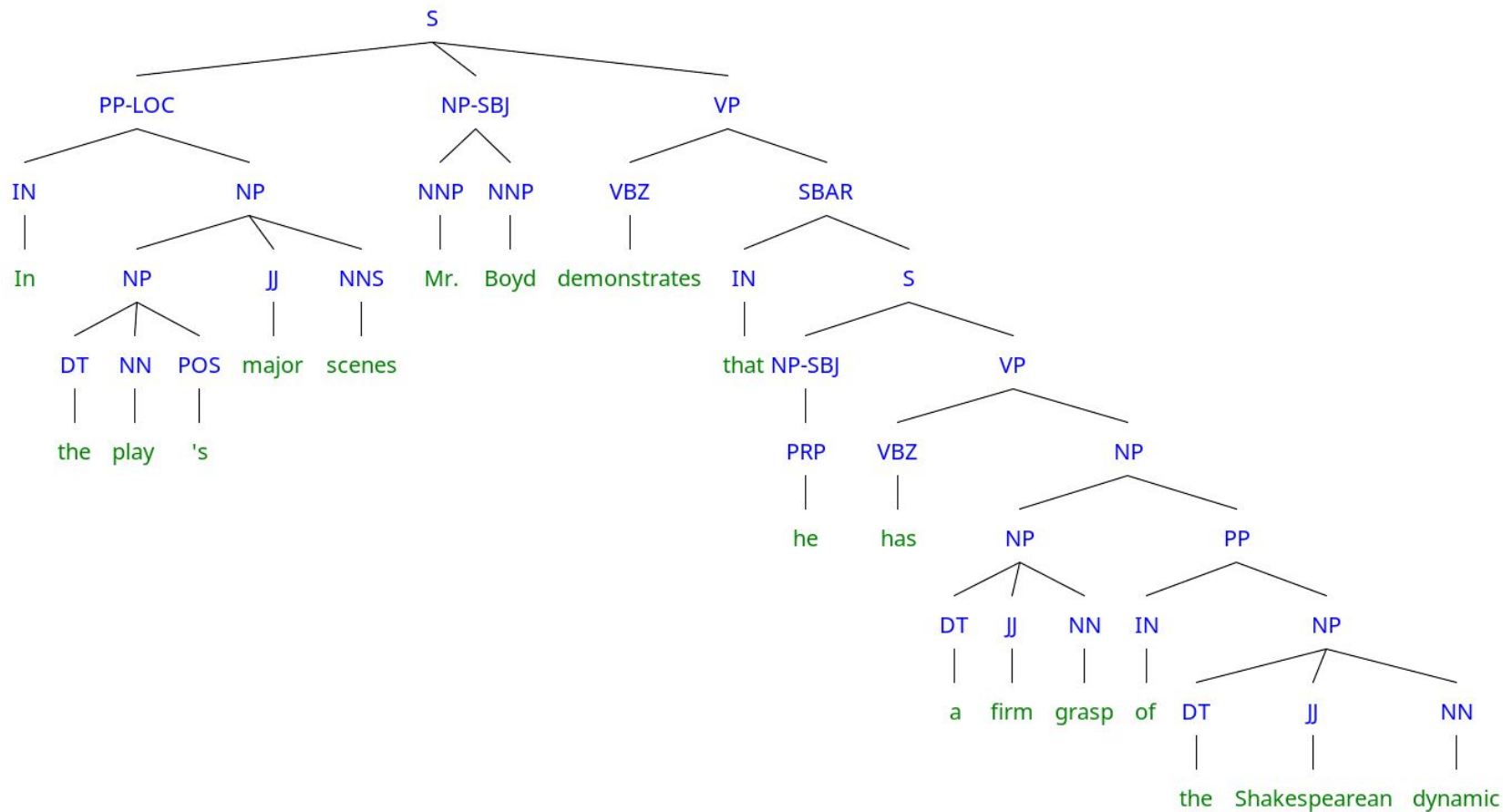


# (Unsupervised) Parsing & Grammar Induction

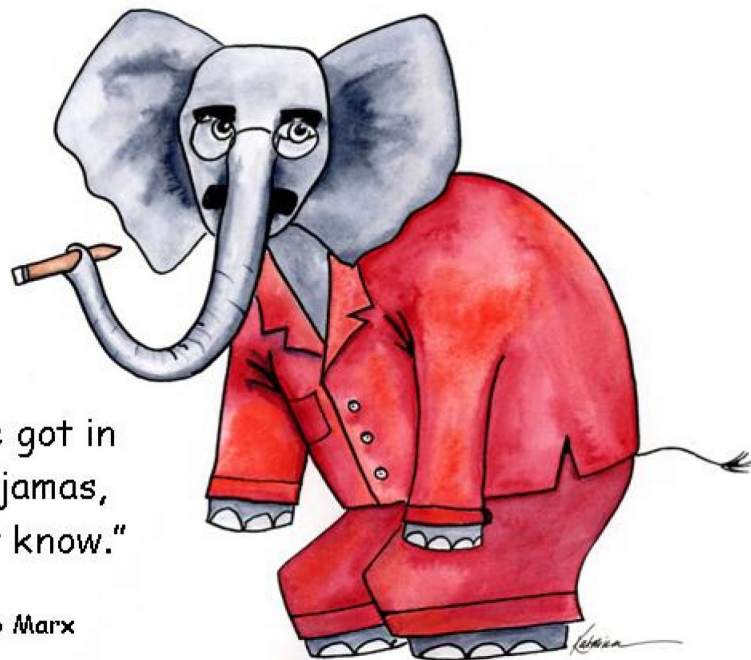
# Outline

- **Motivation**
- Review: Formal grammars
- Probabilistic grammar induction
- Recent approaches for grammar induction & unsupervised parsing
- Conclusion

# Language has hierarchical structure

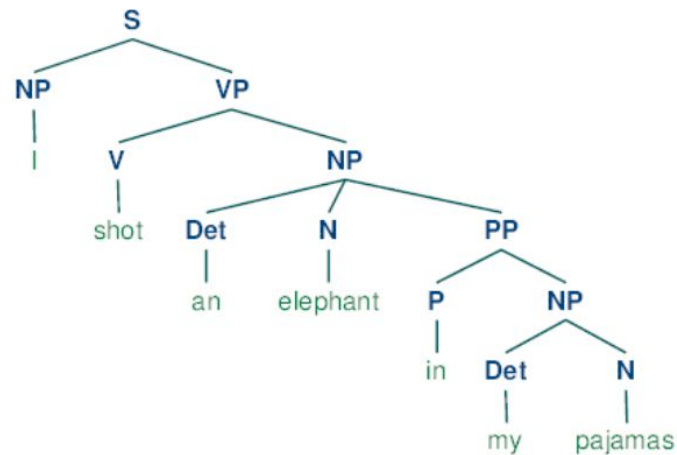
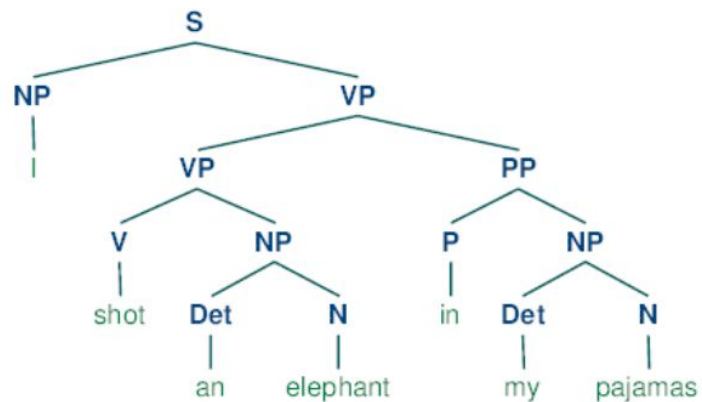


"One morning I shot an elephant in my pajamas.



How he got in  
my pajamas,  
I don't know."

Groucho Marx





**Bojan Tunguz**  
@tunguz



Watching a model train can be very calming and satisfying.

**Watching  
a model train**



**Watching  
a model train**





**Bojan Tunguz**  
@tunguz

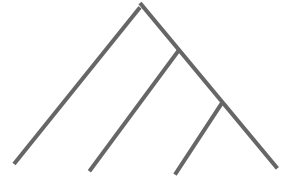


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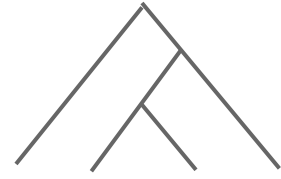
**Watching  
a model train**



**Watching  
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watching a model train



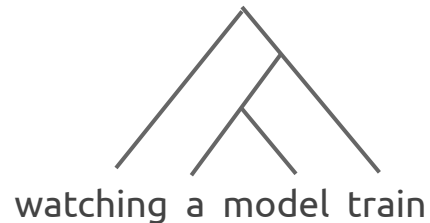
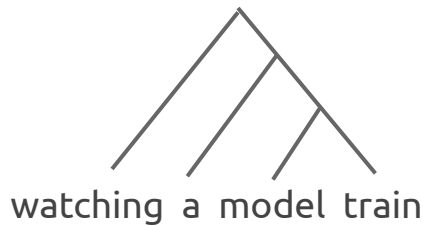
watching a model train

# Human Language Competence

- Robust intuitions about grammaticality of novel but meaningless sentences:
  - ✓ *“colorless green ideas sleep furiously”*
  - ✗ *“furiously sleep ideas green colorless”*
- What is the underlying structure governing human language that allows us to generate/recognize an infinite number well-formed sentences?

# Parse Trees

- **Linguistics:** Human language understanding is mediated by compositional tree-like structures [Chomsky '57].






# Parsing as a step towards meaning

Text

watching a model train

Parsing



watching a model train

Meaning

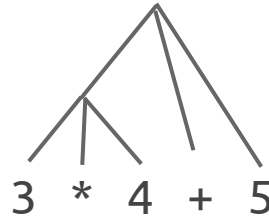


# Parsing as a step towards meaning

Program

3 \* 4 + 5

Parsing

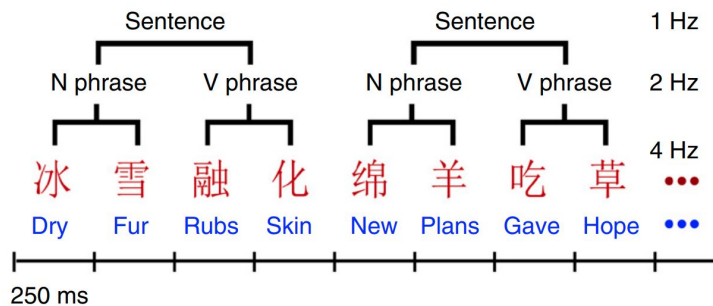


Meaning

17

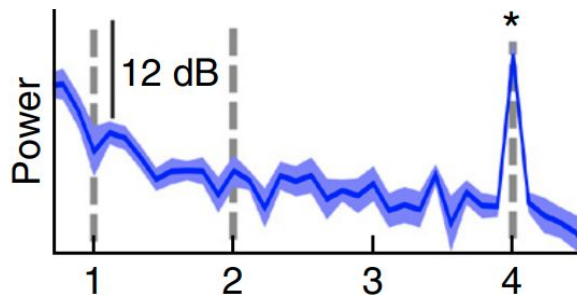
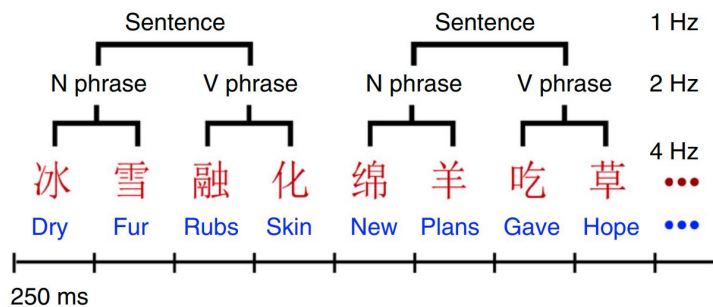
# Human Parsing

