Taller 03

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Matemáticas
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Índice

1. Punto 1: tabla de verdad y función

1.1. $p = \phi$

p
F
T

$$H_\phi(\mathtt{T}) = \mathtt{T}$$
$$H_\phi(\mathtt{F}) = \mathtt{F}$$

1.2. $(p \equiv r) = \phi$

p	r	$(p \equiv r)$
F	F	T
F	T	F
T	F	F
T	T	T

$$H_\phi(\mathtt{F}, \mathtt{T}) = H_\phi(\mathtt{T}, \mathtt{F}) = \mathtt{F}$$
$$H_\phi(\mathtt{F}, \mathtt{F}) = H_\phi(\mathtt{T}, \mathtt{T}) = \mathtt{F}$$

1.3. $((p \to (\neg q)) \to r) = \phi$

p	q	r	$(\neg q)$	$(p \to (\neg q))$	$((p \to (\neg q)) \to r)$
F	F	F	T	T	F
F	F	T	T	T	T
F	T	F	F	T	F
F	T	T	F	T	T
T	F	F	T	T	F
T	F	T	T	T	T
T	T	F	F	F	T
T	T	T	F	F	T

$$H_\phi(\mathtt{F}, \mathtt{F}, \mathtt{T}) = H_\phi(\mathtt{F}, \mathtt{T}, \mathtt{T}) = H_\phi(\mathtt{T}, \mathtt{F}, \mathtt{T}) = H_\phi(\mathtt{T}, \mathtt{T}, \mathtt{F}) = H_\phi(\mathtt{T}, \mathtt{T}, \mathtt{T})$$
$$H_\phi(\mathtt{F}, \mathtt{F}, \mathtt{F}) = H_\phi(\mathtt{F}, \mathtt{T}, \mathtt{F}) = H_\phi(\mathtt{T}, \mathtt{F}, \mathtt{F})$$

1.4. $((p \to q) \vee ((\neg p) \to (\neg q))) = \phi$

p	q	$(\neg p)$	$(\neg q)$	$(p \to q)$	$((\neg p) \to (\neg q))$	$((p \to q) \vee ((\neg p) \to (\neg q)))$
F	F	T	T	T	T	T
F	T	T	F	T	F	T
T	F	F	T	F	T	T
T	T	F	F	T	T	T

$$H_\phi(\mathtt{F}, \mathtt{F}) = H_\phi(\mathtt{F}, \mathtt{T}) = H_\phi(\mathtt{T}, \mathtt{F}) = H_\phi(\mathtt{T}, \mathtt{T}) = \mathtt{T}$$

1.5. $(p \to (q \to p)) = \phi$

p	q	$(q \to p)$	$(p \to (q \to p))$
F	F	T	T
F	T	F	T
T	F	T	T
T	T	T	T

$$H_\phi(\mathtt{F},\mathtt{F}) = H_\phi(\mathtt{F},\mathtt{T}) = H_\phi(\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{T}) = \mathtt{T}$$

1.6. $((p \vee r) \wedge (p \to q)) = \phi$

p	q	r	$(p \vee r)$	$(p \to q)$	$((p \vee r) \wedge (p \to q))$
F	F	F	F	T	F
F	F	T	T	T	T
F	T	F	F	T	F
F	T	T	T	T	T
T	F	F	T	F	F
T	F	T	T	F	F
T	T	F	T	T	T
T	T	T	T	T	T

$$H_\phi(\mathtt{F},\mathtt{F},\mathtt{F}) = H_\phi(\mathtt{F},\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{F},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{F},\mathtt{T}) = \mathtt{F}$$
$$H_\phi(\mathtt{F},\mathtt{F},\mathtt{T}) = H_\phi(\mathtt{F},\mathtt{T},\mathtt{T}) = H_\phi(\mathtt{T},\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{T},\mathtt{T}) = \mathtt{T}$$

1.7. $(\neg((r \to (r \wedge (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r)))))) = \phi$

