Tarea 04

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Matemáticas
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Colombia
4 de septiembre de 2022

Índice

1. Punto 1

1.1. a)

a)

0.	$((\phi \vee (\psi \equiv \tau)) \equiv ((\phi \vee \psi) \equiv (\phi \vee \psi)))$	Enunciado
1.	$\mathbf{v}[(\phi \vee (\psi \equiv \tau))] = \mathbf{v}[((\phi \vee \psi) \equiv (\phi \vee \psi))]$	MT 2.23 ($\equiv$) (p0)
2.	$\mathbf{v}[(\phi \vee (\psi \equiv \tau))] = \mathtt{T}$	Suposición 1
3.	$\mathbf{v}[\phi] = \mathtt{T}$ o $\mathbf{v}[(\psi \equiv \tau)] = \mathtt{T}$	
4.	$\mathbf{v}[\phi] = \mathtt{T}$	Suposición 1.1
5.	$\mathbf{v}[((\phi \vee \psi) \equiv (\phi \vee \psi))] = \mathtt{T}$	MT 2.23 ($\vee$, $\equiv$) (p2, p4)
6.	$\mathbf{v}[\phi] = \mathtt{F}$ y $\mathbf{v}[(\psi \equiv \tau)] = \mathtt{T}$	Suposición 1.2
7.	$\mathbf{v}[\psi] = \mathtt{T}$	Suposición 1.2.1
8.	$\mathbf{v}[\tau] = \mathtt{T}$	MT 2.23 ($\equiv$) (p7)
9.	$\mathbf{v}[((\phi \vee \psi) \equiv (\phi \vee \psi))] = \mathtt{T}$	MT 2.23 ($\equiv$, $\vee$) (p8, p7)
10.	$\mathbf{v}[\psi] = \mathtt{F}$	Suposición 1.2.2
11.	$\mathbf{v}[\tau] = \mathtt{F}$	MT 2.23 ($\equiv$) (p10)
12.	$\mathbf{v}[((\phi \vee \psi) \equiv (\phi \vee \psi))] = \mathtt{F}$	MT 2.23 ($\equiv$, $\vee$) (p10, p11, p6)
13.	$\mathbf{v}[(\phi \vee (\psi \equiv \tau))] = \mathtt{F}$	Suposición 2
14.	$\mathbf{v}[\phi] = \mathtt{F}$ y $\mathbf{v}[(\psi \equiv \tau)] = \mathtt{F}$	MT 2.23 ($\vee$) (p13)
15.	$\mathbf{v}[\psi] = \mathtt{T}$	Suposición 2.1
16.	$\mathbf{v}[\tau] = \mathtt{F}$	MT 2.23 ($\equiv$) (p15, 14)
17.	$\mathbf{v}[((\phi \vee \psi) \equiv (\phi \vee \psi))] = \mathtt{F}$	MT 2.23 ($\equiv$, $\vee$) (p16, p15, p14)
18.	$\mathbf{v}[\psi] = \mathtt{F}$	Suposición 2.2
19.	$\mathbf{v}[\tau] = \mathtt{T}$	MT 2.23 ($\equiv$) (p18, 14)
20.	$\mathbf{v}[((\phi \vee \psi) \equiv (\phi \vee \psi))] = \mathtt{F}$	MT 2.23 ($\equiv$, $\vee$) (p19, p18, p14)
21.	$\therefore \quad ((\phi \vee (\psi \equiv \tau)) \equiv ((\phi \vee \psi) \equiv (\phi \vee \psi)))$	

1.2. b)

b)

0.	$((\phi \to (\psi \to \tau)) \equiv ((\phi \to \psi) \to (\phi \to \tau)))$	Enunciado
1.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))]$	MT 2.23 ($\equiv$)(p0)
2.	Suposición 1	
3.	Suposición 2	
4.	$\therefore \quad ((\phi \to (\psi \to \tau)) \equiv ((\phi \to \psi) \to (\phi \to \tau)))$	