- **Neuroscience:** different neural activity for **English** vs. **Chinese** listeners when listening to Chinese [Ding et al '15]



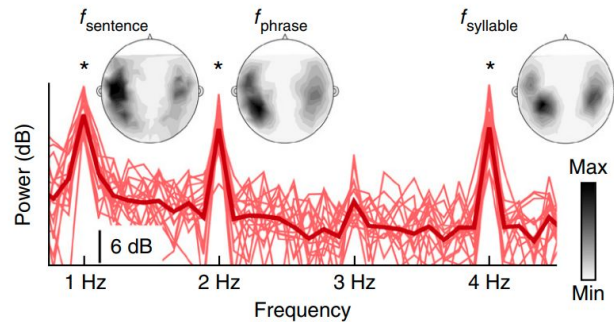
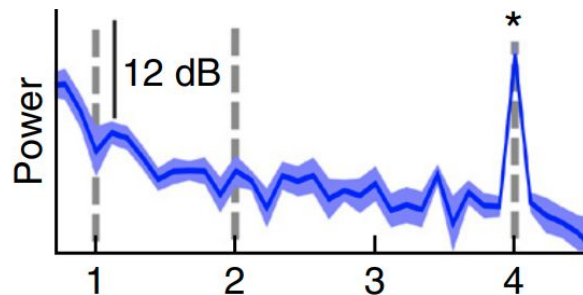
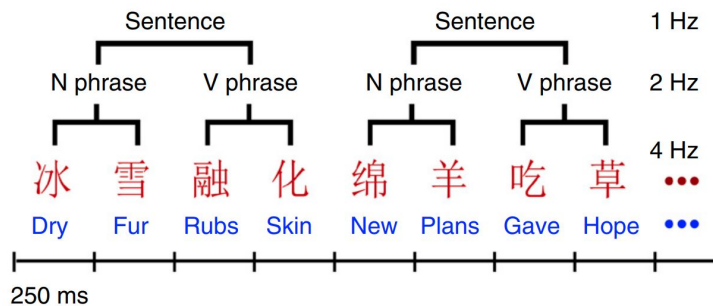
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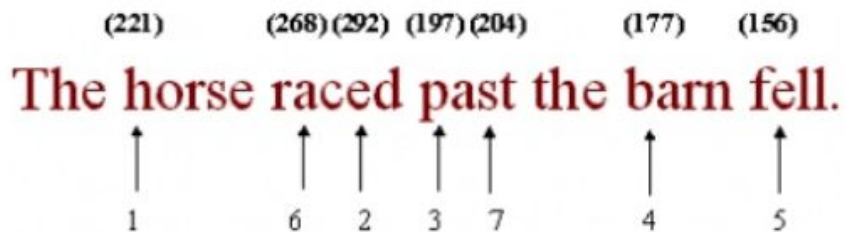
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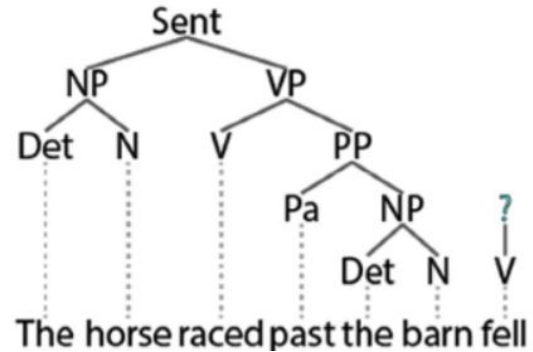
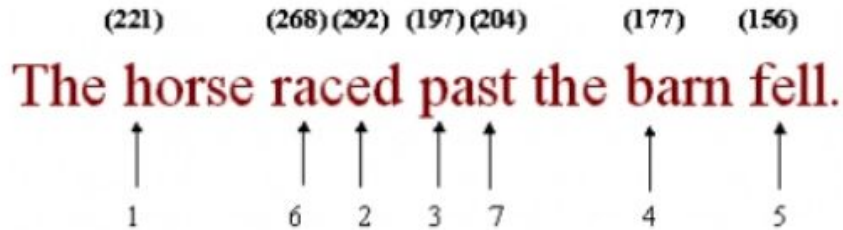
# Human Parsing

- **Psycholinguistics:** Eye movement in garden path sentences



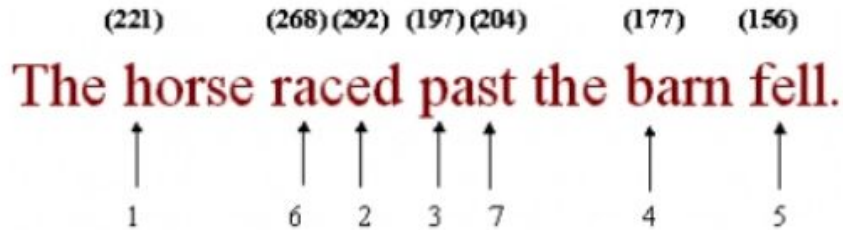
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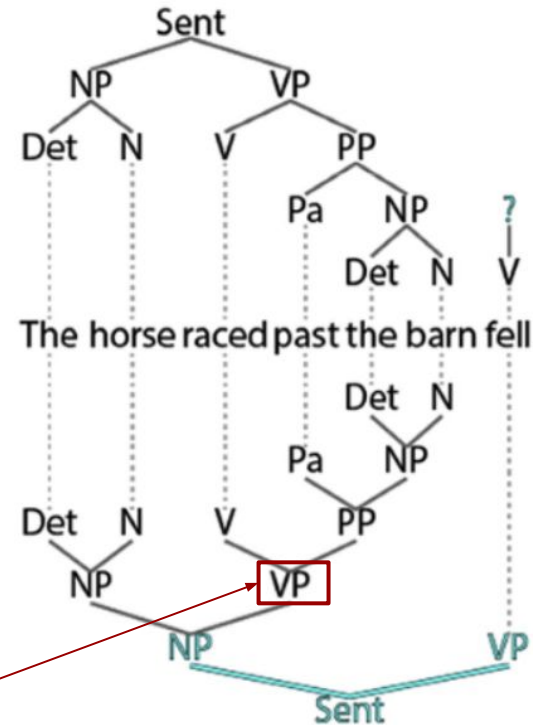


# Human Parsing

- **Psycholinguistics:** Eye movement in garden path sentences



Increased processing time to reanalyze  
this verb phrase into a relative clause





# Computational Approaches to Parsing

- **Rules-based:**

- Ask a really smart linguist to come up rules for parsing (e.g. based on a grammar).
- Hard to capture the complexities of natural language.

- **Statistical:**

- Ask linguists to annotate sentences with their corresponding parse trees.
- Treat it as a supervised learning problem.

# Supervised Parsing

- Core task in NLP with lots of different approaches.
  - Graph-based
  - Transition-based
- Parse trees often used as part of a larger NLP pipeline for a downstream task.
- ... until deep learning came long.

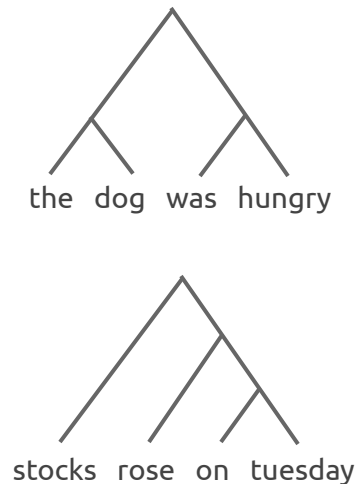
Model	$F_1$
Non-Neural Models	
Collins (1997)	87.8
Charniak (1999)	89.6
Petrov and Klein (2007)	90.1
McClosky et al. (2006)	92.1
Neural Models	
Dyer et al. (2016)	93.3
Fried et al. (2017)	94.7
Kitaev and Klein (2019)	95.8

(on WSJ Penn Treebank)

# Unsupervised Parsing

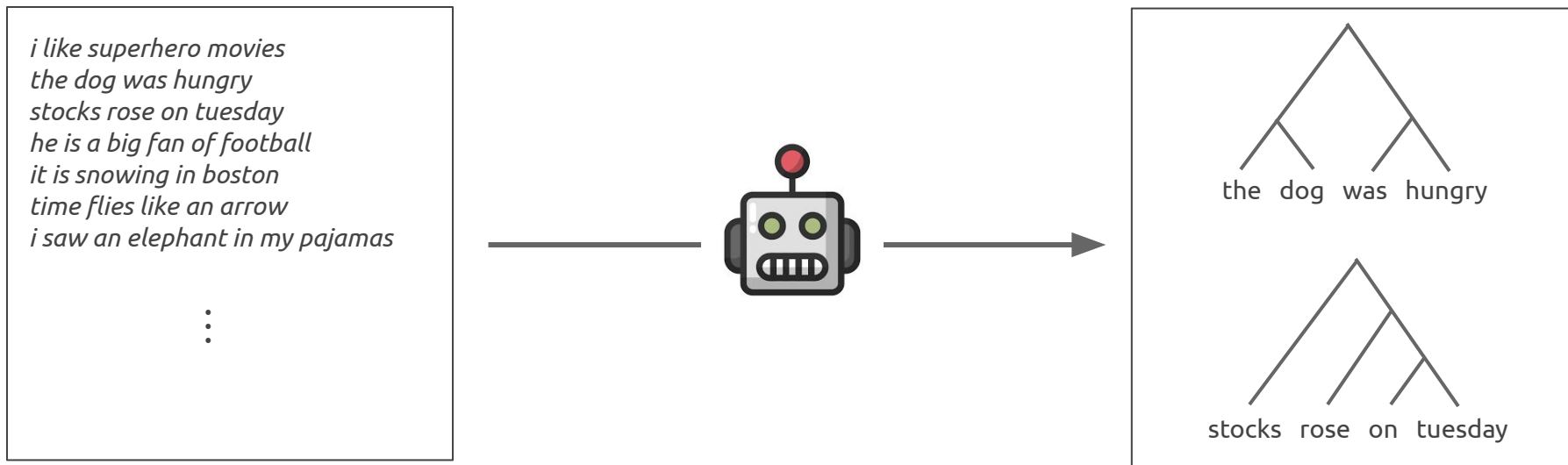
- Parse trees may not be as useful from an engineering perspective, but still interesting from a cognitive science standpoint.
- How do humans learn to parse? (Mostly) Unsupervised!

*i like superhero movies*  
*the dog was hungry*  
*stocks rose on tuesday*  
*he is a big fan of football*  
*it is snowing in boston*  
*time flies like an arrow*  
*i saw an elephant in my pajamas*



# Unsupervised Parsing

- Parse trees may not be as useful from an engineering perspective, but still interesting from a cognitive science standpoint.
- Can we train machine to do the same?



# Grammars for Parsing

- Classic approach: hypothesize a **formal grammar** that generates human language.



(Parse tree structure implied by the grammar.)

