Communications

A Comparison of Adaptive and Nonadaptive Filters for Reduction of Power Line Interference in the ECG

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Abstract—**We have investigated the relative performance of an adaptive and nonadaptive 60-Hz notch filter applied to an ECG signal. We evaluated the performance of the two implementations with respect to adaptation rate (or transient response time), signal distortion, and implementation complexity. We also investigated the relative effect of adaptive and nonadaptive 60-Hz filtering on ECG data compression. With a 360 Hz sample rate and an adaptation time of approximately 0.3 s for a 1 mV 60-Hz signal, the adaptive implementation is less complex and introduces less noise, particularly in the ST-segment, into a typical ECG signal. When applied to ECG signals, prior to data compression by average beat subtraction and residual differencing, the residual signal resulting from the adaptively filtered signal had an average entropy 0.37 bits per sample (bps) lower than the unfiltered signal. The nonadaptive 60-Hz filter produced an average entropy decrease of 0.08 bps relative to the unfiltered ECG.**

I. Introduction

Power line interference (either 60 Hz or 50 Hz) is a significant source of noise in biomedical signal recording [1]. Elimination of power line interference in the ECG by adaptive filtering, using an external reference signal, was first proposed by Widrow *et al.* [2]. A system for adaptive elimination of line interference, using an external reference, in diagnostic ECG's has also been reported by Ider and Koymen [3]. It is often desirable, or necessary, to reduce power line interference when an external reference is not available. Ahlstrom and Tompkins [4] reported on an adaptive 60-Hz filter for ECG signals that used an internally generated reference signal. Glover [5] showed that Ahlstrom and Tompkins' filter is approximately equivalent to a nonadaptive, second order, notch filter, implying that the performance of a nonadaptive 60-Hz notch filter and an adaptive 60-Hz notch filter with an internally generated reference is equivalent.

As the transient response time of a notch filter increases, the rejection bandwidth of the filter decreases. If an adaptive 60-Hz notch filter is adjusted to adapt quickly to changes in noise, the rejection bandwidth will be wider, and there will be more attenuation in signal components at frequencies close to 60 Hz. Conversely, if the bandwidth of the notch filter is reduced, the transient response time will increase, and the filter will adapt more slowly to changes in noise. Because the transient behavior of Ahlstrom and Tompkins' filter is different from the transient behavior of a nonadaptive notch filter, it is hard predict how well the filters will track changes in noise in an actual ECG signal. It is also hard to predict the relative distortion that the two filters will introduce in a typical ECG signal.

For this study, we compared Ahlstrom and Tompkins' algorithm to a nonadaptive 60-Hz notch filter with respect to signal distortion in the absence of noise and noise reduction in the presence of changing noise levels. We also examine the computational complexity of the two filter implementations. Computational complexity is an important issue if the filter will be implemented in a microprocessor system with limited computational resources or if the filter will be implemented directly in silicon. We previously reported preliminary results and implementation details in [6].

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II. Adaptive 60-Hz Notch Filter

Ahlstrom and Tompkins' adaptive 60-Hz notch filter maintains a running estimate of the 60-Hz noise. At time t the present noise estimate can be generated from the previous two noise estimates according to the equation

$$e(t) = Ne(t - nT) - e(t - 2nT)$$

where T is the sample period and $N = 2\cos(2\pi 60T)$. The error in the noise estimate is taken to be

$$f(t) = [x(t) - e(t)] - [x(t - nT) - e(t - nT)]$$

where the second term is an estimate of any dc offset. If $f(t) < 0$, the present noise estimate, $e(t)$, is decreased by an increment d. If $f(t) > 0$ the present noise estimate is decreased by d. The output of the filter is generated by subtracting the noise estimate $e(t)$ from the input signal $x(t)$. As d increases, the filter adapts more quickly, and exhibits a larger bandwidth. As d decreases, the filter adapts more slowly, and has a smaller bandwidth.

