Tarea 04

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1. Sección 2.3

1.1. Punto 1

1.1.1. a) $\models ((\phi \equiv \psi) \equiv (\psi \equiv \phi))$

0.	$\mathbf{v}[((\phi \equiv \psi) \equiv (\psi \equiv \phi))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \equiv \psi) \equiv (\psi \equiv \phi))] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \equiv \psi)] \neq \mathbf{v}[(\psi \equiv \phi)]$	
3.	$(\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \equiv \phi)] = \mathtt{F})$	suposición 1, MT 2.20
4.	$(\mathbf{v}[\phi] = \mathbf{v}[\psi]) \wedge (\mathbf{v}[\psi] \neq \mathbf{v}[\phi])$	MT 2.23 ($\equiv$)
5.	Contradicción	
6.	$(\mathbf{v}[(\phi \equiv \psi)] = \mathtt{F}) \wedge (\mathbf{v}[(\psi \equiv \phi)] = \mathtt{T})$	suposición 2
7.	$(\mathbf{v}[\phi] \neq \mathbf{v}[\psi]) \wedge (\mathbf{v}[\psi] = \mathbf{v}[\phi])$	MT 2.23 ($\equiv$)
8.	Contradicción	
9.	$\therefore \quad \models ((\phi \equiv \psi) \equiv (\psi \equiv \phi))$	

1.1.2. b) $\models ((\phi \equiv \boldsymbol{true}) \equiv \phi)$

0.	$\mathbf{v}[((\phi \equiv true) \equiv \phi)] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \equiv true) \equiv \phi)] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \equiv true)] \neq \mathbf{v}[\phi]$	
3.	$(\mathbf{v}[(\phi \equiv true)] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1, MT 2.20
4.	$(\mathbf{v}[\phi] = \mathbf{v}[true]) \wedge (\mathbf{v}[\phi]) = \mathtt{F}$	MT 2.23
5.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	
6.	Contradicción	
7.	$(\mathbf{v}[(\phi \equiv true)] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	suposición 2, MT 2.20
8.	$(\mathbf{v}[\phi] \neq \mathbf{v}[true]) \wedge (\mathbf{v}[\phi]) = \mathtt{T}$	MT 2.23
9.	$(\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	
10.	Contradicción	
11.	$\therefore \quad \models ((\phi \equiv true) \equiv \phi)$	

1.1.3. f) $\models ((\phi \vee \boldsymbol{false}) \equiv \phi)$

0.	$\mathbf{v}[((\phi \vee \mathit{false}) \equiv \phi)] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \vee \mathit{false}) \equiv \phi)] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \vee \mathit{false})] \neq \mathbf{v}[\phi]$	
3.	$(\mathbf{v}[(\phi \vee \mathit{false})] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1, MT 2.20
4.	$((\mathbf{v}[\phi] = \mathtt{T}) \vee (\mathbf{v}[\mathit{false}] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	MT 2.23
5.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	
6.	Contradicción	
7.	$(\mathbf{v}[(\phi \vee \mathit{false})] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	suposición 2, MT 2.20
8.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\mathit{false}] = \mathtt{F})) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	MT 2.23
9.	$(\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	simplificación (8)
10.	Contradicción	
11.	$\therefore \quad \models ((\phi \vee \mathit{false}) \equiv \phi)$	

1.1.4. g) $\models ((\phi \vee \phi) \equiv \phi)$

0.	$\mathbf{v}[((\phi \vee \phi) \equiv \phi)] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \vee \phi) \equiv \phi)] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \vee \phi)] \neq \mathbf{v}[\phi]$	MT 2.23 ($\equiv$)
3.	$(\mathbf{v}[(\phi \vee \phi)] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1, MT 2.20
4.	$((\mathbf{v}[\phi] = \mathtt{T}) \vee (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	MT 2.20
5.	$((\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1.1
6.	Contradicción	
7.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1.2
8.	Contradicción	
9.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1.3
10.	Contradicción	
11.	$(\mathbf{v}[(\phi \vee \phi)] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	suposición 2, MT 2.20
12.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{F})) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	MT 2.23
13.	Contradicción	
14.	$\therefore \quad \models ((\phi \vee \phi) \equiv \phi)$	

