AMA Midterm Test

10071

JIANG Guanlin

21093962D

1、

(a) $\lim\limits_{x\to-\infty} \dfrac{x+8}{\sqrt{2x^3+3}}$

$= \dfrac{-\infty}{-\infty}$

$\therefore$ Does not exist

(b) $\lim\limits_{x\to+\infty} \dfrac{x+8}{\sqrt{2x^3+3}}$

$= \lim\limits_{x\to+\infty} \dfrac{\frac{1}{x^{\frac{1}{2}}}+\frac{8}{x^{\frac{3}{2}}}}{\sqrt{2+\frac{3}{x^3}}}$

$= \dfrac{\lim\limits_{x\to+\infty}\left(\frac{1}{x^{\frac{1}{2}}}+\frac{8}{x^{\frac{3}{2}}}\right)}{\lim\limits_{x\to+\infty}\sqrt{2+\frac{3}{x^3}}}$

$= \dfrac{0+0}{\sqrt{2}+0}$

$= 0$

2、 ~~$\lim\limits_{x\to0^-} f'(x) = \lim\limits_{x\to0^-} \dfrac{x^2+A\tan x+B-B}{x} = 2B$~~

~~$\lim\limits_{x\to0^+} f'(x) = \lim\limits_{x\to0^+} \dfrac{\frac{\ln(1+2x)}{x}+}{x}$~~

$\lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^-} x^2+A\tan x+B = B$

$\lim\limits_{x\to0^+} f(x) = \lim\limits_{x\to0^+} \dfrac{\ln(1+2x)}{x} = 2$

$\therefore B=2$

$\therefore f'(0^-) = \lim\limits_{x\to0^-} \dfrac{x^2+A\tan x+B-B}{x} = 0$

$f'(0^+) = \lim\limits_{x\to0^+} \dfrac{\frac{\ln(1+2x)}{x}-2}{x} = 0$

$f'(0^-) = f'(0^+) = 0 = A$

$\therefore A=0, B=2$

~~2、~~

~~$f(0) = 0+0+B$~~

~~$f(0)=B$~~

~~$\lim\limits_{x\to0^-} f(x) = \lim\limits_{x\to0^-} x^2+A\tan x+B = B$~~

~~$\lim\limits_{x\to0^+} f(x) = \lim\limits_{x\to0^+} \dfrac{\ln(1+2x)}{x} = \dfrac{\frac{2}{1+2x}}{1} = 2$~~

~~$\therefore B=2$~~

~~$\therefore \lim\limits_{x\to0^-}$~~

3、 $y = (1-3x)^{\cos(x)}$

$y' = e^{\cos(x)\ln(1-3x)} \cdot (\cos(x)\ln(1-3x))'$

$y' = e^{\cos(x)\ln(1-3x)} \cdot -\sin x\ln(1-3x) + \dfrac{\cos(x)}{1-3x}\cdot(-3)$

$y' = e^{\cos(x)\ln(1-3x)} \cdot \left(-\sin(x)\ln(1-3x) - \dfrac{3\cos(x)}{1-3x}\right)$

$y' = (1-3x)^{\cos x} \cdot \left(-\sin(x)\ln(1-3x) - \dfrac{3\cos(x)}{1-3x}\right)$

4、

(a) $\lim\limits_{x\to0^+} [\cos(2x)]^{\frac{1}{x^2}}$

$= \lim\limits_{x\to0^+} \left(e^{\frac{1}{x^2}\ln(\cos(2x))}\right)$

$= \dfrac{1}{e^2}$

(b) $\lim\limits_{x\to0} \dfrac{\tan x - x}{x^3}$

$\because \left(\frac{0}{0}\right)$

$\therefore$ Use L'Hopital's Rule

$\therefore = \lim\limits_{x\to0} \dfrac{\sec^2(x)-1}{3x^2}$

$= \lim\limits_{x\to0} \dfrac{\tan^2(x)}{3x^2}$

$= \lim\limits_{x\to0} \dfrac{2\tan(x)\cdot\sec^2(x)}{6x}$

$= \lim\limits_{x\to0} \dfrac{2(\sec^2(x)\sec^2(x) + 2\sec^2(x)\cdot\tan^2(x))}{6}$

$= \dfrac{2}{6} = \dfrac{1}{3}$

5、

(a) $\int \frac{1}{\sqrt{4-9x^2}}dx$

$u=\frac{3x}{2} \Rightarrow \frac{du}{dx}=\frac{3}{2} \Rightarrow dx=\frac{2}{3}du$

