Intro to AI A2

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2.1

(a) $P(D=1 \mid S_1=1, S_2=1, \cdots S_K=1) = \frac{P(S_1=1, S_2=1, \cdots S_K=1 \mid D=1)\, P(D=1)}{P(S_1=1, S_2=1, \cdots S_K=1)}$

$P(D=0 \mid S_1=1, S_2=1, \cdots S_K=1) = \frac{P(S_1=1, S_2=1, \cdots S_K=1 \mid D=0)\, P(D=0)}{P(S_1=1, S_2=1, \cdots S_K=1)}$

$\gamma_K = \frac{P(D=0 \mid S_1=1, S_2=1, \cdots S_K=1)}{P(D=1 \mid S_1=1, S_2=1, \cdots S_K=1)}$

$= \frac{P(S_1=1, S_2=1, \cdots S_K=1 \mid D=0)\, P(D=0)}{P(S_1=1, S_2=1, \cdots S_K=1 \mid D=1)\, P(D=1)}$

$\because P(D=0) = P(D=1) = \frac{1}{2}$

$\therefore = \frac{P(S_1=1, S_2=1, \cdots S_K=1 \mid D=0)}{P(S_1=1, S_2=1, \cdots S_K=1 \mid D=1)}$

$P(S_1=1, S_2=1, \cdots S_K=1 \mid D=1)$

$\because = \frac{f(K-1)}{f(K)}$

$= \frac{2^{K-1} + (-1)^{K-1}}{2^K + (-1)^K}$

$P(S_1=1, S_2=1, \cdots S_K=1 \mid D=0)$

$= P(S_1=1 \mid D=0) \times P(S_2=1 \mid D=0) \times \cdots \times P(S_K=1 \mid D=0)$

$= (1-\frac{1}{2}) \times (1-\frac{1}{2}) \times \cdots \times (1-\frac{1}{2})$

$= \frac{1}{2^K}$

$\therefore = P(S_1=1 \mid D=1) \times P(S_2=1 \mid D=1) \times \cdots \times P(S_K=1 \mid D=1)$

$= \frac{f(0)}{f(1)} \times \frac{f(1)}{f(2)} \times \frac{f(2)}{f(3)} \times \cdots \times \frac{f(K-1)}{f(K)}$

$= 1 \times \frac{f(1)}{f(2)} \times \frac{f(2)}{f(3)} \times \cdots \times \frac{f(K-1)}{f(K)}$

$= \frac{f(1)}{f(K)}$

$= \frac{1}{2^K + (-1)^K}$

$\therefore \gamma_K = \frac{\frac{1}{2^K}}{\frac{1}{2^K + (-1)^K}} = \frac{2^K + (-1)^K}{2^K} = 1 - \frac{1}{2^K}$

$= 1 - \frac{1}{2^K}$

(b) $\because$ K is odd: $\gamma_K > 1$

K is even: $\gamma_K < 1$

$\therefore$ The doctor diagnoses on odd days: $D=1$; on even days: $D=0$.

follow the K increase, the γ_K more close to 1.

2.2

(a) CPT:

X_1	X_2	X_3	$P(Y=1 \mid X_1, X_2, X_3)$
0	0	0	0
1	0	0	$\frac{1}{6}$
0	1	0	$\frac{1}{3}$
0	0	1	$\frac{2}{5}$
1	1	0	$\frac{4}{9}$
1	0	1	$\frac{1}{2}$
0	1	1	$\frac{4}{5}$
1	1	1	$\frac{5}{6}$

$P(Y=1 \mid X_1=0, X_2=0, X_3=0) = 0$
$P(Y=1 \mid X_1=0, X_2=1, X_3=0) = \frac{1}{3}$
$P(Y=1 \mid X_1=0, X_2=1, X_3=1) = \frac{4}{5}$
$P(Y=1 \mid X_1=1, X_2=1, X_3=1) = \frac{5}{6}$

(b)