1.1.5. k) $\vDash (\neg(\phi \wedge (\neg\phi)))$

0.	$\mathbf{v}[(\neg(\phi \wedge (\neg\phi)))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[(\neg(\phi \wedge (\neg\phi)))] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \wedge (\neg\phi))] = \mathtt{T}$	MT 2.23 ($\neg$)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[(\neg\phi)] = \mathtt{T})$	MT 2.23 ($\wedge$)
4.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	MT 2.23 ($\neg$)
5.	Contradicción	
6.	$\therefore \quad \vDash (\neg(\phi \wedge (\neg\phi)))$	

1.1.6. l) $\vDash (\phi \to (\psi \to \phi))$

0.	$\mathbf{v}[(\phi \to (\psi \to \phi))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[(\phi \to (\psi \to \phi))] = \mathtt{F}$	suposición (intento por contradicción)
2.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \to \phi)] = \mathtt{F})$	MT 2.23 ($\to$)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge ((\mathbf{v}[\psi] = \mathtt{T}) \wedge ((\mathbf{v}[\phi] = \mathtt{F})))$	MT 2.23 ($\to$)
4.	Contradicción	
5.	$\therefore \quad (\phi \to (\psi \to \phi))$	

1.1.7. n) $\vDash ((\phi \to \psi) \equiv ((\neg\psi) \to (\neg\phi)))$

0.	$\mathbf{v}[((\phi \to \psi) \equiv ((\neg\psi) \to (\neg\phi)))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[(\phi \to \psi)] = \mathbf{v}[((\neg\psi) \to (\neg\phi))]$	MT 2.23 ($\equiv$)
2.	Partiendo de $(\phi \to \psi)$	
3.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{F}$	suposición 1
4.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\psi] = \mathtt{F})$	MT 2.23 ($\to$)
5.	$(\mathbf{v}[\psi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	Conmutativa de $\wedge$
6.	$(H_\neg(\mathbf{v}[\psi]) = \mathtt{T}) \wedge (H_\neg(\mathbf{v}[\phi]) = \mathtt{F})$	Doble negación (5)
7.	$(\mathbf{v}[(\neg\psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\neg\phi)]) = \mathtt{F}$	Def. 2.18 ($\neg$)
8.	$\mathbf{v}[((\neg\psi) \to (\neg\phi))] = \mathtt{F}$	MT 2.23 ($\to$)
9.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$	suposición 2
10.	$(\mathbf{v}[\phi] = \mathtt{F}) \vee (\mathbf{v}[\psi] = \mathtt{T})$	MTT 2.23 ($\to$)
11.	$(\mathbf{v}[\psi] = \mathtt{T}) \vee (\mathbf{v}[\phi] = \mathtt{F})$	Conmutativa de $\vee$
12.	$(H_\neg(\mathbf{v}[\psi]) = \mathtt{F}) \vee (H_\neg(\mathbf{v}[\phi]) = \mathtt{T})$	Doble negación (10)
13.	$(\mathbf{v}[(\neg\psi)] = \mathtt{F}) \vee (\mathbf{v}[(\neg\phi)]) = \mathtt{T}$	Def. 2.18 ($\neg$)
14.	$\mathbf{v}[((\neg\psi) \to (\neg\phi))] = \mathtt{T}$	MT 2.23 ($\to$)
15.	$\therefore \quad \vDash ((\phi \to \psi) \equiv ((\neg\psi) \to (\neg\phi)))$	

