Taller 08

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Matemáticas
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5 de octubre de 2022

Índice

1. Punto 1

$\vdash_{\text{DS}} (\neg \mathit{false})$

0. $((\neg \mathit{false}) \equiv \mathit{true})$ Teo 4.15.2
1. $(\neg \mathit{false})$ Identidad (p0)

2. Punto 2

$\vdash_{\text{DS}} ((\phi \not\equiv (\psi \not\equiv \tau)) \equiv ((\phi \not\equiv \psi) \not\equiv \tau))$

0. $((\phi \not\equiv (\psi \not\equiv \tau)) \equiv (\phi \not\equiv (\psi \not\equiv \tau)))$ Teo 4.6.3
1. $((\phi \not\equiv (\psi \not\equiv \tau)) \equiv ((\neg\phi) \equiv (\psi \not\equiv \tau)))$ Def($\not\equiv$)
2. $(((\neg\phi) \equiv (\psi \not\equiv \tau)) \equiv ((\neg\phi) \equiv ((\neg\psi) \equiv \tau)))$ Def($\not\equiv$), Leibniz ($\phi = ((\neg\phi) \equiv p)$)
3. $(((\neg\phi) \equiv ((\neg\psi) \equiv \tau)) \equiv ((\neg\phi) \equiv (\psi \equiv (\neg\tau))))$ Teo 4.15.5, Leibniz ($\phi = ((\neg\phi) \equiv p)$)
4. $(((\neg\phi) \equiv (\psi \equiv (\neg\tau))) \equiv (((\neg\phi) \equiv \psi) \equiv (\neg\tau)))$ Asociativa($\equiv$)
5. $((((\neg\phi) \equiv \psi) \equiv (\neg\tau)) \equiv ((\neg((\neg\phi) \equiv \psi)) \equiv \tau))$ Teo 4.15.5
6. $(((\neg((\neg\phi) \equiv \psi)) \equiv \tau) \equiv ((\neg(\phi \not\equiv \psi)) \equiv \tau))$ Def($\not\equiv$)
7. $(((\neg(\phi \not\equiv \psi)) \equiv \tau) \equiv ((\phi \not\equiv \psi) \equiv \tau))$ Def($\equiv$)
8. $((\phi \not\equiv (\psi \not\equiv \tau)) \equiv ((\phi \not\equiv \psi) \equiv \tau))$ Transitividad(p7,p6,p5,p4,p3,p2,p1,p0)

3. Punto 3

$\vdash_{\text{DS}} ((\phi \not\equiv (\psi \not\equiv \tau)) \equiv ((\phi \not\equiv \psi) \not\equiv \tau))$

$$
\begin{aligned}
& (\phi \not\equiv (\psi \not\equiv \tau)) \\
\equiv \quad & \langle \text{ Def.}(\neg) \rangle \\
& ((\neg\phi) \equiv (\psi \not\equiv \tau)) \\
\equiv \quad & \langle \text{ Def.}(\not\equiv), \text{ Leibniz } (\phi = ((\neg\phi) \equiv p)) \rangle \\
& ((\neg\phi) \equiv ((\neg\psi) \equiv \tau)) \\
\equiv \quad & \langle \text{ Teo 4.15.5, Leibniz } (\phi = ((\neg\phi) \equiv p)) \rangle \\
& ((\neg\phi) \equiv (\psi \equiv (\neg\tau))) \\
\equiv \quad & \langle \text{ Asociativa}(\equiv) \rangle \\
& (((\neg\phi) \equiv \psi) \equiv (\neg\tau)) \\
\equiv \quad & \langle \text{ Teo 4.15.5 } \rangle \\
& ((\neg((\neg\phi) \equiv \psi)) \equiv \tau) \\
\equiv \quad & \langle \text{ Def.}(\not\equiv) \rangle \\
& ((\neg(\phi \not\equiv \psi)) \equiv \tau) \\
\equiv \quad & \langle \text{ Def.}(\not\equiv) \rangle \\
& ((\phi \not\equiv \psi) \not\equiv \tau)
\end{aligned}
$$

