Taller 06

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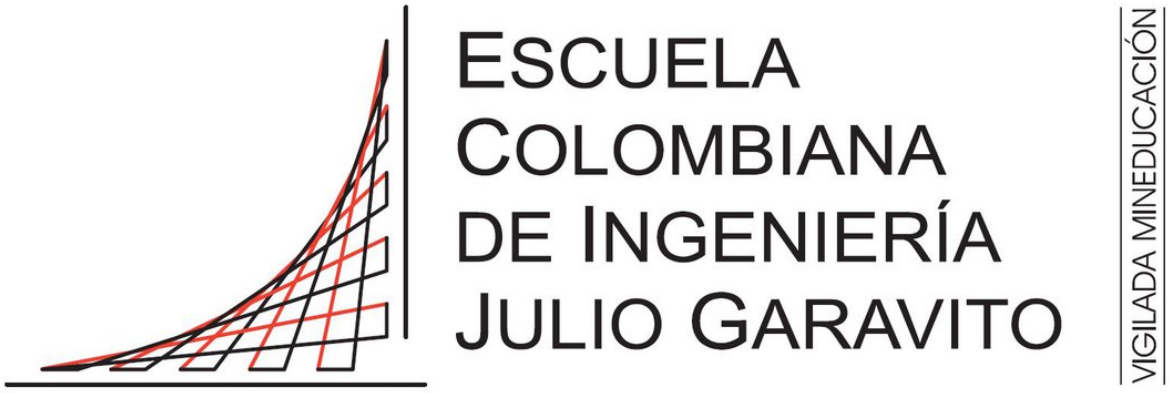

Matemáticas
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Índice

1. Punto 1

- $((p \wedge (\neg q)) \rightarrow r)$

$\overline{\mathbf{F_1}}[\phi]$	$(((p \equiv q) \wedge (\neg(r \rightarrow s)) \rightarrow \mathit{false}))$
$\overline{\mathbf{F_2}}[\phi]$	$(((p \equiv q) \wedge (\neg q)) \rightarrow r)$
$\overline{\mathbf{F_3}}[\phi]$	$((p \wedge (\neg q)) \rightarrow \mathit{false})$
$\overline{\mathbf{F_4}}[\phi]$	$(((p \equiv q) \wedge (\neg(r \rightarrow s))) \rightarrow r)$
$\overline{\mathbf{F_5}}[\phi]$	$((p \wedge (\neg(r \rightarrow s))) \rightarrow \mathit{false})$

- $(p \rightarrow (q \rightarrow p))$

$\overline{\mathbf{F_1}}[\phi]$	$((p \equiv q) \rightarrow ((r \rightarrow s) \rightarrow (p \equiv q)))$
$\overline{\mathbf{F_2}}[\phi]$	$((p \equiv q) \rightarrow (q \rightarrow (p \equiv q)))$
$\overline{\mathbf{F_3}}[\phi]$	$(p \rightarrow (q \rightarrow p))$
$\overline{\mathbf{F_4}}[\phi]$	$((p \equiv q) \rightarrow ((r \rightarrow s) \rightarrow (p \equiv q)))$
$\overline{\mathbf{F_5}}[\phi]$	$(p \rightarrow ((r \rightarrow s) \rightarrow p))$

- $(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \rightarrow q) \vee (r \wedge (\neg r))))))$

$\overline{\mathbf{F_1}}[\phi]$	$(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \rightarrow (r \rightarrow s)) \vee (\mathit{false} \wedge (\neg \mathit{false}))))))$
$\overline{\mathbf{F_2}}[\phi]$	$(\neg((r \wedge (r \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \rightarrow q) \vee (r \wedge (\neg r))))))$
$\overline{\mathbf{F_3}}[\phi]$	$(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow (p \vee s))) \equiv (\neg((p \rightarrow q) \vee (\mathit{false} \wedge (\neg \mathit{false}))))))$
$\overline{\mathbf{F_4}}[\phi]$	$(\neg((r \wedge (r \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \rightarrow (r \rightarrow s)) \vee (r \wedge (\neg r))))))$
$\overline{\mathbf{F_5}}[\phi]$	$(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \vee s))) \equiv (\neg((p \rightarrow (r \rightarrow s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))$

2. Punto 2

- $(\overline{\mathbf{F_1}} \circ \overline{\mathbf{F_2}})[((p \wedge (\neg q)) \to r)]$

  punto 2

  $$\begin{aligned}\overline{\mathbf{F_1}}[\overline{\mathbf{F_2}}[((p \wedge (\neg q)) \to r)]] &= \overline{\mathbf{F_1}}[(((p \equiv q) \wedge (\neg q)) \to r)] \\ &= ((((p \equiv q) \equiv (r \to s)) \wedge (\neg(r \to s)) \to \mathit{false}))\end{aligned}$$