# Outline

- Motivation
- Review: Formal grammars
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- Conclusion

# Grammars

- A set of production rules for deriving strings in a formal language.

$$G = (N, \Sigma, P, S)$$

$N$  : set of nonterminal symbols

$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$S$  : start symbol ( $S \in N$ )

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$S$  : start symbol ( $S \in N$ )

- $L(G) = \{w \in \Sigma^* \mid S \rightarrow_G w\}$

(set of strings that can be generated from  $S$  by applying rules in  $G$ )



# Grammars

$$N = \{S\}$$

$$\Sigma = \{a, b\}$$

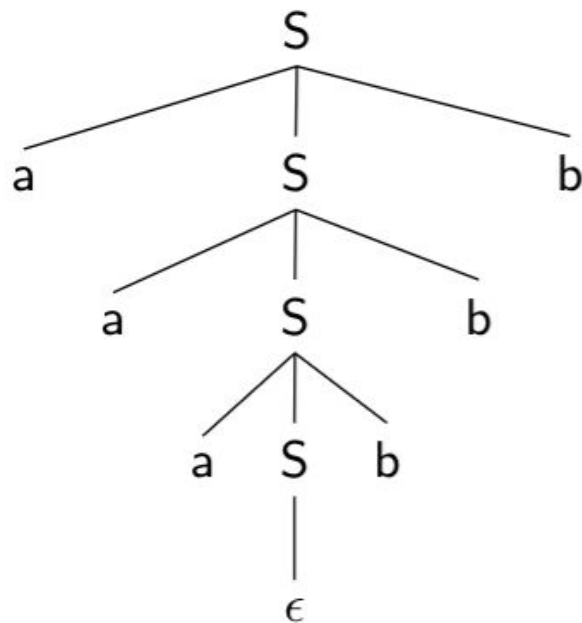
$$P = \{S \rightarrow aSb, S \rightarrow \epsilon\}$$

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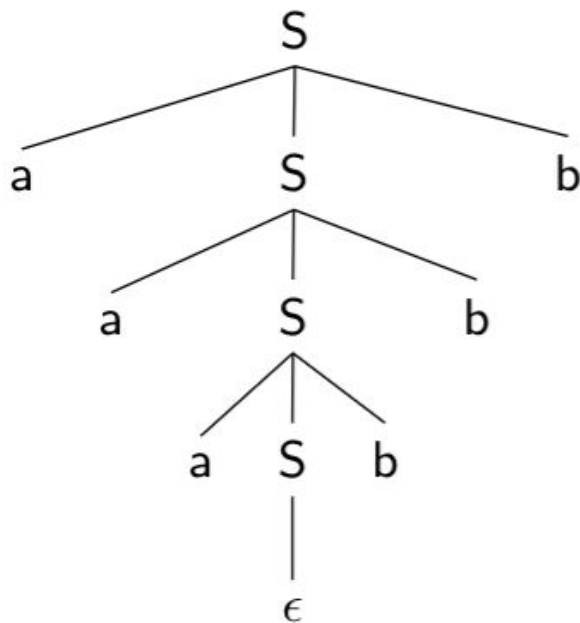
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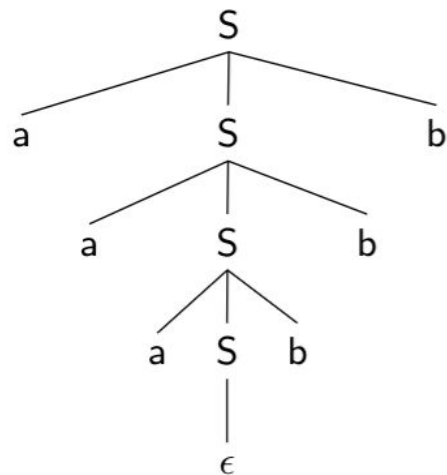
$$P = \{S \rightarrow aSb, S \rightarrow \epsilon\}$$

$$\begin{aligned} L(G) &= \{\epsilon, ab, aabb, aaabbb \dots\} \\ &= \{a^n b^n : n \geq 0\} \end{aligned}$$



# Grammars

- Is “aaaabbbb” in  $L(G)$ ? What about “abbb”?
- Given a grammar  $G$ :
  - Can check if a string belongs to  $L(G)$  by parsing.
  - Parsing also gives gives underlying sentence structure.



# Grammars for Natural Language

$N = \{S, NP, VP, CP, C, N, D, V_t, V_i\}$

$\Sigma = \{a, the, that, said, meows, barks, noticed, cat, dog, zyzzyva\}$

$P = S \rightarrow NP VP$

$VP \rightarrow V_t CP \mid V_i$

$CP \rightarrow C S$

$NP \rightarrow D N$

$N \rightarrow cat \mid dog \mid zyzzyva$

$V_t \rightarrow said \mid noticed$

$V_i \rightarrow meows \mid barks$

$C \rightarrow that$

$D \rightarrow the \mid a$

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$V_i \rightarrow meows \mid barks$

$C \rightarrow that$

$D \rightarrow the \mid a$

$L(G) =$  the cat meows

a zyzzzyva barks

the dog said that a cat barks

the zyzzzyva noticed that a dog meows

$\vdots$

# Grammars for Natural Language

$N = \{S, NP, VP, CP, C, N, D, V_t, V_i\}$

$\Sigma = \{a, the, that, said, meows, barks, noticed, cat, dog, zyzzyva\}$

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$L(G) = \text{the cat meows}$

$VP \rightarrow V_t CP \mid V_i$

$CP \rightarrow C S$

$NP \rightarrow D N$

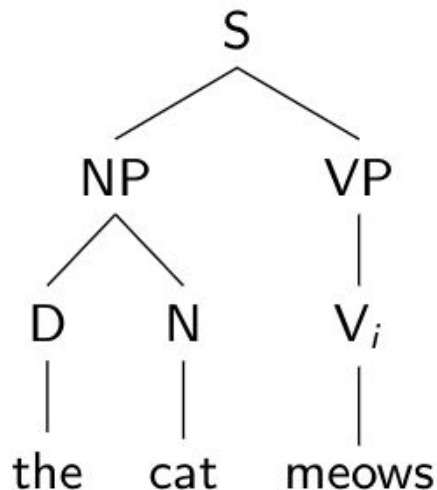
$N \rightarrow \text{cat} \mid \text{dog} \mid \text{zyzzyva}$

$V_t \rightarrow \text{said} \mid \text{noticed}$

$V_i \rightarrow \text{meows} \mid \text{barks}$

$C \rightarrow \text{that}$

$D \rightarrow \text{the} \mid \text{a}$



# Grammar Induction

- What is  $G$  such that  $L(G) = \text{Human Language}$ ?

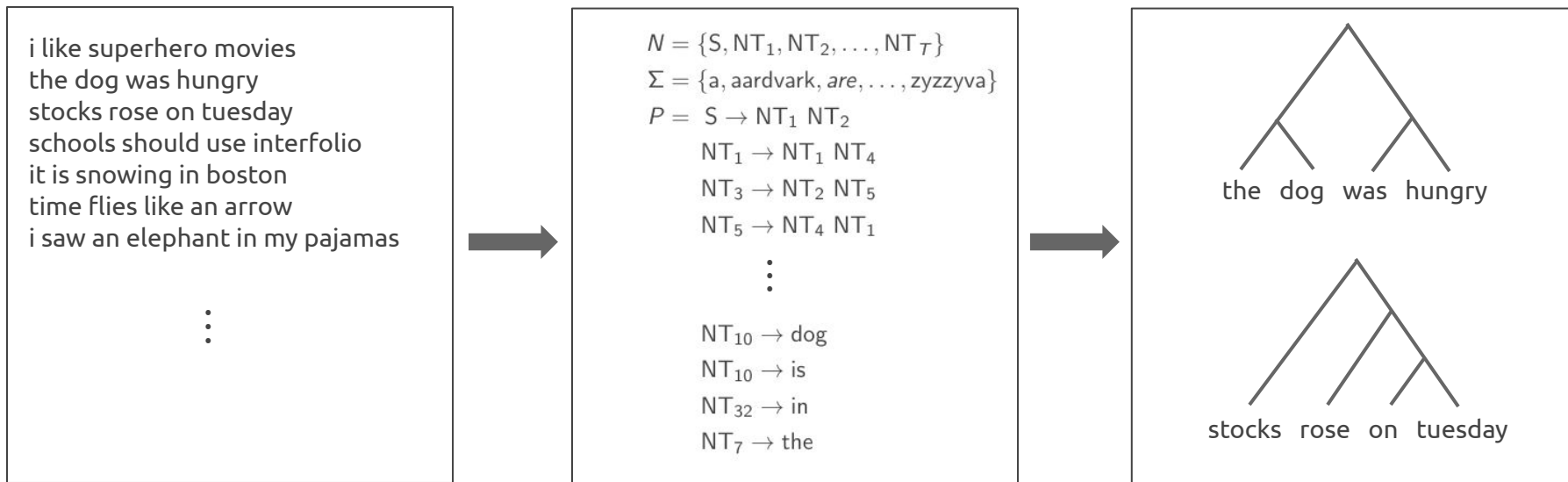
(A huge chunk of linguistics is devoted to finding this  $G$ )

- Data-driven approach: can we learn  $G$  from observed sentences alone?



# Grammar Induction

- Learn an underlying grammar (syntax) from observed sentences alone



# Tangent #1: “The Poverty of the Stimulus”

1. Children acquire the syntax of their native language by 4~5 years and can generalize in sophisticated ways.
2. They have not been exposed to enough data to learn such generalizations.
3. Therefore, there must be an innate “language acquisition device” (“universal grammar”) that children are equipped with at birth.

# Tangent #1: “The Poverty of the Stimulus”

“If a Martian linguist were to visit Earth, he would deduce from the evidence that there was only one language, with a number of local variants.”



# Tangent #1: “The Poverty of the Stimulus”

- Grammar induction  $\Rightarrow$  Data-driven acquisition of syntax is possible!
- Depending on how much bias one builds into the learning system, successful grammar induction can be used as an empirical argument against the poverty of the stimulus.

# Tangent #2: Grammar vs. Parsing

- Grammar  $\Rightarrow$  Parser, but Parser  $\nRightarrow$  Grammar.
- Can learn an unsupervised parser without learning a grammar.  
(most prior work in unsupervised parsing has been in this vein)
- Robust neurobiological evidence for human parsing, much less for grammars.

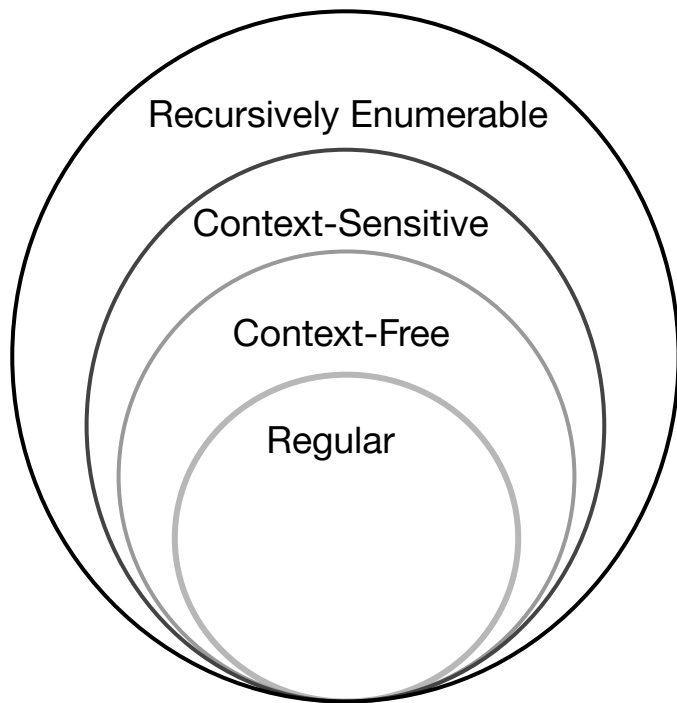
# Tangent #2: Grammar vs. Parsing

Why learn a grammar?

