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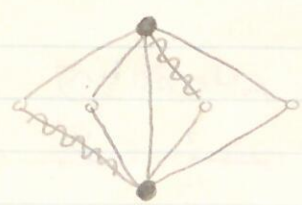
Math 239 - Lecture #33

Theorem: If M is any matching and C is any cover then $|M| \leq |C|$.

Proof: For each edge uv in M, at least one of u or v is in C in order to cover the edge uv. Also, no edges in M share any vertices so we need at least $|M|$ vertices in C. So $|C| \geq |M|$. □



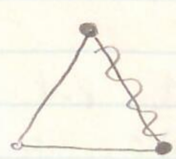
~ Matching
• Cover (could be bigger!)

Corollary: If M is a matching and C is a cover where $|M| = |C|$, then M is a maximum matching and C is a minimum cover.

Proof: Let M' be any matching. By previous theorem, $|M'| \leq |C|$. But $|M| = |C|$, so $|M'| \leq |M|$. So M is a maximum matching. Let C' be any cover. Then $|C'| \geq |M| = |C|$. So C is a minimum cover. □

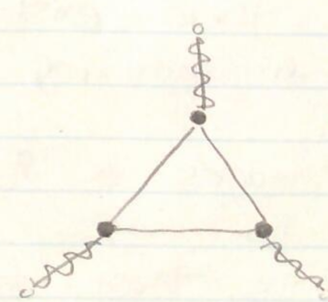
- One way to prove a matching M is maximum is to find a cover of the same size.
- We cannot always find a matching and a cover of the same size.

Ex:



~ M
• C

$|M| \neq |C|$
(cannot)



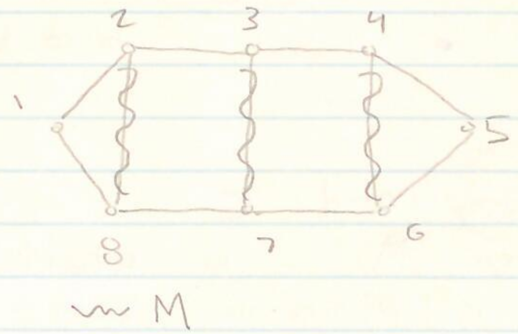
$|M| = |C|$

Konig's Theorem: In a bipartite graph, the size of a maximum matching is equal to the size of a minimum cover.

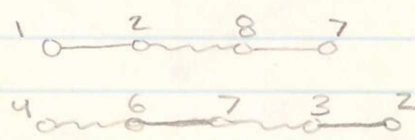
Alternating / Augmenting Paths: An <u>alternating path</u> P with respect to matching M is a path where consecutive edges alternate between being in M and not in M.

An <u>augmenting path</u> is an alternating path that starts and ends at different unsaturated vertices.

Example:



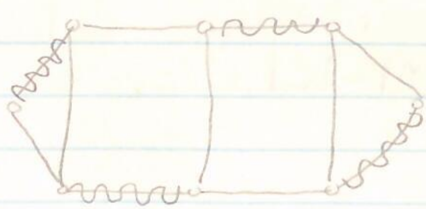
Alternating Paths



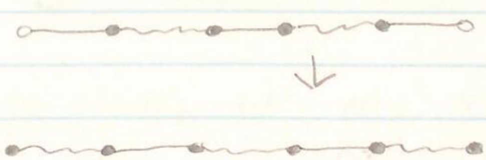
etc
etc

Augmenting Paths

• Saturated



Switch edges on an augmenting path



Theorem: If there exists an augmenting path with respect to a matching M, then M is not maximum.

For edges in P that is not in M, put them in M. For edges not in P that are in M, remove them from M.
Then we get a matching that saturates 2 more vertices.

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Example: