Taller 07

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Matemáticas
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Índice

1. Punto 1

$\vdash_{\mathrm{DS}} ((\neg(\phi \equiv \psi)) \equiv ((\neg\phi) \equiv \psi))$

0. $((\neg(\phi \equiv \psi)) \equiv ((\phi \equiv \psi) \equiv \mathit{false}))$	Ax9 $[\phi := (\phi \equiv \psi)]$
1. $((\neg(\phi \equiv \psi)) \equiv (\mathit{false} \equiv (\phi \equiv \psi)))$	Ax2 $[\phi, \psi := (\phi \equiv \psi), \mathit{false}]$, Leibniz $(\phi = (\neg(\phi \equiv \psi) \equiv p))$, Ecuanimidad
2. $((\neg(\phi \equiv \psi)) \equiv ((\mathit{false} \equiv \phi) \equiv \psi))$	Ax1 $[\phi, \psi, \tau := \mathit{false}, \phi, \psi]$, Leibniz $(\phi = ((\neg(\phi \equiv \psi)) \equiv p))$, Ecuanimidad
3. $((\neg(\phi \equiv \psi)) \equiv ((\neg\phi) \equiv \psi))$	Ax9, Leibniz $(\phi = ((\neg(\phi \equiv \psi)) \equiv (p \equiv \psi)))$, Ecuanimidad

2. Punto 2

$\vdash ((\neg(\neg\phi)) \equiv \phi)$

0. $((\phi \equiv \mathit{true}) \equiv \phi)$	Ax3
1. $((\neg\mathit{false}) \equiv \mathit{true})$	$\vdash_{\mathrm{DS}} ((\neg\mathit{false}) \equiv \mathit{true})$
2. $(((\phi \equiv (\neg\mathit{false})) \equiv \mathit{true}) \equiv ((\phi \equiv \mathit{true}) \equiv \phi))$	Leibniz $(\phi = ((\phi \equiv p) \equiv \phi))$(p1)
3. $((\phi \equiv (\neg\mathit{false})) \equiv \phi)$	Ecuanimidad (p2, p0)
4. $(((\neg\mathit{false}) \equiv \phi) \equiv \phi)$	Ax2$[\psi := (\neg\mathit{false})]$, Leibniz $(\phi = (p \equiv \phi))$, Ecuanimidad
5. $(((\neg\mathit{false}) \equiv \phi) \equiv (\neg(\mathit{false} \equiv \phi)))$	Teo 4.15.4$[\phi, \psi := \mathit{false}, \phi]$
6. $((((\neg\mathit{false}) \equiv \phi) \equiv \phi) \equiv ((\neg(\mathit{false} \equiv \phi)) \equiv \phi))$	Leibniz $(\phi = (p \equiv \phi))$(p5)
7. $((\neg(\mathit{false} \equiv \phi)) \equiv \phi)$	Ecuanimidad (p6, p4)
8. $((\neg(\phi \equiv \mathit{false})) \equiv \phi)$	Ax2, Leibniz $(\phi = ((\neg p) \equiv \phi))$, Ecuanimidad
9. $((\neg(\neg\phi)) \equiv \phi)$	Ax9, Leibniz $(\phi = ((\neg p) \equiv \phi))$, Ecuanimidad

