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Math 239 - Lecture #36 (Final Review)

*This is apparently easier than our final :(

Q1) $A = \{1,2,3,4,5\}$ $\quad w: A \to \mathbb{N} \cup \{0\}$ $\quad$ find $\Phi_A(x)$

$a \to a^2+1$

a	1	2	3	4	5
$w(a)$	2	5	10	17	26

$\therefore \Phi_A(x) = x^2 + x^5 + x^{10} + x^{17} + x^{26}$

Q2) Give an unambiguous expr for the set S of binary strings that do not contain 000111 as a substr.

Use block decomp: $\{0\}^*(\{1\}\{1\}^*\{0\}\{0\}^*)^*\{1\}^* = \{0,1\}^*$
$= \{1\}^*(\{0\}\{0\}^*\{1\}\{1\}^*)^*\{0\}^*$

- Starting $\{1\}^*$ is fine, ending $\{0\}^*$ is fine.
- If we have 3 or more zeros it needs to be followed by one or two 1's: $\{000\}\{0\}^*\{1,11\}$
- If we have 1 or 2 zeros, we can have any number of followings ones: $\{0,00\}\{1\}\{1\}^*$

Final: $\{1\}^*\left(\{000\}\{0\}^*\{1,11\} \cup \{0,00\}\{1\}\{1\}^*\right)^*\{0\}^*$

Q3) Let C be the set of all compositions of length exactly 3, where:

i) The first part is even
ii) The second part is at least 5
iii) The third part is at most 3

Find a generating function for C where the weight function is: $w: C \to \mathbb{N} \cup \{0\}$

$(c_1, c_2, c_3) \to c_1 + c_2 + c_3$

Q3 cont.)

$A_1 = \{2, 4, 6, \dots\}$ For $i = 1,2,3$
$A_2 = \{5, 6, 7, \dots\}$ $w_i : A_i \to \mathbb{N} \cup \{0\}$
$A_3 = \{1, 2, 3\}$ $a \to a$

Notice $C = A_1 \times A_2 \times A_3$ and for all $(a_1, a_2, a_3) \in C$,
$w(a_1, a_2, a_3) = w_1(a_1) + w_2(a_2) + w_3(a_3)$

Then by the product lemma,

$\Phi_c(x) = \Phi_{A_1}(x) \times \Phi_{A_2}(x) \times \Phi_{A_3}(x)$.

$\Phi_{A_1}(x) = x^2 + x^4 + x^6 + \dots = \dfrac{x^2}{1-x^2}$

$\Phi_{A_2}(x) = x^5 + x^6 + x^7 + \dots = \dfrac{x^5}{1-x}$

$\Phi_{A_3}(x) = x + x^2 + x^3 = x(1+x+x^2)$

$\therefore \Phi_c(x) = \dfrac{x^8(1+x+x^2)}{(1-x^2)(1-x)}$

Q4) Solve: $x_n - 2x_{n-1} = 3 \cdot 2^n$ $(n \geq 1)$
$x_0 = 5$

Char eq: $x - 2 = 0 \implies x = 2$
Write $b_n = a_n + c_n$ ← specific So $a_n = A \cdot 2^n$
generic

Normally for c_n we'd guess $\alpha \cdot 2^n$. But this choice of c_n is a solution to a_n already. Instead we'll guess $c_n = \alpha \cdot n \cdot 2^n$.

Plug it in: $c_n - 2c_{n-1} = 3 \cdot 2^n$
$\alpha n \cdot 2^n - 2\alpha(n-1) \cdot 2^{n-1} = 3 \cdot 2^n$
$\cancel{\alpha n 2^n} - \cancel{\alpha n 2^n} + \alpha 2^n = 3 \cdot 2^n$
$\alpha 2^n = 3 \cdot 2^n$
$\alpha = 3$ $\therefore c_n = 3 \cdot 2^n$

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Q4 Cont)

$a_0 = b_0 - c_0 = 5 - 0 = 5$ Recall $a_n = A \cdot 2^n$

$a_n = A \cdot 2^n$

Substituting $a_0 = 5 \Rightarrow A = 5$

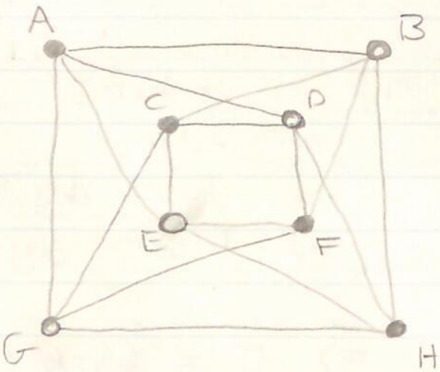
$\therefore \lambda_n = a_n + c_n = (5 + 3n) \cdot 2^n$

Q5)

i) Is J bipartite?

ii) Is J Eularian?

iii) Is J planar?



i) Yes see colouring

ii) Yes (every vertex even deg)

iii) Nope, $K_{3,3}$ exists

(easy to find, don't ruin my bipartition!)

Q6)

Let G be a graph in which each vertex has degree at least t. Prove G has a cycle of length at least t+1.

Start with the longest path P: $v_0 v_1 v_2 \dots v_k$. We know $k \geq t$ since it is the longest path.

Let v_0's neighbours be $\{v_{a_i}\}$ for $1 \leq i \leq \deg v_i$ where $a_1 < a_2 < a_3$.

Claim $a_t \geq t$ (vertices indexs are strictly increasing!)

Observe that $v_0 v_1 v_2 \dots v_{a_t} v_0$ is thus a cycle in G. Its length is $a_t + 1 \geq t+1$.

Done.

Q8) Let G be a 3-regular connected planar graph which has a planar embedding where each face has degree 5 or 6.

a) How many faces of deg 5 are there?

b) If you know the number of faces of deg 6 is 20, how many vertices & edges are there?

a) n vertices By HSL, $2m = 3n$

m edges By HSLFF, $2m = 5x + 6y = 6(x+y) - x$

x faces deg 5 By E.F, $2 = n - m + (x+y)$

y faces deg 6

$\Rightarrow 12 = 6n - 6m + 6(x+y)$

$12 = 4m - 6m + 6(x+y)$

$12 - x = 4m - 6m + 6(x+y) - x$

$12 - x = 4m - 6m + 2m = 0$

$\underline{x = 12}$ 12 faces!

b) $5x + 6y = 2m$

$5(12) + 6(20) = 2m$ $\therefore m = 90$

$2m = 3n$

$2(90) = 3n$ $\therefore n = 60$

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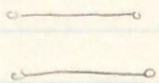
Q11) a) Every graph with no cycle is a tree



G

X

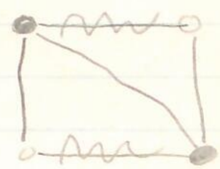
b) Every 3-colourable graph is planar

take $K_{3,3}$ is 3-colourable
but not planar

X

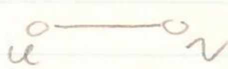
c) If G is not bipartite then the size of a max matching is strictly less than the size of a min cover.

X



max matching
min cover

d) If G has a closed walk of leng ≥ 3 it has a cycle.



G

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