Debido a la longitud de la proposición, decidí añadir subíndices a los conectores, para así no tener que hacer mención de las variables que hacen parte del mismo:

$$(\neg_0((r \to_2 (r \wedge_3 (p \vee_4 s))) \equiv_1 (\neg_2((p \to_4 q) \vee_3 (r \wedge_4 (\neg_5 r))))))$$

Posición	p	q	r	s
0	F	F	F	F
1	F	F	F	T
2	F	F	T	F
3	F	F	T	T
4	F	T	F	F
5	F	T	F	T
6	F	T	T	F
7	F	T	T	T
8	T	F	F	F
9	T	F	F	T
10	T	F	T	F
11	T	F	T	T
12	T	T	F	F
13	T	T	F	T
14	T	T	T	F
15	T	T	T	T

Posición	$\neg_5$	$\vee_4$	$\rightarrow_4$	$\wedge_4$	$\wedge_3$	$\vee_3$	$\rightarrow_2$	$\neg_2$	$\equiv_1$	$\neg_0$
0	T	F	T	F	F	T	T	F	F	T
1	F	T	T	F	F	T	T	F	F	T
2	T	F	T	F	F	T	F	F	T	F
3	F	T	T	F	T	T	T	F	F	T
4	T	F	T	F	F	T	T	F	F	T
5	F	T	T	F	F	T	T	F	F	T
6	T	F	T	F	F	T	F	F	T	F
7	F	T	T	F	T	T	T	F	F	T
8	T	T	F	F	F	F	T	T	T	F
9	F	T	T	F	F	F	T	T	T	F
10	T	T	T	F	T	F	T	T	T	F
11	F	T	F	F	T	F	T	T	T	F
12	T	T	T	F	F	T	T	F	F	T
13	F	T	T	F	F	T	T	F	F	T
14	T	T	T	F	T	T	T	F	F	T
15	F	T	T	F	T	T	T	F	F	T

$$\begin{aligned} H_\phi(\mathtt{F},\mathtt{F},\mathtt{F},\mathtt{F}) &= H_\phi(\mathtt{F},\mathtt{F},\mathtt{F},\mathtt{T}) = H_\phi(\mathtt{F},\mathtt{F},\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{F},\mathtt{T},\mathtt{F},\mathtt{F}) = H_\phi(\mathtt{F},\mathtt{T},\mathtt{F},\mathtt{T}) = H_\phi(\mathtt{F},\mathtt{T},\mathtt{T},\mathtt{T}) \\ &= H_\phi(\mathtt{T},\mathtt{T},\mathtt{F},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{T},\mathtt{F},\mathtt{T}) = H_\phi(\mathtt{T},\mathtt{T},\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{T},\mathtt{T},\mathtt{T}) = \mathtt{T} \end{aligned}$$

$$H_\phi(\mathtt{F},\mathtt{F},\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{F},\mathtt{F},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{T},\mathtt{F},\mathtt{T}) = H_\phi(\mathtt{T},\mathtt{F},\mathtt{T},\mathtt{F}) = H_\phi(\mathtt{T},\mathtt{F},\mathtt{T},\mathtt{T}) = \mathtt{F}$$

2. Punto 2: Tabla de verdad

$((p \vee (q \vee r)) \equiv ((p \vee q) \vee r))$

p	q	r	$(q \vee r)$	$(p \vee q)$	$(p \vee (q \vee r))$	$((p \vee q) \vee r)$	$((p \vee (q \vee r)) \equiv ((p \vee q) \vee r))$
F	F	F	F	F	F	F	T
F	F	T	T	F	T	T	T
F	T	F	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	T	T	T	T	T	T	T

3. Punto 3: Justificar que la implicación no es asociativa

$(\forall \phi, \psi, \tau \in \mathbb{B} : ((\phi \rightarrow \psi) \rightarrow \tau) \equiv (\phi \rightarrow (\psi \rightarrow \tau)))$ propiedad a refutar

