

Taller 07

David Gómez



VIGILADA MINEDUCACIÓN

UNIVERSIDAD

Matemáticas
Escuela Colombiana de Ingeniería Julio Garavito
Colombia
27 de septiembre de 2022

Índice

1. Punto 1	2
2. Punto 2	2
3. Punto 3	2
4. Punto 4	3

1. Punto 1

$\vdash_{DS} ((\neg(\phi \equiv \psi)) \equiv ((\neg\phi) \equiv \psi))$

0. $((\neg(\phi \equiv \psi)) \equiv ((\phi \equiv \psi) \equiv \text{false}))$ Ax9 $[\phi := (\phi \equiv \psi)]$
1. $((\neg(\phi \equiv \psi)) \equiv (\text{false} \equiv (\phi \equiv \psi)))$ Ax2 $[\phi, \psi := (\phi \equiv \psi), \text{false}]$, Leibniz $(\phi = (\neg(\phi \equiv \psi) \equiv p))$, Ecuanimidad
2. $((\neg(\phi \equiv \psi)) \equiv ((\text{false} \equiv \phi) \equiv \psi))$ Ax1 $[\phi, \psi, \tau := \text{false}, \phi, \psi]$, Leibniz $(\phi = ((\neg(\phi \equiv \psi)) \equiv p))$, Ecuanimidad
3. $((\neg(\phi \equiv \psi)) \equiv ((\neg\phi) \equiv \psi))$ Ax9, Leibniz $(\phi = ((\neg(\phi \equiv \psi)) \equiv (p \equiv \psi)))$, Ecuanimidad

2. Punto 2

$\vdash ((\neg(\neg\phi)) \equiv \phi)$

0. $((\phi \equiv \text{true}) \equiv \phi)$ Ax3
1. $((\neg\text{false}) \equiv \text{true})$ $\vdash_{DS} ((\neg\text{false}) \equiv \text{true})$
2. $((\phi \equiv (\neg\text{false})) \equiv \text{true}) \equiv ((\phi \equiv \text{true}) \equiv \phi))$ Leibniz $(\phi = ((\phi \equiv p) \equiv \phi))(p1)$
3. $((\phi \equiv (\neg\text{false})) \equiv \phi)$ Ecuanimidad (p2, p0)
4. $((\neg\text{false}) \equiv \phi) \equiv \phi$ Ax2 $[\psi := (\neg\text{false})]$, Leibniz $(\phi = (p \equiv \phi))$, Ecuanimidad
5. $((\neg\text{false}) \equiv \phi) \equiv (\neg(\text{false} \equiv \phi))$ Teo 4.15.4 $[\phi, \psi := \text{false}, \phi]$
6. $((\neg\text{false}) \equiv \phi) \equiv \phi \equiv ((\neg(\text{false} \equiv \phi)) \equiv \phi)$ Leibniz $(\phi = (p \equiv \phi))(p5)$
7. $((\neg(\text{false} \equiv \phi)) \equiv \phi)$ Ecuanimidad (p6, p4)
8. $((\neg(\phi \equiv \text{false})) \equiv \phi)$ Ax2, Leibniz $(\phi = ((\neg p) \equiv \phi))$, Ecuanimidad
9. $((\neg(\neg\phi)) \equiv \phi)$ Ax9, Leibniz $(\phi = ((\neg p) \equiv \phi))$, Ecuanimidad

