

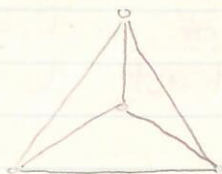
Math 239 - Lecture #32

Theorem:

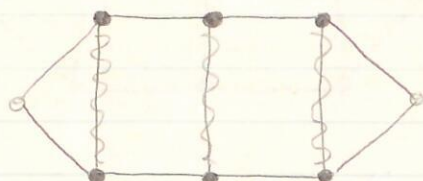
Every planar graph is 4-colourable.

Remember the colour-fill game?

There are planar graphs that are not 3-colourable.

 K_4 

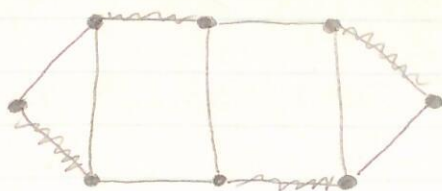
- Needs 4 colours
- Is planar

Matchings

- ~~~~~ = Matching
- = Saturated
 - = Unsaturated

A matching of G is a set of edges in G where no two edges share a common vertex.
(Edges form a subgraph of deg at most 1)

Given a graph, what is the maximum size of a matching?



- ~~~~~ = Matching
- = Saturated (all cause perfect)

Definition:

A matching that spans all vertices is called a perfect matching. (has size $n/2$)

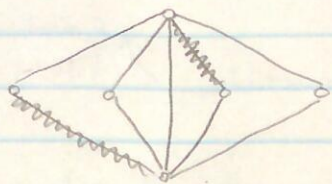
- A vertex that is used in a matching m is called saturated with respect to m , and otherwise it is called unsaturated.

Above graphs have been updated.



Midway

Ex:



~~~~~ Matching size 2

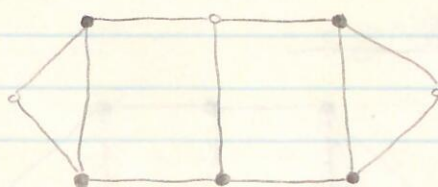
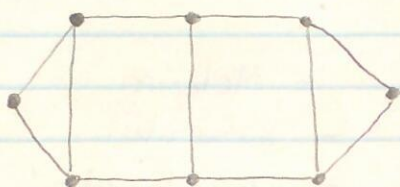
But how do we know  
its maximal?

Vertex  
Covers

A cover  $C$  in  $G$  is a set of vertices in  $G$ ,  
such that every edge has at least 1 endpoint  
in  $C$ .

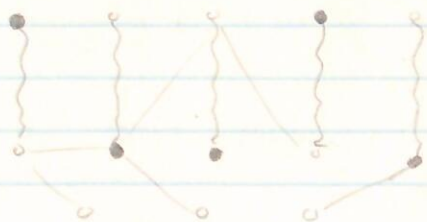
• Cover

Ex:



What is the minimum size of a cover in a graph?

Given a matching  $M$ .



~~~~~  $M$

Let C be any cover.

To cover M , we need at least one vertex from
each edge in M .

• See Posted Slides on Matchings

Update 4/13/16: Pei never posted these in