Cap. 3.1 → Lei de Coulomb e Campo elétrico

1) $m = 10{,}0\,g$ $F = 1{,}00 \times 10^4\,N$

$r = 1{,}00\,cm$

$$F = K \times \frac{qq'}{r^2} \Leftrightarrow 1{,}00 \times 10^4 = 8{,}99 \times 10^9 \times q_1 q_2$$

$$\Leftrightarrow q_1 q_2 = 1{,}11 \times 10^{-6}\ (C^2)$$ $e = 1{,}6022 \times 10^{-19}\,C$

$$\#\ \text{átomos esfera}/\text{gm} = \frac{6{,}022 \times 10^{23}}{107{,}87} = 5{,}58 \times 10^{21}\ \text{átomos/gm}$$

$\underset{\times 10}{\Rightarrow}$ $5{,}58 \times 10^{22}$ átomos $\underset{\times 47}{\Rightarrow}$ $2{,}62 \times 10^{24}$ electrões ✓ (comprimento!)

2) $r = 0{,}30\,m$ $\qquad q_1 = 12{,}0\,\mu C$

$q_2 = -18{,}0\,\mu C$

a)

$$F = k_e \times \frac{q_1 q_2}{r^2} = 8{,}99 \times 10^9 \times \frac{12 \times 18}{0{,}30^2} \times (10^{-9})^2$$

$$= 2{,}16 \times 10^{-5}\ N \quad \text{atrativa}$$

b) $-18 + 12 = -6\,nC$

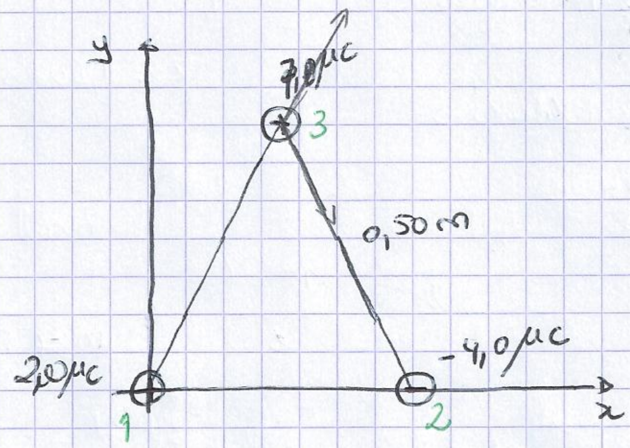
$6 : 2 = 3$

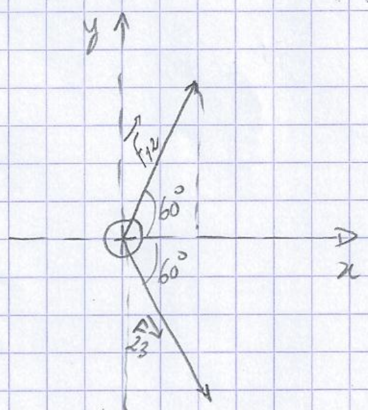
$-3nC \qquad -3nC \qquad$ Repulsiva

$q = -3 \times 10^{-9}\,C$

$$F = k_e \times \frac{q^2}{r^2} = 8{,}99 \times 10^9 \times \frac{(-3\times10^{-9})^2}{0{,}3^2} = 8{,}99 \times 10^{-7}\ N$$

3) Trig. eq.





$$F_{12} = k_e \frac{2\times10^{-6} \times 7\times10^{-6}}{0{,}5^2} = 0{,}503\ J$$

$$F_{23} = k_e \frac{7\times10^{-6} \times 4\times10^{-6}}{0{,}5^2} = 1{,}007\ J$$

logo

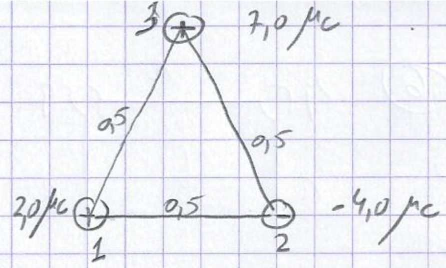
$$F_x = F_{12}\cos 60° + F_{23}\cos 60° = 0{,}755\ J$$

