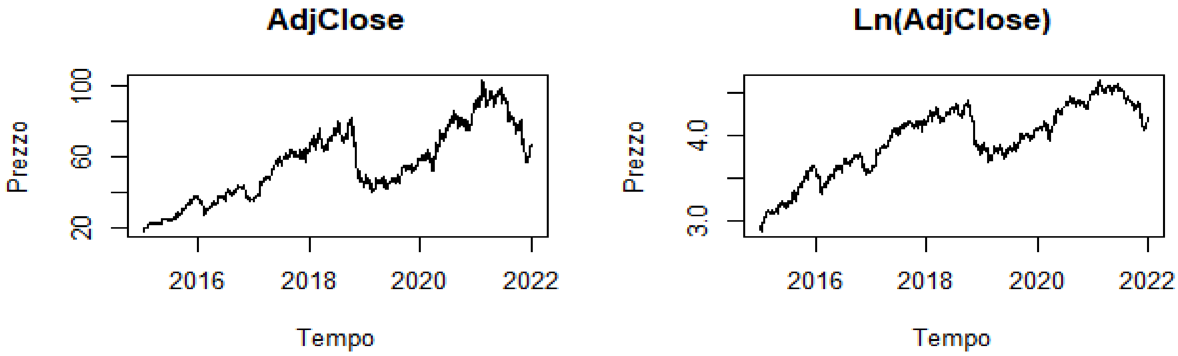
Financial Time Series Analysis:

Activision Blizzard, Inc. (ATVI)

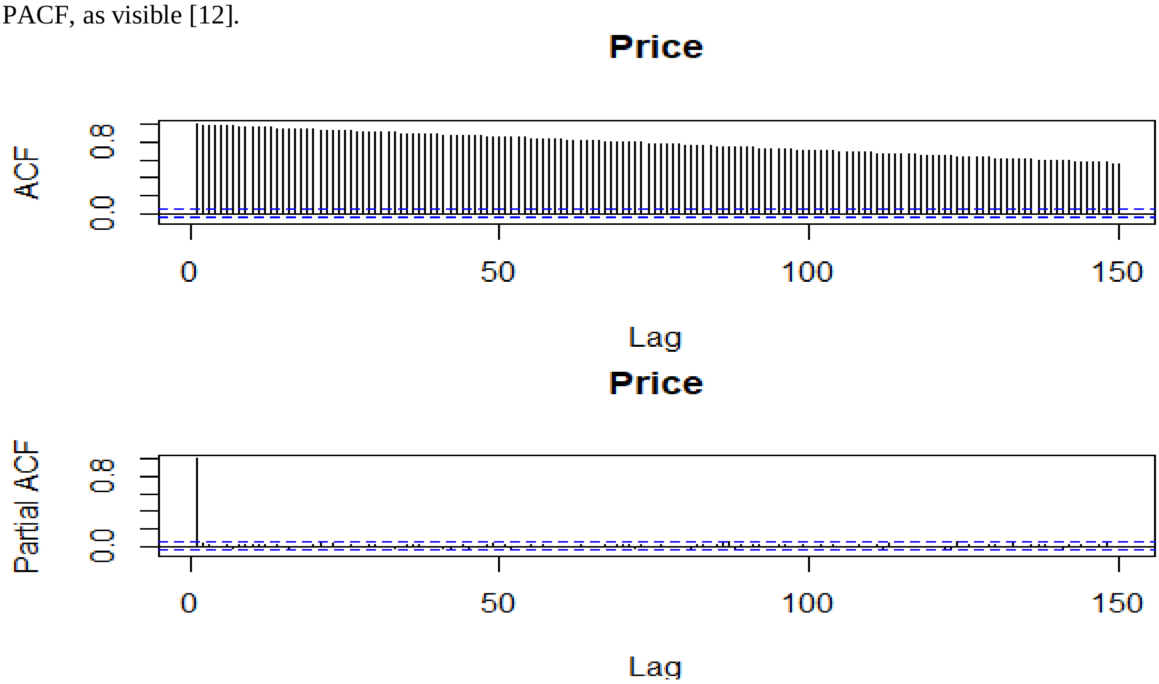
The analysis concerns the listing of Activision Blizzard, Inc. (ATVI), a US company that produces and distributes video games. The data, on a daily basis, can be found on Yahoo Finance (https://it.finance.yahoo.com/quote/ATVI?p=ATVI&.tsrc=fin-srch) range from 5 January 2015 to 31 December 2021 (1762 observations). The available variables are "open", the opening prices, "close", the closing prices, "high", the highest price, "low", the lowest price, "volume", number of shares traded, and "adjusted", i.e. the "close" prices adjusted for dividends. The objective of the analysis is to forecast volatility in the next 10 days from 1 January 2022 to 12 January 2022 (excluding Saturdays and Sundays).