- $(\overline{\mathbf{F_3}} \circ \overline{\mathbf{F_4}})[(p \to (q \to p))]$

  Punto 2

  $$\begin{aligned}\overline{\mathbf{F_3}}[\overline{\mathbf{F_4}}[(p \to (q \to p))]] &= \overline{\mathbf{F_3}}[((p \equiv q) \to ((r \to s) \to (p \equiv q)))] \\ &= ((p \equiv q) \to ((\mathit{false} \to s) \to (p \equiv q)))\end{aligned}$$

- $(\overline{\mathbf{F_5}} \circ \overline{\mathbf{F_1}})[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]$

  punto 2

  $$\begin{aligned}&\overline{\mathbf{F_5}}[\overline{\mathbf{F_1}}[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]] \\ &= \overline{\mathbf{F_5}}[(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \to (r \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false}))))))] \\ &= (\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \to (r \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false))))))\end{aligned}$$

- $(\overline{\mathbf{F_2}} \circ \overline{\mathbf{F_3}})[((p \wedge (\neg q)) \to r)]$

  Punto 2

  $$\begin{aligned}\overline{\mathbf{F_2}}[\overline{\mathbf{F_3}}[((p \wedge (\neg q)) \to r)]] &= \overline{\mathbf{F_2}}[((p \wedge (\neg q)) \to \mathit{false})] \\ &= (((p \equiv q) \wedge (\neg q)) \to \mathit{false})\end{aligned}$$

- $(\overline{\mathbf{F_4}} \circ \overline{\mathbf{F_5}})[(p \to (q \to p))]$

  Punto 2

  $$\begin{aligned}\overline{\mathbf{F_4}}[\overline{\mathbf{F_5}}[(p \to (q \to p))]] &= \overline{\mathbf{F_4}}[(p \to ((r \to s) \to p))] \\ &= ((p \equiv q) \to ((r \to s) \to (p \equiv q)))\end{aligned}$$

- $(\overline{\mathbf{F_1}} \circ \overline{\mathbf{F_3}})[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]$

  Punto 2

  $$\begin{aligned}&\overline{\mathbf{F_1}}[\overline{\mathbf{F_3}}[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]] \\ &= \overline{\mathbf{F_1}}[(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (\mathit{false} \wedge (\neg \mathit{false}))))))] \\ &= (\neg(\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \to (r \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))\end{aligned}$$

3. Punto 3

- $(\overline{\mathbf{F_1}} \circ \overline{\mathbf{F_2}} \circ \overline{\mathbf{F_3}})[((p \wedge (\neg q)) \to r)]$

punto 3

$$\begin{aligned}\overline{\mathbf{F_1}}[\overline{\mathbf{F_2}}[\overline{\mathbf{F_3}}[((p \wedge (\neg q)) \to r)]]] &= \overline{\mathbf{F_1}}[\overline{\mathbf{F_2}}[((p \wedge (\neg q)) \to \mathit{false})]] \\ &= \overline{\mathbf{F_1}}[(((p \equiv q) \wedge (\neg q)) \to \mathit{false})] \\ &= ((((p \equiv q) \equiv (r \to s)) \wedge (\neg(r \to s)) \to \mathit{false}))\end{aligned}$$

- $(\overline{\mathbf{F_4}} \circ \overline{\mathbf{F_5}} \circ \overline{\mathbf{F_1}})[(p \to (q \to p))]$

punto 3

$$\begin{aligned}\overline{\mathbf{F_4}}[\overline{\mathbf{F_5}}[\overline{\mathbf{F_1}}[(p \to (q \to p))]]] &= \overline{\mathbf{F_4}}[\overline{\mathbf{F_5}}[((p \equiv q) \to ((r \to s) \to (p \equiv q)))]] \\ &= \overline{\mathbf{F_4}}[((p \equiv (r \to s)) \to ((\mathit{false} \to s) \to (p \equiv (r \to s))))] \\ &= (((p \equiv q) \equiv (\mathit{false} \to s)) \to ((\mathit{false} \to s) \to (p \equiv (\mathit{false} \to s))))\end{aligned}$$

- $(\overline{\mathbf{F_2}} \circ \overline{\mathbf{F_3}} \circ \overline{\mathbf{F_4}})[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]$

punto 3

$$\begin{aligned}&\overline{\mathbf{F_2}}[\overline{\mathbf{F_3}}[\overline{\mathbf{F_4}}[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]]] \\ &= \overline{\mathbf{F_2}}[\overline{\mathbf{F_3}}[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg(((p \equiv q) \to (r \to s)) \vee (r \wedge (\neg r))))))]] \\ &= \overline{\mathbf{F_2}}[(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \to (\mathit{false} \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))] \\ &= (\neg((\mathit{false} \wedge (\mathit{false} \leftarrow (((p \equiv q) \equiv q) \vee s))) \equiv (\neg((((p \equiv q) \equiv q) \to (\mathit{false} \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))\end{aligned}$$