- Theoretically appealing.
- Grammar can explain how humans can recognize (i.e. parse) \*and\* generate an infinite number of sentences.
- Some experimental evidence that Grammar = Parser.

Children can start using syntactic rules for generation immediately after learning to recognize it [McKee et al '93]

# Tangent #3: Grammars & Formal Languages



$N$  : set of nonterminal symbols

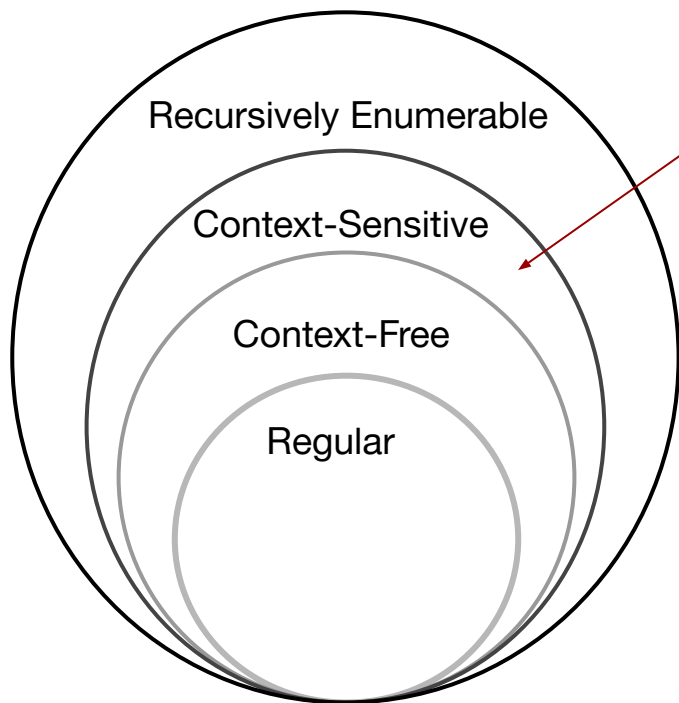
$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$L(G)$	Rules ( $P$ )
Recursively Enumerable	$\gamma \rightarrow \beta$
Context-Sensitive	$\alpha A \beta \rightarrow \alpha \gamma \beta$
Context-Free	$A \rightarrow \alpha$
Regular	$A \rightarrow a, A \rightarrow aB, A \rightarrow \epsilon$

$\gamma \in (N \cup \Sigma)^+$      $\alpha, \beta \in (N \cup \Sigma)^*$      $A, B \in N$      $a \in \Sigma$

# Tangent #3: Grammars & Formal Languages



Human language thought to be here  
(mildly context-sensitive)

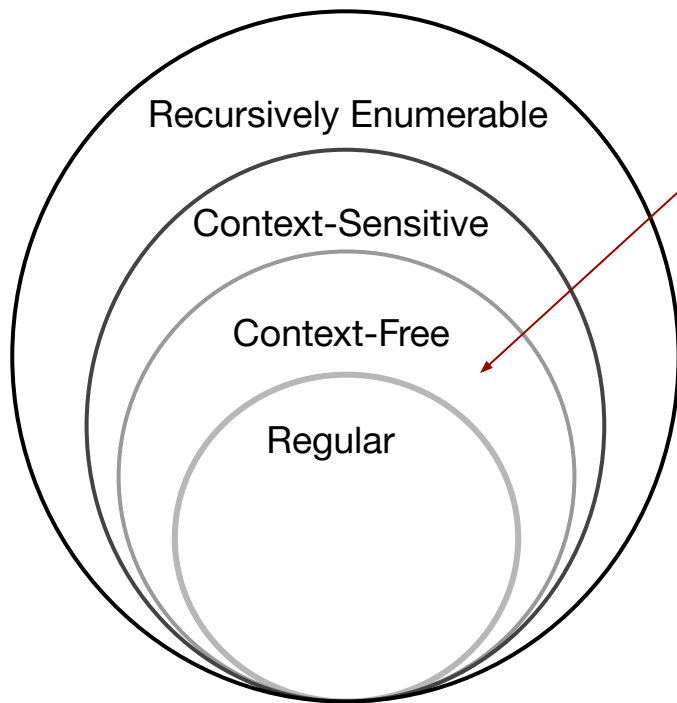
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# Tangent #3: Grammars & Formal Languages

Today's lecture will focus here



$L(G)$	Rules ( $P$ )
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- Review: Formal grammars
- **Probabilistic grammar induction**
- Recent approaches for grammar induction & unsupervised parsing
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# Probabilistic Modeling

- Probabilistic grammars: associate a probability to each rule.

$$S \rightarrow A_5 A_7 \qquad p_{\pi}(S \rightarrow A_5 A_7)$$

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$$x \in \Sigma^* \quad p_\pi(x)$$

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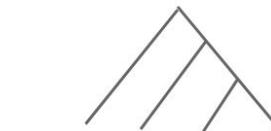
- Induces a distribution over surface strings (language model)

$$x \in \Sigma^* \quad p_{\pi}(x)$$

- $\pi$  := rule probabilities (probabilistic grammars)  
:= RNN / Transformer parameters (neural language models)  
:= n-gram probabilities (count-based language models)

# Probabilistic Modeling

- Why probabilistic grammars?
  - Naturally model uncertainty/ambiguity



watching a model train



watching a model train

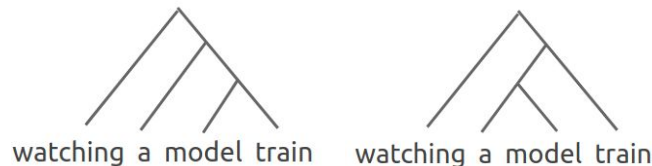
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[Gold '67]: Cannot even learn regular grammars from positive samples alone

[Horning '69]: Probabilistic grammars are learnable from positive samples



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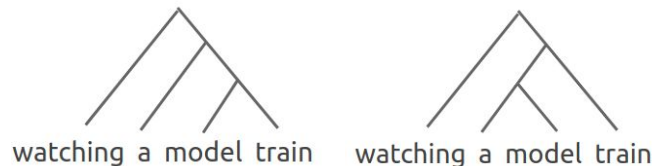
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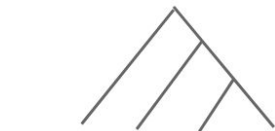
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- Information-theoretic interpretation: grammar induction corresponds to finding a grammar that can best compress the data statistically
- Natural objective to optimize: likelihood of corpus



watching a model train



watching a model train

# PCFG Induction

- Classic approach: assume each sentence is generated from a probabilistic context-free grammar (PCFG).

$$\mathbf{x} \sim \text{PCFG}(N, \Sigma, P, \pi)$$

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Nonterminal symbols (50 - 100)

$$N = \{S, A_1, A_2, \dots, A_T\}$$

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Special “Start” symbol

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Terminal symbols (10K - 100K)

$$\Sigma = \{a, \text{aardvark}, \text{able}, \text{are}, \dots, \text{zyzzyva}\}$$

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Context-free rules

(all possible unary/binary rules:  $O(|N||\Sigma| + |N|^3)$ )

$$\begin{array}{ll} P = S \rightarrow A_1 A_1 & A_1 \rightarrow \text{zyzzyva} \\ S \rightarrow A_1 A_2 & \dots \quad A_2 \rightarrow a \\ S \rightarrow A_1 A_3 & A_2 \rightarrow \text{aardvark} \end{array}$$

# PCFG Induction

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$$P = S \rightarrow A_1 A_1 \quad A_1 \rightarrow \text{zyzzyva}$$

$$S \rightarrow A_1 A_2 \quad \dots \quad A_2 \rightarrow a$$

$$S \rightarrow A_1 A_3 \quad A_3 \rightarrow \text{aardvark}$$

Rule probabilities

$$\pi = \{p_\pi(r) \mid r \in P\}$$

$$p_\pi(S \rightarrow A_5 A_7)$$

$$p_\pi(A_5 \rightarrow \text{John})$$

# PCFG Induction

- Classic approach: assume each sentence is generated from a probabilistic context-free grammar (PCFG).

$$\mathbf{x} \sim \text{PCFG}(N, \Sigma, P, \boxed{\pi})$$

$$N = \{S, A_1, A_2, \dots, A_T\}$$

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$$P = S \rightarrow A_1 A_1 \quad A_1 \rightarrow \text{zyzzyva}$$

$$S \rightarrow A_1 A_2 \quad \dots \quad A_2 \rightarrow a$$

$$S \rightarrow A_1 A_3 \quad A_2 \rightarrow \text{aardvark}$$

**Probabilistic grammar induction**  
 **$\Rightarrow$  Learning rule probabilities from data**

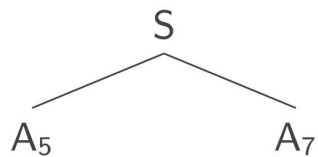
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# PCFG Example

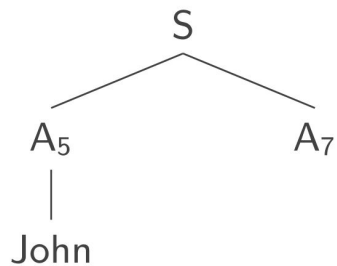
S

# PCFG Example



$$S \rightarrow A_5 A_7$$

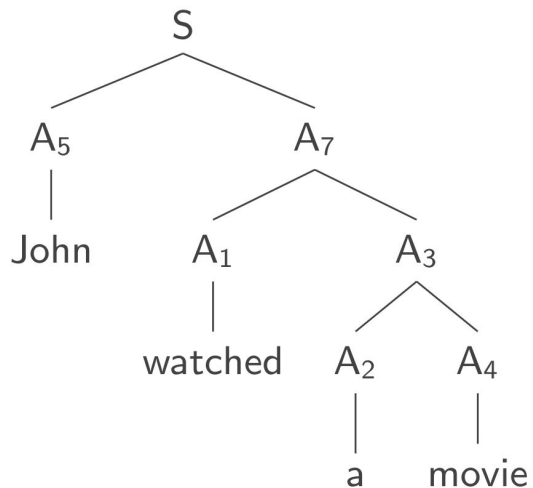
# PCFG Example



$S \rightarrow A_5 A_7$

$A_5 \rightarrow \text{John}$

# PCFG Example



$S \rightarrow A_5 A_7$

$A_5 \rightarrow \text{John}$

$A_7 \rightarrow A_1 A_3$

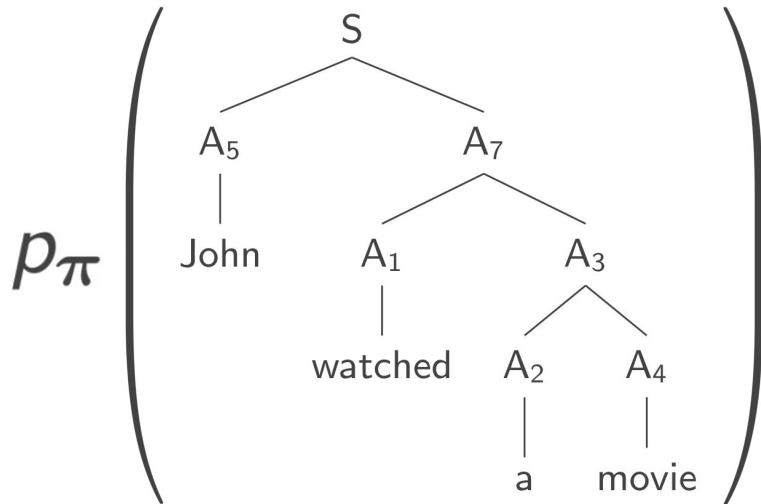
$A_1 \rightarrow \text{watched}$

$A_3 \rightarrow A_2 A_4$

$A_2 \rightarrow a$

$A_4 \rightarrow \text{movie}$

# PCFG Example



Probability of a tree

$p_\pi(t)$

$$\begin{aligned}
 & p_\pi(S \rightarrow A_5 A_7) \times \\
 & p_\pi(A_5 \rightarrow \text{John}) \times \\
 & p_\pi(A_7 \rightarrow A_1 A_3) \times \\
 & p_\pi(A_1 \rightarrow \text{watched}) \times \\
 & p_\pi(A_3 \rightarrow A_2 A_4) \times \\
 & p_\pi(A_2 \rightarrow a) \times \\
 & p_\pi(A_4 \rightarrow \text{movie})
 \end{aligned}$$

