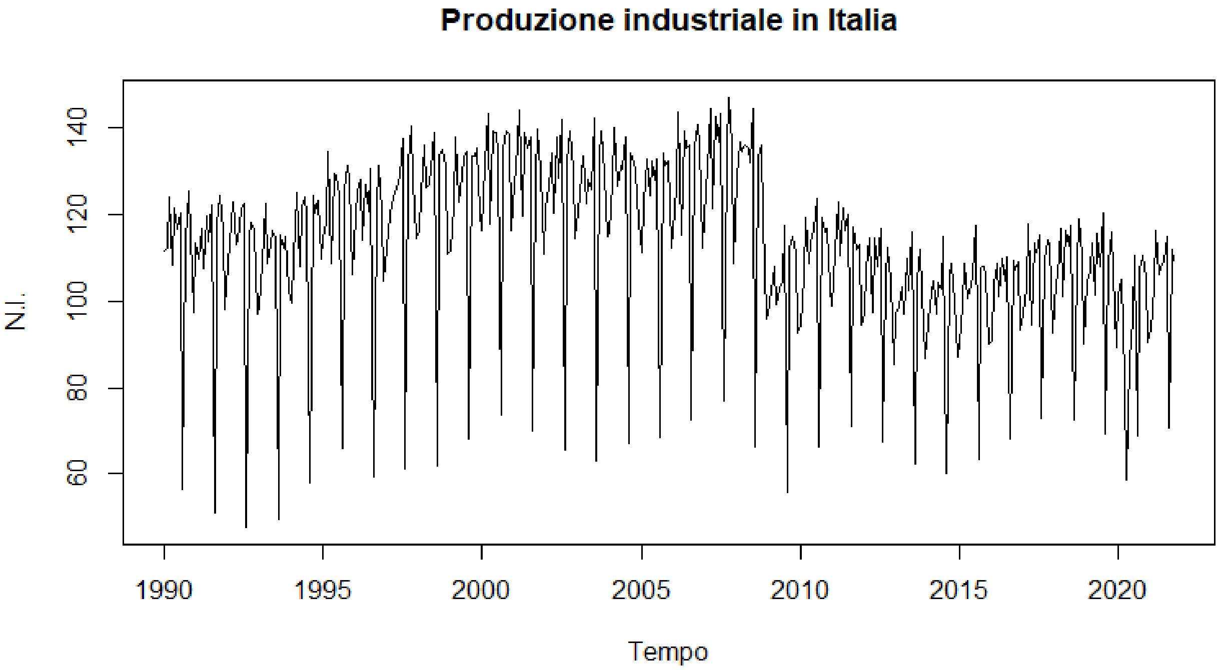
Economic Time Series Analysis

Monthly industrial production in Italy

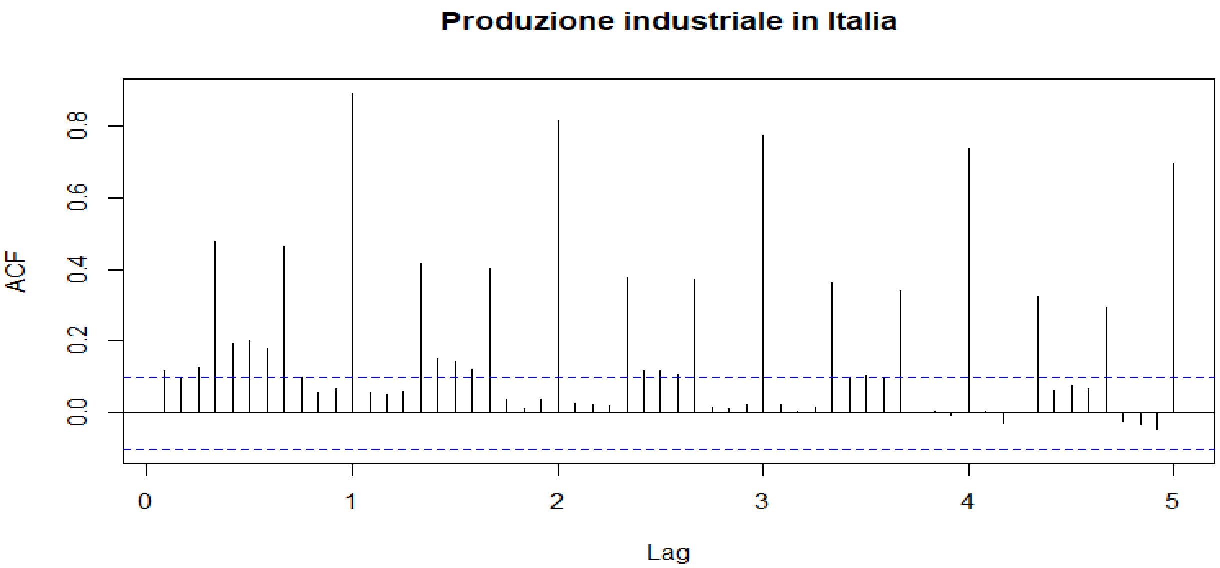
The analysis concerns theindustrial production index (base 2015 = 100) which measures the variation, over time, of the physical volume (output) of production carried out by industry. The monthly value is obtained from a specific sample survey conducted among Italian industrial companies. The time series runs from January 1990 to October 2021 (382 observations). The data can be found on the "Eurostat" website:
https://ec.europa.eu/eurostat/databrowser/view/STS_INPR_M__custom_1879177/default/table?lang=en.
The objective of this analysis is to forecast the change in the industrial production index for the next 12 months, from November 2021 to October 2022.