$(\exists \phi, \psi, \tau \in \mathbb{B} : \mathbf{v}[((\phi \rightarrow \psi) \rightarrow \tau)] \neq \mathbf{v}[(\phi \rightarrow (\psi \rightarrow \tau))])$ negación de la propiedad

$\mathbf{v}[((\phi \rightarrow \psi) \rightarrow \tau)] = \mathtt{F}$ suposición

$\equiv$

$(\mathbf{v}[(\phi \rightarrow \psi)] = \mathtt{T}) \wedge (\mathbf{v}[\tau] = \mathtt{F})$ Metateorema 2.23

$\equiv$

$((\mathbf{v}[\phi] = \mathtt{F}) \vee (\mathbf{v}[\psi] = \mathtt{T})) \wedge (\mathbf{v}[\tau] = \mathtt{F})$

Tomando $\mathbf{v} = \{\phi \mapsto \mathtt{F}, \psi \mapsto \mathtt{T}, \tau \mapsto \mathtt{F}\}$

$$\begin{aligned}
\mathbf{v}[(\phi \rightarrow (\psi \rightarrow \tau))] &= H_{\rightarrow}[\mathtt{F}, \mathbf{v}[(\psi \rightarrow \tau)]] \\
&= H_{\rightarrow}[\mathtt{F}, H_{\rightarrow}(\mathtt{T}, \mathtt{F})] \\
&= H_{\rightarrow}[\mathtt{F}, \mathtt{F}] \\
&= \mathtt{T}
\end{aligned}$$

$\therefore (\exists \phi, \psi, \tau \in \mathbb{B} : \mathbf{v}[((\phi \rightarrow \psi) \rightarrow \tau)] \neq \mathbf{v}[(\phi \rightarrow (\psi \rightarrow \tau))])$ por suposición

4. Punto 4: Tabla de verdad

$(((p \rightarrow q) \wedge (\neg true)) \equiv (r \vee q))$

p	q	r	$true$	$(\neg true)$	$(p \rightarrow q)$	$(r \vee q)$	$((p \rightarrow q) \wedge (\neg true))$	$(((p \rightarrow q) \wedge (\neg true)) \equiv (r \vee q))$
F	F	F	T	F	T	F	F	T
F	F	T	T	F	T	T	F	F
F	T	F	T	F	T	T	F	F
F	T	T	T	F	T	T	F	F
T	F	F	T	F	F	F	F	T
T	F	T	T	F	F	T	F	F
T	T	F	T	F	T	T	F	F
T	T	T	T	F	T	T	F	F

5. Punto 5: Nuevo conector

5.1. Definir $*$ en conectores lógicos clásicos

$$\frac{H_*(\phi, \psi) = H_{\neg}[H_{\vee}(\phi, \psi)]}{(\phi * \psi) \equiv (\neg(\phi \vee \psi))}$$

5.2. Hallar una proposición equivalente a $(\neg p)$ usando únicamente $\{p, *\}$

$(\phi * \psi) \equiv (\neg(\phi \vee \psi))$	sub-punto 1
$(p \vee p) \equiv p$	tabla de verdad
$(p * p) \equiv (\neg p)$	

5.3. Hallar una proposición equivalente a $(p \wedge q)$ usando únicamente $\{p, q, *\}$

$(\phi * \psi) \equiv (\neg(\phi \vee \psi))$	sub-punto 1
$(p * p) \equiv (\neg p)$	sub-punto 2
$(\neg(\phi \vee \psi)) \equiv ((\neg\phi) \wedge (\neg\psi))$	tabla de verdad
$((p * p) * (q * q)) \equiv (p \wedge q)$	

5.4. Justificar o refutar propiedades de $*$

Metateorema 2.23 para $*$

$\mathbf{v}[(\phi * \psi)] = \mathtt{T}$ si y solo si $\mathbf{v}[\phi] = \mathtt{F}$ y $\mathbf{v}[\psi] = \mathtt{F}$; de lo contrario $\mathbf{v}[(\phi * \psi)] = \mathtt{F}$