Suposición 1

0.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathtt{F}$	
1.	$\mathbf{v}[\phi] = \mathtt{T}$ y $\mathbf{v}[(\psi \to \tau)] = \mathtt{F}$	MT 2.23 ($\to$)(p0)
2.	$\mathbf{v}[\psi] = \mathtt{T}$ y $\mathbf{v}[\tau] = \mathtt{F}$	MT 2.23 ($\to$)(p1)
3.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$	MT 2.23 ($\to$)(p2, p1)
4.	$\mathbf{v}[(\phi \to \tau)] = \mathtt{F}$	MT 2.23 ($\to$)(p2, p1)
5.	$\mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))] = \mathtt{F}$	MT 2.23 ($\to$)(p3, p4)
6.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))]$	(p5, p0)

Suposición 2

0.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathtt{T}$	
1.	$\mathbf{v}[\phi] = \mathtt{F}$ o $\mathbf{v}[(\psi \to \tau)] = \mathtt{T}$	MT 2.23 ($\to$)(p0)
2.	Suposición 2.1	
3.	Suposición 2.2	

suposición 2.1

0.	$\mathbf{v}[\phi] = \mathtt{F}$	
1.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$	MT 2.23 ($\to$)(p0)
2.	$\mathbf{v}[(\phi \to \tau)] = \mathtt{T}$	MT 2.23 ($\to$)(p0)
3.	$\mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))] = \mathtt{T}$	MT 2.23 ($\to$)(p2, p1)
4.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))]$	(p0 Suposición 2, p3)

Suposición 2.2

0.	$\mathbf{v}[\phi] = \mathtt{T}$ y $\mathbf{v}[(\psi \to \tau)] = \mathtt{T}$	
1.	$\mathbf{v}[\psi] = \mathtt{F}$ o $\mathbf{v}[\tau] = \mathtt{T}$	MT 2.23 ($\to$)(p0)
2.	Suposición 2.2.1	
3.	Suposición 2.2.2	

Suposición 2.2.1

0.	$\mathbf{v}[\psi] = \mathtt{F}$	
1.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{F}$	MT 2.23 ($\to$)(p0 Suposición 2.2, p0)
2.	$\mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))] = \mathtt{T}$	MT 2.23 ($\to$)(p1)
3.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))]$	(p0 Suposición 2, p2)

Suposición 2.2.2

0.	$\mathbf{v}[\psi] = \mathtt{T}$ y $\mathbf{v}[\tau] = \mathtt{T}$	
1.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$	MT 2.23 ($\to$)(p0 Suposición 2.2, p0)
2.	$\mathbf{v}[(\phi \to \tau)] = \mathtt{T}$	MT 2.23 ($\to$)(p0 Suposición 2.2, p0)
3.	$\mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))] = \mathtt{T}$	MT 2.23 ($\to$)(p2, p1)
4.	$\mathbf{v}[(\phi \to (\psi \to \tau))] = \mathbf{v}[((\phi \to \psi) \to (\phi \to \tau))]$	(p0 Suposición 2, p3)

2. Punto 2

$((\neg(p \vee q)) \to p) = \phi$

0.	ϕ es satisfacible pero no tautología	Enunciado
1.	demostración 1	
2.	demostración 2	
3.	$\therefore$ ϕ es satisfacible pero no tautología	

demostración 1

0.	$(\exists \mathbf{v} \mid \mathbf{v}[\phi] = \mathtt{T})$	Enunciado
1.	$\mathbf{v}[((\neg(p \vee q)) \to p)] = \mathtt{T}$	Def(p0)
2.	$\mathbf{v}[(\neg(p \vee q))] = \mathtt{F}$ o $\mathbf{v}[p] = \mathtt{T}$	MT 2.23 ($\to$)(p1)
3.	$\mathbf{v}[(p \vee q)] = \mathtt{T}$ o $\mathbf{v}[p] = \mathtt{T}$	MT 2.23 ($\neg$)(p2)
4.	$\mathbf{v}[p] = \mathtt{T}$ o $\mathbf{v}[q] = \mathtt{T}$ o $\mathbf{v}[p] = \mathtt{T}$	MT 2.23 ($\vee$)(p3)
5.	$\mathbf{v} = \{p \mapsto \mathtt{T}, q \mapsto \mathtt{T}\}$	Suposición (p4)

