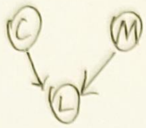
Exam Answer Sheet

Course Name: Introduction to AI - ~~&~~ Probabilistic Reasoning and Decision Making

Course Code: ECON170026

Student Name: JIANG Guanlin Student ID: 22349090012

1、




(a)

| C | M | $P(L=1 \mid C,M)$ |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0.2 |
| 1 | 0 | 0.36 |
| 1 | 1 | 0.44 |

(b)

$P(M=1 \mid C=0, L=1)$

$P(M=1 \mid C=1, L=1)$

$P(M=1) = P(M=1 \mid C=0)$

(c) $P(M=1)=0.05$

$P(C=1)=0.01$

$P(M=1 \mid C=1, L=1)$

$= \dfrac{P(M=1, C=1, L=1)}{P(C=1, L=1)}$

$=$

$P(M=1 \mid C=0, L=1)$

$= \dfrac{P(M=1, C=0, L=1)}{P(C=0, L=1)}$

$=$

2.

(a) A, B, C

(b)

(i) True

(ii) False

(iii) False

(iv) True

(v) True

(c) $P(H|C,D) \neq P(H|C)$ $\quad X=\{H\}\ Y=\{D\}\ E=\{C\}$

~~$P(H|C,D) = \frac{P(H,C,D)}{P(C,D)} = \frac{P(C,D|H)\times P(H)}{P(C,D)}$~~

~~$P(H|C) = \frac{P(H,C)}{P(C)}$~~

~~$\therefore P(H|C,D) \neq P(H|C)$~~

① $H \leftarrow F \leftarrow C \leftarrow B \rightarrow D$

condition 1

② $H \leftarrow E \leftarrow C \leftarrow B \rightarrow D$

condition 1

③ $H \leftarrow E \leftarrow A \rightarrow C \leftarrow B \rightarrow D$

condition 3

$\therefore$ They are not condition independent.

3.

(a) $P(C=c, F=0, E=1)$

~~CPT:~~

| ~~C~~ | ~~F~~ | ~~E~~ | ~~$P(C=c, F=0, E=1)$~~ |
|---|---|---|---|
| ~~0~~ | ~~0~~ | ~~1~~ | |
| ~~1~~ | ~~0~~ | ~~1~~ | |

$P(C=c, F=0, E=1)$

$= \sum_A \sum_B \sum_D P(A=a)\, P(B=b)\, P(C=c|A=a, B=b)\, P(D=d|A=a, B=b)\, P(E=1|C=c)\, P(F=0|C=c, D=d)$

~~$P(C=0, F=0, E=1)$~~

~~$P(C=1, F=0, E=1)$~~

(b)

| #m | #a | #d |
|---|---|---|
| 32 | 4 | 0 |
| 32 | 4 | 0 |
| 0 | 1 | 1 |
| 64 | 9 | 1 |

4、

(a)

(i) let the sum of $P(x_1=x_1^{(t)}, X_2=x_2^{(t)}, \dots, X_n=x_n^{(t)}) = P(data)$

(ii)

(iii) count the sum of $x$ and $\pi$.

(b)

(i)

$P_{ML}(A=1) = \frac{1}{2}$

$P_{ML}(B=1|A=0) = 0.7$

$P_{ML}(B=1|A=1) = 0.7$

(ii) $P(B=1) = P(0,1,0)+P(0,1,1)+P(1,1,0)+P(1,1,1)$

$= \frac{200+150+150+200}{1000} = 0.7$

$\therefore P(B=1) = P(B=1|A=0) = P(B=1|A=1)$

$\therefore$ A & B are marging independent.

(iii)

$P_{ML}(C=1|A=0,B=0) = \frac{1}{3}$

$P_{ML}(C=1|A=0,B=1) = \frac{3}{7}$

$P_{ML}(C=1|A=1,B=0) = \frac{2}{3}$

$P_{ML}(C=1|A=1,B=1) = \frac{4}{7}$

5、

$$P(E|W=w) = \begin{cases} 1 \\ 0 \end{cases}$$

```
evidenceGivenWord(evidence, w):
    for each letter which is a key in evidence:
        let places = evidence[letter]
        if
```

6.

(a) $P(B,C|A,D,E)$

$$= \frac{P(B,C,A,D,E)}{P(A,D,E)}$$

(b) $a_{it} = P(O_1, O_2, \cdots O_t, S_t = i)$

$$a_{it+1} = P(S_t = i | O_1, O_2, O_3, \cdots O_{t+1}) \times P(O_1, O_2, O_3 \cdots, O_{t+1})$$

$$= \frac{P(O_{t+1} | S_t = i, O_1, \cdots O_t) \times P(S_t = i | O_1, O_2 \cdots O_t)}{P(O_{t+1} | O_1, O_2 \cdots O_t)} \times P(O_1, O_2, \cdots O_{t+1})$$

$$= \frac{b_i(O_{t+1}) \overbrace{\cancel{P(S_t = i | O_1, O_2, \cdots O_t) \times P(O_1, O_2 \cdots O_t)}}^{a_{it}}}{\sum_j b_j(O_{t+1}) \times \frac{a_{it}}{P(O_1, \cdots O_t)}}$$

(c)

$$L = \sum_t \log P(A = a_t, D = d_t, E = e_t)$$

$$= \sum_t \log \sum_{b,c} P(a_t, B = b, C = c, d_t, e_t)$$

$$= \sum_t \log \sum_{b,c} P(a_t) P(b) P(c | a_t, b) P(d_t | c) P(e_t | b, c, d_t)$$

(d)

$P(B=b|A=a) =$

$$P(D=d|A=a, B=b) = \frac{\sum_{t=1}^{T} I(a, a_t) P(b, d | a_t, d_t, e_t)}{\sum_{t=1}^{T} I(a, a_t) P(b | a_t, d_t, e_t)}$$

$$P(E=e|B=b, C=c) = \frac{\sum_t I(e, e_t) P(b, c | a_t, d_t, e_t)}{P(a_t, d_t, e_t)}$$

7、

(a) $P(O_1=O_1, S_1=i)$

$a_{ij}=P(S_t=j \mid S_{t-1}=i)$

$a_1^A=0.8\times0.99=0.792$

$a_1^B=0.1\times0.01=0.001$

$a_2^A=0.2\times(0.792\times0.99+0.001\times0.01)=0.144344$

$a_2^B=0.9\times(0.792\times0.01+0.001\times0.99)=0.008019$

$a_3^A=0.8\times(0.144344\times0.99+0.008019\times0.01)=0.1143846$

$a_3^B=0.1\times(0.144344\times0.01+0.008019\times0.99)=0.000938225$

$P(O_1=O_1, S_1=i)=0.1143846+0.000938225=0.115322825$

(b)

8.

$$P(Y=1 \mid X_1, X_2, \cdots X_n)$$

$$= \frac{P(X_1, X_2, \cdots X_n \mid Y=1)\, P(Y=1)}{P(X_1, X_2, \cdots X_n)}$$

$$= \frac{P(Y=1) \times \left(P(X_1 \mid Y=1) \times P(X_2 \mid Y=1) \times \cdots \times P(X_n \mid Y=1)\right)}{P(X_1, X_2, \cdots X_n)}$$

$$= \frac{\pi_i \prod_{t=1}^{T} P(X_t \mid Y=i)}{\sum_{j=1}^{n} \pi_j \prod_{t=1}^{n} P(X_t \mid Y=j)}$$