Sean $z_1 = r_1(\cos(\theta_1) + i\sin(\theta_1))$ y $z_2 = r_2(\cos(\theta_2) + i\sin(\theta_2))$, con $\theta_1, \theta_2 \in (-\pi, \pi]$. Tomando $\arg z_1 + \arg z_2 = \{\theta_1 + \theta_2 + 2k\pi \mid k \in \mathbb{Z}\}$, se quiere ver que $\arg(z_1\, z_2) = \arg z_1 + \arg z_2$.

Sean $\phi_1 \in \arg z_1$ y $\phi_2 \in \arg z_2$.

$z_1\, z_2$

$=$

$r_1(\cos(\theta_1) + i\sin(\theta_1))r_2(\cos(\theta_2) + i\sin(\theta_2))$

$=$

$r_1 r_2[\cos(\theta_1)\cos(\theta_2) - \sin(\theta_1)\sin(\theta_2) + i(\cos(\theta_1)\sin(\theta_2) + \sin(\theta_1)\cos(\theta_2))]$

$=$

$r_1 r_2(\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2))$

Con esto, $\arg(z_1) + \arg(z_2) \subseteq \arg(z_1 z_2)$

Sean (x_1, y_1) las coordenadas cartesianas de z_1, (x_2, y_2) las coordenadas cartesianas de z_2, $\phi \in \arg(z_1 z_2)$ y $\phi_2 \in \arg z_1$. Con esto,

$$z_1 z_2 = (x_1 x_2 - y_1 y_2, x_1 y_2 + x_2 y_1)$$

Por la relación entre coordenadas cartesianas y polares, se sabe que

$$\tan(\phi) = \frac{x_1 y_2 + x_2 y_1}{x_1 x_2 - y_1 y_2}$$

Para ver si $\phi \in \arg(z_1) + \arg(z_2)$, se considerará la siguiente ecuación:

$$\tan(\phi_1 + \alpha) = \frac{x_1 y_2 + x_2 y_1}{x_1 x_2 - y_1 y_2}$$

$$\equiv$$

$$\frac{\tan(\phi_1) + \tan(\alpha)}{1 + \tan(\phi_1)\tan(\alpha)} = \frac{x_1 y_2 + x_2 y_1}{x_1 x_2 - y_1 y_2}$$

$$\equiv$$

$$\frac{x_1 + \tan(\alpha)}{x_1 + y_1 \tan(\alpha)} = \frac{x_1 y_2 + x_2 y_1}{x_1 x_2 - y_1 y_2}$$

$$\equiv$$

$$x_1 x_2 y_1 + x_1^2 x_2 \tan(\alpha) - y_1^2 y_2 - x_1 y_1 y_2 \tan(\alpha)$$

$$=$$

$$x_1^2 y_2 + x_1 x_2 y_1 - x_1 y_1 y_2 \tan(\alpha) - x_2 y_1^2 \tan(\alpha)$$

$$\equiv$$

$$\tan(\alpha)(x_1^2 x_2 + x_2 y_1^2) = x_1^2 y_2 + y_1^2 y_2$$

$$\equiv$$

$$\tan(\alpha) = \frac{y_2}{x_2}$$

$$\equiv$$

$$\alpha \in \arg z_2$$

Con este argumento, se tiene entonces que

$$\arg(z_1 z_2) \subseteq \arg z_2 + \arg z_2$$