Tarea 04

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Índice

1. Justificaciones extra:

Metateorema SS

1. $\mathbf{v}[\phi] = \mathtt{T}$ Es una proposicón X_1
1.1. $\mathbf{v}[\phi] = \mathtt{F}$ es igual a la proposición $(\neg X_1)$
2. $\mathbf{v}[\phi] = \mathtt{F}$ Es una proposicón X_2
2.1. $\mathbf{v}[\phi] = \mathtt{T}$ es igual a la proposición $(\neg X_2)$
3. Los conectores usados en las definiciones y metateoremas sobre valuaciones pueden ser reemplazados por conectores lógicos.

2. Sección 2.3

2.1. Punto 1

2.1.1. a) $\models ((\phi \equiv \psi) \equiv (\psi \equiv \phi))$

0.	$\mathbf{v}[((\phi \equiv \psi) \equiv (\psi \equiv \phi))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \equiv \psi) \equiv (\psi \equiv \phi))] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \equiv \psi)] \neq \mathbf{v}[(\psi \equiv \phi)]$	
3.	$(\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \equiv \phi)] = \mathtt{F})$	suposición 1, MT 2.20
4.	$(\mathbf{v}[\phi] = \mathbf{v}[\psi]) \wedge (\mathbf{v}[\psi] \neq \mathbf{v}[\phi])$	MT 2.23 ($\equiv$)
5.	Contradicción	
6.	$(\mathbf{v}[(\phi \equiv \psi)] = \mathtt{F}) \wedge (\mathbf{v}[(\psi \equiv \phi)] = \mathtt{T})$	suposición 2
7.	$(\mathbf{v}[\phi] \neq \mathbf{v}[\psi]) \wedge (\mathbf{v}[\psi] = \mathbf{v}[\phi])$	MT 2.23 ($\equiv$)
8.	Contradicción	
9.	$\therefore \quad \models ((\phi \equiv \psi) \equiv (\psi \equiv \phi))$	

2.1.2. b) $\models ((\phi \equiv \boldsymbol{true}) \equiv \phi)$

0.	$\mathbf{v}[((\phi \equiv \mathit{true}) \equiv \phi)] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \equiv \mathit{true}) \equiv \phi)] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \equiv \mathit{true})] \neq \mathbf{v}[\phi]$	
3.	$(\mathbf{v}[(\phi \equiv \mathit{true})] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1, MT 2.20
4.	$(\mathbf{v}[\phi] = \mathbf{v}[\mathit{true}]) \wedge (\mathbf{v}[\phi]) = \mathtt{F}$	MT 2.23
5.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	
6.	Contradicción	
7.	$(\mathbf{v}[(\phi \equiv \mathit{true})] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	suposición 2, MT 2.20
8.	$(\mathbf{v}[\phi] \neq \mathbf{v}[\mathit{true}]) \wedge (\mathbf{v}[\phi]) = \mathtt{T}$	MT 2.23
9.	$(\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	
10.	Contradicción	
11.	$\therefore \quad \models ((\phi \equiv \mathit{true}) \equiv \phi)$	

2.1.3. f) $\models ((\phi \vee \boldsymbol{false}) \equiv \phi)$

0.	$\mathbf{v}[((\phi \vee \mathit{false}) \equiv \phi)] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \vee \mathit{false}) \equiv \phi)] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \vee \mathit{false})] \neq \mathbf{v}[\phi]$	
3.	$(\mathbf{v}[(\phi \vee \mathit{false})] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1, MT 2.20
4.	$((\mathbf{v}[\phi] = \mathtt{T}) \vee (\mathbf{v}[\mathit{false}] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	MT 2.23
5.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	
6.	Contradicción	
7.	$(\mathbf{v}[(\phi \vee \mathit{false})] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	suposición 2, MT 2.20
8.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\mathit{false}] = \mathtt{F})) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	MT 2.23
9.	$(\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	simplificación (8)
10.	Contradicción	
11.	$\therefore \quad \models ((\phi \vee \mathit{false}) \equiv \phi)$	

