Pg #1

29/01/16

Math 239 - Lecture #12

Binstr Examples:

① $\{0\}\{00\}^*$ • All binstrs of just 0 of odd length i.e $\{0, 000, 00000, \ldots\}$

② $\{0, 111\}^*$ • Blocks of 1's that are multiples of 3, seperated by 0's. e.g 00 111 111 111 000 111 0 111 111

③ $\{0\}^*(\{1\}\{0\}^*)^*$ • 1 followed by any numbers of zero with any amount of 0's as binstr head.

e.g $\underbrace{000}_{\{0\}^*}\underbrace{1\,10\,1\,1\,1000\,100}_{(\{1\}\{0\}^*)^*}$

- Break the binstr before each 1.

This is a decomposition of strings.

Generating Series of Binstr:

• S = set of strings of certain properties.
• Weight of a string is its length.
• # of strings in S of length n is $[x^n]\,\Phi_S(x)$

Assuming that the concatenation works like the cartesian product.

Example:

$A = \{1, 11\}$ $B = \{00, 000\}$
$\Phi_A(x) = x + x^2$, $\Phi_B(x) = x^2 + x^3$

$w(ab) = w(a) + w(b)$ (since length(ab) = length(a) + length(b))
$\Phi_{AB}(x) = \Phi_A \cdot \Phi_B = (x+x^2)\cdot(x^2+x^3) = \underline{x^3 + 2x^4 + x^5}$

Verify:

$AB = \{100, 1000, 1100, 11000\}$
$x^3 \quad x^4 \quad x^4 \quad x^5 = x^3 + 2x^4 + x^5$

It works!

Example: $\{0,1\}^* = \bigcup_{k\geq 0} \{0,1\}^k$, $\Phi_{\{0,1\}^k}(x) = (\Phi_{\{0,1\}}(x))^k$

$= (2x)^k$ since "0" has length 1, "1" has length 1.

$\Phi_{\{0,1\}^*}(x) = \sum_{k\geq 0} \Phi_{\{0,1\}^k}(x) = \sum_{k\geq 0} (2x)^k = \frac{1}{1-2x}$

Ambiguity of Strings: Compare $A\times B$ with AB (Previous Example)

$A\times B = \{(1,00), (1,000), (11,00), (11,000)\}$
$AB = \{100, 1000, 1100, 11000\}$

We have a mapping!

$f: A\times B \to AB$ by $f(a,b) = ab$
Thus, f is a bijection in this example.
This might not be true in all cases!

Example: $A = \{010, 01\}$, $B = \{01, 001\}$

$A\times B = \{(010,01), (010,001), (01,01), (01,001)\}$

$AB = \{01001, 010001, 0101\}$

No longer a bijection! when brackets and commas are removed, $(010,01) = (01,001)$
$\therefore \Phi_{A\times B}(x) \neq \Phi_{AB}(x)$

Definition: An expression for strings is ambiguous if there is a string that can be generated more than once. Otherwise, it is unambiguous.

AB is ambiguous if there are $a_1 a_2 \in A$, $b_1 b_2 \in B$, where $a_1 \neq b_1$ and $a_2 \neq b_2$, but $a_1 b_1 = a_2 b_2$

$A\cup B$ is ambiguous if $A\cap B \neq \emptyset$

Math 239- Lecture #12 Cont.

Def^{n}: Sum and Product lemma for strings:
- Let A,B be sets of strings

(1) If $A \cap B = \emptyset$, then $\Phi_{A \cup B}(x) = \Phi_A(x) + \Phi_B(x)$

(2) If AB is unambiguous, then $\Phi_{AB}(x) = \Phi_A(x)\,\Phi_B(x)$

(3) If A^* is unambiguous, then $\Phi_{A^*}(x) = \dfrac{1}{1-\Phi_A(x)}$

Proofs:

(1) sum lemma (simple)

(2) If AB is unambiguous, then $F: A \times B \to AB$ where $f(a,b) = ab$ is a bijection. Then $\underset{A \times B}{w(a,b)} = \underset{AB}{w(ab)}$. So $\Phi_{A \times B}(x) = \sum_{(a,b) \in A \times B} x^{w(a,b)} = \sum_{ab \in AB} w^{ab} = \Phi_{AB}(x)$

(3) Using 1 and 2,

$$\Phi_{A^*}(x) = \sum_{k \geq 0} \Phi_{A^k}(x) = \sum_{k \geq 0} \left(\Phi_A(x)\right)^k = \frac{1}{1-\Phi_A(x)}$$

geo series

- We also need to make sure the constant term is 0.
- The only string that can give a non-zero constant term is the empty string, ε.

If $\varepsilon \in A$, then A^* is ambiguous! ($\varepsilon + 010 = 010$!!)
Thus, $\varepsilon \notin A$, so we're safe with our geometric series.