3. Punto 3

$\vdash ((\phi \not\equiv \phi) \equiv \mathit{false})$

0. $((\phi \equiv \phi) \equiv \mathit{true})$	$\vdash_{\mathrm{DS}} ((\phi \equiv \phi) \equiv \mathit{true})$
1. $((\neg\mathit{false}) \equiv \mathit{true})$	$\vdash_{\mathrm{DS}} ((\neg\mathit{false}) \equiv \mathit{true})$
2. $(((\phi \equiv \phi) \equiv (\neg\mathit{false})) \equiv ((\phi \equiv \phi) \equiv \mathit{true}))$	Leibniz $(\phi = ((\phi \equiv \phi) \equiv p))$(p1)
3. $((\phi \equiv \phi) \equiv (\neg\mathit{false}))$	Ecuanimidad (p2, p0)
4. $((\neg\mathit{false}) \equiv (\phi \equiv \phi))$	Ax2$[\phi, \psi := (\phi \equiv \phi), (\neg\mathit{false})]$, Ecuanimidad (p3)
5. $(\neg(\mathit{false} \equiv (\phi \equiv \phi)))$	Teo 4.15.4$[\phi, \psi := (\neg\mathit{false}), (\phi \equiv \phi)]$(p4)
6. $(\neg((\phi \equiv \phi) \equiv (\neg\mathit{false})))$	Ax2$[\phi, \psi := (\phi \equiv \phi), (\neg\mathit{false})]$, Leibniz $(\phi = (\neg p))$, Ecuanimidad (p5)
7. $((\neg(\phi \equiv \phi)) \equiv \mathit{false})$	Teo 4.15.4$[\phi, \psi := (\phi \equiv \phi), (\neg\mathit{false})]$, Ecuanimidad (p6)
8. $(((\neg\phi) \equiv \phi) \equiv \mathit{false})$	Teo 4.15.4$[\psi := \mathit{false}]$, Ecuanimidad (p7)
9. $(((\neg\phi) \equiv \phi) \equiv (\phi \not\equiv \phi))$	Ax10, Ax2$[\phi, \psi := (\phi \not\equiv \phi), ((\neg\phi) \equiv \phi)]$
10. $((((\neg\phi) \equiv \phi) \equiv \mathit{false}) \equiv ((\phi \not\equiv \phi) \equiv \mathit{false}))$	Leibniz $(\phi = (p \equiv \mathit{false}))$(p9)
11. $((\phi \not\equiv \phi) \equiv \mathit{false})$	Ecuanimidad (p10, p8)

4. Punto 4

$\vdash_{\mathrm{DS}} (((\neg \mathit{false}) \equiv \mathit{true}) \equiv (\mathit{false} \equiv (\neg \mathit{true})))$

0.	$((\mathit{false} \equiv \mathit{true}) \equiv \mathit{false})$	Ax3$[\phi := \mathit{true}]$
1.	$(\mathit{false} \equiv (\mathit{false} \equiv \mathit{true}))$	Ax2$[\phi, \psi := (\mathit{false} \equiv \mathit{true}), \mathit{true}]$, Ecuanimidad (p0)
2.	$((\mathit{false} \equiv \mathit{false}) \equiv \mathit{true})$	Ax1$[\phi, \psi, \tau := \mathit{false}, \mathit{false}, \mathit{true}]$, Ecuanimidad (p1)
3.	$((\neg \mathit{false}) \equiv (\mathit{false} \equiv \mathit{false}))$	Ax9$[\phi := \mathit{false}]$
4.	$((\neg \mathit{false}) \equiv \mathit{true})$	Leibniz $(\phi = ((\neg \mathit{false}) \equiv p))$(p2), Ecuanimidad (p3)
5.	$(\mathit{false} \equiv (\mathit{true} \equiv \mathit{false}))$	Ax1$[\phi, \psi, \tau := \mathit{false}, \mathit{true}, \mathit{false}]$, Ax2, Ecuanimidad (p0)
6.	$(\mathit{false} \equiv (\neg \mathit{true}))$	Ax9$[\phi := \mathit{true}]$, Leibniz $(\phi = (\phi \equiv p))$, Ecuanimidad (p5)
7.	$((\mathit{false} \equiv (\neg \mathit{true})) \equiv \mathit{true})$	Identidad (p6)
8.	$(\mathit{true} \equiv (\mathit{false} \equiv (\neg \mathit{true})))$	Ax2$[\phi, \psi := (\mathit{false} \equiv (\neg \mathit{true})), \mathit{true}]$
9.	$((\neg \mathit{false}) \equiv (\mathit{false} \equiv (\neg \mathit{true})))$	Leibniz $(\phi = (p \equiv (\mathit{false} \equiv (\neg \mathit{true}))))$(p4), Ecuanimidad (p8)
10.	$(((\neg \mathit{false}) \equiv (\mathit{false} \equiv (\neg \mathit{true}))) \equiv \mathit{true})$	Identidad (p9)
11.	$(\mathit{true} \equiv ((\neg \mathit{false}) \equiv (\mathit{false} \equiv (\neg \mathit{true}))))$	Ax2$[\phi, \psi := ((\neg \mathit{false}) \equiv (\mathit{false} \equiv (\neg \mathit{true}))), \mathit{true}]$, Ecuanimidad (p10)
12.	$((\mathit{true} \equiv (\neg \mathit{false})) \equiv (\mathit{false} \equiv (\neg \mathit{true})))$	Ax1$[\phi, \psi, \tau := \mathit{true}, (\neg \mathit{false}), (\mathit{false} \equiv (\neg \mathit{true}))]$, Ecuanimidad (p11)
13.	$(((\neg \mathit{false}) \equiv \mathit{true}) \equiv (\mathit{false} \equiv (\neg \mathit{true})))$	Ax2$[\phi, \psi := \mathit{true}, (\neg \mathit{false})]$, Leibniz $(\phi = (p \equiv (\mathit{false} \equiv (\neg \mathit{true}))))$, Ec. (p12)