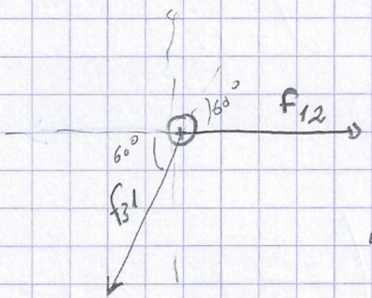
$$F_y = F_{12}\,\mathrm{sen}\,60° - F_{23}\,\mathrm{sen}\,60° = -0{,}436\ J$$

$$\vec{F} = 0{,}755\,\hat{i} - 0{,}436\,\hat{j} \quad (J)$$

②

4) a) $\vec{E} = K_e \sum_{i=2}^{3} \frac{q_i}{r_i^2} \vec{u_{ri}}$





$E_x = 8{,}99 \times 10^9 \left[\frac{+4}{0{,}5^2} - \frac{7}{0{,}5^2} \cos 60° \right] \times 10^{-6}$

$= 17980 \ N/C$

nota

$\frac{N \cdot m^2}{C^2} \times \frac{C}{m^2} = \frac{N}{C}$

$V = \frac{J}{C} \Leftrightarrow C = \frac{J}{V}$

$N = \frac{kg \cdot m}{s^2}$ $\quad J = kg \frac{m^2}{s^2}$

$\frac{N}{C} = \frac{\frac{kg \cdot m}{s^2}}{\frac{kg \, m^2}{s^2}} V = \frac{V}{m}$

$17980 \frac{N}{C} = 17{,}980 \frac{kV}{m}$

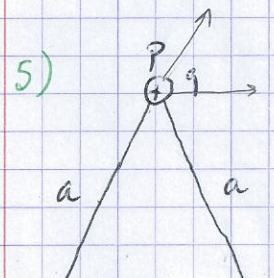
$E_y = 8{,}99 \times 10^9 \left[0 - \frac{7}{0{,}5^2} \operatorname{sen} 60° \right] \times 10^{-6}$

$= -217996 \ N/C$

$\vec{E} = 17980 \hat{i} - 217996 \hat{j} \quad N/C$

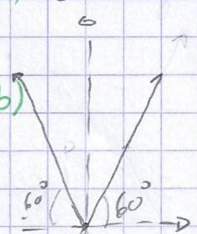
b) $\vec{E} = \frac{\vec{F}}{q_0} \Leftrightarrow \vec{F} = 2 \times 10^{-6} \vec{E} \Leftrightarrow \vec{F} = 0{,}03596 \, i - 0{,}435992 \hat{j} \quad N$

5)



a) Centro geométrico do triângulo

b)



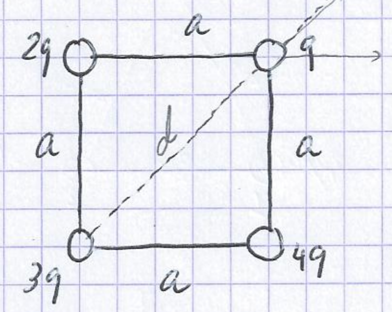
$E_x = K_e \left[\frac{q}{a^2} \cos 60° - \frac{q}{a^2} \cos 60° \right] = 0$

$E_y = K_e \left[\frac{q}{a^2} \operatorname{sen} 60° + \frac{q}{a^2} \operatorname{sen} 60° \right] = K_e \times \frac{q}{a^2} \times \frac{\sqrt{3}}{2} \times 2 =$

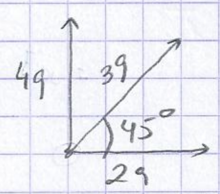
$= \frac{\sqrt{3} q}{a^2} \times \frac{1}{4\pi\varepsilon_0} = \frac{\sqrt{3} q}{4\pi\varepsilon_0 a^2} \Rightarrow \vec{E} = 0\hat{i} + \frac{\sqrt{3} q}{4\pi\varepsilon_0 a^2} \hat{j}$

direcy vertical SN e $|\vec{E}| = \frac{\sqrt{3} q}{4\pi\varepsilon_0 a^2}$

6



a)



