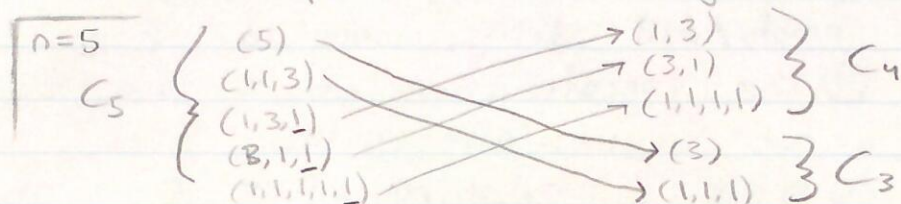


Math 239 - Lecture #11

Cont from:
Last Lecture

Let C_n be compositions of n where every part is odd for $n \geq 3$, $|C_n| = |C_{n-1}| + |C_{n-2}|$

Combinatorial proof: Find a bijection $F: C_n \rightarrow C_{n-1} \cup C_{n-2}$



- If it ends with a 1, remove the one (C_{n-1})
- Otherwise, subtract 2 from the largest int to get something in C_{n-2} .

Let $F: C_n \rightarrow C_{n-1} \cup C_{n-2}$ where for each $(a_1, \dots, a_k) \in C_n$,

$$F(a_1, \dots, a_k) = \begin{cases} (a_1, \dots, a_{k-1}) & \text{if } a_k = 1 \\ (a_1, \dots, a_{k-1}, a_k - 2) & \text{if } a_k > 1 \end{cases}$$

Every part in the output is still odd. In the first case, we get a composition in C_{n-1} , in the second case, we get a composition in C_{n-2} .

So $F(a_1, \dots, a_k) \in C_{n-1} \cup C_{n-2}$.

The inverse is $f^{-1}: C_{n-1} \cup C_{n-2} \rightarrow C_n$ where for each $(b_1, \dots, b_\ell) \in C_{n-1} \cup C_{n-2}$,

$$f^{-1}(b_1, \dots, b_\ell) = \begin{cases} (b_1, \dots, b_\ell, 1) & \text{if } b_1 + \dots + b_\ell = n-1 \\ (b_1, \dots, b_\ell + 2) & \text{if } b_1 + \dots + b_\ell = n-2 \end{cases}$$

So F is a bijection, and, $|C_n| = |C_{n-1} \cup C_{n-2}|$

We can recursively create C_n based on C_{n-1} and C_{n-2} . Add a part 1 to comp in C_{n-1} and 2 to the last part of any comp in C_{n-2} .

Ex $C_6 = \left\{ (5, 1), (1, 1, 3, 1), (1, 3, 1, 1), (3, 1, 1, 1), (1, 1, 1, 1, 1, 1), (1, 5), (3, 3), (1, 1, 1, 3) \right\}$

End of
Int of
Compositions

Binary strings

Define - (1) A binary string is a sequence of 0's and 1's.

(2) The length of a string is the total # of 0's and 1's in the string.

(3) There is only one string of length 0, the empty/null string, denoted ϵ .

(4) The concatenation of a and b is ab .
i.e. $a = 001$ $b = 1110$, $ab = 0011110$.

(5) b is a substring of s if $s = abc$ for some strings a, c (possibly ϵ).

(6) A block is a maximal nonempty substring of all 0's or all 1's.

Ex: 0000111011000101111 blocks of the str \sqcup

Main Q: How many binary strings of length n have certain properties?

$$S = \{\epsilon, 0, 00, 000, 0000, \dots\} \quad \text{no 1's}$$

Define weight of a string to be its length. Find gen series

$$\Phi_S(x) = 1 + x + x^2 + x^3 + \dots = \frac{1}{1-x}$$

Two operations on regular expressions:

(1) The concatenation of two sets of strings A, B , is
 $AB = \{ab \mid a \in A, b \in B\}$

Example: $A = \{0, 11\}$ $B = \{1, 11\}$

$$AB = \{01, 011, 111, 1111\}$$

Math 239 Lecture #11 - Cont

Concatenation is "like" a cartesian product, but not always.

Power of sets - $A^k = \underbrace{A \cdot A \cdot \dots \cdot A}_{k \text{ times}}$

Example: $A = \{0, 1\}$ 00011111110011 $\in A^{10}$

Star Operator - $A^* = A^0 \cup A^1 \cup A^2 \cup A^3 \cup \dots = \bigcup_{k \geq 0} A^k$

* note $A^0 = \{\epsilon\}$

Example: $\{0\}^* = \{\underbrace{\epsilon}_{\{0\}^0}, \underbrace{0}_{\{0\}^1}, \underbrace{00}_{\{0\}^2}, \dots\}$

$\{0, 1\}^*$ is the set of all binary strings
01101 $\in \{0, 1\}^5$, $\{0, 1\}^5 \subseteq \{0, 1\}^*$

$\Rightarrow 01101 \in \{0, 1\}^*$

- ① $\{0\} \subseteq \{00\}^*$
- ② $\{0, 11\}^*$
- ③ $\{0\}^* (\{1\} \{0\}^*)^*$