Preliminary analyses:



Adjusted vs Ln(Adjusted) [11]

Figure [11] shows the trend of "adjusted" prices, the one on the right is in a logarithmic scale often preferred because it better captures the variations. A growing trend emerges interrupted by negative phases in 2019 and 2021.
The data appear to be non-stationary, a fact confirmed by the linear decay of the ACF and by $\phi(1)$ close to 1 in the PACF, as visible [12].



ln(Adjusted) ACF and PACF [12]

The ADF test confirms the presence of a RU (table 11):

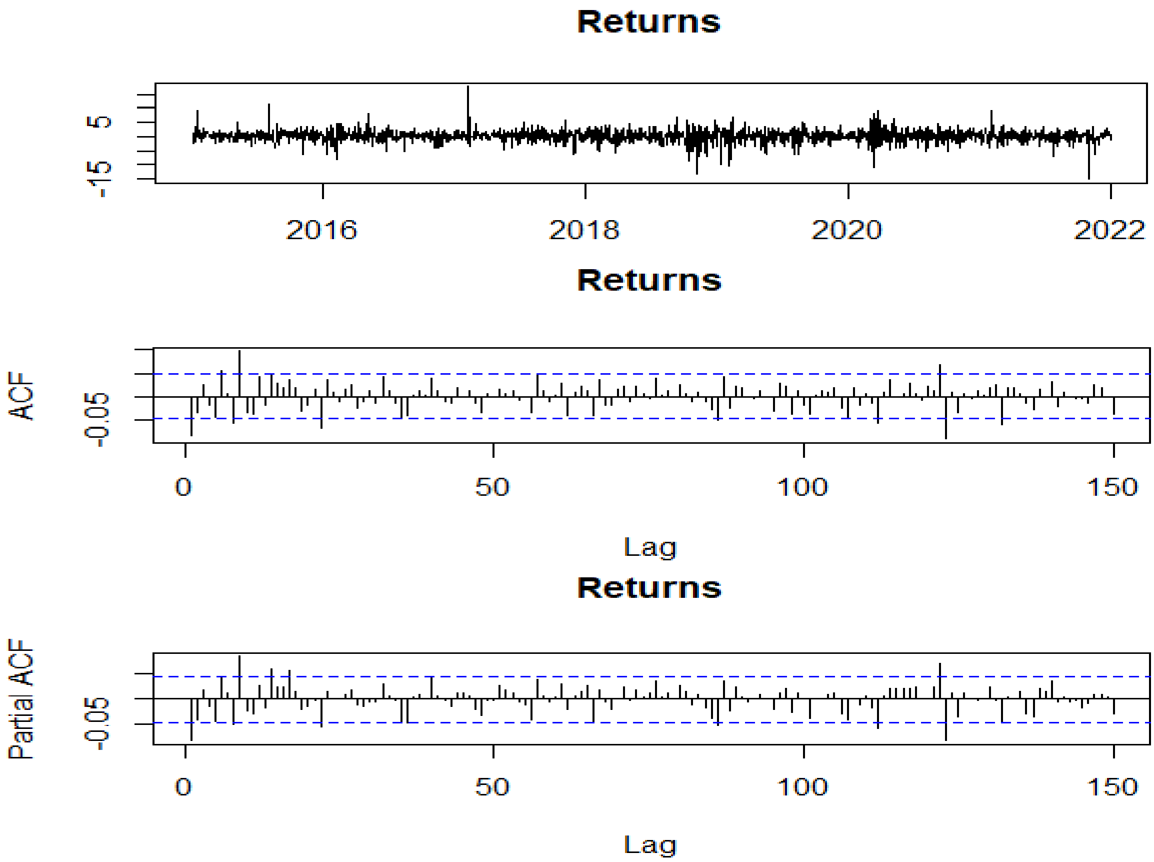
	tau3	phi2	phi3
statistics	-2.191422	2.60629	3.139543