$$= \prod_{r \in P_t} p_\pi(r)$$

Set of rules used to derive the tree

# PCFG as a generative model of language

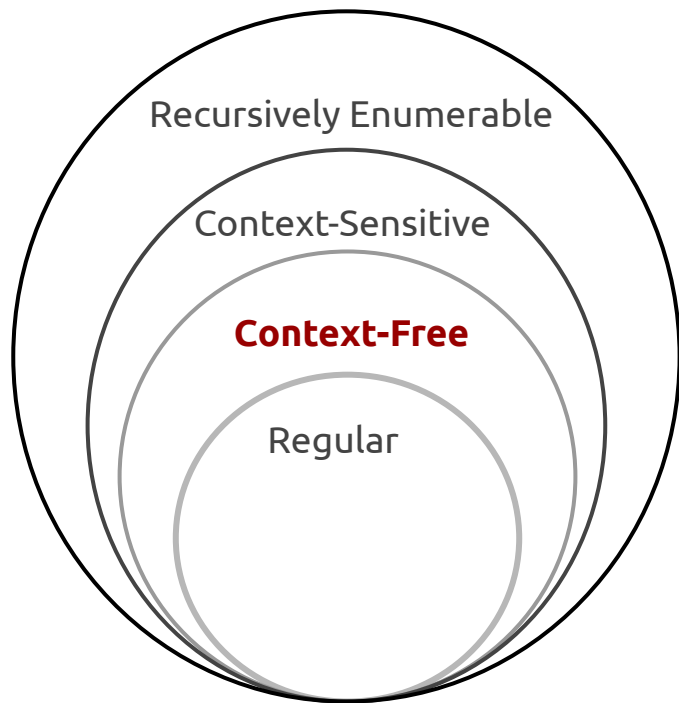
## Autoregressive Language Model

$$p_{\theta} \left( \text{John watched a movie} \right) = p_{\theta}(\text{John}) \times p_{\theta}(\text{watched} \mid \text{John}) \times p_{\theta}(\text{a} \mid \text{John watched}) \times p_{\theta}(\text{movie} \mid \text{John watched a})$$

## PCFG

$$p_{\pi} \left( \begin{array}{c} S \\ \swarrow \quad \searrow \\ A_5 \quad A_7 \\ | \quad \swarrow \quad \searrow \\ \text{John} \quad A_1 \quad A_3 \\ | \quad \quad \swarrow \quad \searrow \\ \text{watched} \quad A_2 \quad A_4 \\ | \quad \quad | \\ \text{a} \quad \text{movie} \end{array} \right) = p_{\pi}(S \rightarrow A_5 A_7) \times p_{\pi}(A_5 \rightarrow \text{John}) \times p_{\pi}(A_7 \rightarrow A_1 A_3) \times p_{\pi}(A_1 \rightarrow \text{watched}) \times p_{\pi}(A_3 \rightarrow A_2 A_4) \times p_{\pi}(A_2 \rightarrow \text{a}) \times p_{\pi}(A_4 \rightarrow \text{movie})$$

# Why context-free grammars?



- Reasonably fast (cubic) algorithms for learning.
- Many human language phenomena can be captured by context-free grammars [Pullum and Gazdar '82].
- ... though not all [Shieber '85].

# PCFG Training

- Supervised case:  $\mathbf{t}$  is observed, so can just maximize likelihood:

$$\sum_{m=1}^M \log p_{\pi}(\mathbf{t}^{(m)})$$

- MLE solution corresponds to just counting and dividing observed rules, as in n-gram language models.



# PCFG Training

- Unsupervised case: only the leaves  $\mathbf{x}$  are observed
- MLE: maximize the likelihood of  $\mathbf{x}$

$$\max_{\pi} \sum_{m=1}^M \log p_{\pi}(x^{(m)})$$

# PCFG Training

- Unsupervised case: only the leaves  $\mathbf{x}$  are observed
- MLE: maximize the likelihood of  $\mathbf{x}$

$$\max_{\pi} \sum_{m=1}^M \log p_{\pi}(x^{(m)}) = \max_{\pi} \sum_{m=1}^M \log \left( \sum_{\mathbf{t} \in \mathcal{T}(x^{(m)})} p_{\pi}(\mathbf{t}) \right)$$

$$\mathcal{T}(x) = \{\mathbf{t} \text{ such that its leaves are } x\}$$

- Marginalize out unseen structure

# PCFG Training

- Marginalization with dynamic programming

$$p_{\pi}(x) = \sum_{t \in \mathcal{T}(x)} p_{\pi}(t)$$

# PCFG Training

- Marginalization with dynamic programming

$$p_{\pi}(x) = \sum_{t \in \mathcal{T}(x)} p_{\pi}(t)$$

Sum over an exponentially-sized with dynamic programming

$$p_{\pi}(\text{John watched a movie}) = p_{\pi} \left( \begin{array}{c} S \\ \swarrow \quad \searrow \\ A_5 \quad A_7 \\ | \quad \swarrow \quad \searrow \\ \text{John} \quad A_1 \quad A_3 \\ | \quad | \quad \swarrow \quad \searrow \\ \text{watched} \quad A_5 \quad A_4 \\ | \quad | \\ a \quad \text{movie} \end{array} \right) + p_{\pi} \left( \begin{array}{c} S \\ \swarrow \quad \searrow \\ A_4 \quad A_7 \\ \swarrow \quad \searrow \quad \swarrow \quad \searrow \\ A_1 \quad A_1 \quad A_2 \quad A_{10} \\ | \quad | \quad | \quad | \\ \text{John} \quad \text{watched} \quad a \quad \text{movie} \end{array} \right) +$$

$$p_{\pi} \left( \begin{array}{c} S \\ \swarrow \quad \searrow \\ A_{12} \quad A_5 \\ | \quad \swarrow \quad \searrow \\ \text{John} \quad A_9 \quad A_3 \\ | \quad \swarrow \quad \searrow \quad | \\ A_8 \quad A_5 \quad \text{movie} \\ | \quad | \\ \text{watched} \quad a \end{array} \right) + p_{\pi} \left( \begin{array}{c} S \\ \swarrow \quad \searrow \\ A_4 \quad A_3 \\ \swarrow \quad \searrow \quad | \\ A_6 \quad A_5 \quad \text{movie} \\ \swarrow \quad \searrow \quad | \\ A_2 \quad A_1 \quad a \\ | \quad | \\ \text{John} \quad \text{watched} \end{array} \right) + \dots$$

# PCFG Training: Inside Algorithm

- Inside algorithm to calculate marginal likelihood (generalization of backward algorithm in HMMs)  $p_{\pi}(x) = \sum_{t \in \mathcal{T}(x)} p_{\pi}(t)$

- Define the “inside” variables as

$$\beta[s, t, A] = \text{Prob}(\text{nonterminal } A \text{ expands to } x_{s:t})$$

- Then marginal likelihood given by

$$p_{\pi}(x) = \beta[1, L, S]$$

# PCFG Training: Inside Algorithm

- Bottom-up dynamic programming

Initialization:

for  $i = 1, \dots, L$

for  $C \in N$

$$\beta[i, i, C] = p_{\pi}(C \rightarrow x_i)$$

$$G = (N, \Sigma, P, S)$$

$N$  : set of nonterminal symbols

$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$S$  : start symbol ( $S \in N$ )

Initialize “width-0” span probabilities (i.e. words)  
ith unary expansion probabilities

# PCFG Training: Inside Algorithm

- Bottom-up dynamic programming

Recursion:

for  $w = 1, \dots, L - 1$

$$G = (N, \Sigma, P, S)$$

$N$  : set of nonterminal symbols

$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$S$  : start symbol ( $S \in N$ )

For all spans with width  $w$

# PCFG Training: Inside Algorithm

- Bottom-up dynamic programming

Recursion:

for  $w = 1, \dots, L - 1$   
  for  $s = 1, \dots, L - w$

$$G = (N, \Sigma, P, S)$$

$N$  : set of nonterminal symbols

$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$S$  : start symbol ( $S \in N$ )

For all spans with width  $w$

For all spans starting at position  $s$



# PCFG Training: Inside Algorithm

- Bottom-up dynamic programming

Recursion:

```
for  $w = 1, \dots, L - 1$ 
  for  $s = 1, \dots, L - w$ 
     $t = s + w$ 
    for  $k = s, \dots, t - 1$ 
```

$G = (N, \Sigma, P, S)$

$N$  : set of nonterminal symbols

$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$S$  : start symbol ( $S \in N$ )

For all spans with width  $w$

For all spans starting at position  $s$

Span end position  $t$

For all possible ways to break up span  $(s, t)$

# PCFG Training: Inside Algorithm

- Bottom-up dynamic programming

Recursion:

for  $w = 1, \dots, L - 1$

for  $s = 1, \dots, L - w$

$t = s + w$

for  $k = s, \dots, t - 1$

for all rules  $A \rightarrow BC \in P$

$$\beta[s, t, A] += \beta[s, k, B]\beta[k + 1, t, C]p_{\pi}(A \rightarrow BC)$$

$$G = (N, \Sigma, P, S)$$

$N$  : set of nonterminal symbols

$\Sigma$  : set of terminal symbols

$P$  : set of production rules

$S$  : start symbol ( $S \in N$ )

For all spans with width  $w$

For all spans starting at position  $s$

Span end position  $t$

For all possible ways to break up span  $(s, t)$

For all possible rules of the form  $A \Rightarrow BC$

# PCFG Training: Inside Algorithm

- A version of the CKY algorithm:  $O(|P|L^3)$
- Each step of the dynamic program to calculate  $p_{\pi}(x) = \beta[1, L, S]$  is just a series of multiplications / additions

$\Rightarrow$  we can easily calculate  $\nabla_{\pi} \log p_{\pi}(x)$

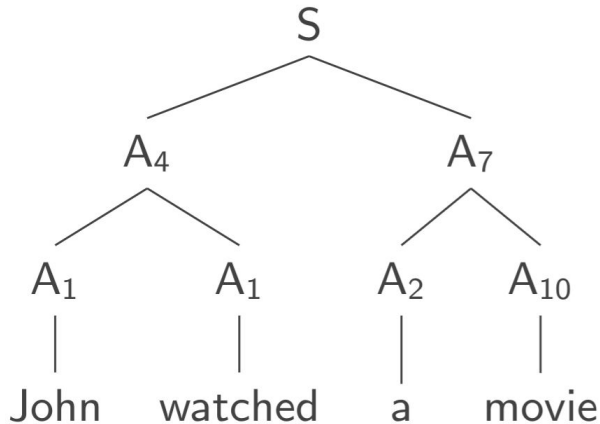
- Can ensure  $\pi$  are probabilities by working in logit space:

$$\pi = \text{softmax}(\theta)$$

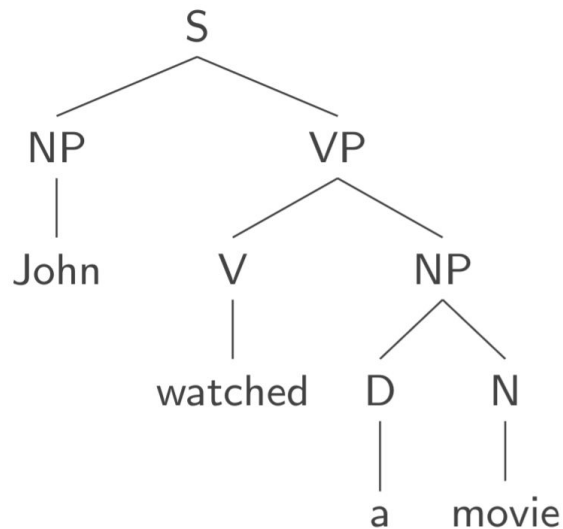
- Gradient ascent on log marginal likelihood = (one version of the) expectation maximization algorithm [Dempster '77].

