AMA10071 Assignment 2

JIANG Guanlin
21093962D

(1)
$$\int_a^b f(x)dx = \int_0^3 (x^2-4x)dx$$
$$\Delta x = \frac{b-a}{n} = \frac{3-0}{6} = \frac{1}{2}$$
$$f(\tfrac{1}{2}) = (\tfrac{1}{2})^2 - \tfrac{1}{2}\times 4 = -1.75$$
$$f(1) = 1-4\times 1 = -3$$
$$f(\tfrac{3}{2}) = (\tfrac{3}{2})^2 - \tfrac{3}{2}\times 4 = -3.75$$
$$f(2) = (2)^2 - \tfrac{3}{2}\times 2 = -4$$
$$f(\tfrac{5}{2}) = (\tfrac{5}{2})^2 - 4\times\tfrac{5}{2} = -3.75$$
$$f(3) = 3^2 - 4\times 3 = -3$$

$$= \tfrac{1}{2}(-1.75-3-3.75-4-3.75-3)$$
$$= -9.625$$

$$\int_0^3 (x^2-4x)dx$$
$$= \frac{x^3-6x^2}{3}\Big|_{x=0}^{x=3}$$
$$= \frac{27-6\times 9}{3}$$
$$= -9$$

right Riemann sum ≠ the value.

(2) $f'(x) = x + \frac{1}{x^3}$

$$f(x) = \int x + \frac{1}{x^3}\Big|_{x=2}^{x>0} = 6$$
$$f(x) = \frac{x^2}{2} + \int \frac{1}{x^3} = \frac{x^2}{2} - \frac{1}{2x^2} + C = 6\Big|_{x=2}^{x>0}$$
$$\frac{4}{2} - \frac{1}{8} + C = 6$$
$$C = \frac{33}{8}$$
$$\therefore f(x) = \frac{x^2}{2} - \frac{1}{2x^2} + \frac{33}{8}$$

(3)

1、 $2\int x\sin(x^2)dx$
$$\Rightarrow u = x^2$$
$$du = 2x\cdot dx \Rightarrow dx = \frac{du}{2}$$
$$= 2\int \frac{\sin(u)}{2}\cdot du$$
$$= \int \sin(u)\cdot du$$
$$= -\cos(u)$$
$$= -\cos(x^2) + C$$

2、 $\int x^3 e^{x^4} dx$
$$\Rightarrow u = x^4$$
$$du = 4x^3\cdot dx = x^3 dx = \frac{du}{4}$$
$$= \int \frac{e^u}{4} du$$
$$= \frac{1}{4}\int e^u\cdot du$$
$$= \frac{1}{4}e^u = \frac{1}{4}e^{x^4} + C$$

3、 $\int (1-2x)^8 dx$
$$\Rightarrow u = 1-2x$$
$$du = -2\cdot dx \Rightarrow dx = -\frac{du}{2}$$
$$= \int (-\frac{u^8}{2})du$$
$$= -\frac{1}{2}\int u^8 du$$
$$= -\frac{1}{2}\times\frac{u^9}{9}$$
$$= -\frac{u^9}{18}$$
$$= C - \frac{(1-2x)^9}{18} = \frac{(2x-1)^9}{18} + C$$

4、 $\frac{1}{2}\int (x+1)\sqrt{2x+x^2}\,dx$
$$\Rightarrow u = \sqrt{2x+x^2}$$
$$du = \frac{x+1}{\sqrt{2x+x^2}}\cdot dx \Rightarrow (x+1)dx = \sqrt{2x+x^2}\cdot dx$$
$$= \frac{1}{2}\int \sqrt{u}\cdot du$$
$$= \frac{1}{2}\times\frac{2}{3}u^{\frac{3}{2}}$$
$$= \frac{u^{\frac{3}{2}}}{3} = \frac{(x^2+2x)^{\frac{3}{4}}}{3} + C$$

5、 $\int \frac{\cos x}{\sin^3 x} dx$

$\Rightarrow u = \sin(x) \Rightarrow du = \cos(x) dx$

$= \int \frac{1}{u^3} du$

$= \cancel{\quad} - \frac{1}{2u^2}$

$= -\frac{1}{2\sin^2(x)} + C$

6. $\int \cot x \, dx$

$= \int \frac{\cos(x)}{\sin(x)} dx$

$\Rightarrow u = \sin(x)$

$du = \cos(x) \cancel{dx}$

$= \int \frac{1}{u} du$

$= \ln(u)$

$= \ln(|\sin(x)|) + C$

(4)

1、 $\int_1^9 3\frac{x-1}{\sqrt{x}} dx$

$= 3\int_1^9 \frac{x-1}{\sqrt{x}} dx$

$= 3\int_1^9 \left(\frac{x}{\sqrt{x}} - \frac{1}{\sqrt{x}}\right) dx$

$= 3\left(\int_1^9 \frac{x}{\sqrt{x}} dx - \int_1^9 \frac{1}{\sqrt{x}} dx\right)$