	1pct	5pct	10pct
tau3	-3.96	-3.41	-3.12
phi2	6.09	4.68	4.03
phi3	8.27	6.25	5.34

	tau2	phi1
statistic	-2.195816	3.180493

	1pct	5pct	10pct
tau2	-3.43	-2.86	-2.57
phi1	6.43	4.59	3.78

ADF Tests (11)



Ts plot, ACF and PACF of log-returns [13]

Figure [13] shows yield plots with the corresponding ACF and PACF.The log-returns values seem to behave like a WN (in fact ACF and PACF are similar) with mean 0 with the difference that the volatility varies over time.The best ARMA on returns is ARMA(1,0) (table 12) with standardized student t distribution of errors [figure 16].

Robust matrix of ARMA(1,0) model coefficients:

	Estimate	Std.Error	t value	Pr(>|t|)	Information Criteria:
mu	0.14496328	0.03715019	3.902087	9.536679e-05	Akaike 4.134797
ar1	-0.08989513	0.02607008	-3.448210	5.643141e-04	Bayes 4.147225
sigma	2.13518543	0.12689134	16.826880	0.000000e+00	
shape	3.59854165	0.35343093	10.181740	0.000000e+00	

WEAPON(1,0) (12)

In order to model volatility, various GARCH-type models are adopted starting from simple-GARCH (sGARCH), the simplest (Table 13).

Robust matrix of sGARCH model coefficients (1, 1):

	Estimate	Std.Error	t value	Pr(>|t|)	Information criteria:
					Akaike 4.083470
mu	0.16031128	0.03538001	4.531126	5.867024e-06	Bayes 4.102111
ar1	-0.06724427	0.02424906	-2.773067	5.553069e-03	
omega	0.32272135	0.11298132	2.856413	4.284569e-03	
alpha1	0.10332267	0.03640286	2.838312	4.535284e-03	
beta1	0.82617634	0.05162429	16.003637	0.000000e+00	
shape	4.27591533	0.47097146	9.078927	0.000000e+00	

sGARCH(1,1) (13)

CIs are better than ARMA(1,0) and all coefficients are highly significant. Before moving on to the GJR-GARCH specification, a model analogous to the sGARCH with the addition of a term that presents a possible asymmetric behavior of the conditional variance, it is necessary to test that the data actually present this asymmetric component with the tests of signs (Table 14)

	t value	prob		t value	prob
Sign bias	0.6318618	0.5275594	Positive sign bias	0.5356643	0.5922583
Negative sign bias	0.3560575	0.7218402	Joint Effect	0.4823431	0.9227531

Sign Tests (14)

Table (14) seems to exclude an asymmetry effect. The estimated GJR-GARCH(1,1) (Table 15) contradicts this result: it has slightly lower CIs than the sGARCH(1,1) model.

