GIRTON SUMMER PROGRAMME

MATHEMATICS FOR ENGINEERING

MATRICES: HOMEWORK QUESTIONS

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The Homework Answers sheet can be downloaded from Moodle. Once completed (for all five homeworks) is must be uploaded as a ".pdf".

Question: Matrix-1

Find, if it exists, the matrix inverse for the following matrices:

(i)
$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$
 (ii)
$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 2 & 6 \\ 6 & 1 & 3 \end{bmatrix}$$

Enter your value for the distance on the **Homework Answers sheet**.

Question: Matrix-2

A two-dimensional image is processed so that the original point $\mathbf{x}_{old} = \begin{bmatrix} x_{old} & y_{old} \end{bmatrix}^t$ is mapped to the new position $\mathbf{x}_{\text{new}} = \begin{bmatrix} x_{\text{new}} & y_{\text{new}} \end{bmatrix}^t$ given by the non-square matrix:

$$\begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} = \begin{bmatrix} \frac{1}{2}(a+b) & \frac{1}{2}(a-b) & c \\ \frac{1}{2}(a-b) & \frac{1}{2}(a+b) & d \end{bmatrix} \begin{bmatrix} x_{\text{old}} \\ y_{\text{old}} \\ 1 \end{bmatrix}$$

where $\begin{bmatrix} x_{old} & y_{old} & 1 \end{bmatrix}^t$ is the "augmented vector". The mapping can be re-written as:

$$\mathbf{x}_{\mathsf{new}} = \mathbf{x}_{\mathsf{0}} + \mathbf{A}\,\mathbf{x}_{\mathsf{old}}$$

- Find expressions for the vector $\boldsymbol{x}_{\scriptscriptstyle 0}$ and the 2x2 matrix \boldsymbol{A} . (i)
- Evaluate the determinant |A| (ii)
- By examining the vectors $\begin{bmatrix} 1 \\ 1 \end{bmatrix}^t$ and $\begin{bmatrix} 1 \\ -1 \end{bmatrix}^t$ describe the mapping. (iii)

Enter your description in the box on the **Homework Answers sheet**.