

$$= 0.2044$$

(b) $P(B=1|A=1,E=0)$

$$= \frac{P(B=1,A=1,E=0)}{P(A=1,E=0)}$$

$$= \frac{P(E=0)\times P(B=1)\times P(A=1|E=1,B=0)}{P(E=0)\times P(B=1)\times P(A=1|E=1,B=0)+P(E=0)\times P(B=0)\times P(A=1|E=0,B=0)}$$

$$= \frac{0.995\times0.001\times0.96}{0.001\times0.995\times0.96+0.995\times0.999\times0.002}$$

$$= 0.3245$$

(c) $P(A=1|J=1)$

$$= \frac{P(A=1,J=1)}{P(J=1)}$$

$$= \frac{P(J=1|A=1)P(A=1)}{P(J=1)}$$

$$= \frac{0.93\times0.00469636}{P(A=1)\times P(J=1|A=1)+P(A=0)\times P(J=1|A=0)}$$

$$= \frac{0.93\times0.00469636}{0.93\times0.00469636+(1-0.00469636)\times0.1}$$

$$= 0.0420$$

(d) $P(A=1|J=1,M=1)$

$$= \frac{P(A=1,J=1,M=1)}{P(J=1,M=1)}$$

$$= \frac{P(A=1)\times P(J=1|A=1)\times P(M=1|A=1)}{P(A=1)\times P(J=1|A=1)\times P(M=1|A=1)+P(A=0)\times P(J=1|A=0)\times P(M=1|A=0)}$$

$$= \frac{0.00469636\times0.93\times0.65}{0.00469636\times0.93\times0.65+(1-0.00469636)\times0.1\times0.01}$$

$$= 0.7404$$

(e) $P(A=1|M=0)$

$$= \frac{P(A=1, M=0)}{P(M=0)}$$

$$= \frac{P(A=1)\times(1-P(M=1|A=1))}{P(A=1)\times(1-P(M=1|A=1))+(1-P(A=1))\times(1-P(M=1|A=0))}$$

$$= \frac{0.00469636\times(1-0.65)}{0.00469636\times(1-0.65)+(1-0.00469636)\times(1-0.01)}$$

$$= 0.001665385$$

(f) $P(A=1|M=0, E=1)$

$$= \frac{P(A=1, M=0, E=1)}{P(M=0, E=1)}$$

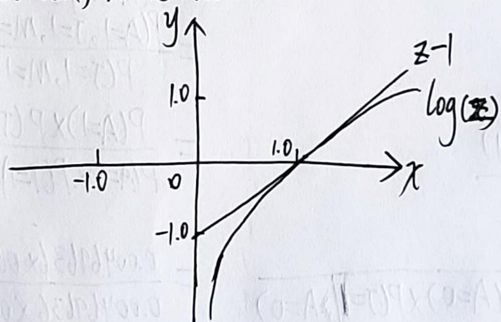
$$= \frac{P(A=1)\times(1-P(M=1|A=1))\times P(E=1|A=1)}{P(A=1)\times(1-P(M=1|A=1))\times P(E=1|A=1)+P(A=0)\times P(M=0|A=0)\times P(E=1|A=0)}$$

$$= \frac{0.00469636\times(1-0.65)\times\frac{0.001\times0.005\times0.98+0.999\times0.005\times0.35}{0.00469636}}{0.00469636\times(1-0.65)\times\frac{0.001\times0.005\times0.98+0.999\times0.005\times0.35}{0.00469636}+(1-0.00469636)\times(1-0.01)\times3.26217\times10^{-3}}$$

$$=0.1602939175$$

Those answer some are related, some not.

1.6.



(a) $\log z \le z-1$

$f(z)=\log(z)-(z-1)$

$f'(z)=\frac{1}{z}-1 \Rightarrow f'(z=1)=0$

$f''(z)=-\frac{1}{z^2} \Rightarrow f''(z=1)=-1,\ f''<0$

$\therefore$ The $f'(z)=0$.

(b) $KL(p,q)\ge 0$

$p_i = q_i$

$z=\frac{q_i}{p_i}$

$$KL(p,q)=\sum_i p_i \log\left(\frac{p_i}{q_i}\right)$$
$$=\sum_i -p_i \log\left(\frac{q_i}{p_i}\right) \ge 0 = \sum_i -p_i\left(\frac{q_i}{p_i}-1\right)$$
$$=-\sum_i q_i+\sum_i p_i = 1-1$$

$z=1$

$\Downarrow$

$\therefore \frac{q_i}{p_i}=1$ and $p_i=q_i$ $\therefore KL(p,q)\ge 0$

(c) $KL(p,q) \neq KL(q,p)$

Let $p = [\frac{1}{4}, \frac{1}{4}, \frac{1}{2}]$, $q = [\frac{1}{6}, \frac{1}{3}, \frac{1}{2}]$

$$KL(p,q) = \frac{1}{4} \times \log\left(\frac{\frac{1}{4}}{\frac{1}{6}}\right) + \frac{1}{4} \times \log\left(\frac{\frac{1}{4}}{\frac{1}{3}}\right) + \frac{1}{2} \times \log\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right)$$

$$= 0.01278813$$

$$KL(q,p) = \frac{1}{6} \times \log\left(\frac{\frac{1}{6}}{\frac{1}{4}}\right) + \frac{1}{3} \times \log\left(\frac{\frac{1}{3}}{\frac{1}{4}}\right) + \frac{1}{2} \times \log\left(\frac{\frac{1}{2}}{\frac{1}{2}}\right)$$

$$= 0.0122977$$

$\therefore KL(p,q) \neq KL(q,p)$