1.2. Punto 2

1.2.1. b) $((\neg p) \vee q)$

0.	$(\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[((\neg p) \vee q)] = \mathtt{T}) \wedge (\mathbf{w}[((\neg p) \vee q)] = \mathtt{F}))$	Enunciado
1.	$\mathbf{v}[((\neg p) \vee q)] = \mathtt{T}$	suposición 1
2.	$(\mathbf{v}[(\neg p)] = \mathtt{T}) \vee (\mathbf{v}[q] = \mathtt{T})$	MT 2.23 $(\vee)$
3.	$(\mathbf{v}[p] = \mathtt{F}) \vee (\mathbf{v}[q] = \mathtt{T})$	Doble negación (2)
4.	$\mathbf{v} = \{p \mapsto \mathtt{F}, q \mapsto \mathtt{T}\}$	
5.	$(\exists \mathbf{v} \mid \mathbf{v}[((\neg p) \vee q)] = \mathtt{T})$	
6.	$\mathbf{w}[((\neg p) \vee q)] = \mathtt{F}$	suposición 2
7.	$(\mathbf{w}[(\neg p)] = \mathtt{F}) \wedge (\mathbf{w}[q] = \mathtt{F})$	MT 2.23 $(\vee)$
8.	$(\mathbf{w}[p] = \mathtt{T}) \wedge (\mathbf{w}[q] = \mathtt{F})$	Doble negación (2)
9.	$\mathbf{w} = \{p \mapsto \mathtt{T}, q \mapsto \mathtt{F}\}$	
10.	$(\exists \mathbf{w} \mid \mathbf{w}[((\neg p) \vee q)] = \mathtt{f})$	
11.	$\therefore \quad (\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[((\neg p) \vee q)] = \mathtt{T}) \wedge (\mathbf{w}[((\neg p) \vee q)] = \mathtt{F}))$	

1.2.2. d) $(\neg(p \wedge (\neg q)))$

0.	$(\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{T}) \wedge (\mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{F}))$	Enunciado
1.	$\mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{F}$	suposición 1
2.	$\mathbf{v}[(p \wedge (\neg q))] = \mathtt{T}$	Def. 2.18 $(\neg)$
3.	$(\mathbf{v}[p] = \mathtt{T}) \wedge (\mathbf{v}[(\neg q)] = \mathtt{T})$	MT 2.23 $(\wedge)$
4.	$(\mathbf{v}[p] = \mathtt{T}) \wedge (\mathbf{v}[q] = \mathtt{F})$	Def. 2.18 $(\neg)$
5.	$\mathbf{v} = \{p \mapsto \mathtt{T}, q \mapsto \mathtt{F}\}$	
6.	$(\exists \mathbf{v} \mid \mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{T})$	
7.	$\mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{T}$	suposición 2
8.	$\mathbf{w}[(p \wedge (\neg q))] = \mathtt{F}$	Def. 2.18 $(\neg)$
9.	$(\mathbf{w}[p] = \mathtt{F}) \vee (\mathbf{w}[(\neg q)] = \mathtt{F})$	
10.	$(\mathbf{w}[p] = \mathtt{F}) \vee (\mathbf{w}[q] = \mathtt{T})$	Def. 2.18 $(\neg)$
11.	$\mathbf{w} = \{p \mapsto \mathtt{F}, q \mapsto \mathtt{T}\}$	
12.	$(\exists \mathbf{w} \mid \mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{T})$	
13.	$\therefore \quad (\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{T}) \wedge (\mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{F}))$	

1.3. Punto 3

$$\bullet(\phi \wedge (\neg\phi))$$
$$\bullet((\phi \rightarrow \psi) \equiv (\phi \wedge (\neg\psi)))$$
$$\bullet((\phi \leftarrow \psi) \equiv ((\neg\phi) \wedge \psi))$$

1.4. Punto 6

1.4.1. b)

Se tiene de entrada que *false*, al ser una constante, no hace de operador entre 2 proposiciones, por lo que la única opción es:

0.	$\models (\mathit{false})$	suposición/ enunciado
1.	$\mathbf{v}[\mathit{false}] = \mathtt{T}$	Def.(p0)
2.	$\mathtt{F} = \mathtt{T}$	Def. 2.18 (*false*) (p1)
3.	Contradicción	
4.	$\therefore$ No existe dicha proposición mencionada en el enunciado	

1.4.2. d)

