Tarea 09

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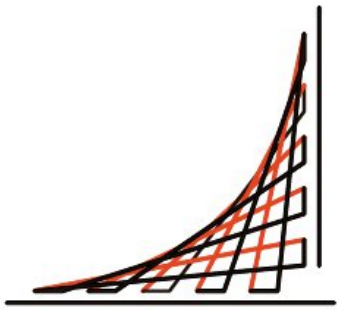

ESCUELA COLOMBIANA DE INGENIERÍA JULIO GARAVITO

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Matemáticas
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1. Sección 4.6

1.1. Punto 4

Teo 4.24.3

$$
\begin{array}{ll}
 & (\phi \wedge \mathit{true}) \\
\equiv & \langle\ \text{Def.}(\wedge)\ \rangle \\
 & (\phi \equiv (\mathit{true} \equiv (\phi \vee \mathit{true}))) \\
\equiv & \langle\ \text{Teo 4.19.2, Leibniz}(\phi = (\phi \equiv (\mathit{true} \equiv p)))\ \rangle \\
 & (\phi \equiv (\mathit{true} \equiv \mathit{true})) \\
\equiv & \langle\ \text{Teo 4.6.2, Leibniz}(\phi = (\phi \equiv p))\ \rangle \\
 & (\phi \equiv \mathit{true}) \\
\equiv & \langle\ \text{Identidad}(\equiv)\ \rangle \\
 & \phi
\end{array}
$$

Por MT 4.21 se demuestra que
$\vdash_{\text{DS}} ((\phi \wedge \mathit{true}) \equiv \phi)$

1.2. Punto 5

Teo 4.24.4

$$
\begin{array}{ll}
 & (\phi \wedge \mathit{false}) \\
\equiv & \langle\ \text{Def.}(\wedge)\ \rangle \\
 & (\phi \equiv (\mathit{false} \equiv (\phi \vee \mathit{false}))) \\
\equiv & \langle\ \text{Identidad}(\vee), \text{Conmutativa}(\equiv), \text{Leibniz}(\phi = (\phi \equiv (\mathit{false} \equiv p)))\ \rangle \\
 & (\phi \equiv (\mathit{false} \equiv \phi)) \\
\equiv & \langle\ \text{Def.}(\neg), \text{Conmutativa}(\equiv), \text{Leibniz}(\phi = (\phi \equiv p))\ \rangle \\
 & (\phi \equiv (\neg\phi)) \\
\equiv & \langle\ \text{Teo 4.15.7}\ \rangle \\
 & \mathit{false}
\end{array}
$$

1.3. Punto 6

Teo 4.25.5

$$
\begin{array}{ll}
 & (\phi \wedge \phi) \\
\equiv & \langle \text{ Def.}(\wedge) \rangle \\
 & (\phi \equiv (\phi \equiv (\phi \vee \phi))) \\
\equiv & \langle \text{ Asociativa}(\equiv) \rangle \\
 & ((\phi \equiv \phi) \equiv (\phi \vee \phi)) \\
\equiv & \langle \text{ Idempotencia}(\vee), \text{ Leibniz}(\phi = ((\phi \equiv \phi) \equiv p)) \rangle \\
 & ((\phi \equiv \phi) \equiv \phi) \\
\equiv & \langle \text{ Teo 4.6.3, Conmutativa}(\equiv) \rangle \\
 & (\phi \equiv \mathit{true}) \\
\equiv & \langle \text{ Identidad} \rangle \\
 & \phi
\end{array}
$$