Preliminary analyses:

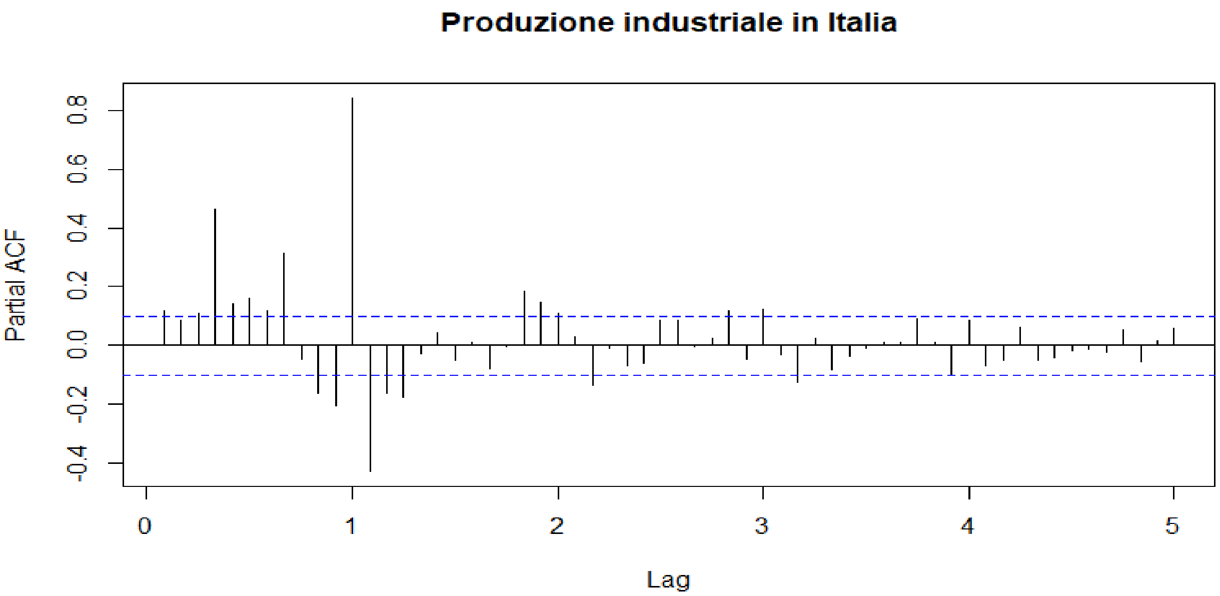


Time series plot [1]

Graph [1] depicts the time series: there is a strong annual seasonality and an increasing trend until 2008, the year of the economic crisis. Following it, the industrial production index maintained an almost constant level until the period of Covid-19.



ACF [2]



PACF [3]

Given the monthly seasonality, a seasonal ARIMA model (p, d, q)x(P, D, Q)[S] with period S=12 is used for the analysis. Graphs [2] and [3] show respectively ACF and PACF, i.e. the autocorrelation and the partial autocorrelation to the different lags which are indicated in years. Graph [2] confirms the presence of seasonality, given the decline of the ACF to lag multiples of S, while graph [3] denotes that$\phi(12)$ is close to 1: it would seem that the stochastic process is not stationary due to the seasonal part (D>0).
To be sure of this non-stationarity, the ADF (Augmented Dickey-Fuller) test is performed (Table 1).

tau3 phi2 phi3 tau2 phi1
statistic -1.762103 1.656208 2.408001 statistic -0.7810373 0.3805903
1pct 5pct 10pct 1pct 5pct 10pct
tau3 -3.98 -3.42 -3.13 tau2 -3.44 -2.87 -2.57
phi2 6.15 4.71 4.05 phi1 6.47 4.61 3.79
phi3 8.34 6.30 5.3 ADF test on raw data (1)

