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Math 239 - Lecture #13

Recall ambiguity within the context of binary strings.
Three unambiguous expressions for the set of all strings:

All binstr: (1) $\{0,1\}^*$ Every string can be uniquely generated by decomposing after each bit. I.E $\underbrace{0/1/1/0/1}_{\in\{0,1\}}$

(2) $\{0\}^*(\{1\}\{0\}^*)^*$ Every string with K 1's has a form $\underbrace{0^{a_1}}_{\in\{0\}^*}\underbrace{1\,0^{a_2}\,1\cdots 1\,0^{a_{k+1}}}_{\in\{1\}\{0\}^*}$; break just before each 1.

(3) $\{0\}^*(\{1\}\{1\}^*\{0\}\{0\}^*)^*\{1\}^*$ block decomposition

$\underbrace{000}_{\in\{0\}^*}/\underbrace{111100/100/1110/100}_{\in\{1\}\{1\}^*\{0\}\{0\}^*}/\underbrace{11}_{\in\{1\}^*}$

- Break up each block of 1's and 0's.
- Leading 0's and trailing 1's could be empty.

Problems on restriction of substrings. Take one of the unambiguous decompositions for all strings, and remove/restrict parts of it. The resulting expression is still unambiguous.

Example: Let S be the set of all strings with no 3 consecutive 0's. (no 000 as a substring)

Start with $\{0\}^*(\{1\}\{0\}^*)^*$
- Where can we find '000' in this expression?

Remove instances of '000' from head. So
$\{0\}^* = \{\varepsilon, 0, 00, 000, 0000, 00000, \ldots\}$
So replace $\{0\}^*$ with $\{\varepsilon, 0, 00\}$.
Do the exact same thing to the inner $\{0\}^*$.

$$\therefore\ S = \{\varepsilon, 0, 00\}(\{1\}\{\varepsilon, 0, 00\})^*$$

Cont

$\Phi_{\{\varepsilon,0,00\}}(x) = 1+x+x^2$, $\Phi_{\{1\}}(x) = x$

$$\Phi_S = (1+x+x^2) \cdot \frac{1}{1-(x\cdot(1+x+x^2))} = \frac{1+x+x^2}{1-x-x^2-x^3}$$

The # of str of length n with no 000 is $[x^n]\dfrac{1+x+x^2}{1-x-x^2-x^3}$

Example: Let S be the set of all strings where every block has length of atleast 2.

Start with $\{0\}^*(\{1\}\{1\}^*\{0\}\{0\}^*)^*\{1\}^*$.

$\{0\}^* = \{\varepsilon, \cancel{0}, 00, 000, 0000, \ldots\}$ remove blocks length < 2.

- we keep ε since it isn't a block, its just empty!
- Similar for tail $\{1\}^*$.

$\{1\}\{1\}^* = \{\cancel{1}, 11, 111, 1111, \ldots\}$ just remove 1. No epsilon!

- Similar for $\{0\}\{0\}^*$.

So $S = (\{\varepsilon\} \cup \{00\}\{0\}^*)(\{11\}\{1\}^*\{00\}\{1\}^*)^*(\{\varepsilon\} \cup \{11\}\{1\}^*)$

$$\therefore \Phi_S(x) = \left(1+x^2\cdot\frac{1}{1-x}\right)\left(\frac{1}{1-\left(x^2\frac{1}{1-x}\cdot x^2\frac{1}{1-x}\right)}\right)\left(1+x^2\cdot\frac{1}{1-x}\right)$$

$$= \frac{1-x+x^2}{1-x-x^2}$$

Example: Let S be the set of all str where an even block of 0's cannot be followed by an odd block of 1's.

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Example: cont

Start with $\{1\}^* (\{0\}\{0\}^* \{1\}\{1\}^*)^* \{0\}^*$

- Modified block - swapped the 0s and 1's.

We can only find what we're looking to exclude in the middle partition. Break into 2 cases.

① Even # of 0's : $\{00\}\{00\}^*$ + even 1's, $\{11\}\{11\}^*$

② Odd # of 0's : $\{0\}\{00\}^*$ + any 1's, $\{1\}\{1\}^*$

So $S = \{1\}^* \left(\{00\}\{00\}^* \{11\}\{11\}^* \cup \{0\}\{00\}^* \{1\}\{1\}^* \right)^* \{0\}^*$

$$\Phi_S(x) = \frac{1}{1-x} \left(\frac{1}{1 - \left(x^2 \frac{1}{1-x^2} \cdot x^2 \frac{1}{1-x^2} + x \frac{1}{1-x^2} \cdot x \frac{1}{1-x} \right)} \right) \frac{1}{1-x}$$

$$= \frac{1+2x+x^2}{1-3x^2-x^3}$$

$$x - x^3 + x^2 - x^3 = \frac{x + x^2 - 2x^3}{(1-x)(1-x^2)}$$

$$\frac{1 - x - x^2 + x^3 - x - x^2 + 2x^3}{(1-x)(1-x^2)}$$

$$= \frac{1 - 2x - 2x^2 - 2x^3}{(1-x)(1-x^2)}$$