2.1.4. **g)** $\models ((\phi \vee \phi) \equiv \phi)$

0.	$\mathbf{v}[((\phi \vee \phi) \equiv \phi)] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[((\phi \vee \phi) \equiv \phi)] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \vee \phi)] \neq \mathbf{v}[\phi]$	MT 2.23 ($\equiv$)
3.	$(\mathbf{v}[(\phi \vee \phi)] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1, MT 2.20
4.	$((\mathbf{v}[\phi] = \mathtt{T}) \vee (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	MT 2.20
5.	$((\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1.1
6.	Contradicción	
7.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1.2
8.	Contradicción	
9.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	suposición 1.3
10.	Contradicción	
11.	$(\mathbf{v}[(\phi \vee \phi)] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	suposición 2, MT 2.20
12.	$((\mathbf{v}[\phi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{F})) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	MT 2.23
13.	Contradicción	
14.	$\therefore \quad \models ((\phi \vee \phi) \equiv \phi)$	

2.1.5. **k)** $\models (\neg(\phi \wedge (\neg\phi)))$

0.	$\mathbf{v}[(\neg(\phi \wedge (\neg\phi)))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[(\neg(\phi \wedge (\neg\phi)))] = \mathtt{F}$	suposición (intento por contradicción)
2.	$\mathbf{v}[(\phi \wedge (\neg\phi))] = \mathtt{T}$	MT 2.23 ($\neg$)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[(\neg\phi)] = \mathtt{T})$	MT 2.23 ($\wedge$)
4.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\phi] = \mathtt{F})$	MT 2.23 ($\neg$)
5.	Contradicción	
6.	$\therefore \quad \models (\neg(\phi \wedge (\neg\phi)))$	

2.1.6. **l)** $\models (\phi \to (\psi \to \phi))$

0.	$\mathbf{v}[(\phi \to (\psi \to \phi))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[(\phi \to (\psi \to \phi))] = \mathtt{F}$	suposición (intento por contradicción)
2.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \to \phi)] = \mathtt{F})$	MT 2.23 ($\to$)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge ((\mathbf{v}[\psi] = \mathtt{T}) \wedge ((\mathbf{v}[\phi] = \mathtt{F})))$	MT 2.23 ($\to$)
4.	Contradicción	
5.	$\therefore \quad (\phi \to (\psi \to \phi))$	

2.1.7. n) $\models ((\phi \to \psi) \equiv ((\neg\psi) \to (\neg\phi)))$

0.	$\mathbf{v}[((\phi \to \psi) \equiv ((\neg\psi) \to (\neg\phi)))] = \mathtt{T}$	Enunciado
1.	$\mathbf{v}[(\phi \to \psi)] = \mathbf{v}[((\neg\psi) \to (\neg\phi))]$	MT 2.23 ($\equiv$)
2.	Partiendo de $(\phi \to \psi)$	
3.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{F}$	suposición 1
4.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\psi] = \mathtt{F})$	MT 2.23 ($\to$)
5.	$(\mathbf{v}[\psi] = \mathtt{F}) \wedge (\mathbf{v}[\phi] = \mathtt{T})$	Conmutativa de $\wedge$
6.	$(H_\neg(\mathbf{v}[\psi]) = \mathtt{T}) \wedge (H_\neg(\mathbf{v}[\phi]) = \mathtt{F})$	Doble negación (5)
7.	$(\mathbf{v}[(\neg\psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\neg\phi)]) = \mathtt{F}$	Def. 2.18 ($\neg$)
8.	$\mathbf{v}[((\neg\psi) \to (\neg\phi))] = \mathtt{F}$	MT 2.23 ($\to$)
9.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$	suposición 2
10.	$(\mathbf{v}[\phi] = \mathtt{F}) \vee (\mathbf{v}[\psi] = \mathtt{T})$	MTT 2.23 ($\to$)
11.	$(\mathbf{v}[\psi] = \mathtt{T}) \vee (\mathbf{v}[\phi] = \mathtt{F})$	Conmutativa de $\vee$
12.	$(H_\neg(\mathbf{v}[\psi]) = \mathtt{F}) \vee (H_\neg(\mathbf{v}[\phi]) = \mathtt{T})$	Doble negación (10)
13.	$(\mathbf{v}[(\neg\psi)] = \mathtt{F}) \vee (\mathbf{v}[(\neg\phi)]) = \mathtt{T}$	Def. 2.18 ($\neg$)
14.	$\mathbf{v}[((\neg\psi) \to (\neg\phi))] = \mathtt{T}$	MT 2.23 ($\to$)
15.	$\therefore \quad \models ((\phi \to \psi) \equiv ((\neg\psi) \to (\neg\phi)))$	