1.4. Punto 8

Teo 4.25.1

$$
\begin{array}{ll}
 & (\phi \wedge (\neg\phi)) \\
\equiv & \langle \text{ Def.}(\wedge) \rangle \\
 & (\phi \equiv ((\neg\phi) \equiv (\phi \vee (\neg\phi)))) \\
\equiv & \langle \text{ Asociativa}(\equiv) \rangle \\
 & ((\phi \equiv (\neg\phi)) \equiv (\phi \vee (\neg\phi))) \\
\equiv & \langle \text{ Teo 4.19.1, Identidad, Leibniz}(\phi = ((\phi \equiv (\neg\phi)) \equiv p)) \rangle \\
 & ((\phi \equiv (\neg\phi)) \equiv \mathit{true}) \\
\equiv & \langle \text{ Teo 4.15.7, Conmutativa}(\equiv), \text{ Leibniz}(\phi = (p \equiv \mathit{true})) \rangle \\
 & (\mathit{false} \equiv \mathit{true}) \\
\equiv & \langle \text{ Identidad} \rangle \\
 & \mathit{false}
\end{array}
$$

1.5. Punto 9

Teo 4.25.2

$$
\begin{aligned}
& (\neg(\phi \wedge \psi)) \\
\equiv \quad & \langle \text{ Def.}(\wedge), \text{ Leibniz}(\phi = (\neg p)) \rangle \\
& (\neg(\phi \equiv (\psi \equiv (\phi \vee \psi)))) \\
\equiv \quad & \langle \text{ Conmutativa}(\vee), \text{ Leibniz}(\phi = (\neg(\phi \equiv p))) \rangle \\
& (\neg(\phi \equiv (\psi \equiv (\psi \vee \phi)))) \\
\equiv \quad & \langle \text{ Conmutativa}(\equiv), \text{ Leibniz}(\phi = (\neg(\phi \equiv p))) \rangle \\
& (\neg(\phi \equiv ((\psi \vee \phi) \equiv \psi))) \\
\equiv \quad & \langle \text{ Teo 4.15.6, Leibniz}(\phi = (\neg(\phi \equiv ((\psi \vee p) \equiv \psi)))) \rangle \\
& (\neg(\phi \equiv ((\psi \vee (\neg(\neg\phi))) \equiv \psi))) \\
\equiv \quad & \langle \text{ Teo 4.19.4, Leibniz}(\phi = (\neg(\phi \equiv p))) \rangle \\
& (\neg(\phi \equiv (\psi \vee (\neg\phi)))) \\
\equiv \quad & \langle \text{ Conmutativa 4.15.4} \rangle \\
& ((\neg\phi) \equiv (\psi \vee (\neg\phi))) \\
\equiv \quad & \langle \text{ Conmutativa}(\vee), \text{ Leibniz}(\phi = ((\neg\phi) \equiv p)) \rangle \\
& ((\neg\phi) \equiv ((\neg\phi) \vee \psi)) \\
\equiv \quad & \langle \text{ Conmutativa}(\equiv) \rangle \\
& (((\neg\phi) \vee \psi) \equiv (\neg\phi)) \\
\equiv \quad & \langle \text{ Teo 4.15.6, Leibniz}(\phi = (((\neg\phi) \vee p) \equiv (\neg\phi))) \rangle \\
& (((\neg\phi) \vee (\neg(\neg\psi))) \equiv (\neg\phi)) \\
\equiv \quad & \langle \text{ Teo 4.19.4} \rangle \\
& ((\neg\phi) \vee (\neg\psi))
\end{aligned}
$$