$P(X_2=1)$
$P(X_2=1 \mid Y=0)$
$P(X_2=1 \mid Y=1)$
$P(X_2=1 \mid Y=1, X_1=0, X_3=0)$
$P(X_2=1 \mid Y=1, X_1=1, X_3=1)$

$\Rightarrow$ (big $\rightarrow$ small)

$P(X_2=1 \mid Y=1, X_1=0, X_3=0)$
$P(X_2=1 \mid Y=1)$
$P(X_2=1 \mid Y=1, X_1=1, X_3=1)$
$P(X_2=1)$
$P(X_2=1 \mid Y=0)$

(c) $P(X_i=1)=\frac{1}{3}$ $i \in \{1,2,3\}$

$P(X_2=1 \mid Y=0) = 0$

$P(X_2=1 \mid Y=1)$

$= \frac{P(X_2=1, Y=1)}{P(Y=1)}$

$= \frac{P(X_2=1)}{P(Y=1)}$

$= \frac{\frac{1}{3}}{(1-(\frac{2}{3})^3)}$

$= \frac{9}{19}$

$P(X_2=1 \mid Y=1, X_1=0, X_3=0)$

$= \frac{P(X_2=1, Y=1, X_1=0, X_3=0)}{P(Y=1, X_1=0, X_3=0)}$

$= 1$

$P(X_2=1 \mid Y=1, X_1=1, X_3=1)$

$= \frac{P(X_2=1, Y=1, X_1=1, X_3=1)}{P(Y=1, X_1=1, X_3=1)}$

$= \frac{1}{3}$

$\therefore$ The values are match.

2.3

(a)

In file hw2_2-3_code.py.

(b)

correctly guessed	incorrectly guessed	best next guess ℓ	$P(L_i=\ell$ for some $i \in \{1,2,3,4,5,6\})$
_ _ _ _ _	{ }	E	0.5394
_ _ _ _ _	{E, O}	I	0.6366
Q _ _ _ _	{ }	U	0.9867
Q _ _ _ _	{A}	A	0.9999 ...
_ _ Z E _	{A, D, I, R}	O	0.8803
_ _ _ _ _	{E, O}	I	0.6366
D _ _ I _	{ }	A	0.8207
D _ _ I _	{A}	E	0.7521
_ U _ _ _	{A, E, I, O, S}	Y	0.6270

2.4.

(a) False $P(F|H) = P(F|C,H)$

(b) True $P(E|A,B) = P(E|A,B,F)$

(c) False $P(E,F|B,G) = P(E|B,G)P(F|B,G)$

(d) True $P(F|B,C,G,H) = P(F|B,C,E,G,H)$

(e) True $P(A,B|D,E,F) = P(A,B|D,E,F,G,H)$

(f) False $P(D,E,F) = P(D)P(E|D)P(F|E)$

(g) False $P(A|F) = P(A)$

(h) True $P(E,F) = P(E)P(F)$

(i) False $P(D|A) = P(D|A,E)$

(j) True $P(B,C) = P(B)P(C)$

2.5

(a) $P(A) = P(A|S)$

$\therefore S = \{B, C, D, E, F\}$

(b) $P(A|C) = P(A|S)$

$\therefore S = \{C, D, E, F\}$

(c) $P(A|B, C) = P(A|S)$

$\therefore S = \{C, D, E, F\}$

(d) $P(B) = P(B|S)$

$\therefore S = \{D, E, F\}$

(e) $P(B|A, E) = P(B|S)$

$\therefore S = \{D, F\}$

(f) $P(B|A, C, E) = P(B|S)$

$\therefore S = \{D, F\}$

(g) $P(D) = P(D|S)$

$\therefore S = \{E, F\}$

(h) $P(D|A) = P(D|S)$

$\therefore S = \{C, E, F\}$

(i) $P(D|C, E) = P(D|S)$

$\therefore S = \{F\}$

(j) $P(D|F) = P(D|S)$

$\therefore S = \{\}$