1. (30 points) Multiple-choice questions.

   (a) Finding a minimum element from a binary search tree takes time.   ___

   A. $\Theta(1)$ B. $\Theta(\log n)$ C. $O(\log n)$ D. $O(n)$

   (b) What is the height (depth) of a complete binary tree with 2011 nodes?   ___

   A. 8   B. 9   C. 10   D. 11

   (c) The number of nodes in a binary tree of height (depth) 7 *cannot* be?   ___

   A. 8   B. 88   C. 126   D. 288

   (d) In a binary tree on n nodes $(n > 0)$ and ℓ leaves, the number of nodes with two children *cannot* be?   ___

   A. $n - 2\ell$ B. $n - \ell - 1$ C. ℓ D. None of them

   (e) After inserting elements with keys [56, 54, 50, 20, 45, 55, 47, 41, 40, 70, 30, 35, 65] into an empty binary search tree, its depth is   ___

   A. ≤ 7 B. 8   C. 9   D. ≥ 10

   (f) In the AVL tree below, the balance factors of nodes containing elements 50 and 71 are?   ___
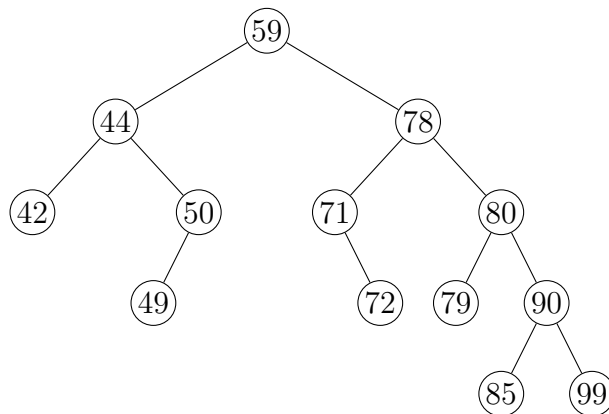
   A. Both -1 B. Both 0   C. Both 1   D. They are different

   (g) Which of the following arrays represents a heap.   ___

   A. [2432, 2411, 2422, 2322, 2022, 2222, 2421, 2021, 2012, 2121, 2011]

   B. [2432, 2411, 2422, 2322, 2022, 2012, 2421, 2222, 2121, 2021, 2011]

   C. [2432, 2411, 2422, 2322, 2022, 2421, 2012, 2222, 2021, 2121, 2011]

   D. None of them

   (h) After inserting elements with keys [56, 54, 50, 20, 45, 55, 47, 41, 40, 70, 30, 35, 65] into an empty binary heap, its depth is   ___

   A. ≤ 3 B. 4   C. 5   D. ≥ 6

   (i) Is heapsort stable when sorting arrays [9, 6, 6] and [6, 6, 9]?   ___

   A. Only [9, 6, 6]   B. Only [6, 6, 9]   C. Neither of them   D. Both of them

(j) Is heapsort stable when sorting arrays [2, 0, 2, 1] and [2, 0, 1, 1]?   ___

    A. Only [2, 0, 2, 1]

    B. Only [2, 0, 1, 1]

    C. Neither of them

    D. Both of them

2. (15 points) Fill in the blank space.

    (a) (4 points) A binary tree of characters has the preorder sequence "algorithm" and the inorder sequence "gloraihtm." In the level-wise traversal of this tree, the fourth element is (start counting from 1):_____.

    (b) (3 points) There are _____ `null` references in a binary tree of n nodes. There are _____ `null` references in a linked list of n nodes. There are _____ `null` references in a doubly linked list of n nodes.

    (c) (8 points) Insert the ten letters of the word "ALGORITHMS" into an empty binary search tree and an empty heap. Draw the results.

    (a) binary search tree   (b) heap

3. (9 points) Given a binary tree, calculate the total number of nodes with only one child. The `Node` class is given.

```
private class Node {
    int i;
    public Node leftChild, rightChild;
}
Node root;

int nodesWithOneChild()
```

4. (6 points) Check whether an array represents a *minimum* heap.

```
boolean isMinHeap(int[] a)
```

                —The End—