# Unsupervised Parsing Evaluation

- Predicted trees compared against linguistic trees ignoring label alignment.



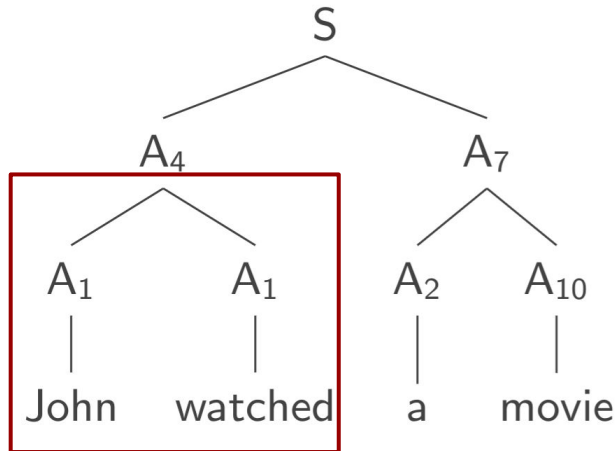
**Predicted**



**Linguistic Annotation**

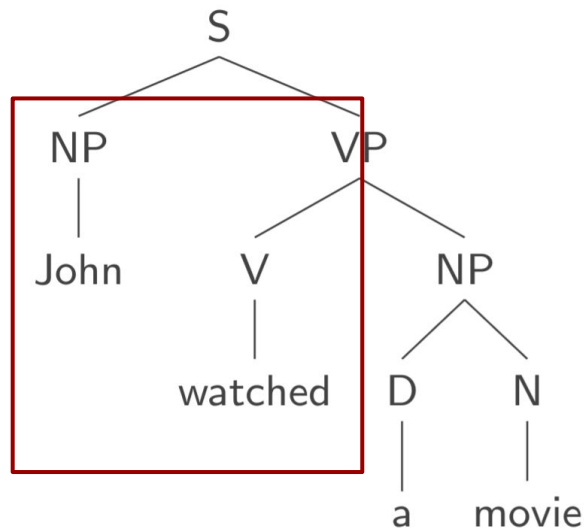
# Unsupervised Parsing Evaluation

- Predicted trees compared against linguistic trees ignoring label alignment.



**Predicted**

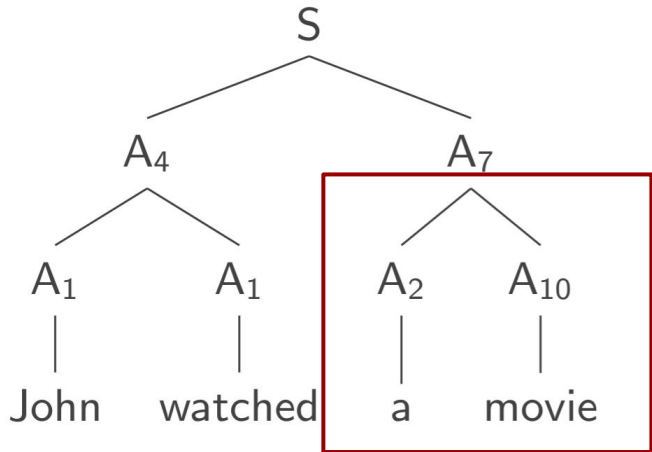
False Positive



**Linguistic Annotation**

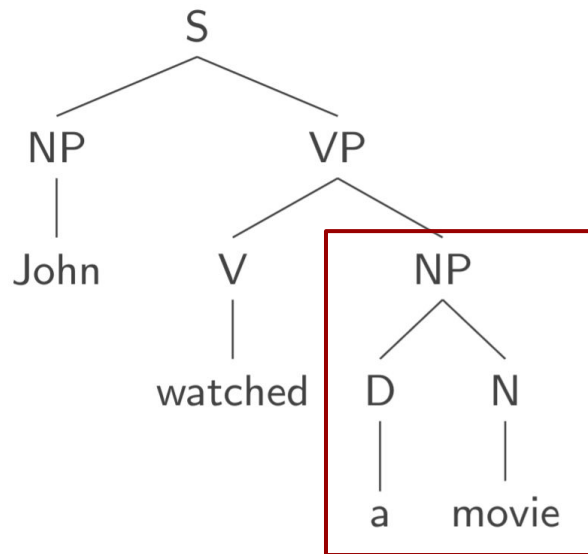
# Unsupervised Parsing Evaluation

- Predicted trees compared against linguistic trees ignoring label alignment.



True Positive

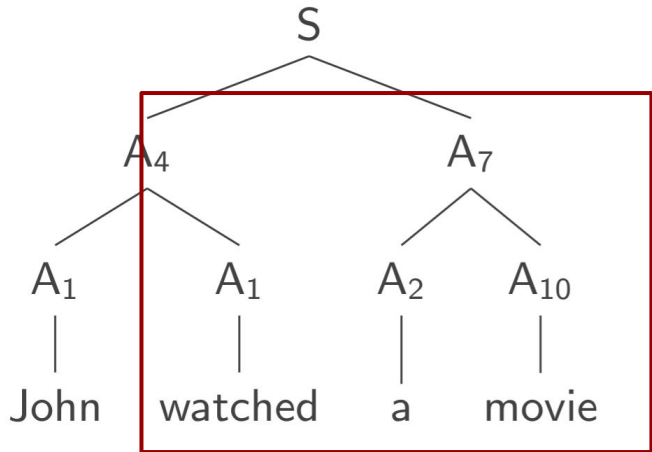
**Predicted**



**Linguistic Annotation**

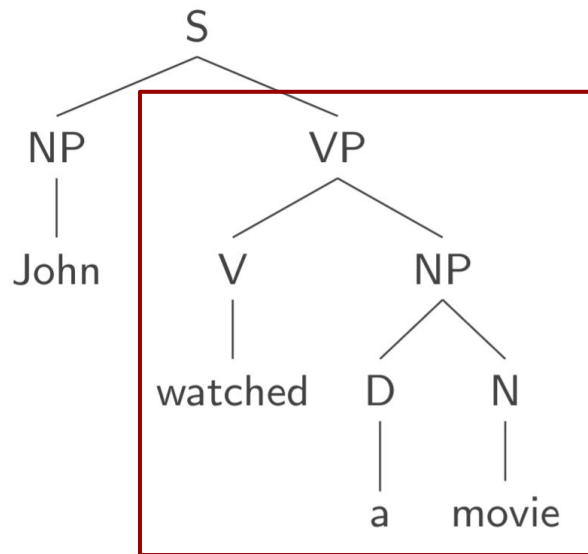
# Unsupervised Parsing Evaluation

- Predicted trees compared against linguistic trees ignoring label alignment.



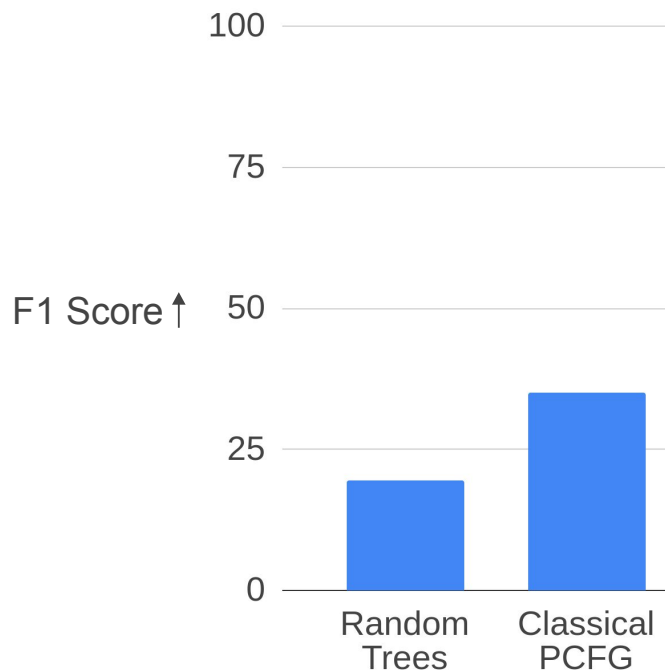
**Predicted**

False Negative



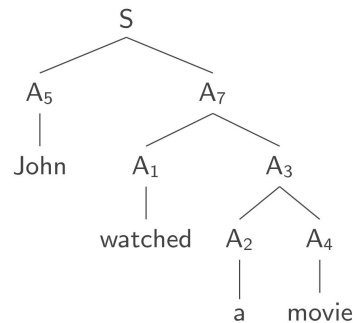
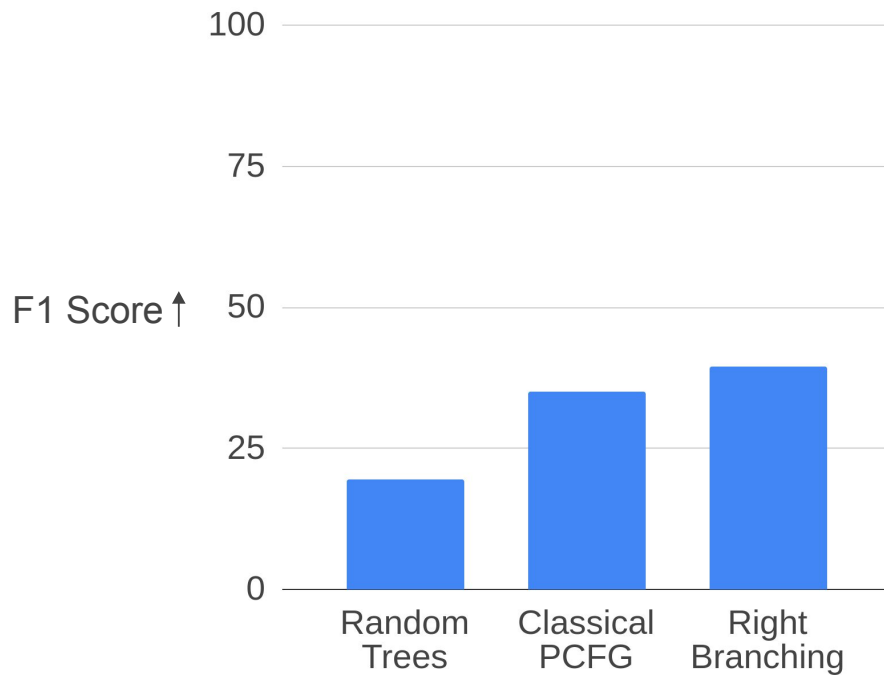
**Linguistic Annotation**