Accepting all tested H0, H0(1):γ=0, H0(2):ξ=γ=0, H0(3):ω=ξ=γ=0, H0(5):γ=0 , H0(6):ω=γ=0 (the last 2 H0 obtained by switching to DGP=RW eξ=0 in the model) concludes that at least one unit root is definitely present. From the data it seems that the root unit found is due to seasonality. Then, to identify other possible RUs, a new ADF test is performed on the twelfth differences (table 2):

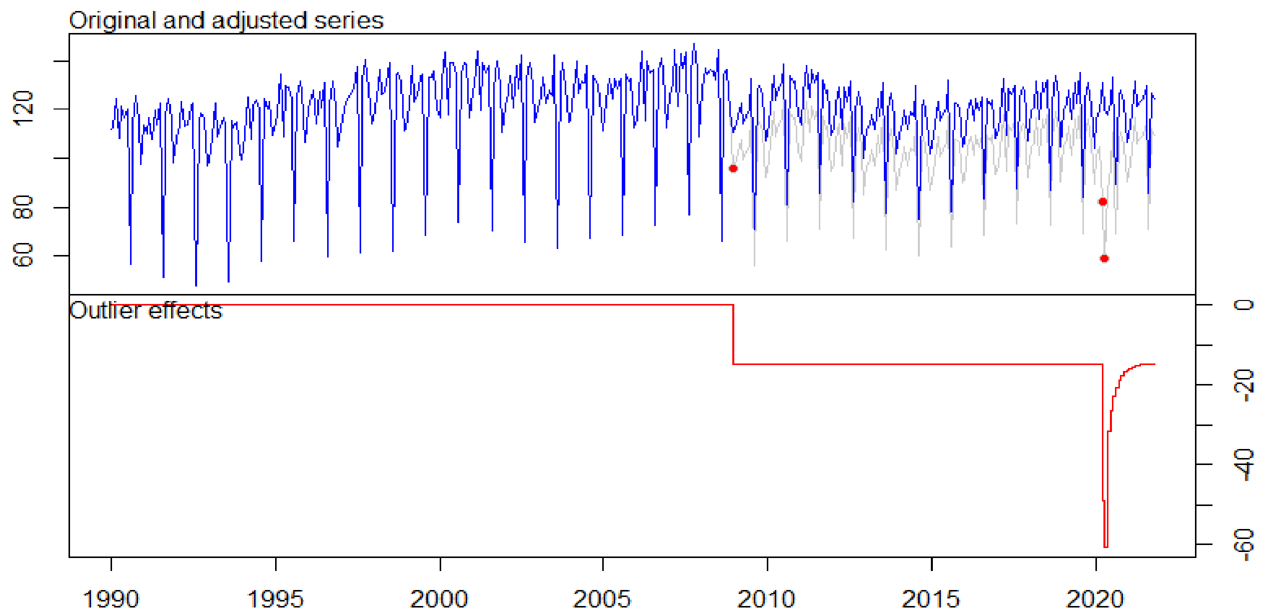
tau3 phi2 phi3
statistics -4.53685 6.879441 10.30642
1pct 5pct 10pct
tau3 -3.98 -3.42 -3.13
phi2 6.15 4.71 4.05
phi3 8.34 6.30 5.36 *ADF test on twelfth differences (2)*
From which it emerges that there are no other RUs.

ARIMA model estimation:
Based on the preliminary analyses, the starting point for ARIMA modeling is the setting of d=0 and D=1, after which we proceed by formulating p, q, P and Q. From here, based on Information Criteria, analysis of the residuals, values of the likelihood and estimation of the coefficients it is concluded that the best model without external regressors is (Table 3):
ARIMA(3,0,0)(2,1,2)[12]
coefficients:
ar1 ar2 ar3 sar1 sar2 sma1 sma2 AIC=2247.26 AICc=2247.65 BIC=2278.56
0.5646 0.1237 0.1856 0.7893 -0.7183 -1.2769 0.6847 sigma^2 estimated as 23.43: log likelihood=-1115.63
if 0.0546 0.0598 0.0531 0.0512 0.0593 0.0593 0.0518
ARIMA without external regressors (3)
By inserting external regressors, in particular calendar variables and possible anomalies, the selected model is (table 4):

Regression with ARIMA(3,0,0)(0,1,1)[12] errors
coefficients:
ar1 ar2 ar3 sma1 wd lh heh
0.8104 -0.1207 0.2034 -0.5580 0.9470 1.7404 1.7385
if 0.0520 0.0675 0.0510 0.0425 0.0429 0.4125 0.2261
sigma^2 estimated as 14.71: log likelihood=-1021.56
AIC=2059.12 AICc=2059.52 BIC=2090.43