2.2. Punto 2

2.2.1. b) $((\neg p) \vee q)$

0.	$(\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[((\neg p) \vee q)] = \mathtt{T}) \wedge (\mathbf{w}[((\neg p) \vee q)] = \mathtt{F}))$	Enunciado
1.	$\mathbf{v}[((\neg p) \vee q)] = \mathtt{T}$	suposición 1
2.	$(\mathbf{v}[(\neg p)] = \mathtt{T}) \vee (\mathbf{v}[q] = \mathtt{T})$	MT 2.23 ($\vee$)
3.	$(\mathbf{v}[p] = \mathtt{F}) \vee (\mathbf{v}[q] = \mathtt{T})$	Doble negación (2)
4.	$\mathbf{v} = \{p \mapsto \mathtt{F}, q \mapsto \mathtt{T}\}$	
5.	$(\exists \mathbf{v} \mid \mathbf{v}[((\neg p) \vee q)] = \mathtt{T})$	
6.	$\mathbf{w}[((\neg p) \vee q)] = \mathtt{F}$	suposición 2
7.	$(\mathbf{w}[(\neg p)] = \mathtt{F}) \wedge (\mathbf{w}[q] = \mathtt{F})$	MT 2.23 ($\vee$)
8.	$(\mathbf{w}[p] = \mathtt{T}) \wedge (\mathbf{w}[q] = \mathtt{F})$	Doble negación (2)
9.	$\mathbf{w} = \{p \mapsto \mathtt{T}, q \mapsto \mathtt{F}\}$	
10.	$(\exists \mathbf{w} \mid \mathbf{w}[((\neg p) \vee q)] = \mathtt{f})$	
11.	$\therefore \quad (\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[((\neg p) \vee q)] = \mathtt{T}) \wedge (\mathbf{w}[((\neg p) \vee q)] = \mathtt{F}))$	

2.2.2. d) $(\neg(p \wedge (\neg q)))$

0.	$(\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{T}) \wedge (\mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{F}))$	Enunciado
1.	$\mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{F}$	suposición 1
2.	$\mathbf{v}[(p \wedge (\neg q))] = \mathtt{T}$	Def. 2.18 ($\neg$)
3.	$(\mathbf{v}[p] = \mathtt{T}) \wedge (\mathbf{v}[(\neg q)] = \mathtt{T})$	MT 2.23 ($\wedge$)
4.	$(\mathbf{v}[p] = \mathtt{T}) \wedge (\mathbf{v}[q] = \mathtt{F})$	Def. 2.18 ($\neg$)
5.	$\mathbf{v} = \{p \mapsto \mathtt{T}, q \mapsto \mathtt{F}\}$	
6.	$(\exists \mathbf{v} \mid \mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{T})$	
7.	$\mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{T}$	suposición 2
8.	$\mathbf{w}[(p \wedge (\neg q))] = \mathtt{F}$	Def. 2.18 ($\neg$)
9.	$(\mathbf{w}[p] = \mathtt{F}) \vee (\mathbf{w}[(\neg q)] = \mathtt{F})$	
10.	$(\mathbf{w}[p] = \mathtt{F}) \vee (\mathbf{w}[q] = \mathtt{T})$	Def. 2.18 ($\neg$)
11.	$\mathbf{w} = \{p \mapsto \mathtt{F}, q \mapsto \mathtt{T}\}$	
12.	$(\exists \mathbf{w} \mid \mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{T})$	
13.	$\therefore \quad (\exists \mathbf{v}, \mathbf{w} \mid (\mathbf{v}[(\neg(p \wedge (\neg q)))] = \mathtt{T}) \wedge (\mathbf{w}[(\neg(p \wedge (\neg q)))] = \mathtt{F}))$	

2.3. Punto 3

$$\bullet(\phi \wedge (\neg\phi))$$
$$\bullet((\phi \to \psi) \equiv (\phi \wedge (\neg\psi)))$$
$$\bullet((\phi \leftarrow \psi) \equiv ((\neg\phi) \wedge \psi))$$

