21093962D

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AMA1007I FINAL EXAM

1、

(a) $\lim\limits_{x \to 0} \frac{1-\cos(2x)}{x^2}$

$= \frac{\lim\limits_{x \to 0}(1-\cos(2x))}{\lim\limits_{x \to 0}(x^2)}$

$= \frac{0}{0}$

use L'Hopital Rule

$\therefore \frac{\lim\limits_{x \to 0} 1+\sin(2x)\cdot 2}{2x}$

$= \lim\limits_{x \to 0} \frac{1+\cos(2x)\cdot 4}{2} = \frac{5}{2}$

(b) $\lim\limits_{x \to +\infty} \frac{\frac{\pi}{2} - \arctan x}{\sin(\frac{1}{x})}$

$= \frac{-\infty}{+\infty}$ $\therefore$ Because $\frac{-\infty}{+\infty}$.

~~∴ use L'Hopital Rule~~

$\cancel{= \lim\limits_{x \to +\infty} \frac{-\frac{1}{1+x^2}}{\cos(\frac{1}{x}) \cdot \frac{1}{x^2}}}$

2、

(a)

$x > 2: f'(x) = 2x$

$x \leq 2: f'(x) = a$

$2x = a$

$a = 2 \times 2 = 4$

$f(x) = 4x+3, x \leq 2$

$f'(x) = 4$

(b) $e^{x+y} - xy - e = 0$

3、

$\int \sec x \, dx = \int \frac{1}{\cos x} dx$

$= \ln|\cos x| + C$

4、

(a) $y = \tan x$

$V = \pi \int_0^1 \tan^2(x) \cdot dx$

(b)

$V = \pi \cdot \frac{\tan^3(x)}{3} \cdot \ln|\sec x| \Big|_{x=0}^{x=1}$

$V \approx$

5、

$\int_0^1 e^{6x} \sin(e^{3x}) dx$

$u = e^{6x}$ $\frac{du}{dx} = e^{3x}$ $dv = \sin(e^{3x})$

$V = -\frac{\cos(e^{3x})}{3e^{3x}}$

$\because \int u dv = uv - \int v du$

$\therefore = e^{6x} \cdot -\frac{\cos(e^{3x})}{3e^{3x}} - \int -\frac{\cos(e^{3x})}{3e^{3x}} \cdot 3x \cdot dx$

$= -e^{3x} \cdot \frac{\cos(e^{3x})}{3} - \int$

6、

(a) $y=0$, x-int: $0 = \sqrt[3]{x^3 - x^2 - x + 1} \Rightarrow x^3 - x^2 - x + 1 = 0 \Rightarrow x = \pm 1 \Rightarrow (1,0)\ (-1,0)$

$x=0$, y-int: $f(0) = \sqrt[3]{1} \Rightarrow f(0) = 1 \Rightarrow (0,1)$

domain: $(-\infty, +\infty)$

(b)

(c)

$$f'(x)=\frac{1}{3}\cdot(x^3-x^2-x+1)^{-\frac{2}{3}}\cdot(3x^2-2x-1)$$

$$f''(x)=\frac{(x^3-x^2-x+1)\cdot(18x-6)-2}{9(x^3-x^2-x+1)^{\frac{5}{3}}}$$

$f'(x)=0$

$3x^2-2x-1=0$

$x_1=-\frac{1}{3}$

$x_2=1$

x	$-\infty<x<-\frac{1}{3}$	$-\frac{1}{3}\leq x\leq 1$	$1<x<+\infty$
$f(x)$	Increase	decrease ~~Increase~~	~~de~~ Increase
	up	down	up

(d) $\begin{cases} x=-\frac{1}{3},\ f(-\frac{1}{3})=1.058267368 \\ x=1,\ f(1)=0 \end{cases}$

$\therefore$ local min: $f(1)=0$

local max: $f(-\frac{1}{3})=1.058267368$

inflection:

$f''(x)=0$

$(x^3-x^2-x+1)\cdot(18x-6)-2=0$

7、

(a) $y=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$

$\cancel{y=(2\pi)^{-\frac{1}{2}}\cdot e^{-\frac{x^2}{2}}}$

$$y'=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2}}\cdot(-x)$$

$$y'=-\frac{x}{\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2}}$$

$$y''=-\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2}}+\left(-\frac{x}{\sqrt{2\pi}}\cdot e^{-\frac{x^2}{2}}\cdot(-x)\right)$$

$$y''=\cancel{\frac{x+x}{\sqrt{2\pi}}}\ \frac{x^2\cdot e^{-\frac{x^2}{2}}-e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}=\frac{(x^2-1)\cdot e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}$$

$y'=0$

$-\frac{x\cdot e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}=0$

$x=0$

$y''=0$

$\frac{(x^2-1)\cdot e^{-\frac{x^2}{2}}}{\sqrt{2\pi}}=0$

$x^2-1=0$

$x=\pm 1$

$\begin{cases} x=1:\ f(1)=0.24197 \\ x=0:\ f(0)=0.39894 \\ x=-1:\ f(-1)=0.24197 \end{cases}$