Implementation of this filter is particularly efficient for a sample rate of 360 sps because $N = 1$, all the equation coefficients are equal to 1, and no multiplications are required. At sample rates other than 360 sps, Ahlstrom and Tompkins' filter requires a single multiplication with reasonable precision [6].

III. Nonadaptive 60-Hz Notch Filter

The transfer function for a digital 60-Hz notch filter can be represented in the z domain as

$$H(z) = \frac{1 - 2\cos(2\pi \cdot 60 \cdot T)z^{-1} + z^{-2}}{1 - 2r\cos(2\pi \cdot 60 \cdot T)z^{-1} + r^2 z^{-2}}$$

where T is the sample interval. This filter will have a zero on the unit circle at 60 Hz and a pole at the same angle with a radius of r. As r increases, the pole approaches the unit circle, the bandwidth of the notch decreases, and the transient response time of the filter increases. The dc gain of the filter will be $(1 - r + r^2)^{-1}$.

The nonadaptive 60-Hz notch filter can be implemented with the difference equation

$$\begin{aligned} y(t) = {} & rN \cdot y(t - nT) - r^2 y(t - 2nT) \\ & + x(t) - N \cdot x(t - nT) + x(t - 2nT) \end{aligned}$$

where $N = 2\cos(2\pi 60T)$.

As in the adaptive case, for a sample rate of 360 sps, $N = 1$, reducing the number of multiplications required to implement the

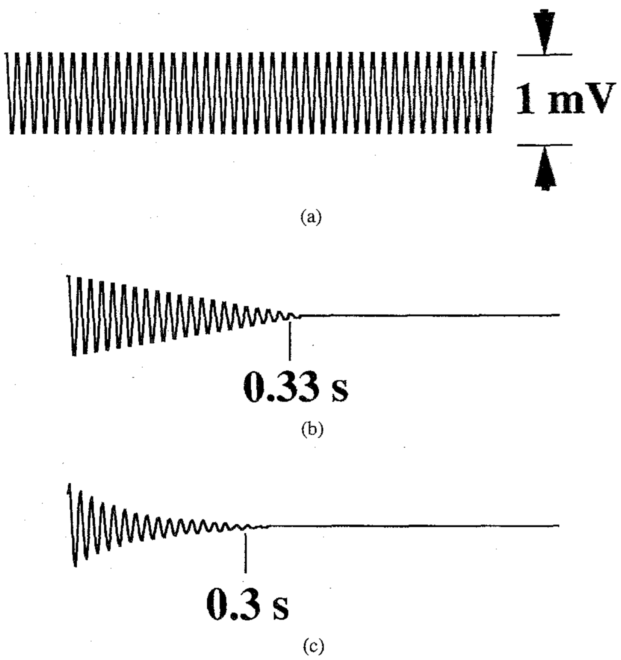

Fig. 1. (a) 0.5 mV 60-Hz input, (b) adaptive filter response with $d = 10\ \mu$V, and (c) nonadaptive 60-Hz filter response.

filter. However, because of the r and r^2 coefficients, the filter requires two multiplications of reasonable precision. We implemented the 60-Hz notch filter using integer arithmetic, as would be done in a microprocessor based system with limited computational resources. Consequently, we used 32-bit fixed point numbers, with the upper 16 bits representing the integer portion of the number and the lower 16 bits representing the fractional portion of the number. At sample rates other than 360 sps, the nonadaptive notch filter requires three multiplications with reasonable precision [6].

IV. Transient Filter Responses

Despite similar frequency responses, the adaptive and nonadaptive 60-Hz notch filters have significantly different transient response characteristics. For the nonadaptive notch filter, it can be shown that the transient response, $g(t)$, to a step input of 60 Hz is [7]

$$g(nT) = u(t)\sin(2\pi \cdot 60 \cdot nT)r^{n-1}.$$

The output of the nonadaptive 60-Hz filter decays exponentially with each sample. Because the noise estimate in the adaptive filter is increased or decreased by a fixed increment each sample period, the response to a step change in 60-Hz input is a sinusoid with an amplitude that decreases linearly with time.

