Intro to AI A3

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3.1.

(a) $P(T=0|S=1,R=1)$ and $P(T=1|S=1,R=1)$

$$= \frac{P(T=0,S=1,R=1)}{P(S=1,R=1)}$$

$f_0(T)$
$f_1(F)$
$f_2(T,F,A)$
$f_3(F)$
$f_4(A,C)$
$f_5(C)$

$f_6(A) = \sum_L f_4(A,L) \times f_5(L)$

$f_7(T,F) = \sum_A f_0(A) \times f_2(T,F,A)$

$f_8(T) = \sum_F f_1(F) \times f_3(F) \times f_7(T,F)$

$f_9(T) = f_8(T) \times f_0(T)$

f_6:

A	Values
0	0.01074
1	0.66120

f_7:

T	F	Values
0	0	0.010805
0	1	0.654695
1	0	0.563631
1	1	0.335970

f_8:

T	Values
0	0.005999
1	0.008604

f_9:

T	Values
0	0.005879
1	0.000172

$P(T|S=1,R=1)$:

T	Values
0	$P(T=0\|S=1,R=1) \Rightarrow P(A=0\|T=0,F=1) = 0.971564$
1	$P(T=1\|S=1,R=1) \Rightarrow P(A=1\|T=1,F=1) = 0.028436$

(b)

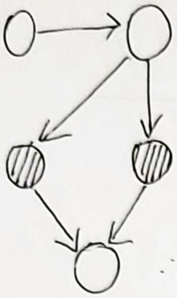
Phase of Algorithm	# multiplications	# additions	# divisions
E S	0	0	0
E R	0	0	0
E L	4	2	0
E A	8	4	0
E F	6	2	0
C T factors	2	0	0
ND over T	0	1	2
total	20	9	2

(c)

Phase of algorithm	# multiplications	# addititons	# divisions
compute $P(T=0, S=1, R=1)$	1	1	0
compute $P(T=1, S=1, R=1)$	1	1	0
ND over T	0	1	2
total	2	3	2

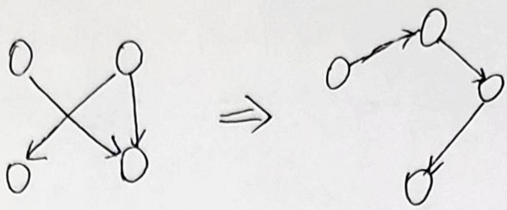
3.2、

(a)



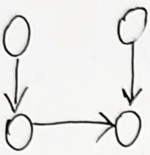
∴ Tree

(b)



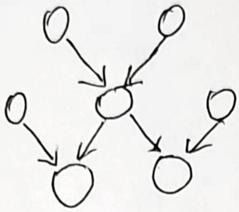
∴ Tree and polytree

(c)



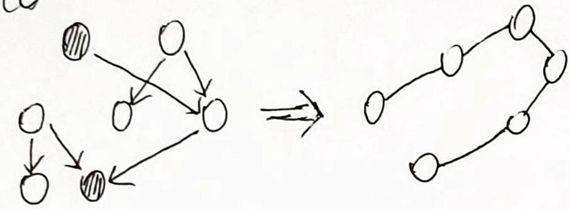
∴ Tree and ~~polytree~~ polytree

(d)



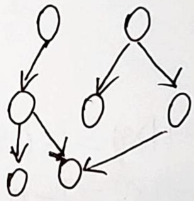
∴ Polytree

(e)



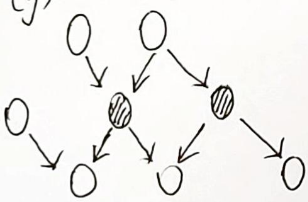
∴ Tree

(f)



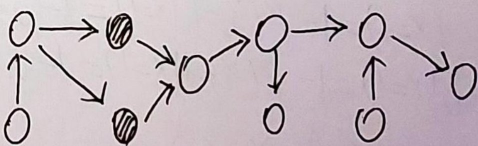
∴ Polytree

(g)



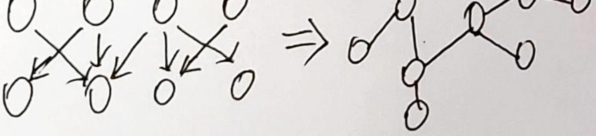
∴ Polytree

(h)