$d^2 = a^2 + a^2 \Rightarrow d^2 = 2a^2$

$d = 2\sqrt{a}$

$$E_x = K_e\left[\frac{2q}{a^2} + \frac{3q}{2a^2}\cos 45°\right] = K_e\left(\underset{(4)}{\frac{2q}{a^2}} + \frac{3\sqrt{2}\,q}{4a^2}\right) =$$

$$= K_e \times \frac{8+3\sqrt{2}}{4a^2}\,q \qquad N/C$$

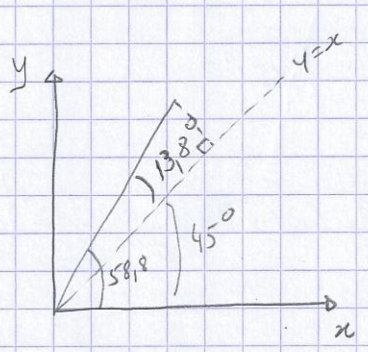
$$E_y = K_e\left[\underset{(4)}{\frac{4q}{a^2}} + \frac{3q}{2a^2}\times\frac{\sqrt{2}}{2}\right] = K_e \times \frac{16+3\sqrt{2}}{4a^2}\,q \quad N/C$$

$$tg\,\theta = \frac{\cancel{K_e}\,\frac{16+3\sqrt{2}}{\cancel{4a^2}}\,\cancel{q}}{\cancel{K_e}\,\frac{8+3\sqrt{2}}{\cancel{4a^2}}\,\cancel{q}} = \frac{16+3\sqrt{2}}{8+3\sqrt{2}} \Rightarrow \theta = 58{,}8° \text{ com a horizontal}$$

$$|\vec{E}| = \sqrt{K_e^2 \times \frac{(8+3\sqrt{2})^2}{(4a^2)^2}\times q^2 + K_e^2 \times \frac{(16+3\sqrt{2})^2}{(4a^2)^2}\times q^2}$$

$$= \frac{K_e \times q}{a^2}\times 5{,}91 = 5{,}91\,\frac{q}{4\pi\varepsilon_0 a^2}$$

b)



$$F = E \cdot \cos 13{,}8°$$

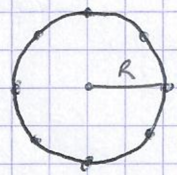
$$= 5{,}91 \times \cos 13{,}8° \times \frac{q}{4\pi\varepsilon_0 a^2}$$

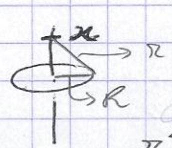
$$= 5{,}74\,\frac{q}{4\pi\varepsilon_0 a^2} \quad N$$

(3)

7) n cargas

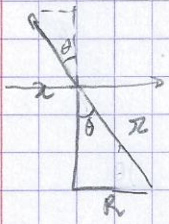
$\frac{Q}{n}$





$r^2 = x^2 + R^2$

$E_x = 0$ por serem simétricas ambas duas a duas



$$E = E_y = Ke \sum_{i=1}^{n} \frac{\frac{Q}{n}}{x^2+R^2} \cos\theta \Leftrightarrow$$

$$\cos\theta = \frac{x}{r} = \frac{x}{\sqrt{x^2+R^2}}$$

$$\Leftrightarrow E = Ke \times \frac{Q}{\cancel{n}} \times \frac{1}{x^2+R^2} \times \frac{x}{\sqrt{x^2+R^2}} \cancel{\sum_{i=1}^{n} 1}$$

$$\Leftrightarrow E = \frac{KeQx}{(x^2+R^2)^{3/2}}$$

8) $\lambda = \lambda_0 + 2x$ $\qquad 0 \leq x \leq L$

$$dq = \lambda\, dL \Leftrightarrow q = \int_0^L (\lambda_0 + 2x)\, dx \qquad \frac{dL}{dx} = 1$$

$$\Leftrightarrow q = \left[\lambda_0 x + x^2\right]_0^L$$

$$\Leftrightarrow q = \lambda_0 L + L^2 \quad (e)$$

9)