Figs. 1 and 2 show the transient response of the two filter implementations to 60-Hz signals with 0.5 mV and 1 mV amplitudes. With $d = 10\ \mu$V and $r = 0.97$, for a 0.5 mV amplitude 60-Hz signal, the filter outputs settle to a steady state level in roughly the same amount of time. When the amplitude of the input transient is doubled, the adaptive filter requires twice as long to reach steady state as the nonadaptive notch filter.

Because of its exponential transient behavior, the nonadaptive filter has a transient response time that is not particularly sensitive to signal amplitude. The nonadaptive filter's transient response time is nearly the same for a large range of input signal amplitudes. In contrast, the transient response time of the adaptive notch filter is linearly dependent on the input signal amplitude. The adaptive filter adapts more quickly to small amplitude signals and more slowly to large amplitude signals. This difference in transient response characteristics produces significant performance differences when the filters are applied to ECG signals.

TABLE I
MSE Signal Distortion for a Typical ECG

Adaptive		Nonadaptive	
d (μV)	mse (μV^2)	r	mse (μV^2)
		0.965	63.90
10	34.00	0.97	60.40
		0.9725	53.65
		0.975	40.00
		0.9775	30.75
5	7.35	0.98	23.95
		0.9825	21.60
		0.985	53.05
		0.99	29.45

V. Methods and Results

A. ECG Signal Distortion

We evaluated the relative distortion produced by the two filter implementations in the presence of a typical ECG signal without noise. We constructed a test signal by averaging normal beats from tape 106 of the MIT/BIH arrhythmia data base [8]. Data from the MIT/BIH data base was recorded at 360 sps with an A-to-D resolution of 5μV/LSB. We repeated the beat at a rate of 60 beats per minute and examined the distortion after four beats, allowing initial filter transients to settle. Table I lists the mean squared error (mse) produced by the two filters over a range of values for r and d.

The transient response times for a 1 mV 60-Hz signal were roughly equivalent with $r = 0.97$ and $d = 10\ \mu$V and $r = 0.98$ and $d = 5\ \mu$V. With the filters adjusted for relatively equivalent transient response to a 1 mV 60-Hz signal, which represents a significant noise signal, the adaptive filter produces consistently lower MSE distortion. The MSE distortion level generally decreases as the transient response time increases and the bandwidth of the notch narrows. The increase in distortion for the nonadaptive filter from $r = 0.9825$ to 0.985 can be attributed to variation in how accurately the coefficients are represented using only 16 bits.

Fig. 3 compares the original signal, the output from the adaptive filter with $d = 10\ \mu$V, and the output from the nonadaptive filter with $r = 0.97$. Fig. 4 shows the absolute values of the errors introduced by each filter over the course of the beat. Figs. 3 and 4 show that the nonadaptive notch filter produces significant distortion in the QRS and ST-segment portion of the beat as a result of filter ringing. As the bandwidth of the filter is reduced, the ringing decreases. The nonadaptive filter also produces a low level of distortion throughout the beat. The adaptive filter produces less distortion in the QRS and ST-segments, exhibiting very little ringing, and more distortion in the P-wave and T-wave.

