AMA2111 Mathematics I

2022-23 Semester 1 Homework 1

Due Date: 17:00, Sunday 16th October 2022

- Put the following information on the top right corner of the front page of your homework.
  - Your name and student number
  - Subject code: AMA2111
  - Subject lecturer: Dr Bob He
- Photograph your solutions onto a PDF file named YourName_StuID, otherwise the marker (not the lecturer) cannot write on your solution, then you cannot see the marking but only the score.
- You may using the app "CamScanner" or other softwares. Make sure that the file is complete, legible, in correct order and orientation.
- Upload/attach your homework solution pdf file at the same place you've downloaded this homework by pressing the "Browse My Computer", then choose your pdf file, and then press Submit. You may re-submit the homework again, to a maximum of twice, before the due time. After submitting, check and make sure your submission is successful.
- No late submission is allowed. It may not be marked.

1. Find all the cubic roots of $\dfrac{(\sqrt{3}-i)^{10}}{(-1+\sqrt{3}i)^7}$ and plot them in the complex plane.

2. Consider the linear system

$$\begin{bmatrix} 1 & 3 & 2 & -1 \\ -2 & -5 & -2 & 3 \\ -1 & 0 & a & 5 \\ 3 & 7 & 2 & a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ b \\ 2 \end{bmatrix}.$$

   (a) Find the conditions satisfied by a and b such that the system has

      i. no solutions;

      ii. infinitely many solutions;

      iii. a unique solution.

   Also solve the system when it has infinitely many solutions.

(b) Consider the vectors $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ -2 \\ -1 \\ 3 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 3 \\ -5 \\ 0 \\ 7 \end{bmatrix}$, $\boldsymbol{v}_3 = \begin{bmatrix} 2 \\ -2 \\ a \\ 2 \end{bmatrix}$, $\boldsymbol{v}_4 = \begin{bmatrix} -1 \\ 3 \\ 5 \\ a \end{bmatrix}$ and $\boldsymbol{v}_5 = \begin{bmatrix} 1 \\ 0 \\ b \\ 2 \end{bmatrix}$ in $\mathbb{R}_4$.

i. Find the conditions satisfied by a and b such that the span of $\boldsymbol{v}_1$,$\boldsymbol{v}_2$,$\boldsymbol{v}_4$ and $\boldsymbol{v}_5$ is $\mathbb{R}_4$.

ii. Find the conditions satisfied by a and b such that $\boldsymbol{v}_3$,$\boldsymbol{v}_4$ and $\boldsymbol{v}_5$ are linearly dependent and write $\boldsymbol{v}_3$ as a linear combination of $\boldsymbol{v}_4$ and $\boldsymbol{v}_5$, if possible.

3. Let $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 1 & -1 \\ -2 & 4 & 5 \end{bmatrix}$. Find a nonsingular matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

4. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) Find the eigenvalues and eigenvectors of A.

(b) Find an orthogonal matrix Q and a diagonal matrix D such that $A = QDQ^T$.

5. Find the general solution of the differential equation

$$y' \cos x + \sin^2 y = \cos^2 y.$$

6. Find the general solution of the differential equation

$$-xy' + 2y = \ln x.$$

7. Find the explicit function $y(x)$ of the initial value problem

$$y' - \frac{y}{x} = \frac{(x+y)^2}{x^2} \ (x > 0), \ y(1) = -2.$$