STAT 6910: HW 7

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```
## Warning: package 'emmeans' was built under R version 3.4.4

## NOTE: As of emmeans versions > 1.2.3,
##       The 'cld' function will be deprecated in favor of 'CLD'.
##       You may use 'cld' only if you have package:multcomp attached.

## Warning: package 'dae' was built under R version 3.4.4

## Loading required package: ggplot2

## Warning: package 'ggplot2' was built under R version 3.4.4
```

Problem 8

Burt Beiter, Doug Fairchild, Leo Russo, and Jim Wirtley, in 1990, ran an experiment to compare the relative strengths of two similarly priced brands of paper towel under varying levels of moisture saturation and liquid type. The treatment factors were "amount of liquid" (factor A, with levels 5 and 10 drops coded 1 and 2), "brand of towel" (factor B, with levels coded 1 and 2), and "type of liquid" (factor C, with levels "beer" and "water" coded 1 and 2). A $2 \times 2 \times 2$ factorial experiment with $r = 3$ was run in a completely randomized design.

(a) The experimenters assumed only factors A and B would interact. Specify the corresponding model.

Solution

Let A, B and C be as described with the associated levels $a = 2$, $b = 2$, and $c = 2$ respectively with $r = 3$. Then we have that the corresponding model is

$$Y_{ijkt} = \mu + \alpha_i + \beta_j + \gamma_k + (\alpha\beta)_{ij} + \epsilon_{ijkt}$$

where $i = 1, 2$, $j = 2$, $k = 2$, and $t = 3$.

(b) Assume there is only one contrast of primary interest: the one comparing brands of paper towels.

Solution Assuming that the contrast of primary interest is the one that compares the brands of paper towels, then the following contracts can be used:

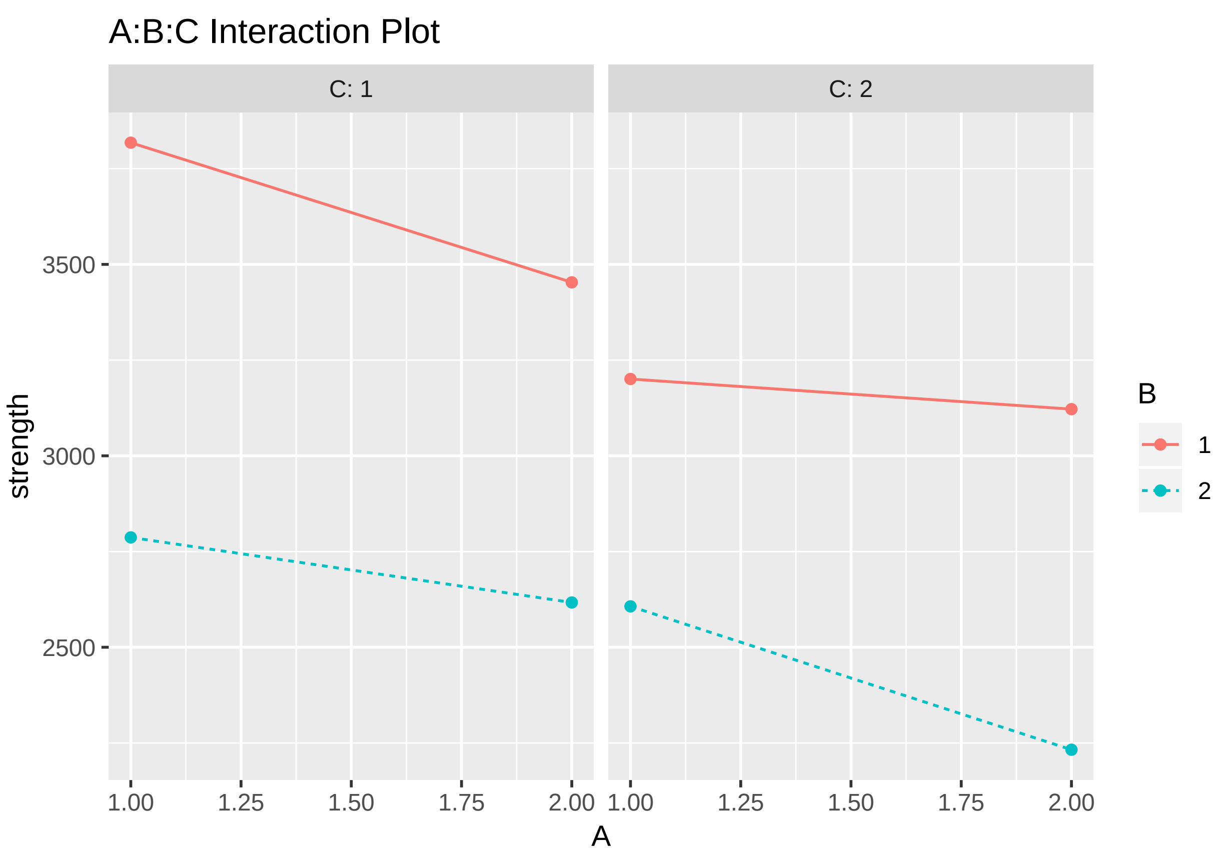
$$\bar{\tau}_{\bullet 1} - \bar{\tau}_{\bullet 2} = \{\beta_1 + (\bar{\alpha\beta})_{\bullet 1}\} - \{\beta_2 + (\bar{\alpha\beta})_{\bullet 2}\}).$$

(c) Use residual plots to evaluate the adequacy of the model specified in part (a).

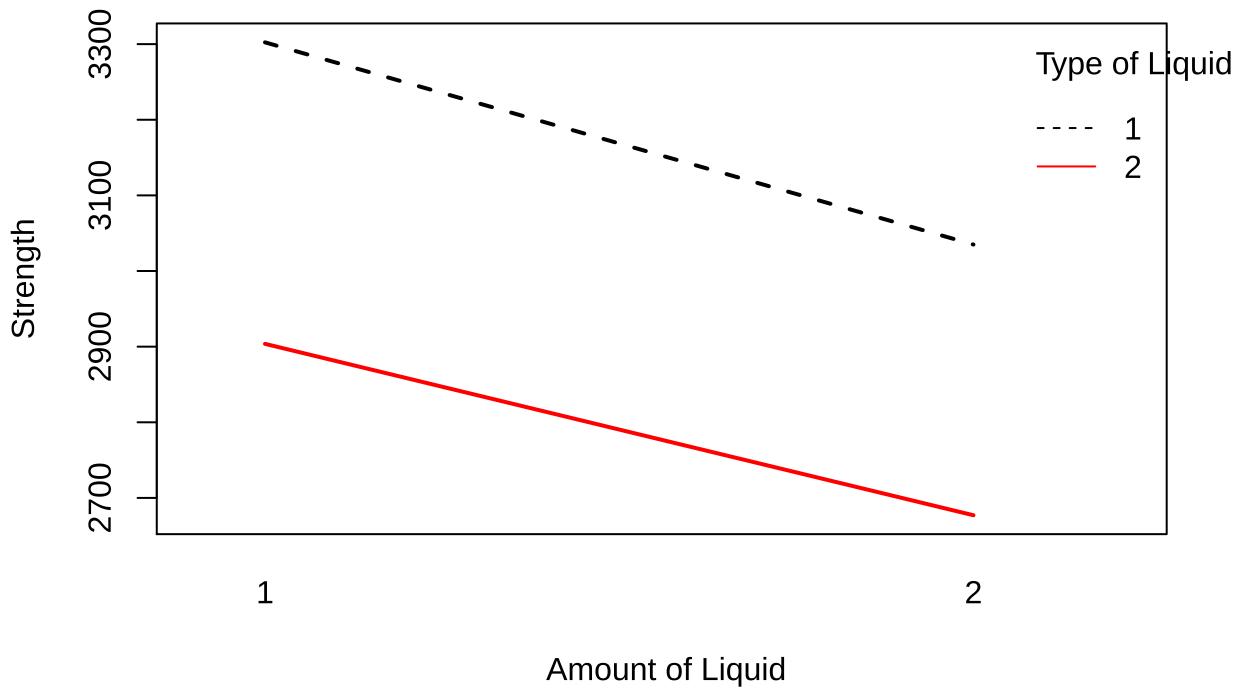
For part (c), comment on the potential for interactions using the graphing tool in the dae R package; you can just use these plots (and/or two-factor interaction plots) to assess whether the model for the mean is reasonable (rather than producing further residual plots to check that assumption).

Solution

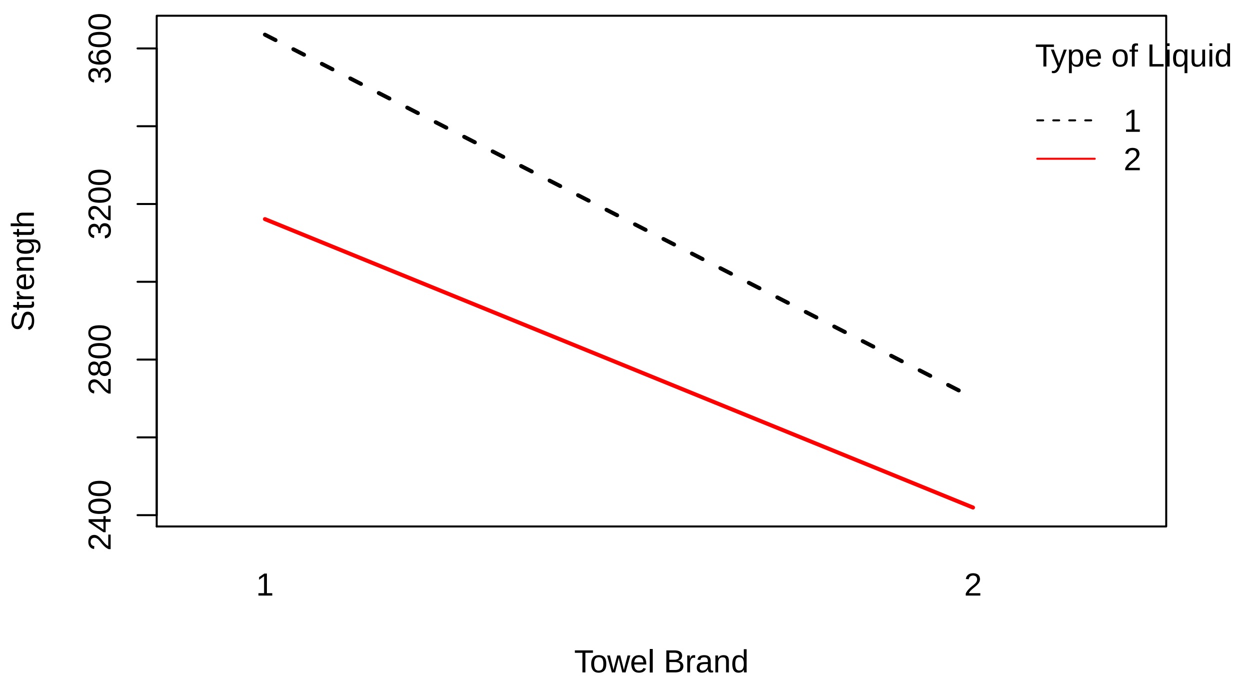
```r
paper.towel.strength <- within(paper.towel.strength,{
  A = factor(A); B = factor(B); C = factor(C); ABC = factor(ABC)})





