TP1

1.1

$V(t) = a + bt^4$
$a = 6$
$b = 2$

a) $a(t) = \frac{dv}{dt} = (a + bt^4) = 4bt^3 \xrightarrow{b=2} 8t^3$ //

b) $\cdot\, t = 0$ $\qquad\qquad$ $\cdot\, t = 1$

$a(0) = 8(0)^3 = 0$ $\qquad\qquad$ $a(1) = 8(1)^3 = 8$ //

c) $s(t) = \int v(t)\, dt$

$s(t) = \int a + bt^4\, dt = at + b\frac{t^5}{5} + C$

$\cdot\, s(0) = 0$

$0 = a(0) + b\frac{(0)^5}{5} + C \Leftrightarrow C = 0$

$s(t) = at + b\frac{t^5}{5} \xrightarrow{a=6,\, b=2} s(t) = 6t + \frac{2}{5}t^5$ //

d)

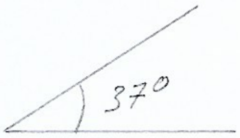
$\Delta s = \int_{t_i}^{t_f} v(t)\, dt \rightarrow \int_2^4 a + bt^4\, dt =$

$= \left[at + b\frac{t^5}{5}\right]_2^4 = \left(a(4) + b\frac{4^5}{5}\right) - \left(a(2) + b\frac{2^5}{5}\right)$

$a = 6$
$b = 2$ $\;= \left(6(4) + 2\frac{4^5}{5}\right) - \left(6(2) + 2\frac{2^5}{5}\right)$

$= 24 + 409.6 - (12 + 12.8) = 433.6 - 24.8 = 408.8$

1.2



$\cos(37) = 0.8$ $\quad h = 125\ m$

$sen(37) = 0,6$ $\quad V(0) = 105\ m/s$

a) $\vec{V}(t) = \|V_0\| \left(\cos(37)_{\hat{e}_x} + sen(37)_{\hat{e}_y} \right)$

$= 105 \cdot \cos(37)_{\hat{e}_x} + 105 \cdot sen(37)_{\hat{e}_y}$

$= 84_{\hat{e}_x} + (63 - 9{,}8t)_{e_y}$ → aceleração gravítica, apenas existe na vertical

$\vec{V}(0) = 84_{\hat{e}_x} + (63 - 9{,}8(0))_{e_y}$

$= 84_{\hat{e}_x} + 63_{\hat{e}_y}$ //

b) Tempo de voo → quanto tempo até atingir o solo

$y(t) = 125 + 63t - \frac{1}{2} 9{,}8t^2$

125 ↓ altura em $t=0$

63t ↓ velocidade em $t=0$

$y(t) = 0 \rightarrow$ solo

$0 = 125 + 63t - 4{,}9t^2 \Leftrightarrow t = \dfrac{-63 \pm \sqrt{63^2 - 4 \cdot 125 \cdot (-4{,}9)}}{2 \cdot (-4{,}9)}$

$\ldots \Leftrightarrow t = 14{,}6\ s$

c) Alcance → Quanto o projetil se desloca na horizontal ante de embater no solo

$x(t) = x_0 + V_0 t$ (o deslocamento na horizontal é uniforme)

$\Leftrightarrow x(t) = 84t$

$x(14{,}6) = 84(14{,}6) = 1226{,}4\ m$

d) Na altura maxima, a velocidade do componente vertical é zero ($V_{y\,max} = 0$)

$V_y(t) = 0 \Leftrightarrow V_y = 63 - 9{,}8t = 0 \Leftrightarrow t = 6{,}43$

$y(t) = 125 + 63t - 4{,}9t^2 \rightarrow t = 6{,}43 \rightarrow y(6{,}43) = 125 + 63(6{,}43) - 4{,}9(6{,}43)^2$