$= 3\left(\int_1^9 \sqrt{x}\, dx - \cancel{\quad}\ \sqrt{x}\Big|_{x=1}^{x=9}\right)$

$= 3\left(\frac{52}{3} - 4\right)$

$= 52 - 12$

$= 40$

2、 $\int_1^3 (2x - 3^x) dx$

$= \cancel{\int_1^3 2x}\ \int_1^3 2x \cdot dx - \int_1^3 3^x \cdot dx$

$= \cancel{\quad}\ x^2\Big|_{x=1}^{x=3} - \frac{3^x}{\ln(3)}\Big|_{x=1}^{x=3}$

$= (9-1) - \frac{24}{\ln(3)}$

$= 8 - \frac{24}{\ln(3)}$

3、 $\int_0^2 \frac{x^4+10x^3+35x^2+52x+34}{(x+1)(x+3)^2} dx$

$\cancel{= \frac{\int_0^2 (x^4+10x^3+35x^2+52x+34)dx}{\int_0^2 (x+1)(x+3)^2 dx}}$

$\cancel{= \frac{\left(\frac{x^5}{5} + \frac{5x^4}{2} + \frac{35x^3}{3} + 26x^2 + 34x\right)\Big|_{x=0}^{x=2}}{\int_0^2 (x^3+7x^2+15x+9)dx}}$

$\cancel{= \frac{\frac{4676}{15}}{\left(\frac{x^4}{4} + \frac{7x^3}{3} + \frac{15x^2}{2} + 9x\right)\Big|_{x=0}^{x=2}}}$

$\cancel{= \frac{\frac{4676}{15}}{\frac{212}{3}} = \frac{1169}{265}}$

$= \int_0^2 \left(\frac{-x^2-2x+7}{(x+1)(x+3)^2} + x + 3\right) dx$

$= -\int_0^2 \frac{x^2+2x-7}{(x+1)(x+3)^2} dx + \int_0^2 x\,dx + 3\int_0^2 1\,dx$

$= 3\int_0^2 \frac{1}{x+3} dx + 2\int \frac{1}{(x+3)^2} dx - 2\int \frac{1}{x+1} dx + \int_0^2 x\,dx + 3\int_0^2 1\,dx$

$= -3\ln(x+3) + 2\ln(x+1) + \frac{2}{x+3} + \frac{x^2}{2} + 3x\Big|_{x=0}^{x=2}$

$= \frac{116}{15} - 3\ln(5) + 5\ln(3)$

4、 $\int_0^{12} \frac{dx}{\sqrt[3]{(1+2x)^2}}$

$= \int_0^{12} (1+2x)^{-\frac{2}{3}} dx$

$u = 1+2x,\ du = 2 \cdot dx$

$= \frac{1}{2}\int_1^{25} u^{-\frac{2}{3}} dx$

$= \frac{1}{2} \times 3u^{\frac{1}{3}}\Big|_1^{25}$

$= \frac{1}{2} \times (3 \times 25^{\frac{1}{3}} - 3)$

$= \frac{3 \times 25^{\frac{1}{3}} - 3}{2}$

5. $\int_1^e \ln x \, dx$ $\quad dv = dx$, $v = x$

$\Rightarrow u = \ln x \Rightarrow du = \frac{1}{x}dx \Rightarrow dx = x\,du$

$= (\ln x \cdot x - \int 1 dx)\Big|_{x=1}^{x=e}$

$= (x\ln(x) - x)\Big|_{x=1}^{x=e}$

$= e\ln(e) - e - \ln(1) + 1$

$= 1$

6. $\int_1^2 x \cdot \ln x \, dx$ $\quad u = \ln(x)$, $du = \frac{1}{x}dx$, $dv = x\,dx$, $V = \frac{x^2}{2}$

$= (\ln(x)\frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx)\Big|_{x=1}^{x=2}$

$= (\frac{x^2\ln(x)}{2} - \frac{(\frac{x^2}{2})}{2})\Big|_{x=1}^{x=2}$

$= (\frac{x^2\ln(x)}{2} - \frac{x^2}{4})\Big|_{x=1}^{x=2}$

$= \frac{4\ln(2)}{2} - \frac{4}{4} - \frac{\ln(1)}{2} + \frac{1}{4}$

$= -\frac{3}{4} + 2\ln(2)$

7. $\int_0^{10} x \cdot e^x dx$

$\Rightarrow u = x$, $du = dx$, $dv = e^x \cdot dx$, $v = e^x$

$\therefore = (x \cdot e^x - \int e^x \cdot dx)\Big|_{x=0}^{x=10}$

$= (x \cdot e^x - e^x)\Big|_{x=0}^{x=10}$

$= 10 \cdot e^{10} - e^{10} - 0 + 1$

$= 9e^{10} + 1$

(5)