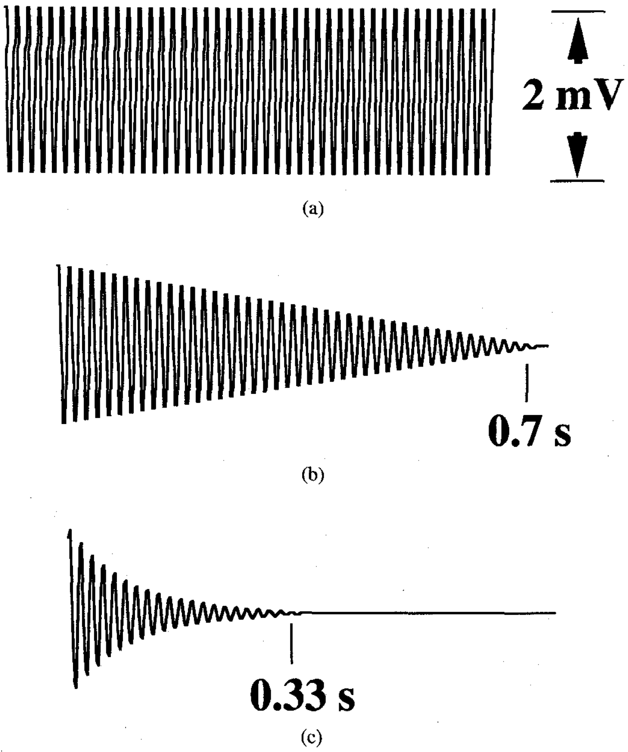

Fig. 2. (a) 1.0 mV 60-Hz input, (b) adaptive filter response with $d = 10\ \mu$V, and (c) nonadadaptive 60-Hz filter response with $r = 0.97$.

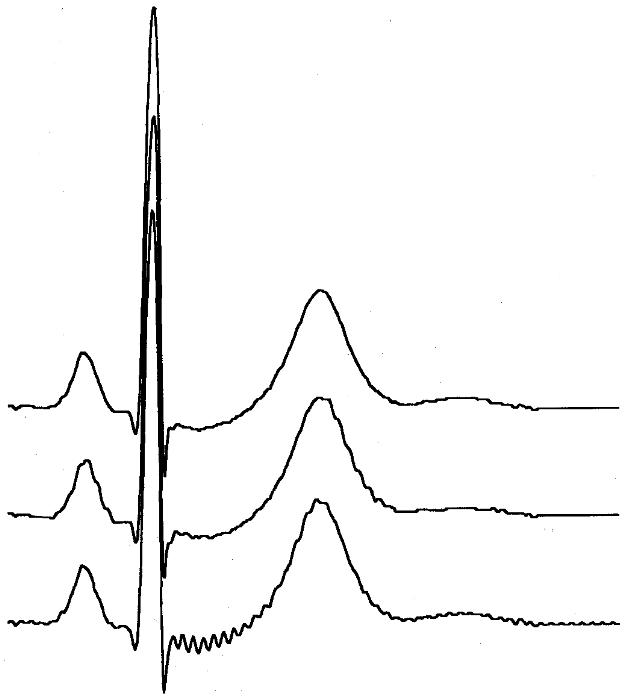

Fig. 3. (top) Original ECG signal (middle) after adaptive 60-Hz filter, and (bottom) after fixed 60-Hz notch filter.

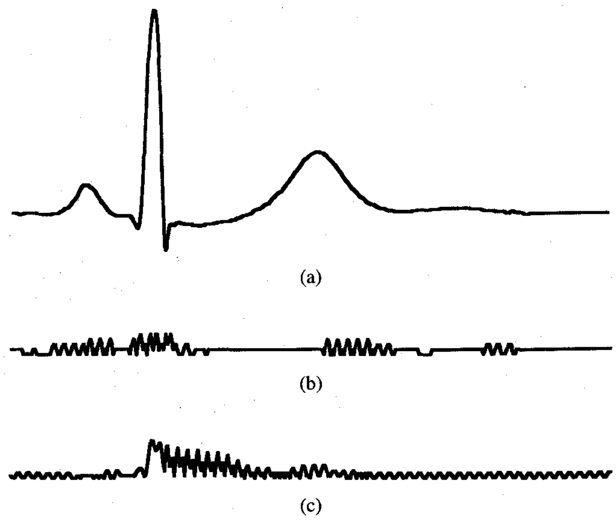

Fig. 4. (a) Scaled ECG signal, (b) adaptive 60-Hz filter absolute error, and (c) nonadaptive notch filter error.

B. Residual Signal Entropy

From the results stated above, we can conclude that the signal distortion decreases as the transient response time increases and the notch bandwidth decreases. In the nonadaptive filter, this reduces the ringing from the QRS complex. However, in a practical situation, a long transient response time will reduce the filter's ability to track changes in the noise level. We have attempted to evaluate relative performance of the two filters on actual ECG signals and noise by measuring the relative effect of the two filter implementations on the entropy of the residual ECG signal. As the noise is reduced the entropy of the residual signal should decrease.

