Pg #1

27/01/16

Math 239 - Lecture #11

Cont from: Last Lecture

Let C_n be compositions of n where every part is odd for $n \geq 3$, $|C_n| = |C_{n-1}| + |C_{n-2}|$

Combinatorial proof: Find a bijection $f: C_n \to C_{n-1} \cup C_{n-2}$

$n=5$

$C_5 = \{(5), (1,1,3), (1,3,\underline{1}), (3,1,\underline{1}), (1,1,1,1,\underline{1})\}$

$C_4 = \{(1,3), (3,1), (1,1,1,1)\}$

$C_3 = \{(3), (1,1,1)\}$

$(5) \to (3)$
$(1,1,3) \to (1,1,1)$
$(1,3,1) \to (1,3)$
$(3,1,1) \to (3,1)$
$(1,1,1,1,1) \to (1,1,1,1)$

- If it ends with a 1, remove the one. (C_{n-1})
- Otherwise, subtract 2 from the largest int to get something is C_{n-2}.

Let $f: C_n \to C_{n-1} \cup C_{n-2}$ where for each $(a_1, \ldots, a_k) \in C_n$,

$$f(a_1, \ldots, a_k) = \begin{cases} (a_1, \ldots, a_{k-1}) & \text{if } a_k = 1 \\ (a_1, \ldots, a_{k-1}, a_{k-2}) & \text{if } a_k > 1 \end{cases}$$

Every part in the output is still odd. In the first case, we get a composition in C_{n-1}, in the second case, we get a composition in C_{n-2}.

So $f(a_1, \ldots, a_k) \in C_{n-1} \cup C_{n-2}$.

The inverse is f^{-1} $C_{n-1} \cup C_{n-2} \to C_n$ where for each $(b_1, \ldots, b_\ell) \in C_{n-1} \cup C_{n-2}$,

End of Int Compositions

$$f^{-1}(b_1, \ldots, b_\ell) = \begin{cases} (b_1, \ldots, b_\ell, 1) & \text{if } b_1 + \ldots + b_\ell = n-1 \\ (b_1, \ldots, b_\ell + 2) & \text{if } b_1 + \ldots + b_\ell = n-2 \end{cases}$$

So f is a bijection, and, $|C_n| = |C_{n-1} \cup C_{n-2}|$

We can recursively create C_n based on C_{n-1} and C_{n-2}. Add a part 1 to comp in C_{n-1} and 2 to the last part of any comp in C_{n-2}.

Ex $C_6 = \{(5,1), (1,1,3,1), (1,3,1,1), (3,1,1,1), (1,1,1,1,1,1), (1,5), (3,3), (1,1,1,3)\}$

Binary strings: Define – (1) A <u>binary string</u> is a sequence of 0's and 1's.

(2) The <u>length</u> of a string is the total # of 0's and 1's in the string.

(3) There is only one string of <u>length 0</u>, the empty/null string, denoted ε.

(4) The concatenation of a and b is ab.

i.e $a = 001$ $b = 1110$, $ab = 0011110$.

(5) b is a <u>substring</u> of s if $s = abc$ for some strings a, c (possibly ε).

(6) A block is a maximal nonempty substring of all 0's or all 1's.

Ex: 0000111011000101111 blocks of the str

Main Q: How many binary strings of length n have certain properties?

$S = \{\varepsilon, 0, 00, 000, 0000, \ldots\}$ no 1's

Define weight of a string to be its length. Find gen serie

$$\Phi_S(x) = 1 + x + x^2 + x^3 + \ldots = \frac{1}{1-x}$$

Two operations on regular expressions:

(1) The concatenation of two sets of strings A, B, is $AB = \{ab \mid a \in A, b \in B\}$

Example: $A = \{0, 11\}$ $B = \{1, 11\}$

$AB = \{01, 011, 111, 1111\}$

Math 239 Lecture #11 - Cont

Concatenation is "like" a cartesian product, but not always.

Power of sets - $A^k = \underbrace{A \cdot A \cdot \ldots \cdot A}_{k \text{ times}}$

Example: $A = \{0, 11\}$ $\quad 0\,0\,0\,11\,11\,11\,11\,0\,0\,11 \in A^{10}$

Star Operator - $A^* = A^0 \cup A^1 \cup A^2 \cup A^3 \cup \ldots = \bigcup_{k \geq 0} A^k$

* note $A^0 = \{\varepsilon\}$

Example: $\{0\}^* = \{\underbrace{\varepsilon}_{\{0\}^0}, \underbrace{0}_{\{0\}^1}, \underbrace{00}_{\{0\}^2}, 000, \ldots\}$

$\{0,1\}^*$ is the set of all binary Strings

$0\,1\,1\,0\,1 \in \{0,1\}^5$, $\quad \{0,1\}^5 \subseteq \{0,1\}^*$

$\Rightarrow 01101 \in \{0,1\}^*$

1. $\{0\}\{00\}^*$
2. $\{0, 111\}^*$
3. $\{0\}^* (\{1\}\{0\}^*)^*$