Pg #1

10/01/16

Math 239 - Lecture #17

Nonhomogeneous cont.

Example: $\{a_n\}$ $a_0 = 0$, $a_1 = 1$, $a_n - a_{n-1} - 2a_{n-2} = 3 \cdot 2^n$ for $n \geq 2$.
Let b_n be a specific solution.

$x^2 - x - 2$

Guess $b_n = \alpha \cdot 2^n$.
Then $b_n - b_{n-1} - 2b_{n-2} = \alpha \cdot 2^n - \alpha \cdot 2^{n-1} - 2 \cdot \alpha \cdot 2^{n-2}$
$= \alpha \cdot 2^{n-2}(2^2 - 2 - 2) = 0 \neq 3 \cdot 2^n$
This never equals $3 \cdot 2^n$. Our guess failed.
Introduce a factor of n. Guess $b_n = \alpha \cdot n \cdot 2^n$
$b_n - b_{n-1} - 2b_{n-2} = \alpha n 2^n - \alpha(n-1)2^{n-1} - 2\alpha(n-2)2^{n-2}$
$= [\alpha n 2^n - \alpha n 2^{n-1} - 2\alpha n 2^{n-2}] + [\alpha 2^{n-1} + 4\alpha 2^{n-2}]$
$= 0 + \alpha 2^n [\frac{1}{2} + 1] = \underline{\frac{3}{2}\alpha 2^n}$

This equals $3 \cdot 2^n$, so $\alpha = 2$, and $b_n = 2n \cdot 2^n = n \cdot 2^{n+1}$
Char poly is $x^2 - x - 2 = (x-2)(x+1)$. So $x = 2, -1$.

So $a_n = A \cdot 2^n + B \cdot (-1)^n + n \cdot 2^{n+1}$ for some const A, B.

$$\left.\begin{array}{l} a_0 = 0 = A + B + 0 \\ a_1 = 1 = 2A - B + 4 \end{array}\right) \Rightarrow A = -1,\ B = 1$$

So $a_n = -2^n + (-1)^n + n \cdot 2^{n+1}$.
Our earlier guess failed because we have 2^n on the RHS, and 2 is a root of the char poly.

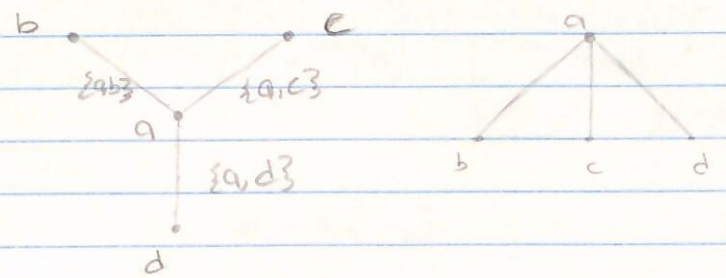
General Rule: If the RHS has some constant $C \cdot r^n$ in the recurrence and r is a root of the char poly with multiplicity k, then we should guess $b_n = \alpha \cdot n^k \cdot r^n$ for some constant α.

We are finished enumeration.
On to graph theory.

Definitions: A graph is a pair $G = (V(G), E(G))$ where $V(G)$ is a set of objects called vertices, and $E(G)$ is a set of certain unordered pairs of $V(G)$ called edges. (Subsets of $V(G)$ of size 2)

Example: Define G where $V(G) = \{a, b, c, d\}$, and then $E(G) = \{\{a,b\}, \{a,c\}, \{a,d\}\}$

Graphical representation of G



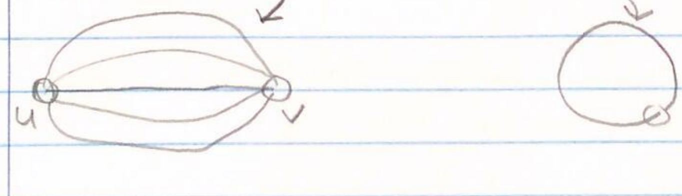
etc. etc. (any drawing)

Graph Vocabulary:

1. Two vertices u, v are adjacent if $\{u, v\}$ is an edge.
2. Vertex u is a neighbour of v if u, v are adjacent. The set of all neighbours of v in G is called the neighbourhood of v in G, $N_G(v)$
   - Example above: $N_G(a) = \{b, c, d\}$, $N_G(b) = \{a\}$
3. An edge $e = \{u, v\}$ is incident with its two endpoints u and v. We can also say e joins u and v.

Notes:

1. A shorthand for $e = \{u, v\}$ is $e = uv$.
2. Edges are unordered, so $uv = vu$.
3. We mostly consider "simple" graphs; there are no multiple edges or loops.



4. We only consider finite graphs.