# Taller 05

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# Índice

# 1. Punto 1

Punto 1 , ($A$ , $B$)

$A$ dice: "nosotros tenemos la misma naturaleza"

$\Gamma_0 = \{(a \equiv (a \equiv b)), a\}$

$\Gamma_1 = \{(a \equiv (a \equiv b)), (\neg a)\}$

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No es posible determinar la naturaleza de $A$ y $B$

Con $\Gamma_0$

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(a \equiv (a \equiv b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(a \equiv b)] = \mathtt{T}$ MT 2.23($\equiv$)(p1, p2)
4. $\mathbf{v}[b] = \mathtt{T}$ MTT 2.23 ($\equiv$)(p3, p2)

Con $\Gamma_1$

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_1)$
1. $\mathbf{v}[(a \equiv (a \equiv b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(a \equiv b)] = \mathtt{F}$ MT 2.23($\equiv$)(p2, p1)
4. $\mathbf{v}[a] \neq \mathbf{v}[b]$ MT 2.23($\equiv$)(p3)
5. $\mathbf{v}[b] = \mathtt{T}$ (p4, p2)

# 2. Punto 2

Punto 2 , ($A$ , $B$)

$A$ dice: "al menos uno de nosotros es caballero"

$\Gamma_0 = \{(a \equiv (a \vee b)), a\}$

$\Gamma_1 = \{(a \equiv (a \vee b)), (\neg a)\}$

---

No se puede determinar la naturaleza de $A$ y $B$

Con $\Gamma_0$

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(a \equiv (a \vee b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)

Con $\Gamma_1$

0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$
1. $\mathbf{v}[(a \equiv (a \vee b))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[a] = \mathtt{F}$ y $\mathbf{v}[b] = \mathtt{F}$

## 3. Punto 3

punto 3, $(A\ ,\ B\ ,\ C)$

$A$ dice: "$B$ es escudero"
$B$ dice: "$A$ y $C$ son del mismo tipo"
$\Gamma_0 = \{(a \equiv (\neg b)), (b \equiv (a \equiv c)), a\}$
$\Gamma_1 = \{(a \equiv (\neg b)), (b \equiv (a \equiv c)), (\neg a)\}$

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$C$ es escudero

Con $\Gamma_0$

| | | |
|---|---|---|
| 0. | $(\exists \mathbf{v}, \mid \mathbf{v}$ satisface $\Gamma_0)$ | |
| 1. | $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$ | Def.(p0) |
| 2. | $\mathbf{v}[(b \equiv (a \equiv c))] = \mathtt{T}$ | Def.(p0) |
| 3. | $\mathbf{v}[a] = \mathtt{T}$ | Def.(p0) |
| 4. | $\mathbf{v}[a] = \mathbf{v}[(\neg b)]$ | MT 2.23($\equiv$)(p2) |
| 5. | $\mathbf{v}[b] = \mathbf{v}[(a \equiv c)]$ | MT 2.23($\equiv$)(p3) |
| 6. | $\mathbf{v}[b] = \mathtt{F}$ | MT 2.23($\equiv$, $\neg$)(p4, p3) |
| 7. | $\mathbf{v}[a] \neq \mathbf{v}[c]$ | MT 2.23($\equiv$)(p6, p5) |
| 8. | $\mathbf{v}[c] = \mathtt{F}$ | (p7, p3) |

Con $\Gamma_1$

| | | |
|---|---|---|
| 0. | $(\exists \mathbf{v}, \mid \mathbf{v}$ satisface $\Gamma_1)$ | |
| 1. | $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$ | Def.(p0) |
| 2. | $\mathbf{v}[(b \equiv (a \equiv c))] = \mathtt{T}$ | Def.(p0) |
| 3. | $\mathbf{v}[(\neg a)] = \mathtt{T}$ | Def.(p0) |
| 4. | $\mathbf{v}[b] = \mathtt{T}$ | MT 2.23($\equiv$, $\neg$)(p3, p1) |
| 5. | $\mathbf{v}[a] = \mathbf{v}[c]$ | MT 2.23($\equiv$)(p4, p2) |
| 6. | $\mathbf{v}[c] = \mathtt{F}$ | MT 2.23($\neg$)(p5, p3) |

## 4. Punto 4

Punto 4, $(A, B, C)$

$A$ dice: "$B$ y $C$ son de la misma naturaleza"
$\Gamma_0 = \{(a \equiv (b \equiv c)), a, b\}$
$\Gamma_1 = \{(a \equiv (b \equiv c)), a, (\neg b)\}$
$\Gamma_2 = \{(a \equiv (b \equiv c)), (\neg a), b\}$
$\Gamma_0 = \{(a \equiv (b \equiv c)), (\neg a), (\neg b)\}$