$\sigma = 3r$

$S = \pi R^2$

$$dq = \sigma\, dS \qquad \frac{dS}{dr} = 2\pi r$$

$$q = \int_{0}^{R} \sigma \cdot 2\pi r\, dr$$

$$q = \int_0^R 6\pi r^2\, dr$$

$$q = 6\pi \left[\frac{r^3}{3}\right]_0^R = 6\pi \frac{R^3}{3} = 2\pi R^3 \quad (c)$$

10)

$V = \frac{4}{3}\pi r^3$

$dV = 4\pi r^2\, dr$

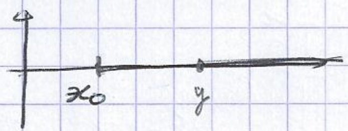
$$Q = \int_{r_1}^{r_2} \rho \cdot 4\pi r^2\, dr \qquad \rho = \frac{K}{r}$$

$$Q = \int_{r_1}^{r_2} K \cdot 4\pi r\, dr \Leftrightarrow Q = K \cdot 4\pi \left[\frac{r^2}{2}\right]_{r_1}^{r_2}$$

$$Q = K 2\pi (r_2^2 - r_1^2) \Leftrightarrow \frac{Q}{2\pi(r_2^2 - r_1^2)} = K$$

$$\Leftrightarrow \rho r = \frac{Q}{2\pi(r_2^2 - r_1^2)} \Leftrightarrow \rho = \frac{Q}{2\pi r(r_2^2 - r_1^2)}$$

11)



$\lambda = \lambda_0 \qquad dq = \lambda_0\, dx$

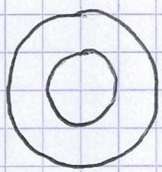
$E_y = 0$

$$E_x = k_e \int_{x_0}^{+\infty} \frac{\lambda_0}{x^2}\, dx = k_e \times \left(-\int_0^{x_0} \frac{\lambda_0}{x_0^2}\, dx\right)$$

$$= -k_e \frac{\lambda_0}{x_0^2} \Big[x \Big]_0^{x_0} = -k_e \frac{\lambda_0}{x_0^2} \times x_0 = -k_e \frac{\lambda_0}{x_0}$$

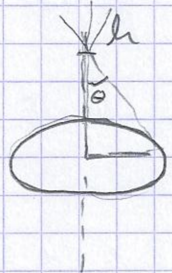
dirigé horizontal

12)



$r = a$

Q



$r^2 = h^2 + a^2$

$Q = \lambda 2\pi r$

$dq = \lambda 2\pi\, dr$

$\cos\theta = \frac{h}{\sqrt{h^2+a^2}}$

$E_x = 0$

$$E_y = k_e \int \frac{1}{r^2} \vec{e_r}\, dq$$

$$= k_e \int_0^a \frac{1}{h^2+a^2} \cos\theta \times \lambda 2\pi\, dr$$

$$= k_e \times \frac{1}{h^2+a^2} \times \cos\theta \times \lambda \times 2\pi \times \Big[r \Big]_0^a$$

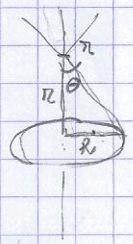
$$= k_e \times \frac{1}{h^2+a^2} \times \frac{h}{\sqrt{h^2+a^2}} \times \underbrace{2\pi\lambda a}_{Q}$$

$$= k_e \times \frac{h}{(h^2+a^2)^{3/2}} \times Q$$

ou

$$E = k_e \times \frac{Q}{h^2+a^2} \cos\theta = k_e \frac{Q}{h^2+a^2} \times \frac{h}{\sqrt{h^2+a^2}} \quad ✓$$

(4) 13) R ~~[illegible]~~ $Q = \pi R^2 \sigma$

a) 

Pelo princípio de sobreposição, o campo produzido no ponto pretendido é a soma (integral) dos campos produzidos por aneis de raio x, com $0 \leq x \leq R$

$$E_{anel} = k_e \frac{Q\, r}{(r^2 + R^2)^{3/2}}$$

$$E = k_e \int \frac{r}{(r^2 + x^2)^{3/2}}\, dq$$

$Q = \pi x^2 \sigma$

$dq = 2\pi x \sigma\, dx$

$$= k_e \times 2\pi r \sigma \int_0^R \frac{x}{(r^2 + x^2)^{3/2}}\, dx$$

$$= k_e * 2\pi r \sigma \left[-\frac{1}{\sqrt{r^2 + x^2}} \right]_0^R$$

$$= k_e \times 2\pi r \sigma \times \left[-\frac{1}{\sqrt{r^2 + R^2}} + \frac{1}{r} \right]$$