$\Leftrightarrow y(6{,}43) = 327$

1.3 $\vec{v}(t) = (t^2-1)\hat{\imath} + (-t)\hat{\jmath}$ $\qquad s(0) = 0$

a)

$$\vec{v}(t) = \frac{ds}{dt}$$

$$s(t) = \int \vec{v}(t)\, dt = \int (t^2-1)_{\hat{\imath}} + (-t)_{\hat{\jmath}}\, dt$$

$$= \left(\frac{t^3}{3} - t\right)_{\hat{\imath}} + \left(-\frac{t^2}{2}\right)_{\hat{\jmath}}$$

$$s(2) = \left(\frac{2^3}{3} - 2\right)_{\hat{\imath}} + \left(-\frac{(2)^2}{2}\right)_{\hat{\jmath}}$$

$$= \frac{2}{3}\,\hat{\imath} + (-2)_{\hat{\imath}}$$

b) $\vec{a} = \frac{dv}{dt}$

$$\vec{a}(t) = (2t)_{\hat{\imath}} + (-1)_{\hat{\jmath}}$$

c) $a_t = \frac{dv}{dt}$ $\qquad v = \sqrt{(t^2-1)^2 + (-t)^2}$

$$dv = \left(\sqrt{(t^2-1)^2 + (-t)^2}\right)'$$

$$= \frac{1}{2}\left((t^2-1)^2 + (-t)^2\right)^{-\frac{1}{2}} \cdot (4t^3 - 2t)$$

$$= \frac{4t^3 - 2t}{\sqrt{t^4 - t^2 + 1}}$$

$$a_t(t) = \frac{4t^3 - 2t}{\sqrt{t^4 - t^2 + 1}}$$

$$a_t(1) = \frac{2}{\sqrt{1}}$$

$$= 2\,\Delta t$$

d)

$a_t = a\cos(\theta)$ $\qquad \|\vec{a}(t)\| = \sqrt{(2(1))^2 + (-1)^2}$

$a_n = a\sin(\theta)$ $\qquad = \sqrt{4+1}$

$\qquad = \sqrt{5}$

$$2 = \sqrt{5} \cdot \cos(\theta)$$

$$\Leftrightarrow \theta \cong 26{,}6^\circ$$

$$a_n = \sqrt{5} \cdot \sin(26{,}6) \cong 1$$

1.4.

a) $\vec{v}(t) = v(t)\,\vec{a}_t$

$v(t) = \dfrac{ds}{dt}$, $\quad s(t) = R\,\theta(t)$

$v(t) = R\dfrac{d\theta}{dt} \Leftrightarrow v(t) = R\,w(t)$

$w(0) = \dfrac{v(0)}{R} = \dfrac{25}{4} = 6{,}25$ rad/s

b) $\vec{\alpha}$, aceleração angular constante

$$\alpha = \frac{dw}{dt} \Leftrightarrow \int_{t_0}^{t} \alpha\, dt = \int_{w_0}^{w(t)} dw \Leftrightarrow \alpha(t - t_0) = w(t) - w_0$$

$\Rightarrow w(t) = 6{,}25 + \alpha t$ rad/s

$$\cdot\ w(t) = \frac{d\theta}{dt} \Leftrightarrow \int_{t_0}^{t} w(t)\, dt = \int_{\theta_0}^{\theta(t)} d\theta \Leftrightarrow \int_{t_0}^{t} 6{,}25 + \alpha t\ dt = \theta(t) - \theta_0$$

$$\Leftrightarrow \theta(t) = \theta_0 + \left(6{,}25(t - t_0) + \frac{\alpha}{2}(t^2 - t_0^2)\right)$$

$\cdot\ \theta_0 = 0$ e $t_0 = 0$

$\theta(t) = \left(6{,}25t + \frac{\alpha}{2}t^2\right)$ rad

$\cdot$ Como $\theta(0{,}3) = \pi$ rad

$$\pi = 6{,}25(0{,}3) + \frac{\alpha}{2}(0{,}3)^2 \Leftrightarrow \alpha \simeq 28{,}1 \text{ rad/s}$$

$\cdot\ \theta(t) \simeq 6{,}2t + \dfrac{28{,}1}{2}t^2$ rad

c) $F_n = m a_n$

$a_n(0{,}3) = \dfrac{v_B^2}{R} = \dfrac{w_B^2 R^2}{R} = w_B^2 R \qquad w(0{,}3) = w_B$

$w(0{,}3) \simeq 6{,}25 + 28{,}1 \times 0{,}3 \simeq 14{,}68$ rad/s

$F_n = 0{,}1 \times (14{,}68)^2 \times 4 \Leftrightarrow F_n \simeq 86{,}2$ (N)