$\therefore = \int \frac{2}{3\sqrt{4-4u^2}}du$

$= \int \frac{2}{3\times 2\times\sqrt{1-u^2}}du$

$= \frac{1}{3}\int \frac{1}{\sqrt{1-u^2}}du$

$= \frac{\arcsin(u)}{3}$

$= \frac{\arcsin(\frac{3x}{2})}{3}+C$

(b) $\int x^7(8+3x^4)^8dx$

$u=3x^4+8 \Rightarrow \frac{du}{dx}=12x^3 \Rightarrow dx=\frac{1}{12x^3}du$

$\therefore = \frac{1}{36}\int (u-8)u^8du$

$= \frac{1}{36}\left(\int u^9du - 8\int u^8du\right)$

$= \frac{1}{36}\times\left(\frac{u^{10}}{10}-\frac{8u^9}{9}\right)$

$= \frac{1}{36}\times\left(\frac{9u^{10}}{90}-\frac{80u^9}{90}\right)$

$= \frac{(3x^4+8)^9(27x^4-8)}{3240}+C$

(c) $\int 4x\cos(2-3x)dx$

$= 4\int x\cos(2-3x)dx$

$f=x,\ dg=\cos(2-3x)dx$

$df=dx,\ g=-\frac{1}{3}\sin(2-3x)$

$= (-\frac{4}{3}x\sin(2-3x)+\frac{4}{3}\int\sin(2-3x)dx$

$= -\frac{4}{3}x\sin(2-3x)+\frac{4}{9}\cos(2-3x)+C$

6、

(a) $\int_{-2}^{0} x\sqrt{3+x^2}+\frac{3}{(6x-1)^2}dx$

$= \int_{-2}^{0} x\sqrt{3+x^2}dx + 3\int_{-2}^{0}\frac{1}{(6x-1)^2}dx$

$u_1=x^2+3,\ \frac{du_1}{dx}=2x,\ dx=\frac{1}{2x}du_1$ | $u_2=6x-1,\ \frac{du_2}{dx}=6,\ dx=\frac{1}{6}du$

$= \frac{\int\sqrt{u_1}du}{2}+\frac{\int\frac{1}{u_2^2}du}{6}\Big|_{x=-2}^{x=0}$

$= \frac{3\int\sqrt{u_1}du+\int\frac{1}{u_2^2}du}{6}\Big|_{x=-2}^{x=0}$

$= \frac{u_1^{\frac{3}{2}}+(-\frac{1}{u_2})}{6}\Big|_{x=-2}^{x=0}$

$= \frac{(x^2+3)^{\frac{3}{2}}+(-\frac{1}{(6x-1)^2})}{6}\Big|_{x=-2}^{x=0} = -\frac{7^{\frac{3}{2}}}{3}+\sqrt{3}+\frac{6}{13}$

(b) $\int_6^0 (2+5x)e^{\frac{1}{3}x}dx$

$f=5x+2,\ \frac{dg}{dx}=e^{\frac{1}{3}x}dx$

$\frac{df}{dx}=5,\ g=3e^{\frac{x}{3}}$

$=3(5x+2)e^{\frac{1}{3}x}\Big|_{x=6}^{x=0}-\int_6^0 15e^{\frac{x}{3}}dx$

$u=\frac{x}{3},\ \frac{du}{dx}=\frac{1}{3},\ dx=3du$

$=3(5x+2)e^{\frac{1}{3}x}\Big|_{x=6}^{x=0}-45\int e^u du\Big|_{x=6}^{x=0}$

$=3(5x+2)e^{\frac{1}{3}x}\Big|_{x=6}^{x=0}-45e^u\Big|_{x=6}^{x=0}$

$=\left[3(5x+2)e^{\frac{1}{3}x}-45e^{\frac{x}{3}}\right]_{x=6}^{x=0}$

$=(15x-39)e^{\frac{x}{3}}\Big|_{x=6}^{x=0}$

$=-51e^2-39$

7.

$A(n)=2000-10ne^{5-\frac{n^2}{8}}$

$A'(n)=-10\left(e^{5-\frac{n^2}{8}}-\frac{e^{\frac{40-n^2}{8}}n^2}{4}\right)$

$-10\left(e^{5-\frac{n^2}{8}}-\frac{e^{\frac{40-n^2}{8}}n^2}{4}\right)=0$

∴ no solution

∴

$A(1)=\cancel{2000}\ 690.26$

$A(10)=\cancel{2000}\ 1999.94$

∴ The ~~lowest~~ min is 690.26 in $n=1$, max is 1999.94 in $n=10$.