$-\frac{3}{2} + \frac{2}{2} =$

$$= k_e \times 2\pi \sigma \left(1 - \frac{r}{\sqrt{r^2 + R^2}} \right)$$

$k_e = \frac{1}{4\pi \varepsilon_0}$

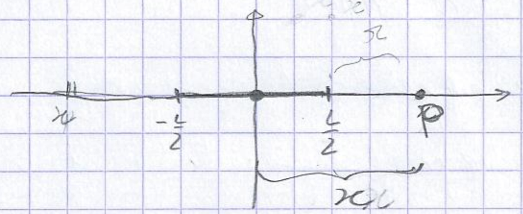
$$= \frac{1}{4\cancel{\pi}\varepsilon_0} \times \cancel{2\pi} \sigma \left(1 - \frac{r}{\sqrt{r^2 + R^2}} \right)$$

$$= \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{r}{\sqrt{r^2 + R^2}} \right)$$

b) $r << R \Rightarrow \frac{r}{\sqrt{r^2 + R^2}} \cong 0$ logo $E \cong \frac{\sigma}{2\varepsilon_0}$

c) $r >> R$? $E \; \frac{\sigma}{2\varepsilon_0} \left(1 - \frac{r}{\sqrt{r^2 + R^2}} \right)$ fica igual!

14) L
λ



a)

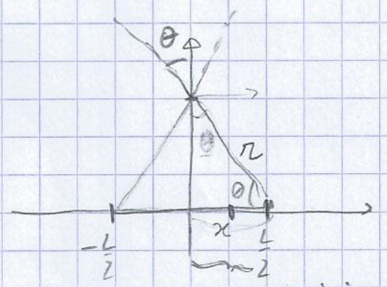
$$E = k_e \int \frac{\vec{e_r}}{r^2}\, dq \qquad Q = \lambda\, l \qquad dq = \lambda\, dl$$

$$= k_e \int_{x-\frac{L}{2}}^{x+\frac{L}{2}} \frac{\lambda}{r^2}\, dr \qquad r = x - \frac{L}{2} \qquad dx = -\frac{1}{2}\, dl$$

$$= k_e \cdot \lambda \left[-\frac{1}{r} \right]_{x-\frac{L}{2}}^{x+\frac{L}{2}}$$

$$= k_e \cdot \lambda \left[-\frac{1}{x+\frac{L}{2}} + \frac{1}{x-\frac{L}{2}} \right] = k_e \cdot \lambda \cdot \left(\frac{1}{x-\frac{L}{2}} - \frac{1}{x+\frac{L}{2}} \right)$$

b)



Simetria

$r = \sqrt{x^2 + y^2}$

$$E = k_e \int \frac{\vec{e_r}}{r^2}\, dq \qquad dq = \lambda\, dl$$

$$= k_e \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{r^2} \operatorname{sen}\theta\, \lambda\, dl$$

$$= k_e \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{x^2+y^2} \times \frac{y}{\sqrt{x^2+y^2}}\, \lambda\, dx$$

$x = y \operatorname{sen} t$
$dx = y \cos t\, dt$

$$= k_e \lambda y \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{1}{(x^2+y^2)^{3/2}}\, dx$$

integrar

(5) 15) $\vec{E} = -6{,}00 \times 10^5\ \hat{\imath}\ N/C$ 7,00 cm

$q_0 = 1{,}602 \times 10^{-19}\ C$

a) $F = q_0 E = 1{,}602 \times 10^{-19} \times (-6{,}00 \times 10^5) = -9{,}612 \times 10^{-14}\ N$

$F = m \times a \Leftrightarrow -9{,}612 \times 10^{-14} = 1{,}673 \times 10^{-27} \times a$