1.6. Punto 11

Teo 4.25.4

$$
\begin{array}{ll}
 & (((\phi \wedge \psi)) \equiv ((\phi \wedge \tau)) \equiv \phi) \\
\equiv & \langle \text{ Def.}(\wedge) \rangle \\
 & (((\phi \equiv (\psi \equiv (\phi \vee \psi))) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau)))) \equiv \phi) \\
\equiv & \langle \text{ Conmutativa}(\equiv) \rangle \\
 & (\phi \equiv ((\phi \equiv (\psi \equiv (\phi \vee \psi))) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau))))) \\
\equiv & \langle \text{ Asociativa}(\equiv), \text{Leibniz}(\phi = (\phi \equiv p)) \rangle \\
 & (\phi \equiv (\phi \equiv ((\psi \equiv (\phi \vee \psi))) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau))))) \\
\equiv & \langle \text{ Asociativa}(\equiv) \rangle \\
 & ((\phi \equiv \phi) \equiv ((\psi \equiv (\phi \vee \psi)) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau))))) \\
\equiv & \langle \text{ Teo 4.6.2, Leibniz}(\phi = (p \equiv ((\psi \equiv (\phi \vee \psi)) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau)))))) \rangle \\
 & (\mathit{true} \equiv ((\psi \equiv (\phi \vee \psi)) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau))))) \\
\equiv & \langle \text{ Conmutativa}(\equiv), \text{Identidad}(\equiv) \rangle \\
 & ((\psi \equiv (\phi \vee \psi)) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau)))) \\
\equiv & \langle \text{ Conmutativa}(\equiv), \text{Leibniz}(\phi = ((\psi \equiv (\phi \vee \psi)) \equiv p)) \rangle \\
 & ((\psi \equiv (\phi \vee \psi)) \equiv ((\tau \equiv (\phi \vee \psi)) \equiv \phi)) \\
\equiv & \langle \text{ Asociativa}(\equiv) \rangle \\
 & (((\psi \equiv (\phi \vee \psi)) \equiv (\tau \equiv (\phi \vee \psi))) \equiv \phi) \\
\equiv & \langle \text{ Conmutativa}(\equiv) \rangle \\
 & (\phi \equiv ((\psi \equiv (\phi \vee \psi)) \equiv (\tau \equiv (\phi \vee \psi)))) \\
\equiv & \langle \text{ Asociativa}(\equiv), \text{Leibniz}(\phi = (\phi \equiv (\phi \equiv p))) \rangle \\
 & (\phi \equiv (\psi \equiv ((\phi \vee \psi) \equiv (\tau \equiv (\phi \vee \psi))))) \\
\equiv & \langle \text{ Conmutativa}(\equiv), \text{Leibniz}(\phi = (\phi \equiv (\psi \equiv p))) \rangle \\
 & (\phi \equiv (\psi \equiv ((\tau \equiv (\phi \vee \psi)) \equiv (\phi \vee \psi)))) \\
\equiv & \langle \text{ Asociativa}(\equiv), \text{Leibniz}(\phi = (\phi \equiv p)) \rangle \\
 & (\phi \equiv ((\phi \equiv \tau) \equiv ((\phi \vee \tau) \equiv (\phi \vee \psi)))) \\
\equiv & \langle \text{ Distribución}(\vee, \equiv), \text{Leibniz}(\phi = (\phi \equiv ((\psi \equiv \tau) \equiv p))) \rangle \\
 & (\phi \equiv ((\psi \equiv \tau) \equiv (\phi \vee (\tau \equiv \psi)))) \\
\equiv & \langle \text{ Conmutativa}(\vee), \text{Leibniz}(\phi = (\phi \equiv ((\psi \equiv \tau) \equiv (\phi \vee p)))) \rangle \\
 & (\phi \equiv ((\psi \equiv \tau) \equiv (\phi \vee (\psi \equiv \tau)))) \\
\equiv & \langle \text{ Def.}(\wedge) \rangle \\
 & (\phi \wedge (\psi \equiv \tau))
\end{array}
$$

2. Sección 4.7

2.1. Punto 3

Teo 4.28.2

$$
\begin{array}{ll}
 & ((\phi \wedge \psi) \equiv \phi) \\
\equiv & \langle \text{ Def.}(\wedge), \text{ Leibniz}(\phi = (p \equiv \phi)) \rangle \\
 & ((\phi \equiv (\psi \equiv (\phi \vee \psi))) \equiv \phi) \\
\equiv & \langle \text{ Conmutativa}(\equiv), \text{ Asociativa}(\equiv), \text{ Identidad}(\equiv) \rangle \\
 & (\psi \equiv (\phi \vee \psi)) \\
\equiv & \langle \text{ Conmutativa}(\equiv), \text{ Def.}(\rightarrow) \rangle \\
 & (\phi \rightarrow \psi)
\end{array}
$$