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$C$ responderá "sí"

Con $\Gamma_0$

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[b] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[c] = \mathtt{T}$ MT 2.23($\equiv$)(p3, p2, p1)
5. $C$ respondería sí

Con $\Gamma_1$

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_1)$
1. $\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[a] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(\neg b)] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[c] = \mathtt{F}$ MT 2.23($\equiv$, $\neg$)(p3, p2, p1)
5. $C$ respondería sí

Con $\Gamma_2$

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[b] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[c] = \mathtt{F}$ MT 2.23($\equiv$, $\neg$)(p3, p2, p1)
5. $C$ respondería sí

Con $\Gamma_3$

0. $(\exists \mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$
1. $\mathbf{v}[(a \equiv (b \equiv c))] = \mathtt{T}$ Def.(p0)
2. $\mathbf{v}[(\neg a)] = \mathtt{T}$ Def.(p0)
3. $\mathbf{v}[(\neg b)] = \mathtt{T}$ Def.(p0)
4. $\mathbf{v}[c] = \mathtt{T}$ MT 2.23($\equiv$, $\neg$)(p3, p2, p1)
5. $C$ respondería sí

# 5. Punto 5

El habitante $A$ dice "Yo dije que si no soy caballero entonces soy escudero, y si soy caballero entonces no soy escudero"

## 6. Punto 6

$A$, $B$, $(a \equiv (\neg b)), (b \equiv (a \vee b))$

$A$ dice: "$B$ es escudero"
$B$ dice: "al menos uno de nosotros es caballero"
$\Gamma_0 = \{(a \equiv (\neg b)), (b \equiv (a \vee b)), a\}$
$\Gamma_1 = \{(a \equiv (\neg b)), (b \equiv (a \vee b)), (\neg a)\}$

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$A$ es escudero y $B$ es caballero

Con $\Gamma_0$

| | |
|---|---|
| 0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_0)$ | |
| 1. $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$ | Def.(p0) |
| 2. $\mathbf{v}[(b \equiv (a \vee b))] = \mathtt{T}$ | Def.(p0) |
| 3. $\mathbf{v}[a] = \mathtt{T}$ | Def.(p0) |
| 4. $\mathbf{v}[b] = \mathtt{F}$ | MTT 2.23 $(\equiv, \neg)$(p3, p1) |
| 5. $\mathbf{v}[(a \vee b)] = \mathtt{T}$ | MT 2.23 $(\equiv)$(p4, p2) |
| 6. $\mathbf{v}[a] = \mathtt{F}$ y $\mathbf{v}[b] = \mathtt{F}$ | MT 2.23$(\vee)$(p5) |
| 7. $\mathbf{v}[a] = \mathtt{F}$ y $\mathbf{v}[a] = \mathtt{T}$ | Contradicción (p6, p3) |

Con $\Gamma_1$

| | |
|---|---|
| 0. $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma_1)$ | |
| 1. $\mathbf{v}[(a \equiv (\neg b))] = \mathtt{T}$ | Def.(p0) |
| 2. $\mathbf{v}[(b \equiv (a \vee b))] = \mathtt{T}$ | Def.(p0) |
| 3. $\mathbf{v}[(\neg a)] = \mathtt{T}$ | Def.(p0) |
| 4. $\mathbf{v}[a] = \mathtt{F}$ | MT 2.23$(\neg)$(p3) |
| 5. $\mathbf{v}[b] = \mathtt{T}$ | MT 2.23 $(\equiv, \neg)$(p4, p1) |

## 7. Punto 7

"Soy caballero" , debido a que $\models (\phi \equiv \phi)$

## 8. Punto 8

"Soy escudero" , debido a que $(\phi \equiv (\neg\phi))$ es una contradicción

## 9. Punto 9

Punto 9, $A$, $B$, $C$

$A$ dice: "Yo soy hombre lobo"
$B$ dice: "Yo soy hombre lobo"
$C$ dice: "A lo sumo, uno de nosotros es caballero"
$\Gamma = \{(c \equiv ((a \wedge (\neg b) \wedge (\neg c)) \vee ((\neg a) \wedge b \wedge (\neg c)) \vee ((\neg a) \wedge (\neg b) \wedge c) \vee ((\neg a) \wedge (\neg b) \wedge (\neg c))))\}$
$\Gamma_0 = \Gamma \cup \{c\}$
$\Gamma_1 = \Gamma \cup \{(\neg c), a\}$
$\Gamma_2 = \Gamma \cup \{(\neg c), (\neg a)\}$

. . . Tomando lo que dijo $C$ como $\phi$. . .