Por MT 4.21 se demuestra que
$\vdash_{\text{DS}} ((\phi \not\equiv (\psi \not\equiv \tau)) \equiv ((\phi \not\equiv \psi) \not\equiv \tau))$

4. Punto 4

$\vdash_{\text{DS}} ((\phi \vee \mathit{true}) \equiv \mathit{true})$

$$\begin{array}{ll} & (\phi \vee \mathit{true}) \\ \equiv & \langle \text{ Teo 4.6.2 } \rangle \\ & (\phi \vee (\mathit{true} \equiv \mathit{true})) \\ \equiv & \langle \text{ Distribución } (\vee, \equiv) \rangle \\ & ((\phi \vee \mathit{true}) \equiv (\phi \vee \mathit{true})) \\ \equiv & \langle \text{ Teo 4.6.2 } \rangle \\ & \mathit{true} \end{array}$$

Por MT 4.21 se demuestra que
$\vdash_{\text{DS}} ((\phi \vee \mathit{true}) \equiv \mathit{true})$

5. Punto 5

$\vdash_{\text{DS}} ((\phi \vee \psi) \equiv ((\phi \vee (\neg\psi)) \equiv \phi))$

$$\begin{array}{ll} & (\phi \vee \psi) \\ \equiv & \langle \text{ Identidad } \rangle \\ & ((\phi \vee \psi) \equiv \mathit{true}) \\ \equiv & \langle \text{ Teo 4.6.2 } \rangle \\ & ((\phi \vee \psi) \equiv (\phi \equiv \phi)) \\ \equiv & \langle \text{ Asociativa}(\equiv) \rangle \\ & (((\phi \vee \psi) \equiv \phi) \equiv \phi) \\ \equiv & \langle \text{ Identidad}(\vee) \rangle \\ & (((\phi \vee \psi) \equiv (\phi \vee \mathit{false})) \equiv \phi) \\ \equiv & \langle \text{ Distribución}(\vee, \equiv) \rangle \\ & ((\phi \vee (\psi \equiv \mathit{false})) \equiv \phi) \\ \equiv & \langle \text{ Def.}(\neg) \rangle \\ & ((\phi \vee (\neg\psi)) \equiv \phi) \end{array}$$

Por MT 4.21 se demuestra que
$\vdash_{\text{DS}} ((\phi \vee \psi) \equiv ((\phi \vee (\neg\psi)) \equiv \phi))$

6. Punto 6

$\vdash_{\mathrm{DS}} ((\neg(\phi \vee \psi)) \equiv ((\neg\phi) \wedge (\neg\psi)))$

$$
\begin{aligned}
& (\neg(\phi \vee \psi)) \\
\equiv \quad & \langle \text{ Teo 4.19.4 } \rangle \\
& (\neg((\phi \vee (\neg\psi)) \equiv \phi)) \\
\equiv \quad & \langle \text{ Teo 4.15.4 } \rangle \\
& ((\neg(\phi \vee (\neg\psi))) \equiv \phi) \\
\equiv \quad & \langle \text{ Teo 4.15.5 } \rangle \\
& ((\phi \vee (\neg\psi)) \equiv (\neg\phi)) \\
\equiv \quad & \langle \text{ Conmutativa}(\equiv) \rangle \\
& ((\neg\phi) \equiv (\phi \vee (\neg\psi))) \\
\equiv \quad & \langle \text{ Conmutativa}(\equiv)\text{, Leibniz } (\phi = ((\neg\phi) \equiv \phi)) \rangle \\
& ((\neg\phi) \equiv ((\neg\psi) \vee \phi)) \\
\equiv \quad & \langle \text{ Teo 4.19.4, Leibniz } (\phi = ((\neg\phi) \equiv p)) \rangle \\
& ((\neg\phi) \equiv (((\neg\psi) \vee (\neg\phi)) \equiv (\neg\psi))) \\
\equiv \quad & \langle \text{ Conmutativa}(\equiv)\text{, Leibniz } (\phi = ((\neg\phi) \equiv p)) \rangle \\
& ((\neg\phi) \equiv ((\neg\psi) \equiv ((\neg\psi) \vee (\neg\phi)))) \\
\equiv \quad & \langle \text{ Conmutativa}(\equiv)\text{, Leibniz } (\phi = (\phi = ((\neg\phi) \equiv ((\neg\psi) \equiv p)))) \rangle \\
& ((\neg\phi) \equiv ((\neg\psi) \equiv ((\neg\phi) \vee (\neg\psi)))) \\
\equiv \quad & \langle \text{ Def.}(\wedge) \rangle \\
& ((\neg\phi) \wedge (\neg\psi))
\end{aligned}
$$

