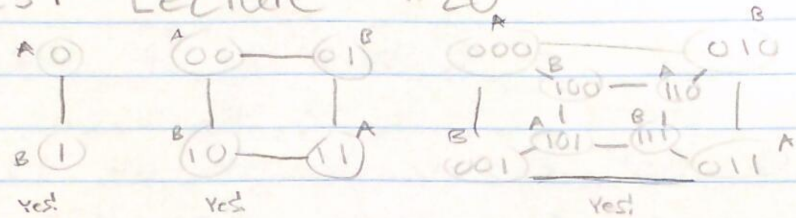
Pg #1

24/02/16

Math 239 - Lecture #20

Recall: N-cubes

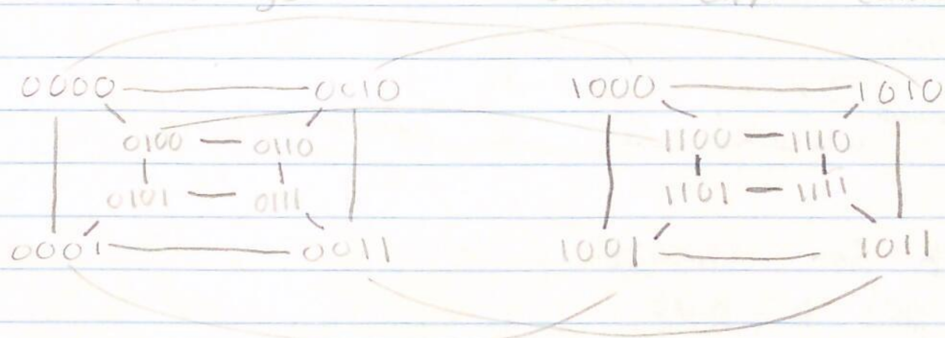


Are these cubes bipartite? Yup!
Are all n-cubes bipartite? Yup!

Let A be the binstr of length n with an even number of 1's, let B be the binstr of length n with an odd number of 1's. Suppose s, t are adjacent. We get t from s by either changing $0 \to 1$, or $1 \to 0$. The number of 1's increases or decreases by one respectively. So the parities of the # of 1's between s and t are different; so one is in A, and the other is in B. Hence the n-cube is bipartite.

Recursive construction of the n-cube

1. Take 2 copies of the (n-1)-cube.
2. Add 0 to all string in one copy, and 1 to all strings in the other copy. (infront).



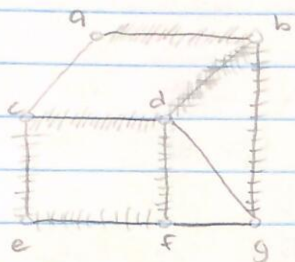
3. Join vertices of corresponding copies with an edge; 0's → 1's.

The 4-cube is a cube within a cube!

Midterm Cutoff

Walks and Paths: A <u>walk</u> is a sequence of alternating vertices and edges $v_0, e_1, v_1, e_2, v_2, \ldots, e_k, v_k$ such that $e_i = v_{i-1}v_i$ for all $1 \le i \le k$.
If $u = v_0$ and $v = v_k$, then this is a <u>u,v-walk</u>.
The <u>length</u> is k, the "# of edges"

Ex:



an a,g-walk:
a, ab, b, bd, d, dc, c, ce, e, ef, f, fd, d, db, b, bg, g.

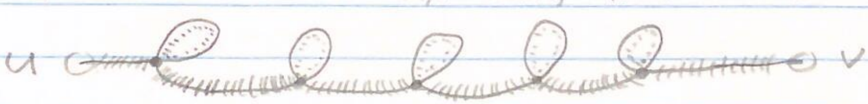
Length is 8.

For a simple graph (no multiple edges), we can describe a walk by only the sequence of vertices visited. walk above can be represented as a, b, d, c, e, f, d, b, g.

A <u>path</u> is a u-v walk with no repeated vertices and edges. (A trivial path from v to v has length 0).

Ex: a, d, f, g is an a-g path of length 3.

Theorem: If there is a u,v-walk, then there is a u,v-path.



- Look for repeated parts (the ⋰⋱) and remove them to get a path.

i.e repeated parts are between repeated vertices.

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Proof. Since a u,v walk exists and all walks have length ≥ 0, pick a walk W of the shortest length. If W has no repeated vertices, then W is a u,v-path, and we are done.

Suppose W is $v_0, v_1, \ldots, v_k$ and suppose $v_i = v_j$ where $i<j$. Then $v_0, \ldots, v_i, v_{j+1}, \ldots, v_k$ is a u,v-walk of shorter length than W. But W is a shortest u,v-walk, so this is a contradiction.

$\square$