5.4.1. Conmutativa

$(\forall \phi, \psi \in \mathbb{B} : \mathbf{v}[(\phi * \psi)] = \mathbf{v}[(\psi * \phi)])$	def. propiedad conmutativa
$\mathbf{v}[(\phi * \psi)] = \mathtt{T}$	suposición
$(\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\psi] = \mathtt{F})$	
$(\mathbf{v}[\psi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	Propiedad conmutativa de $\wedge$
$\mathbf{v}[(\psi * \phi)] = \mathtt{T}$	Metateorema 2.23
$(\forall \phi, \psi \in \mathbb{B} : \mathbf{v}[(\phi * \psi)] = \mathbf{v}[(\psi * \phi)])$	Es conmutativa

Se puede ver que cualquier otra valuación cumple la propiedad (negar el metatorema y aplicar propiedad conmutativa sobre $\vee$)

5.4.2. Asociativa

$(\forall \phi, \psi, \tau \in \mathbb{B} : \mathbf{v}[(\phi * \psi) * \tau] = \mathbf{v}[(\phi * (\psi * \tau))])$	def. propiedad asociativa
$\mathbf{v}[(\phi * \psi) * \tau] = \mathtt{T}$	suposición
$(\mathbf{v}[(\phi * \psi)] = \mathtt{F}) \wedge (\mathbf{v}[\tau] = \mathtt{F})$	
$(\mathbf{v}[\phi] = \mathtt{T} \vee \mathbf{v}[\psi] = \mathtt{T}) \wedge (\mathbf{v}[\tau] = \mathtt{F})$	

Tomando $\mathbf{v} = \{\phi \mapsto \texttt{T}, \psi \mapsto \texttt{T}, \tau \mapsto \texttt{F}\}$

$$\begin{aligned}
\mathbf{v}[(\phi * (\psi * \tau))] &= H_*[\phi, (\psi * \tau)] \\
&= H_*[\texttt{T}, H_*(\texttt{T}, \texttt{F})] \\
&= H_*[\texttt{T}, \texttt{F}] \\
&= \texttt{F}
\end{aligned}$$

$$\therefore (\exists \phi, \psi, \tau \in \mathbb{B} : \mathbf{v}[(\phi * \psi) * \tau] \neq \mathbf{v}[(\phi * (\psi * \tau))]) \qquad \text{No es asociativa}$$

6. Punto 6: Nuevo conector

6.1. Definir $*$ en conectores lógicos clásicos

$$H_*(\phi, \psi) = H_\neg[H_\leftarrow(\phi, \psi)]$$

$$(\phi * \psi) \equiv (\neg(\phi \leftarrow \psi))$$

6.2. Justificar o refutar propiedades de $*$

Metateorema 2.23 para $*$

$\mathbf{v}[(\phi * \psi) = \texttt{T}$ si y solo si $\mathbf{v}[\phi] = \texttt{F}$ y $\mathbf{v}[\psi] = \texttt{T}$; de lo contrario $\mathbf{v}[(\phi * \psi)] = \texttt{F}]$

6.2.1. Asociativa

$$\begin{array}{ll}
(\forall \phi, \psi, \tau \in \mathbb{B} : \mathbf{v}[(\phi * \psi) * \tau] = \mathbf{v}[(\phi * (\psi * \tau))]) & \text{def. propiedad asociativa} \\
\mathbf{v}[(\phi * \psi) * \tau] = \texttt{T} & \text{suposición} \\
\quad (\mathbf{v}[\phi] = \texttt{F}) \wedge (\mathbf{v}[\psi] = \texttt{T}) & \\
\quad (\mathbf{v}[\psi] = \texttt{T}) \wedge (\mathbf{v}[\phi] = \texttt{F}) & \text{Propiedad conmutativa de } \wedge \\
\quad \mathbf{v}[(\psi * \phi)] = \texttt{F} & \\
\hline
\therefore (\exists \phi, \psi, \tau \in \mathbb{B} : \mathbf{v}[(\phi * \psi) * \tau] \neq \mathbf{v}[(\phi * (\psi * \tau))]) & \text{No es asociativa}
\end{array}$$