# Results with Simple PCFG induction





# Results with Simple PCFG induction



# Why doesn't PCFG induction “work”?

- Complex optimization landscape (non-convex)
- PCFG model is too simple
- But no one really knows... ㄟ\_(ツ)\_/

# History of Unsupervised Constituency Parsing

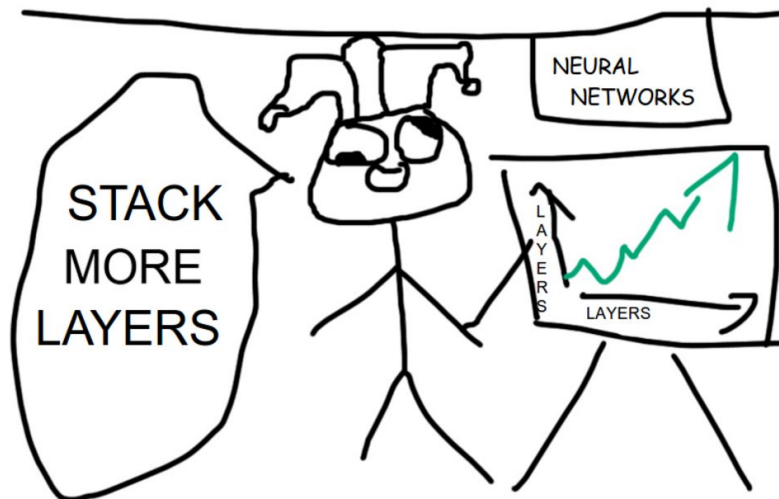
- Decades of negative results from 90s [Carroll and Charniak '92]
- Inspired rich line of work on alternative approaches to unsupervised parsing [Clark '01, Klein and Manning '02, Bod '06, Seginer '07]...
- Some success on unsupervised parsing with simple setups:
  - Part-of-speech tags as inputs
  - Train/evaluate on short sentences (<20 words)
  - Incorporate heuristics (e.g. based on punctuation)

# Outline

- Motivation
- Review: Formal grammars
- Probabilistic grammar induction
- **Recent approaches for grammar induction & unsupervised parsing**
- Conclusion

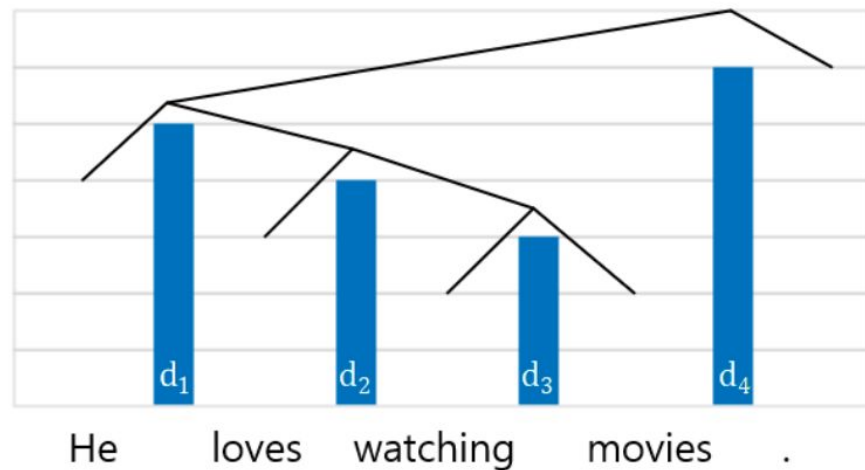
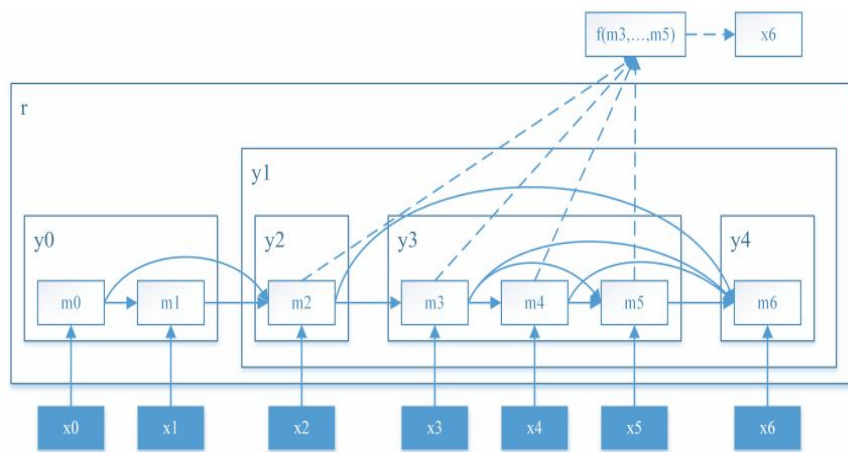
# Recent Work

- Induce parse trees directly from words on full-length sentences.
- Employ neural networks “somewhere” in the pipeline



# Gating Functions within Neural Language Models

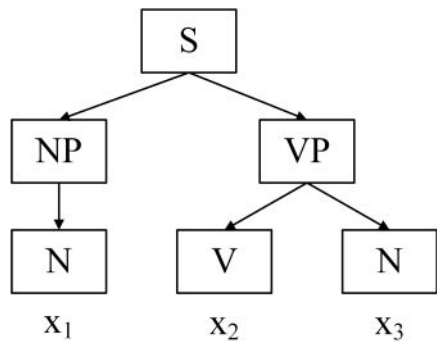
- Parsing-Reading-Predict Network [Shen et al '18]



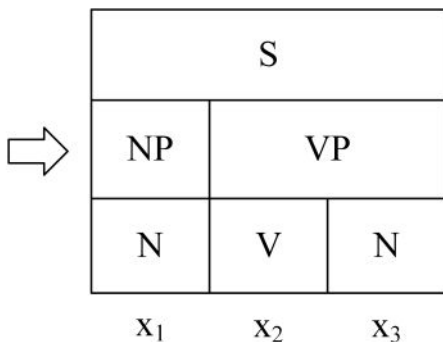
Attention over previous words mediated by (predicted) “syntactic distance” between two words  $\Rightarrow$  use syntactic distance to derive trees

# Gating Functions within Neural Language Models

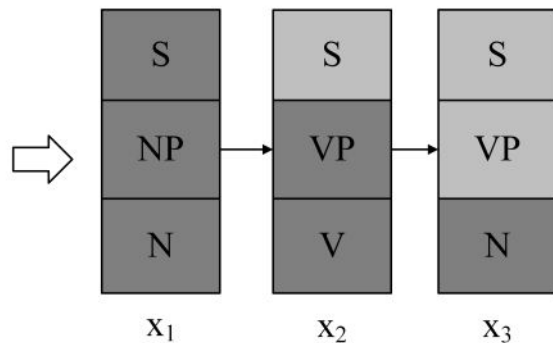
- Ordered Neurons [Shen et al '19]



(a) Constituency tree



(b) Block view

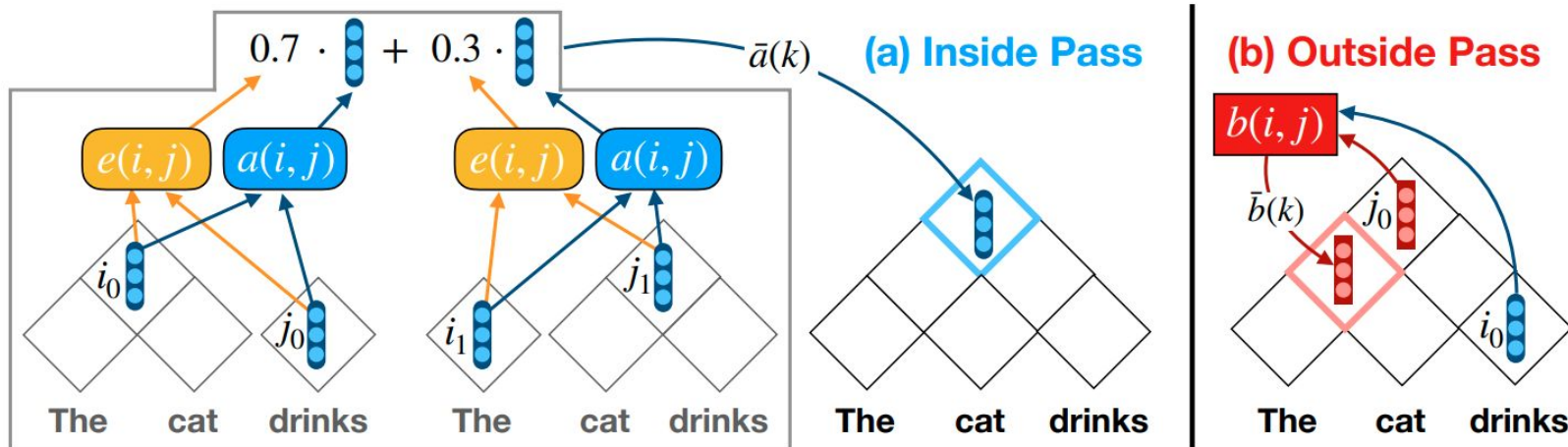


(c) ON-LSTM cell states

Softly partition the hidden states of an LSTM into “blocks” which represent constituents

# Structured Autoencoder

- Deep Inside-Outside Recursive Autoencoder [Drozdo et al '19]

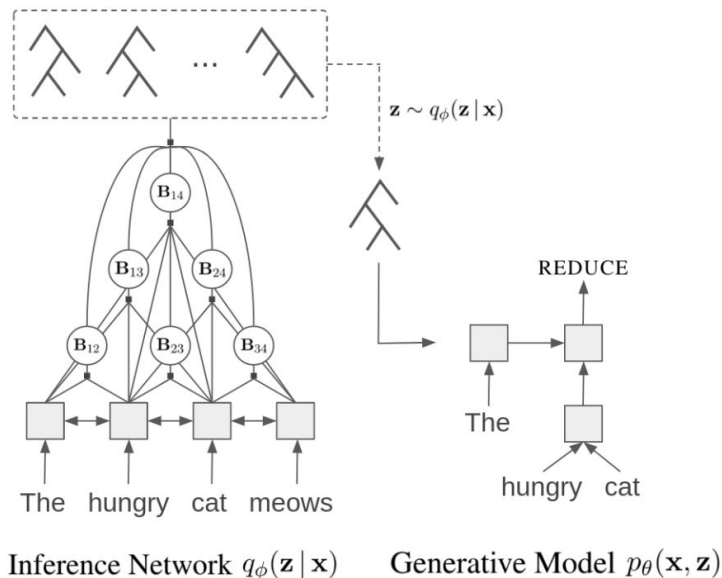


Autoencoder tries to obtain (soft) spans that best reconstruct the leaf word embeddings (pretrained)



# Structured Autoencoder

- Unsupervised Recurrent Neural Network Grammars [Kim et al '19]



Variational autoencoder where the latent variable is a parse tree, and the generative model is a syntax-aware language model.