- $(\overline{\mathbf{F_5}} \circ \overline{\mathbf{F_1}} \circ \overline{\mathbf{F_2}})[((p \wedge (\neg q)) \to r)]$

punto 3

$$\begin{aligned}\overline{\mathbf{F_5}}[\overline{\mathbf{F_1}}[\overline{\mathbf{F_2}}[((p \wedge (\neg q)) \to r)]]] &= \overline{\mathbf{F_5}}[\overline{\mathbf{F_1}}[(((p \equiv q) \wedge (\neg q)) \to r)]] \\ &= \overline{\mathbf{F_5}}[((((p \equiv q) \equiv (r \to s)) \wedge (\neg(r \to s))) \to \mathit{false})] \\ &= ((((p \equiv (r \to s)) \equiv (\mathit{false} \to s)) \wedge (\neg(\mathit{false} \to s))) \to \mathit{false})\end{aligned}$$

- $(\overline{\mathbf{F_3}} \circ \overline{\mathbf{F_4}} \circ \overline{\mathbf{F_5}})[(p \to (q \to p))]$

punto 3

$$\begin{aligned}\overline{\mathbf{F_3}}[\overline{\mathbf{F_4}}[\overline{\mathbf{F_5}}[(p \to (q \to p))]]] &= \overline{\mathbf{F_3}}[\overline{\mathbf{F_4}}[(p \to ((r \to s) \to p))]] \\ &= \overline{\mathbf{F_3}}[((p \equiv q) \to ((r \to s) \to (p \equiv q)))] \\ &= ((p \equiv q) \to ((\mathit{false} \to s) \to (p \equiv q)))\end{aligned}$$

- $(\overline{\mathbf{F_5}} \circ \overline{\mathbf{F_3}} \circ \overline{\mathbf{F_1}})[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]$

punto 3

$$\begin{aligned}&\overline{\mathbf{F_5}}[\overline{\mathbf{F_3}}[\overline{\mathbf{F_1}}[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r))))))]]] \\ &= \overline{\mathbf{F_5}}[\overline{\mathbf{F_3}}[(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \to (r \to s))) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))]] \\ &= \overline{\mathbf{F_5}}[(\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv q) \vee s))) \equiv (\neg(((p \equiv q) \to (r \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))] \\ &= (\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \equiv (r \to s)) \vee s))) \equiv (\neg(((p \equiv (r \to s)) \to (\mathit{false} \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))\end{aligned}$$

4. Punto 4

$\overline{\mathbf{F}} = \{p \mapsto (p \equiv q), q \mapsto (r \to s), r \mapsto \mathit{false}\}$

- $\overline{\mathbf{F}}[((p \wedge (\neg q)) \to r)] = (((p \equiv q) \wedge (\neg(r \to s)) \to \mathit{false}))$
- $\overline{\mathbf{F}}[(p \to (q \to p))] = ((p \equiv q) \to ((r \to s) \to (p \equiv q)))$
- $\overline{\mathbf{F}}[(\neg((r \wedge (r \leftarrow (p \vee s))) \equiv (\neg((p \to q) \vee (r \wedge (\neg r)))))))] = (\neg((\mathit{false} \wedge (\mathit{false} \leftarrow ((p \vee s))) \equiv (\neg((p \to (r \to s)) \vee (\mathit{false} \wedge (\neg \mathit{false})))))))$

5. Punto 5

punto 5

- $(p \equiv r)[p, r := t, t] = (p \equiv p)\ ,\ \vDash (t \equiv t)$
- $((p \wedge q) \vee ((\neg p) \wedge (\neg q)))[p, q := t, t] = ((t \wedge t) \vee ((\neg t) \wedge (\neg t)))\ ,\ \vDash ((t \wedge t) \vee ((\neg t) \wedge (\neg t)))$
- $((p \vee r) \leftarrow (p \wedge q))[p, q, r := t, t, t] = ((t \vee t) \leftarrow (t \wedge t))\ ,\ \vDash ((t \vee t) \leftarrow (t \wedge t))$

6. Punto 6

punto 6

0.	$\gamma = \psi[q := r]$	suposición
1.	p, q, r son variables proposicionales distintas	Enunciado
2.	r no aparece en ϕ ni en ψ	Enunciado
3.	$\phi[p := \gamma] = \phi[p := \psi][q := r]$	(p0, p2)
4.	$\phi[p := \psi][q := r][q := \tau][r := q] = \phi[p, q := \psi, \tau]$	(p2, p3)