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Normalizing flows: an introduction and review of current methods

Normalizing flows are a family of generative models. It is a transfomration of a simple probability distribution into a more complicated one via a series of invertible and differentiable maps. We can evaluate the density of a sample from the complex distribution by looking at the density of its pre-image in the simple distribution, multiplied by the appropriate change in volume induced by the transformations: this is given by the aboslute values of the determinant of the Jacobians of each transformation.

Standard applications of normalizing flows include density estimation and prior construction, but th eone that interests us in is oulier detection.

Let $Z \in \mathbb{R}^D$ be a random variable with probability density function (pdf):

$$p_Z : \mathbb{R}^D \to \mathbb{R}$$

Then, for an invertible function g, we can let $Y := g(Z)$.

We then have:

$$\begin{aligned} F_Y(y) &= P(Y \leq y) && (1) \\ &= P(g(Z) \leq y) && (2) \\ &= P(Z \leq g^{-1}(y)) && (3) \\ &= F_Z(g^{-1}(y)) && (4) \end{aligned}$$

Thus:

$$p_Y(y) = p_Z(g^{-1}(y))|detDf(y)| \quad (5)$$