AMA2111 Mathematics I

2022-23 Semester 1 Homework 2

Due Date: 17:00, Sunday, November 20, 2022

- Put the following information on the top right corner of the front page of your homework.
  - Your name and student number
  - Subject code: AMA2111
  - Subject lecturer: Dr. Bob He
- Photograph your solutions onto a PDF file named "YourName_StudentID", otherwise the marker (not the lecturer) cannot write on your solution, then you cannot see the marking but only the score.
- You may use the app "CamScanner" or other softwares. Make sure that the file is complete, legible, in correct order and orientation.
- Upload/attach your homework solution pdf file at the same place you've downloaded this homework by pressing "Browse My Computer", then choose your pdf file, and then press "Submit". You may re-submit the homework again, to a maximum of twice, before the due time. After submitting, check and make sure your submission is successful.
- No late submission is allowed. It may not be marked.

1. Solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} -1 & 4 & 2 \\ -2 & 5 & 2 \\ 1 & -2 & 0 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 7 \\ 5 \\ -1 \end{bmatrix}.$$

2. Find the general solution of

$$y'' - 2y' + 5y = 10\cos x$$

by method of undetermined coefficients.

3. Solve the initial value problem

$$y'' - 3y' + 2y = 4x + e^{2x}, \quad y(0) = 0,\ y'(0) = 1$$

by method of undetermined coefficients.

4. Find the general solution of

$$y'' + y' = \frac{1}{e^x + e^{-x}}$$

by variation of parameters.

5. Solve the initial value problem

$$y'' - 4y' + 4y = \frac{e^{2x}}{(x+1)^2}, \quad y(0) = 1,\ y'(0) = 0$$

by variation of parameters.

6. Use the Laplace transform to solve the initial value problem

$$y'' - 2y' + y = t^2 e^t + 2 \sin \frac{t}{2} \cos \frac{t}{2}, \quad y(0) = 1, \quad y'(0) = 0.$$

7. Use the Laplace transform to solve the initial value problem

$$y'' + y = f(t), \quad y(0) = 0, \quad y'(0) = 0$$

where

$$f(t) = \begin{cases} 0, & t < 1, \\ 2t, & 1 \le t < 3, \\ -t, & t \ge 3. \end{cases} .$$

8. Find the partial derivatives $z_x(x, y)$, $z_y(x, y)$, and $z_{xy}(x, y)$, where $z(x, y)$ is defined by $x^2 z - xy^2 = \sin z$.

9. Let f be a differentiable function of one variable, and $w = f(e^x + 2xy)$.

   i) Verify that the function w satisfies the equation $(e^x + 2y)w_y - 2xw_x = 0$.

   ii) If $f(u) = \cos u$, calculate w_{xy}.

10. For an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, its area can be calculated by $A = \pi ab$. Assume that a and b are measured with the relative error of no more than 1.2% and 0.4%, respectively. Estimate the error in the calculation of area A of the ellipse $4x^2 + 9y^2 = 36$, using linear approximation.

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