6.2.2. Transitiva

$$\boxed{\begin{array}{l}(\forall\phi,\psi,\tau\in\mathbb{B}:((\mathbf{v}[(\phi*\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi*\tau)]=\mathtt{T}))\Rightarrow(\mathbf{v}[(\phi*\tau)]=\mathtt{T}))\\ \qquad\qquad\qquad\wedge\\ (\forall\phi,\psi,\tau\in\mathbb{B}:((\mathbf{v}[(\phi*\psi)]=\mathtt{F})\wedge(\mathbf{v}[(\psi*\tau)]=\mathtt{F}))\Rightarrow(\mathbf{v}[(\phi*\tau)]=\mathtt{F}))\end{array}} \quad \text{def. propiedad transitiva}$$

$$\begin{array}{cr}(\mathbf{v}[(\phi*\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi*\tau)]=\mathtt{T}) & \text{suposición}\\ ((\mathbf{v}[\phi]=\mathtt{F})\wedge(\mathbf{v}[\psi]=\mathtt{T}))\wedge((\mathbf{v}[\psi]=\mathtt{F})\wedge(\mathbf{v}[\tau]=\mathtt{T})) & \\ (\mathbf{v}[\phi]=\mathtt{F})\wedge \mathit{false}\wedge(\mathbf{v}[\tau]=\mathtt{T}) & \text{Propiedad asociativa de }\wedge\\ \mathit{false} & \end{array}$$

$$\boxed{\begin{array}{l}(\exists\phi,\psi,\tau\in\mathbb{B}:((\mathbf{v}[(\phi*\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi*\tau)]=\mathtt{T}))\not\Rightarrow(\mathbf{v}[(\phi*\tau)]=\mathtt{T}))\\ \qquad\qquad\qquad\vee\\ (\exists\phi,\psi,\tau\in\mathbb{B}:((\mathbf{v}[(\phi*\psi)]=\mathtt{F})\wedge(\mathbf{v}[(\psi*\tau)]=\mathtt{F}))\not\Rightarrow(\mathbf{v}[(\phi*\tau)]=\mathtt{F}))\end{array}} \quad \text{No es transitiva}$$

7. Punto 7: considere las valuaciones v y w ...

$$\begin{aligned}\mathbf{v}&=\{p\mapsto\mathtt{T},\ q\mapsto\mathtt{F},\ r\mapsto\mathtt{F},\ \ldots\}\\ \mathbf{w}&=\{p\mapsto\mathtt{T},\ q\mapsto\mathtt{F},\ r\mapsto\mathtt{T},\ \ldots\}\end{aligned}$$

Demostrar que $\mathbf{v}[(p\equiv(\neg q))]=\mathbf{w}[(p\equiv(\neg q))]$

Tomando **v** ...

$$\begin{aligned}\mathbf{v}[(p\equiv(\neg q))]&=H_{\equiv}[p,(\neg q)]\\ &=H_{\equiv}[\mathbf{v}[p],H_{\neg}(\mathbf{v}[q])]\\ &=H_{\equiv}[\mathbf{w}[p],H_{\neg}(\mathbf{w}[q])]\\ &=\mathbf{w}[(p\equiv(\neg q))]\end{aligned}$$

8. Punto 8: Demuestre que $\mathbf{v}[\phi]\neq\mathbf{v}[(\neg\phi)]$ para cualquier valuación v

$$\begin{array}{rr}\mathbf{v}[(\phi\equiv(\neg\phi))]=\mathtt{F} & \text{Enunciado/ proposición a probar}\\ \underline{\qquad\qquad\mathbf{v}[\phi]\neq\mathbf{v}[(\neg\phi)]} & \text{negación de Metateorema 2.23}\\ \therefore\ \mathbf{v}[(\phi\equiv(\neg\phi))]=\mathtt{F} & \end{array}$$

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