Por MT 4.21 se demuestra que
$\vdash_{\mathrm{DS}} ((\neg(\phi \vee \psi)) \equiv ((\neg\phi) \wedge (\neg\psi)))$

7. Punto 7

$\vdash_{DS} ((\phi \wedge (\psi \not\equiv \tau)) \equiv ((\phi \wedge \psi) \not\equiv (\phi \wedge \tau)))$

$((\phi \wedge \psi) \not\equiv (\phi \wedge \tau))$

$\equiv \langle$ Def.($\not\equiv$) $\rangle$

$((\neg(\phi \wedge \psi)) \equiv (\phi \wedge \tau))$

$\equiv \langle$ Teo 4.15.4 $\rangle$

$(\neg((\phi \wedge \psi) \equiv (\phi \wedge \psi)))$

$\equiv \langle$ Def.($\wedge$), Lbz($\phi = (\neg(p \equiv (\phi \wedge \tau))))$, Lbz($\phi = (\neg((\phi \equiv (\psi \equiv (\phi \vee \psi))) \equiv p)))$ $\rangle$

$(\neg((\phi \equiv (\psi \equiv (\phi \vee \psi))) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau)))))$

$\equiv \langle$ Asociativa($\equiv$), Leibniz($\phi = (\neg p)$) $\rangle$

$(\neg(\phi \equiv ((\psi \equiv (\phi \vee \psi)) \equiv (\phi \equiv (\tau \equiv (\phi \vee \tau))))))$

$\equiv \langle$ Conmutativa($\equiv$), Leibniz($\phi = (\neg(\phi \equiv p))$) $\rangle$

$(\neg(\phi \equiv ((\phi \equiv (\tau \equiv (\phi \vee \tau))) \equiv (\psi \equiv (\phi \vee \psi)))))$

$\equiv \langle$ Asociativa($\equiv$), Leibniz($\phi = (\neg(\phi \equiv p))$) $\rangle$

$(\neg(\phi \equiv (\phi \equiv ((\tau \equiv (\phi \vee \tau)) \equiv (\psi \equiv (\phi \vee \psi))))))$

$\equiv \langle$ Asociativa($\equiv$), Leibniz($\phi = (\neg p)$) $\rangle$

$(\neg((\phi \equiv \phi) \equiv ((\tau \equiv (\phi \vee \tau)) \equiv (\psi \equiv (\phi \vee \psi)))))$

$\equiv \langle$ Teo 4.15.6, Teo 4.6.4 $\rangle$

$(\neg(true \equiv ((\tau \equiv ((\neg(\neg\phi)) \vee \tau)) \equiv (\psi \equiv (\neg(\neg\phi)) \vee \psi))))$

$\equiv \langle$ Distribución($\vee, \equiv$), Leibniz($\phi = (\neg(true \equiv p))$) $\rangle$