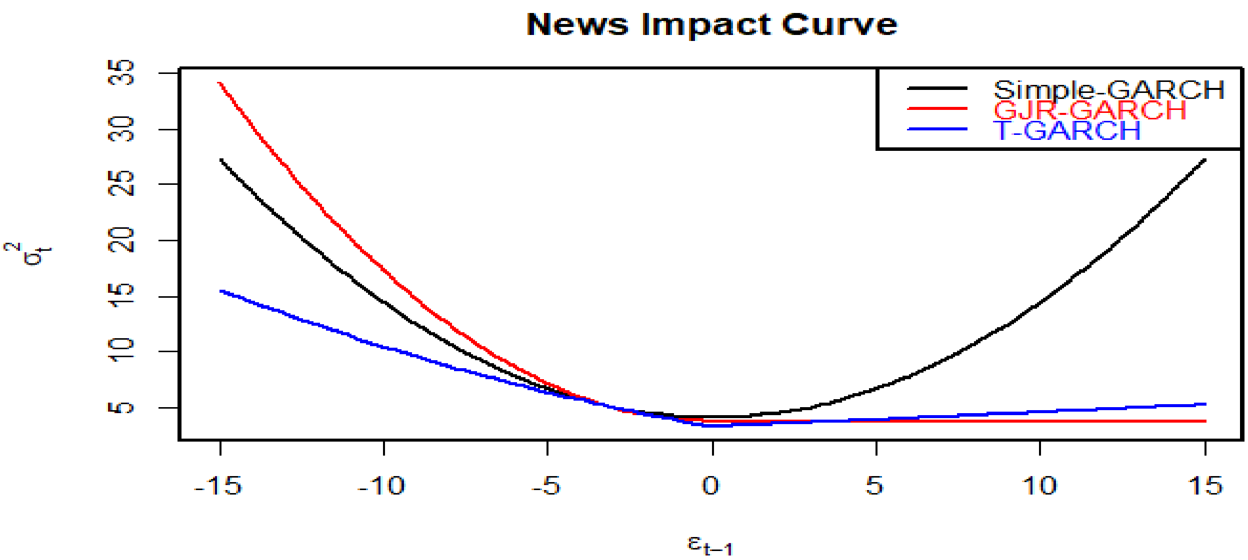
	Estimate	Std.Error	t value	Pr(>|t|)	Information criteria:
mu	1.408501e-01	3.572732e-02	3.9423650	8.068206e-05	Akaike 4.068428
ar1	-7.918403e-02	2.495840e-02	-3.1726398	1.510598e-03	Bayes 4.090175
omega	2.219963e-01	9.691699e-02	2.2905815	2.198763e-02	
alpha1	3.424963e-08	2.505195e-07	0.1367144	8.912565e-01	
beta1	8.773602e-01	3.837333e-02	22.8637996	0.000000e+00	
range1	1.352935e-01	4.131032e-02	3.2750540	1.056418e-03	
shape	4.326810e+00	4.538131e-01	9.5343427	0.000000e+00	GJR-*GARCH(1,1) (15)*

The coefficient associated with the "leverage" effect (gamma1) is highly significant (see also the NIC (News Impact Curve) in figure [14]).

Obtained as a special case of the Family-GARCH, the T-GARCH model is estimated (Table 16):

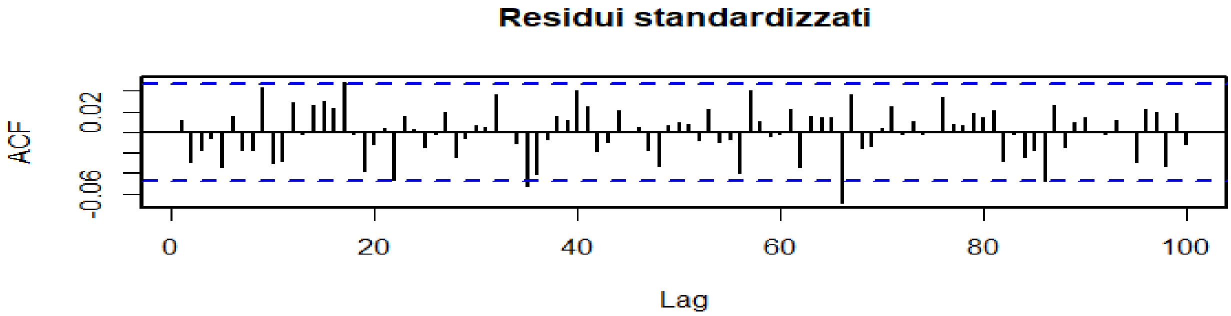
	Estimate	Std.Error	t value	Pr(>|t|)	Information criteria:
mu	0.13383993	0.03560369	3.759159	0.0001704856	Akaike 4.062516
ar1	-0.07171283	0.02209787	-3.245237	0.0011735280	Bayes 4.084263
omega	0.10139953	0.03300704	3.072058	0.0021258859	
alpha1	0.03208596	0.02409900	1.331423	0.1830499650	
beta1	0.88464996	0.02909388	30.406738	0.0000000000	
range1	0.11006211	0.02859328	3.849230	0.0001184897	
shape	4.46090036	0.48836320	9.134391	0.0000000000	*T-GARCH(1,1) (16)*