ARIMA + calendar variables (3)

The inclusion of the "working days" (wd) variable is motivated by the fact that only Sundays and single holidays were not significant in the model with all calendar variables; the ARMA orders, on the other hand, have been changed due to refinements. Using the tso() function in the models specified so far results in the anomalies.
The model in table (4) shows ARIMA with outliers:
Regression with ARIMA(3,0,0)(2,1,2)[12] errors
coefficients:
ar1 ar2 ar3 sar1 sar2 sma1 sma2 LS228 TC363 AO364
0.2137 0.2486 0.3726 0.8482 -0.8565 -1.1612 0.7936 -14.7659 -34.0485 -22.0353
if 0.0510 0.0490 0.0525 0.0447 0.0735 0.0474 0.1045 2.2063 2.7729 3.0110
sigma^2 estimated as 13.01: log likelihood=-1006.45
AIC=2034.91 AICc=2035.64 BIC=2077.96 *ARIMA + Outliers (4)*



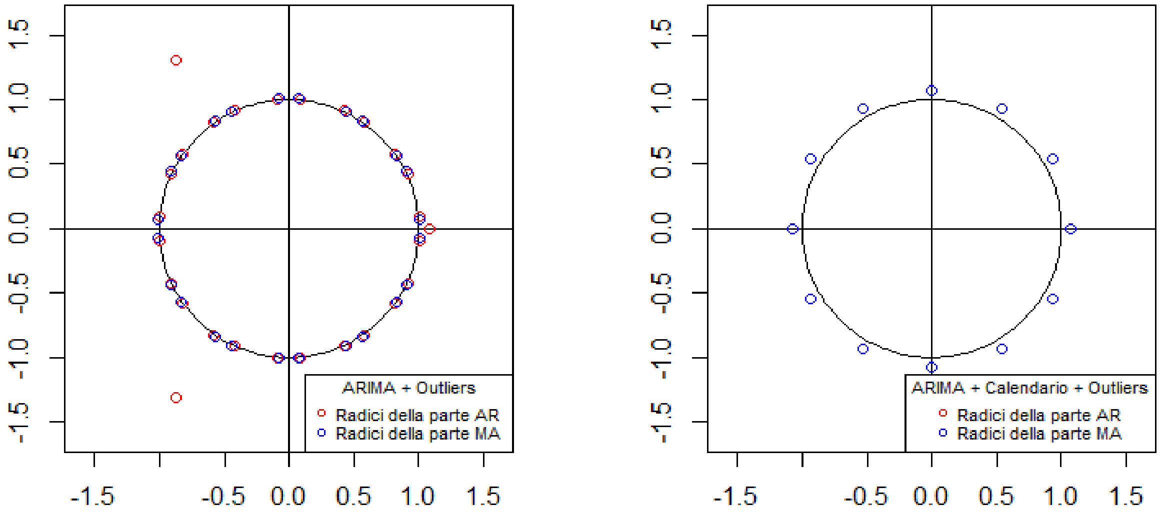
Outliers plot (ARIMA + Outliers) [4]

The graph [4] shows the presence of 3 outliers: a LS in December 2008, and a TC and an AO at the beginning of the Covid-19 pandemic (March-April 2020). Anomalies have a substantial impact because the CIs of simple ARIMA integrated with anomalies are much smaller than the model with calendar variables. Following the considerations made, it can be conjectured that the best model is ARIMA integrated with calendar variables and anomalies (Table 5):

Regression with ARIMA(3,0,0)(0,1,1)[12] errors
coefficients:
ar1 ar2 ar3 sma1 wd lh heh LS228 TC363 AO364
0.5058 0.1597 0.2329 -0.4220 0.9500 1.7838 1.7255 -13.1688 -34.7474 -21.8548
if 0.0528 0.0582 0.0519 0.0546 0.0364 0.3108 0.1873 2.0820 2.1791 2.0552
sigma^2 estimated as 7.168: log likelihood=-885.97
AIC=1793.94 AICc=1794.68 BIC=1836.99

ARIMA + calendar variables + Outliers (5)