demostración 2

0.	$(\exists \mathbf{v} \mid \mathbf{v}[\phi] = \mathtt{F})$	Enunciado
1.	$\mathbf{v}[((\neg(p \vee q)) \to p)] = \mathtt{F}$	Def. (p0)
2.	$\mathbf{v}[(\neg(p \vee q))] = \mathtt{T}$ y $\mathbf{v}[p] = \mathtt{F}$	MT 2.23 ($\to$)(p1)
3.	$\mathbf{v}[(p \vee q)] = \mathtt{F}$ y $\mathbf{v}[p] = \mathtt{F}$	MT 2.23 ($\neg$)(p2)
4.	$\mathbf{v}[p] = \mathtt{F}$ y $\mathbf{v}[q] = \mathtt{F}$ y $\mathbf{v}[p] = \mathtt{F}$	MT 2.23 ($\vee$)(p3)
5.	$\mathbf{v} = \{p \mapsto \mathtt{F}, q \mapsto \mathtt{F}\}$	

3. Punto 3

- ϕ tiene a $\vee$ como único conector lógico.

primer inciso

0.	$\phi = (\psi \vee \tau)$	Enunciado
1.	$\models \phi$	Enunciado
2.	$(\exists \mathbf{v} \mid \mathbf{v}[\phi] = \mathtt{F})$	Suposición
3.	$\mathbf{v}[(\psi \vee \tau)] = \mathtt{F}$	Def.(p2)
4.	$\mathbf{v}[\psi] = \mathtt{F}$ o $\mathbf{v}[\tau] = \mathtt{F}$	MT 2.23 ($\vee$)(p3)
5.	$\not\models \phi$	(p4, p2)

- ϕ tiene a $\wedge$ como único conector lógico.

primer inciso

0.	$\phi = (\psi \wedge \tau)$	Enunciado
1.	$\models \phi$	Enunciado
2.	$(\exists \mathbf{v} \mid \mathbf{v}[\phi] = \mathtt{F})$	Suposición
3.	$\mathbf{v}[(\psi \wedge \tau)] = \mathtt{F}$	Def.(p2)
4.	$\mathbf{v}[\psi] = \mathtt{F}$ y $\mathbf{v}[\tau] = \mathtt{F}$	MT 2.23 ($\wedge$)(p3)
5.	$\not\models \phi$	(p4, p2)

4. Punto 4

$\models (\phi \equiv \psi)$ sii $(\phi \rightarrow \psi)$ y $(\phi \leftarrow \psi)$

0.	demostración 1
1.	demostración 2
2.	$\therefore$ $\models (\phi \equiv \psi)$ sii $(\phi \rightarrow \psi)$ y $(\phi \leftarrow \psi)$

demostración 1

0.	Si $\models (\phi \equiv \psi)$,entonces $\models (\phi \rightarrow \psi)$ y $\models (\phi \leftarrow \psi)$	
1.	$\models (\phi \equiv \psi)$	Suposición
2.	$\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}$	Def.(p1)
3.	$\mathbf{v}[\phi] = \mathbf{v}[\psi]$	MT 2.23 ($\equiv$)(p2)
4.	Suposición 1	
5.	Suposición 2	
6.	$\therefore$ Si $\models (\phi \equiv \psi)$,entonces $\models (\phi \rightarrow \psi)$ y $\models (\phi \leftarrow \psi)$	

Suposición 1

0.	$\mathbf{v}[\phi] = \mathtt{T}$	
1.	$\mathbf{v}[\psi] = \mathtt{T}$	(p0, p3 demostración 1)
2.	$\mathbf{v}[(\phi \rightarrow \psi)] = \mathtt{T}$	MTT 2.23 ($\rightarrow$)(p1, p0)
3.	$\mathbf{v}[(\phi \leftarrow \psi)] = \mathtt{T}$	MTT 2.23 ($\leftarrow$)(p1, p0)