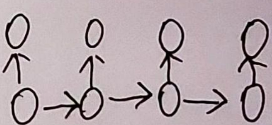
∴ Not Polytree

(i)



∴ Polytree and tree

(j)



∴ Polytree

3.3、

Y_1	Y_2	Y_3	Y	$P(Y\mid X=0)$	$P(Y\mid X=1)$	$P(Z_1=1\mid Y)$	$P(Z_2=1\mid Y)$
0	0	0	1	$0.15\times0.70\times0.5$ $=0.0525$	$0.75\times0.65\times0.3$ $=0.14625$	0.8	0.2
1	0	0	2	0.2075	0.04075	0.7	0.3
0	1	0	3	0.0225	0.07075	0.6	0.4
0	0	1	4	0.0525	0.34125	0.5	0.5
1	1	0	5	0.1275	0.02625	0.4	0.6
1	0	1	6	0.2975	0.11375	0.3	0.7
0	1	1	7	0.0225	0.10375	0.2	0.8
1	1	1	8	0.1275	0.06125	0.1	0.9

$P(Y_1=0\mid X=0)=1-P(Y_1=1\mid X=0)=1-0.85=0.15$

$P(Y_2=0\mid X=0)=1-P(Y_2=1\mid X=0)=1-0.3=0.7$

$P(Y_3=0\mid X=0)=1-P(Y_3=1\mid X=0)=1-0.5=0.5$

$P(Y_1=0\mid X=1)=1-P(Y_1=1\mid X=1)=1-0.25=0.75$

$P(Y_2=0\mid X=1)=1-P(Y_2=1\mid X=1)=1-0.35=0.65$

$P(Y_3=0\mid X=1)=1-P(Y_3=1\mid X=1)=1-0.7=0.3$

$\because P(Z_1=1\mid Y)=P(Z_1=1\mid Y_1,Y_2,Y_3)$

$P(Z_2=1\mid Y)=P(Z_2=1\mid Y_1,Y_2,Y_3)$

3.4

(a)

$$\mathcal{L}(p)=\sum_{k=1}^{n} C_k \log p_k$$

$$R(P)=\prod_{k=1}^{T} P_r(X^{(1)}=n^{(1)})=\prod_{k=1}^{n}(P_k)^{C_k}$$

$$\log R(P)=\log\left(\prod_{k=1}^{n}(P_k)^{C_k}\right)$$

$$\mathcal{L}(p)=\sum_{k=1}^{n}\log(P_k)^{C_k}=\sum_{k=1}^{n} C_k\log(P_k)$$

(b)

$$q_k=\frac{C_k}{T}$$

$$T=\Sigma_k C_k$$

$$KL(q,p)=\sum_{k=1}^{n} q_k\log q_k-\frac{\mathcal{L}(p)}{T}$$

$$KL(q,p)=\sum_{x\in X} q^{(x)}\log\left(\frac{q(x)}{p(x)}\right)$$

$$=\sum_{k=1}^{n} q_k\log\left(\frac{q_k}{p_k}\right)$$

$$=\sum_{k=1}^{n} q_k(\log(q_k)-\log(P_k))$$

$$=\sum_{k=1}^{n} q_k\log(q_k)-\sum_{k=1}^{n} q_k\log(P_k)$$

$$=\sum_{k=1}^{n} q_k\log q_k-\frac{1}{T}\sum_{k=1}^{n} C_k\log(P_k)$$

$$\sum_{k=1}^{n} C_k\log(P_k)=\mathcal{L}(p)$$

$$\therefore KL(q,p)=\sum_{k=1}^{n} q_k\log q_k-\frac{\mathcal{L}(p)}{T}$$

to minimizing $KL(p,q)$
need to maximizing $\mathcal{L}(p)$

(c) $KL(q,p)\geq 0$ if $q_k=P_k$

$P_k=C_k/T$

for Maximize $\mathcal{L}(p)$, we need $p_k=q_k$

Maximum likelihood estimate of P_k is $P_k=q_k=\frac{C_k}{T}$.