$\therefore$ The global max is $x=0$, $y=$ ~~0.39894~~ $\frac{1}{\sqrt{2\pi}}$,

global min are $x=1$ and -1, $y=\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}}$

(b) $\int_{-\infty}^{+\infty}x^{101}y(x)\,dx$

8、

$$\int \frac{x^5+1}{x^3(x+2)}dx$$

$$= \int \frac{x^5}{x^3(x+2)}dx + \int \frac{1}{x^3(x+2)}dx$$

$$= \int \frac{x^2}{x+2}dx + \int \frac{1}{x^3(x+2)}dx$$

$u = x^2$, $\frac{du}{dx} = 2x$ $\quad dx = \frac{du}{2x}$

$$= \int \frac{u}{}$$

9、

(a)

Inverse: $A^{-1} = \begin{bmatrix} 1 & 3 & 5 \\ -1 & 4 & -8 \\ 1 & 5 & 12 \end{bmatrix}^{-1} = $

$$\det(A) = \det\begin{bmatrix} 1 & 3 & 5 \\ -1 & 4 & -8 \\ 1 & 5 & 12 \end{bmatrix}$$

$$= a_{11}\det(A_{11}) + a_{21}\det(A_{21}) + a_{31}\det(A_{31})$$

$$= 1\times\det(A_{11}) - 1\times\det(A_{21}) + 1\times\det(A_{31})$$

$$= C_{11} - C_{21} + C_{31}$$

$$= \begin{bmatrix} 4 & -8 \\ 5 & 12 \end{bmatrix} - \begin{bmatrix} 3 & 5 \\ 5 & 12 \end{bmatrix} + \det\begin{bmatrix} 3 & 5 \\ 4 & -8 \end{bmatrix}$$

$$= (4\times12-(-8)\times5) - (3\times12-5\times5) + (3\times(-8)-5\times4)$$

$= 33$ $\quad \therefore$ determinant is 33.

(b)

$$\begin{matrix} 1 & 3 & 5 & 3 \\ -1 & 4 & -8 & 0 \\ 1 & 5 & 12 & 1 \end{matrix}$$

$\Downarrow R_2 = R_1 + R_2$

$$\begin{matrix} 1 & 3 & 5 & 3 \\ 0 & 7 & -3 & 3 \\ 1 & 5 & 12 & 1 \end{matrix}$$

$\Downarrow R_3 = R_3 - R_1$

$$\begin{matrix} 1 & 3 & 5 & 3 \\ 0 & 7 & -3 & 3 \\ 0 & 2 & 7 & -2 \end{matrix}$$

$\Downarrow R_1 = R_1 - R_3$

$$\begin{matrix} 1 & 1 & -2 & 1 \\ 0 & 7 & -3 & 3 \\ 0 & 2 & 7 & -2 \end{matrix}$$

$\Downarrow R_2 = R_2 - R_3$

$$\begin{matrix} 1 & 1 & -2 & 1 \\ 0 & 5 & -10 & 5 \\ 0 & 2 & 7 & -2 \end{matrix}$$

$\Downarrow R_2 = R_2/5$

$$\begin{matrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 2 & 7 & -2 \end{matrix}$$

$\Downarrow R_3 = R_3 - 2R_2$

$$\begin{matrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & -11 & -4 \end{matrix}$$

$\Downarrow R_3 = R_3/-11$

$$\begin{matrix} 1 & 1 & -2 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & \frac{4}{11} \end{matrix} \xRightarrow{R_1 = R_1 - R_2} \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & \frac{4}{11} \end{matrix} \xRightarrow{R_2 = R_2 + 2R_3} \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{19}{11} \\ 0 & 0 & 1 & \frac{4}{11} \end{matrix}$$

$$\therefore \begin{cases} \cancel{x_1 + x_2 - 2x_3 = 1} \\ \cancel{x_2 - 2x_3 = 1} \\ \cancel{x_3 = \frac{4}{11}} \end{cases}$$

$$\therefore \begin{cases} x_1 = 0 \\ x_2 = \frac{19}{11} \\ x_3 = \frac{4}{11} \end{cases}$$

10、

(a)

$$\begin{matrix} 3 & -1 & 0 & a \\ a & 2 & 0 & 4 \\ 2 & 0 & 1 & 2 \\ 13 & a & -2 & -7 \end{matrix}$$

(b) $Ax = b$

$$\begin{bmatrix} 1 & 3 & 1 & 1 \\ -1 & -3 & -4 & 2 \\ 5 & 15 & 1 & 9 \end{bmatrix} \cdot x = \begin{bmatrix} 2 \\ 1 \\ 14 \end{bmatrix}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{cases} x_1 + 3x_1 + x_1 + x_1 = 2 \\ -x_2 - 3x_2 - 4x_2 + 2x_2 = 1 \\ 5x_3 + 15x_3 + x_3 + 9x_3 = 14 \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{3} \\ x_2 = -\frac{1}{6} \\ x_3 = \frac{7}{15} \end{cases}$$

$$\therefore \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{6} \\ \frac{7}{15} \end{bmatrix}$$