$A = \left|\int_{-2}^1 [(x^3 - x^2 + x - 2) - (5x - 6)]dx\right| + \int_1^2 [(5x-6) - (x^3 - x^2 + x - 2)]dx$

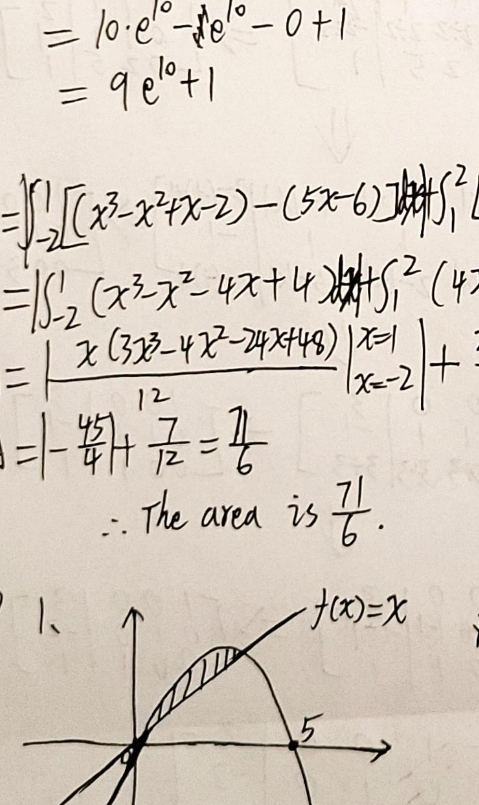
$A = \left|\int_{-2}^1 (x^3 - x^2 - 4x + 4)dx\right| + \int_1^2 (4x - x^3 + x^2 - 4)dx$

$A = \left| \frac{x(3x^3 - 4x^2 - 24x + 48)}{12}\Big|_{x=-2}^{x=1} \right| + \frac{x(-3x^3 + 4x^2 + 24x - 48)}{12}\Big|_{x=1}^{x=2}$

$A = \left|-\frac{45}{4}\right| + \frac{7}{12} = \frac{71}{6}$

$\therefore$ The area is $\frac{71}{6}$.

(6) 1.



$x = 5x - x^2$

$x^2 - 4x = 0$

$x = 4, x = 0$

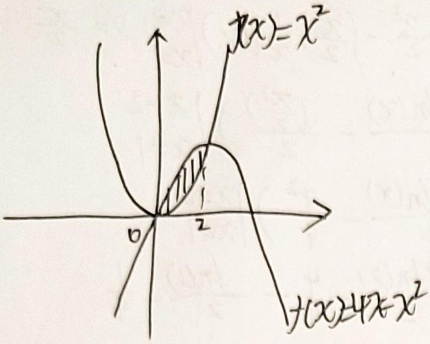
$\int_0^4 ((5x - x^2) - x)dx$

$= \int_0^4 (4x - x^2)dx$

$= \frac{6x^2 - x^3}{3}\Big|_{x=0}^{x=4} = \frac{32}{3}$

$\therefore$ Area is $\frac{32}{3}$.

2.



$$x^2=4x-x^2$$
$$2x^2=4x$$
$$x=2,0$$

$$A=\int_0^2(4x-x^2-x^2)\,dx$$
$$A=\int_0^2(4x-2x^2)\,dx$$
$$A=\frac{6x^2-2x^3}{3}\Big|_{x=0}^{x=2}$$
$$A=\frac{8}{3}-0=\frac{8}{3}$$
$\therefore$ Area is $\frac{8}{3}$.

(7)

(a)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1 & 3 & 3 & 0 \\ 1 & 3 & 6 & 3 \end{array}\right]$$

(b)

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 1-1\times1 & 3-1\times1 & 3-1\times1 & 0-2\times1 \\ 1-1\times1 & 3-1\times1 & 6-1\times1 & 3-2\times1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 2 & 2 & -2 \\ 0 & 2 & 5 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0\div2 & 2\div2 & 2\div2 & -2\div2 \\ 0 & 2 & 5 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 2 & 5 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1-0\times1 & 1-1\times1 & 1-1\times1 & 2-(-1)\times1 \\ 0 & 1 & 1 & -1 \\ 0-0\times1 & 2-1\times1 & 5-1\times1 & 1-(-1)\times1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 3 & 3 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & -1 \\ 0\div3 & 0\div3 & 3\div3 & 3\div3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\Downarrow$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0-0 & 1-0 & 1-1 & -1-1 \\ 0 & 0 & 1 & 1 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$$\therefore \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

(c)
$$\begin{cases} x=3 \\ y=-2 \\ z=1 \end{cases}$$