2.4. Punto 6

2.4.1. b)

Se tiene de entrada que *false*, al ser una constante, no hace de operador entre 2 proposiciones, por lo que la única opción es:

0.	$\models (\mathit{false})$	suposición/ enunciado
1.	$\mathbf{v}[\mathit{false}] = \mathtt{T}$	Def.(p0)
2.	$\mathtt{F} = \mathtt{T}$	Def. 2.18 (*false*) (p1)
3.	Contradicción	
4.	$\therefore$ No existe dicha proposición mencionada en el enunciado	

2.4.2. d)

0.	$\vDash (\phi \not\equiv \psi)$	suposición/ enunciado
1.	$\mathbf{v}[(\phi \not\equiv \psi)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\phi \not\equiv \psi)] = \mathtt{F}$	suposición (intento por contradicción)
3.	$\mathbf{v}[\phi] = \mathbf{v}[\psi]$	MT 2.23 ($\not\equiv$) (p2)
4.	$\mathbf{v}[\phi] = \mathtt{T}$	suposición 1 , MT 2.19 N 2.20
5.	$\mathbf{v}[\psi] = \mathtt{T}$	p(3, 4)
6.	$\therefore \quad (\exists \mathbf{v} \mid \mathbf{v}[(\phi \not\equiv \psi)] = \mathtt{F})$	

2.4.3. f)

0.	$\vDash (\phi \wedge \psi)$	suposición/ enunciado
1.	$\mathbf{v}[(\phi \wedge \psi)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\phi \wedge \psi)] = \mathtt{F}$	suposición (intento por contradicción)
3.	$(\mathbf{v}[\phi] = \mathtt{F}) \vee (\mathbf{v}[\psi] = \mathtt{F})$	MT 2.23 (p2)
4.	$\mathbf{v}[\phi] = \mathtt{F}$	suposición 1, MT 2.19 N 2.20
5.	$\mathbf{v}[(\phi \wedge \psi)] = \mathtt{F}$	MT 2.23, p4
6.	$\therefore \quad (\exists \mathbf{v} \mid \mathbf{v}[(\phi \wedge \psi)] = \mathtt{F})$	

2.4.4. g)

0.	$\vDash (\phi \rightarrow \psi)$	suposición/ enunciado
1.	$\mathbf{v}[(\phi \rightarrow \psi)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\phi \rightarrow \psi)] = \mathtt{F}$	suposición (intento por contradicción)
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\psi] = \mathtt{F})$	
4.	$\therefore \quad (\exists \mathbf{v} \mid \mathbf{v}[(\phi \rightarrow \psi)] = \mathtt{F})$	

2.5. Punto 8

2.5.1. a)

0.	Si $\vDash(\phi\to\psi)$ y $\vDash(\psi\to\tau)$, entonces $\vDash(\phi\to\tau)$	Enunciado
1.	$((\mathbf{v}[(\phi\to\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi\to\tau)]=\mathtt{T}))\implies(\mathbf{v}[(\phi\to\tau)]=\mathtt{T})$	Def. (p0)
2.	$(\exists\mathbf{v}\mid\mathbf{v}[(\phi\to\tau)]=\mathtt{F})$	suposición , MT 2.19 N 2.20 ($\to$) (p1)
3.	$(\mathbf{v}[\phi]=\mathtt{T})\wedge(\mathbf{v}[\tau]=\mathtt{F})$	MT 2.23 ($\to$) (p2)
4.	se tiene que $((\mathbf{v}[(\phi\to\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi\to\tau)]=\mathtt{T}))$	
5.	$\mathbf{v}[\phi]=\mathtt{T}$	Caso ($true\to\xi$) (p4 , p3)
6.	$\mathbf{v}[\tau]=\mathtt{T}$	Caso ($true\to\xi$) (p4 , p5)
7.	Contradicción	(p6, p3)
8.	$\therefore$ Si $\vDash(\phi\to\psi)$ y $\vDash(\psi\to\tau)$, entonces $\vDash(\phi\to\tau)$	Enunciado

2.5.2. b)