2.2. punto 7

Teo 4.29.4

$$
\begin{array}{ll}
 & (\phi \rightarrow \mathit{false}) \\
\equiv & \langle \text{ Teo 4.28.1} \rangle \\
 & ((\neg\phi) \vee \mathit{false}) \\
\equiv & \langle \text{ Identidad}(\vee) \rangle \\
 & (\neg\phi)
\end{array}
$$

2.3. punto 10

Teo 4.30.3

$$
\begin{array}{ll}
 & (\phi \rightarrow (\psi \wedge \tau)) \\
\equiv & \langle \text{ Teo 4.28.1} \rangle \\
 & ((\neg\phi) \vee (\psi \wedge \tau)) \\
\equiv & \langle \text{ Dist.}(\vee, \wedge) \rangle \\
 & (((\neg\phi) \vee \psi) \wedge ((\neg\phi) \vee \tau)) \\
\equiv & \langle \text{ Def.}(\rightarrow) \rangle \\
 & ((\phi \rightarrow \psi) \wedge (\phi \rightarrow \tau))
\end{array}
$$

2.4. Punto 18

Teo 4.31.6

$$
\begin{array}{ll}
 & (\phi \vee (\phi \to \psi)) \\
\equiv & \langle \text{ Teo 4.28.1, Leibniz } (\phi = (\phi \vee p)) \rangle \\
 & (\phi \vee ((\neg\phi) \vee \psi)) \\
\equiv & \langle \text{ Asociativa}(\vee) \rangle \\
 & ((\phi \vee (\neg\phi)) \vee \psi) \\
\equiv & \langle \text{ Teo 4.19.1, Identidad}(\equiv) \rangle \\
 & (\mathit{true} \vee \psi) \\
\equiv & \langle \text{ Teo 4.19.2} \rangle \\
 & \mathit{true}
\end{array}
$$

2.5. Punto 17

Teo 4.31.5

$$
\begin{array}{ll}
 & (\phi \to (\psi \to \tau)) \\
\equiv & \langle \text{ Teo 4.18.1} \rangle \\
 & ((\neg\phi) \vee (\psi \to \tau)) \\
\equiv & \langle \text{ Teo 4.28.1, Leibniz}(\phi = ((\neg\phi) \vee p)) \rangle \\
 & ((\neg\phi) \vee ((\neg\psi) \vee \tau)) \\
\equiv & \langle \text{ Asociativa}(\vee) \rangle \\
 & (((\neg\phi) \vee (\neg\psi)) \vee \tau) \\
\equiv & \langle \text{ De Morgan, Leibniz}(\phi = (p \vee \tau)) \rangle \\
 & ((\neg(\phi \wedge \psi)) \vee \tau) \\
\equiv & \langle \text{ Teo 4.28.1} \rangle \\
 & ((\phi \wedge \psi) \to \tau)
\end{array}
$$

2.6. Punto 18

Teo 4.31.6

$$
\begin{array}{ll}
 & (\phi \vee (\phi \to \psi)) \\
\equiv & \langle \text{ Teo 4.28.1, Leibniz}(\phi = (\phi \vee p)) \rangle \\
 & (\phi \vee ((\neg\phi) \vee \psi)) \\
\equiv & \langle \text{ Asociativa}(\vee) \rangle \\
 & ((\phi \vee (\neg\phi)) \vee \psi) \\
\equiv & \langle \text{ Teo 4.19.1, Identidad}(\equiv) \rangle \\
 & (\mathit{true} \vee \psi) \\
\equiv & \langle \text{ Teo 4.19.2} \rangle \\
 & \mathit{true}
\end{array}
$$