1:30 - 2:30
Off tomorrow

02/03/16

Math 239 - Lecture #23

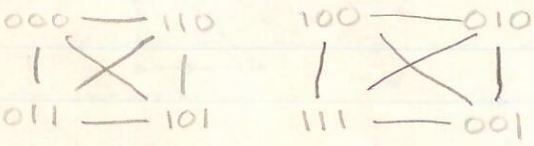
Recall: G is not connected if and only if there exists a nonempty proper subset X of $V(G)$ that induces an empty cut.

Example: Let G_n be a graph where vertices are all binary strings of length n, where two strings are adjacent if and only if they differ in 2 bits (similar to n-cubes).

G_1: 0 — 1

G_2: 00 — 11, 10 — 01

G_3:



- They're all disconnected!

Prove that G_n is connected.

- Each half is either even parity of 1's or odd parity.

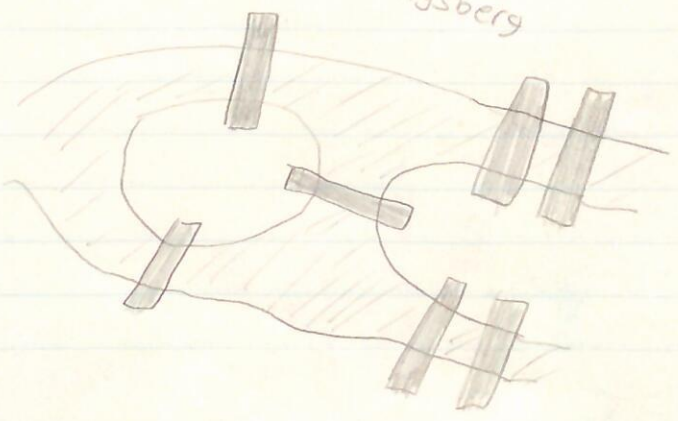
Proof: Let X be the set of all strings of length n with an even # of 1's. This is a nonempty proper subset of $V(G_n)$.

Suppose uv is an edge where $u \in X$. We get v from u by changing two bits. Changing one bit alters the parity of the # of 1's, so by changing two bits, the parity of 1's in v is the same as u.

So $v \in X$, and no edge is in the cut induced by X. Thus, G_n is not connected. □

Eulerian Circuits:

Konigsberg

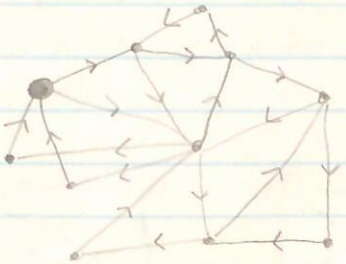


- Can you cross every bridge only once and get back to where you started?

Nope!

Definition: An Eulerian circuit of G is a closed walk that uses each edge of G exactly once.

Assume G is connected. Suppose G has an E.C.



- Every time you visit a vertex, you must exit the vertex as well!

Every vertex has an even degree: we must enter and leave a vertex using distinct edges.

$\Rightarrow$ Impossible to get an E.C in Konigsberg.

Theorem: Suppose G is connected. Then G has an E.C if and only if every vertex of G has even degree.

See Eularian circuit example slides.