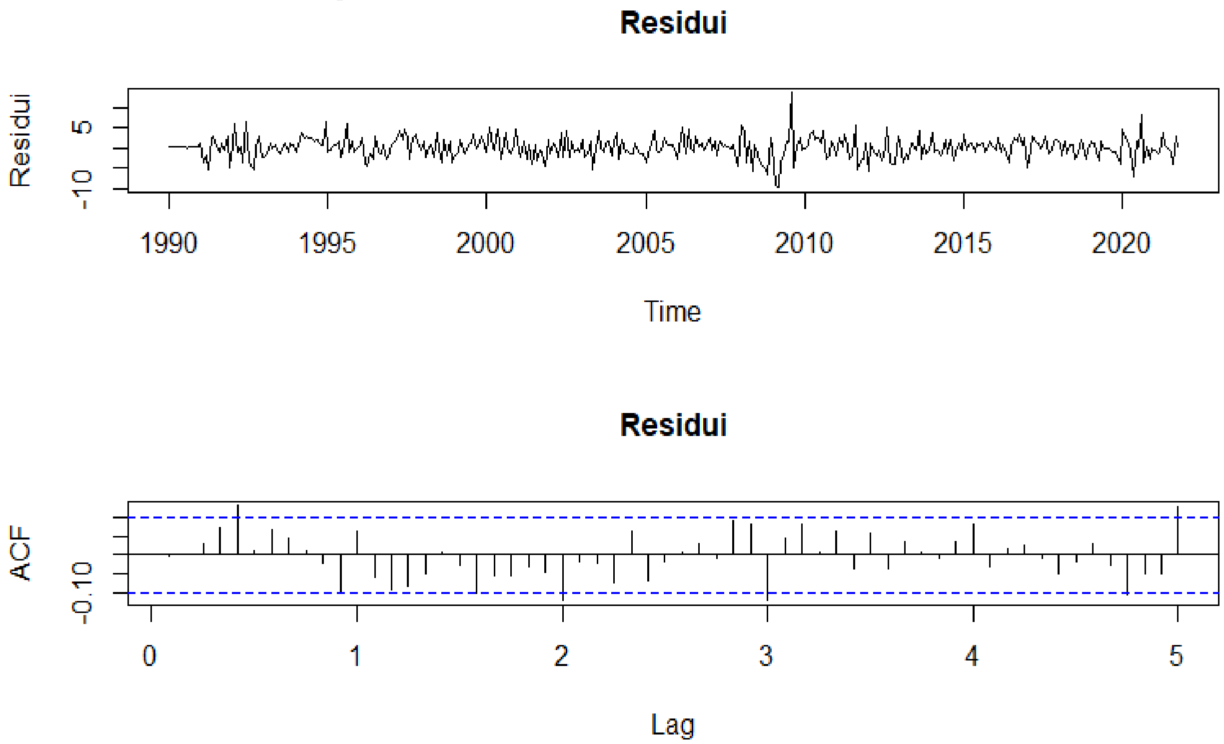
As expected, the model that fits the data best is the latter, which has the same anomalies as the model without calendar variables.



Root analysis [5]

Graph [5] shows the roots of the characteristic equations AR and MA. In the first model they are very close to each other: remodeling ARMA orders solves this problem (second plot of graph [5]).

Diagnostics:
Graph [6] shows the plot and ACF of the residuals of the integrated model with calendar variables and anomalies. It is noted that their behavior is compatible with a White Noise.



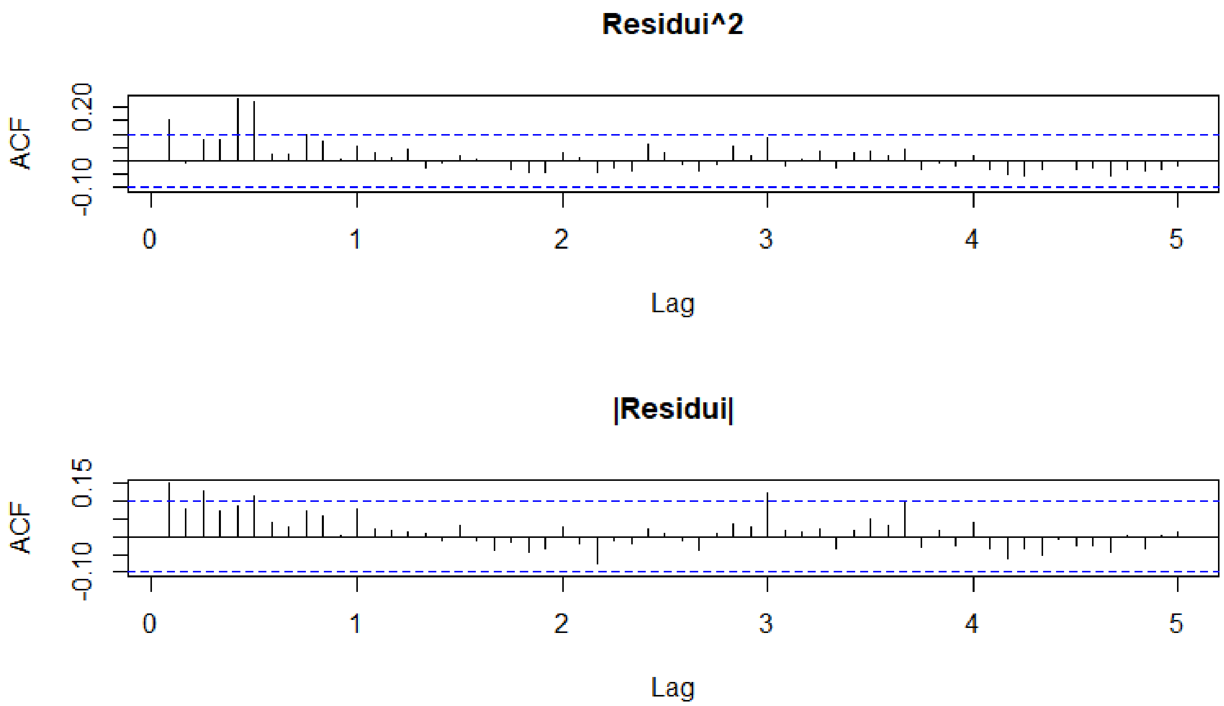
Plot and ACF of residuals [6]

