Hong Kong Polytechnic University

Department of Applied Mathematics

AMA1104 quiz1 Date: 21st September, 2021 Time: 0920 to 1020 a.m.

Answer all questions (Total marks=35)

1. A family has 3 boys and 2 girls.

(a) Find the number of ways they can sit in a row.

(b) How many ways are there if the boys and girls are each to sit together? (4marks)

2. A class contains 10 students with 6 men and 4 women. Find the number ***n*** of ways:

(a) a 4-member committee can be selected from the students,

(b) a 4-member committee with 2 men and 2 women can selected,

(c) the class can elect a president, vice-president, treasurer, and secretary. (6marks)

3. In a game, the probability that a player wins the game is 0.6. Suppose 5 players play the game independently. Find the probability that

(a) only 4 of them win the game,

(b) at least 4 of them win the game,

(c) only 4 of them win the game, given that at least 4 of them win the game.

(Give the answer correct to 4 decimal places.) (6marks)

4. A medical test is used to test for a certain illness. The test is 85% accurate if the patient has such illness. However, if the patient does not have such illness, the test also has a 5% probability to show a positive result. Suppose 8% of the population have such illness.

(a) Find the probability that the test shows a positive result.

(b) If the test is conducted for 10 people, find the probability that

(i) at least one of them shows a positive result;

(ii) exactly two of them show positive results, given that at least one of them shows a positive result. (7marks)

(Give the answers correct to 4 decimal places if necessary.)

5. A factory has three machines that make watches. The daily production and percentage of defective watches are given in the following table.

Machine	Daily production	Defective percentage
A	250	2%
B	400	5%
C	350	4%

(a) If a watch is selected at random, find the probability that

(i) the watch is defective,

(ii) the watch is made by machine *A*, given that the watch is defective,

(iii) the watch is made by machine *B*, given that the watch is not defective.

(b) If three watches are selected at random, find the probability that

(i) all of them are not defective;

(ii) at least one of them is defective;

(iii) exactly one watch is defective.

(Give the answers correct to 4 decimal places.) (12marks)

End of paper

1.

(a) The five children can sit in a row in 5(4)(3)(2)(1)=5!=120 ways

(b) There are two ways to distribute them according to sex: BBBGG or GGBBB. In each case, the boys can sit in 3(2)(1)=3!=6 ways, and the girls can sit in 2(1)=2! =2 ways. Thus, altogether, there are 2(3!)(2!)=2(6)(2)= 24 ways.

2.

(a) This concerns combinations, not permutations, since order does not count. There are "10 choose 4" such committees. That is,

n= 10C4 =210

(b) The 2 men can be chosen from the 6 men in 6C2 ways, and the 2 women can be chosen from the 4 women in 4C2 ways. Thus, by the product rule,

n=6C2 x 4C2 = 90 ways.

(c) This concerns permutations, not combinations, since order does count. Thus

n= 10P4 = 5040.

3. (a) Let A be the event that only 4 of them win the game.

$$P(A) = 0.6^4 \times (1-0.6) \times C_4^5$$
$$= \underline{\underline{0.2592}}$$

(b) Let B be the event that at least 4 of them win the game.

$$P(B) = 0.2592 + 0.6^5$$
$$= \underline{\underline{0.33696}}$$

(c) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

$= \dfrac{P(A)}{P(B)}$

$= \dfrac{0.2592}{0.33696}$

$= \underline{\underline{0.7692}}$ *(cor. to 4 d. p.)*

4. Let I be the event that a person has such illness and T be the event of showing a positive result in the test.

(a) $P(T)$

$= P(T \mid I)P(I) + P(T \mid I')P(I')$

$= 0.08 \times 0.85 + (1 - 0.08) \times 0.05$

$= \underline{\underline{0.114}}$

(b) **(i)** Let E be the event that at least one of them shows a positive result.

$P(E)$

$= 1 - P$ (none of them shows a positive result)

$= 1 - (1 - 0.114)^{10}$

≈ 0.701917

$= \underline{\underline{0.7019}}$ *(cor. to 4 d. p.)*

(ii) Let F be the event that exactly two of them show positive results.

$P(F) = (1 - 0.114)^8 \times 0.114^2 \times C_2^{10}$

≈ 0.22207

$P(F \mid E) = \dfrac{P(F \cap E)}{P(E)}$

$= \dfrac{P(F)}{P(E)}$

$\approx \dfrac{0.22207}{0.701917}$

$= \underline{\underline{0.3164}}$ *(cor. to 4 d. p.)*

5. Let A, B and C denote machines A, B and C respectively, and let D be the event that a defective watch is selected.

(a) **(i)** $P(D)$

$= P(D \mid A)P(A) + P(D \mid B)P(B) + P(D \mid C)P(C)$

$= \dfrac{250}{1000} \times 0.02 + \dfrac{400}{1000} \times 0.05 + \dfrac{350}{1000} \times 0.04$

$= \underline{\underline{0.039}}$

(ii) $P(A \mid D)$

$= \dfrac{P(D \mid A)P(A)}{P(D \mid A)P(A) + P(D \mid B)P(B) + P(D \mid C)P(C)}$

$= \dfrac{\frac{250}{1000} \times 0.02}{0.039}$

$= \underline{\underline{\dfrac{5}{39}}}$

(iii) $P(B \mid D') = \dfrac{P(D' \mid B)P(B)}{1 - P(D)}$

$$= \frac{\frac{400}{1000} \times (1 - 0.05)}{1 - 0.039}$$

$$= \underline{\underline{\frac{380}{961}}}$$

(b) **(i)** P (all of them are not defective)

$= (1 - 0.039)^3$

≈ 0.887504

$= \underline{\underline{0.8875}}$ *(cor. to 4 d. p.)*

(ii) P (at least one of them is defective)

$= 1 - P$ (none of them is defective)

$= 1 - P$ (all of them are not defective)

$\approx 1 - 0.887504$

$= \underline{\underline{0.1125}}$ *(cor. to 4 d. p.)*

(iii) P (exactly one watch is defective)

$= 0.039 \times (1 - 0.039)^2 \times 3$

$= \underline{\underline{0.1081}}$ *(cor. to 4 d. p.)*