Taller 05

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1. Punto 1

Punto 1 , $(A\ ,\ B)$

A dice: "nosotros tenemos la misma naturaleza"

$\Gamma_0 = \{(a \equiv (a \equiv b)), a\}$

$\Gamma_1 = \{(a \equiv (a \equiv b)), (\neg a)\}$

No es posible determinar la naturaleza de A y B

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv (a \equiv b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(a \equiv b)] = \mathtt{T}$ MT 2.23($\equiv$)(p1, p2)
4. $\mathbf{v}[b] = \mathtt{T}$ MTT 2.23 ($\equiv$)(p3, p2)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_1)$
1. $\mathbf{v}[(a \equiv (a \equiv b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(a \equiv b)] = \mathtt{F}$ MT 2.23($\equiv$)(p2, p1)
4. $\mathbf{v}[a] \neq \mathbf{v}[b]$ MT 2.23($\equiv$)(p3)
5. $\mathbf{v}[b] = \mathtt{T}$ (p4, p2)

2. Punto 2

Punto 2 , $(A\ ,\ B)$

A dice: "al menos uno de nosotros es caballero"

$\Gamma_0 = \{(a \equiv (a \vee b)), a\}$

$\Gamma_1 = \{(a \equiv (a \vee b)), (\neg a)\}$

No se puede determinar la naturaleza de A y B

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv (a \vee b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv (a \vee b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[a] = \mathtt{F}$ y $\mathbf{v}[b] = \mathtt{F}$

3. Punto 3

punto 3, $(A\ ,\ B\ ,\ C)$

A dice: "B es escudero"
B dice: "A y C son del mismo tipo"
$\Gamma_0 = \{(a \equiv (\neg b)), (b \equiv (a \equiv c)), a\}$
$\Gamma_1 = \{(a \equiv (\neg b)), (b \equiv (a \equiv c)), (\neg a)\}$

C es escudero

Con Γ_0

0. $(\exists \mathbf{v}, \mid \mathbf{v}$ satisface $\Gamma_0)$	
1. $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$	Def.(p0)
2. $\mathbf{v}[(b \equiv (a \equiv c))] = \mathtt{T}$	Def.(p0)
3. $\mathbf{v}[a] = \mathtt{T}$	Def.(p0)
4. $\mathbf{v}[a] = \mathbf{v}[(\neg b)]$	MT 2.23($\equiv$)(p2)
5. $\mathbf{v}[b] = \mathbf{v}[(a \equiv c)]$	MT 2.23($\equiv$)(p3)
6. $\mathbf{v}[b] = \mathtt{F}$	MT 2.23($\equiv$, $\neg$)(p4, p3)
7. $\mathbf{v}[a] \neq \mathbf{v}[c]$	MT 2.23($\equiv$)(p6, p5)
8. $\mathbf{v}[c] = \mathtt{F}$	(p7, p3)

Con Γ_1

0. $(\exists \mathbf{v}, \mid \mathbf{v}$ satisface $\Gamma_1)$	
1. $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$	Def.(p0)
2. $\mathbf{v}[(b \equiv (a \equiv c))] = \mathtt{T}$	Def.(p0)
3. $\mathbf{v}[(\neg a)] = \mathtt{T}$	Def.(p0)
4. $\mathbf{v}[b] = \mathtt{T}$	MT 2.23($\equiv$, $\neg$)(p3, p1)
5. $\mathbf{v}[a] = \mathbf{v}[c]$	MT 2.23($\equiv$)(p4, p2)
6. $\mathbf{v}[c] = \mathtt{F}$	MT 2.23($\neg$)(p5, p3)

4. Punto 4

Punto 4, (A, B, C)

A dice: "B y C son de la misma naturaleza"
$\Gamma_0 = \{(a \equiv (b \equiv c)), a, b\}$
$\Gamma_1 = \{(a \equiv (b \equiv c)), a, (\neg b)\}$
$\Gamma_2 = \{(a \equiv (b \equiv c)), (\neg a), b\}$
$\Gamma_0 = \{(a \equiv (b \equiv c)), (\neg a), (\neg b)\}$

C responderá "sí"

Con Γ_0

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1.	$\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[a] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[b] = \mathtt{T}$	Def.(p0)
4.	$\mathbf{v}[c] = \mathtt{T}$	MT 2.23($\equiv$)(p3, p2, p1)
5.	C respondería sí	