$\vdash_{\mathrm{DS}} ((\phi \equiv \psi) \not\equiv ((\neg \phi) \equiv \psi))$

0.	$((((\phi \equiv \mathit{false}) \equiv \psi) \equiv \mathit{true}) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax3$[\phi := ((\phi \equiv \mathit{false}) \equiv \psi)]$
1.	$((\mathit{true} \equiv ((\phi \equiv \mathit{false}) \equiv \psi)) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax2$[\phi, \psi := (\phi \equiv \mathit{false}), \mathit{true}]$, Lbz. $(\phi = (p \equiv ((\phi \equiv \mathit{false}) \equiv \psi)))$, Ec. (p0)
2.	$(\mathit{true} \equiv (((\phi \equiv \mathit{false}) \equiv \psi) \equiv ((\phi \equiv \mathit{false}) \equiv \psi)))$	Ax1$[\phi, \psi, \tau := \mathit{true}, ((\phi \equiv \mathit{false}) \equiv \psi), ((\phi \equiv \mathit{false}) \equiv \psi)]$, Ec. (p1)
3.	$((((\phi \equiv \mathit{false}) \equiv \psi) \equiv ((\phi \equiv \mathit{false}) \equiv \psi)) \equiv \mathit{true})$	Ax2$[\phi, \psi := \mathit{true}, (((\phi \equiv \mathit{false}) \equiv \psi) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))]$, Ec. (p2)
4.	$(((\phi \equiv \mathit{false}) \equiv \psi) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Identidad (p3)
5.	$(((\mathit{false} \equiv \phi) \equiv \psi) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax2$[\psi := \mathit{false}]$, Lbz. $(\phi = ((p \equiv \psi) \equiv ((\phi \equiv \mathit{false}) \equiv \psi)))$, Ec. (p4)
6.	$((\mathit{false} \equiv (\phi \equiv \psi)) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax1$[\phi, \psi, \tau := \mathit{false}, \phi, \psi]$, Lbz. $(\phi = (p \equiv ((\phi \equiv \mathit{false}) \equiv \psi)))$, Ec. (p5)
7.	$(((\phi \equiv \psi) \equiv \mathit{false}) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax2$[\phi, \psi := \mathit{false}, (\phi \equiv \psi)]$, Lbz. $(\phi = (p \equiv ((\phi \equiv \mathit{false}) \equiv \psi)))$, Ec. (p6)
8.	$((\neg(\phi \equiv \psi)) \equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax9$[\phi := (\phi \equiv \psi)]$, Lbz. $(\phi = (p \equiv ((\phi \equiv \mathit{false})) \equiv \psi))$ Ec. (p7)
9.	$((\phi \equiv \psi) \not\equiv ((\phi \equiv \mathit{false}) \equiv \psi))$	Ax10$[\phi, \psi := (\phi \equiv \psi), ((\phi \equiv \mathit{false}) \equiv \psi)]$, Ecuanimidad (p8)
10.	$((\phi \equiv \psi) \not\equiv ((\neg \phi) \equiv \psi))$	Ax9, Leibniz $(\phi = ((\phi \equiv \psi) \not\equiv (p \equiv \psi)))$, Ecuanimidad (p9)

$\vdash_{\mathrm{DS}} ((\neg(\neg \phi)) \equiv (\mathit{false} \equiv (\neg \phi)))$

0.	$((\neg(\neg \phi)) \equiv ((\neg \phi) \equiv \mathit{false}))$	Ax9$[\phi := (\neg \phi)]$
1.	$((\neg(\neg \phi)) \equiv (\mathit{false} \equiv (\neg \phi)))$	Ax2$[\phi, \psi := (\neg \phi), \mathit{false}]$, Leibniz $(\phi = ((\neg(\neg \phi)) \equiv p))$, Ecuanimidad (p0)