3. Punto 3

$\vdash ((\phi \neq \phi) \equiv \text{false})$

0. $((\phi \equiv \phi) \equiv \text{true})$ $\vdash_{DS} ((\phi \equiv \phi) \equiv \text{true})$
1. $((\neg\text{false}) \equiv \text{true})$ $\vdash_{DS} ((\neg\text{false}) \equiv \text{true})$
2. $((\phi \equiv \phi) \equiv (\neg\text{false})) \equiv ((\phi \equiv \phi) \equiv \text{true})$ Leibniz $(\phi = ((\phi \equiv \phi) \equiv p))(p1)$
3. $((\phi \equiv \phi) \equiv (\neg\text{false}))$ Ecuanimidad (p2, p0)
4. $((\neg\text{false}) \equiv (\phi \equiv \phi))$ Ax2 $[\phi, \psi := (\phi \equiv \phi), (\neg\text{false})]$, Ecuanimidad (p3)
5. $(\neg(\text{false} \equiv (\phi \equiv \phi)))$ Teo 4.15.4 $[\phi, \psi := (\neg\text{false}), (\phi \equiv \phi)](p4)$
6. $(\neg((\phi \equiv \phi) \equiv (\neg\text{false})))$ Ax2 $[\phi, \psi := (\phi \equiv \phi), (\neg\text{false})]$, Leibniz $(\phi = (\neg p))$, Ecuanimidad (p5)
7. $((\neg(\phi \equiv \phi)) \equiv \text{false})$ Teo 4.15.4 $[\phi, \psi := (\phi \equiv \phi), (\neg\text{false})]$, Ecuanimidad (p6)
8. $((\neg\phi) \equiv \phi) \equiv \text{false}$ Teo 4.15.4 $[\psi := \text{false}]$, Ecuanimidad (p7)
9. $((\neg\phi) \equiv \phi) \equiv (\phi \neq \phi)$ Ax10, Ax2 $[\phi, \psi := (\phi \neq \phi), ((\neg\phi) \equiv \phi)]$
10. $((\neg\phi) \equiv \phi) \equiv \text{false} \equiv ((\phi \neq \phi) \equiv \text{false})$ Leibniz $(\phi = (p \equiv \text{false}))(p9)$
11. $((\phi \neq \phi) \equiv \text{false})$ Ecuanimidad (p10, p8)

4. Punto 4

$\vdash_{DS} (((\neg false) \equiv true) \equiv (false \equiv (\neg true)))$

- | | |
|--|---|
| 0. $((false \equiv true) \equiv false)$ | $Ax3[\phi := true]$ |
| 1. $(false \equiv (false \equiv true))$ | $Ax2[\phi, \psi := (false \equiv true), true], \text{Ecuanimidad (p0)}$ |
| 2. $((false \equiv false) \equiv true)$ | $Ax1[\phi, \psi, \tau := false, false, true], \text{Ecuanimidad (p1)}$ |
| 3. $((\neg false) \equiv (false \equiv false))$ | $Ax9[\phi := false]$ |
| 4. $((\neg false) \equiv true)$ | $\text{Leibniz } (\phi = ((\neg false) \equiv p))(p2), \text{Ecuanimidad (p3)}$ |
| 5. $(false \equiv (true \equiv false))$ | $Ax1[\phi, \psi, \tau := false, true, false], Ax2, \text{Ecuanimidad (p0)}$ |
| 6. $(false \equiv (\neg true))$ | $Ax9[\phi := true], \text{Leibniz } (\phi = (\phi \equiv p)), \text{Ecuanimidad (p5)}$ |
| 7. $((false \equiv (\neg true)) \equiv true)$ | Identidad (p6) |
| 8. $(true \equiv (false \equiv (\neg true)))$ | $Ax2[\phi, \psi := (false \equiv (\neg true)), true]$ |
| 9. $((\neg false) \equiv (false \equiv (\neg true)))$ | $\text{Leibniz } (\phi = (p \equiv (false \equiv (\neg true))))(p4), \text{Ecuanimidad (p8)}$ |
| 10. $((\neg false) \equiv (false \equiv (\neg true))) \equiv true$ | Identidad (p9) |
| 11. $(true \equiv ((\neg false) \equiv (false \equiv (\neg true))))$ | $Ax2[\phi, \psi := ((\neg false) \equiv (false \equiv (\neg true))), true], \text{Ecuanimidad (p10)}$ |
| 12. $((true \equiv (\neg false)) \equiv (false \equiv (\neg true)))$ | $Ax1[\phi, \psi, \tau := true, (\neg false), (false \equiv (\neg true))], \text{Ecuanimidad (p11)}$ |
| 13. $((\neg false) \equiv true) \equiv (false \equiv (\neg true))$ | $Ax2[\phi, \psi := true, (\neg false)], \text{Leibniz } (\phi = (p \equiv (false \equiv (\neg true)))) \text{ Ec. (p12)}$ |

