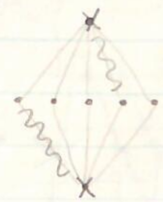
Pg #1

28/03/16

Math 239 - Lecture #34

- Recall matchings & covers

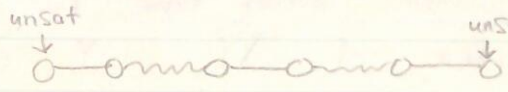


~~ : Matching
X : cover

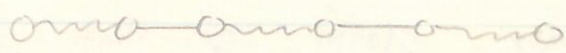
$|M| \le |C|$. If $|M| = |C|$, then M is max, C is min.

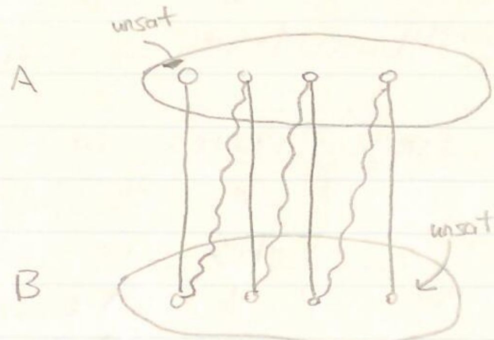
Bipartite: Matching Algorithm

Recall augmenting path :



Switch to get a larger matching :





Augmenting path in a bipartite graph. Start with a unsaturated vertex in A.

The other end of the augmenting path <u>must</u> be in B.

- This is because every time you reach A, you use a matching edge, so it must be saturated.

See attached handout for the formal algorithm.

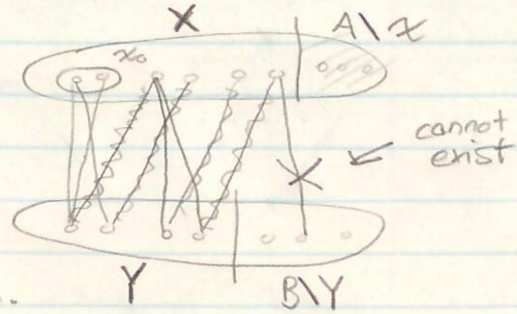
So for T_2 on the handout,
$x = \{a, b, c, d, e\}$
$Y = \{1, 2, 3, 4\}$
Matching is maximum, cover is $Y \cup (A \setminus x) = \{1, 2, 3, 4, f, g\}$

König's : Theorem

In a bipartite graph, the size of a max matching is equal to the size of a min cover.

↩

Proof: Let M, X_0, X, Y be the sets at the end of the algorithm. We claim that $Y \cup (A \setminus X)$ is a cover.



So if an edge joining X and $B\setminus Y$ exists, then the algorithm would have put the vertex of $B\setminus Y$ in Y, contradiction.

Our next claim is that every vertex in Y is saturated. Otherwise, we'd have an augmenting path and the algorithm would continue.

Our next claim is that every vertex in $A\setminus X$ is saturated. Since it is not in X_0, where the unsaturated vertices in X go, they are saturated. No matching edge goes from Y to $A\setminus X$, so the matching edges saturating Y and $A\setminus X$ are distinct. So the size of M is $|Y| + |A\setminus X|$, which is the size of our cover. $\square$

Lecture #34

Math 239 Bipartite Matching Algorithm

XY-Construction. We are given a bipartite graph G with bipartition (A, B). Let M be a matching in G.

1. Let X_0 be the set of all unsaturated vertices in A. Put these vertices into X.
2. Find all neighbours of X in B currently not in Y.
   (a) If one of these vertices is unsaturated, then we have found an augmenting path. Update the matching and repeat from step 1.
   (b) If all such vertices are saturated, put them in Y and add their matching neighbours to X, repeat step 2.
   (c) If no such vertices exist, then STOP, our matching is maximum with vertex cover $Y \cup (A \setminus X)$.

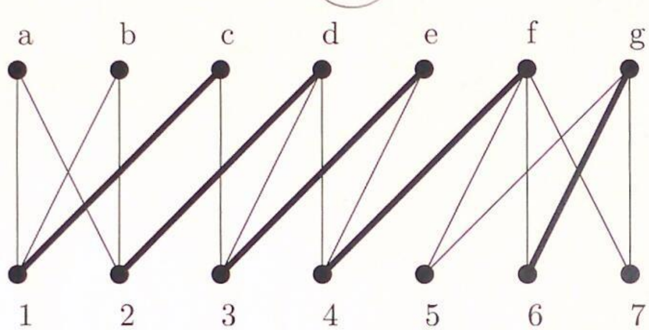
By the end of the algorithm...

Go $P_1 \Rightarrow T_1 \Rightarrow P_2 \Rightarrow T_2$

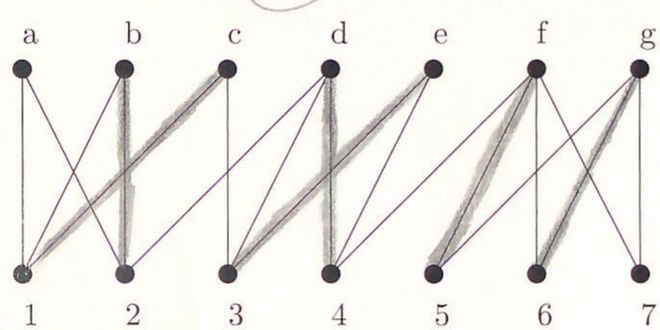
1. X_0 is the set of unsaturated vertices in A.
2. X is the set of vertices in A reachable via an alternating path starting with a vertex in X_0.
3. Y is the set of vertices in B reachable via an alternating path starting with a vertex in X_0.

Example.

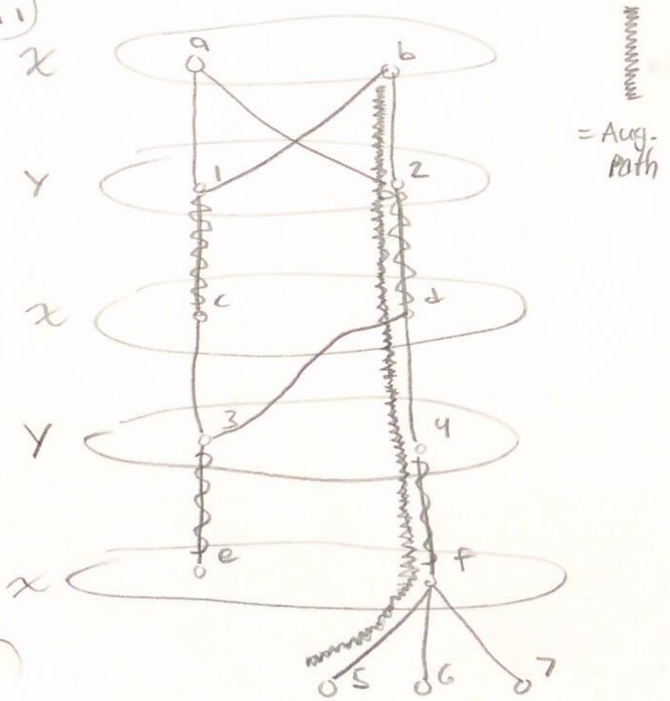
P_1



P_2



T_1



T_2