COMP 2012 Quiz 1
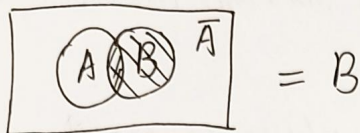## JIANG Guanlin (21093962D)

Q1) $(A \cap B) \cup (\bar{A} \cap B) = B$

 $= B$

consider LHS $= (A \cap B) \cup (\bar{A} \cap B)$

$\underset{\text{by distributive}}{=} B \cap (A \cup \bar{A}) \Rightarrow B \cap \cancel{A \cup B}$

$= B = RHS = B$

$\therefore$ True $\Rightarrow (A \cap B) \cup (\bar{A} \cap B) = B$
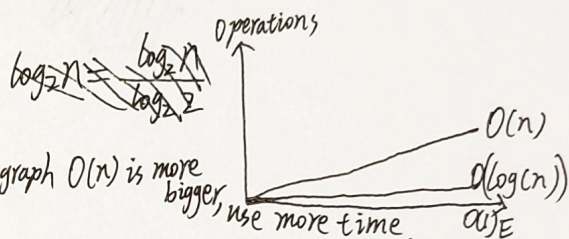
Q2)

$f(n) = 25 + 8n + \log_2 n$

$f(n) = O(1) + O(n) + O(\log n) = O(n)$

$\because O(n) > O(\log n) \Rightarrow$ according to graph $O(n)$ is more bigger, use more time.

$\therefore O(n) = O(n)$

$\log_2 n = \dfrac{\log_2 n}{\log_2 2}$

$\therefore$ True $\Rightarrow f(n) = 25 + 8n + \log_2 n$ is $O(n)$.

$\rightarrow$ also: $\log_2 n < n \Rightarrow \log_b n = \dfrac{\log_2 n}{\log_2 b} < \dfrac{1}{\log_2 b} n$.

operations graph: $O(n)$, $O(\log(n))$, axis

Q3) let $n = 1$:

$5^n - 1 = 5^1 - 1 = 4$ $\dfrac{4}{4} = 1$ $\therefore$ True

let $n = k$:

$k \in \mathbb{N}$

$5^n - 1 = 5^k - 1 = 4m$ $(m \in \mathbb{Z})$

$\dfrac{4m}{4} = m$ $\therefore$ True.

$\therefore 5^n - 1$ is divisible by 4 is True.

let $n = k+1$:

$5^n - 1 = 5^{(k+1)} - 1 = 5^k \times 5^1 - 1 = 5^k \times (4+1) - 1 = 4 \times 5^k + \cancel{5^k} - 1 \Rightarrow$

$\because 5^k - 1 = 4m$

$\therefore 4 \times 5^k + 4m = 4(5^k + m)$ $\therefore \dfrac{4(5^k + m)}{4} = 5^k + m$

$\therefore$ True

Q4).

let $A \leftrightarrow B$

＃ let if in any group of 7 people, there is a person who knows the odd number of people.

use point to be the people, use edges to be their relation, so it need 2 points 1 edge,

it must be even number of people. for example:

for odd number. ~~~~~~ use (A, B) to display the point and edge.

need $2 \times (A, B)$

∴ need even number of people.