Assignment 1 AMA1104

JIANG Guanlin 21093962D

1. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(a) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= \frac{3}{5} + \frac{1}{6} - \frac{7}{10} = \frac{1}{15}$

≈ 0.0667

(b) $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$= \frac{\frac{1}{15}}{\frac{1}{6}}$

$= 0.4$

2.

(a) $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$= \frac{P(B) \times P(A|B)}{P(A)}$

$= \frac{\frac{1}{2} \times \frac{1}{5}}{\frac{2}{7}} = \frac{7}{20} = 0.35$

(b) $P(B|A') = \frac{P(B \cap \bar{A})}{1 - P(A)}$

$= \frac{(1 - \frac{1}{5}) \times \frac{1}{2}}{1 - \frac{2}{7}}$

$= 0.56$

(c) $P(B'|A') = 1 - 0.56$

$= 0.44$

3. (a)

$P(\text{first yellow and last two green})$

$= \frac{C_1^6}{C_1^{15}} \times \frac{C_2^9}{C_2^{14}} \times \frac{C_1^8}{C_1^{13}}$

$= 0.1582$

(b) $\frac{C_2^6}{C_3^{15}} + \frac{C_2^9}{C_2^{15}} \frac{P_2^6 P_1^9}{P_3^{15}} \times 2 + \frac{P_2^9 P_1^6}{P_3^{15}}$

$= 0.4857$

$= 0.2571$

4.

(a) drawn

$P(2 \text{ black}) = (C_2^{26} \times C_2^{26}) \div C_4^{52} = 0.3902$

(b)

$P(\text{at least 3 Kings drawn}) = P(3) + P(4)$

$= \frac{C_3^4 \times C_1^{48}}{C_4^{52}} + \frac{C_4^4 \times C_0^{48}}{C_4^{52}} = 0.0007$

(c) $P(2 \text{ black drawn} | \text{at least 3 kings drawn})$

$= \frac{\frac{C_4^4 \times C_0^{48}}{C_4^{52}} + \frac{C_2^2 \times C_1^2 \times C_1^{48}}{C_4^{52}} + \frac{C_2^2 \times C_1^2 \times C_1^{48}}{C_4^{52}}}{0.0007}$

$= 0.0004 / 0.0007 = 0.5714$

5.

(a) A: 1, 3, 5

$P(A) = \frac{3}{6} = \frac{1}{2}$

B: 2, 4, 6, but greater than 5, only 6

$P(B) = \frac{1}{6}$

$P(A \cap B) = 0$

$P(A \cap B) \neq P(A), P(B)$

$\therefore$ They are dependent.

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6.

(a) $P(\text{age between 30-50} \mid \text{can\_swim}) = \frac{P(\text{age between 30-50} \cap \text{can\_swim})}{P(\text{can\_swim})}$

$= \frac{0.30 \times 0.60}{0.30 \times 0.60 + 0.90 \times 0.45 + 0.25 \times 0.70}$

$= 0.2368 \approx 0.237$

(b) $P(\text{age below 30} \mid \text{cannot\_swim}) = \frac{P(\text{age below 30} \cap \text{cannot\_swim})}{P(\text{cannot\_swim})}$

$= \frac{0.45 \times 0.10}{0.45 \times 0.10 + 0.30 \times 0.40 + 0.25 \times 0.30} = 0.1875$

≈ 0.188

~~$= 0.187 \approx 0.187$~~

5.

(b) $P(A) = \frac{3}{6} = \frac{1}{2}$

$P(B^C) = \frac{4}{6} = \frac{2}{3}$

~~$P(A \cap B)$~~

$P(A \cap B) = P(A) \times P(B)$

∴ They are independent.

(c) $P(B) = \frac{1}{6}$

$P(D) = \frac{5}{36}$

$P(B \cap D) = \frac{1}{36}$

$P(B \cap D) \neq P(B) \times P(D)$

∴ They are dependent

(2,6)
(3,5)
(4,4)
(5,3)
(6,2)

(d) $P(C) = \frac{4}{6} = \frac{2}{3}$

$P(D) = \frac{5}{36}$

$P(C \cap D) = \frac{3}{36} = \frac{1}{12}$

$P(C \cap D) \neq P(C) \times P(D)$

∴ They are dependent.