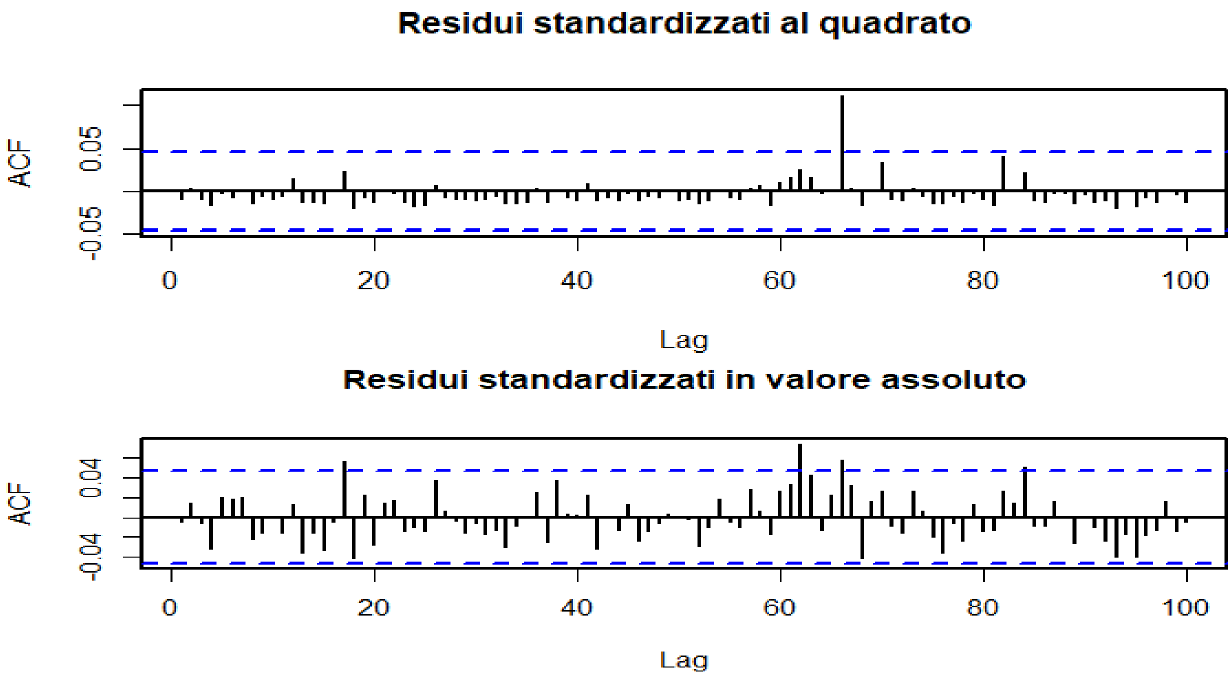
Usually this model is the one that best fits the data: in this case we observe even lower CIs than the GJR-GARCH. Graph [14] shows the NIC of the estimated GARCH models: the curves show the impact of Ut-1 news versus the conditional variance. In sGARCH the conditional variance does not react asymmetrically with respect to Ut-1, the opposite of what happens in the other two models.



NIC (News Impact Curve) [14]

Figure [15] shows the ACF of the standardized residuals of the T-GARCH: it is noted that they are not correlated and the autocorrelations are always almost all within the acceptance bands.





Residue ACF [15]

The outcome of the Ljung-Box and ARCH tests can be predicted from the ACFs in figure [15] (first H autocorrelations essentially null at all lags and very stable variance), since the autocorrelations are found within the bands except for some lags: facts confirmed by table (17).

