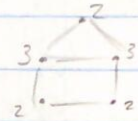
pg #1

22/02/16

Math 239 - Lecture #19

Classes of Graphs:

Recall the HSL: $\sum_{v \in V(G)} \deg(v) = 2|E(G)|$



• 6 E(G)
• 12 Σdeg

① **Complete Graphs:** A graph on n vertices is a complete graph if every pair of vertices is in an edge, denoted by K_n.

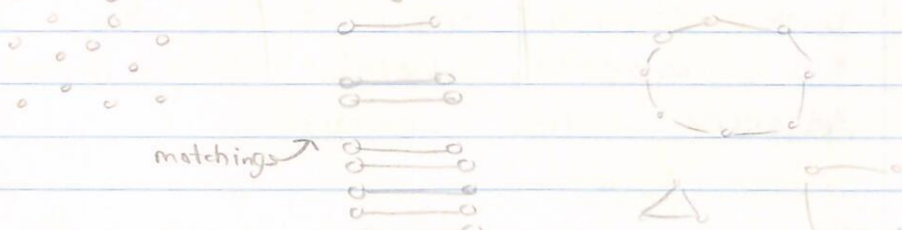
Examples: K_1 K_2 K_3 K_4 K_5 . . . .

• These can be drawn in many different ways.

Q: How many edges are there in K_n? There are $\binom{n}{2}$ edges. There are n vertices and each pair forms an edge.

② A graph is **K-regular** if every vertex has degree K. (All the same!)

Examples: 0-regular | 1-regular | 2-regular



isolated vertices

matchings

cycles (think K loops)

3-regular

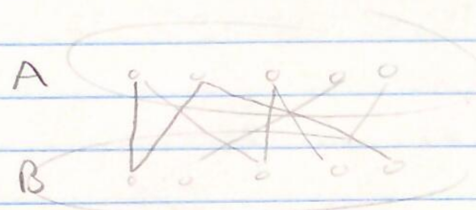
• K_n is $(n-1)$-regular

Q: How many edges are in a K-regular graph with n vertices? $\frac{n \cdot K}{2}$!

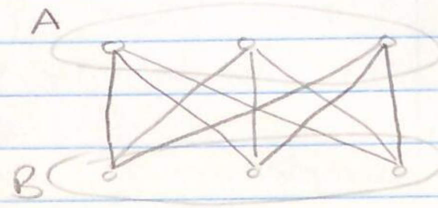
The sum of degrees in $n \cdot K$, so by the handshaking lemma,* the edges in a K-regular graph is $\frac{n \cdot K}{2}$! $|E| = \frac{nK}{2}$

*since $2|E| = nK$

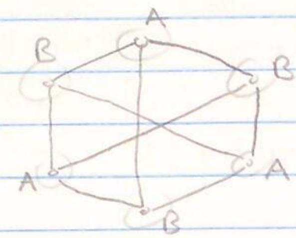
(3) A graph G is <u>bipartite</u> if there exists a partition (A,B) of the vertices where every edge in G joins one vertex is A with one vertex in B.



Ex:

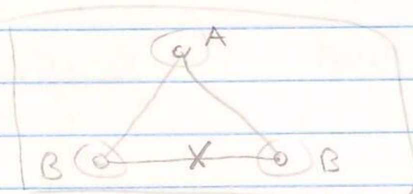


Ex:



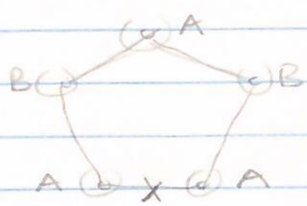
• Start with one vertex in A, then all neighbours are in B. Then the neighbours of these neighbours must be in A, etc.

<u>Non-bipartite graphs</u>:



• If one vertex is in A, both neighbours are in B. But those neighbours are adjacent. Therefor, not bipartite.
Adjacent $\Rightarrow$ Not bipartite.

Ex



• A cycle with an odd number of vertices is non-bipartite.

(4) For m, n, the complete bipartite graph $K_{m,n}$ with bipartition (A,B) satisfies $|A| = m$, $|B| = n$, and every vertex in A is joined with every vertex in B.

Pg#2

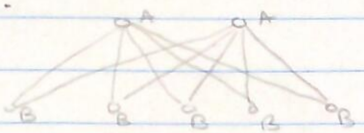
22/02/16

Math 239 - Lecture #19 Cont.

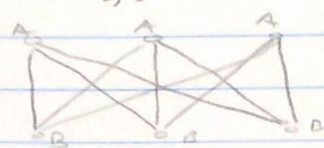
(4) Cont.

Examples:

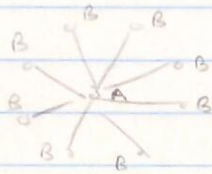
$K_{2,5}$



$K_{3,3}$



$K_{1,8}$



Q: How many edges in $K_{m,n}$?

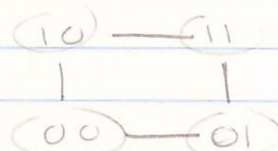
- Each of the m vertices in A can be paired up with n vertices in B, so we get $(m \cdot n)$

n-cube: The n-cube is a graph where the vertices are all binary strings of length n, and two strings are adjacent if and only if they differ in exactly one bit. (Also known as hypercube).

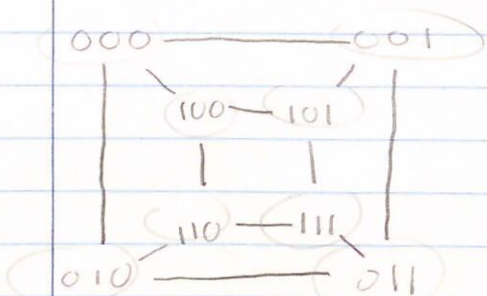
Examples:

1-cube

2-cube



3-cube



it's a 3D cube!! woah!

Properties:
1. 2^n vertices
2. n-regular; For each string changing any 1 of the n-bits gives a neighbour. So it has deg n.
3. $\frac{n \cdot 2^n}{2} = n \cdot 2^{n-1}$ edges! (use HSC)

FIVE STAR