COMP2012 (Fall 2022) Discrete Mathematics

Individual Assignment 1 Due Date: 23:59, 28^{th} October, 2022

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Notes:

- This is an **individual** assignment.
- Please submit the **soft copy** of your answer to Blackboard (as a doc/docx/pdf file).
- You just need to write your answer. There is no need to copy questions.

Question 1.

1(a) **[10 marks]**

You are given n integers $a_1, a_2, \ldots, a_n$.
Let $m = (a_1 + a_2 + \ldots + a_n)/n$.
Prove that there exists some number in $a_1, a_2, \ldots, a_n$
such that it is smaller than or equal to m.

1(b) **[10 marks]**

Prove that $\neg (p \vee q \vee r)$ and $\neg p \wedge \neg q \wedge \neg r$
are logically equivalent by using a truth table.

1(c) **[10 marks]**

Let Dom be the domain $\{2,4,6,8,10,12,14,16,18\}$.
Suggest two propositional functions $P(x)$ and $Q(y)$ so that all the following statements are true at the same time:

- $\exists\, x \in Dom\ P(x)$
- $\exists\, y \in Dom\ Q(y)$
- $\neg (\exists\, z \in Dom\ P(z) \wedge Q(z)\ )$

Prove that your proposed functions P and Q satisfy the above conditions.

Question 2.

2(a) **[10 marks]**

You are given the following sets: *A, B, C*.
Draw the Venn diagram
and **shade** the region of $(A - C) - B$.

2(b) **[10 marks]**

Let $A = \{a, b, c, d, e\}$ be a set with 5 elements.

(I) How many relations are there on A?

(II) How many equivalence relations $\sim$ are there on A that satisfy $d \sim e$ and $d \sim a$?

(*Hint: you may count the number of partitions*.)

Question 3.

3(a) **[10 marks]**

What is the asymptotic relationship between $f(n)$ and $g(n)$, where $f(n) = log_2 n^{log_2 23}$, and $g(n) = log_2 23^{log_2 n}$? You can determine whether $f(n) = O(g(n))$, $\Omega(g(n))$, and / or $\Theta(g(n))$.

3(b) **[10 marks]**

Show the relation $\prec$ is a transitive relation on the set of all real-valued functions on $(0, \infty)$. That is, **show** that $f(n) \prec g(n)$ and $g(n) \prec h(n)$ implies $f(n) \prec h(n)$. *(Hint: You may use the Big-O notation to rewrite the statement.)*

3(c) **[10 marks]**

You are given an array *A* of *n* integers.
Assume that *n* is odd and all *n* integers in the array *A* are different.
Write an algorithm to find the median of the input array *A*.

```
FindMedian ( Array A[1..n])
1. ???
2. ???
3. …
…
return ???
```

Question 4.

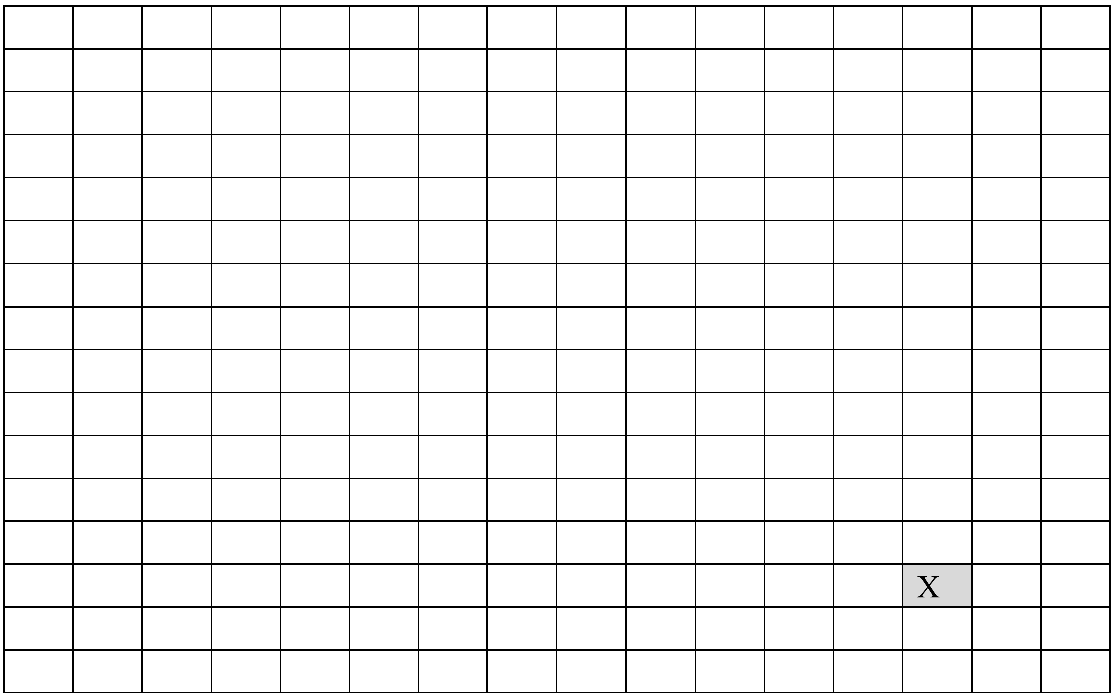
4(a) **[10 marks]**

You are given the following sets: S_1, S_2, …, S_n, T.
Prove that the following statement is true for all integers n
by using mathematical induction.

$$(S_1 \cup T) \cap (S_2 \cup T) \cap \ldots \cap (S_n \cup T) = (S_1 \cap S_2 \ldots \cap S_n) \cup T$$

4(b) **[10 marks]**

We are referring to the tiling problem in "MI: example 3" in the slides of lecture #5.
Now, we are given the following 16×16 checkboard with a missing square (X).



A triomino looks like one of the followings.

Draw the above checkboard after tiling it with triominoes.
Adjacent triominoes should be filled in different colours (like in the slides).

End of Assignment 1