In ECG compression, it is common practice to construct an average beat, subtract the average beat from the original signal, creating a residual signal, first difference the residual signal, and encode and store the first differenced residual signal along with the average beat [8]–[12]. If the beat subtraction is done well, the residual signal will consist primarily of low level noise that has been added to the signal. The entropy, H, of the differenced residual signal is calculated as

$$H = \sum_{\text{all } x} P_x \log_2 \left(\frac{1}{P_x} \right)$$

and represents the minimum, average bits per sample (bps) required to store the signal [13].

For our test data, we used the first 10 seconds from twenty-four MIT/BIH data base records. We used only records that contained a single predominant beat morphology in the first 10 seconds and included 60-Hz noise. The 10-second segments were filtered prior to compression and compressed by average beat subtraction and residual differencing. Fig. 5 compares the original signal from tape 106 to the adaptively filtered signal with $d = 5\ \mu$V and the nonadaptively filtered signal with $r = 0.97$. Visually, the adaptive filter appears to do a better job of reducing the low level 60-Hz noise in the signal.

We calculated the residual signal entropy for the twenty-four unfiltered records, for the adaptively filtered records with $d = 2.5\ \mu$V, $d = 5.0\ \mu$V, and $d = 10.0\ \mu$V, and for the nonadaptively filtered records with $r = 0.97, r = 0.98$, and $r = 0.99$. Table II summarizes the average results. We achieved minimum average residual signal entropies with $d = 5.0\ \mu$V for the adaptive filter and $r = 0.97$ for the nonadaptive filter. Fig. 6 shows the residual signal entropies for the

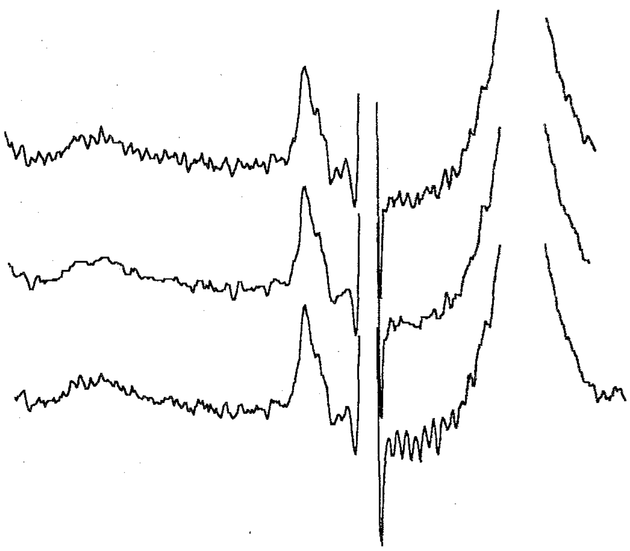

Fig. 5. (top) ECG signal from MIT/BIH record 106, (middle) adaptive filter output ($d = 10\ \mu$V), and (bottom) nonadaptive filter output ($r = 0.97$).

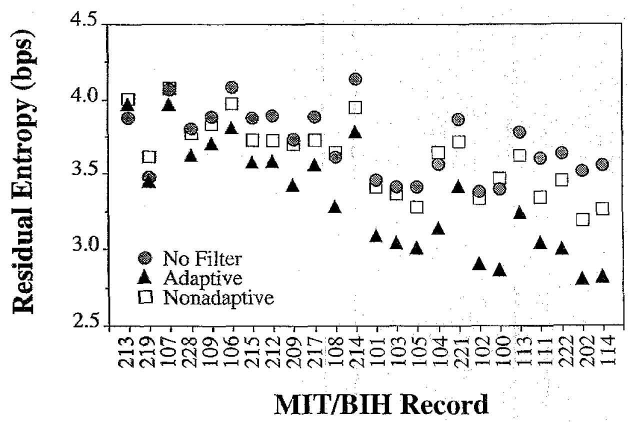

Fig. 6. Comparison of unfiltered, adaptively filtered ($d = 5\ \mu$V), and nonadaptively filtered ($r = 0.97$) residual ECG signal entropies for 24 10-second segments from the MIT/BIH arrhythmia data base.