# “Neuralizing” Classic PCFGs

- Neural PCFG [Kim et al '19]: Use neural networks over symbol embeddings parameterize rule probabilities

## “Neural” Language Models

$$\begin{array}{l} a \\ aardvark \\ able \\ are \\ \vdots \\ zyzzyva \end{array} \begin{bmatrix} 1.2 & -0.1 & 0.3 & \dots & 0.1 \\ 0.2 & 0.7 & -0.4 & \dots & 1.1 \\ -0.7 & 0.5 & 0.6 & \dots & -0.8 \\ 0.1 & 0.9 & 0.8 & \dots & 0.7 \\ & & \vdots & & \\ 0.3 & -0.2 & 0.7 & \dots & 0.4 \end{bmatrix}$$

$$\mathbf{e} = \text{EMBED}(\text{trading})$$

$$\mathbf{h} = \text{NEURALNET}(\mathbf{e})$$

$$p_{\theta}(\text{high} \mid \text{trading}) \propto \exp(\mathbf{h}^{\top} \mathbf{w}_{\text{high}})$$

## “Neural” PCFG

$$\begin{array}{l} S \\ A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_T \end{array} \begin{bmatrix} 0.2 & -0.6 & 0.3 & \dots & 1.8 \\ 1.4 & 0.7 & -0.4 & \dots & 0.1 \\ -0.7 & 0.3 & 0.6 & \dots & -0.9 \\ 0.4 & 0.9 & 0.8 & \dots & 0.7 \\ & & \vdots & & \\ 0.5 & -0.1 & 0.3 & \dots & 0.8 \end{bmatrix}$$

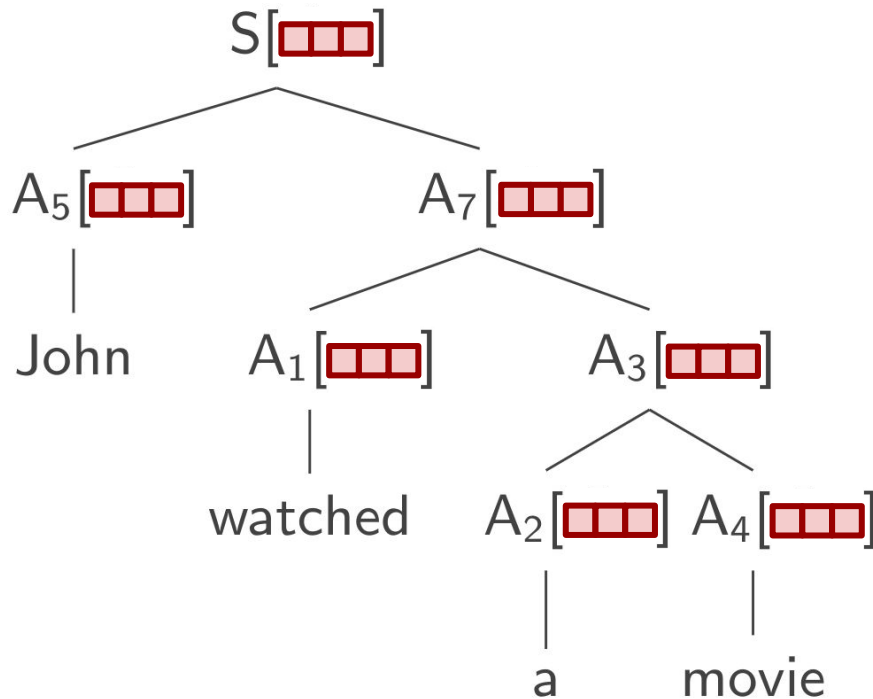
$$\mathbf{e} = \text{EMBED}(S)$$

$$\mathbf{h} = \text{NEURALNET}(\mathbf{e})$$

$$p_{\pi}(S \rightarrow A_1 A_4) \propto \exp(\mathbf{h}^{\top} \mathbf{w}_{A_1, A_4})$$

# “Neuralizing” Classic PCFGs

- Compound PCFG [Kim et al '19]: Learn richer grammars with neural variational inference



  $z \sim p_{\gamma}(z)$

Sentence-level latent vector

# Linguistically Motivated Grammaticality Tests

- Create new sentences via “constituency tests” [Cao et al '20]

The quick brown fox jumped over **the lazy dog**.

The quick brown fox jumped over **it**.

**The lazy dog**, the quick brown fox jumped over.

# Linguistically Motivated Grammaticality Tests

- Create new sentences via “constituency tests” [Cao et al '20]

The quick brown fox jumped over the lazy dog.

\*The quick brown fox it the lazy dog.

\*Jumped over the, the quick brown fox lazy dog.

- These tests indicate that “the lazy dog” is a constituent while “jumped over the” is not.

# Linguistically Motivated Grammaticality Tests

- Create new sentences via these transformations [Cao et al '20]
- Derive a score for each span by performing these tests and giving it to a pretrained LM

$score(\text{"the lazy dog"}) = model(\text{"The quick brown fox jumped over it"})$

- $model()$  is a “grammaticality” model pretrained on real/fake sentences (outputs higher score for real sentences)
- Use these scores as input into the CKY algorithm:

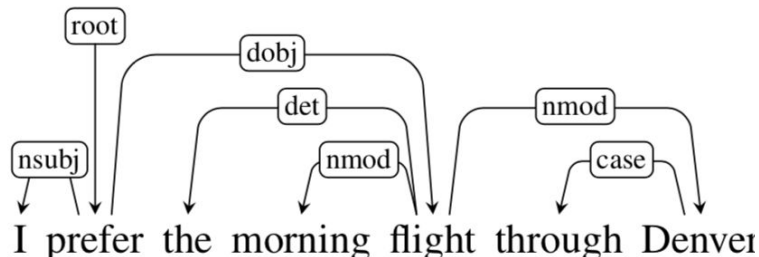
$$\beta[s, t] = \sum_{k=s}^{t-1} \beta[s, k] \beta[k+1, t] score(x_{s:t})$$

# Comparison of recent work

Approach	F1 Score↑
Random Trees	19.5
Right Branching Trees	39.5
Classic PCFG	35.0
Parsing Reading Predict Network [Shen et al. '18]	47.9
Ordered Neurons [Shen et al. '19]	50.0
Unsupervised RNNG [Kim et al. '19]	45.4
Deep Inside-Outside Autoencoders [Drozdov et al. '19]	58.9
Neural PCFG [Kim et al '19]	52.6
Compound PCFG [Kim et al '19]	60.1
Linguistic Constituency Tests [Cao et al '20]	<b>65.9</b>
Supervised Neural Binary Parser	71.9
Binary Tree Oracle (upper bound)	84.3

# What we haven't covered today

- Other formalisms: dependency grammars, tree substitution grammars



- Non-probabilistic approaches



Why should we care about grammars/trees in modern NLP?

# Grammar Induction / Parsing in Contemporary NLP

- “We assume that the goal of learning a context-free grammar needs no justification.” [Carroll and Charniak '92]
- Parsing becoming less important in deep learning-based NLP



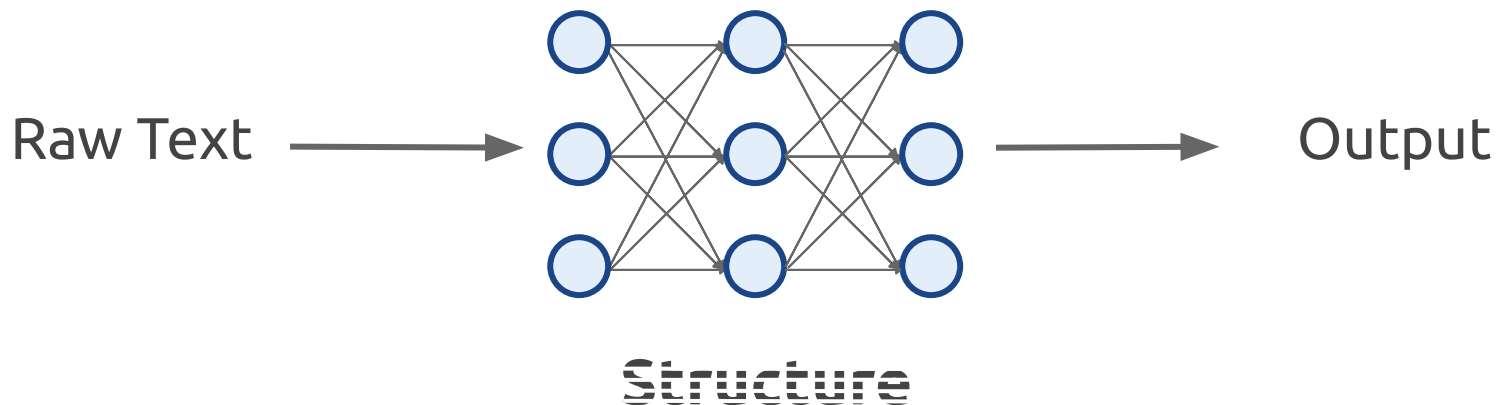
# Grammar Induction / Parsing in Contemporary NLP

- Much evidence that ELMo/BERT etc. capture many language phenomena (including syntax!) implicitly in their hidden layers [Liu et al. '19, Tenney et al. '19].



# Grammar Induction / Parsing in Contemporary NLP

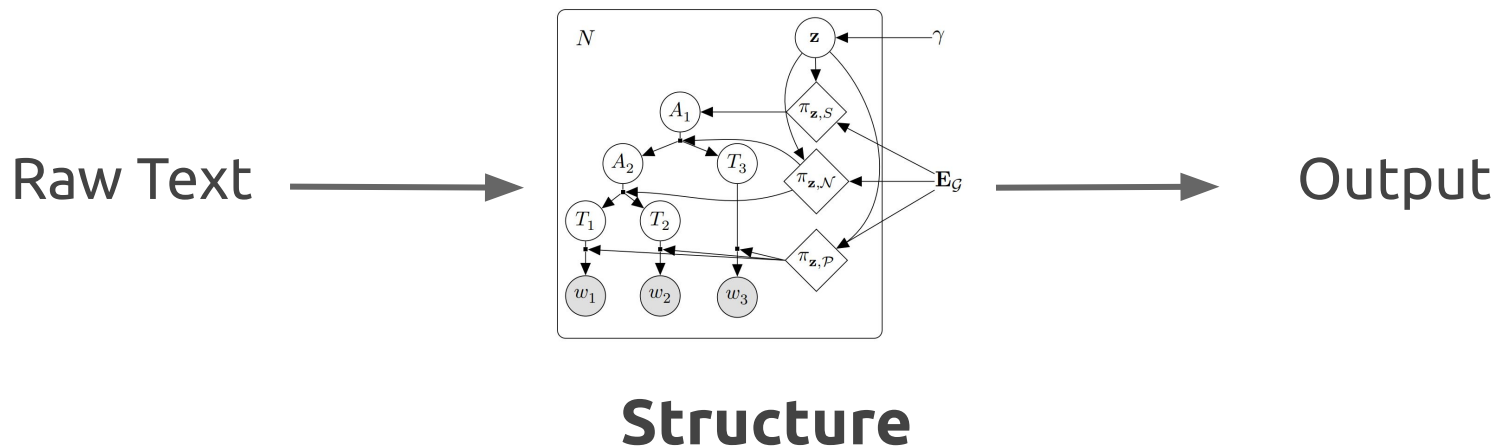
- A case for latent variables: **implicit** vs. explicit modeling of structure



("sort of" captured through hidden layers)

# Grammar Induction / Parsing in Contemporary NLP

- A case for latent variables: implicit vs. **explicit** modeling of structure



(explicitly captured through latent variables)

# Grammar Induction / Parsing in Contemporary NLP

- Would be ideal to have explicit access to such structures from the perspective of
  - Controllability
  - Interpretability
  - Transfer learning
- Grammars can readily operationalize various notions of compositionality.