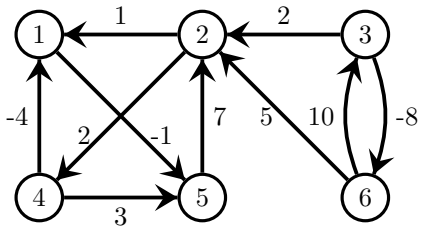
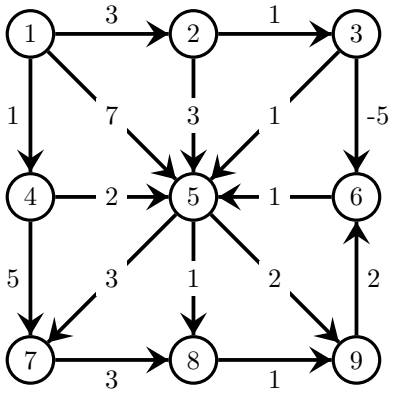
Aula Prática 5

ASA 2020/2021

Ex. 25.2-1 Run the Floyd-Warshall algorithm on the weighted, directed graph. Show the matrix $D^{(k)}$ that results for each iteration of the outer loop.



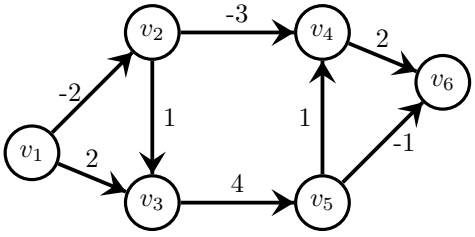
T1 16/17 I.d Considere o grafo dirigido e pesado da figura.



Considere o algoritmo Floyd-Warshall. Calcule os seguintes valores:

$d^{(2)}(1,5)$	$d^{(2)}(1,9)$	$d^{(3)}(1,5)$	$d^{(4)}(1,5)$	$d^{(5)}(1,9)$	$d^{(6)}(1,9)$

R1 08/09 II.3 Considere a execução do algoritmo de Johnson, sobre o grafo dirigido e pesado da figura abaixo. Indique o valor dos pesos dos arcos, após o procedimento de repesagem.



Ex. 25.2-4 As it appears above, the Floyd-Warshall algorithm requires $\Theta(n^3)$ space, since we compute $d_{ij}^{(k)}$ for $i, j, k = 1, \ldots, n$. Show that the following procedure, which simply drops all the superscripts, is correct, and thus only $\Theta(n^2)$ space is required.

FLOYD-WARSHALL'(W)
```
n ← rows[W]
D ← W
for k ← 1 to n do
  for i ← 1 to n do
    for j ← 1 to n do
      d_ij = min(d_ij, d_ik + d_kj)
    end for
  end for
end for
```

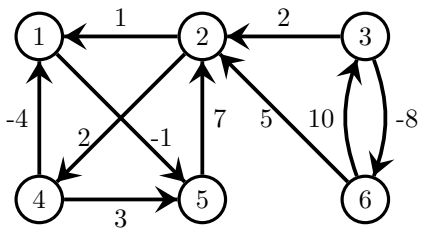
Ex. 25.2-6 How can we use the output of the Floyd-Warshall algorithm to detect the presence of a negative-weight cycle?

Ex. 25.3-3 Suppose that $w(u, v) \geq 0$ for all edges $(u, v) \in E$. What is the relationship between the weight functions w and $\hat{w}$.

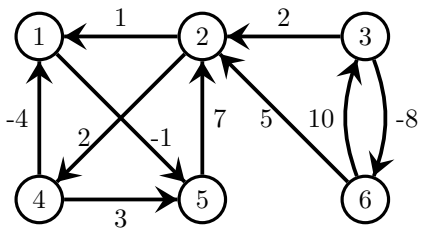
T1 08/09 II.2 Considere os algoritmos para o cálculo de caminhos mais curtos entre todos os pares de vértices. Indique se cada uma das seguintes afirmações é verdadeira (V) ou falsa (F).

1. É possível implementar o algoritmo de Floyd-Warshall por forma a que a memória necessária à sua execução seja $O(V^2)$.
2. No algoritmo de Floyd-Warshall, a matriz $D^{(k)}$ contém os custos dos caminhos mais curtos, entre todos os pares de vértices, que contenham no máximo $k-1$ arcos.
3. No algoritmo de Johnson, o valor de $h(u)$ é o mínimo entre 0 e o custo do caminho mais curto que termina no vértice u.
4. O tempo de execução do algoritmo de Johnson é $O(V^5)$.
5. Devido ao procedimento de repesagem utilizado no algoritmo de Johnson, um caminho mais curto entre dois vértices pode deixar de o ser.
6. O tempo de execução do algoritmo de Floyd-Warshall é $O(V^3)$.

Ex. 25.1-1 Run SLOW-ALL-PAIRS-SHORTEST-PATHS on the following weighted, directed graph, showing the matrices that result for each iteration of the loop. Then do the same for FASTER-ALL-PAIRS-SHORTEST-PATHS.



Ex. 25.3-1 Use Johnson's algorithm to find the shortest paths between all pairs of vertices in the graph. Show the values of h and $\hat{w}$ computed by the algorithm.



Ex. 25.3-6 Professor Michener claims that there is no need to create a new source vertex in line 1 of JOHNSON. He claims that instead we can just use $G' = G$ and let s be any vertex. Give an example of a weighted, directed graph G for which incorporating the professor's idea into JOHNSON causes incorrect answers. Then show that if G is strongly connected (every vertex is reachable from every other vertex), the results returned by JOHNSON with the professor's modification are correct.