

$$\hat{y} = mx + b$$

$$\text{Initial } m_0 = 0$$

$$\text{Initial } b_0 = 0$$

$$\text{learning rate } \alpha = 0.1$$

Data points

$$(x_1, y_1) = (1, 2) \text{ and } (x_2, y_2) = (2, 3)$$

$$\text{Loss: MSE, } J(m, b) = \frac{1}{2} \sum_{i=1}^n (y_i - (mx_i + b))^2$$

$$\hat{y} = mx + b$$

$$\frac{\partial J}{\partial m} = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i) x_i$$

$$\frac{\partial J}{\partial b} = \frac{1}{n} \sum_{i=1}^n (\hat{y}_i - y_i)$$

$$m_{\text{new}} = m_{\text{old}} - \frac{\partial J}{\partial m}, \quad b_{\text{new}} = b_{\text{old}} - \frac{\partial J}{\partial b}$$

$$\hat{y} \text{ for } (1, 2) = 0(1) + 0 = 0 \quad \begin{array}{l} \text{Error } 2 - 0 = 2 \\ \text{Error } 3 - 0 = 3 \end{array}$$

$$\frac{\partial J}{\partial m} = \frac{1}{2} (2 + 3 + 2 \cdot 3) = -1(2 + 6) = -8$$

$$\frac{\partial J}{\partial b} = \frac{1}{2} (2 + 3) = -1.3 = -5$$

update m and b

$$m_1 = 0 - 0.1(-8) = 0.8, \quad b_1 = 0 - 0 \cdot 1(-5) = 0.5$$

$$\text{Predicted } \hat{y}_1 = 0.8 \cdot 1 + 0.5 = 1.3 \quad \begin{array}{l} \text{Error } 1 - 1.3 = 0.7 \\ \text{Error } 1 - 1.3 = 0.7 \end{array}$$

$$\hat{y}_2 = 0.8(2) + 0.5 = 2.1 \quad \begin{array}{l} \text{Error } 3 - 2.1 = 0.9 \\ \text{Error } 3 - 2.1 = 0.9 \end{array}$$