attach(paper.towel.strength)

interaction.ABC.plot(strength, A, B, C, data = paper.towel.strength)
```



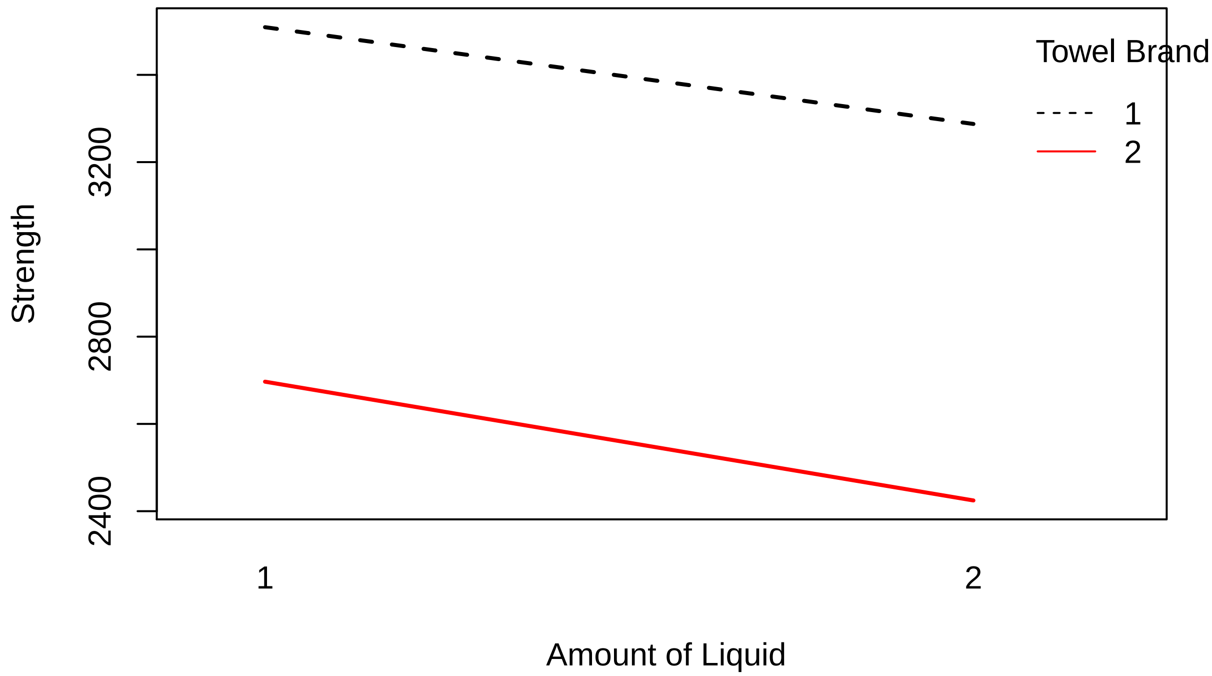