Con $\Gamma_0$

| | | |
|---|---|---|
| 0. | $(\exists\mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$ | |
| 1. | $\mathbf{v}[(c \equiv \phi)] = \mathtt{T}$ | Def.(p0) |
| 2. | $\mathbf{v}[c] = \mathtt{T}$ | Def.(p0) |
| 3. | $\mathbf{v}[\phi] = \mathtt{T}$ | MT 2.23($\equiv$)(p2, p1) |
| 4. | $\mathbf{v}[((\neg a) \wedge (\neg b) \wedge c)] = \mathtt{T}$ | MT 2.23($\vee$)(p3, p1) |
| 5. | $\mathbf{v}_0 = \{a \mapsto \mathtt{F}, b \mapsto \mathtt{F}, c \mapsto \mathtt{T}\}$ | |

Con $\Gamma_1$

| | | |
|---|---|---|
| 0. | $(\exists\mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$ | |
| 1. | $\mathbf{v}[(c \equiv \phi)] = \mathtt{T}$ | Def.(p0) |
| 2. | $\mathbf{v}[(\neg c)] = \mathtt{T}$ | Def.(p0) |
| 3. | $\mathbf{v}[a] = \mathtt{T}$ | Def.(p0) |
| 4. | $\mathbf{v}[(\neg b)] = \mathtt{F}$ | MT 2.23($\wedge$)(p3, p2, p1) |
| 5. | $\mathbf{v}[b] = \mathtt{T}$ | MT 2.23($\neg$)(p4) |
| 6. | $\mathbf{v}_1 = \{a \mapsto \mathtt{T}, b \mapsto \mathtt{T}, c \mapsto \mathtt{F}\}$ | |

Con $\Gamma_2$

| | | |
|---|---|---|
| 0. | $(\exists\mathbf{v} \mid \mathbf{v} \text{ satisface } \Gamma_0)$ | |
| 1. | $\mathbf{v}[(c \equiv \phi)] = \mathtt{T}$ | Def.(p0) |
| 2. | $\mathbf{v}[(\neg c)] = \mathtt{T}$ | Def.(p0) |
| 3. | $\mathbf{v}[(\neg a)] = \mathtt{T}$ | Def.(p0) |
| 4. | $\mathbf{v}[a] = \mathtt{F}$ | MT 2.23($\neg$)(p3) |
| 5. | $\mathbf{v}[b] = \mathtt{F}$ | MT 2.23($\vee$, $\wedge$)(p3, p2, p1) |
| 6. | $\mathbf{v}[(\neg b)] = \mathtt{F}$ | MT 2.23($\vee$, $\wedge$)(p3, p2, p1) |
| 7. | $\mathbf{v}[b] = \mathtt{F}$ y $\mathbf{v}[b] = \mathtt{T}$ | Contradicción(p6, p5) |

Debido a la suposición del enunciado "exáctamente uno entre $A$, $B$ y $C$ es hombre lobo", hace que $\mathbf{v}_1$ no sea una respuesta posible.
Por consiguiente, $C$ es el único caballero, y el único hombre lobo.

## 10. Punto 10

Suponga que $\Gamma$ es un conjunto de proposiciones que especifica información dada acerca de un acertijo de la isla de caballeros y escuderos. Además, suponga que la variable proposicional $a$ modela la naturaleza de un habitante $A$ de la isla. Demuestre o refute:

## 10.1. a)

Si $A$ es caballero , entonces $\Gamma \models a$

| | | |
|---|---|---|
| 0. | $A$ es caballero | suposición |
| 1. | $\mathbf{v}[a] = \mathtt{T}$ | Def.(p0) |
| 2. | $\mathbf{v}$ satisface $\Gamma$ | (p1)(contexto C & E) |
| 3. | $\Gamma \models a$ | |

## 10.2. b)

Si $\Gamma \models a$ es caballero , entonces $A$

| | | |
|---|---|---|
| 0. | $(\exists \mathbf{v} \mid \mathbf{v}$ satisface $\Gamma)$ | suposición |
| 1. | $\Gamma \models a$ | suposición |
| 2. | $\mathbf{v}[a] = \mathtt{T}$ | Def.(p1, p0) |
| 3. | $A$ es caballero | (p2)(contexto C & E) |