TABLE II
AVERAGE RESIDUAL SIGNAL ENTROPIES

Unfiltered	Adaptive		Nonadaptive	
Entropy (bps)	d (μV)	Entropy (bps)	r	Entropy (bps)
3.708	10	3.388	0.97	3.621
	5	3.337	0.98	3.629
	2.5	3.414	0.99	3.691

unfiltered signal, the adaptively filtered signal with $d = 5.0\ \mu$V, and the nonadaptively filtered signal with $r = 0.97$. Adaptive filtering reduced the residual signal entropy in all but one record, while nonadaptive filtering increased the residual signal entropy in six of the records.

VI. DISCUSSION

We have compared the complexity and performance of two 60-Hz notch filter implementations, an adaptive filter with an internally generated reference and a nonadaptive second order notch filter. Both filters are most efficiently implemented with a sample rate of 360 sps. In this case, the adaptive filter requires no multiplications and can be effectively implemented with 16-bit integers. A reasonable implementation of the nonadaptive filter requires two 16-bit multiplications, and performance will vary with the precision of the implementation. At other sample rates, the adaptive and nonadaptive filters can be implemented with one and three 16-bit multiplications respectively. In any case, Ahlstrom and Tompkins' filter represents a more computationally efficient implementation of a 60-Hz notch filter.

With a 360 sps sampling rate, in the absence of noise, the adaptive filter introduces less distortion in a typical ECG signal. The most significant distortion produced by the nonadaptive filter can be characterized as filter ringing following the QRS complex. Because of the incremental adaptation used in Ahlstrom and Tompkins' filter, the adaptive filter output is not as significantly effected by the QRS complex, which has a large amplitude but relatively short duration, and no ringing is observed.

The ECG signal distortion observed in the absence of noise tended to decrease as the transient response time of the filter increased. This decrease in signal distortion must be traded off with the ability of the filter to track changes in noise amplitude. We evaluated the effectiveness of the two filters on real signals by determining their effect on the residual signal entropy of a compressed ECG signal containing low level 60-Hz noise. Though severe power line interference can be avoided by shielding and careful amplifier design, low level power line interference can still be present in ECG signals. We found that this low level noise has a significant effect on the bits per sample required to store a compressed ECG signal in applications such as diagnostic ECG's.

The adaptive filter produced the largest average residual signal entropy decrease with an adaptive increment of 5 μV, a larger decrease than the filter produced with increments of 10 μV or 2.5 μV, which produce longer and shorter transient response times respectively. The nonadaptive filter reduced the residual signal entropy most with a pole radius of 0.97, performing better than with pole radii of 0.98 or 0.99, both of which produce longer transient filter responses. These results confirm long transient response times can hinder the filters' abilities to track changes in the noise amplitude.

With our test data, the Ahlstrom and Tompkins' filter was much more effective in reducing residual signal entropy than the nonadaptive filter. In the best case, the adaptive filter reduced the residual signal entropy by an average of 10%. The nonadaptive filter decreased the entropy of the residual signal by an average of only 2.3%, and, in a number of cases, the entropy actually increased. These results indicate that, for low level power line interference, the reduction in 60-Hz noise resulting from nonadaptive filtering does not always compensate for the distortion introduced by the filter. With a low level of 60-Hz noise, the better performance of the adaptive filter implementation may also result from the filter's rapid adaptation to small signal changes.

In summary, we have shown that, compared to a nonadaptive implementation of a power line frequency filter, the Ahlstrom and Tompkins' adaptive filter with an internally generated reference is less complex, produces less distortion in a typical ECG, and is more effective in removing low level 60-Hz noise. These performance differences are due primarily to the different transient behavior of the two filters.

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