8.

(a) $f(x)=\frac{x^3}{x^2+1}$

$\frac{x^3}{x^2+1}=0$, $f(x)=\frac{0^3}{0^2+1}$

$x=0$, $f(x)=0$

domain: $(-\infty,+\infty)$

Vertical Asymptotes: None

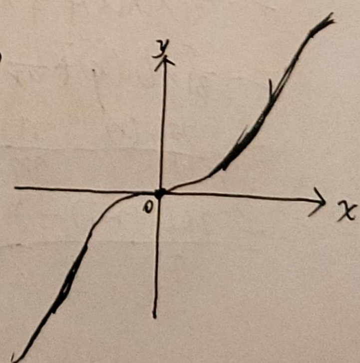
Horizontal Asymptotes: $y=x$

Symmetry: None

x-int: (0,0)

y-int: (0,0)

(c)




(b)

$f(x)=\frac{x^3}{x^2+1}$

$f'(x)=\frac{3x^2(x^2+1)-x^3(2x)}{(x^2+1)^2}$

$f'(x)=\frac{x^4+3x^2}{(x^2+1)^2}$

$\frac{x^4+3x^2}{(x^2+1)^2}=0$

$x=0$

$f''(x)=\left(\frac{x^4+3x^2}{(x^2+1)^2}\right)'$

$f''(x)=\frac{2x(-x^2+3)}{(x^2+1)^3}$

$\frac{2x(-x^2+3)}{(x^2+1)^3}=0$

$x=0,\pm\sqrt{3}$

| $f'(x)$ | $>0$ | $<0$ |
|---|---|---|
| $f(x)$ | increasing | decreasing |

| $f''(x)$ | $-\infty<x<-\sqrt{3}$ | $-\sqrt{3}<x<0$ | $0<x<\sqrt{3}$ | $\sqrt{3}<x<+\infty$ |
|---|---|---|---|---|
| $f(x)$ | concave upward | concave down | concave upward | concave down |

9、

$$f(x) = 3x(x+4)^{\frac{2}{3}}$$

$$f'(x) = 3(x+4)^{\frac{2}{3}} + 3x\left(\frac{2}{3(x+4)^{\frac{1}{3}}}\right)$$

$$f'(x) = \frac{5x+12}{(x+4)^{\frac{1}{3}}}$$

$$f''(x) = \left(\frac{5x+12}{(x+4)^{\frac{1}{3}}}\right)'$$

$$f''(x) = \frac{10x+48}{3(x+4)^{\frac{4}{3}}}$$

$$\frac{5x+12}{(x+4)^{\frac{1}{3}}} = 0$$

$$x = -\frac{12}{5}$$

$$\frac{10x+48}{3(x+4)^{\frac{4}{3}}} = 0$$

$$x = -\frac{12}{5}$$

$$f(-5) = -15$$

$$f\left(-\frac{12}{5}\right) = -9.849461453$$

$$f(-1) = -6.24$$

∴ The global max: $-6.24$ at $x=-1$,
global min: $-15$ at $x=-5$.

10、

$$\int \frac{4x-11}{x^3-9x^2}\,dx$$

$$= \int \frac{4x-11}{x^2(x-9)}\,dx$$

$$= \int \left(-\frac{25}{81x} + \frac{11}{9x^2} + \frac{25}{81(x-9)}\right)dx$$

$$= -\frac{25}{81}\int \frac{1}{x}\,dx + \frac{11}{9}\int \frac{1}{x^2}\,dx + \frac{25}{81}\int \frac{1}{x-9}\,dx \qquad u = x-9,\ \frac{du}{dx} = 1,\ dx = du$$

$$= -\frac{25}{81} \times \ln(x) + \frac{11}{9} \times -\frac{1}{x} + \frac{25}{81}\int \frac{1}{u}\,du$$

$$= -\frac{25}{81} \times \ln(x) - \frac{11}{9x} + \frac{25}{81} \times \ln(x-9) + C$$

$$= -\frac{25\ln(x)}{81} - \frac{11}{9x} + \frac{25\ln(x-9)}{81} + C$$

$$= \frac{25\ln(x-9) - 25\ln(x)}{81} - \frac{11}{9x} + C$$