Table (6) reports the Ljung-Box test:

lag1	11	12	15	20	25	30
statistics	15.90651	17.43224	24.71936	30.9941	39.22983	44.97053
parameter	1	2	5	10	15	20
p.value	6.654938e-05	0.0001639222	0.000157837	0.0005880431	0.0005920806	0.001113668

Ljung-Box test (6)

The test is highly significant at all lags, although the ACF does not show much room for improvement [6].



ACF of residues^2 and of |residues| [7]

Figure [7], which reports the ACF of squared residuals and |residuals|, leads us to think that they are generated by independent random variables. The autocorrelations are almost all within the bands.

Table (7) shows the ARCH test on |residues|:

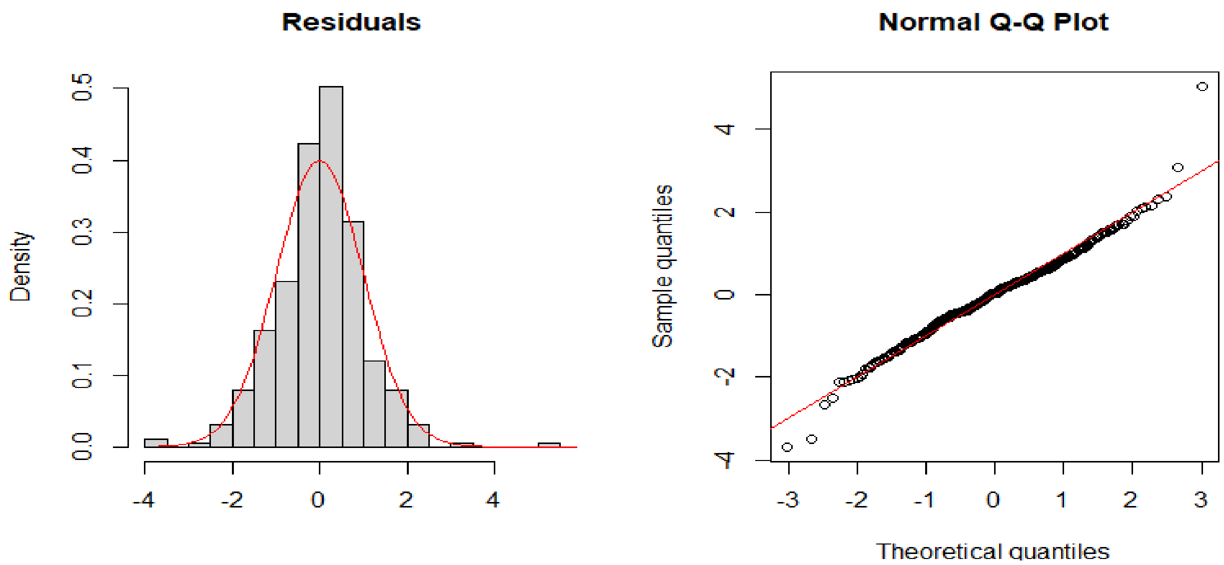
statistics	6.231912	6.62628	7.107674	52.16441	60.62085	61.81133
parameter	1	2	3	6	12	24
p.value	0.01254682	0.0364017	0.06854389	1.728112e-09	1.739086e-08	3.52958e-05

ARCH tests (7)

Rejecting the null hypothesis for almost all M past values chosen, one concludes that the conditional variance is not entirely stable; this indicates that perhaps a data transformation is needed to stabilize it. Table (8) indicates the transformation test which suggests not to transform the data:

Estimate	self	H0	if(HAC)	tstat(HAC)
-0.008419042	0.005419369	fit = 0	0.005873673	-1.433352
-0.008419042	0.005419369	fit = 1	0.005873673	-171.684578
-0.008419042	0.005419369	fit = 2	0.005873673	-341.935804

Transformation tests (8)



Normality of standardized residues [8]

Figure [8] shows the unconditioned distribution of the standardized residuals: a certain leptocurticity is denoted and therefore the Gaussian distribution does not fit the data well. In fact, the null hypothesis of normality of the residuals (W = 0.97978, p-value = 3.464e-05) is rejected in the Shapiro-Wilk normality test.