Ljung-Box on standardized residues:

lag	8	9	12	17	22	27
statistics	5.79413	9.192854	13.66523	21.8104	28.4878	30.04746
parameter	1	2	5	10	15	20
p.value	0.01607977	0.01008781	0.01788173	0.01610019	0.01870755	0.06908835

Ljung-Box on standardized squared residuals:

lag	8	9	12	17	22	27
statistics	1.273935	1.320364	1.868456	3.820555	4.885224	6.459754
parameter	1	2	5	10	15	20
p.value	0.2590297	0.5167572	0.8670308	0.9550791	0.9930544	0.9981168

Ljung-Box on standardized residuals in absolute value:

lag	8	9	12	17	22	27
statistics	5.073308	5.492713	6.245992	16.56302	22.65442	26.12174
parameter	1	2	5	10	15	20
p.value	0.02429697	0.06416119	0.2830134	0.08460998	0.09175785	0.1618287

ARCH tests:

statistics	0.7946968	1.23498	1.852772	2.864086
parameter	4	8	12	16
p.value	0.9391576	0.9962842	0.9996005	0.9998758

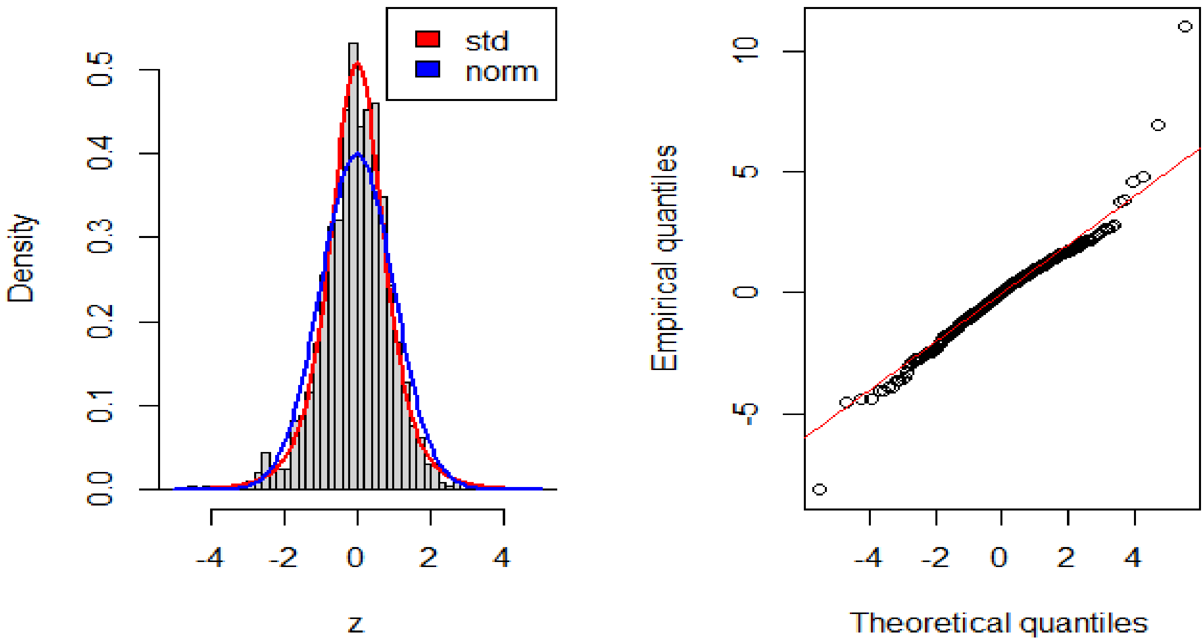
Ljung-Box and ARCH test (17)

In table (18) the Nyblom test, to verify the stability of the parameters throughout the period considered:

Individual test statistics:		Individual critical values:	Joint test statistic:	Joint Critical Values:
mu	0.0765550	10% 5% 1%	1.289422	10% 5% 1%
ar1	0.1128225	0.353 0.470 0.748		1.69 1.90 2.35
omega	0.3434348			
alpha1	0.2408769			
beta1	0.3300493			
age11	0.6133600			
shape	0.2500094			

Nyblom test (18)

The test, obtained after shortening the series from 2011-2021 (unstable parameters were obtained) to 2015-2021, shows that in general all the parameters are stable.Figure [16] shows the unconditioned distribution of the standardized residuals, from which it can be seen that they fit better to a standardized student t than to a normal one.