0.	$\models (\phi \not\equiv \psi)$	suposición/ enunciado
1.	$\mathbf{v}[(\phi \not\equiv \psi)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\phi \not\equiv \psi)] = \mathtt{F}$	suposición (intento por contradicción)
3.	$\mathbf{v}[\phi] = \mathbf{v}[\psi]$	MT 2.23 ($\not\equiv$) (p2)
4.	$\mathbf{v}[\phi] = \mathtt{T}$	suposición 1 , MT 2.19 N 2.20
5.	$\mathbf{v}[\psi] = \mathtt{T}$	p(3, 4)
6.	$\therefore$ $(\exists \mathbf{v} \mid \mathbf{v}[(\phi \not\equiv \psi)] = \mathtt{F})$	

1.4.3. f)

0.	$\models (\phi \wedge \psi)$	suposición/ enunciado
1.	$\mathbf{v}[(\phi \wedge \psi)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\phi \wedge \psi)] = \mathtt{F}$	suposición (intento por contradicción)
3.	$(\mathbf{v}[\phi] = \mathtt{F}) \vee (\mathbf{v}[\psi] = \mathtt{F})$	MT 2.23 (p2)
4.	$\mathbf{v}[\phi] = \mathtt{F}$	suposición 1, MT 2.19 N 2.20
5.	$\mathbf{v}[(\phi \wedge \psi)] = \mathtt{F}$	MT 2.23, p4
6.	$\therefore$ $(\exists \mathbf{v} \mid \mathbf{v}[(\phi \wedge \psi)] = \mathtt{F})$	

1.4.4. g)

0.	$\vDash (\phi \to \psi)$	suposición/ enunciado
1.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{F}$	suposición (intento por contradicción)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\psi] = \mathtt{F})$	
4.	$\therefore \quad (\exists \mathbf{v} \mid \mathbf{v}[(\phi \to \psi)] = \mathtt{F})$	

1.5. Punto 8

1.6. a)

0.	Si $\vDash (\phi \to \psi)$ y $\vDash (\psi \to \tau)$, entonces $\vDash (\phi \to \tau)$	Enunciado
1.	$((\mathbf{v}[(\phi \to \psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \to \tau)] = \mathtt{T})) \implies (\mathbf{v}[(\phi \to \tau)] = \mathtt{T})$	Def. (p0)
2.	$(\exists \mathbf{v} \mid \mathbf{v}[(\phi \to \tau)] = \mathtt{F})$	suposición , MT 2.19 N 2.20 $(\to)$ (p1)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\tau] = \mathtt{F})$	MT 2.23 $(\to)$ (p2)
4.	se tiene que $((\mathbf{v}[(\phi \to \psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \to \tau)] = \mathtt{T}))$	
5.	$\mathbf{v}[\phi] = \mathtt{T}$	Caso $(\mathit{true} \to \xi)$ (p4 , p3)
6.	$\mathbf{v}[\tau] = \mathtt{T}$	Caso $(\mathit{true} \to \xi)$ (p4 , p5)
7.	Contradicción	(p6, p3)
8.	$\therefore$ Si $\vDash (\phi \to \psi)$ y $\vDash (\psi \to \tau)$, entonces $\vDash (\phi \to \tau)$	Enunciado

1.7. b)

0.	Si $\vDash (\phi \to \psi)$ y $\vDash (\phi)$, entonces $\vDash (\psi)$	Enunciado
1.	$((\mathbf{v}[(\phi \to \psi)] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \implies (\mathbf{v}[\psi] = \mathtt{T})$	Def.(p0)
2.	$(\exists \mathbf{v} \mid \mathbf{v}[\psi] = \mathtt{F})$	suposición, MT 2.19 N 2.20
3.	Se tiene que $((\mathbf{v}[(\phi \to \psi)] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{T}))$	
4.	$((\mathbf{v}[\phi] = \mathtt{F}) \vee (\mathbf{v}[\psi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	MT 2.23 $(\to)$ (p3)
5.	Nótese que por un lado (mediante $\vee$) $(\mathbf{v}[\phi] = \mathtt{F})$ y por otro $(\mathbf{v}[\phi] = \mathtt{T})$	
6.	$\mathbf{v}[\psi] = \mathtt{T}$	Caso $(\mathit{false} \vee \xi)$
7.	Contradicción	(p6, p2)
8.	$\therefore$ Si $\vDash (\phi \to \psi)$ y $\vDash (\phi)$, entonces $\vDash (\psi)$	Enunciado