Con Γ_1

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_1)$	
1.	$\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[a] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[(\neg b)] = \mathtt{T}$	Def.(p0)
4.	$\mathbf{v}[c] = \mathtt{F}$	MT 2.23($\equiv$, $\neg$)(p3, p2, p1)
5.	C respondería sí	

Con Γ_2

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1.	$\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\neg a)] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[b] = \mathtt{T}$	Def.(p0)
4.	$\mathbf{v}[c] = \mathtt{F}$	MT 2.23($\equiv$, $\neg$)(p3, p2, p1)
5.	C respondería sí	

Con Γ_3

0.	$(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1.	$\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\neg a)] = \mathtt{T}$	Def.(p0)
3.	$\mathbf{v}[(\neg b)] = \mathtt{T}$	Def.(p0)
4.	$\mathbf{v}[c] = \mathtt{T}$	MT 2.23($\equiv$, $\neg$)(p3, p2, p1)
5.	C respondería sí	

5. Punto 5

El habitante A dice "Yo dije que si no soy caballero entonces soy escudero, y si soy caballero entonces no soy escudero"

6. Punto 6

A, B, $(a \equiv (\neg b)), (b \equiv (a \vee b))$

A dice: "B es escudero"
B dice: "al menos uno de nosotros es caballero"
$\Gamma_0 = \{(a \equiv (\neg b)), (b \equiv (a \vee b)), a\}$
$\Gamma_1 = \{(a \equiv (\neg b)), (b \equiv (a \vee b)), (\neg a)\}$

A es escudero y B es caballero

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$	
1. $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$	Def.(p0)
2. $\mathbf{v}[(b \equiv (a \vee b))] = \mathtt{T}$	Def.(p0)
3. $\mathbf{v}[a] = \mathtt{T}$	Def.(p0)
4. $\mathbf{v}[b] = \mathtt{F}$	MTT 2.23 $(\equiv, \neg)$(p3, p1)
5. $\mathbf{v}[(a \vee b)] = \mathtt{T}$	MT 2.23 $(\equiv)$(p4, p2)
6. $\mathbf{v}[a] = \mathtt{F}$ y $\mathbf{v}[b] = \mathtt{F}$	MT 2.23$(\vee)$(p5)
7. $\mathbf{v}[a] = \mathtt{F}$ y $\mathbf{v}[a] = \mathtt{T}$	Contradicción (p6, p3)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_1)$	
1. $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$	Def.(p0)
2. $\mathbf{v}[(b \equiv (a \vee b))] = \mathtt{T}$	Def.(p0)
3. $\mathbf{v}[(\neg a)] = \mathtt{T}$	Def.(p0)
4. $\mathbf{v}[a] = \mathtt{F}$	MT 2.23$(\neg)$(p3)
5. $\mathbf{v}[b] = \mathtt{T}$	MT 2.23 $(\equiv,\ \neg)$(p4, p1)

7. Punto 7

"Soy caballero" , debido a que $\models (\phi \equiv \phi)$

8. Punto 8

"Soy escudero" , debido a que $(\phi \equiv (\neg \phi))$ es una contradicción

9. Punto 9

Punto 23

C dijo: "A lo sumo uno de nosotros es caballero"
$\Gamma_0 = \{(c \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c) \vee ((\neg a) \wedge (\neg b) \wedge (\neg c))), c\}$
$\Gamma_1 = \{(c \equiv (a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c) \vee ((\neg a) \wedge (\neg b) \wedge (\neg c))), (\neg c)\}$

C es el único caballero, y por ende el único hombre lobo (tomando la suposición dada por el enunciado).

. . . Tomando lo que dijo C como ϕ. . .

Con Γ_0

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(c \equiv \phi)] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[c] = \mathtt{T}$ Def(p0)
3. $\mathbf{v}[((\neg a) \wedge (\neg b) \wedge c)] = \mathtt{T}$ MT 2.23($\equiv$, $\wedge$)(p2, p1)

Con Γ_1

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(c \equiv \phi)] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg c)] = \mathtt{T}$ Def(p0)
3. $\mathbf{v}[(a \wedge (\neg b) \wedge (\neg c))] = \mathtt{F}$ MT 2.23($\equiv$, $\wedge$) (p2, p1)
4. $\mathbf{v}[(\neg c)] = \mathtt{F}$ MTT 2.23 ($\wedge$)(p3)
5. $\mathbf{v}[(\neg c)] = \mathtt{F}$ y $\mathbf{v}[(\neg c)] = \mathtt{T}$ Contradicción (p4, p2)