$(\neg(true \equiv ((\tau \vee (\neg\phi)) \equiv (\psi \vee (\neg\phi)))))$

$\equiv \langle$ Teo 4.15.2, Teo 4.15.4 $\rangle$

$(false \equiv ((\tau \vee (\neg\phi)) \equiv (\psi \vee (\neg\phi))))$

$\equiv \langle$ Def($\neg$), Conmutativa($\equiv$) $\rangle$

$(\neg((\tau \vee (\neg\phi)) \equiv (\psi \vee (\neg\phi))))$

$\equiv \langle$ Distribución($\vee, \equiv$) $\rangle$

$(\neg((\neg\phi) \vee (\tau \equiv \psi)))$

$\equiv \langle$ Teo 4.25.4, Teo 4.16.6 $\rangle$

$(\phi \wedge (\neg(\tau \equiv \psi)))$

$\equiv \langle$ Conmutativa($\equiv$), Leibniz($\phi = (\phi \wedge (\neg p))$) $\rangle$

$(\phi \wedge (\neg(\psi \equiv \tau)))$

$\equiv \langle$ Def.($\equiv$), Teo 4.15.4 $\rangle$

$(\phi \wedge (\psi \not\equiv \tau))$

Por MT 4.21 y por conmutatividad de ($\equiv$) se demuestra que
$\vdash_{DS} ((\phi \wedge (\psi \not\equiv \tau)) \equiv ((\phi \wedge \psi) \not\equiv (\phi \wedge \tau)))$

8. Punto 8

$\vdash_{\mathrm{DS}} ((\phi \vee (\psi \wedge \tau)) \equiv ((\phi \vee \psi) \wedge (\phi \vee \tau)))$

$$
\begin{aligned}
& (\phi \vee (\psi \wedge \tau)) \\
\equiv \quad & \langle \text{ Def.}(\wedge)\text{, Leibniz}(\phi = (\phi \vee p)) \ \rangle \\
& (\phi \vee (\psi \equiv (\tau \equiv (\psi \vee \tau)))) \\
\equiv \quad & \langle \text{ Distribución}(\vee, \equiv) \ \rangle \\
& ((\phi \vee \psi) \equiv (\phi \vee (\tau \equiv (\psi \vee \tau)))) \\
\equiv \quad & \langle \text{ Idempotencia}(\vee)\text{, Leibniz}(\phi = ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv (p \vee (\psi \vee \tau))))) \ \rangle \\
& ((\phi \vee \psi) \equiv ((\phi \vee \tau) \equiv ((\phi \vee \phi) \vee (\psi \vee \tau)))) \\
\equiv \quad & \langle \text{ Asociativa}(\vee)\text{, Leibniz}(\phi = ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv p))) \ \rangle \\
& ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv (\phi \vee (\phi \vee (\psi \vee \tau))))) \\
\equiv \quad & \langle \text{ Asociativa}(\vee)\text{, Leibniz}(\phi = ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv (\phi \vee p)))) \ \rangle \\
& ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv (\phi \vee ((\phi \vee \psi) \vee \tau)))) \\
\equiv \quad & \langle \text{ Conmutativa}(\vee)\text{, Leibniz}(\phi = ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv (\phi \vee p)))) \ \rangle \\
& ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv (\phi \vee (\tau \vee (\phi \vee \psi))))) \\
\equiv \quad & \langle \text{ Asociativa}(\vee)\text{, Leibniz}(\phi = ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv p))) \ \rangle \\
& ((\phi \vee \psi) \equiv ((\phi \vee \psi) \equiv ((\phi \vee \tau) \vee (\phi \vee \psi)))) \\
\equiv \quad & \langle \text{ Def.}(\wedge) \ \rangle \\
& ((\phi \vee \psi) \wedge (\phi \vee \psi))
\end{aligned}
$$

Por MT 4.21 se demuestra que
$\vdash_{\mathrm{DS}} ((\phi \vee (\psi \wedge \tau)) \equiv ((\phi \vee \psi) \wedge (\phi \vee \tau)))$