Tarea 04

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Matemáticas
Escuela Colombiana de Ingeniería Julio Garavito
Colombia
11 de septiembre de 2022

Sección 2.6

Índice

1. Punto 2

Punto 2

A dice: "Soy escudero o B es un caballero"
$\Gamma_0 = \{(a \equiv ((\neg a) \vee b)), a\}$
$\Gamma_1 = \{(a \equiv ((\neg a) \vee b)), (\neg a)\}$

$\therefore$ A es caballero y B es escudero

Con Γ_0

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1.	$\mathbf{v}[(a \equiv ((\neg a) \vee b))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[a] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[((\neg a) \vee b)] = \mathtt{T}$	MT 2.23($\equiv$)(p2, p1)
4.	$\mathbf{v}[(\neg a)] = \mathtt{T}$ o $\mathbf{v}[b] = \mathtt{T}$	MT 2.23 ($\vee$)(p3)
5.	$\mathbf{v}[b] = \mathtt{T}$	MT 2.23($\vee$)(p4, p2)

Con Γ_1

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1.	$\mathbf{v}[(a \equiv ((\neg a) \vee b))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\neg a)] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[((\neg a) \vee b)] = \mathtt{F}$	MT 2.23($\equiv$)(p2, p1)
4.	$\mathbf{v}[(\neg a)] = \mathtt{F}$ y $\mathbf{v}[b] = \mathtt{F}$	MT 2.23($\vee$)(p3)
5.	$\mathbf{v}[(\neg a)] = \mathtt{F}$ y $\mathbf{v}[(\neg a)] = \mathtt{T}$	Contradicción (p4, p2)

2. Punto 11

Punto 11

B dijo: "A dijo que es escudero"
C dijo: "No le crea a B porque está mintiendo"
$\Gamma_0 = \{(b \equiv (a \equiv (\neg a))), (c \equiv (\neg b)), b\}$
$\Gamma_1 = \{(b \equiv (a \equiv (\neg a))), (c \equiv (\neg b)), (\neg b)\}$

$\therefore$ B es escudero y C es caballero

Con Γ_0

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1.	$\mathbf{v}[(b \equiv (a \equiv (\neg a)))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[b] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[(a \equiv (\neg a))] = \mathtt{F}$	MT 2.23 ($\equiv$)
4.	$\mathbf{v}[b] = \mathtt{F}$	MT 2.23($\equiv$)(p1)
5.	$\mathbf{v}[b] = \mathtt{T}$ y $\mathbf{v}[b] = \mathtt{F}$	Contradicción (p4, p2)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(b \equiv (a \equiv (\neg a)))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[b] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[c] = \mathtt{T}$

3. Punto 12

Punto 12

B dijo: "A dijo que hay al menos un caballero entre nosotros"
C dijo: "B miente"
$\Gamma_0 = \{(b \equiv (a \equiv (a \vee b \vee c))), (c \equiv (\neg b)), b\}$
$\Gamma_1 = \{(b \equiv (a \equiv (a \vee b \vee c))), (c \equiv (\neg b)), (\neg b)\}$

$\therefore$ No es posible determinar su naturaleza

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(b \equiv (a \equiv (a \vee b \vee c)))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(c \equiv (\neg b))] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[b] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[(\neg b)] = \mathtt{F}$ MT 2.23($\neg$)(p3)
5. $\mathbf{v}[c] = \mathtt{F}$ MT 2.23($\equiv$)(p4,p2)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(b \equiv (a \equiv (a \vee b \vee c)))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(c \equiv (\neg b))] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(\neg b)] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[c] = \mathtt{T}$ MT 2.23($\equiv$)(p3, p2)

4. Punto 13

Punto 13

A dice: "Todos nosotros somos escuderos"
B dice: "Exactamente uno de nosotros es caballero"

$$\begin{aligned}
\Gamma_0 = \{&(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c))),\\
&(b \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c)), ((\neg a) \wedge (\neg b))\}\\
\Gamma_1 = \{&(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c))),\\
&(b \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c)), b\}\\
\Gamma_2 = \{&(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c))),\\
&(b \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c)), (\neg b)\}
\end{aligned}$$

$\therefore$ A es escudero, no se puede determinar la naturaleza de los demás

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c)))] = \mathtt{T}$ — Def.(p0)
2. $\mathbf{v}[(b \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c))]$ — Def.(p0)
3. $\mathbf{v}[((\neg a) \wedge (\neg b))]$
4. $\mathbf{v}[(\neg c)] = \mathtt{F}$ — MT 2.23($\equiv$, $\wedge$)(p3, p1)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_1)$
1. $\mathbf{v}[(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c)))] = \mathtt{T}$ — Def.(p0)
2. $\mathbf{v}[(b \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c))]$ — Def.(p0)
3. $\mathbf{v}[b] = \mathtt{T}$
4. $\mathbf{v}[((\neg a) \wedge b \wedge (\neg c))] = \mathtt{F}$ — MT 2.23($\equiv$, $\wedge$) (p2)
5. $\mathbf{v}[a] = \mathtt{F}$ — MT 2.23($\wedge$, $\neg$)
6. $\mathbf{v}[c] = \mathtt{F}$ — MT 2.23($\wedge$, $\neg$)