```r
interaction.plot(A,C , strength , col = c(1,2,3), lwd = 2, xlab = " Amount of Liquid",
                 ylab = "Strength", trace.label = "Type of Liquid")
```




```r
interaction.plot(B,C , strength , col = c(1,2,3), lwd = 2, xlab = "Towel Brand",
                 ylab = "Strength", trace.label = "Type of Liquid")
```




```r
interaction.plot(A,B , strength , col = c(1,2,3), lwd = 2, xlab = " Amount of Liquid",
                 ylab = "Strength", trace.label = "Towel Brand")
```



The interaction plot of A and C, Amounts of Liquid vs. Strength with respect to Type of Liquid, and the interaction plot of B and C, Towel Brand vs. Strength with respect to Type of Liquid, both appear to have parallel lines. Furthermore, the three way interaction plot appears to have parallel lines as well but a minor downward shift of the factor B when changing from level 1 to level 2 of factor C. However, it still might not be cause for concern. These two results would indicate that factors A and C probably don't interact. Similarly, we can say that factors B and C probably don't interact. From the last interaction plot which shows the interactions between A and B, we can see that the lines appear to be very parallel which would suggest that the assumptions that A and B interact is likely to be false and therefore the model in part (a) might not be the best model to represent the data.

(d) Provide an analysis of variance table for this experiment, test the various effects, show plots of significant main effects and interactions, and draw conclusions.

For part (d), proceed with the typical ANOVA, without variable transformation, etc., regardless of your opinion on the residual plots in (c). Test each effect in the ANOVA table at 0.01 (so you maintain a FWER of 0.04). You do not need to produce plots of significant effects.

```
model_AB <- aov(strength ~ A + B + C + A:B, data= paper.towel.strength)
anova(model_AB)
```

```
## Analysis of Variance Table
##
## Response: strength
##           Df  Sum Sq Mean Sq F value    Pr(>F)
## A          1  365931  365931  3.7322  0.068426 .
## B          1 4209359 4209359 42.9318 2.836e-06 ***
## C          1  858552  858552  8.7565  0.008058 **
```

```
## A:B         1    3823    3823  0.0390  0.845566
## Residuals 19 1862906   98048
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

From the Anova table above we get a p-value of $.8456 > .01$. So we can see that our data shows negligible interactions between amount of liquid and towel brand. The effects of Type of Liquid and Paper Towel averaged over Amount of Liquid do appear to have some effect on mean strength since we obtain a p-value of $.008 < .01$ and $.0000028 < .01$ respectively. Lastly, the Amount of Liquid averaged over Paper Towel does not seem to have an effect on mean strength since we obtain a p-value of $.068 > .01$.

(e) Construct confidence intervals for each of the treatment contrasts that you listed in part (b), using an appropriate method of multiple comparisons. Discuss the results.

For part (e), use a confidence level of 99% for the one primary contrast of interest

```
em.B <- emmeans(model_AB, specs = ~B)
```

```
## NOTE: Results may be misleading due to involvement in interactions
```

```
summary(contrast(em.B, method = "tukey"), infer = c(TRUE,TRUE), level=.99)
```

```
##  contrast estimate      SE df lower.CL upper.CL t.ratio p.value
##  1 - 2    837.5917 127.833 19 471.8698 1203.314   6.552  <.0001
##
## Results are averaged over the levels of: A, C
## Confidence level used: 0.99
```