0.	Si $\vDash(\phi\to\psi)$ y $\vDash(\phi)$, entonces $\vDash(\psi)$	Enunciado
1.	$((\mathbf{v}[(\phi\to\psi)]=\mathtt{T})\wedge(\mathbf{v}[\phi]=\mathtt{T}))\implies(\mathbf{v}[\psi]=\mathtt{T})$	Def.(p0)
2.	$(\exists\mathbf{v}\mid\mathbf{v}[\psi]=\mathtt{F})$	suposición, MT 2.19 N 2.20
3.	Se tiene que $((\mathbf{v}[(\phi\to\psi)]=\mathtt{T})\wedge(\mathbf{v}[\phi]=\mathtt{T}))$	
4.	$((\mathbf{v}[\phi]=\mathtt{F})\vee(\mathbf{v}[\psi]=\mathtt{T}))\wedge(\mathbf{v}[\phi]=\mathtt{T})$	MT 2.23 ($\to$) (p3)
5.	Nótese que por un lado (mediante $\vee$) $(\mathbf{v}[\phi]=\mathtt{F})$ y por otro $(\mathbf{v}[\phi]=\mathtt{T})$	
6.	$\mathbf{v}[\psi]=\mathtt{T}$	Caso ($false\vee\xi$)
7.	Contradicción	(p6, p2)
8.	$\therefore$ Si $\vDash(\phi\to\psi)$ y $\vDash(\phi)$, entonces $\vDash(\psi)$	Enunciado

2.5.3. c)

0.	Si $\vDash(\phi\to\psi)$ y $\vDash(\psi\equiv\tau)$, entonces $\vDash(\phi\to\tau)$	Enunciado
1.	$((\mathbf{v}[(\phi\to\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi\equiv\tau)]=\mathtt{T}))\implies(\mathbf{v}[(\phi\to\tau)]=\mathtt{T})$	Def. (p0)
2.	$(\exists\mathbf{v}\mid\mathbf{v}[(\phi\to\tau)]=\mathtt{T})$	suposición, MT 2.19 N 2.20
3.	$(\mathbf{v}[\phi]=\mathtt{T})\wedge(\mathbf{v}[\tau]=\mathtt{F})$	MT 2.23 ($\to$) (p2)
4.	Se tiene que $((\mathbf{v}[(\phi\to\psi)]=\mathtt{T})\wedge(\mathbf{v}[(\psi\equiv\tau)]=\mathtt{T}))$	
5.	$\mathbf{v}[\psi]=\mathtt{F}$	MT 2.23 ($\equiv$) (p4, p3)
6.	$\mathbf{v}[(\phi\to\psi)]=\mathtt{F}$	MT 2.23 ($\to$) (p5, p3)
7.	Contradicción	(p6, p3)
8.	$\therefore$ Si $\vDash(\phi\to\psi)$ y $\vDash(\psi\equiv\tau)$, entonces $\vDash(\phi\to\tau)$	

2.5.4. d)

0.	Si $\models (\phi \equiv \psi)$ y $\models (\psi \to \tau)$, entonces $\models (\phi \to \tau)$	Enunciado
1.	$((\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \to \tau)] = \mathtt{T})) \implies (\mathbf{v}[(\phi \to \tau)] = \mathtt{T})$	Def. (p0)
2.	$(\exists \mathbf{v} \mid \mathbf{v}[(\phi \to \tau)] = \mathtt{F})$	suposición, MT 2.19 N 2.20
3.	$(\mathbf{v}[\phi] = \mathtt{T}) \wedge (\mathbf{v}[\tau] = \mathtt{F})$	MT 2.23 ($\to$) (p2)
4.	Se tiene que $((\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}) \wedge (\mathbf{v}[(\psi \to \tau)] = \mathtt{T}))$	
5.	$\mathbf{v}[\psi] = \mathtt{T}$	MT 2.23 ($\equiv$), (p4, p3)
6.	$\mathbf{v}[(\psi \to \tau)] = \mathtt{F}$	MT 2.23 ($\to$), (p5, p3)
7.	Contradicción	(p6, p3)
8.	$\therefore$ Si $\models (\phi \equiv \psi)$ y $\models (\psi \to \tau)$, entonces $\models (\phi \to \tau)$	

2.6. Punto 9

2.6.1. a)

