Considerando P^* con un punto más que P. Sea $x^* \in [x_{k-1}, x_k]$ el punto extra mencionado. Sean

$$w_1 = \text{ínf}\,\{f(x)\,|\,x \in [x_{k-1}, x^*]\}$$
$$w_2 = \text{ínf}\,\{f(x)\,|\,x \in [x^*, x_k]\}$$

Se tiene entonces que:

$$\begin{aligned}
& L(P^*, f, \alpha) - L(P, f, \alpha) \\
= \\
& [\alpha(x^*) - \alpha(x_{k-1})]w_1 + [\alpha(x_k) - \alpha(x^*)]w_2 - [\alpha(x_k) - \alpha(x_{k-1})]m_k^f \\
= \\
& [\alpha(x^*) - \alpha(x_{k-1})]w_1 + [\alpha(x_k) - \alpha(x^*)]w_2 \\
& -[\alpha(x_k) - \alpha(x^*)]m_k^f - [\alpha(x^*) - \alpha(x_{k-1})]m_k^f \\
= \\
& [\alpha(x_k) - \alpha(x^*)](w_2 - m_k^f)[\alpha(x^*) - \alpha(x_{k-1})](w_1 - m_k^f) \\
\geq \quad & \left\langle\, m_k^f = \text{mín}\{w_1, w_2\} \,\right\rangle \\
& 0
\end{aligned}$$

Para la segunda proposición, la demostración sigue la misma idea, teniendo en cuenta que:

$$M_k^f = \text{máx}\,\{\sup\,\{f(x)\,|\,x \in [x_{k-1}, x^*]\}\,, \sup\,\{f(x)\,|\,x \in [x^*, x_k]\}\}$$