$\vdash_{DS} ((\phi \equiv \psi) \neq ((\neg \phi) \equiv \psi))$

- | | |
|---|---|
| 0. $((((\phi \equiv false) \equiv \psi) \equiv true) \equiv ((\phi \equiv false) \equiv \psi))$ | $Ax3[\phi := ((\phi \equiv false) \equiv \psi)]$ |
| 1. $((true \equiv ((\phi \equiv false) \equiv \psi)) \equiv ((\phi \equiv false) \equiv \psi))$ | $Ax2[\phi, \psi := (\phi \equiv false), true], \text{Lbz. } (\phi = (p \equiv ((\phi \equiv false) \equiv \psi))), \text{Ec. (p0)}$ |
| 2. $(true \equiv (((\phi \equiv false) \equiv \psi) \equiv ((\phi \equiv false) \equiv \psi)))$ | $Ax1[\phi, \psi, \tau := true, ((\phi \equiv false) \equiv \psi), ((\phi \equiv false) \equiv \psi)], \text{Ec. (p1)}$ |
| 3. $((((\phi \equiv false) \equiv \psi) \equiv ((\phi \equiv false) \equiv \psi)) \equiv true)$ | $Ax2[\phi, \psi := true, (((\phi \equiv false) \equiv \psi) \equiv ((\phi \equiv false) \equiv \psi))], \text{Ec. (p2)}$ |
| 4. $((\phi \equiv false) \equiv \psi) \equiv ((\phi \equiv false) \equiv \psi)$ | Identidad (p3) |
| 5. $((false \equiv \phi) \equiv \psi) \equiv ((\phi \equiv false) \equiv \psi)$ | $Ax2[\psi := false], \text{Lbz. } (\phi = ((p \equiv \psi) \equiv ((\phi \equiv false) \equiv \psi))), \text{Ec. (p4)}$ |
| 6. $((false \equiv (\phi \equiv \psi)) \equiv ((\phi \equiv false) \equiv \psi))$ | $Ax1[\phi, \psi, \tau := false, \phi, \psi], \text{Lbz. } (\phi = (p \equiv ((\phi \equiv false) \equiv \psi))), \text{Ec. (p5)}$ |
| 7. $((\phi \equiv \psi) \equiv false) \equiv ((\phi \equiv false) \equiv \psi)$ | $Ax2[\phi, \psi := false, (\phi \equiv \psi)], \text{Lbz. } (\phi = (p \equiv ((\phi \equiv false) \equiv \psi))), \text{Ec. (p6)}$ |
| 8. $((\neg(\phi \equiv \psi)) \equiv ((\phi \equiv false) \equiv \psi))$ | $Ax9[\phi := (\phi \equiv \psi)], \text{Lbz. } (\phi = (p \equiv ((\phi \equiv false) \equiv \psi))) \text{ Ec. (p7)}$ |
| 9. $((\phi \equiv \psi) \neq ((\phi \equiv false) \equiv \psi))$ | $Ax10[\phi, \psi := (\phi \equiv \psi), ((\phi \equiv false) \equiv \psi)], \text{Ecuanimidad (p8)}$ |
| 10. $((\phi \equiv \psi) \neq ((\neg \phi) \equiv \psi))$ | $Ax9, \text{Leibniz } (\phi = ((\phi \equiv \psi) \neq (p \equiv \psi))), \text{Ecuanimidad (p9)}$ |

$\vdash_{DS} ((\neg(\neg \phi)) \equiv (false \equiv (\neg \phi)))$

- | | |
|--|---|
| 0. $((\neg(\neg \phi)) \equiv ((\neg \phi) \equiv false))$ | $Ax9[\phi := (\neg \phi)]$ |
| 1. $((\neg(\neg \phi)) \equiv (false \equiv (\neg \phi)))$ | $Ax2[\phi, \psi := (\neg \phi), false], \text{Leibniz } (\phi = ((\neg(\neg \phi)) \equiv p)), \text{Ecuanimidad (p0)}$ |