0.	ϕ Es insatisfacible si y solo si $\models (\phi \equiv \mathit{false})$	Enunciado
1.	Si ϕ Es insatisfacible, entonces $\models (\phi \equiv \mathit{false})$	demostración 1
2.	$\mathbf{v}[\phi] = \mathtt{F}$	MT 2.19 N 2.20
3.	$\mathbf{v}[\mathit{false}] = \mathtt{F}$	Def 2.18 (*false*)
4.	$\mathbf{v}[\phi] = \mathbf{v}[\mathit{false}]$	(p3, p2)
5.	$\mathbf{v}[(\phi \equiv \mathit{false})] = \mathtt{T}$	MT 2.23 ($\equiv$) (p4)
6.	Si $\models (\phi \equiv \mathit{false})$ entonces ϕ Es insatisfacible	demostración 2
7.	$\mathbf{v}[(\phi \equiv \mathit{false})] = \mathtt{T}$	MT 2.19 N 2.20
8.	$\mathbf{v}[\phi] = \mathbf{v}[\mathit{false}]$	MT 2.23 ($\equiv$) (p7)
9.	$\mathbf{v}[\phi] = \mathtt{F}$	Def. 2.18 (*false*)
10.	$\therefore$ ϕ Es insatisfacible si y solo si $\models (\phi \equiv \mathit{false})$	

2.6.2. b)

0.	ϕ Es insatisfacible si y solo si $\models (\neg\phi)$	Enunciado
1.	Si ϕ es insatisfacible, entonces $\models (\neg\phi)$	demostración 1
2.	$\mathbf{v}[\phi] = \texttt{F}$	MT 2.19 N 2.20
3.	$H_\neg()\mathbf{v}[\phi] = \texttt{T}$	doble negación (p2)
4.	$\mathbf{v}[(\neg\phi)] = \texttt{T}$	
5.	Si $\models (\neg\phi)$, entonces ϕ es insatisfacible	demostración 2
6.	$\mathbf{v}[(\neg\phi)] = \texttt{T}$	MT 2.19 N 2.20
7.	$\mathbf{v}[\phi] = \texttt{F}$	MT 2.23 ($\neg$) (p6)
8.	$\therefore$ ϕ Es insatisfacible si y solo si $\models (\neg\phi)$	

2.7. Punto 10

2.7.1. a)

0.	$\models (\phi \equiv \psi)$ si y solo si $\models (\phi)$ y $\models (\psi)$	Enunciado
1.	Si $\models (\phi \equiv \psi)$ entonces $\models (\phi)$ y $\models (\psi)$	demostración 1
2.	$\mathbf{v}[(\phi \equiv \psi)] = \texttt{T}$	MT 2.19 N 2.20
3.	$\mathbf{v}[\phi] = \mathbf{v}[\psi]$	MT 2.23 ($\equiv$) (p2)
4.	Tomando $\mathbf{v} \mid \mathbf{v}[\phi] = \texttt{F}$	MT 2.19 N 2.20
5.	$\mathbf{v}[\psi] = \texttt{F}$	MT 2.23 ($\equiv$) (p4, p3)
6.	$\therefore$ No se cumple el enunciado	

2.7.2. b)

0.	$\models (\phi \wedge \psi)$ si y solo si $\models (\phi)$ y $\models (\psi)$	Enunciado
1.	Si $\models (\phi \wedge \psi)$, entonces $\models (\phi)$ y $\models (\psi)$	demostración 1
2.	$\mathbf{v}[(\phi \wedge \psi)] = \texttt{T}$	MT 2.19 N 2.20
3.	$(\mathbf{v}[\phi] = \texttt{T}) \wedge (\mathbf{v}[\phi] = \texttt{T})$	MT 2.23 ($\wedge$) (p2)
4.	$\models (\phi)$ y $\models (\psi)$	Def. (p4)
5.	Si $\models (\phi)$ y $\models (\psi)$, entonces $\models (\phi \wedge \psi)$	demostración 2
6.	$(\mathbf{v}[\phi] = \texttt{T}) \wedge (\mathbf{v}[\psi] = \texttt{T})$	Def. (p5)
7.	$(\mathbf{v}[(\phi \wedge \psi)] = \texttt{T})$	MT 2.23 ($\wedge$) (p6)
8.	$\therefore$ $\models (\phi \wedge \psi)$ si y solo si $\models (\phi)$ y $\models (\psi)$	Enunciado