Unconditional distribution of the standardized residuals [16]

The BDS test (Table 19) tests the null hypothesis that the residuals are generated by random variables IID:

Standard Normal					p-value				
	[0.5798]	[1.1596]	[1.7394]	[2.3191]		[0.5798]	[1.1596]	[1.7394]	[2.3191]
[2]	-1.3038	-0.6705	-0.1822	-0.2915	[2]	0.19230.5026		0.8554	0.7707
[3]	-1.1255	-0.3828	0.0683	-0.2773	[3]	0.26040.7019		0.9455	0.7815
[4]	-0.6237	-0.0215	0.3061	-0.1418	[4]	0.53290.9829		0.7595	0.8872

BDS Tests (19)

The null hypothesis of the test is confirmed.

Finally the I-GARCH (1, 1), which by construction is stationary in the strong sense but not in the weak sense.

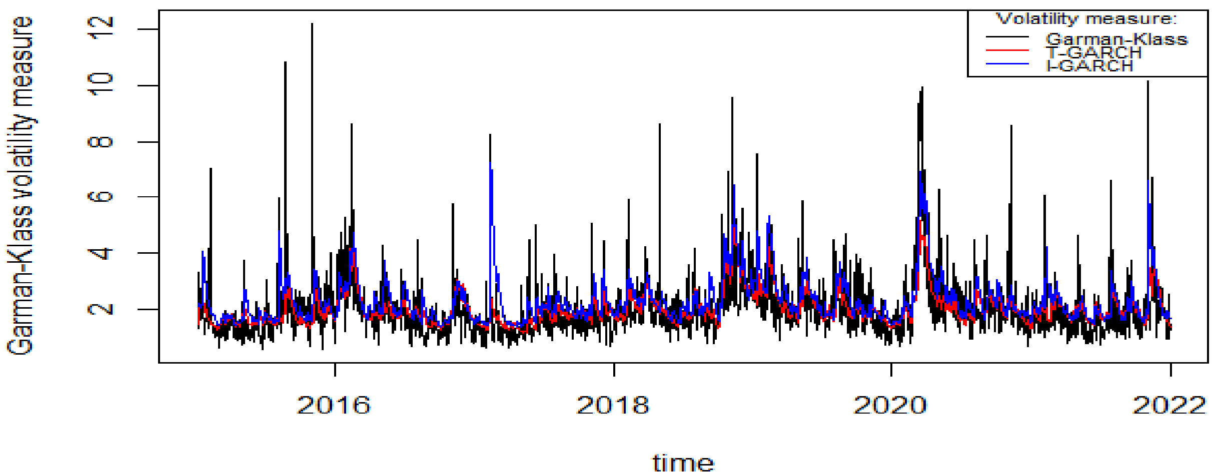
	Estimate	Std.Error	t value	Pr(>|t|)	Information Criteria
mu	0.1668748	0.03861495	4.321508	1.549665e-05	Akaike 4.093164
omega	0.2455451	0.10569714	2.323100	2.017376e-02	Bayes 4.105591
alpha1	0.1698579	0.04670338	3.636951	2.758840e-04	
beta1	0.8301421	NA	NA	NA	
shape	3.5209391	0.31478619	11.185176	0.000000e+00	

I-GARCH(1,1) (20)

Table (20) shows slightly higher ICs than the three GARCH models presented. The I-GARCH has a persistence of alpha1 + beta1 = 0.8301421 + 0.1698579 = 1.

Ex-post forecast:

To compare the results, the Garman-Klass measure is used as a benchmark, a measure of volatility that uses the four price measures "open", "close", "high" and "low".



