12/02/16

Graph Theory cont.

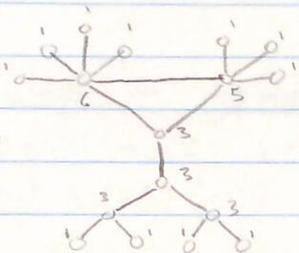
Math 239 - Lecture #18

Degrees: The degree of a vertex v in graph G is the number of edges incident with v, denoted $\deg_G(v)$ or $\deg(v)$.

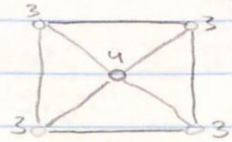
Examples:

deg(v)

Ⓐ



Ⓑ



• Must be even ö

$\sum_{v \in V(G)} \deg(v) = 34$ $\qquad = 16$

Handshaking Lemma: For any graph G, $\sum_{v \in V(G)} \deg(v) = 2|E(G)|$ ← 2 · # of edges

Proof - Every edge uv in G contributes 2 to the sum, 1 for deg(u), 1 for deg(v). □

For above, # of odd degree vertices:
Ⓐ = 16, Ⓑ = 4.

Corollary: For any graph, the number of odd degree vertices is even.

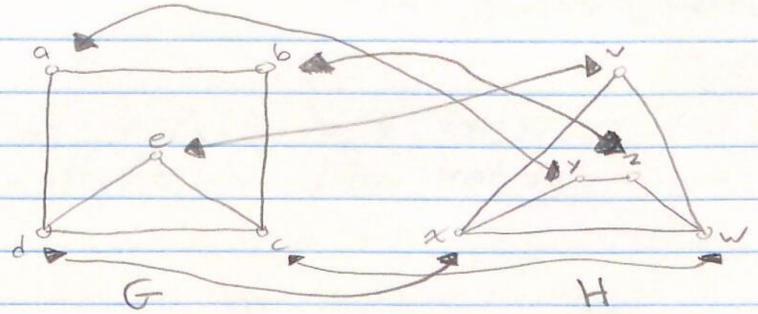
Proof - Let E, O be vertices in G of even, odd degrees, respectively.

Then $\sum_{v \in V(G)} \deg(v) = \sum_{v \in E} \deg(v) + \sum_{v \in O} \deg(v)$

The LHS is even by the handshaking lemma, and the first half of the RHS is even since its a sum of even numbers. Therefor, the second half of the RHS is even; so an even number of odd degrees.

Hence, the number of odd degree vertices is even. □

Isomorphism:



They're the same!

"suck a,b in, pull e up"

Def^{n}: Two graphs G_1, G_2 are <u>isomorphic</u> if there exists a bijection $f: V(G_1) \to V(G_2)$ such that $uv \in E(G_1)$ if and only if $f(u)f(v) \in E(G_2)$. Such a bijection is called an <u>isomorphism</u>.

Adjacency structure is preserved. edges $\to$ edges, non-edges $\to$ non-edges

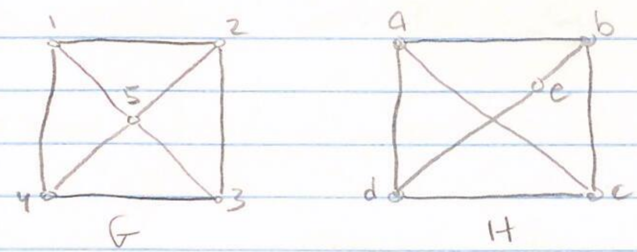
Back to previous example —

v	a	b	c	d	e
$f(v)$	y	z	w	x	v

edges in G		edges in H
ab	$\longleftrightarrow$	yz
bc	$\longleftrightarrow$	zw
cd	$\longleftrightarrow$	wx
da	$\longleftrightarrow$	xy
de	$\longleftrightarrow$	xv
ec	$\longleftrightarrow$	vw

$\Rightarrow f$ is an isomorphism.
$\Rightarrow G, H$ are isomorphic.

Example:



G, H are not isomorphic since G has a vertex of degree 4, but H does not. So no bijection that preserves the degree of vertex '5'.

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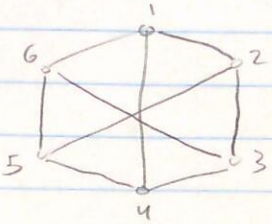
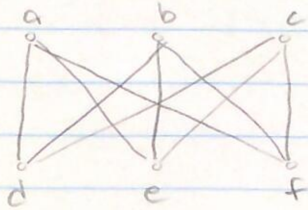
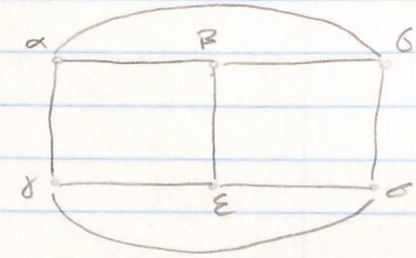
Example:



G_1



G_2



G_3

G_1, G_2 are isomorphic with isomorphism $f: V(G_1) \to V(G_2)$ where

v	1	2	3	4	5	6
$f(v)$	e	c	f	b	d	a

visualize: "flip" the b and e and pull things out into a hexagon.

G_1, G_3 are <u>not</u> isomorphic since G_3 contains 3 mutually adjacent vertices $\gamma, \varepsilon, \sigma$, but such a structure does not exist in G_1.
i.e 4 connects to 5,3 while ε connects to γ, σ, <u>BUT</u> 5 doesn't connect to 3 while γ does connect to σ.

Summary: To show two graphs are isomorphic, give an isomorphism. To show two graphs are not isomorphic, find an adjacency structure that is in only 1 of the 2 graphs.