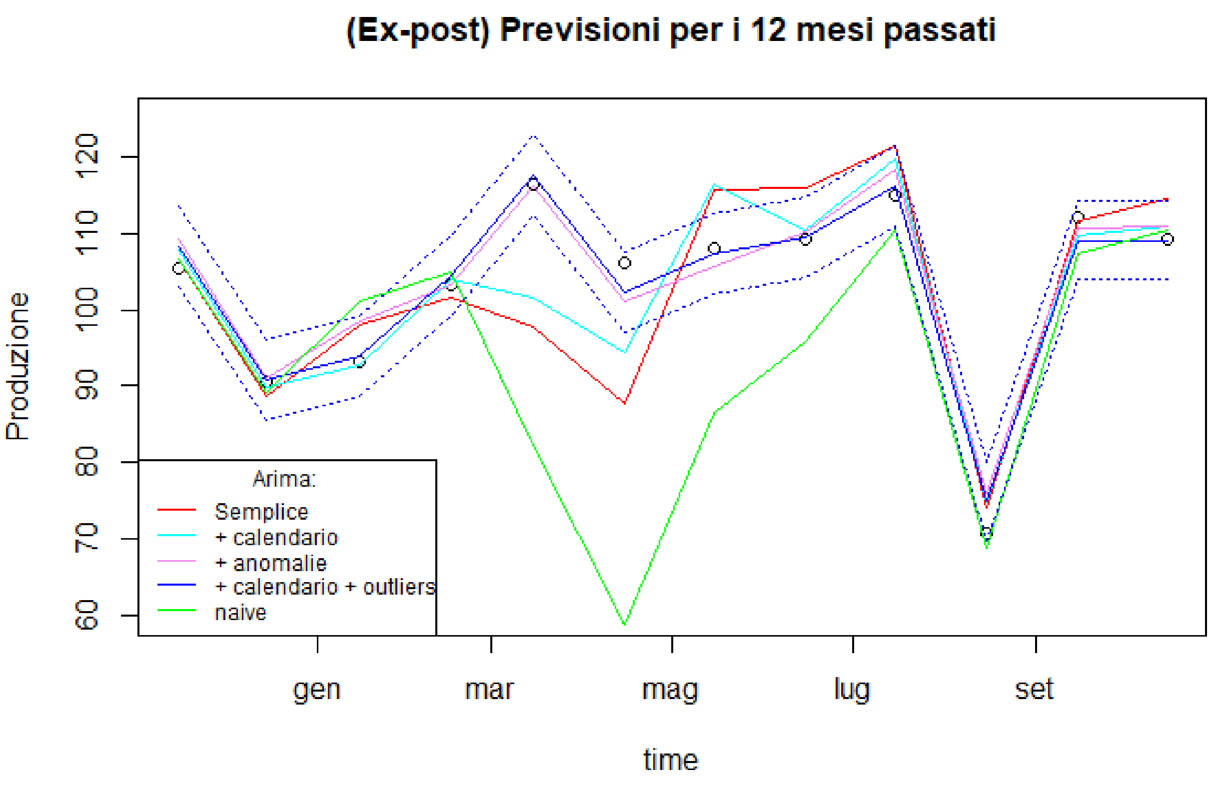
Ex-post forecast:
Measurements of forecast errors (Table 9):

model	h	ME	MAE	RMSE	MPE	MAPE	RMSPE	ScMA	ScRMSE
Arima	1	0.4965	6.3404	8.6414	0.0026	0.0598	0.0792	0.5400	0.4639
Arima + calendar	1	0.5994	4.4186	6.3374	0.0036	0.0418	0.0582	0.3763	0.3402
Arima + outliers	1	-0.9643	2.4918	3.1140	-0.0116	0.0257	0.0338	0.2122	0.1672
Arima + calendar + outliers	1	-0.3327	1.6130	2.1033	-0.0049	0.0166	0.0232	0.1374	0.1129
Naïve	1	9.7583	11.7417	18.6263	0.0886	0.1091	0.1712	1.0000	1.0000

Forecast error measures (9)