$\Leftrightarrow a = -5{,}75 \times 10^{13}\ m/s^2$

Nota: $F = q_0 E \Leftrightarrow -ma = q_0 E \Leftrightarrow a = \frac{-q_0 E}{m}$

b) 7cm = 0,07 m

$v^2 = v_0^2 - 2a\Delta x$

$0 = v_0^2 - 2 \times (5{,}75 \times 10^{13}) \times 0{,}07$

$v_0 = 2837252\ m/s$

Nota: $v = \sqrt{0{,}14 \frac{q_0 E}{m}}$

c) $a = \frac{|v_f - v_i|}{t}$

$t = \frac{a}{2837252} = 20266088\ s$

Nota $t = \frac{\sqrt{0{,}14 \frac{q_0 E}{m}}}{\frac{q_0 E}{m}} = \sqrt{\frac{0{,}14 \cdot \frac{q_0 E}{m}}{\left(\frac{q_0 E}{m}\right)^2}} = \sqrt{0{,}14 \frac{m}{q_0 E}}$

16) $E_c = K$

$\frac{1}{2} m v^2 = K$

$v^2 = \frac{2K}{m}$

$v_f^2 = v_i^2 - 2a\Delta x$

$0 = \frac{2K}{m} - 2ad$

$a = \frac{K}{md}$

$E = \frac{F}{q} = \frac{\cancel{m} \times \frac{K}{\cancel{m}d}}{q}$

$E = \frac{K}{qd}$ //

17) v_p E

a) h $E = \frac{F}{q} \Leftrightarrow ma = Eq \Leftrightarrow a = \frac{Eq}{m}$

$$h = v_p t - \frac{1}{2}at^2$$

$$2h = v_p t - at^2$$

$$at^2 - 2v_p t - 2h = 0$$

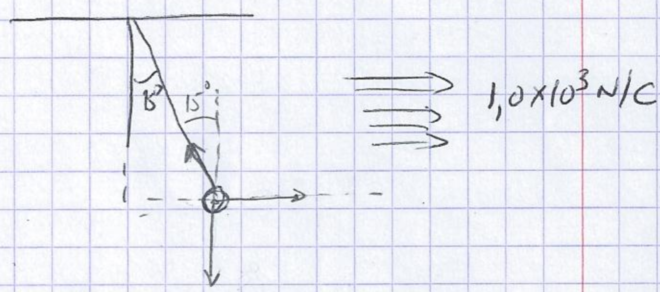
$$t = \frac{2v_p \pm \sqrt{4v_p^2 + 8ah}}{2a}$$

$$t = \frac{v_p \pm \sqrt{v_p^2 + 2ah}}{a}$$

mas $\sqrt{v_p^2+2ah} > v_p$ logo $t = \frac{v_p + \sqrt{v_p^2 + 2ah}}{a}$ com $a = \frac{Eq}{m}$

18) $m = 2g$

$l = 20cm$



$$\begin{cases} T\cos 15^\circ = mg \\ T\,\text{sen}\,15^\circ = F_E \end{cases} \Leftrightarrow$$

$$tg\,15^\circ = \frac{Eq}{mg} \Leftrightarrow q = \frac{mg\,tg\,15}{E} \Leftrightarrow$$

$$\Leftrightarrow q = \frac{2\times10^{-3} \times 9{,}8 \times tg\,15}{1{,}0\times10^{3}} = 5{,}25\times10^{-6}\ C$$

19) a) $d = 7cm = 0{,}07\ m$ repulsiva (ambas negativas)

nº das linhas de campo é proporcional à q

linhas forças $q_1 = 8$

linhas forças $q_2 = 18$

$$\frac{8}{r^2} = \frac{18}{(7-r)^2}$$

$$18r^2 = 8(7^2 - 14r + r^2)$$

$$10r^2 + 112r - 392 = 0$$

$$r = 2{,}8\ ✓\ \cancel{r = 14}$$

2,8 cm de eq.

b) $-8\ nC$

$$\frac{n_1}{q_1} = \frac{n_2}{q_2}$$

$$\frac{8}{q_1} = \frac{18}{-8}$$

$$q_1 = \frac{-8\times8}{18}$$

$$q_1 = -3{,}56\ nC$$