Suposición 2

0. $\mathbf{v}[\phi] = \mathtt{F}$
1. $\mathbf{v}[\psi] = \mathtt{F}$ (p0, p3 demostración 1)
2. $\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$ MTT 2.23 ($\to$)(p1, p0)
3. $\mathbf{v}[(\phi \leftarrow \psi)] = \mathtt{T}$ MTT 2.23 ($\leftarrow$)(p1, p0)

demostración 2

0.	Si $\models (\phi \to \psi)$ y $\models (\phi \leftarrow \psi)$, entonces $\models (\phi \equiv \psi)$	
1.	$\models (\phi \to \psi)$ y $\models (\phi \leftarrow \psi)$	Suposición
2.	$\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$ y $\mathbf{v}[(\phi \leftarrow \psi)] = \mathtt{T}$	Def.(p1)
3.	$(\mathbf{v}[\phi] = \mathtt{F}$ o $\mathbf{v}[\psi] = \mathtt{T})$ y $(\mathbf{v}[\phi] = \mathtt{T}$ o $\mathbf{v}[\psi] = \mathtt{F})$	MT 2.23 ($\to$, $\leftarrow$)(p2)
4.	Suposición 1	
5.	Suposición 2	
6.	$\therefore$ Si $\models (\phi \to \psi)$ y $\models (\phi \leftarrow \psi)$, entonces $\models (\phi \equiv \psi)$	

Suposición 1

0. $\mathbf{v}[\phi] = \mathtt{T}$
1. $\mathbf{v}[\psi] = \mathtt{T}$ MT 2.23 ($\leftarrow$)(p0, p3 demostración 2)
2. $\mathbf{v}[\phi] = \mathbf{v}[\psi]$ (p1, p0)
3. $\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}$ MT 2.23 ($\equiv$)(p2)

Suposición 2

0. $\mathbf{v}[\phi] = \mathtt{F}$
1. $\mathbf{v}[\psi] = \mathtt{F}$ MT 2.23 ($\to$)(p0, p3 demostración 2)
2. $\mathbf{v}[\phi] = \mathbf{v}[\psi]$ (p1, p0)
3. $\mathbf{v}[(\phi \equiv \psi)] = \mathtt{T}$ MT 2.23 ($\equiv$)(p2)

5. Punto 5

$\models (\phi \wedge \psi)$ sii $\models \phi$ y $\models \psi$

0. demostración 1
1. demostración 2
2. $\therefore$ $\models (\phi \wedge \psi)$ sii $\models \phi$ y $\models \psi$

demostración 1

0.	Si $\models (\phi \wedge \psi)$, entonces $\models \phi$ y $\models \psi$	
1.	$\models (\phi \wedge \psi)$	Suposición
2.	$\mathbf{v}[(\phi \wedge \psi)]$	Def.(p1)
3.	$\mathbf{v}[\phi] = \mathtt{T}$ y $\mathbf{v}[\psi] = \mathtt{T}$	MT 2.23 ($\wedge$)(p2)
4.	$\models \phi$ y $\models \psi$	Def.(p3)
5.	$\therefore$ Si $\models (\phi \wedge \psi)$, entonces $\models \phi$ y $\models \psi$	

demostración 2

0. Si $\models \phi$ y $\models \psi$, entonces $\models (\phi \wedge \psi)$
1. $\models \phi$ y $\models \psi$ — Suposición
2. $\mathbf{v}[\phi] = \mathtt{T}$ y $\mathbf{v}[\psi] = \mathtt{T}$ — Def.(p1)
3. $\mathbf{v}[(\phi \wedge \psi)] = \mathtt{T}$ — MTT 2.23 ($\wedge$)(p2)
4. $\models (\phi \wedge \psi)$ — Def.(p3)

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5. Si $\models \phi$ y $\models \psi$, entonces $\models (\phi \wedge \psi)$

