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Math 239 - Lecture #14

String Recursion: Idea of recursion for a set S of strings: Within a string in S, find another copy of a string S. (strings within themselves!)

Example: Let S be the set of all strings.
Any string $\sigma \in S$ consists of 1 or 0, followed by another string in S; or ε. (cannot apply recursion to ε)

$S = \{0,1\}S$, $\sigma = \underbrace{0}_{\{0,1\}}\underbrace{110}_{\in S}$

Then $\Phi_S(x) = (2x) \cdot \Phi_S(x) + 1$ ie $\Phi_S(x) = \dfrac{1}{1-2x}$

Example: Let S be the set of all strings with no 000.

$\underbrace{01}_{\{1,01,001\}} / \underbrace{0011110100111 01}_{\text{no '000', } \therefore \in S}$. Break the string just after the first 1.

$S = \{1, 01, 001\}S \cup \{\varepsilon, 0, 00\}$

Recursion does not apply to strings with no 1's $\in S$.

Then $\Phi_S(x) = (x + x^2 + x^3)\Phi_S(x) + (1 + x + x^2)$

$\therefore \Phi_S(x) = \dfrac{1+x+x^2}{1-x-x^2-x^3}$ divide $\Phi_S(x)$

Example: Let S be the set of all strings with no 1010.
Let T be the set of all strings with exactly one copy of 1010, at the right end.

Equation ①: $\{\varepsilon\} \cup S\{0,1\} = S \cup T$

Prove the sets are equal.

Proof: ($\subseteq$) $\varepsilon \in S$, it has no 1010. For any $\sigma \in S$, $\sigma\{1,0\}$ either has no 1010 (in which case it is in S), or it contains 1010, it would have only one copy of 1010 at the right (in T). So it is in $S \cup T$.

($\supseteq$) A string in S is either ε, or removing the rightmost bit results in another string in S. So it is in $\{\varepsilon\} \cup S\{0,1\}$.

A string $\in T$ ends with 1010, and by removing the 0 at the rightmost end, we destroy the only copy of 1010 in T. So the remaining string is in S. So the original string is in $S\{0,1\}$.

Equation ② - $S\{1010\} = T \cup T\{1,0\}$

i.e $\underbrace{\rule{3em}{0.4pt}}_{\text{no 1010}}1010$, or possibly $\overbrace{\underbrace{\rule{3em}{0.4pt}}_{\text{no 1010}}10\,10}^{\in T}\overbrace{10}^{\text{2 copies}}$

· Could have 2 copies of 1010!

Proof: ($\subseteq$) Let $\sigma \in S$. Then $\sigma 1010$ has atleast 1 copy of 1010 at the right end. If σ does not end with 10, then only 1 copy of 1010 exists, so $\sigma 1010 \in T$. If σ ends with 10, then it has 2 copies of $\underbrace{1010\ldots(1010)}_{\in T}10$. In this case, it is in $T\{10\}$.

($\supseteq$) Any string in $T \cup T\{1,0\}$ ends with 1010.

[T: ______ 1010
T{10}: ______ 1010 10]

By removing 1010 at the right end, we destroy any copy of 1010 in the string, so the resulting string is in S.

So a string in $T \cup T\{10\}$ is in $S\{1010\}$

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Cont.

EQ ①: $1 + \Phi_S(x) \cdot (2x) = \Phi_S(x) + \Phi_T(x)$

EQ ②: $\Phi_S(x) \cdot (x^4) = \Phi_T(x) + \Phi_T(x) \cdot (x^2)$

Equation 2 gives $\quad \Phi_T(x) = \Phi_S(x) \cdot \dfrac{x^4}{1+x^2}$

Equation 1 (sub 2 in) $\quad 1 + \Phi_S(x)(2x) = \Phi_S(x) + \Phi_S(x) \cdot \dfrac{x^4}{1+x^2}$

gives

$$\Phi_S(x) = \frac{1}{1 + \frac{x^4}{1+x^2} - 2x}$$

$$= \frac{1+x^2}{1-2x+x^2-2x^3+x^4}$$

FIVE STAR