Con Γ_2

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_2)$
1. $\mathbf{v}[(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c)))] = \mathtt{T}$ — Def.(p0)
2. $\mathbf{v}[(b \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c))]$ — Def.(p0)
3. $\mathbf{v}[(\neg b)] = \mathtt{T}$ — Def.(p0)
4. $\mathbf{v}[(a \wedge (\neg b) \wedge (\neg c))] = \mathtt{F}$ — MT 2.23($\neg$, $\vee$)(p3, p2)
5. $\mathbf{v}[(\neg b)] = \mathtt{F}$ — MT 2.23($\wedge$)(4)
6. $\mathbf{v}[(\neg b)] = \mathtt{F}$ y $\mathbf{v}[(\neg b)] = \mathtt{T}$ — Contradicción (p5, p3)

5. Punto 14

Punto 14

A dice: "Todos somos escuderos"
B dice: "Exactamente uno de nosotros es escudero"
$\Gamma_0 = \{(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c))),$
$(b \equiv (((\neg a) \wedge b \wedge c) \vee (a \wedge (\neg b) \wedge c) \vee (a \wedge b \wedge (\neg c)))), (\neg a)\}$
$\Gamma_1 = \{(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c))),$
$(b \equiv (((\neg a) \wedge b \wedge c) \vee (a \wedge (\neg b) \wedge c) \vee (a \wedge b \wedge (\neg c)))), a\}$

El ejercicio está mal planteado

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c)))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(b \equiv (((\neg a) \wedge b \wedge c) \vee (a \wedge (\neg b) \wedge c) \vee (a \wedge b \wedge (\neg c))))] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[a] = \mathtt{F}$ MTT 2.23($\neg$)(p3)
5. $\mathbf{v}[(\neg a)] = \mathtt{F}$ y $\mathbf{v}[(\neg b)] = \mathtt{F}$ y $\mathbf{v}[(\neg c)] = \mathtt{F}$ MT 2.23 ($\equiv$, $\wedge$)(p4, p1)
6. $\mathbf{v}[(\neg a)] = \mathtt{F}$ y $\mathbf{v}[(\neg a)] = \mathtt{T}$ Contradicción (p5, p3)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv ((\neg a) \wedge (\neg b) \wedge (\neg c)))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(b \equiv (((\neg a) \wedge b \wedge c) \vee (a \wedge (\neg b) \wedge c) \vee (a \wedge b \wedge (\neg c))))] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[(\neg a)] = \mathtt{T}$ y $\mathbf{v}[(\neg b)] = \mathtt{T}$ y $\mathbf{v}[(\neg c)] = \mathtt{T}$ MT 2.23($\equiv$, $\wedge$)
5. $\mathbf{v}[a] = \mathtt{F}$ MT 2.23($\neg$)(p4)
6. $\mathbf{v}[a] = \mathtt{T}$ y $\mathbf{v}[a] = \mathtt{F}$ (p5, p3)

6. Punto 18

El habitante A dice "Yo dije que si soy caballero entonces soy escudero, y si soy escudero entonces soy caballero"

7. Punto 19

El habitante A dice "Si B es caballero, entonces soy caballero"
El habitante B dice "Si soy caballero, entonces A es caballero"

8. Punto 23

Punto 23

C dijo: "A lo sumo uno de nosotros es caballero"

$\Gamma_0 = \{(c \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c) \vee ((\neg a) \wedge (\neg b) \wedge (\neg c))), c\}$

$\Gamma_1 = \{(c \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c) \vee ((\neg a) \wedge (\neg b) \wedge (\neg c))), (\neg c)\}$

C es el único caballero, y por ende el único hombre lobo (tomando la suposición dada por el enunciado).

... Tomando lo que dijo C como ϕ...

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(c \equiv \phi] = \mathtt{T}$ — Def.(p0)
2. $\mathbf{v}[c] = \mathtt{T}$ — Def(p0)
3. $\mathbf{v}[((\neg a) \wedge (\neg b) \wedge c)] = \mathtt{T}$ — MT 2.23($\equiv$, $\wedge$)(p2, p1)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(c \equiv \phi] = \mathtt{T}$ — Def.(p0)
2. $\mathbf{v}[(\neg c)] = \mathtt{T}$ — Def(p0)
3. $\mathbf{v}[(a \wedge (\neg b) \wedge (\neg c))] = \mathtt{F}$ — MT 2.23($\equiv$, $\wedge$) (p2, p1)
4. $\mathbf{v}[(\neg c)] = \mathtt{F}$ — MTT 2.23 ($\wedge$)(p3)
5. $\mathbf{v}[(\neg c)] = \mathtt{F}$ y $\mathbf{v}[(\neg c)] = \mathtt{T}$ — Contradicción (p4, p2)