6. Punto 6

7. Punto 7

$\Gamma \cup \{\phi\} \models \psi$ sii $\Gamma \models (\phi \to \psi)$

0. demostración 1
1. demostración 2

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2. $\therefore$ $\Gamma \cup \{\phi\} \models \psi$ sii $\Gamma \models (\phi \to \psi)$

demostración 1

0. Si $\Gamma \cup \{\phi\} \models \psi$, entonces $\Gamma \models (\phi \to \psi)$
1. $\Gamma \cup \{\phi\} \models \psi$ — Suposición
2. Existe $\mathbf{v}$ tal que$\mathbf{v} satisface \Gamma \cup \{\phi\}$ — Def(p1)
3. $\mathbf{v}$ satisface Γ y a $\{\phi\}$ — $\Gamma \cup \{\phi\} := (\exists \xi \,|\, \xi \in \Gamma \wedge \xi \in \{\phi\})$
4. $\mathbf{v}[\psi] = \mathtt{T}$ — Def.(p2, p1)
5. $\mathbf{v}[\phi] = \mathtt{T}$ — Def.(p3)
6. $\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$ — MT 2.23 ($\to$)(p5, p4)
7. $\mathbf{v}$ satisface Γ y a $(\phi \to \psi)$ — (p6, p3)

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8. $\therefore$ Si $\Gamma \cup \{\phi\} \models \psi$, entonces $\Gamma \models (\phi \to \psi)$

demostración 2

0. Si $\Gamma \models (\phi \to \psi)$, entonces $\Gamma \cup \{\phi\} \models \psi$
1. $\Gamma \models (\phi \to \psi)$ — Suposición
2. Existe $\mathbf{v}$ tal que $\mathbf{v}$ satisface Γ
3. $\mathbf{v}[(\phi \to \psi)] = \mathtt{T}$ — Def.(p2, p1)
4. $\mathbf{v}[\phi] = \mathtt{F}$ o $\mathbf{v}[\psi] = \mathtt{T}$ — MT 2.23 ($\to$)(p3)
5. $\mathbf{v}[\phi] = \mathtt{T}$ y $\mathbf{v}[\psi] = \mathtt{T}$ — Suposición
6. $\mathbf{v}$ satisface Γy a$\{\phi\}$

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7. $\therefore$ Si $\Gamma \models (\phi \to \psi)$, entonces $\Gamma \cup \{\phi\} \models \psi$

8. Punto 8

Si $\Delta \not\models \phi$ y $\Gamma \subset \Delta$, entonces $\Gamma \not\models \phi$

0.	Existe $\mathbf{v}$ tal que $\mathbf{v}$ satisface Δ y $\mathbf{v}[\phi] = \mathtt{F}$	
1.	$(\forall \xi \mid \xi \in \Delta : \xi \in \Gamma)$	Def. ($\subset$)
2.	$(\forall \xi \mid \xi \in \Delta : \mathbf{v}[\xi] = \mathtt{T})$	Def. (p0)
3.	$(\forall \xi \mid \xi \in \Gamma : \mathbf{v}[\xi] = \mathtt{T})$	(p2, p1)
4.	$\mathbf{v}$ satisface Γ y $\mathbf{v}[\phi] = \mathtt{F}$	
5.	$\therefore$ Si $\Delta \not\models \phi$ y $\Gamma \subset \Delta$, entonces $\Gamma \not\models \phi$	

9. Punto 9

- Si Pedro entiende matemáticas, entonces puede entender lógica. Pedro no entiende lógica. Consecuentemente, Pedro no entiende matemáticas.

Si Pedro...

p : Pedro entiende matemáticas

q : Pedro puede entender lógica

$$\Gamma = \{(p \to q), (\neg q)\}, \phi = (\neg p)$$

0.	Existe $\mathbf{v}$ tal que $\mathbf{v}$ satisface Γ	Suposición
1.	$\mathbf{v}[(p \to q)] = \mathtt{T}$	Def.(p0)
2.	$\mathbf{v}[(\neg q)] = \mathtt{T}$	Def(p0)
3.	$\mathbf{v}[q] = \mathtt{F}$	MT 2.23 ($\neg$)(p2)
4.	$\mathbf{v}[p] = \mathtt{F}$	MT 2.23 ($\to$)(p3, p1)
5.	$\mathbf{v}[(\neg p)] = \mathtt{T}$	MTT 2.23 ($\neg$)(p4)
6.	$\Gamma \models \phi$	