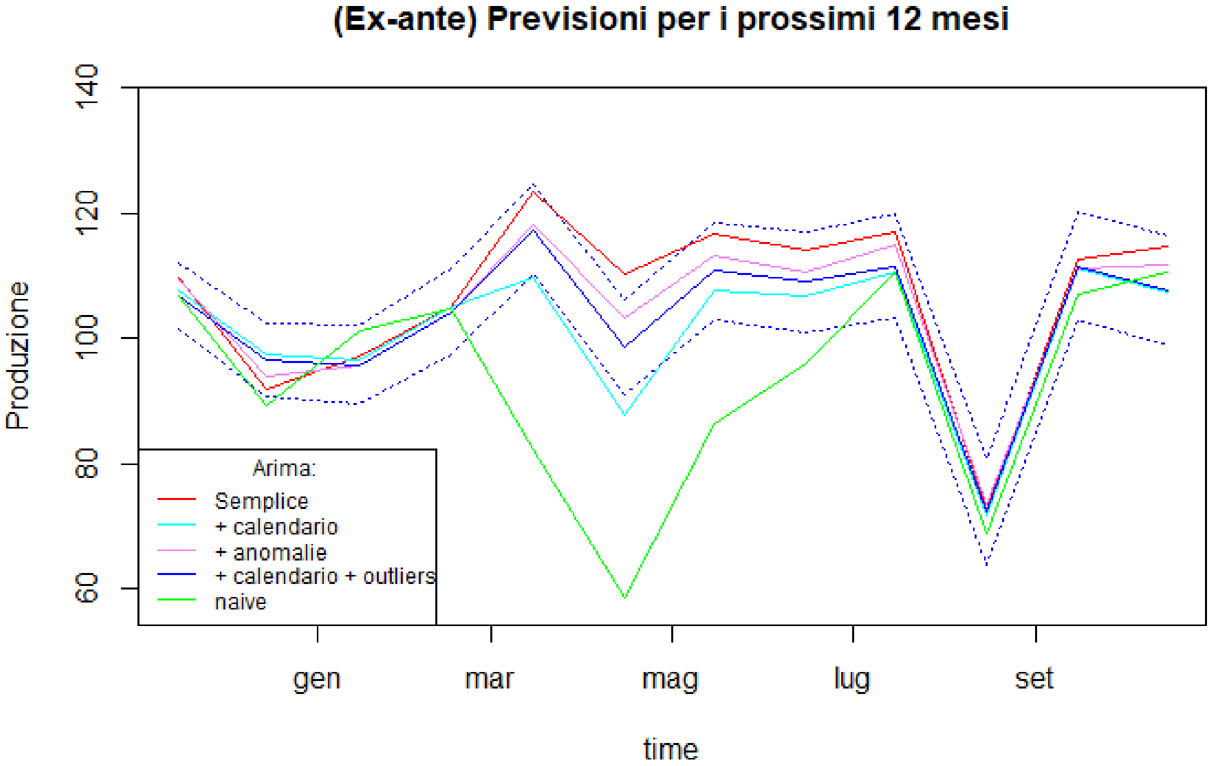
The naive in the table is a benchmark model that we take as a reference for the comparisons: in the case of the data considered, the naive behaves like a seasonal RW being the process with seasonal integration. The model that has the greatest gain in forecasting terms is the ARIMA with calendar effects and outliers because, comparing all the specified models with the "naive" one, it presents the smaller scaled measurement errors.

In the graph [9] the ex-post forecasts. It turns out that the integrated model with calendar variables and outliers is the best since the true observations are all within the prediction bands of this model.



Ex-post forecasts [9]

Ex-ante forecasts:
Graph [10] shows the ex-ante forecasts with the 5 models listed:



Ex-ante forecasts [10]

The ex-ante forecasts [10] follow similar trends (Table 10). In particular, a negative peak in production is observed around August 2022 which will probably occur.

T	pred1	pred2	pred3	pred4	naïve
383	109.64	107.69	109.37	106.81	106.7
384	92.05	97.59	94.12	96.67	89.2
385	97.24	96.46	95.71	95.74	101.2
386	104.99	105.18	104.17	104.28	104.9
387	123.54	109.83	118.21	117.46	82.2
388	110.36	87.77	103.28	98.54	58.8
389	116.63	107.65	113.22	110.77	86.5
390	114.03	106.94	110.75	109.05	95.9
391	117.02	110.50	115.01	111.61	110.5
392	72.75	71.65	73.50	72.33	68.8
393	112.57	111.16	111.28	111.54	107.2
394	114.80	107.53	111.89	107.62	110.5

Ex ante forecasts (10)

Legend:
pred1: Simple ARIMA
pred2: ARIMA + calendar
pred3: simple ARIMA + anomalies
pred4: ARIMA + calendar + outliers