Garman-Klass volatility measure [17]

In general, the two models seem to follow the proxy in figure [17] quite well, there is some slight more marked difference only towards mid-2017 and at the peak of 2020. To evaluate which model is the best among the four, in the table (21) the comparison of the error measures for both volatility and variance:

Measure	model	MYSELF	BUT IT IS	RMS extension	MPE extension	MAPS	RMSPE extension	ScMAE	scRMSE
Volatility	GARCH	-0.0309	0.6638	1.0431	-0.1882	0.3620	0.5066	0.8659	0.9116
Volatility	GJRGARCH	-0.0284	0.6610	1.0496	-0.1748	0.3520	0.4884	0.8623	0.9173
Volatility	TGARCH extension	-0.0040	0.6401	1.0156	-0.1595	0.3386	0.4569	0.8350	0.8876
Volatility	IGARCH extension	-0.2819	0.7640	1.1214	-0.3151	0.4415	0.6295	0.9966	0.9800
Variance	GARCH	0.8663	3.4716	9.0957	-0.6330	0.9181	1.6171	0.9096	0.9438
Variance	GJRGARCH	0.7742	3.5075	9.1830	-0.5880	0.8782	1.5050	0.9190	0.9529
Variance	TGARCH extension	0.9480	3.3568	8.9925	-0.5277	0.8209	1.3027	0.8795	0.9331
Variance	IGARCH extension	-0.5523	4.0805	9.2493	-1.0265	1.2352	2.3172	1.0691	0.9597

Error Measurements (21)

With respect to volatility, the T-GARCH seems to be the best (it is a model built specifically on volatility), while with respect to variance things are not very clear. It is noted that many ME and all MPE are negative therefore forecasts will probably tend to be higher than the true values. Having so many observations available, it is possible to make other checks such as the Diebold-Mariano test which compares two models at a time, evaluating which is the best among them.

Volatility:
GJR-GARCH vs T-GARCH -> Horiz: 1 , Loss fct pow: 1 , Stat (L1-L2): 4.585386, p-value = 4.851e-06
GARCH vs T-GARCH -> Horiz: 1 , Loss fct pow: 2 , Stat (L1-L2): 2.998648, p-value = 0.00275

Conditional variance:
GJR-GARCH vs T-GARCH -> Horiz: 1 , Loss fct pow: 1 , Stat (L1-L2): 4.769405, p-value = 2e-06
GARCH vs T-GARCH -> Horiz: 1 , Loss fct pow: 2 , Stat (L1-L2): 1.984773, p-value = 0.04732
It is concluded that the best model is the T-GARCH.

The Mincer-Zarnowitz test in table (22) checks whether an estimated model produces unbiased predictions. For the GARCH, GJR-GARCH and T-GARCH models the predictions do not appear biased, while for the IGARCH the F statistic in the Joint test is rejected.

T-GARCH:

	estimates	HAC.se	HAC.tstat	HAC.pvalue
Intercept	0.1983835	0.2758210	0.7192473	4.720840e-01
fit	0.8998397	0.1452298	6.1959723	7.196606e-10

(HAC) F stat: 0.285455 , df: (2 , 1760), p-value: 0.75170

IGARCH:

	Estimate	HAC.se	HAC.tstat	HAC.pvalue
Intercept	0.6468953	0.2787662	2.320565	2.042411e-02
fit	0.5959210	0.1299054	4.587348	4.806193e-06

(HAC) F stat: 27.92452 , df: (2 , 1760), p-value: 1.150737e-12

Mincer-Zarnowitz test (22)

Ex-ante forecasts:

Figure [18] shows the forecast of returns and volatility for the 10 days following 31 December 2021 in which the financial markets are open. In table (23) the forecast values.

	Return forecasts (%):	Volatility forecasts:
T[0] = 2021-12-31		
T+1	0.2462	1,524
T+2	0.1258	1,545
T+3	0.1344	1,566
T+4	0.1338	1,585
T+5	0.1338	1.603
T+6	0.1338	1,621
T+7	0.1338	1,637
T+8	0.1338	1,653
T+9	0.1338	1,667
T+10	0.1338	1,681

Ex-ante forecast values (23)