TP1

1.1

$V(t) = a + bt^4$

$a = 6$

$b = 2$

a) $a(t) = \frac{dv}{dt} = (a + bt^4) = 4bt^3 \xrightarrow{b=2} 8t^3$

b) $\cdot\ t = 0$ $\qquad\qquad \cdot\ t = 1$

$a(0) = 8(0)^3 = 0 \qquad\qquad a(1) = 8(1)^3 = 8$

c) $s(t) = \int v(t)\, dt$

$s(t) = \int a + bt^4\, dt = at + \frac{bt^5}{5} + C$

$\cdot\ s(0) = 0$

$0 = a(0) + b\frac{(0)^5}{5} + C \Leftrightarrow C = 0$

$s(t) = at + b\frac{t^5}{5} \xrightarrow{a=6,\ b=2} s(t) = 6t + \frac{2}{5}t^5$

d)

$\Delta s = \int_{t_i}^{t_f} v(t)\, dt \rightarrow \int_2^4 a + bt^4\, dt =$

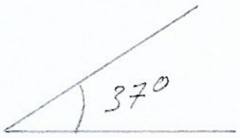
$= \left[at + b\frac{t^5}{5}\right]_2^4 = \left(a(4) + b\frac{4^5}{5}\right) - \left(a(2) + b\frac{2^5}{5}\right)$

$a = 6$

$b = 2$ $= \left(6(4) + 2\frac{4^5}{5}\right) - \left(6(2) + 2\frac{2^5}{5}\right)$

$= 24 + 409.6 - (12 + 12.8) = 433.6 - 24.8 = 408.8$

1.2



$\cos(37) = 0.8$ $\quad h = 125\,m$

$\text{sen}(37) = 0,6$ $\quad V(0) = 105\,m/s$

a) $\vec{V}(t) = \|V_0\|\left(\cos(37)_{\hat{e}_x} + \text{sen}(37)_{\hat{e}_y}\right)$

$= 105 \cdot \cos(37)\,\hat{e}_x + 105 \cdot \text{sen}(37)\,\hat{e}_y$

$= 84\,\hat{e}_x + (63 - 9,8t)\,e_y$ → aceleração gravítica, apenas existe na vertical

$\vec{V}(0) = 84\,\hat{e}_x + (63 - 9,8(0))\,e_y$

$= 84\,\hat{e}_x + 63\,\hat{e}_y$ //

b) Tempo de voo → quanto tempo até atingir o solo

$$y(t) = \underbrace{125}_{\substack{\text{altura}\\ \text{em } t=0}} + \underbrace{63t}_{\substack{\text{velocidade}\\ \text{em } t=0}} - \frac{1}{2}9,8t^2$$

$y(t) = 0 \longrightarrow$ solo

$$0 = 125 + 63t - 4,9t^2 \Leftrightarrow t = \frac{-63 \pm \sqrt{63^2 - 4 \cdot 125 \cdot (-4,9)}}{2 \cdot (-4,9)}$$

$\ldots \Leftrightarrow t = 14,6\,s$

c) Alcance → Quanto o projetil se desloca na horizontal ante de embater no solo

$x(t) = x_0 + V_0 t$ (o deslocamento na horizontal é uniforme)

$\Leftrightarrow x(t) = 84t$

$x(14,6) = 84(14,6) = 1226,4\,m$

d) Na altura maxima, a velocidade do componente vertical é zero ($V_{y\,max} = 0$)

$V_y(t) = 0 \Leftrightarrow V_y = 63 - 9,8t = 0 \Leftrightarrow t = 6,43$

$y(t) = 125 + 63t - 4,9t^2 \to t = 6,43 \to y(6,43) = 125 + 63(6,43) - 4,9(6,43)^2$

$\Leftrightarrow y(6,43) = 327$

1.3 $\vec{v}(t) = (t^2-1)_{\hat{\imath}} + (-t)_{\hat{\jmath}}$ $s(0) = 0$

a)

$$\vec{v}(t) = \frac{ds}{dt}$$

$$s(t) = \int \vec{v}(t)\, dt = \int (t^2-1)_{\hat{\imath}} + (-t)_{\hat{\jmath}}\, dt$$

$$= \left(\frac{t^3}{3} - t\right)_{\hat{\imath}} + \left(-\frac{t^2}{2}\right)_{\hat{\jmath}}$$

$$s(2) = \left(\frac{2^3}{3} - 2\right)_{\hat{\imath}} + \left(-\frac{(2)^2}{2}\right)_{\hat{\jmath}}$$

$$= \frac{2}{3}\hat{\imath} + (-2)_{\hat{\imath}}$$

b) $\vec{a} = \frac{dv}{dt}$

$$\vec{a}(t) = (2t)_{\hat{\imath}} + (-1)_{\hat{\jmath}}$$

c) $a_t = \frac{dv}{dt}$ $v = \sqrt{(t^2-1)^2 + (-t)^2}$

$$a_t(t) = \frac{4t^3 - 2t}{\sqrt{t^4 - t^2 + 1}}$$

$$dv = \left(\sqrt{(t^2-1)^2 + (-t)^2}\right)'$$

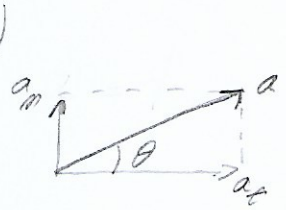
$$= \frac{1}{2}\left((t^2-1)^2 + (-t)^2\right)^{-\frac{1}{2}} \cdot (4t^3 - 2t)$$

$$= \frac{4t^3 - 2t}{\sqrt{t^4 - t^2 + 1}}$$

$$a_t(1) = \frac{2}{\sqrt{1}}$$

$$= 2_{\Delta t}$$

d)



$a_t = a\cos(\theta)$

$a_n = a\sin(\theta)$

$$\|\vec{a}(t)\| = \sqrt{(2(1))^2 + (-1)^2}$$

$$= \sqrt{4+1}$$

$$= \sqrt{5}$$

$$2 = \sqrt{5} \cdot \cos(\theta)$$

$$\ominus\ \theta \cong 26{,}6^\circ$$

$$a_n = \sqrt{5} \cdot \sin(26{,}6) \cong 1$$

1.4

$m = 0{,}1\,Kg \qquad R = 4m \qquad v_0 = 25\ m/s$

a) $w = \frac{v}{R} \Leftrightarrow w = \frac{25}{4} = 6{,}25\ rad/s$

b) $\theta(t) = \frac{s(t)}{R}$

$s(t) = 0 + 25t + \frac{1}{2}at^2$

$s(0{,}3) = 1 \qquad s(0) = 0$

$s(0{,}3) = 25(0{,}3) + \frac{1}{2}a(0{,}3)^2 = 1$

$\Leftrightarrow 7{,}5 + \frac{1}{2}a\,0.09 = 1 \Leftrightarrow a \cdot 0{,}09 = 13 \Leftrightarrow a = 144{,}4$

$\theta(t) = \frac{25t + 72{,}2t^2}{4} = 6{,}25t + 18.1t^2$

c)

$w(t) = \frac{d\theta}{dt} = 6{,}25 + 36{,}1t$

$w(0{,}3) = 6.25 + 10.83 = 17{,}08\ rad/s$

$a_n = w^2 \cdot R = 17{,}08 \times 4 = 68.32\ rad/s^2$

$F_N = m \times a_N = 0{,}1 \times 68.32 = 6{,}832\ N$